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## Potential surprise and portfolio theory

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POTENTIAL SURPRISE  
AND PORTFOLIO THEORY

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Serie A - Nr. 144

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# I

This essay was prompted by the recent appearance of Douglas Vickers's Financial Markets in the Capitalist Process. This thought-provoking book represents both a critique of the established theory of financial markets, and its implications for decision-making by companies and by investors, and an attempt to construct an alternative structure for the analysis of portfolio selection and corporate investment decisions. At the core of both the critique and the suggested reconstruction lie two principal sets of ideas. The first consists of Vickers's own conception of the firm as developed in his previous writings.<sup>(1)</sup> The second is a view of the nature of time and uncertainty, and of their role in the economic process, based on the work of G.L.S. Shackle.

The main purpose of this essay is to examine the way in which Vickers has employed Shackle's framework as a foundation for his theory of portfolio choice and to suggest an approach whereby it could be generalised further.

In view of the close connection between Vickers's model and Shackle's theory of decision, it seemed appropriate to begin with an exposition of those elements of the latter which relate to the present discussion. This is the subject of the first of the two main sections of this essay. The second is devoted to an appraisal of Vickers's use of the 'potential surprise' model as a basis for his own theory, and to the outline of a proposal for generalising this approach to portfolio selection. The final section contains a summary of the main conclusions arrived at in the discussion.

## II

Professor Shackle's work covers a variety of areas of economic theory but much of it is devoted to developing and promoting a theoretical framework for decision-making which contrasts sharply with the Bernouillian one which lies at the heart of the neoclassical approach to decisions taken in conditions of risk. It is this part of his work which I wish to outline here. For my exposition, I draw on mainly two of Shackle's many writings on this subject.<sup>(2)</sup> These sources are his 1957 Amsterdam lectures, published under the title Time in Economics, and the 1969 edition of his Decision, Order and Time in Human Affairs.

Central to an understanding of Shackle's approach to decisions under uncertainty is an appreciation of his conception of time and of the place and role in it of the individual as a decision-maker. He characterises the economist's conventional treatment of time as being one which views it as "a space, or as one dimension of such a space, wherein distinct points do not, a priori, differ from each other in their general essential nature

and properties"<sup>(3)</sup>. In contrast, he regards time, as perceived by the individual, as consisting of a continuous series of unique but related 'moments' each evolving into the next. A moment is not a distinct and separable point in time but a transitory interval, the immediate present, "whose very essence and also whose very existence involves a continuous movement and a continuous evolution"<sup>(4)</sup>. "Like the phase of a wave the moment is individual but continuous with the preceding and the following phases."<sup>(5)</sup>

It is in such a moment, which Shackle calls the 'solitary moment' or 'moment in being', that the individual is placed as he surveys his world and it is for him "the locus of every actual sense-experience, every thought, feeling, decision and action"<sup>(6)</sup>. Even as he stands in it, one moment is continually transforming itself into the next and his position at a particular point in calendar time is one station, as it were, in the progress of the moment in being, the individual's present.

Placed in this way in time, the decision-maker considers his possibilities for choice and action.

In so doing, he must take a view both of the opportunities available to him and of the possible consequences of selecting each. Shackle posits that the way in which the individual performs this task is by constructing imaginary scenarios representing possible actualisations, at some future time, of a decision made now. Imagination is his link with the future as memory is his link with the past the latter providing him with much of the 'objective knowledge' of the world on which he bases his view of the future. These imagined scenarios or sequences, provided they appear to the decision-maker to be consistent with the information available to him in his present moment, and are not mere fantasies, Shackle refers to as 'expectations'. "The decision-maker forms expectations, by which I mean conceptions of outcomes looked on as possible. What seems to him possible, from the viewpoint of a particular, unique moment, depends not only on the supposed laws of nature but on the particular detailed state of affairs which is the consequence of previous decisions and actions of others. Each decision of any individual does something to shape the background of beliefs and assumptions, and of data for

judging possibility, against which the later decisions of others and himself will be taken." (7)

Imagination plays two important and related roles in Shackle's conception. Firstly, it is the vehicle whereby anticipations concerning future paths of history may be experienced in the present. It is through the use of this faculty that the decision-maker can conceive the possible consequences of each course of action and thus compare them in his mind at the point of choice. It provides the only means whereby the present and the future can 'co-exist' since, otherwise, "two moments cannot be 'in being' together, the actuality of the one denies the actuality of the other, there is no 'common ground' on which they can be brought face to face" (8).

Secondly, the faculty of imagination and the potential for exercising it in 'creating' opportunities, in innovating, opens the door to decisions which are not 'empty' or 'meaningless', terms which Shackle uses to describe decision-taking in the deterministic framework of neoclassical economics. "Non-empty decision we define as choice which cannot be

explained as in every respect the inevitable consequence of what went before; as choice which is not structurally identical with the cosmos that pre-existed this choice, no matter how comprehensive our knowledge of that cosmos, its nature and principles, may be."<sup>(9)</sup>

Another essential aspect of decision thus defined is the presence of uncertainty. There are two senses in which the term uncertainty is used here. The first refers to the simultaneous entertainment, in the decision-maker's mind, of a number of possible and mutually exclusive hypotheses regarding the future outcome of any choice made now. The second sense in which Shackle sees uncertainty as pertinent to non-empty decision is in creating the opportunity and incentive for innovative thought. "Decision, as we mean the word, is creative and is able to be so through the freedom which uncertainty gives for the creation of unpredictable hypotheses."<sup>(10)</sup>

So, for Shackle, time is an evolutionary process to which change and uncertainty are inextricably linked. Man finds himself in a central position

in this process with an active part to play in creating the future through his own decisions, taken in the context of his subjective reality which determines his expectations. This subjective reality is essentially unknowable to any outside observer, there being "no assurance that any one can in advance say what set of hypotheses a decision maker will entertain concerning any specified act available to him"<sup>(11)</sup>. <sup>(12)</sup>

Against this background, Shackle attempts to construct a theory of decision faithful to his conception of the decision-maker's place in time. He addresses himself first to the question of whether the probability calculus provides a suitable framework for such a theory of decision. In this essay, I do not intend to join the lists of those who have commented on his rejection of the use of probability theory as a tool in economic science.<sup>(13)</sup> It will suffice here to note that he dismisses probabilities, however determined, as inadequate to the task of accounting for uncertainty when a decision is essentially a unique and 'self-destructive' experiment in the sense that "in being performed, (it) irreversibly destroys some circumstances which are essential to it"<sup>(14)</sup>.

More generally, his view is that: "Estimation, judgement, inference, the exploitation of suggestions which the visible present and records of the past supply, are worthy forms of language,, but they must not be allowed to disguise the essential non-observability of the future."<sup>(15)</sup>

What Shackle seeks, as a foundation for his theory of decision, is a 'non-distributional' uncertainty variable which conveys the notion, in the decision-maker's mind, of possibility rather than relative likelihood on the grounds that "if several distinct outcomes are all regarded as perfectly possible, there can be no sense in which a man can feel any degree of positive confidence in any one of them"<sup>(16)</sup>. More specifically, he requires an index of expectation which "can concede full and perfect possibility to each of an unlimited number of rival and diverse hypotheses"<sup>(17)</sup> and which can also "retain the power to indicate imperfect, obstructed, strained or tenuous possibility for other hypotheses"<sup>(18)</sup>.

These considerations lead him to the concept of 'potential surprise' as a measure of the decision-maker's assessment of the possibility of a

particular imagined outcome being realised. What is indicated by the term potential surprise is the extent to which the individual feels now that he would be surprised if a given outcome were to occur in the future. The transition from possibility to potential surprise stems from Shackle's supposition that the degree of plausibility<sup>(19)</sup> which an individual has attributed to a particular outcome is manifested by the intensity of his psychic experience of shock or surprise when it does in fact occur. However, since he cannot know the actual surprise he will feel in future, particularly as his expectations may alter in the interim, the index refers to his expected or potential surprise as assessed at the point of decision.

Accordingly, the idea, in the decision-maker's mind, of a hypothesis being perfectly possible is translated into that of his not being at all surprised if it did prove true; or, if  $y$  is the index of potential surprise, into  $y = 0$ . At the other end of the scale, outcomes which are regarded as totally impossible are represented by the highest available degree of surprise, some arbitrarily selected value of  $y$ ,  $\bar{y}$ . Between

these extremes, gradations of possibility are expressed by values of  $y$  between 0 and  $\bar{y}$ . (20)

Supposing that the choice in question relates to an investment programme, each of its imagined outcomes may be expressed as a gross profit figure (positive or negative) represented by values of the variable  $x$ . The investor then assigns degrees of potential surprise to values of  $x$ . Assuming continuity of the function  $y(x)$ , Shackle suggests that when plotted it will typically assume a shape of the kind shown in Figure 1. On this graph,  $x = 0$  is also used to denote  $x_n$ , the 'neutral outcome' or the hypothesis that adoption of the investment proposal will leave the decision-maker's situation unchanged. In the present context, it refers to the possibility of the project breaking even. It is regarded as the common reference point around which all other outcomes are ranked and  $y(x_n)$  is assigned zero potential surprise. (21) Around  $x_n$  is an 'inner range' of outcomes all regarded as perfectly possible and assigned values  $y(x) = 0$ . Outside this range and up to the cut-off value  $y(x) = \bar{y}$ ,  $dy/dx > 0$  for  $x > 0$  and  $dy/dx < 0$  for  $x < 0$ .

It is important to remember that a particular potential surprise curve is unique to (a) the opportunity whose imagined consequences it portrays, (b) the decision-maker who has constructed it, and (c) the moment in being in which it is conceived. It is a completely subjective construct although, of course, that does not preclude its being based principally on generally available information and knowledge.

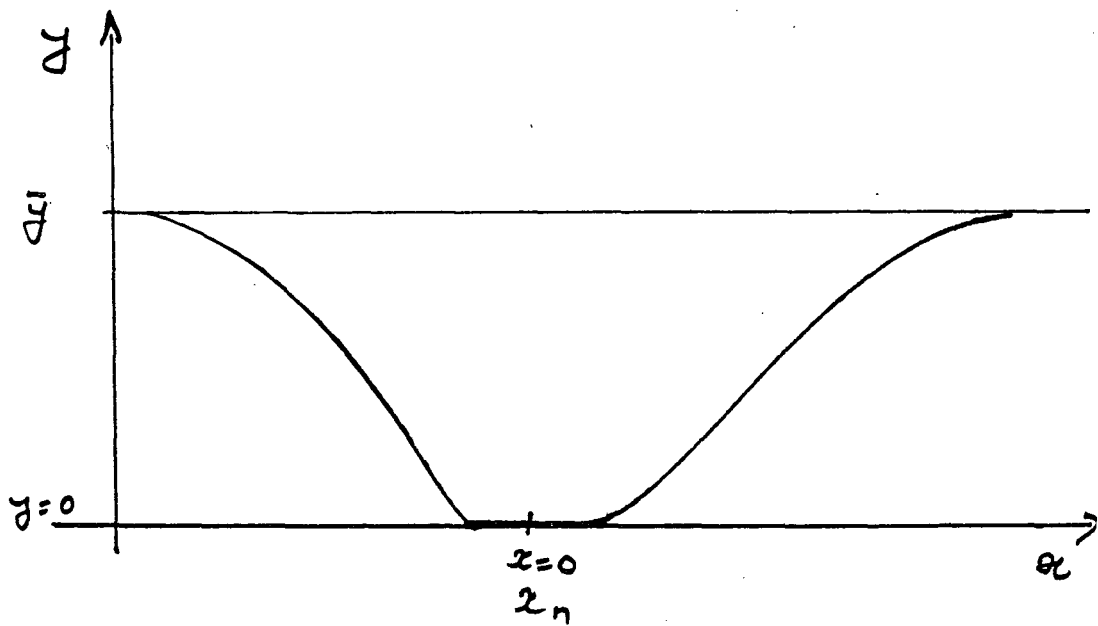


Figure 1

The decision-maker then constructs a potential surprise curve for each of his investment options. The next stage in Shackle's procedure is to assume that, in considering each of them, he restricts his attention to only two of its outcomes. Shackle argues that, if all the imagined outcomes of an investment bore zero potential surprise, the individual would be concerned almost entirely with the best and with the worst situations which its acceptance exposed him to. The most desired outcome would possess 'ascendancy', in terms of power to attract attention, over all other positive values of  $x$  and the least desired would similarly dominate all other negative values of  $x$ . More generally, where outcomes bear differing degrees of potential surprise, "ascendancy will be the combined effect of the desiredness or distastefulness of the hypothetical outcome and of the potential surprise associated with it. A hypothetical outcome loses ascendancy by seeming to the decision-maker less than perfectly possible, it gains ascendancy by representing a high degree of success or of misfortune."<sup>(22)</sup>

This subjective trade-off between the magnitude

(positive or negative) of an outcome and the degree of potential surprise assigned to it is expressed by an ascendancy function,  $A = A(x,y) = A[x, y(x)]$ . This is usually represented as a family of indifference curves or 'equal-ascendancy contours' of the type depicted in Figure 2. The function is assumed to possess the following properties:

(a) the greater the absolute magnitude of the outcome, the more attention it attracts, or  $\delta A / \delta x > 0$  for  $x > 0$  and  $\delta A / \delta x < 0$  for  $x < 0$ , and

(b) the higher the potential surprise assigned to an outcome, the less interesting it is and both the neutral outcomes and impossible outcomes are of no concern, or  $\delta A / \delta y < 0$  for  $0 \leq y \leq \bar{y}$  and for  $x \neq 0$ .<sup>(23)</sup>

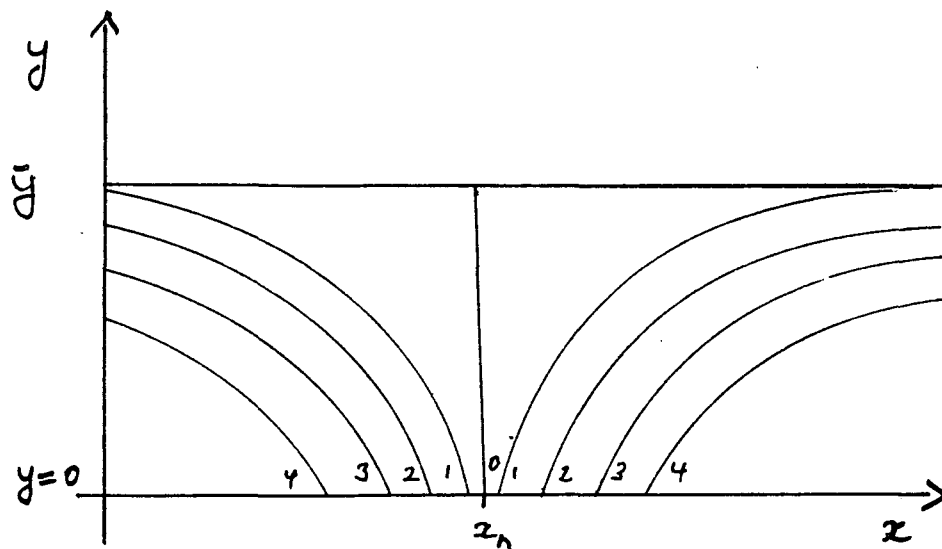


Figure 2

The focus values of an investment opportunity, the two elements which are presumed to dominate the decision-maker's attention, are given by the two maxima of the ascendancy function constrained by the surprise function in question. Graphically, when, as in Figure 3, equal-ascendancy contours are superimposed on the relevant potential surprise curve the resulting tangencies indicate the focus elements,  $x_G$  and  $x_L$ , and their respective degrees of surprise,  $y_G$  and  $y_L$ . Shackle refers to these as the 'primary' focus gain and focus loss of the investment. To enable comparison between investments with primary focal elements bearing different degrees of potential surprise, these primary values are standardised by relating each to an equivalent outcome carrying zero potential surprise. The notion of equivalence is interpreted here in terms of equal ascendancy so that the standardised focus gain,  $x_G^S$ , is to be found at the point  $(x_i, 0)$  on the same indifference curve as  $(x_G, y_G)$ , and similarly for the standardised focus loss,  $x_L^S$ .

By this procedure the investor isolates the standardised focus elements of each proposal

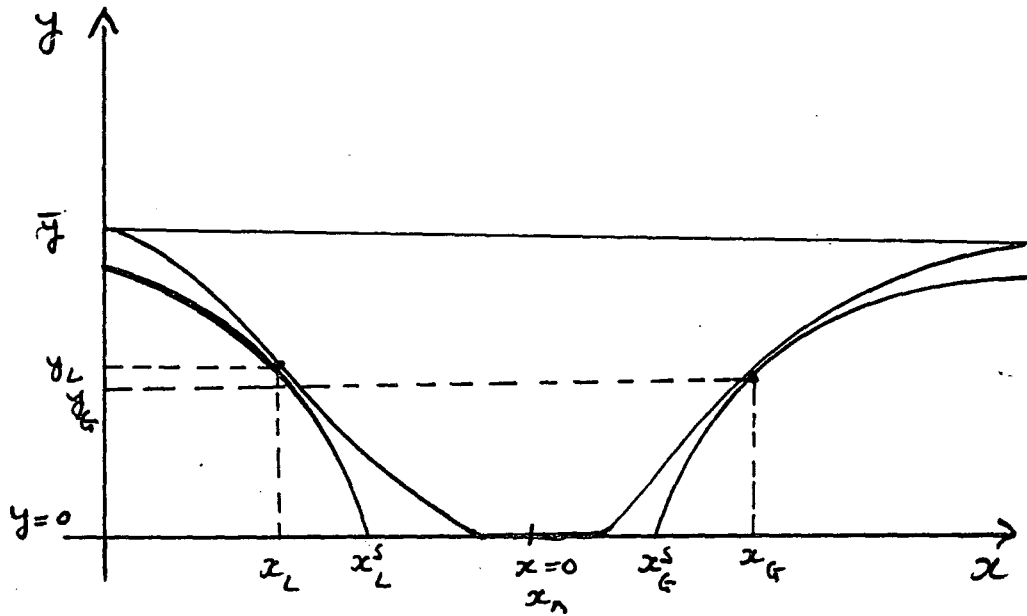


Figure 3.

under review, thus allowing him to compare them. The course of action which he selects from among these will depend also upon the terms on which he is willing to trade standardised focus losses and gains. To express this aspect of the investor's preferences, Shackle assumes that he is able to rank  $(x_G^S, x_L^S)$  pairs in such a way as to produce an indifference map of the kind depicted in Figure 4.

A special characteristic of this construction is the role performed by the constraint  $k$ , denoting the decision-maker's 'total investible resources'. The extent to which the family of indifference curves 'bow' towards this constraint reflects the degree of cautiousness of the investor. So, for example, for a relatively conservative investor indifference curves will never meet the constraint indicating that on no account will he consider a project whose standardised focus loss is equal to his total resources. Conversely, if there do **exist** terms upon which he is willing to accept a focal loss of  $Ok$  then his indifference curves will intersect the constraint.

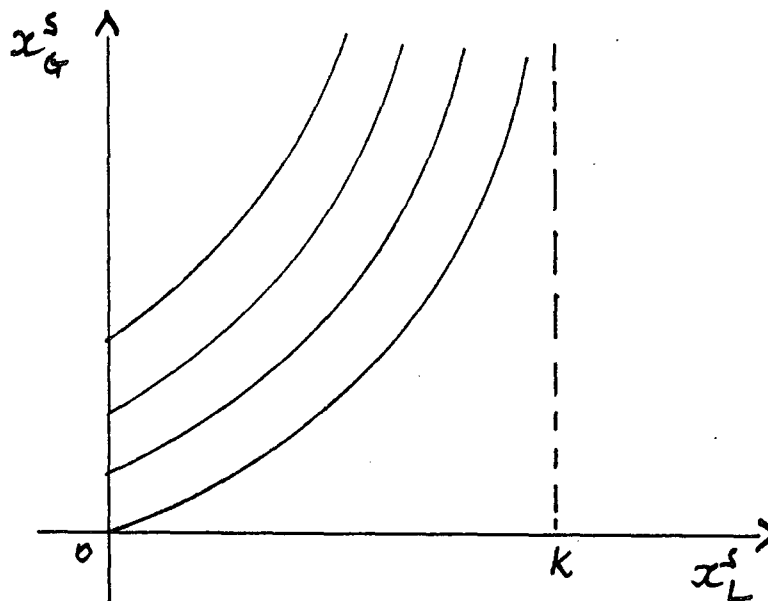


Figure 4

In the focus loss-focus gain space of figure 4 we can also plot each of the proposed projects using their respective  $x_G^S$  and  $x_L^S$  values as co-ordinates. If the indifference curve emanating from the origin is regarded in the same light as the neutral outcome was then it defines a set of situations thought by the investor to be equivalent to that which would prevail if he were to refrain from investing. It then divides the space into two parts, investment options lying on or below the origin indifference curve being 'inefficient' in the sense that on balance they offer no prospect of improvement in the investor's wealth position. From that set of projects lying above the origin curve, the decision-maker is then presumed to select the one lying on the highest indifference curve. (24)

Shackle's theory of decision consists then of three main elements:

- (a) a potential surprise function assigning degrees of possibility to each imagined outcome of a decision,
- (b) an ascendancy function reflecting the decision-maker's trade-off between the desirability of

outcomes and their respective degrees of potential surprise, and,

(c) a gambler indifference map depicting his trade-off between standardised focus gains and standardised focus losses.

Of these elements, the first encompasses the individual's 'description' of his uncertain environment as he perceives it from the stand - point of his present moment. The second covers both his preference ranking of imagined outcomes and his attitude to uncertainty, whereas the third reflects further his attitude to uncertainty.<sup>(25)</sup>

The model I have outlined in the preceding pages is what Shackle labels as his 'refined' model. His 'basic' model reflects the same scheme of thought but does not incorporate the assumptions necessary to obtain continuous functions of the kind used here. However, to summarise the principal components of even the refined model as I did just above is to lose much of one underlying thesis which is that "in economics, we are forced to deal with concepts characterised by an irreducible degree of imprecision"<sup>(26)</sup> and that "the essential

components of economic understanding can be held together only by the deliberate and careful cultivation of a language for imprecision"<sup>(27)</sup>. Both his formulations of the potential surprise model are attempts to develop such a language.

### III

The reason for this quite lengthy primer on Shackle is that his perspectives on the nature of time, uncertainty and decision lie at the root of much of what is contained in Douglas Vickers's Financial Markets in the Capitalist Process. Most of this book is devoted to an outline of the equilibrium approach to the theory of financial markets, as exemplified by Fama and Miller (1972) or by Mossin (1973), and to an extensive critique of its basic presuppositions. In the remainder, Vickers uses the potential surprise model to try to lay the foundations of a new theory of investment and finance.

I do not intend here to discuss or comment on his appraisal of existing theory. The main strands of his critical argument are drawn from Shackle,<sup>(28)</sup> and I can offer no more concise statement of Vickers's position than that contained in his opening manifesto which I reproduce below.

" The contemporary theory of financial markets and the place of capital in the capitalist process are deeply committed to two interdependent sets of assumptions. These are related to the notion of equilibrium, particularly as that comes to expression in a neo-Walrasian theory of generalized market equilibrium; and the idea of probability, or what I shall refer to as probabilistic reduction methods by which uncertainty is reduced to certainty or certainty-equivalents. The first of these sets of assumptions has meant, moreover, that the theory has been cast in a timeless, static mold, and the richness of market behavior and the stance of economic decision-makers in the flow of historic time have been largely unexplored. Equilibrium, probability and abstraction from historic time have together formed a crust of orthodoxy, and it is necessary to ask whether the real-world relevance of financial theory has not been seriously impaired.

In this proposition the principal thesis emerges. An examination is now required of the ways in which financial theory assumes a markedly different shape and a heightened potential relevance when

its assumptions and objectives are regarded in a sharply different light; when the assumption of equilibrium, for example, is replaced by that of disequilibrium and by an explicit consideration of market behavioral reactions; when the assumption of risk or of a probabilistically quantifiable uncertainty variable is replaced by that of residual uncertainty and a non-distributional variable approach to the understanding of it; and when characteristically atemporal analyses, or the preoccupation with 'logical time', are replaced by an examination of actual transactions in calendar or historic time." (29)

It is as part of this programme for the reconstruction of financial theory that Vickers turns to the potential surprise model. In the remainder of this essay, I wish to evaluate his proposal on its own terms, as an attempt to apply Shackle's theory, rather than in relation to the established theory. In what follows, I shall briefly outline the main aspects of his application before commenting on those elements which I regard as questionable.

Vickers begins his exposition by assuming that the investor will take a view as to the possible rates of return which could be attained on each financial asset during a specified period of time defined as his 'decision horizon'. This period is not the same as a holding period for a portfolio of assets and the investor is free to trade at any time during his decision horizon.<sup>(30)</sup> He defines the rate of return on an asset as "the expectation that the decision maker holds at his decision moment regarding the various possible levels of rate of return that might be realized on an asset during his horizon period, taking account of the possibility that the market value of the asset may rise to a pre-specified critical level during that period, and that if it does his corresponding portfolio position will be liquidated and the capital gain will be taken"<sup>(31)</sup>.

Thus, the investor must form hypotheses concerning both the dividend paid on an asset and the capital gain from holding it which he can then transform into estimates of net rates of return on the asset during the horizon period. He then constructs a

potential surprise curve of the kind illustrated in Figure 1 which expresses his assessment of the degree of possibility to be attached to each of these rate of return hypotheses. Vickers's treatment of the potential surprise curve involves assuming the existence of a risk-free asset, a bond not subject to fluctuations in capital value, whose rate of return,  $r_f$ , is represented by the origin point.  $r_f$  then performs the function of the neutral outcome referred to earlier and other possible rates are ranged on either side of it according to whether or not they exceed  $r_f$ . A potential surprise curve of this kind is made up for each asset under consideration.

As before, the investor also possesses an ascendancy function which, when it is combined with his potential surprise function for the asset  $i$ ,  $y_i = y(r_i)$ , yields its primary focus gain and loss,  $r_{iG}$  and  $r_{iL}$ , and its standardised focus elements,  $r_{iG}^s$  and  $r_{iL}^s$ .<sup>(32)</sup> Thus, the standardised focus elements are derived for each asset and these can then be entered in the focus loss-focus gain space of the investor's gambler indifference map, as shown in Figure 5.

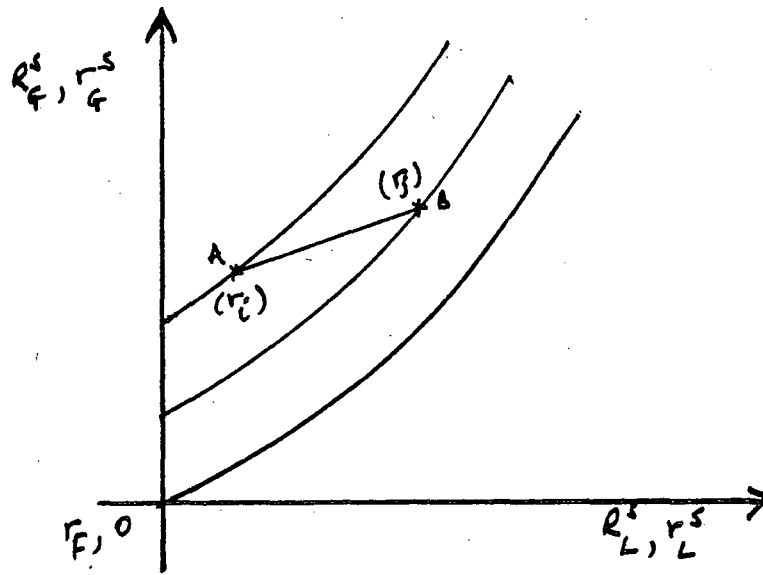


Figure 5.

The investor's choice of a single asset among those available to him is straightforward. He merely selects the one whose standardised focus elements are such as to place it on a higher indifference curve than any other.

Concerning portfolio combinations of assets, Vickers asserts the following. "Rates of return generated by asset A may be independent of those realized from holding asset B. Alternatively,

the economic nature of assets A and B may be such that their fortunes could well fluctuate in opposite directions, so that the advantages of diversification may follow from a portfolio combination of the two assets. In that case an investor may achieve a standardised focus gain and loss combination on the linear locus AB by investing in a weighted combination of the assets." (33) (34)

This argument is then generalised to define portfolio standardised focus gain and loss,  $R_{jG}^S$ ,  $R_{jL}^S$ , as:

$$R_{jG}^S = \sum_i w_i^j r_{iG}^S \quad (1)$$

$$R_{jL}^S = \sum_i w_i^j r_{iL}^S \quad (2)$$

for:  $i = 1, 2, \dots, n$   
 $j = 1, 2, \dots, m,$

and where  $w_i^j$  is the weight attached to the  $i$ th. asset in the  $j$ th. portfolio and  $\sum_i w_i^j = 1$ .

Cautioning the reader that portfolio diversification is not as widespread a phenomenon as is sometimes suggested, he adds that it "must be based on a broader range of considerations, many of them

involving qualitative rather than quantitative variables, and not on a severely, mechanistically quantified set of posited statistical relationships!"<sup>(35)</sup>.

Finally, Vickers offers a criterion whereby the investor decides whether or not to amend his portfolio. If  $j$  is his existing portfolio and  $k$  is the alternative under review, he defines changes in portfolio standardised focus values as:

$$A = R_{kG}^S - R_{jG}^S \quad (3)$$

$$K = R_{kL}^S - R_{jL}^S \quad (4)$$

If  $\beta$  is the slope of the indifference curve at  $j$ , then portfolio  $k$  is preferred to portfolio  $j$  when,

for  $K > 0$ ,  $A/K > \beta$ ,

for  $K = 0$ ,  $A > 0$ ,

for  $K < 0$ ,  $A/K < \beta$ .

Accepting his definitions of rates of return and of a decision horizon, Vickers's application seems quite straightforward until he introduces asset

portfolios to his analysis. My misgivings concerning his treatment of asset combinations are threefold. In the first place, he seems to have overlooked the fact that, in the context of the theory he adopts, the investor would be expected to define potential surprise functions for the returns on combinations of assets as well as for the returns on individual assets.<sup>(36)</sup> My second objection is that, contrary to the assertion reproduced above, he has in fact dealt only with the special case of independence between asset returns. Finally, he does not explain that in his formulation the term 'independence' has a very particular meaning which makes the case he generalises a very special one indeed.

Consider a portfolio  $k$  consisting of two assets,  $i$  and  $j$ , held in the proportions  $w_i^k$  and  $(1 - w_i^k)$  respectively. The investor is assumed to have potential surprise functions,  $y(r_i)$  and  $y(r_j)$ , defined over imagined rates of return,  $r_i$  and  $r_j$ . A similar function,  $y(R_k)$ , expresses his expectations concerning portfolio returns,  $R_k$ ; indeed, a separate function of this kind would be associated with each value of  $w_i$  defining a

distinct portfolio. The first requirement for definitions (1) and (2) to hold is that  $y(r_i)$  and  $y(r_j)$  be independent. In terms of investor expectations, this means that the investor must be unable to perceive of any opportunity for improving patterns of returns by combined holding of  $i$  and  $j$ . So, for example, taking some degree of potential surprise  $y^* > 0$  and considering the values  $R_k^*$ ,  $r_i^*$  and  $r_j^*$  associated with it in the  $dy/dx > 0$  range of each function, it must be the case that  $R_k^* = w_i^k r_i^* + (1 - w_i^k) r_j^*$ ; and this must be so throughout.

In addition to this, to guarantee (1) and (2), the investor's ascendancy function must be such that, when combined with  $y(R_k)$ , it generates standardised focus outcomes that are weighted sums of those obtained when it was used in conjunction with  $y(r_i)$  and  $y(r_j)$ . Graphically, this means that the indifference map of his ascendancy function must consist of curves which are 'identical'. Vickers subsumes both of these requirements in the notion of independence which he uses to obtain definitions (1) and (2).

Such assumptions are singularly restrictive as

a basis for a theory which claims descriptive realism as one of its principal properties. They preclude all possibility of systematically examining investor diversification motivated by the desire to spread risks. They can only be sustained in the context of Shackle's methodology if opportunities for risk spreading are not generally available or if investors cannot perceive them or regard them as uninteresting. The first of these possibilities would suggest that markets are in equilibrium and that investor expectations concerning future asset price movements are reflected in current market prices. This justification runs counter to Vickers's own critique of the neoclassical theory whereas the second seems seems unduly arbitrary. The problem of finding an appropriate basis for the treatment diversification of asset holdings is not a new one, however, and it seems to have bedevilled all attempts to incorporate such behaviour in Shackle's formulation. (37)

It is not difficult to suggest informal lines of approach to this problem. The issue of whether or not there is scope, in the estimation of a

particular investor, for spreading risks by joint holding of assets  $i$  and  $j$  rests on the relationship between the investor's expectations concerning their individual returns, expressed by  $y(r_i)$  and  $y(r_j)$  and his expectations concerning their joint returns, expressed by  $y(R_k)$ . Possibilities of this kind would exist if the relevant potential surprise functions were such that, when combined with the investor's ascendancy function, they yielded either

$$(a) R_{kG}^S > w_i^k r_{iG}^S + (1 - w_i^k) r_{jG}^S \text{ and}$$

$$R_{kL}^S \leq w_i^k r_{iL}^S + (1 - w_i^k) r_{jL}^S,$$

$$\text{or (b) } R_{kL}^S < w_i^k r_{iL}^S + (1 - w_i^k) r_{jL}^S \text{ and}$$

$$R_{kG}^S \geq w_i^k r_{iG}^S + (1 - w_i^k) r_{jG}^S.$$

Diagrammatically, that means that the point representing such a portfolio would lie above the line segment  $r_i r_j$  in Figure 6. If a result of this kind obtained for every value of  $w_i$ , then all portfolios of  $i$  and  $j$  would lie on the curve  $r_i R_k r_j$  convex to the focus gain axis.

Similarly, all  $n$  assets under review, and all  $m$  portfolios composed of them, would be assigned

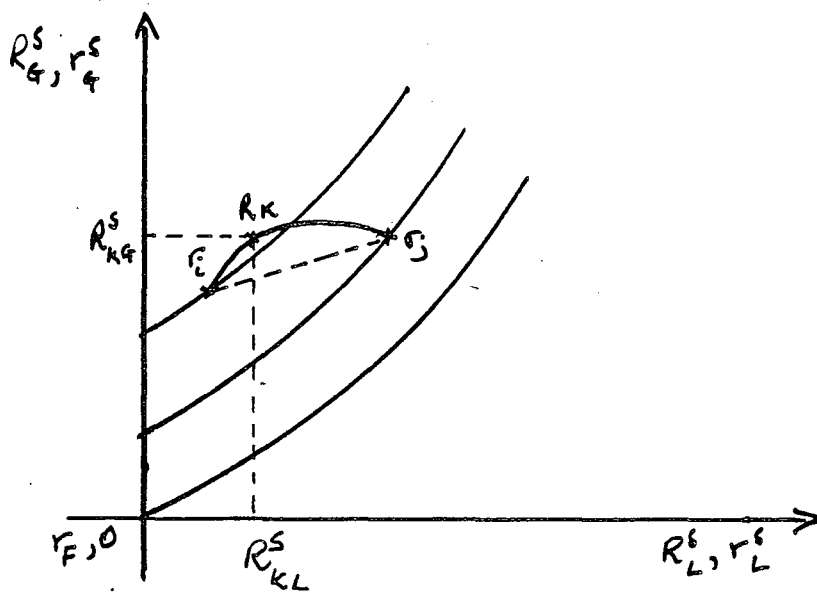


Figure 6

individual potential surprise functions, and each would be represented in the space of the gambler indifference map by its respective standardised focus gain and standardised focus loss. The set of such points would constitute the investor's opportunity set. The 'efficient' frontier of that set would consist of portfolios which, among those available, yielded the highest standardised focus gain for a given focus loss. That means that if  $k$  is to lie on the frontier there must exist no other portfolio  $j$  whose standardised elements are such that either,

$$(a) R_{jG}^S > R_{kG}^S \text{ and } R_{jL}^S \leq R_{kL}^S;$$

$$\text{or } (b) R_{jL}^S < R_{kL}^S \text{ and } R_{jG}^S \geq R_{kG}^S.$$

To the extent that the investor conceived of opportunities for risk spreading that frontier would be a curve convex to the focus gain axis; however, it need not be strictly convex throughout. The investor would then select a portfolio from those on the frontier to achieve his most desired focus loss-focus gain combination. This is illustrated in Figure 7 below.

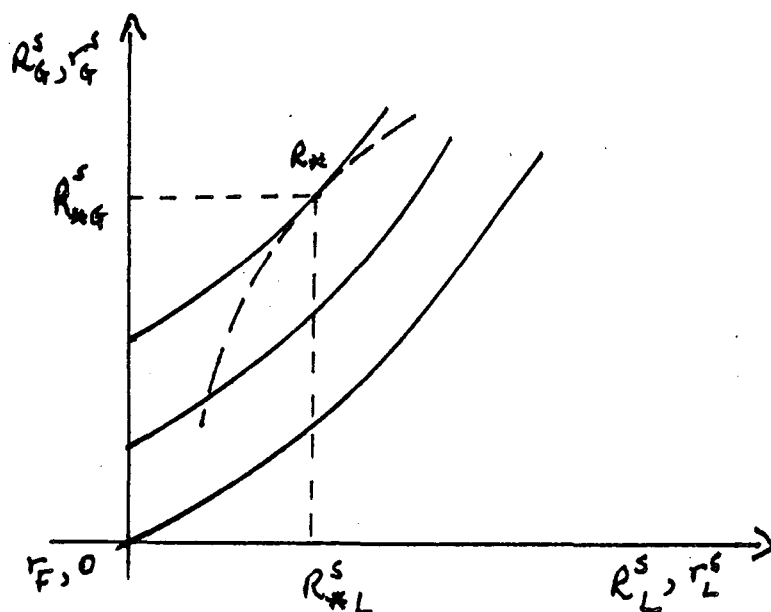


Figure 7

The parallels between the geometry of this formulation and the neoclassical one are quite

striking. They become even more so if it is considered appropriate to introduce an asset with a riskless return,  $r_F$ , of the kind employed by Vickers. It would be represented by the origin point of the focus gain-focus loss space and could be mixed with any asset or portfolio from the investor's opportunity set of 'uncertain portfolios'. In this case, any such overall portfolio, consisting of  $\beta$  invested in the riskless asset and  $(1 - \beta)$  in the uncertain portfolio  $j$ , would yield a certain return of  $\beta r_F$  and would have standardised focus elements  $(1 - \beta)R_{jG}^S$  and  $(1 - \beta)R_{jL}^S$ , represented by a point on the ray  $OR_j$  in Figure 8. Of all such overall portfolios, the preferred combinations would be with portfolios on the efficiency frontier and, among these, the dominant one would be with the portfolio at the tangency of the frontier with the steepest ray from the origin,  $OR^*$  in Figure 8.

Then, if the decision-maker regards the return on some asset as certain for the length of his decision horizon, his portfolio selection routine can be divided into two separate stages.

First, he identifies that portfolio with which it is most efficient to combine the riskless asset,  $R^*$  in this example.<sup>(38)</sup> Then he chooses that overall portfolio on  $OR^*$ , effectively his new efficiency frontier, which yields the most desired combination of standardised focus gain and standardised focus loss,  $P^*$  in Figure 8. This is a result analogous to the familiar separation theorem in the standard theory.

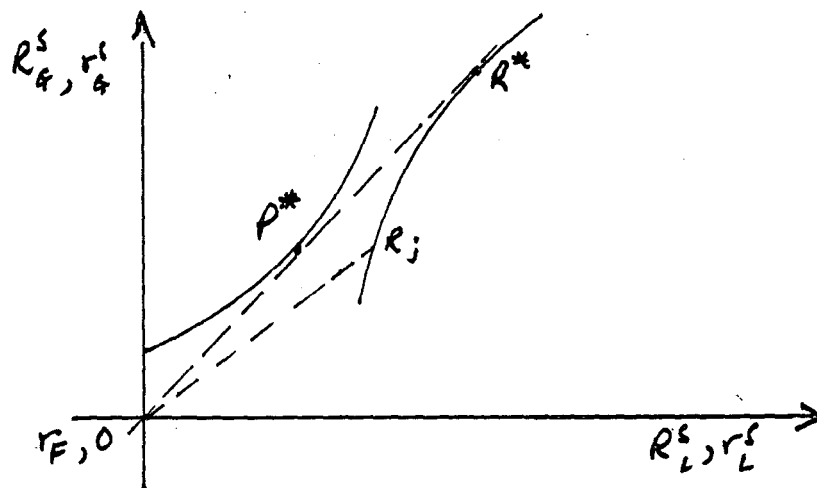


Figure 8

The close analogy between the arguments and graphical representations employed here and those found in standard theory should not be allowed to disguise the fact that standardised focal elements and moments of a distribution of returns

are very different entities, even when the latter refer to subjectively derived distributions. It should be remembered that the preceding analysis refers to the individual investor and to his expectations formed at a particular moment and on the basis of whatever information and logical procedures he considers appropriate.

The assumptions employed in this discussion of portfolio selection were that the investor assigns separate potential surprise functions to his estimates of portfolio returns and that he regards standardised focus gain as a 'good' and standardised focus loss as a 'bad', his trade-off between them being characterised by a diminishing marginal rate of substitution. These are already embodied in Shackle's model and do not represent additional requirements.

To proceed from an informal analysis of the kind employed here to a more rigorous derivation of these propositions is a more daunting task. It would entail the establishment of a procedure for combining the surprise functions for returns on single assets to produce a potential surprise

function for portfolio returns. This involves two basic requirements. First, to generate, for example, the portfolio function  $y(R_k)$  on the basis of  $y(r_i)$  and  $y(r_j)$  alone the latter functions would have to reflect the investor's expectations not only about  $r_i$  or  $r_j$  but also about joint returns when the two assets are held together. More generally, each asset's surprise function would have to contain information concerning the investor's expectations about joint returns when it is held in conjunction with any of the other assets under review. Secondly, it would also be necessary to formulate a rule whereby such information could be extracted and processed in arriving at potential surprise functions for portfolio returns.

The potential surprise model, as currently formulated, can meet neither of these requirements. It contains no specification of the procedures the decision-maker uses, nor of the information he bases his judgements on, to determine the degree of surprise to attach to an outcome. Neither does it explain the precise nature of the relationship between portfolio and individual

asset surprise functions. Like 'tastes' in a utility function, imagined outcomes and the degree of surprise associated with them are not susceptible to observation and analysis by anyone other than the decision-maker himself.

Perhaps, with sufficient ingenuity, the model could be specified in such a way as to meet these requirements and to allow for more formal analysis. Whether such a development would be consistent with the concepts underlying the model and with the methodology of Shackle's economics is a different question, however, and one which must pose itself to anyone working within this framework.

#### IV

This essay began with an outline of the main concepts which form the basis of Shackle's theory of decision, and of its apparatus of potential surprise functions, ascendancy functions and gambler indifference maps. This was followed by a review of the way in which these ideas are employed by Vickers to question the validity of the equilibrium theory of financial markets and to construct an alternative theory of the individual's portfolio selection decision.

Vickers's formulation was shown to be based on a particular interpretation of the notion of independence between asset returns, and to be unable to deal with investor diversification motivated by the desire to spread risks.

The outlines of a more general theory, which does incorporate diversification of this kind, were then sketched out on the assumption that the investor derives separate potential surprise

functions for portfolios as well as for individual assets. The framework presented here, with its convex efficiency frontiers and its 'separation theorem', bears a close but deceptive resemblance to the neoclassical one.

The reasoning employed in this exposition was relatively informal and it was suggested that the main obstacle to providing it with a more rigorous underpinning was the current specification of the potential surprise concept. However, it is not self-evident that, in the context of this theory at least, the search for more formal methods of expression and argument is an appropriate one. "We can, of course, construct theoretical models where the functions and their variables are such as to make the model self-sufficient and determinate. But the self-sufficiency is then that of an abstraction, and does not of itself make the model a safe policy guide." (39)

Footnotes:

1. cf. Vickers (1970, 1972)
2. Shackle's theory of decision was developed in a series of essays published from 1939 to 1945 and beginning with his 'Expectations and Employment'. In 1949, the first edition of his monograph, Expectation in Economics, appeared and contained a detailed and integrated account of his thesis. The most recent re-statement of it is in Decision, Order and Time in Human Affairs, published in its second edition in 1969.
3. Shackle (1958), p. 13, emphasis in original text.
4. *ibid.*, p. 14.
5. Shackle (1969), p. 42.
6. Shackle (1958), p. 13.
7. Shackle (1969), p. 32, emphasis in original text.
8. Shackle (1958), p. 18.
9. Shackle (1969), p. 6, emphasis in original text.
10. *ibid.*, p. 6, emphasis in original text.
11. *ibid.*, p. 6.
12. These ideas bear a close resemblance to those underlying the concept of opportunity cost, cf. Matthews (1980).
13. For an introduction to the debate, see Shackle (1969), Part II, where he details his objections to the use of probabilistic methods in economics and where he replies to some of his critics.

14. Shackle (1969), p. 57.
15. Shackle (1970), p. 111.
16. Shackle (1958), p. 44, emphasis in original text.
17. *ibid.*, p. 44, emphasis in original text.
18. *ibid.*, p. 44, emphasis in original text.
19. It is important to note that Shackle uses the terms 'plausibility' and 'possibility' synonymously, or, rather, that he uses the latter to indicate the plausibility of a hypothesis. See, for example, Shackle (1969), pp. 74-75: "We must, therefore, distinguish sharply between the notions, on the one hand, of a man's degree of actual belief in some hypothesis, and on the other hand, of the degree of 'believability' which he accords it. To believe positively that some hypothesis is right is pro tanto to dismiss all rival hypotheses as wrong. But to regard a hypothesis as credible or plausible is not necessarily to pass judgement on the credibility or plausibility of any other hypotheses: the insufficiency of any man's knowledge, of principles and of facts and perhaps of essentially unknowable things, leaves room for him to accept as believable very many diverse and mutually contradictory hypotheses about any one matter. .... Thus we may say that potential surprise indicates plausibility or possibility, understood as a judgement made by the particular individual, and further, that a hypothesis attracts such a judgement by virtue of its own special features and their relation with the general features of the world at large and with those of the existing situation in

particular, and not by virtue of its having few rivals in mind." (emphasis in original text)

20. An axiom system to underpin the concept of potential surprise is proposed in Shackle (1949). It is reprinted in a slightly revised form in Shackle (1969).
21. For a discussion of the reasons for using the notion of neutral outcome in this way, see Shackle (1969), chap. XV.
22. Shackle (1969), pp. 144-145.
23. "Since we are in effect assuming that interest or relevance attaches to the possibility of change in the decision-maker's situation, and if a given positive or negative gain which in the decision-maker's judgement cannot happen is, for that reason, of no interest, will not a zero gain or loss which can happen be equally of no interest? Change is equally excluded, whether he thinks of change which cannot happen or of no-change which can happen." Shackle (1969), pp. 147-148, emphasis in original text.
24. It is assumed here that the investor is confronted with a discrete number of investment opportunities. The question of mixed portfolios of investments raises problems similar to those discussed in the latter part of section III of this essay.
25. Shackle considers both the potential surprise function and the ascendancy function as functions which are cardinal up to a linear transformation, cf. Shackle (1969), chaps. XVI - XVIII.
26. Coddington (1975), p. 158.

27. Coddington (1975), p. 151.
28. Shackle's work is quoted extensively throughout Vickers's text. The main source for the critique seems to be Shackle (1972).
29. Vickers (1978), p. 3.
30. "... the decision horizon as initially specified is to be understood as a real time span as seen by the decision maker at the time of decision, and ... if trading occurs during that interval the decision horizon has by that fact been terminated and a new decision moment with its own decision horizon has been specified. The specification at each decision moment of a corresponding decision horizon is a way of tentatively tying down time in order to estimate what might be the rates of return earned on the holding of various assets during that period." Vickers (1978), p. 163, emphasis in original text. What is not clear here is whether the investor forms estimates of rates of return for each relevant time period within his decision horizon. The use, elsewhere, of the notion of a 'critical' level of prices or capital gains which, if attained, induce the investor to liquidate his holdings of an asset would suggest that he does.
31. Vickers (1978), p. 163, emphasis in original text.
32. My notation differs from that employed in the text.
33. Vickers (1978), p. 166.
34. My figure 5 is the same as that appearing in the

text as figure 19, p. 166. According to that construction, the investor would, in fact, continue to hold a single asset, A, as no portfolio of assets dominates it.

35. Vickers (1978), p. 169.
36. This is implied by the discussion in Shackle (1952), chap. IV, see particularly pp. 78-79 and pp. 90-93. Nevertheless, in that chapter, Shackle also assumes independence between potential surprise functions.
37. cf. Egerton (1955, 1956) on Shackle (1952) and the analysis of speculation presented in chapter 4. For example:

" The Shackle theory cannot explain risk spreading because of the way in which it assumes an investor to look at the total focus outcome of a composite holding of assets. An investor is assumed to think that either all the focus gains or all the focus losses will occur; and, if he does think in this way, there is no advantage to him in holding more than two assets. But it is doubtful if an investor would think in this way unless he believed there to be a high degree of positive correlation between the price movements of the assets. If an investor believed that there was no, or only slight, correlation between the price movements of certain assets it would be curious for him to consider the outcome of a composite holding of them as being simply the sum of their separate focus outcomes, because it would presumably cause him considerable surprise if they all did yield either their

focus gain, or if they all yielded their focus loss. Where any two assets have a high degree of positive correlation in their price movements, Professor Shackle's method of aggregation is appropriate; but when two assets have a high degree of negative correlation in their price movements, the total focus outcome of a composite holding will be found - not by adding focus gains and focus losses - but by adding each focus gain to the focus loss of the other. But either method of aggregation is strictly proper only when the correlation coefficient (either positive or negative) is unity." Egerton, 1956, pp. 53-54, emphasis in original text.

38. As the frontier need not be strictly convex, the ray from the origin may be tangential to a linear segment of it, indicating that several uncertain portfolios are efficient for the purposes of joint holding with the riskless asset.
39. Shackle (1972), p. 8.

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