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Working Paper

The borrower-lender relationship, competitive equilibrium, and the theory of hedonic prices

Diskussionsbeiträge, No. 66

Provided in Cooperation with:

Department of Economics, University of Konstanz

Suggested Citation: Baltensperger, Ernst (1975) : The borrower-lender relationship, competitive equilibrium, and the theory of hedonic prices, Diskussionsbeiträge, No. 66, Universität Konstanz, Fachbereich Wirtschaftswissenschaften, Konstanz

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The Borrower-Lender Relationship,
Competitive Equilibrium, and the
Theory of Hedonic Prices

by
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Nr. 66

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August 1975

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In a series of recent articles, Vernon Smith (1972b; see also 1971a, 1971b, 1972a) has presented a very interesting analysis of the lender-borrower relationship, which emphasizes that the debtor's equity capital plays the role of an external economy to the lender, with the result that the traditional competitive equilibrium solution is not Pareto optimal. Smith considers a situation where a lender with initial wealth w_0 has the option of either buying government bonds with a certain return r , or giving a risky loan (z) at a contractual rate of interest r^* to an individual borrower or firm. The borrower uses his loan to finance a risky investment project yielding an uncertain return θ (assumed independent of the amount invested). The borrower also has an initial wealth W_0 , which he invests partly in his risky investment project (his "firm"), and partly in the form of riskless government securities at rate r . Since the borrower's risky investment activity (his firm) is assumed to be incorporated, with limited liability, so that his private holdings of riskless bonds are protected from lender claims, the lender's expected utility from extending a loan clearly becomes dependent on the debtors' equity decision, i.e., on the way in which the debtor divides his personal wealth W_0 into investment in the risky project y (his equity in his firm) and investment in riskless bonds (his personal, nonliable assets). The higher y , the lesser is, given the size of the loan z , the risk that the debtor will default on his loan, and thus the higher is the lender's expected utility. The debtor's equity capital acts,

as Smith expresses it, as an external economy to the lender. The existence of this external effect implies that the standard competitive equilibrium solution, as summarized by his equations (12) and (13) (1972b), is not Pareto efficient. Therefore, he suggests that free competitive negotiations between borrowers and lenders may produce arrangements which look quite different from what is normally expected to characterize a "competitive equilibrium". More specifically, he links his analysis to the discussion about credit rationing (see Jaffee-Modigliani, 1969, and Jaffee, 1971), and points out that a quota, or rationing, system can generate a solution which is consistent with the conditions for Pareto efficiency (If the quotas are set such that their shadow values are equal to the appropriate tax or subsidy. But he does not have a theory which explains how these quotas are determined as the outcome of a market process).

Fundamental to Smith's whole analysis is the behavioral assumption that both lender and borrower optimize based on the premise that the borrower's interest payment to the lender is independent of the debtor's equity capital y , and thus of his risk of default. The borrower takes the contractual interest rate r^* as a datum and optimizes based on the assumption that he can obtain any amount of credit at this rate. The borrower has no inducement to increase his equity capital y towards the Pareto optimal level, since his interest payment r^*z depends, for any parametrically given value of the price r^* , on z only, independent of y .

This paper discusses a model of the competitive process based on an alternative assumption about competitive behavior, one which is, in our opinion, more justified, and which leads to a Pareto efficient solution which, in contrast to Smith's rationing scheme, is perfectly consistent with "what is usually expected to characterize a competitive equilibrium". It is argued that, in the context of a credit market system as discussed here, the assumption that the transactors optimize subject to a parametrically given contractual interest rate r^* does not seem justified, even in an atomistic environment. The assumption of price taking is, of course, a very standard behavioral assumption in economic theory, usually used as equivalent with "competitive behavior", and normally justified by assuming that the seller or buyer in question is but one among a large number of transactors, so that his individual behavior does not appreciably affect the market as a whole and thus the market price. Clearly, this justification for price taking, if applied to the contractual interest rate r^* as in Smith's model, is invalid. Every borrower, however small and insignificant he is relative to the market as a whole, knows that, given his equity capital, y , he cannot borrow unlimited amounts at the same (contractual) interest rate, since an increase in his loan (or, alternatively, a change in his equity capital), *ceteris paribus*, will change his risk of default and therefore the "quality" of the loan. Recognition of this has nothing to do with presence or absence of competition. Even in an atomistic environment, a borrower is not reasonable if he does not

take into account that the contractual rate r^* must contain a risk premium which reflects the characteristics and riskiness of the individual borrower, and thus his equity decision, rather than general demand and supply conditions for the market as a whole. A credit transaction has two dimensions or "characteristics": dollar amount (or "quantity") and riskiness (or "quality", depending on y , relative to z). Both parties are perfectly well aware of this (and they know that the other side is aware of it), and thus should not be expected to optimize on the premise that the other side is not aware of it. Consequently, we suggest to treat the loan as a "good" with two characteristics. There is a perfect formal analogy between the problem under discussion here and the model of hedonic prices employed by Rosen (1974) to discuss product differentiation in a competitive environment (which is based on the well known work on consumer theory by Lancaster (1966), Becker (1965), and Houthakker (1952), where goods are treated as collections of utility generating characteristics).

Thus, we object to Smith's description of the conventional competitive model, since it is equivalent to assuming that the market participants optimize based on the assumption that price is independent not only of quantity, but also of quality. There is nothing in the traditional competitive model requiring this. It is shown that if this assumption is abandoned, and the loan is treated as a good with two characteristics, a competitive (atomistic) environment can be expected to produce a Pareto efficient solution quite consistent with what

is usually expected to characterize a competitive equilibrium (in other words: a situation where the appropriate "charges" or "subsidies" are imposed quite naturally as the outcome of a market process).

In section I below, Smith's model will be briefly summarized. In section II, a competitive model which does take account of the preceding discussion will be presented. Section III contains a brief summary.

I.

In the Smith model, the borrower's terminal wealth W is

$$(1+r)(W_0-y) + (1+\theta)(y+z) - (1+r^*)z$$

in the event of nondefault, i.e., as long as $(1+\theta)(y+z)$, the end of period value of his investment project (or firm), exceeds his debt to the lender $(1+r^*)z$, or in other words, as long as $\theta > [(1+r^*)z / (y+z)] - 1 \equiv \theta^*$, but only

$$(1+r)(W_0-y),$$

the end of period value of his personal holdings of nonliable bonds, in the event of default, i.e. when $\theta < \theta^*$. The borrower is assumed to maximize his expected utility

$$(1) \quad B = \int_{\theta^*}^{\infty} U[(1+r)(W_0-y) + (1+\theta)(y+z) - (1+r^*)z] f(\theta) d\theta \\ + \int_{-1}^{\theta^*} U[(1+r)(W_0-y)] f(\theta) d\theta,$$

where $U(W)$ denotes his (strictly concave) utility function for terminal wealth, and $f(\theta)$ his subjective probability density over the rate of return θ . Maximizing B with respect to y and z , for given r^* , r and W_0 , determines the borrower's optimal equity

capital y and his loan demand z , as a function of r^* , r and w_0 .

The lender's terminal wealth w , on the other hand, is equal to

$$(1+r)(w_0-z) + (1+r^*)z$$

in the event of nondefault ($\theta > 0^*$), but only

$$(1+r)(w_0-z) + (1+\theta)(y+z)$$

in the case of default ($\theta < 0^*$)¹.

The lender is assumed to maximize his expected utility

$$(2) \quad L = \int_{\theta^*}^{\infty} V[(1+r)(w_0-z) + (1+r^*)z] g(\theta) d\theta \\ + \int_{-1}^{\theta^*} V[(1+r)(w_0-z) + (1+\theta)(y+z)] g(\theta) d\theta,$$

where $V(w)$ denotes his utility function for terminal wealth, and $g(\theta)$ his subjective density function over θ . Maximization of L with respect to z , subject to given values of r^* , r , w_0 and y , yields the lender's optimal loan supply z .

Clearly, a parametric change in y changes the value of the lender's expected utility L :

$$(3) \quad \frac{\partial L}{\partial y} = \int_{-1}^{\theta^*} V'(\cdot)(1+\theta) g(\theta) d\theta > 0,$$

and thus leads, ceteris paribus, to a change in the lender's loan supply. "The borrower's equity in an investment is an external economy to the lender", as Smith expresses it (his proposition 1, p. 479, 1972b). The existence of this external effect leads to the conclusion that the traditional competitive market equilibrium is not Pareto efficient, as noted in the introduction.

II.

As pointed out, this result is obtained because of the special (and, as argued here, objectionable) way in which the behavior of the market participants is specified. More specifically, because it is assumed that both borrower and lender optimize based on the premise that the contractual interest rate r^* and thus the borrower's interest payment to the lender, is independent of his equity capital y , and thus of his risk of default. This is equivalent with assuming that price is independent of quality, and does not seem justified, even in an atomistic environment. I may expect that I can buy "any" amount of Coca Cola in the supermarket (i.e., any amount within a very large range around my current purchase, e.g., 100 bottles instead of my normal purchase of one), without exerting any noticable effect on the market price, because I am but one of millions of purchasers of Coca Cola. However, even if I am but one among millions of (similar) credit takers, I will never expect that I can get "any" amount of credit at an unchanged contractual rate of interest, unless I keep default risk (the "quality" of the loan) constant by appropriately adjusting equity capital y (or in some other way²). To assume competitive behavior in the sense of having many relatively small market participants whose behavior, taken individually, does not appreciably effect total market excess demand ("general market conditions") should not be equalized with price taking in the sense of accepting r^* as a datum (independent of y).

This paper suggests to treat the loan as a good with two characteristics, z and y ("size" and "quality"). Consequently, we specify a price function $R=R(z,y)$, which relates the borrowers payment to the lender R (defined as including the repayment of capital) to the two characteristics z and y .³ At any point in time, the market reveals an $R(z,y)$ function, which can be inferred from observed prices for different z/y packages. Presumably, $R_z > 0$ and $R_y < 0$: An increase in z implies a larger total payment R , while an increase in y , given z , represents an increase in the quality of the loan, and thus implies a lower compensation R . The transactors treat this price function as parametric to their decisions. This is the meaning of competition in this context. In other words, each transactor is small enough relative to the market as a whole that his individual actions have no recognizable effect on the $R(z,y)$ function exhibited by the market. An equilibrium price function is determined by market forces in the same way as equilibrium price is determined in the usual competitive model: An equilibrium value for $R(z,y)$ is reached if buyers and sellers are perfectly matched at each z/y combination. For a discussion of the determination of such an equilibrium price function, see Rosen, and Court (1941).

The borrower's and lender's expected utility function now are

$$(4) \quad B = \int_{\theta^*}^{\infty} U[(1+r)(W_0 - y) + (1+\theta) - R(z,y)] f(\theta) d\theta + \int_{-1}^{\theta^*} U[(1+r)(W_0 - y)] f(\theta) d\theta$$

and

$$(5) \quad L = \int_{\theta^*}^{\infty} V[(1+r)(w_0 - z) + R(z, y)] g(\theta) d\theta + \int_{-1}^{\theta^*} [V(1+r)(w_0 - z) + (1+\theta)(y+z)] g(\theta) d\theta,$$

where $\theta^* = [R(z, y)/(y+z)] - 1$ again is the value of θ below which the borrower defaults on his loan.

Treating $R(z, y)$ parametrically, the borrower determines his optimal z and y (size and "quality" of loan). His optimality conditions are

$$(6) \quad \frac{\partial B}{\partial z} = \int_{\theta^*}^{\infty} U'(\cdot) [\theta - R_z] f(\theta) d\theta = 0$$

$$(7) \quad \frac{\partial B}{\partial y} = \int_{\theta^*}^{\infty} U'(\cdot) [\theta - r - R_y] f(\theta) d\theta - \int_{-1}^{\theta^*} U'(\cdot) (1+r) f(\theta) d\theta = 0.$$

Similarly, the lender determines, conditional upon $R(z, y)$, his desired values of z and y , i.e., the size and "quality" of the loan he would like to make, given the market's $R(z, y)$:

$$(8) \quad \frac{\partial L}{\partial z} = \int_{\theta^*}^{\infty} V'(\cdot) [R_z - r] g(\theta) d\theta + \int_{-1}^{\theta^*} V'(\cdot) [\theta - r] g(\theta) d\theta = 0$$

$$(9) \quad \frac{\partial L}{\partial y} = \int_{\theta^*}^{\infty} V'(\cdot) R_y g(\theta) d\theta + \int_{-1}^{\theta^*} V'(\cdot) (1-\theta) g(\theta) d\theta = 0.^4$$

A competitive market equilibrium is reached if the $R(z, y)$ function is such that there results complete consistency between borrowers and lenders. See Rosen for a discussion of the price function as the joint envelope of a family of indifference surfaces for borrowers and lenders. Of course, the existence of a whole equilibrium price function, i.e., the coexistence of different z/y combinations in equilibrium, requires the existence of heterogeneity among borrowers and/or lenders

(e.g. a distribution of U-function, and/or distribution of V-functions). If all borrowers, as well as all lenders, were identical, all borrowers and all lenders would always end up with identical decisions, so that there would be only one equilibrium z/y combination. Consequently, the market then could not provide the information necessary to infer a complete $R(z,y)$ function. A discussion of how an equilibrium price function is reached, starting from a disequilibrium situation, is clearly beyond the scope of this paper. However, it can be easily seen that such an equilibrium, if obtained, satisfies all the conditions for Pareto optimality. Using the shorthand notation employed by Smith (p. 481, 1972b), we can rewrite the borrower's and lender's objective functions (4) and (5) as $B=B(u,z,y/R(z,y),r)$ and $L=L(v,z,y/R(z,y),r)$, with $u=W_0-y$ and $v=w_0-z$. Independent maximization by both sides yields the following marginal conditions (equivalent to (6)-(9))

$$(10) \quad B_u = B_y, \quad B_z = 0, \text{ and}$$

$$(11) \quad L_v = L_z, \quad L_y = 0,$$

which is perfectly consistent with the condition for Pareto efficiency (as summarized by Smith's (14), p. 481, 1972b).

III.

We have discussed a competitive model of the credit market which does explicitly take into account that the credit transaction has two characteristics: size and riskiness, or "quality". It has been argued that it is reasonable to assume that market participants do explicitly take this into account in their optimization procedures; and it has been shown that a model of the competitive process where this is the case does lead to a Pareto efficient solution which is perfectly consistent with what is usually expected to characterize a competitive equilibrium (in contrast to the competitive model discussed by Smith).

Footnotes

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¹To simplify the presentation, it is assumed that the lowest possible value of θ is not below -1 , i.e., the lowest possible terminal value of the lender's investment project not less than zero. If lower values for θ were admitted, we would have to include a third term in (2), since in this case, the lender's terminal wealth would be given by $(1+r)(w_0 - z)$ alone (because he is not liable for the borrower's debt to somebody else).

²For example, via putting up otherwise nonliable wealth as collateral. This would be an obvious extension of the model.

³Note that z and y together determine θ^* , the critical value of θ below which the debtor defaults on his loan. The "quality" of the loan, of course, does also depend on the density functions over θ , which we treat as given here. In principle, we can think of the parameters of these functions as additional characteristics of the loan.

⁴In addition, the appropriate second order conditions for a maximum must be satisfied, of course.

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