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Asset reserve requirements and the control of the money supply (An extended analysis)

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Asset Reserve
Requirements and the
Control of the Money Supply
(An Extended Analysis)

by

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DISKUSSIONSBEITRÄGE

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I n t r o d u c t i o n

So far, asset reserve requirements as an instrument of monetary policy have not been available in Germany. The addition of this instrument to the set of tools of the German Bundesbank has been discussed recently. I do not intend to give a survey of this discussion. Instead, I try to provide some further theoretical results by analyzing this instrument in the framework of the non-linear money-supply hypothesis as developed by K.BRUNNER and A.H.MELTZER.¹⁾ In this hypothesis both the money and the bank credit supply are conceived to be the algebraic product of a multiplier and the monetary base.

It is important to note that asset reserve requirements are not related to the monetary base, they only effect the multipliers. Therefore, assuming that the controllability of the monetary base is not altered by introducing or operating on asset reserve requirements, prospective changes in the effectiveness of monetary policy may be judged on the basis of (prospective) changes in the controllability of the multipliers.

Now, the controllability of multipliers is uniquely related to the elasticities of the multipliers with respect to the various parameters forming these multipliers, and changes in the elasticities reflect changes in controllability. By an analysis of changes in elasticities, it will be possible to show that asset reserve requirements weaken the potency of monetary policy in

controlling the money supply, the result being more favorable if bank credits are the target of monetary policy. A more detailed summary statement of results is given at the end of part A and B.

In section I of part A, I restate some elements of the BRUNNER-MELTZER model which I need in section II, where four models of asset reserve requirements are introduced. In section III of that part multipliers are developed, some properties of which are stated in section IV. Elasticities for the multipliers and their reaction to asset reserve requirement changes are presented in section V, while section VI gives a summary statement of results.

In part B, section I, I shall give an explicit demonstration of how asset reserves (in absolute or percentage rate terms) required for increments of assets may be reformulated as reserve requirements for the level or the total volume of assets.

For each of the four models of asset reserve requirements introduced in part A three different but rather general regimes of incremental asset reserve requirements will be considered. This is to demonstrate that the analysis covers an extremely large variety of schemes of asset reserve requirements that is conceivably relevant in theory and policy. In section II of part B, analytical steps are taken which in section III allow to show that the conclusion drawn in part A carry over to the case of incremental asset reserve requirements. Section IV contains a summary statement of the results of part B and an intuitively appealing interpretation of some of the results.

At the end of the paper some conclusions are drawn.

In order to limit possible misinterpretations four remarks may be helpful.

First, in order to determine the equilibrium price on any market, it is necessary in the first stage to determine the position of both demand and supply curves. Once these positions are given, prices (interest rates) may be determined in the second stage. Both on the money market and on the market for bank credit the position of the supply curves is changed whenever there is an act of monetary policy.

The following analysis is focused on the shifts of both the money supply and the credit supply function due to the introduction of new or due to a change in already existing asset reserve requirements. A shift in the credit supply function will of course affect interest rates as determined on the bank credit market. These interest rate changes have feedback effects on the money market. In particular, they partially offset the former shift of the money supply function. These feedback effects are explicitly taken into account in part B, as may be seen from dashed elasticity expressions, for example (62), while they need not to be considered in part A, where interest rates are assumed as given and held constant in order to derive partial derivatives with respect to credit ceilings and other non-interest rate variables.

Second, what will be called the effects of asset reserves on the effectiveness of (particular instruments of) monetary policy refers to alterations in the shiftability (by acts of monetary policy) of both the money and the credit supply curve. Changes in shiftability will be measured by changes in certain elasticities of the money and/or credit multipliers.

Third, with asset reserve requirements it is necessary to distinguish between gross yields and net yields on earning assets. On the other hand, liability reserve requirements do not imply a divergence between gross and net yields. Thus, a basic difference obtains between the two types of reserve requirements.

In addition, net yields may change by two ways. First, by a change in gross yields that are determined on the credit market. Second, for given gross rates net rates will change by an increase in marginal asset reserve requirements. Such a change in net yields may be called a direct change, while, by definition, an indirect change in net yields is due to a change in gross yields. In this paper, all interest rates are to be interpreted as net rates and therefore possibly as direct functions of the marginal rate of asset reserve requirements. Whether a particular interest rate is or is not directly affected by asset reserves depends on the particular model of asset reserve requirements. Four different models will be distinguished below.

If asset reserves are required for loans (model 1) then an increase in the marginal rate of required asset reserves reduces liquidity and lowers net yields on loans both absolutely and relatively (compared with net yields^{on} investments). Of course, a symmetric statement holds for asset reserve requirements on investments instead of loans. Asset reserve requirements on total earning assets (model 2) affect the marginal net yield on both loans and investments equally such that relative marginal net yields are unchanged while absolute marginal net yields are reduced.

Since borrowings depend both on marginal net yields of earning assets and on the discount rate the borrowing ratio would be directly affected by asset reserve requirements, too. If earning assets of an amount equal to borrowings from the central bank (model 4) or, simply, if borrowings (B_0) are subject to "asset" reserve requirements, then it is necessary to distinguish between a gross rate and a net rate of discount. The net rate depends on the gross rate that is fixed by the central bank and on the marginal rate of required reserves on borrowings. Thus, in model 4 all behavioural ratios that depend on the (net) discount rate also depend on reserve requirements on borrowings.

The case of (model 3) asset reserve requirements for "unborrowed" total earning assets ($K - B_0$) may be interpreted as a combination of positive reserve requirements on total earning assets (K) and of negative reserve requirements (a liquidity bonus) on borrowings from the central bank. Here, increasing marginal "asset" reserve requirements implies both a reduction of the marginal net yield on total earning assets and a reduction of the net discount rate.

Thus, it is obvious that any change in asset reserve requirements ^{directly} alters one, several or all of the net yields that determine the behavioural ratios described in equations (3)-(6). Therefore, if asset reserve requirements are changed, net yields ^{are} changed even if gross yields remain unchanged. No feedback from credit markets is necessary to change marginal net yield.

Of course, holding net yields fixed while changing the marginal rate of required asset reserves is not necessarily a contradictory

procedure of analysis. Two ways exist to avoid contradiction. One is to assume a parallel parametric change in gross yields. The other way is to stress the partial character of the analysis. The second way is followed in part A as already pointed out. In part B, the analysis will be less partial. Both direct and indirect effects of asset reserves on net yields are explicitly taken into account (see equation (62)).

Fourth, initially it will prove economical to formulate asset reserve requirements for total volumes of loans, investments, general earning assets etc. As long as changes in these requirements are the only changing determinant of the behaviour of commercial banks a change in reserves required for the total volume of assets is analytically equivalent to a change in the reserves required for increments or additions to the asset volume. However, if either the monetary base and/or other determinants (in particular interest rates) change - either independently or as a consequence (feedback) of the primary asset reserve requirement changes - then this analytical equivalence disappears and some problems arise. Fortunately, these problems are not difficult to solve.

Part A

I.

I start out with a budget restraint, i.e. a consolidated balance sheet of the commercial banking sector as expressed by equation (1)

$$L + I + R = D + T + S + B_o. \quad (1)$$

Total assets are allocated between loans (L), investments (I) and reserves (R). Total liabilities consist of demand deposits (D), time deposits (T), savings deposits (S) and borrowings from the central bank (B_o). Dividing both sides of the balance sheet by the sum of all deposits, (D + T + S), gives an equation in ratios:

$$l + i + r_1 = 1 + b \quad (2)$$

The loan ratio (l), the investment ratio (i), the total reserve ratio (r₁) and the borrowing ratio (b) describe the behaviour of the banking sector.

The ratios are behavioural parameters. They are functions and depend on an array of determinants, the complete statement of which is not necessary for my purposes. Yet, it will be illustrative to draw attention to some of the determinants and to give a more detailed description of the reserve ratio.

In equations (3), (4) and (5)

$$l = l (i_G, i_L, \rho_o, \dots) \quad (3)$$

$$i = i (i_G, i_L, \rho_o, \dots) \quad (4)$$

$$b = b (i_G, i_L, \rho_0, \dots) \quad (5)$$

+ + -

i_G designates the average yield on government securities, i_L denotes an index of loan rates and ρ_0 is the discount rate.

The total reserve ratio equals the sum of the required reserve ratio (r^r) and the excess reserve ratio (r^e). The excess reserve ratio is a true behavioural parameter of the banking sector as illustrated by equation (6):

$$r^e = r^e (i_G, \rho_0, i_L, \dots) \quad (6)$$

- + -

The required reserve ratio is given by equation (7):

$$r^r = \frac{1 \cdot r^d + t \cdot r^t + s \cdot r^s}{1 + t + s} \quad (7)$$

r^d , r^t and r^s represent reserve requirements as determined by the central bank respectively for demand deposits, time deposits and savings deposits. t and s - not in their role as indices - are symbols for time and saving deposits scaled by demand deposits thus representing part of the allocative behaviour of the non-bank public as described by equation (8) and (9):

$$t = t (i_T, i_S, i_G, i_L, \dots) \quad (8)$$

$$s = s (i_S, i_T, i_G, i_L, \dots) \quad (9)$$

By virtue of (6), the total reserve ratio may be considered to be a behavioural function of the banking sector, too:

$$r_1 = r^r + r^e = r_1 (r^d, r^t, r^s, i_G, i_L, i_T, i_S, \rho_0, \dots) \quad (10)$$

There is no point in further detailing these behavioural functions, since in the following I shall usually omit all the arguments of the behavioural parameters. For the same reason it may be justified to introduce without specifying details the currency ratio as an additional behavioural parameter of the public, which describes the allocation of its money holdings between currency and demand deposits.

Some additional terminology will be used:

$$K = L + I = \text{earning assets}$$

$$e = l + i = \text{earning assets ratio}$$

$$r_2 = r_1 - b = \text{adjusted reserve ratio}$$

By the definition of the monetary base from the user's side I have

$$B^1 = B = C^P + R^r + R^e \quad (11)$$

where B stands for monetary base, C^P is a symbol for currency (of the public) in circulation outside commercial banks, R^r designates required reserves and R^e denotes excess reserves. The concept of the adjusted base will be used, likewise, and may be represented by

$$B^2 = B^a = B - B_0 \quad (12)$$

where B designates the adjusted base, and B_0 stands for borrowings of the commercial banking sector from the central bank.

Both the money supply (M) as defined by

$$M = C^P + D \quad (13)$$

and the supply of credits or the demand for earning assets by banks, $(L + I)$, are determined by transforming the monetary base by a suitable multiplier incorporating behavioural parameters of the commercial banking sector and the nonbank public.

$$M = m_i \cdot B^i \quad i = 1, 2$$

$$K = L + I = a_i B^i$$

The money multipliers for the different concepts are:

$$m_1 = \frac{1 + k}{(r^r + r^e)(1 + t + s) + k} \quad (\text{unadj. base model}) \quad (14)$$

$$m_2 = \frac{1 + k}{(r^r + r^e - b)(1 + t + s) + k} \quad (\text{adj. base model}) \quad (15)$$

The credit or earning asset multipliers are:

$$a_1 = \frac{(1 - r^r - r^e + b)(1 + t + s)}{(r^r + r^e)(1 + t + s) + k} \quad (\text{unadj. base model}) \quad (16)$$

$$a_2 = \frac{(1 - r^r - r^e + b)(1 + t + s)}{(r^r + r^e - b)(1 + t + s) + k} \quad (\text{adj. base model}) \quad (17)$$

II.

This section introduces requirements of asset reserves, in addition to requirements of liability reserves.

I shall distinguish five broad categories of assets:

- a) Loans (L)
- b) investments (I)
- c) total earning assets ($K = L + I$)
- d) "unborrowed" total earning assets ($K - B_o$)
- e) "borrowed" total earning assets (B_o)

For reasons of symmetry, I may again neglect either of the first two categories. I choose to avoid the longer word "investments", but (again) it is understood that in the following the word "loan" may be replaced by the word "investment" without affecting the formal results, general statements and conclusions of my analysis of loan reserves.

I shall distinguish four types or models of asset reserves, according to the asset category in proportion to which asset reserves are required. Formally, the chosen types are characterized by the following equations:

$$R_1^a = r_A L \quad (\text{model 1} = m 1) \quad (18)$$

$$R_2^a = r_A K \quad (\text{model 2} = m 2) \quad (19)$$

$$R_3^a = r_A (K - B_o) \quad (\text{model 3} = m 3) \quad (20)$$

$$R_4^a = r_A B_o \quad (\text{model 4} = m 4) \quad (21)$$

r_A is the percentage rate of required asset reserves.

In order to derive multipliers, it is appropriate to restate the user's side of the monetary base (B^i) in each of the four cases:

$$B_j^i = B^i + R_j^a \quad (\text{model } j, j = 1, \dots, 4) \quad (22)$$

$$(i = 1, 2)$$

B^1 is equal to the sum of currency and all reserves except asset reserves; B^2 is equal to B^1 minus borrowings, B_0 ,

$$B^i = k \cdot D + (r_i^r + r^e) (1 + t + s) D, \quad (23)$$

$$(i = 1, 2)$$

and R_j^a equals asset reserves of type j , while r_i^r and R^r in (24) refer to liability reserves.

By a similar restatement of the consolidated balance sheet of the banking sector, I now have

$$L + I + R^r + R^e + R_j^a = (D + T + S) + B_0 \quad (24)$$

Alternative rearrangements of the balance sheet equation

$$L + R_1^a = (D + T + S) + (B_0 - I - R^r - R^e) = (1 + r_A) L \quad (\text{m } 1) \quad (25)$$

$$K + R_2^a = (D + T + S) + (B_0 - R^r - R^e) = (1 + r_A) K \quad (\text{m } 2) \quad (26)$$

$$(K - B_0) + R_3^a = (D + T + S) - (R^r + R^e) = (1 + r_A) (K - B_0) \quad (\text{m } 3) \quad (27)$$

$$B_0 + R_4^a = (1 + r_A) B_0 \quad (\text{m } 4) \quad (28)$$

together with equations (18)-(21) and appropriate scaling procedures give expressions for R_j^a

$$R_1^a = \frac{r_A}{(1 + r_A)} (1 + b - i - r^r - r^e) (1 + t + s) D \quad (\text{m } 1) \quad (29)$$

$$R_2^a = \frac{r_A}{(1 + r_A)} (1 + b - r^r - r^e) (1 + t + s) D \quad (\text{m } 2) \quad (30)$$

$$R_3^a = \frac{r_A}{(1 + r_A)} (1 - r^r - r^e) (1 + t + s) D \quad (\text{m } 3) \quad (31)$$

$$R_4^a = r_A b (1 + t + s) D \quad (\text{m } 4) \quad (32)$$

These expressions may be used to fully specify B_j^i in terms of demand deposits (D) and behavioural parameters. The details of this specification need not to be stated here explicitly.

III.

My next step is to redefine the total reserve ratio for both the adj. and the unadj. base version of each of the four models. Scaling all reserves to total deposits, $(1 + t + s) D$, the total reserve ratio may be represented by

$$r_1^a = r_1 + r^a \quad (33)$$

where

$$r_1 = r^r + r^e \quad (34)$$

Here asset reserves are captured by the ratio r^a , which should be sharply distinguished from the ratio r_A , the latter being associated with different scales. Index 1 still refers to the unadjusted base version, while (below) index 2 is again chosen to indicate the adjusted base version:

$$r_2^a = r_1^a - b . \quad (35)$$

In table 1 reserve ratios are defined in columns 1 to 4, each column relating to an equally numbered model.

Using these definitions for r_i^a ($i = 1, 2$), numerators (Z), denominators (N_i), and multipliers may be represented in a general fashion covering all four models by:

$$\left. \begin{aligned} Z &= (1 - r_2^a) (1 + t + s) \\ N_i &= r_i^a (1 + t + s) + k \\ m_i^a &= \frac{1 + k}{N_i} \\ a_i^a &= \frac{Z}{N_i} \end{aligned} \right\} \quad i = 1, 2 \quad (36)$$

where again

$$Z = (1 + t + s) + k - N_2 \quad (37)$$

IV.

From table 1 it may be seen that

$$r_i^a = r_i^a(r_A), \quad \frac{\partial r_i^a}{\partial r_A} > 0. \quad i = 1, 2 \quad (38)$$

Therefore, both the money and the credit multiplier is reduced by introducing or increasing an effective rate of asset reserves (r_A). This result is of course independent of the base version used. Thus, the restrictive effect of asset reserves on the supply of money and credit is obvious and may be stated as follows:

$$\begin{aligned} m_i^a &< m_i \\ a_i^a &< a_i \end{aligned} \quad i = 1, 2 \quad \text{if and only if } r_A > 0 \quad (39)$$

$$\frac{\partial m_i^a}{\partial r_A} < 0 \quad \frac{\partial a_i^a}{\partial r_A} < 0 \quad i = 1, 2 \quad (40)$$

(m_i and a_i have been defined in (14) - (17).)

Since required asset reserves primarily affect the total reserves ratio, it is obvious that equally contractive effects can be

reached by an appropriate change in liability reserve requirements. Consequently, a central bank in possession of liability reserve requirements as a policy instrument does not increase its aggregate effectiveness by introducing required asset reserves as an additional policy instrument. Clearly, asset reserves required for particular components of earning assets only also have allocative effects on the asset supply side and consequently on the structure of interest rates. But these effects are not at stake in my present analysis concentrating on stabilization issues.

Table 2 contains general definitions of elasticities of both the money and the credit multipliers. These definitions state the elasticity of a multiplier as the difference between the elasticity of the numerator and the elasticity of the denominator. The money and the credit multiplier have the same denominator, while the numerators differ.

Elasticities of the numerator of the credit multiplier are given in table 3. The elasticity of the numerator of the money multiplier is zero, for all ratios except k . Therefore, no separate table is required. Elasticities of the denominator of both the credit and the money multiplier are contained in table 4.

V.

The main results of this part are contained in tables 5 and 6. Table 5 gives signs and values of the money multiplier elasticities

and states their reaction to a change in asset reserve requirements. In table 6 the same is given for the credit multiplier elasticities. The effects of asset reserves on the elasticities depend on the parameter with respect to which the elasticity is taken. It is meaningful to distinguish between two classes of parameters. The one class contains parameters under control of the authorities, such as r^r , r^d , r^t , r^s and r_A . The other class contains parameters not under control of the authorities, but under control of the bank and nonbank public, such as k , t , s , i , b and r^e .

In each of the four asset reserve models introducing or increasing asset reserve requirements with a few exceptions increases the elasticities of the money multiplier with respect to parameters not under control of the authorities. The exceptions relate to k , b and i . While a decrease is possible for k , the elasticity with respect to i simply cannot change in three of the four models. A less weak exception is b in the adjusted base version with a decrease of the elasticity, while the elasticity with respect to b in the unadjusted base version of model 3, again, simply cannot change.

With one exception, the elasticities of the money multiplier with respect to parameters controlled by the authorities are reduced in each of the four models, if asset reserve requirements are introduced or increased. For r_A this does not happen before r_A has become sufficiently large.

In general, it is not possible to indicate unequivocally the response of credit multiplier elasticities to an increase in asset reserve requirements. But it may be seen from table 6, that the elasticity with respect to k definitely decreases while the elasticities with respect to t and s definitely decrease only if the ratios of liability reserves required for time and savings deposits do not differ. The elasticity with respect to the investment ratio i increases in model 1 (asset reserves on loans), while this elasticity cannot change in all other models. The elasticity with respect to b in the unadjusted base version of model 3 cannot change either. The elasticity of the credit multiplier with respect to asset reserve requirements will also increase, if these requirements are sufficiently small. The reaction is undetermined for sufficiently large r_A except for model 4, where the reaction definitely remains positive.

Thus, I find that in those cases where definite statements about reactions of credit multiplier elasticities can be made, except with respect to i and b (unadjusted base version, model 3), an increase in asset reserve requirements reduces the sensitivity of the credit multiplier with respect to parameters controlled by the bank and nonbank public, while it increases the sensitivity of the credit multiplier with respect to the asset reserve parameter controlled by the authorities.

VI.

The main results of part A of this paper dealing with four models of required asset reserves may be summarized by the following arguments against asset reserve requirements:

1. Asset reserves are not necessary for an effective control of the aggregate supply of money and credit, particularly if the instrument of liability reserves is available.
2. Applying asset reserves in addition to liability reserves reduces the effectiveness of control of the money supply by liability reserves of a given amount.
3. Applying asset reserves in addition to liability reserves increases the effects on the money multiplier of given relative changes in the behaviour of the (private) nonbank public and thus weakens the potency of monetary policy as far as shifts in the time and savings deposit ratio are concerned, while the direction of change in the effects of shifts in the currency ratio is uncertain.
4. For given relative changes in the behaviour of commercial banks this predominance of increases in elasticities does not necessarily prevail. Whether there exists a balance between increases and decreases or a dominance of increases over decreases depends on the precise type of asset reserve model considered.

With respect to the goal of an effective control of the money supply by central bank authorities, these results clearly point towards the existence of undesirable consequences of asset reserve requirements.

In the case of Germany there are even signs of a dominance of these undesirable consequences.²⁾

If the policy goal consists of an effective control of the credit supply instead of the money supply, this evaluation of asset reserves slightly changes.

Part B

I.

In this part B I shall demonstrate how to reduce incremental asset reserve requirements to the types underlying the analysis in part A. This task is twofold.

First, it is a formal task. It is necessary to give a technical treatment of incremental asset reserve requirements. Second, it is necessary to adjust or qualify the nonformal results derived in part A in order to cover incremental types of asset reserve requirements. The second task will be dealt with in section II of this part B.

In a purely incremental model current asset reserve requirements, R_{jt}^a , are related to the series of current and past changes in the volume of the asset in question. In case of model 1:

$$R_{1t}^a = \sum_{\tau=t}^{-\infty} r_{A,\tau}^* (L_{\tau} - L_{\tau-1}) = r_{A,t}^* (L_t - L_{t-1}) + R_{1,t-1}^a \quad (41)$$

$r_{A,\tau}^*$ are exogenously given incremental rates of required asset reserves. Dividing the currently required asset reserves, R_{1t}^a , by the corresponding current asset volume, L_t (in model 1), one obtains the (endogenous) average rate of required asset reserves.

$$r_{A,t} = \frac{r_{A,t}^* (L_t - L_{t-1})}{L_t} + \frac{1}{L_t} \sum_{\tau=t-1}^{-\infty} r_{A,\tau}^* (L_{\tau} - L_{\tau-1}) \quad (42)$$

The recursive structure of (41) allows to rewrite (42) as

$$r_{A,t} = \left[r_{A,t}^* \quad g(L_t) \quad + \quad r_{A,t-1} \right] \frac{L_{t-1}}{L_t} \quad (43)$$

with the growth rate g defined by (44)

$$g(L_t) \equiv \frac{L_t - L_{t-1}}{L_{t-1}} \quad (44)$$

Obviously, while $r_{A,t}^*$ and $r_{A,t-1}$ are exogenous, $r_{A,t}$ is an endogenous variable for which an important elasticity expression may be derived. Given that all L_τ for $\tau < t$ are predetermined, that elasticity is given by (45):

$$\varepsilon(r_{A,t}, x) \equiv y + E(x) \quad (45)$$

where

$$y = \begin{cases} 0 & \text{if } x \neq r_{A,t}^*, r_{A,t-1} \\ w_1 & \text{if } x = r_{A,t}^* \\ w_2 & \text{if } x = r_{A,t-1} \end{cases} \quad (46)$$

and

$$E(x) \equiv F(L_t) \cdot \varepsilon(L_t, x) \quad (47)$$

with

$$F(L_t) \equiv \frac{r_{A,t}^* - r_{A,t-1}}{r_{A,t}} \cdot \frac{L_{t-1}}{L_t} \quad (48)$$

$$= \frac{r_{A,t}^* - r_{A,t}}{r_{A,t}} \quad (49)$$

and

$$w_1 = \frac{r_{A,t}^* \cdot g(L_t)}{r_{A,t}^* \cdot g(L_t) + r_{A,t-1}} \quad (50)$$

$$w_2 = \frac{r_{A,t-1}}{r_{A,t}^* \cdot g(L_t) + r_{A,t-1}} \quad (51)$$

By means of

$$L = \alpha_i () B^i \quad (52)$$

where

$$\alpha_i = \frac{1}{1+i} a_i \quad (53)$$

one obtains

$$E(x) = F(L_t) \left\{ \frac{i}{1+i} \left[\epsilon(1,x) - \epsilon(i,x) \right] + \epsilon(a_i,x) + \epsilon(B^i,x) \right\} . \quad (54)$$

These formulas and elasticity expressions, (41) - (54), may be adjusted to the case of model 2 by substituting K for L and by setting the investment ratio i equal to zero. In (52) this substitution implies replacement of α_i by a_i and in (54) it implies a vanishing of the coefficient for the square bracket term.

For model 3 corresponding formulas are obtained by substitution of $K - B_0$ for L in (41) - (54). This model requires replacement of (52) by (55):

$$K - B_0 = a_i B^i - \frac{b(1+t+s)}{r_i(1+t+s) + k} B^i \quad (55)$$

$$(i = 1, 2)$$

The associated elasticities are again given by (45) and (46) if (54) is replaced by (56):

$$E(x) = F(K_t - B_{0t}) \left\{ \varepsilon \left[(1 - r_1)(1+t+s), x \right] - \varepsilon \left[r_i(1+t+s) + k, x \right] + \varepsilon(B^i, x) \right\} \quad (56)$$

$$(i = 1, 2)$$

For the purposes of model 4 in (41) - (54) B_0 should be substituted for L . This type of model requires replacement of (52) by (57)

$$B_0 = \frac{b(1+t+s)}{r_i(1+t+s) + k} B^i \quad (57)$$

$$(i = 1, 2)$$

while the elasticities are again given by (45) and (46) if (54) is replaced by (58):

$$E(x) = F(B_{0t}) \left[\varepsilon(b(1+t+s), x) - \varepsilon(r_i(1+t+s) + k, x) + \varepsilon(B^i, x) \right]$$

$$(i = 1, 2)$$

Let me introduce the generalizing ^{term} "asset reserve base" for L in model 1, K in model 2, K - B₀ in model 3 and B₀ in model 4.

The model of incremental asset reserve requirements considered so far appears to have a purely incremental structure.

However it is easy to introduce mixed types with both incremental and level related elements. For example, one may think of a model where current increments are subject to an incremental rate, $r_{A,t}^*$, while the level of the asset reserve base at the end of the preceding period (in model 1: L_{t-1}) is subjected as a whole to a level or average rate of required asset reserves, $r_{A,t}^{**}$.

This model is not really an alternative to (41) but a special case of the purely incremental model. If all the incremental rates of required reserves for past increments are equal, then the whole series of asset reserve requirements for past increments may be expressed as a single level related asset reserve requirement with a level or average rate, $r_{A,t}^{**}$, equal to the incremental rates:

$$r_{A,t}^{**} = r_{A,t-1}^* = r_{A,t-2}^* = r_{A,t-3}^* = \dots \quad (59)$$

Substituting $r_{A,t}^{**}$ for $r_{A,t-1}^*$, $r_{A,t-2}^*$ etc. in (42), one has

$$r_{A,t} = \frac{r_{A,t}^*(L_t - L_{t-1})}{L_t} + \frac{r_{A,t}^{**} L_{t-1}}{L_t} \quad (60)$$

Therefore, in equation (43) $r_{A,t-1}$ simply has to be replaced by $r_{A,t}^{**}$ in order to cover this mixed model of asset reserve requirements.

II.

In this section analytical steps are taken that prepare for section III where it will be examined how incremental asset reserve requirements affect the effectiveness of monetary policy.

I start out with a general discription of the elasticity of the money multiplier with asset reserve requirements

$$\begin{aligned}
 \bar{\epsilon}(m_i^a, x) = & \epsilon(m_i^a, k) \bar{\epsilon}(k, x) \\
 & + \epsilon(m_i^a, t) \bar{\epsilon}(t, x) \\
 & + \epsilon(m_i^a, s) \bar{\epsilon}(s, x) \\
 & \text{-----} \\
 & + \epsilon(m_i^a, i) \bar{\epsilon}(i, x) \\
 & + \epsilon(m_i^a, b) \bar{\epsilon}(b, x) \\
 & + \epsilon(m_i^a, r^e) \bar{\epsilon}(r^e, x) \\
 & \text{-----} \\
 & + \epsilon(m_i^a, r^r) \bar{\epsilon}(r^r, x) \\
 & + \epsilon(m_i^a, r_A) \bar{\epsilon}(r_A, x)
 \end{aligned}
 \tag{61}$$

(i = 1,2)

Dashed elasticity expressions are total as opposed to partial elasticities. The last additive term on the right hand side of (61) is due to asset reserve requirements.

The list of elements is delineated into three groups. The first group relates to parameters of the (private) nonbank public, the second group to parameters of the commercial banks and the third group to parameters of the monetary authorities. In case of models 2, 3 and 4 the fourth line in (61), counting from above, vanishes. If, in the case of model 3, the unadjusted base version is considered, then the fifth line of (61) vanishes, too.

The elements on the right of (61) are products of two elasticities. All the tables in this paper relate to the first of the two elasticities, forming these products. The second factors require some further explanations. As an example consider the elasticity of excess reserves:

$$\bar{\epsilon}(r^e, x) = \epsilon(r^e, i_L) \epsilon(i_L, x) + \epsilon(r^e, i_G) \epsilon(i_G, x) + \epsilon(r^e, x) \quad (62)$$

This is a total elasticity decomposed into elementary partial elasticities on the right hand side. The first two products in (62) capture interest rate effects. If these effects are neglected the total elasticity reduces to the partial one, the last term in (62). If variables in addition to i_L and i_G would be allowed to become arguments of the ratios k , t , s , r^e , etc.,

then there would be additional product terms in (62).

An expression similar to (61) holds for the credit multiplier

$$\begin{aligned}
 \bar{\epsilon}(a_i^a, x) &= \epsilon(a_i^a, k) \bar{\epsilon}(k, x) \\
 &\quad \dots \\
 &\quad + \epsilon(a_i^a, b) \bar{\epsilon}(b, x) \\
 &\quad \dots \\
 &\quad + \epsilon(a_i^a, r^r) \bar{\epsilon}(r^r, x) \\
 &\quad + \epsilon(a_i^a, r_A) \bar{\epsilon}(r_A, x)
 \end{aligned}
 \tag{i = 1, 2} \tag{63}$$

Again the last term is due to asset reserve requirements. There is a sharp distinction to be drawn between the total elasticity in (63) for the credit multiplier with asset reserve requirements and the total elasticity for the credit multiplier without asset reserves in (64):

$$\begin{aligned}
 \bar{\epsilon}(a_i, x) &= \epsilon(a_i, k) \bar{\epsilon}(k, x) \\
 &\quad \dots \\
 &\quad + \epsilon(a_i, b) \bar{\epsilon}(b, x) \\
 &\quad \dots \\
 &\quad + \epsilon(a_i, r^r) \bar{\epsilon}(r^r, x)
 \end{aligned}
 \tag{i = 1, 2} \tag{64}$$

In (64) there exists no term with r_A .

In order to determine the effects of asset reserve requirements on various elasticities when r_A is endogenous, it is no longer possible to proceed as in part A. With an endogenous r_A ,

$\bar{\epsilon}(r_A, x)$ is not always zero for $x \neq r_A$. For model 1, one may substitute for $\bar{\epsilon}(r_A, x)$ in (61) by exploiting equations³⁾ (45) - (54). In (54) the elasticity of the credit multiplier without asset reserves, as stated in (64), is of crucial importance and it enters the elasticity expression for the money multiplier with asset reserve requirements.

This is obvious from (65) where (61) is rewritten by ans of (45) - (54) and (64).

model 1:

$$\begin{aligned}
 \bar{\epsilon}(m_i^a, x) = & \left[\begin{array}{c} ? \\ \epsilon(m_i^a, k) + H \epsilon(a_i, k) \\ ? \end{array} \right] \bar{\epsilon}(k, x) \\
 & + \left[\begin{array}{c} + \\ \epsilon(m_i^a, t) + H \epsilon(a_i, t) \\ - \end{array} \right] \bar{\epsilon}(t, x) \\
 & + \left[\begin{array}{c} + \\ \epsilon(m_i^a, s) + H \epsilon(a_i, s) \\ - \end{array} \right] \bar{\epsilon}(s, x) \\
 & \text{-----} \\
 & + \left[\begin{array}{c} + \\ \epsilon(m_i^a, i) + H \underbrace{\epsilon(a_i, i)}_{=0} \\ + \end{array} \right] \bar{\epsilon}(i, x) \\
 & \left. \begin{array}{l} \left\{ \begin{array}{l} + \text{ unadj. base model} \\ - \text{ adj. base model} \end{array} \right\} \\ + \left[\begin{array}{c} \epsilon(m_i^a, b) + H \epsilon(a_i, b) \\ - \text{ unadj. base model } (\frac{?}{?}) \\ + \text{ adj. base model} \end{array} \right] \bar{\epsilon}(b, x) \\ \left. \begin{array}{l} \left\{ \begin{array}{l} - \text{ unadj. base model } (\frac{?}{?}) \\ + \text{ adj. base model} \end{array} \right\} \end{array} \right\} \\
 & + \left[\begin{array}{c} - \\ \epsilon(m_i^a, r^e) + H \epsilon(a_i, r^e) \\ - \end{array} \right] \bar{\epsilon}(r^e, x) \\
 & \text{-----}
 \end{aligned} \tag{65}$$

$$\begin{aligned}
& + \left[\epsilon(m_{\underline{i}}^{\bar{a}}, r^r) + \frac{H}{\bar{t}} \epsilon(a_{\underline{i}}, r^r) \right] \bar{\epsilon}(r^r, x) \\
& \quad (\bar{t}) \\
& + \epsilon(m_{\underline{i}}^a, r_A) F(L) \left\{ \frac{i}{1+i} \left[\bar{\epsilon}(1, x) - \bar{\epsilon}(i, x) \right] + \bar{\epsilon}(B^i, x) \right\} \\
& \text{-----} \\
& \quad (\bar{t}) \\
& + \epsilon(m_{\underline{i}}^a, r_A) y
\end{aligned}$$

where

$$H \equiv F(L_t) \epsilon(m_{\underline{i}}^a, r_A) . \quad (66)$$

$$(i = 1, 2)$$

y and F were defined in (46) and (48)

The corresponding elasticity expression for model 2 differs only slightly from (65) by dropping the fourth line, counted from above, substituting K for L and setting $i/(1+i) = 0$ in the second line, counted from below. Also, for model 2 K has to be substituted for L in (66) and (48).

The corresponding elasticity expressions for model 3 and model 4 are stated in (67) - (68) and (69) - (70), respectively. Using (55) and (56) in case of model 3 and (57) and (58) in case of model 4 these elasticities may be derived by the same procedure that led to (65).

model 3:

$$\begin{aligned}
 \bar{\varepsilon}(m_i^a, x) &= \left[\begin{array}{c} (\bar{+}) \\ \varepsilon(m_i^a, k) - H'k \\ (\bar{+}) \\ \bar{+} \end{array} \right] \bar{\varepsilon}(k, x) \\
 &+ \left[\begin{array}{c} + \\ \varepsilon(m_i^a, t) + H'k \frac{t}{(1+t+s)} \\ - \\ \bar{+} \end{array} \right] \bar{\varepsilon}(t, x) \\
 &+ \left[\begin{array}{c} + \\ \varepsilon(m_i^a, s) + H'k \frac{s}{(1+t+s)} \\ - \\ \bar{+} \end{array} \right] \bar{\varepsilon}(s, x) \\
 &----- \\
 &\left\{ \begin{array}{l} \circ \text{ unadj. base model (o=no change)} \\ - \text{ adj. base model} \end{array} \right. \\
 &+ \left[\begin{array}{c} \varepsilon(m_i^a, b) + H' \frac{(1+t+s)b}{\bar{+}} \\ \circ \text{ if } i=1 \\ + \text{ if } i=2 \end{array} \right] \bar{\varepsilon}(b, x) \quad (67) \\
 &\left\{ \begin{array}{l} \circ \text{ unadj. base model} \\ + \text{ adj. base model} \end{array} \right. \\
 &+ \left[\begin{array}{c} - \\ \varepsilon(m_i^a, r^e) - H' \frac{(1-r_1+r_i)(1+t+s)+k}{\bar{+}} \cdot r^e \\ - \\ \bar{+} \end{array} \right] \bar{\varepsilon}(r^e, x) \\
 &----- \\
 &+ \left[\begin{array}{c} - \\ \varepsilon(m_i^a, r^r) - H' \frac{(1-r_1+r_i)(1+t+s)+k}{\bar{+}} \cdot r^r \\ - \\ \bar{+} \end{array} \right] \bar{\varepsilon}(r^r, x) \\
 &+ \begin{array}{c} (\bar{+}) \\ \varepsilon(m_i^a, r_A) F(K-Bo) \\ - \\ \bar{+} \end{array} \bar{\varepsilon}(B^i, x) \\
 &----- \\
 &+ \begin{array}{c} (\bar{+}) \\ \varepsilon(m_i^a, r_A) y \\ - \\ \bar{+} \end{array}
 \end{aligned}$$

In (67)

$$H' = \frac{F(K_t - Bo_t) \varepsilon(m_i^a, r_A)}{r_i(1+t+s)+k} \quad (68)$$

$F(K_t - Bo_t)$ is obtained from (48) by substituting $K_t - Bo_t$ for L_t and y was defined in (46).

model 4:

$$\begin{aligned}
 \bar{\epsilon}(m_i^a, x) &= \left[\begin{array}{c} \bar{+} \\ \epsilon(m_i^a, k) - H''k \\ \bar{+} \end{array} \right] \bar{\epsilon}(k, x) \\
 &+ \left[\begin{array}{c} \bar{+} \\ \epsilon(m_i^a, t) + H''k \frac{t}{(1+t+s)} \\ \bar{+} \end{array} \right] \bar{\epsilon}(t, x) \\
 &+ \left[\begin{array}{c} \bar{+} \\ \epsilon(m_i^a, s) + H''k \frac{s}{(1+t+s)} \\ \bar{+} \end{array} \right] \bar{\epsilon}(s, x) \\
 &----- \\
 &\left\{ \begin{array}{l} + \text{ unadj. base model} \\ - \text{ adj. base model} \end{array} \right\} \\
 &+ \left\{ \begin{array}{l} \epsilon(m_i^a, b) + H'' \left[\frac{(1+t+s)b + r_i (1+t+s) + k}{\bar{+}} \right] \\ - \text{ unadj. base model} \\ + \text{ adj. base model} \end{array} \right\} \bar{\epsilon}(b, x) \\
 &----- \\
 &+ \left[\begin{array}{c} \bar{-} \\ \epsilon(m_i^a, r^e) - H'' \frac{(1+t+s) r^e}{\bar{+}} \\ \bar{-} \end{array} \right] \bar{\epsilon}(r^e, x) \\
 &----- \\
 &+ \left[\begin{array}{c} \bar{-} \\ \epsilon(m_i^a, r^r) - H'' \frac{(1+t+s) r^r}{\bar{+}} \\ \bar{-} \end{array} \right] \bar{\epsilon}(r^r, x) \\
 &+ \\
 &+ \epsilon(m_i^a, r_A) F(Bo) \cdot \bar{\epsilon}(B^i, x) \\
 &----- \\
 &+ \epsilon(m_i^a, r_A) y
 \end{aligned}
 \tag{69}$$

In (69)

$$H'' = \frac{F(Bo_t) \epsilon(m_i^a, r_A)}{r_i (1+t+s) + k} \tag{70}$$

$F(Bo_t)$ is obtained from (48) by substituting Bo_t for L_t and y was defined in (46).

If the unadjusted base version is used then $\varepsilon(m_i^a, b) =$
 $\varepsilon(m_1^a, b) = 0$.

In (65), (67) and (69) a plus or a minus sign below an elasticity term indicates its being greater or smaller than zero. In (7) the minus sign holds for "small" values of r_A , the "plus" sign for "large" r_A .

It is profitable to distinguish several regimes of incremental asset reserve requirements.

Regime 1: Incremental rates of asset reserves required are nonnegative, $r_{A,\tau}^* \geq 0$, and may be positive even if the asset reserve base falls (in model 1: $L_\tau - L_{\tau-1} < 0$).

Regime 2: Incremental rates of asset reserves required are nonnegative, $r_{A,\tau}^* \geq 0$, and may be positive only if the asset reserve base rises (in model 1: $L_\tau - L_{\tau-1} > 0$).

Regime 3: Incremental rates are as in regime 1. In addition, whenever the calculation of required asset reserves leads to a negative value then required asset reserves are set to zero.

Regime 1 is a completely symmetric system of incremental reserve requirements. It treats increases of the reserve base potentially like decreases. Here, negative asset reserves may be required. Negative required reserves may be interpreted as a required borrowing from (debts to) the central bank, positive required reserves as a required borrowing (credits) to the central bank.

In contrast to regime 1 required asset reserves and therefore $r_{A,\tau}$, the average rate of required asset reserves for period τ , ^{cannot} become negative in regimes 2 and 3.

What are the signs of H , H' and H'' ? It is easy to verify that, given a nonnegative $r_{A,t}^*$, F is negative if $r_{A,t}$ is negative. As noted already, in regimes 2 and 3

$r_{A,t}$ cannot become negative, while in regime 1 a negative $r_{A,t}$ is possible. On the other hand, with $r_{A,t} > 0$, F is positive if and only if $r_{A,t}^* > r_{A,t}$. Thus, provided positive (greater than zero) holdings of asset reserves are required, F is positive if and only if the incremental (marginal) rate of required asset reserves is larger than the average rate of required asset reserves. Thus, it is sufficient for H , H' and H'' to be negative that the average rate of required asset reserves is both positive and smaller than the incremental rate. Under regimes 2 and 3 this condition is also necessary. In other regimes necessity does not obtain.

Now, it is possible to inquire into the changes of the effectiveness of monetary policy. For that purpose the terms in square brackets of (65), (67) and (69) are compared with elasticities $\epsilon(m_i, k)$ $\epsilon(m_i, t)$ etc. where $r_A = 0$.

III.

In part A, results bearing on the issue of changing effectiveness of monetary policy were derived by **exclusive** comparison of the elasticities $\epsilon(m_i^a, x)$, i.e. the first terms inside the square brackets of (65), (67) and (69) with the elasticities holding for the case $r_A = 0$. The second terms inside the square brackets are products of elasticities with factors H , H' and H'' . These product terms vanish if r_A is truly exogenous or predetermined as it was assumed in part A.

With incremental asset reserve requirements r_A is endogenous. An endogenous r_A changes the coefficients of $\bar{\epsilon}(k, x)$, $\bar{\epsilon}(t, x)$ etc. in two ways. The first way of change is the one considered in part A. It is given by the difference of $\epsilon(m_i^a, k)$, $\epsilon(m_i^a, t)$ etc. against their counterparts $\epsilon(m_i, k)$, $\epsilon(m_i, t)$ etc., the latter representing the case $r_A = 0$. This way of change is indicated in (65), (67) and (69) by a plus or a minus sign placed above the respective elasticities. There, a minus sign signals a fall of the absolute value of an elasticity, i.e. of $|\epsilon(\quad, \quad)|$ etc.

The second way of change is given by the addition or subtraction of the products containing the factor H , H' or H'' . A careful examination of how the terms in square brackets change reveals that the two changes may additively reinforce each other. We shall distinguish three patterns, two of which are patterns of reinforced changes and one is a pattern of conflicting changes.

Pattern 1 : A negative elasticity $\epsilon(m_1^a, k)$ is smaller in absolute value than its counterpart $\epsilon(m_1, k)$. The addition of a positive term $H \cdot \epsilon(a_1, k)$ reinforces a fall in the absolute value of the coefficient of $\bar{\epsilon}(k, x)$. (In this example one cannot exclude the possibility of this coefficient turning from^a negative to a positive value. In such cases of sign changes the possibility of an increase in the absolute value of an elasticity is neglected.) The reinforcement of the decrease of the positive elasticity $\epsilon(m_2^a, b)$ by the addition of a negative term $H \epsilon(a_2, b)$ may be subsumed with pattern 1.

Pattern 2 : The negative elasticity $\epsilon(m_1^a, t)$ is larger in absolute value than its counterpart $\epsilon(m_1, t)$. The addition of a negative term $H \cdot \epsilon(a_1, t)$ reinforces the rise in the absolute value of the coefficient of $\bar{\epsilon}(t, x)$.

The reinforcement of the increase of a positive elasticity $\epsilon(m_1^a, k)$ by the addition of a positive term $-H' k$ or $-H'' k$ may be subsumed with pattern 2.

Pattern 3 : With H, H' or H'' positive the addition of terms containing H etc. either partially or fully or more than fully compensates the decrease or increase of an elasticity as observed in part A.

Changes according to pattern 1 may also be observed for the coefficients of the elasticity for excess reserves, $\bar{\epsilon}(r^e, x)$, for the coefficient of the elasticity of required reserves,

$\bar{\epsilon}(r^F, x)$, for the coefficient of $\bar{\epsilon}(k, x)$ if $\epsilon(m_i^a, k) < 0$, and for the coefficient of $\bar{\epsilon}(b, x)$ in the adjusted base model. Changes according to pattern 2 may also be observed for the coefficient of $\bar{\epsilon}(k, x)$ if $\epsilon(m_i^a, k) > 0$, for the coefficient of $\bar{\epsilon}(s, x)$ and for the coefficient of $\bar{\epsilon}(b, x)$ in the unadjusted base model. Changes according to pattern 3 occur if $F < 0$. A summary of these findings is exhibited in table 7.

Thus, if H, H' and H'' are negative and as far as those elasticities are concerned for which, in part A, it was possible to make definite statements about the direction of change, with one exception, the results are not only confirmed but even significantly strengthened by considering incremental asset reserve requirements. The exception is the change of the coefficient of $\bar{\epsilon}(i, x)$, with either no change at all or no modification of the change found in part A. However, if H, H' and H'' are positive, then there are two components of changes which work in opposite direction. The component that is due to the incremental form of the asset reserve requirements predominantly either weakens or balances or overcompensates the changes observed in part A. Remember that H etc. cannot become positive or zero if positive asset reserve holdings are required and if the incremental (marginal) rate of required asset reserves is larger than the average rate of required asset reserves.

IV.

In summarizing part B and as far as the control of the money supply is concerned, it is possible to state that the results of part A are qualitatively confirmed and quantitatively reinforced if, with positive asset reserve requirements, the incremental rate of required asset reserves is larger than the average rate. On the other hand, if the incremental rate of required asset reserves is lower than the average rate, then instead of reinforcement there will be attenuation or even reversal of the results of part A with respect to the control of the money supply. Since, in part A, it was possible only for a very few parameters to make definite statements about reactions of credit multiplier elasticities the analysis in part B is limited to money multiplier elasticities.

An intuitively appealing explanation of some results will be attempted next. A decrease in the elasticity of the money multiplier elasticity with respect to any of the rates of required liability reserves may be explained by noting that an increase in liability reserve requirements reduces credits and thus liberates asset reserves which may be used to cover liability reserve requirements. Thus, increasing the rate of required liability reserves when asset reserves exist will, to some extent, simply induce a relabelling of asset reserves as liability reserves.

The liberation of asset reserves allows to meet increased liability reserve requirements by a weaker contraction of credits and the money supply than would be necessary if no asset reserves were required and held. This reduction in the effectiveness of given relative changes in liability reserve requirements with respect to the money supply is stronger the higher the effective incremental or marginal rate of required asset reserves. (The weaker contraction of both credits and the money supply does not imply a reduction of the credit supply elasticity as it does for the money supply elasticity (see table 6) for the following reason. By an introduction or increase of asset reserve requirements the credit supply is reduced more than the money supply. Therefore there is a difference in the basis (denominator) from which relative changes are measured for given absolute changes. This difference allows the elasticity of the credit multiplier with respect to liability reserve requirements to increase while the elasticity of the money multiplier with respect to the same parameter definitely decreases by the introduction or an increase of asset reserve requirements.)

Excess reserves may be interpreted as reserves required by commercial banks - instead of the central bank - and held by commercial banks. In the adjusted base model an increase in the borrowing ratio, b , is equivalent to a decline of a reserve ratio. Thus, the intuitive explanation of the elasticity changes given for required reserves in the usual sense, r^F , similarly applies both to the decline of the money multiplier elasticity with respect to

to the excess reserves ratio and to the decline of that elasticity with respect to the borrowings ratio (in the adjusted base model).

Obviously, currency holdings of the public may be viewed as reserve holdings required and held by the public. Therefore, a shift in the currency ratio, k , is equivalent to a change in reserve requirements.

If $r^a (1 + t + s)$, the sum of excess reserves of commercial banks and ^{required} reserves by the central bank, expressed as a ratio per demand deposits is smaller than 1, a rise (fall) in k is equivalent to an increase (decrease) in average required reserves. If the change in k is equivalent to an increase (decrease) in average required reserves then $\epsilon(m_1^a, k) < 0 (> 0)$ and the foregoing interpretation applies to the decline (rise) of the money multiplier elasticity (in absolute terms) with respect to the currency ratio, k , when that elasticity is negative (positive).

An increasing elasticity of the money multiplier with respect to the time deposit ratio, t , and with respect to the savings deposit ratio, s , may be explained in a similar intuitive fashion. The "normal" case of negative elasticities is considered only. An increase in time deposits implies for commercial banks either an influx of currency or an increase in earning assets or a combination of both elements. The less the element "increase in earning assets" is contained in this combination the higher will be the relative reduction in money supply for a given relative increase of t .

Now, asset reserve requirements change that combination in favor of additions to currency holdings of banks.

This implies that asset reserve requirements increase the contractive effect of a rise in t . Obviously, the line of reasoning for s is symmetric to that for t .

C o n c l u s i o n

From the present paper the following results emerge. Asset reserve requirements tend to reduce the effectiveness of money supply control in several ways. Liability reserve requirements lose part of their effectiveness as a control instrument. With no change in the variance of shifts in the behaviour of the (private) nonbank public the variance of the money multiplier is likely to increase. That part of the multiplier variance which is related to commercial banks is likely to decrease. As a net result, and as is shown by the German case, it is not unlikely that asset reserve requirements raise the volatility of the money multiplier and thus increase parameter uncertainty. However, whether or not they do is an empirical question.

By the theory of economic policy under conditions of parameter uncertainty more instruments usually are considered as preferable to less when the goal of minimizing the variance of a target variable around a chosen target value is pursued.

In that argument the degree of parameter uncertainty of a particular instrument usually is taken as given and assumed to be independent of the number of instruments available. However, in the problem considered here, that assumption is violated with probability one. A higher volatility and greater uncertainty in movements of the money multiplier as it may be caused by asset reserve requirements raises parameter uncertainty of open market operations and other monetary control instruments.

This weakens arguments in favor of asset reserves and clearly points towards the existence of a trade-off between the degree of parameter uncertainty and the number of instruments.

On the other hand, the selective controllability of particular⁴⁾ types of earning assets may be improved considerably. Thus, the final evaluation of asset reserve requirements as an additional tool of monetary policy depends on the kind of chosen goals. If aggregative control of the money stock or of the volume of credit is to be attained then asset reserve requirements in addition to liability reserve requirements do not add to but⁵⁾ rather reduce the effectiveness of monetary policy. However, if particular types of assets are to be selectively controlled, i.e. if the allocation of a given total volume of credit among alternative kinds of earning assets is to be controlled. Then reserve requirements geared to particular earning assets may serve the purpose.

With respect to the money stock as an aggregative control target and as far as selective goals of credit allocation are concerned these results show a strong similarity with results found for parallel questions in an analysis concerned with credit ceilings, - another instrument not yet available in Germany, so far.⁶⁾

A dissimilarity exists with respect to the total volume of credits (earning assets) as a control target. Here, "realistic" types of credit ceilings appear to be considerably more effective than asset reserve requirements.

Footnotes

1) See K. BRUNNER and A. H. MELTZER, Some Further Investigations of Demand and Supply Functions for Money, *Journal of Finance*, 19/1964, pp. 240 - 283,
—— and ——, A Credit Theory of the Money Supply and an Explanation of Two Puzzles in U.S. Monetary Policy, *Rivist. Intern. di Science Econ. e Comm.*, 13/1966, pp. 405 - 430,
—— and ——, Liquidity Traps for Money, Bank Credit, and Interest Rates, *Journal of Political Economy*, 76/1968, S.1-37.

2) This judgement of the German situation is based on rough calculations that take into account differences in the variability of parameters and corresponding variations in the weights of the respective elasticities.

3) The reader should bear in mind that from now on dashed versions of (45), (47 and (54) are used. The dashed version of (45) is

$$\bar{\varepsilon}(r_{A,t}, x) \equiv y + \bar{E}(x) \quad (45')$$

which contains a dashed expression of (54)

$$\bar{E}(x) = F(L_t) \left\{ \frac{i}{1+i} \left[\bar{\varepsilon}(1, x) - \bar{\varepsilon}(i, x) \right] + \bar{\varepsilon}(a_1, x) + \bar{\varepsilon}(B^1, x) \right\} \quad (54')$$

4) The extent of selective controllability is exclusively related to the direct and indirect effects of asset reserve requirements on relative net yields of the various kinds of earning assets.

5) If liability reserve requirements are compared with asset reserve requirements as an alternative, then the latter appear to be more effective. Liquidity effects of asset reserves are accompanied by direct effects on net rates of interest (on earning assets) while liability reserves have liquidity effects only and are not supported by direct effects on net yields

6. See my "Credit Ceilings and the Control of the Money Supply (An Extended Analysis)", Diskussionsbeiträge des Fachbereichs Wirtschaftswissenschaften der Universität Konstanz, Nr.87

Table 1

Asset reserves

Definition of reserve ratios scaled to total deposits

	m 1	m 2	m 3	m 4
μ	$(1+b)-(r^r+r^e+i)$	$(1+b)-(r^r+r^e)$	$(1 - r^r - r^e)$	
r^a	$\mu \frac{r_A}{1 + r_A}$			$r_A \cdot b$
r_1^a un- adj. base model	$r^r + r^e + \mu \frac{r_A}{1+r_A}$			$r^r + r^e + r_A \cdot b$
r_2^a adj. base model	$r_1^a - b$			

Table 2

Elasticities of the money multiplier = $\epsilon (m_i^a, x)$

x	$\epsilon (m_i^a, x)$
k	$\frac{k}{1+k} - \epsilon(N_i, k) = \frac{k}{1+k} - \frac{k}{N_i}$
all x (except k)	$-\epsilon(N_i, x)$

Elasticities of the credit multiplier

x	$\epsilon (a_i^a, x)$
k	$-\epsilon(N_i, k)$
t, s, k	$\frac{x}{Z} - \frac{N_2}{Z} \cdot \epsilon(N_2, x) - \epsilon(N_i, x)$
all x (except t, s, k)	$-\frac{N_2}{Z} \cdot \epsilon(N_2, x) - \epsilon(N_i, x)$

1. legend for

i as an index :

i = 1 : unadjusted base model

i = 2 : adjusted base model

2. $x \in \{ k, t, s, i, b, r^e, r^r, r^d, r^t, r^s, r_A \}$

Table 3

Elasticities of the numerator of the credit multiplier

x	$e(Z, x)$
k	$\frac{k}{Z} - \frac{N_2}{Z} e(N_2, k) = 0$
t, s	$\frac{x}{Z} - \frac{N_2}{Z} e(N_2, x)$
all x (except k, t, s)	$-\frac{N_2}{Z} e(N_2, x)$

$x \in \{k, t, s, i, b, r^e, r^r, r^d, r^t, r^s, r_A\}$

Table 4

Asset reserves

Elasticities of the denominator of both the money and the credit multiplier

$$\epsilon (N_i, x)$$

x	m 1	m 2	m 3	m 4
k	$\frac{k}{N_i}$			
t	$\frac{t(r^t - r^F)}{N_i(1+r_A)} + \frac{t(N_i - k)}{(1+t+s)N_i}$		$\frac{t(r^t - r^F)}{N_i} + \frac{t(N_i - k)}{(1+t+s)N_i}$	
s	$\frac{s(r^S - r^F)}{N_i(1+r_A)} + \frac{s(N_i - k)}{(1+t+s)N_i}$		$\frac{s(r^S - r^F)}{N_i} + \frac{s(N_i - k)}{(1+t+s)N_i}$	
b unadj. base model	$\frac{b(1+t+s)r_A}{N_1(1+r_A)}$		0	$\frac{b(1+t+s)r_A}{N_1}$
b adj. base model	$\frac{-b(1+t+s)}{N_2(1+r_A)}$		$\frac{-b(1+t+s)}{N_2}$	$\frac{-b(1+t+s)(1-r_A)}{N_2}$
i	$-\frac{i(1+t+s)r_A}{N_i(1+r_A)}$	0		
r ^e	$\frac{r^e(1+t+s)}{N_i(1+r_A)}$		$\frac{r^e(1+t+s)}{N_i}$	
r ^r	$\frac{r^r(1+t+s)}{N_i(1+r_A)}$		$\frac{r^r(1+t+s)}{N_i}$	
r ^d	$\frac{r^d}{N_i(1+r_A)}$		$\frac{r^d}{N_i}$	

- cont'd -

Table 4 continued

x	m 1	m 2	m 3	m 4
r^t		$\frac{r^t t}{N_i (1 + r_A)}$		$\frac{r^t t}{N_i}$
r^s		$\frac{r^s s}{N_i (1 + r_A)}$		$\frac{r^s s}{N_i}$
r_A		$\frac{\mu (1 + t + s) r_A}{N_i (1 + r_A)^2}$		$\frac{b (1 + t + s) r_A}{N_i}$

μ is defined in table 1

i in column 1 is the investment ratio, in all other columns i is an index

$i = 1$: unadj. base model

$i = 2$: adj. base model

Table 5

Money multiplier elasticities and their reaction to an increase in required asset reserves

(r_A exogenous)

x	adj. or unadj. base model	sign or value of $\epsilon(m_i^a, x)$ for $r_A \geq 0$				$\frac{\delta \epsilon(m_i^a, x) }{\delta r_A} =$ change of $ \epsilon(m_i^a, x) $ due to an increase of r_A				
		m1	m2	m3	m4	m 1	m 2	m 3	m 4	
k	either	?	?	?	?	?	?	?	?	decrease as long as $\epsilon(\)$ is negative, increase otherwise
t	either	-	-	-	-	+	+	+	+	increase if negative and if $r^t \leq r^r$; for Germany increase even though $r^t > r^r$
s	either	-	-	-	-	+	+	+	+	increase if negative and if $r^s \leq r^r$ as for Germany
i	either	$r_A > 0$	+	o	o	o	+	no change		
		$r_A = 0$	o	o	o	o	+	no change		
b	unadj.	$r_A > 0$	-	-	o	-	+	+	no change	+
		$r_A = 0$	o	o	o	o	+	+	no change	+
b	adj.	+	+	+	+	-	-	-	-	
r^e	either	-	-	-	-	-	-	-	-	
r^r	either	-	-	-	-	-	-	-	-	
r^d	either	-	-	-	-	-	-	-	-	
r^t	either	-	-	-	-	-	-	-	-	
r^s	either	-	-	-	-	-	-	-	-	
r_A	either	$r_A > 0$	-	-	-	-	(\pm)	(\pm)	(\pm)	+
		$r_A = 0$	o	o	o	o	(\pm)	(\pm)	(\pm)	+

Table 5 continued

legend:

1. + positive if sign (or value)
increase if change (or reaction)
2. - negative if sign (or value)
decrease if change (or reaction)
3. (\pm) positive or increase for small r_A
negative or decrease for large r_A
4. (\mp) negative or decrease for small r_A
positive or increase for large r_A

Table 6

Credit multiplier elasticities and their reaction to an increase in required asset reserves (r_A exogenous)

x	adj. or unadj. base model	sign or value of $\epsilon(a_i^a, x)$ for $r_A \geq 0$				$\frac{\delta \epsilon(a_i^a, x) }{\delta r_A} = \frac{\text{change of } \epsilon(a_i^a, x) }{\text{due to an increase of } r_A}$				
		m1	m2	m3	m4	m1	m2	m3	m4	
k	either	-	-	-	-	-	-	-	-	
t	either	+	+	+	+	?	-	?	-	
		positive if $r^t \leq r^r$; true for Germany for $r_A = 0$ even though $r^t > r^s$				indeterminate decrease if $r^t = r^s$				
s	either	+	+	+	+	?	-	?	-	
		positive if $r^s \leq r^r$ as for Germany				indeterminate decrease if $r^t = r^s$				
i	either	$r_A > 0$	+	o	o	o	+	no change		
		$r_A = 0$	o	o	o	o	+	no change		
b	unadj.	(?)	(?)	o	(?)	?	?	no change	?	
b	adj.	+	+	+	+	?	?	?	?	
r^e	either	-	-	-	-	?	?	?	?	
r^r	either	-	-	-	-	?	?	?	?	
r^d	either	-	-	-	-	?	?	?	?	
r^t	either	-	-	-	-	?	?	?	?	
r^s	either	-	-	-	-	?	?	?	?	
r_A	either	$r_A > 0$	-	-	-	-	(?)	(?)	(?)	(+)
		$r_A = 0$	o	o	o	o	(?)	(?)	(?)	(+)

legend:

1. + : positive if sign (or value), increase if change (or reaction)
2. - : negative if sign, decrease if change
3. ? : indeterminate
4. (?) : positive or increase for small r_A indeterminate for large r_A

Table 7

Changes of money supply elasticities, $\left| \epsilon(m_i^a, x) \right|$,
 due to an increase of required asset reserves (r_A endogenous)

		$H, H', H'' < 0$ [$F > 0$]				$H, H', H'' > 0$ [$F < 0$]			
x	$\epsilon(m_i^a, x)$	m 1	m 2	m 3	m 4	m 1	m 2	m 3	m 4
k	$\left. \begin{matrix} m1 : ? \\ m2 \\ m3 \\ m4 \end{matrix} \right\} : (\bar{+})$?	reinforced: decrease if $\epsilon < 0$ increase if $\epsilon > 0$?	partial-, full- or over- compensation of <u>de-</u> <u>crease</u> if $\epsilon < 0$, increase if $\epsilon > 0$		
t	-	reinforced <u>increase</u>				partial-, full- or overcom- pensation of <u>increase</u>			
s	-	reinforced <u>increase</u>				partial-, full- or overcom- pensation of <u>increase</u>			
i	$\begin{matrix} + \\ (m2, m3, \\ m4: 0) \end{matrix}$	<u>increase</u> * without re- inforcement			/	<u>increase</u> * without re- inforcement			/
b (unadj)	$\begin{matrix} - \\ (m3: 0) \end{matrix}$	<u>(reinforced</u> <u>increase</u> <u>?)</u>		/	reinf. <u>in-</u> <u>crease</u>	<u>(part.-, full-</u> <u>or overcom-</u> <u>pensation</u> <u>of increase</u> <u>?)</u>		/	<u>(p-, f-o</u> <u>comp.</u> <u>of incr.</u> <u>?)</u>
b (adj.)	+	reinforced <u>decrease</u> (if sign does not change)				partial-, full- or overcom- pensation of <u>decrease</u>			
r^e	-	reinforced <u>decrease</u> (if sign does not change)				partial-, full- or overcom- pensation of <u>decrease</u>			
r^r	-	reinforced <u>decrease</u> (if sign does not change)				partial-, full- or overcom- pensation of <u>decrease</u>			

In $\left(\begin{matrix} \dots \\ ? \end{matrix} \right)$... holds for "small" r_A (r_A^*)
 ? holds for "large" r_A (r_A^*)

*) "increase without reinforcement" refers to column m1 only.