Läufer, Nikolaus K. A.

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(An extended analysis)

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Credit Ceilings and the Control of the Money Supply
(An Extended Analysis)

by

Nikolaus K.A. Läufer

DISKUSSIONSB安东GE

D-775 Konstanz
Postfach 733
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Introduction

In Germany the recent discussion on the reform of monetary policy has not yet come to an end, even though the chances for legislative innovations have greatly diminished. It has become obvious that the German government is still attempting to combine such a reform with restrictions of the autonomy of the German Bundesbank. I do not intend to give a survey of this discussion. Instead, I try to provide some further results concerning credit ceilings which is among the proposed instruments of monetary policy so far unavailable in Germany.

The analysis is built on the non-linear money-supply hypothesis developed by K. BRUNNER and A.H. MELTZER \(^1\). In this hypothesis both the money and the bank credit supply are conceived to be the algebraic product of a multiplier and the monetary base.

It is important to note that credit ceilings are not related to the monetary base, they only affect the multipliers. Therefore, assuming that the controllability of the monetary base is not altered by introducing or operating on credit ceilings, prospective changes in the effectiveness of monetary policy may be judged on the basis of (prospective) changes in the controllability of the multipliers.

Now, the controllability of multipliers is uniquely related to the elasticities of the multipliers with respect to the various parameters forming these multipliers, and changes in the elasticities
reflect changes in controllability. By an analysis of changes in
elasticities, it will be possible to show that credit ceilings
weaken the potency of monetary policy in controlling the money
supply, the result being less unfavourable if bank credits are
the target of monetary policy. A more detailed summary statement
of results is given at the end of part A and B.

In order to limit possible misinterpretations three remarks may
be helpful.
First, in order to determine the equilibrium price on any market,
it is necessary in the first stage to determine the position of
both demand and supply curves. Once these positions are given,
prices (interest rates) may be determined in the second stage.
Both on the money market and on the market for bank credit the
position of the supply curves is changed whenever there is an act
of monetary policy.
The following analysis is focused on the shifts of both the money
supply and the credit supply function due to the introduction of
new or due to a change in already existing credit ceilings.
A shift in the credit supply function will of course affect interest
rates as determined on the bank credit market. These interest
rate changes have feedback effects on the money market. In parti-
cular, they partially offset the former shift of the money supply
function. These feedback effects are explicitly taken into account
in part B, as may be seen from dashed elasticity expressions, for
example (78), while they need not to be considered in part A, where
interest rates are assumed as given and held constant in order to
derive partial derivatives with respect to credit ceilings and other
non-interest rate variables.
Second, what will be called the effects of credit ceilings on the effectiveness of (particular instruments of) monetary policy refers to alterations in the shiftability (by acts of monetary policy) of both the money and the credit supply curve. Changes in shiftability will be measured by changes in certain elasticities of the money and/or credit multipliers.

Third, initially it will prove economical to formulate ceilings on loans (or total earning assets) in the form of a tolerated percentage rate of the volume desired by the commercial banking system for a given structure and level of interest rates in contrast to the actual or past volume of loans.

Other more "realistic" types of ceilings - such as a ceiling on the (absolute or relative) change or on the total of the actual or past as opposed to desired volume - are not excluded by this procedure but merely require some additional treatment. In fact, all the four types of credit ceilings considered are analytically equivalent as long as the ceiling is the only changing determinant of the behaviour of commercial banks. However, if either the monetary base and/or other variables (in particular interest rates) are allowed to change, then this analytical equivalence disappears and some problems arise. Fortunately these problems are not difficult to solve.

It is my aim to analyze a variety of ceilings, i.e. ceilings on the total of bank's earning assets as well as on single components of them. For reasons of symmetry it is sufficient to concentrate
on loan ceilings and on ceilings for total earning assets. Since in the following the word loan may be replaced by the word investment without affecting any of the formal results, general statements and conclusions of our analysis, a separate analysis of investment ceilings is not necessary. As it turns out, even ceilings on total earning assets do not require separate treatment. My technical handling of the loan ceiling will be general enough to include ceilings on total earning assets as a special case of loan ceilings.

Part A deals with credit ceilings. In section I of that part I restate some elements of the BRUNNER-MELTZER model, which I need in section II, where various forms of ceilings (loan, investment and general earning asset ceilings) are cast in model form. In section III "constrained" multipliers are derived. Elasticities of multipliers and their reaction to ceilings changes are dealt with in section IV. Section V contains summary statements of the results.

In part B, section I, I shall give an explicit demonstration of how to reduce all more "realistic" types of ceilings to the type analyzed in part A. In section II of part B analytical steps are taken that in section III allow to show that the conclusions drawn in part A carry over to the case of realistically formulated credit ceilings as far as the control of the money supply is concerned. Section IV contains a summary statement of the results of part B and an intuitively appealing interpretation of some of the results. At the end of the paper some conclusions are drawn.
I start out with a budget restraint, i.e. a consolidated balance sheet of the commercial banking sector as expressed by equation (1)

\[ L + I + R = D + T + S + B_0. \]  

Total assets are allocated between loans (L), investments (I) and reserves (R). Total liabilities consist of demand deposits (D), time deposits (T), savings deposits (S) and borrowings from the central bank (B_0). Dividing both sides of the balance sheet by the sum of all deposits, (D + T + S), gives an equation in ratios:

\[ 1 + i + r_1 = 1 + b \]  

The loan ratio (l), the investment ratio (i), the total reserve ratio (r_1) and the borrowing ratio (b) describe the behaviour of the banking sector.

The ratios are behavioural parameters. They are functions and depend on an array of determinants, the complete statement of which is not necessary for my purposes. Yet, it will be illustrative to draw attention to some of the determinants and to give a more detailed description of the reserve ratio.

In equations (3), (4) and (5)

\[ l = l \left( i_G, i_L, \rho_2 \ldots \right) \]  

(3)
\[ i = i_G, i_L, p, \ldots \] 

\[ b = b_G, i_L, p, \ldots \] 

\( i_G \) designates the average yield on government securities, \( i_L \) denotes an index of loan rates, and \( p \) is the discount rate.

The total reserve ratio equals the sum of the required reserve ratio \( (r^R) \) and the excess reserve ratio \( (r^E) \). The excess reserve ratio is a true behavioural parameter of the banking sector as illustrated by equation (6):

\[ r^e = r^e_G, p, i_L, \ldots \] 

The required reserve ratio is given by equation (7):

\[ r^R = \frac{1 \cdot r^d + t \cdot r^t + s \cdot r^s}{1 + t + s} \] 

\( r^d, r^t \) and \( r^s \) represent reserve requirements as determined by the central bank respectively for demand deposits, time deposits and savings deposits. \( t \) and \( s \) - not in their role as indices - are symbols for time and saving deposits scaled by demand deposits, thus representing part of the allocative behaviour of the nonbank public as described by equation (8) and (9):

\[ t = t_G, i_S, i_G, i_L, \ldots \] 

\[ s = s_G, i_T, i_G, i_L, \ldots \]
By virtue of (6), the total reserve ratio may be considered to be a behavioural function of the banking sector, too:

$$r_1 = r^e + r^e = r_1 \left(r^d, r^t, r^s, i_C, i_L, i_T, i_S, \rho_o, \ldots\right)$$ (10)

There is no point in further detailing these behavioural functions, since in the following I shall usually omit all the arguments of the behavioural parameters. For the same reason it may be justified to introduce without specifying details the currency ratio as an additional behavioural parameter of the public which describes the allocation of its money holdings between currency and demand deposits.

Some additional terminology will be required in what follows:

$$K = L + I = \text{earning assets}$$
$$e = 1 + i = \text{earning assets ratio}$$
$$r_2 = r_1 - b = \text{adjusted reserve ratio}$$

II.

Formally, I shall represent a loan ceiling by the parameter $\rho_1$ restricting the desired loan ratio (1) to a constrained loan ratio ($l^c$) as shown by equation (11) and (12).

$$l^c = \rho_1 l$$  \hspace{1cm} (constrained loan behaviour) (11)

$$\rho_1 = \rho_1 \quad 0 \leq \rho_1 \leq 1$$  \hspace{1cm} (autonomous restriction) (12)
Since a budget constraint is assumed to be always effective, a restriction on loans induces restrictions on the other behavioural parameters of the banking sector.

\[ i^c = p_2 i \]  
(constrained investment behaviour) (13)

\[ r^{e,c} = p_2 r^e \]  
(constrained excess reserve behaviour) (14)

\[ b^c = p_3 b \]  
(constrained borrowing behaviour) (15)

\[ p_2 = p_2 (p_1,...) \]  
(induced restriction) (16)

\[ p_3 = p_3 (p_1,...) \]  
(induced restriction) (17)

If \( p_1 < 1 \), then there are two different budget constraints holding simultaneously for the commercial banking sector: a "free" budget constraint as given by equation (2) and a "restricted" budget constraint as given by equation (18).

\[ p_1 l + r^r + p_2 (1 + r^e) = 1 + p_3 b \]  
(18)

The two constraints are noncontradictory for all values of \( p_1 \) and they are identical for \( p_1 = 1 \).

That is to say:

\[ p_2 = p_3 = 1 \text{ if and only if } p_1 = 1 \]  
(19)
To call $\rho_1$ an **autonomous restriction** does not require or imply $\rho_1$ to be an **exogenous** variable. A ceiling may be an upper limit to the total or to the absolute or relative change of the actual or past as opposed to desired volume of loans or total earning assets. In all these cases, as shown in part B, $\rho_1$ is an **endogenous** variable.

It is a truely **exogenous** variable if and only if the ceiling is formulated as an admissible percentage rate of the **desired** as opposed to the actual or past volume of loans (or total earning assets).

In part B, it is shown that the analytical framework developed in this paper is suitable to all types of ceilings mentioned. However, in cases with $\rho_1$ endogenous, equation (12) will have to be replaced by equation (64). Whether endogenous or exogenous, $\rho_1$ may be considered to be an **autonomous** restriction in the sense of being imposed by the monetary authorities. Induced restrictions on borrowings, excess reserve holdings etc. form direct consequences of the autonomous restriction derived by commercial banks for their behaviour.

Induced restrictions represent spillover effects of autonomous restrictions. Obviously an induced "restriction" may actually be an induced **extension**.

With respect to equation (18) one may ask: why are induced restrictions on investments and excess reserve holdings assumed to be equal? (In (18) $\rho_2$ is a coefficient both of $i$ and $r^s$). This assumption represents my belief that the restrictions on investments and excess reserves induced by the restrictions on loans --
as long as interest rates are unchanged - does not affect the distribution of asset holdings between investments and excess reserves but merely affects their level. The restriction on loans is assumed to be equivalent to a change in the scale factor, \((D + T + S)\), leaving unaffected the distribution between investments and excess reserves. This distribution is assumed to change only if there is a change in relative prices (interest rates). It is well known that this separation of scale and distributional aspects of allocation (of bank deposits among alternative assets) is consistent with the assumption of a homogeneous utility function underlying the behaviour of the decision makers of commercial banks. The problem is similar to a problem in consumption theory. With a homogeneous utility function and rationing of demand for a particular commodity (equivalent to the present restriction) the utility maximizing revision of demand for all other commodities (equivalent to the induced restriction on investments and excess reserves) is performed in proportionately equal amounts (equal percentage rates of increase).

The framework of BRUNNER-MELTZER is built on the separability of scale and distributional aspects. Their framework is consistent with the assumption of a homogenous utility function of both the decision makers in commercial banks and of the money holding public. Therefore, it corresponds to the basic spirit of the BRUNNER-MELTZER framework if excess reserves and investments (in the loan ceiling case) are restricted by the same coefficient, \(\rho_2\), i.e. if they are revised by the same percentage rate. However, revising investments and excess reserves by the same percentage rate is not equivalent to treating investments and excess reserves as perfect
substitutes. Perfect substitutability of commodities or assets should not be confounded with the homogeneity of a utility function.

It is necessary to find the functions \( \rho_2 \) and \( \rho_3 \). As soon as (17) is specified, one may substitute for \( \rho_3 \) in (18). From that, (16) may be derived by way of isolating \( \rho_2 \). In order to specify (17), I assume that the borrowing ratio (5) may be described as a sum of two components:

\[
b = b_1 + b_i
\]  \hspace{1cm} \text{(total borrowing behaviour)}  \hspace{1cm} (20)

\[
b_1 = b_1 (i_L, \rho_o, \ldots )
\]  \hspace{1cm} \text{(borrowing for loans behaviour)}  \hspace{1cm} (21)

\[
b_i = b_i (i_G, \rho_o, \ldots )
\]  \hspace{1cm} \text{(borrowing for investments behaviour)}  \hspace{1cm} (22)

Thus I associate each component of the earning asset ratio with a component of the borrowing ratio and vice versa.

I further assume that loan restrictions affect the borrowing behaviour only by a parallel revision of its borrowing-for-loans component. The borrowing-for-loans ratio is constrained by a percentage rate equal to the percentage rate of the autonomous loan restriction.

Thus the constrained borrowing ratio may be written as:

\[
b^c = \rho_1 b_1 + b_i
\]  \hspace{1cm} (23)

from which by virtue of (15) it immediately follows that
\[ \rho_3 = \frac{1}{b} ( \rho_1 b_1 + b_1 ) = \frac{b^c}{b} \]  

\[ \rho_3 = \rho_1 \beta + (1 - \beta) \]  

where

\[ 0 \leq \beta \leq \frac{b_1}{b} \leq 1 \]  

\[ 0 \leq (1 - \beta) \leq \frac{b_1}{b} \leq 1 \]  

\[ \beta = \beta (i_L, i_G, \rho_o, \ldots). \]

The relation between \( \rho_1 \) and \( \rho_3 \) requires some comment. On the one hand, banks borrow from the central bank in order to increase loans and investments on account of their profit motive. On the other hand, loans and investments are the collateral necessary for borrowings from the central bank. Borrowings are clearly not only related to loans but also to investments. In my analysis this feature of bank behaviour is taken into account by a splitting of the total borrowing ratio into a borrowing-for-loans ratio and into a borrowing-for-investments ratio. My hypotheses about banking behaviour then is to assume the restrictions on loans to be exclusively related to the borrowing-for-loans ratio and not to the borrowing-for-investments ratio.

Clearly then, by assumption, the spillover effect of restrictions on loans does not affect the borrowing-for-investments ratio, in spite of an (induced) extension in the investment ratio.
In addition, I assume that the borrowing-for-loans ratio is restricted by a percentage rate equal to the percentage rate of the (autonomous) loan restriction. In the case of a ceiling on total earning assets (a general nondifferentiated earning assets ceiling) the corresponding assumption is that of an equality between the autonomous restriction on the ratio for total earning assets, and the induced restriction on the ratio for total borrowings from the central bank. These equality assumptions are consistent with the assumed separability of distributional and scale aspects of behaviour.

They are equivalent to a treatment of the respective borrowing component as a negative asset. Stated differently, in the analysis whenever there is an autonomous restriction on loans (or on total earning assets) then this restriction is applied to the difference between the loan ratio and the borrowing-for-loans ratio (or to the difference between the ratios for total earning assets and total borrowings), i.e. the restriction is treated as equivalent to a restriction on loans adjusted for borrowings for loans (or on total earning assets adjusted for total borrowings).

As shown by (25), $\rho_3$ may be interpreted as a weighed average of autonomous restrictions placed on components of the earning assets ratio. The autonomous restriction on loans is captured by $\rho_4$, the assumed lack of an autonomous restriction on investments is represented by the coefficient of $(1 - \beta)$, which is equal to 1, and the weights are given by equations (26) - (28). Generally, $\rho_3$ is a function as characterized by equation (29), (30) and (31).
\[ \rho_3 = \rho_3 \left( \rho_1, i_L, i_G, \rho_o, \ldots \right) . \]  

(29)

This function may be formulated more explicitly by means of equation (25) and (28). From (25) it also follows that

\[ \rho_3 < 1 \text{ if and only if } \rho_1 < 1 \]  

(30)

provided \( \rho_1 > 0 \) and \( B > 0 \). In addition, if \( B > 0 \), it is obvious that

\[ \frac{\delta \rho_3}{\delta \rho_1} > 0 \]  

(31)

In the special case of a general nondifferentiated earning assets ceiling the autonomous restriction is of equal size for all components of the earning asset ratio. Therefore \( \rho_3 \) is a weighed average of identical restrictions:

\[ \rho_3 = \rho_1 \beta + \rho_1 \left( 1 - \beta \right) \]  

(32)

and (29) degenerates to

\[ \rho_3 = \rho_1 . \]  

(32a)

Thus, as far as \( \rho_3 \) is concerned, the case of a general earning assets ceiling is also contained in (25). Formally, this general ceiling merely requires the insertion of identity (33) into (25):

\[ \delta \equiv 1 . \]  

(33)

(33) is a degenerate form of function (28).
Finally the function $p_2$ may be derived from (18) and (25).

$$p_2 = p_1 + \frac{1}{1 + r^e} \left[ 1 + b (1 - \beta) - r^e \right] (1 - p_1)$$ (34)

(34) is a specification of (16).

The following assumption will be used throughout the discussion of loan ceilings:

$$(1 - b\beta) > 0 \quad \text{for} \quad 0 < \beta < 1$$ (35)

This assumption merely claims that loans are higher than borrowings for loans. It is a correct assumption for Germany, and it is very likely to hold for many countries.

Now, it is easy to derive from (2), (20), (26), (27) and (35) that

$$p_2 > 1 \quad \text{if and only if} \quad p_1 < 1$$ (36)

regardless of the size of $\beta$. In addition, it follows that

$$\frac{3p_2}{\partial p_1} < 0$$ (37)

As far as $p_2$ is concerned, the case of a general earning assets ceiling is also contained in (34). Formally, this general ceiling merely requires the use of both (33) and (38) in order
to substitute for $\beta$ and $i$ in (34).

In the case of a general earning assets ceiling the analogue of assumption (35) is, of course:

$$ (1 + i - b) > 0 $$

(39) may formally be derived from (35) by adding (38) to (35) and by taking $\beta$ as given in (33).

Under this rather weak assumption, both (36) and (37) hold for the case of a general earning assets ceiling, too.

III.

By the definition of the monetary base from the user's side I have

$$ B^1 = B = C^P + R^r + R^e $$

where $B$ stands for monetary base, $C^P$ is a symbol for currency (of the public) in circulation outside commercial banks, $R^r$ designates required reserves and $R^e$ denotes excess reserves. The concept of the adjusted base will be used, likewise, and may be represented by

$$ B^2 = B^a = B - Bo $$
where $B^a$ designates the adjusted base, and $B_0$ stands for borrowings of the commercial banking sector from the central bank.

Both the money supply ($M$) as defined by

$$M = C^P + D$$

(42)

and the supply of credits or the demand for earning assets by banks, $(L+I)$, are determined by transforming the monetary base by a suitable multiplier incorporating behavioural parameters of the commercial banking sector and the nonbank public.

$$M = m_i \cdot B^i \quad i = 1,2$$

$$K = L + I = a_i B^i$$

The money multipliers for the different concepts are:

$$m_1 = \frac{1 + k}{(r^r + r^e)(1 + t + s) + k} \quad \text{(unadj. base model)} \quad (43)$$

$$m_2 = \frac{1 + k}{(r^r + r^e - b)(1 + t + s) + k} \quad \text{(adj. base model)} \quad (44)$$

The credit or earning assets multipliers are:

$$a_1 = \frac{(1 - r^r - r^e + b)(1 + t + s)}{(r^r + r^e)(1 + t + s) + k} \quad \text{(unadj. base model)} \quad (45)$$

$$a_2 = \frac{(1 - r^r - r^e + b)(1 + t + s)}{(r^r + r^e - b)(1 + t + s) + k} \quad \text{(adj. base model)} \quad (46)$$
A proper treatment of the effects of loan and credit ceilings requires the derivation of constrained multipliers. This goal can easily be reached by replacing the unconstrained behavioural parameters of the multiplier by their constrained (c) counterparts.

Introducing the following notation for constrained reserve ratios

\[ r_1^c = (r^r + r^e) \quad \rho_2^c = (r^r + r^e - \rho_3^c) \]  
\[ r_2^c = (r^r + r^e - b^c) = (r^r + \rho_2^c r^e - \rho_3 b) \]

I can write the constrained multipliers as

\[ m_1^c = \frac{1 + k}{r_1^c (1 + t + s) + k} \quad \text{(unadj. base model)} \]  
\[ a_1^c = \frac{(1 - r_2^c)(1 + t + s)}{r_2^c (1 + t + s) + k} \]  
\[ m_2^c = \frac{1 + k}{r_2^c (1 + t + s) + k} \quad \text{(adj. base model)} \]  
\[ a_2^c = \frac{(1 - r_2^c)(1 + t + s)}{r_2^c (1 + t + s) + k} \]

Exploiting the definitions of \( \rho_2 \) and \( \rho_3 \), as given by (25) and (34), I find

\[ r_1^c = (r^r + r^e) (1: \rho_1 + r^r_1 + [1 + b(1 - \beta)] \frac{r^e}{i + r^e} \) \]  
\[ r_2^c = (r^r + r^e - b) \rho_1 + [\frac{r^r - b(1 - \beta)}{i + r^e} \frac{i + r^e}{1 - \rho_1} \]
(53) and (54) are more explicit statements of (47) and (48). They show that the constrained reserve ratio may be interpreted as a weighted average of two different reserve ratios, the weights being \( \rho_1 \) and \((1 - \rho_1)\). The first of the two ratios is the one prevailing if credit ceilings are absent. I shall call it the unconstrained reserve ratio. The other ratio which I shall call a forced reserve ratio is itself a weighted average of two different reserve ratios. The first of the two ratios forming the forced ratio is related to investments, the second one is related to excess reserves and the weights are given by the relative size of unconstrained (or, equivalently, constrained) investments and excess reserves.

The forced reserve ratio may further be described as follows. To the extent that loans are constrained, commercial banks are forced to allocate deposits and borrowings either to investments or to excess reserves or to both of them. Such a forced allocation implies different and additional reserve holdings as compared with an unforced allocation. In the unadjusted base model, forced allocation to investments absorbs borrowed and unborrowed reserves to an extent expressed by the ratio \( r^i \), while forced allocation to excess reserves absorbs borrowed and unborrowed reserves to an extent expressed as a ratio by the sum of (the ratios) \( b \cdot (1 - \beta) \) and 1.

Similarly, in the adjusted base model, forced allocation to investments requires unborrowed reserves to an extent indicated by the ratio \( r^e - b (1 - \beta) \), while forced allocation to excess reserves is absorbing unborrowed reserves to an extent indicated by a ratio equal to 1.
The case of general earning assets ceilings does not require a separate treatment. Constrained reserve ratios for this case merely require the use of (33) and (38) in order to substitute for $\beta$ and $i$ in (53) and (54). The interpretation of constrained ratios in this case is analogous to the one given for loan ceilings.

Recalling (30) and (36), it is easy to derive from (47) and (48) that
\[
\begin{align*}
    r_1^c > r_1 \\
    r_2^c > r_2 = (r_1 - b)
\end{align*}
\]
if and only if $\rho_1 < 1$, \hspace{1cm} (55)

regardless of the size of $\beta$.

In addition, it may be shown that
\[
\frac{\partial r_i^c}{\partial \rho_1} > 0 \hspace{1cm} i = 1,2
\]
\hspace{1cm} (56)

Since the "forced" reserve ratio is larger than the unconstrained reserve ratio, their average, the constrained reserve ratio, turns out to be larger than the unconstrained reserve ratio. Therefore, the constrained reserve ratio really is an augmented reserve ratio. Lowering the loan or total earning assets ceiling increases the constrained reserve ratio.

From (55) it follows at once that:
\[
\begin{align*}
    m_i^c < m_i \\
    a_i^c < a_i
\end{align*}
\]
if and only if $\rho_1 < 1$, \hspace{1cm} (57)

regardless of the size of $\beta$. 
In addition, from (56) we have

\[- \frac{\partial m^c_1}{\partial \rho} < 0 \quad \text{and} \quad - \frac{\partial a^c_i}{\partial \rho} < 0 \quad i = 1, 2 .\]

Thus, both the money and the credit multiplier is reduced by lowering an effective loan ceiling or by lowering an effective total earning assets ceiling. This result is independent of the base version used.

Since both types of ceilings primarily affect the total reserve ratios, it is obvious that equally contractive effects can be reached by an appropriate change in reserve requirements. Consequently, a central bank in possession of reserve requirements as a policy instrument does not increase its aggregate effectiveness by introducing loan or general credit ceilings as an additional policy instrument. Clearly, loan ceilings or ceilings on other components of earning assets in contrast to ceilings on total earning assets also have allocative effects on the asset supply side and consequently on the structure of interest rates. But these effects are not at stake in my present analysis concentrating on stabilization issues.

IV.

My next step is to present elasticities of the multipliers incorporating loan and earning assets ceilings. There are three tables to begin with. The first table defines the elasticities of the money
multiplier and the asset multiplier
both for the unadjusted \((i = 1)\) and the adjusted \((i = 2)\) base
model \(^3\). Generally, these elasticities are differences of respec-
tive elasticities for the numerator \((Z)\) and the denominator \((N)\)
of a multiplier.

For numerator and denominator elasticities are given in tables 2
and 3, where use is made of the following notation:

\[
Z = (1 - r^c_2) (1 + t + s)
\]

\[
N_i = r^c_i (1 + t + s) + k \quad i = 1, 2
\]  

\[
\frac{1 + k}{N_i}
\]

\[
a^c_i = \frac{Z}{N_i}
\]

and the fact that:

\[
Z = (1 + t + s) + k - N_2 .
\]  

Now, there are two further tables. They contain the main results
of the first part of this paper. Table 4 gives signs and values
of the money multiplier elasticities and states their reaction
to a change in credit ceilings. In table 5 the same is given for
the credit multiplier elasticities.

The effects of credit ceilings on the elasticities depend on the
parameter with respect to which an elasticity is taken. Yet, it is
obvious that the effects may be classified according to whether the parameter is controlled by the central bank authorities, as is true for $r^r$, $r^d$, $r^t$, $r^s$ and $\rho_1$, or whether it is controlled by the bank and non-bank public, as is true for $k$, $t$, $s$, $i$, $b$, $1-\beta$ and $r^e$.

Introducing or lowering existing ceilings (reducing $\rho_1$ below 1 or further below 1), increases the elasticities of the money multiplier with respect to parameters not under control of the authorities; however, this does not hold for $k$, $b$ and $r^e$. For $k$ there may be an increase or a decrease, while for $b$ (adj. base model) there is a decrease in the elasticity. The elasticity of the money multiplier with respect to $r^e$ decreases if ceilings are placed on total earning assets, while it may increase or decrease if only loans are restricted by ceilings. The elasticities of the money multiplier with respect to those parameters that are under (direct) control of the authorities are reduced without exception, if credit ceilings are introduced or lowered.

The reactions of the credit multiplier elasticities differ from those of the money multiplier elasticities. It is only for ceilings on total earning assets as opposed to loan ceilings that I can give definite statements about the reaction of the credit multiplier elasticities. In that case, the reduction of the elasticities is paramount with exceptions relating to $1-\beta$ and $b$ (unadj.). Ceilings on total earning assets thus reduce the sensitivity of the credit multiplier with respect to parameters controlled by private banks and the nonbanks public, as well as with respect to parameters.
controlled by monetary authorities.

V.

The main results of part A of this paper dealing with two models of credit ceilings may be summarized by the following arguments against credit ceilings:

1. Credit ceilings are not necessary for an effective control of the aggregate supply of money and credit, in particular if the instrument of minimum required reserves is available.
2. Applying credit ceilings in addition to minimum required reserves reduces the effectiveness of control of the money supply by minimum required reserves of a given amount.
3. Applying credit ceilings (in addition to minimum required reserves) increases the effects on the money multiplier of given relative changes in the behaviour of the (private) nonbank public and thus weakens the potency of monetary policy as far as shifts in the time and in the savings deposits ratios are concerned, while the direction of change in the effects of shifts in the currency ratio is uncertain.
4. For given relative changes in the behaviour of commercial banks in the case of loan ceilings a predominance of increases in elasticities of the money multiplier does not necessarily prevail. However, in the case of a ceiling on total earning assets all the respective elasticities decrease.

With respect to the goal of an effective control of the money supply by central bank authorities, these results clearly point towards the existence of undesirable consequences of loan or
general earning asset ceilings. In the case of Germany there are even signs of a dominance of these undesirable consequences. 4) As far as ceilings on total earning assets are concerned and not just loan ceilings, the evaluation slightly changes in favor of ceilings, if the policy goal consists of an effective control of the credit supply instead of the money supply.
Part B.

I.

In this part B I shall demonstrate how to reduce more "realistic" types of ceilings to the types underlying the analysis in part A, which in the introduction were termed less "realistic". This task is twofold. First, it is a formal task. It is necessary to give a technical treatment of alternative more realistic types of ceilings. Second, it is necessary to adjust or qualify the nonformal results derived in part A in order to cover the more "realistic" types of credit ceilings. The second task will be dealt with in section II of this part B.

The formal task covers two aspects. Ceilings may be expressed either in absolute terms or in percentage rate terms (first aspect). In addition, they may be imposed either on total volumes or on changes of total volumes (second aspect). The following discussion covers both aspects.

In part A $\rho_1$ is related to $L_t$ and $L_t^c$ as shown by the following equations

$$L_t^c = \rho_1 L_t$$  \hspace{1cm} (60)

$$\rho_1 = \frac{L_t^c}{L_t}$$  \hspace{1cm} (61)

where

- $c =$ constrained
- $L_t =$ total volume of desired loans at time $t$
- $L_t^c =$ constrained volume of loans at time $t$
- $\rho_1 =$ predetermined (exogenous) parameter
shall distinguish three "realistic" types of a ceiling formulation. **Type 1:** if the ceiling imposed is formulated as a percentage rate of admissible growth, $\rho^*_1$, then

$$\Delta L_t^C = \rho^*_1 L_{t-1}$$

in account of

$$L_t^C = \Delta L_t^C + L_{t-1}$$

identity (63), $\rho_1$ may be derived as an endogenous variable yielding (64):

$$\rho_1 = \frac{(1 + \rho^*_1) L_{t-1}}{L_t}$$

(64)

The denominator of (64) is formed by the desired volume of loans. Therefore, $\rho_1$ is endogenous. On the other hand, **type 2**, if the ceiling imposed is formulated as a value for the admissible absolute increase in actual volume, then

$$\Delta L_t^C = \text{exogenous}$$

(65)

Transforming this ceiling into percentage rate terms as stated in (66),

$$\rho^*_1 = \frac{\Delta L_t^C}{L_{t-1}}$$

(66)

may be derived as an endogenous variable as in (64). Finally, **type 3**, if the ceiling imposed is formulated as a value of the admissible total of the actual volume, then

$$L_t^C = \text{exogenous}$$

(67)

and

$$\rho^*_1 = \frac{L_t^C - L_{t-1}}{L_{t-1}}$$

(68)

while $\rho_1$ may be determined as an endogenous variable again by means of (64).
Thus, the following conclusion is arrived at. Since a ceiling on actual increments expressed in absolute terms may immediately be rephrased in percentage rate terms, and since a limit to total actual volume may be reformulated equivalently as a limit to actual incremental volume - either in absolute terms or in percentage rate terms - all three types of ceilings considered coincide as far as equation (64) is concerned. For all three types considered, $\rho_1$ is determined as an endogenous variable, while $\rho^*_1$, $\Delta L^C_t$ and $L^C_t$ are exogenous.

If, on the other hand, the ceiling is formulated as a percentage rate of the desired volume $L_t$, then $\rho_1$ is an exogenous variable, while $L^C_t$ is an endogenous variable.

Part A seems to be exclusively related to this latter case. But this would be a false impression. Since, in part A, interest rates were taken as given and were not allowed to change, the procedure - followed there - of treating $\rho_1$ as autonomous indeed covers all of the different forms of ceiling formulations, and the procedure is not selfcontradictory. But as soon as feedbacks from the credit market are taken into consideration, $\rho_1$, though in a sense still being autonomous, no longer is exogenous and equation (12) should be replaced by equation (64).

For an endogenous $\rho_1$ important elasticity expressions may be derived. Recalling

$$L = a_i \left( B^i \right) \quad (B^1 = B, B^2 = B^a) \quad (69)$$

where

$$a_i = \frac{1}{1 + \bar{a}_i} \quad (i = 1,2) \quad (70)$$
equation (64) may be restated as

\[ \rho_1 = \frac{L_t^e}{L_t^c} = \frac{(1+\rho_1^*) L_t - 1}{\alpha_1 B^j} \quad (i = 1, 2) \]  

(71)

Then, from (71), the following elasticity expressions are obtained:

\[ \varepsilon(\rho_1, x) = -\varepsilon(L, x) \quad (x \neq \rho_1^*) \]

(72)

\[ \varepsilon(\rho_1, \rho_1^*) = \frac{\rho_1^*}{1 + \rho_1^*} - \varepsilon(L, \rho_1^*) \quad (x = \rho_1^*) \]

(73)

where \[ \varepsilon(L, x) = \varepsilon(a_i, x) + \varepsilon(b^i, x) \quad (L = L_t) \quad (i = 1, 2) \]

(74)

and

\[ \varepsilon(a_i, x) = \frac{i}{1+i} \left[ \varepsilon(1, x) - \varepsilon(i, x) \right] + \varepsilon(a_i, x) \quad (i = 1, 2) \]

(75)

In the case of a ceiling on total earning assets (as opposed to a ceiling on loans) (75) is reduced to:

\[ \varepsilon(a_i, x) = \varepsilon(a_i, x), \]

(76)

while in the preceding formulas \( L \) should be replaced by the symbol \( K \). If a ceiling on investments is considered, instead of a ceiling on loans, the appropriate formulas are immediately obtained by substituting \( i \) for \( L \) and \( I \) for \( L \).
It remains to determine whether the introduction of these more realistic types of credit ceilings requires a modification of the former results as to the changes in the effectiveness of monetary policy caused by credit ceilings. As it turns out, surprisingly or not, all the arguments against credit ceilings are strengthened by allowing more realistic types of ceilings. As a next step this will be demonstrated rigorously. A more intuitive explanation and interpretation of these results will then follow.

I start out with a general description of the elasticity of the constrained money multiplier.

\[
\bar{\epsilon}(m^C_1, x) = \epsilon(m^C_1, k) \bar{\epsilon}(k, x) \\
- \epsilon(m^C_1, t) \bar{\epsilon}(t, x) \\
+ \epsilon(m^C_1, s) \bar{\epsilon}(s, x) \\
- \epsilon(m^C_1, i) \bar{\epsilon}(i, x) \\
+ \epsilon(m^C_1, 1-\beta) \bar{\epsilon}(1-\beta, x) \\
+ \epsilon(m^C_1, b) \bar{\epsilon}(b, x) \\
+ \epsilon(m^C_1, r^C) \bar{\epsilon}(r^C, x) \\
- \epsilon(m^C_1, r^F) \bar{\epsilon}(r^F, x) \\
+ \epsilon(m^C_1, \rho_1) \bar{\epsilon}(\rho_1, x)
\]

Dashed elasticity expressions are total as opposed to partial elasticities. The last additive term on the right hand side is due to credit ceilings.
The list of elements is delineated into three groups. The first group relates to parameters of the (private) nonbank public, the second group to parameters of the commercial banks and the third group to parameters of the monetary authorities.

If ceilings on total earning assets are considered \((i = 0, \beta = 1)\), then the fourth and fifth line vanish. If, in this case, the model with the nonadjusted base is considered, then the sixth element, counting from above vanishes, too.

The elements on the right of (77) are products of two elasticities. All the tables in this paper relate to the first of the two elasticities forming these products. The second factors require some further explanation. As an example consider the elasticity of excess reserves:

\[
\bar{\varepsilon}(r^e, x) = \varepsilon(r^e, i_L) \varepsilon(i_L, x) + \varepsilon(r^e, i_G) \varepsilon(i_G, x) + \varepsilon(r^e, x) \tag{78}
\]

This is a total elasticity decomposed into elementary partial elasticities on the right hand side. The first two products in (78) capture interest rate effects. If these effects are neglected the total elasticity reduces to the partial one, the last term in (78). If variables in addition to \(i_L\) and \(i_G\) would be allowed to become arguments of the ratios \(k, t, s, r^e\), etc. then there would be additional product terms in (78).

An expression similar to (77) holds for the constrained credit multiplier

\[
\bar{\varepsilon}(a_i^C, x) = \varepsilon(a_i^C, k) \bar{\varepsilon}(k, x) \tag{79}
\]

...
Again the last term is due to credit ceilings. In contrast to (77) the elasticity of the credit multiplier with respect to the borrowing ratio does not vanish when the adjusted base version is used. If ceilings on total earning assets are considered \((i = 0, \beta = 1)\), then similar to (77) certain elements in (79) vanish. However, in this case the elasticity of the credit multiplier with respect to the borrowing ratio does not vanish if the unadjusted base version is used.

There is a sharp distinction to be drawn between the total elasticity in (79) for the constrained credit multiplier and the total elasticity for the unconstrained multiplier in (80):

\[
\bar{\varepsilon}(a_i, x) = \varepsilon(a_i, k) \bar{\varepsilon}(k, x)
\]

\[
\ldots
\]

\[
+ \varepsilon(a_i, b) \bar{\varepsilon}(b, x)
\]

\[
\ldots
\]

\[
+ \varepsilon(a_i, r^r) \bar{\varepsilon}(r^r, x)
\]

In (80) there exists no term with \(\rho_1\).

In order to determine the effects of credit ceilings on various elasticities when \(\rho_1\) is endogenous, it is no longer possible to proceed as in part \(\ell\). With an endogenous \(\rho_1\), \(\bar{\varepsilon}(\rho_1, x)\) is not always zero for \(x \neq \rho_1\). One may substitute for \(\bar{\varepsilon}(\rho_1, x)\) in (77) by exploiting equations (72') - (75').
\( \varepsilon(p, x) = - \varepsilon(L, x) \) \hspace{1cm} (72')

\( (x \neq p^*) \)

\[ \frac{\rho^*}{1 + \rho^*_1} - \frac{\varepsilon(L, p^*)}{1 + \rho^*_1} \] \hspace{1cm} (73')

\( (x = p^*) \)

\( \varepsilon(L, x) = \varepsilon(a^*_1, x) + \varepsilon(B^1, x) \) \hspace{1cm} (74')

\[ \frac{1}{1 + i} \left[ \varepsilon(1, x) - \varepsilon(i, x) \right] + \varepsilon(a^*_1, x) \] \hspace{1cm} (75')

(72') - (75') are dashed versions of equations (72) - (75). The dashed versions represent total in contrast to partial elasticities. In (75') the elasticity of the unconstrained credit multiplier, as stated in (80), is of crucial importance and it enters the elasticity expression for the constrained money multiplier. This is obvious from (81) where (77) is rewritten by means of both (72') - (75') and (80).
\[ \bar{e}(m_i^c, x) = \left[ \varepsilon(m_i^c, k) - \varepsilon(m_i^c, \rho_1) \varepsilon(a_i, k) \right] \bar{e}(k, x) + \left[ \varepsilon(m_i^c, t) - \varepsilon(m_i^c, \rho_1) \varepsilon(a_i, t) \right] \bar{e}(t, x) + \left[ \varepsilon(m_i^c, s) - \varepsilon(m_i^c, \rho_1) \varepsilon(a_i, s) \right] \bar{e}(s, x) + \left[ \varepsilon(m_i^c, i) - \varepsilon(m_i^c, \rho_1) \varepsilon(a_i, i) \right] \bar{e}(i, x) + \left[ \varepsilon(m_i^c, 1-\beta) - \varepsilon(m_i^c, \rho_1) \varepsilon(a_i, 1-\beta) \right] \bar{e}(1-\beta, x) \] (81)

adj. base model - unadj. base model

adj. base model - unadj. base model

? loan ceilings - total earning assets ceilings

\[ \bar{e}(m_i^c, \rho_1) \varepsilon(a_i, r^e) \bar{e}(r^e, x) \]

\[ \bar{e}(m_i^c, \rho_1) \varepsilon(a_i, r^r) \bar{e}(r^r, x) \]

\[ \varepsilon(m_i^c, \rho_1) \left\{ \frac{i}{1 + i} \left[ \bar{e}(i, x) - \bar{e}(1, x) \right] - \bar{e}(B^i, x) \right\} \]
If $x = \rho_1$ then a further term has to be added:

$$\varepsilon(m_i^C, \rho_1^*) = \ldots + \varepsilon(m_i^C, \rho_1) \frac{\rho_1^*}{1 + \rho_1^*}$$

If a ceiling on total earning assets ($i = 0, \beta = 1$) is considered, then the fourth, fifth and the square bracketed part in the ninth line of (81) vanish. If, in that case, the unadjusted base version is used, then $\varepsilon(m_i^C, b) = \varepsilon(m_i^C, b) = 0$ in the sixth line.

In (81) a plus or a minus sign below an elasticity term indicates its being greater or smaller than zero.

With (81) available, it is possible to inquire into the changes in the effectiveness of monetary policy. For that purpose the terms in square brackets in line 1 to 8 of (81) are compared with the unconstrained elasticities $\varepsilon(m_i, \lambda), \varepsilon(m_i, \eta)$ etc.

III.

In part A, results bearing on the issue of changing effectiveness of monetary policy were derived by exclusive comparison of the first terms inside the square brackets of (81) with their unconstrained counterparts. The second terms inside the square brackets are products of elasticities. They vanish, if $\rho_1$ is truly exogenous or predetermined as it was assumed, preliminarily, in part A. But now $\rho_1$ is assumed to be endogenous since more realistic types of credit ceilings are considered.
An endogenous \( \rho \) changes the coefficients of \( \bar{e}(k, x) \), \( \bar{e}(t, x) \) etc. in two ways. The first way of change is the one considered in part A. It is given by the difference of \( \epsilon(m_c^1, k), \epsilon(m_c^2, t) \) etc. against their unconstrained counterparts \( \epsilon(m_1^1, k), \epsilon(m_1^2, t) \) etc. and indicated in (31) by a plus or a minus sign placed above the respective elasticities.

The second way of change is given by adding or subtracting the products \( \epsilon(m_1^c, \rho_1) \epsilon(a_1^c, k), \epsilon(m_1^c, \rho_1) \epsilon(a_1^c, t) \) etc.

A careful examination of how the terms in square brackets change, reveals that the two changes may additively reinforce each other. There are two patterns of reinforcement to be exemplified.

**Pattern 1:** A negative elasticity \( \epsilon(m_1^c, k) \) is smaller in absolute value than its unconstrained counterpart \( \epsilon(m_1, k) \). An addition of a positive term \( -\epsilon(m_1^c, \rho_1) \epsilon(a_1^c, k) \) reinforces a fall in the absolute value of the coefficient of \( \bar{e}(k, x) \). (In this example one cannot exclude the possibility of this coefficient turning from a negative to a positive value. In such cases of sign changes we neglect (in the following) the possibility of an elasticity to increase in absolute value.)

**Pattern 2:** The negative elasticity \( \epsilon(m_1^c, t) \) is larger in absolute value than its unconstrained counterpart \( \epsilon(m_1, t) \). The addition of the negative term \( -\epsilon(m_1^c, \rho_1) \epsilon(a_1^c, t) \) reinforces the rise in the absolute value of the coefficient of \( \bar{e}(t, x) \).

Changes according to pattern 1 may also be observed for the coefficient of the elasticity for excess reserves, \( \bar{e}(r_e, x) \), in the case of loan ceilings, for the coefficient of the elasticity of required reserves \( \bar{e}(r_v, x) \), for the coefficient of \( \bar{e}(i, x) \) and
for the coefficient of \(\bar{e}(b, x)\) in the adjusted base model. Changes according to pattern 2 may also be observed for the coefficient of \(\bar{e}(b, x)\) in the unadjusted base model, for the coefficient of \(\bar{e}(s, x)\) and for the coefficient of \(\bar{e}(k, x)\) in case of a positive elasticity \(\varepsilon(m^c, k)\).

Thus, as far as those elasticities are concerned for which it was possible in part A to make definite statements about the direction of change, with one exception, the results are not only confirmed but even significantly strengthened by allowing for endogeneity of credit ceilings. The exception is the change of the coefficient of \(\bar{e}(i, x)\) in (81).

IV.

As a summary of part B it is possible to state that the results of part A are qualitatively confirmed and quantitatively considerably reinforced by assuming realistically formulated credit ceilings as far as the control of the money supply is concerned. With respect to the effectiveness of control of the credit volume the case for ceilings on total earning assets is extremely strengthened. Obviously, excluding illegal evasions and formal tricks on the side of commercial banks, the volume of total credit may be completely controlled by formulating ceilings with respect to actual or past as opposed to desired credit volumes.

At this point an intuitively appealing interpretation of some results may be welcome. The decrease in the elasticity of the
money multiplier with respect to any of the rates of required reserves may be explained by noting that a partial or total ceiling on credits will lead both to a contraction of credits and to an extension of excess reserves. Increasing the rate of required reserves when an effective ceiling on credits exists, will, to some extent, simply induce a relabeling of excess reserves and a movement of the volume of credit as desired when effective ceilings do not exist towards the actually existing volume as effectively restricted by ceilings on credits. Of course, the extent to which this movement takes place increases with the quantity of the reserves required additionally.

The instrument of required reserves regains its former effectiveness in the absolute sense (as opposed to the relative sense as measured by elasticity expressions) as soon as the pool of "forced" excess reserves held in consequence of credit ceilings is fully absorbed by the additional reserve requirements.

Excess reserves may be interpreted as reserves required by commercial banks - instead of the central bank - and held by commercial banks. In the adjusted base model an increase in the borrowing ratio, b, is equivalent to a decline of a reserve ratio. Thus the intuitive explanation of the elasticity changes given for required reserves, in the usual sense, r, similarly applies both to the decline of the money multiplier elasticity with respect to the excess reserves ratio (in cases of ceilings on total earnings assets) and to the decline of that elasticity with respect to the borrowings ratio (in the adjusted base model).
Obviously, currency holdings of the public may be viewed as reserve holdings required and held by the public. Therefore, a shift in the currency ratio, $k$, is equivalent to a change in reserve requirements.

If $r_c^C (1 + t + s)$, the sum of excess reserves of commercial banks and reserves required by the central bank, expressed as a ratio per demand deposits, is larger than 1, a rise (fall) in $k$ is equivalent to a decrease (increase) in average required reserves. If the change in $k$ is equivalent to an increase (decrease) in average required reserves then the foregoing interpretation applies to the decline (rise) of the money multiplier elasticity (in absolute terms) with respect to the currency ratio, $k$, when that elasticity is negative (positive).

An increasing elasticity of the money multiplier with respect to the time deposit ratio, $t$, and with respect to the savings deposit ratios, $s$, may be explained in a similar intuitive fashion. The "normal" case of negative elasticities is considered only. An increase in time deposits implies for commercial banks either an influx of currency or an increase in earning assets or a combination of both elements. The less the element "increase in earning assets" is contained in this combination the higher will be the relative reduction in money supply for a given relative increase of $t$.

Now, credit ceilings change that combination in favor of additions to currency holdings of banks. Credit ceilings thus obstruct a weakening of the reduction in money supply via expansion of earning assets.
This implies that credit ceilings increase the contractive effect of a rise in $t$. Obviously, the line of reasoning for $s$ is symmetric to that for $t$. 
Conclusion

From the present paper the following results emerge. Credit ceilings tend to reduce the effectiveness of money supply control in several ways. Minimum required reserves loose part of their effectiveness as a control instrument. With an unchanged variance of shifts in the behaviour of the (private) nonbank public the variance of the money multiplier is likely to increase. That part of the multiplier variance which is related to commercial banks is likely to decrease.

As a net result, and as is shown by the German case, it is not unlikely that credit ceilings raise the volatility of the money multiplier and thus increase parameter uncertainty. However, whether or not they do is an empirical question.

By the theory of economic policy under conditions of parameter uncertainty it has become customary to consider more instruments as preferable to less when the goal of minimizing the variance of a target variable around a chosen target value is pursued.

In that literature the degree of parameter uncertainty of a particular instrument usually is taken as given and assumed to be independent of the number of instruments available. However in the problem considered here, that assumption is violated with probability one. A higher volatility and greater uncertainty in movements of the money multiplier as it may be caused by effective credit ceilings raises parameter uncertainty of open market operations and other monetary control instruments.
This weakens arguments in favor of credit ceilings and clearly points towards the existence of a trade-off between the degree of parameter uncertainty and the number of instruments.

On the other hand, the controllability of the credit volume may be extremely improved by credit ceilings. Thus the final evaluation of credit ceilings in general, and of ceilings on total earning assets in particular, depends on the choice of an intermediary target of monetary policy among the following alternatives: money stock or volume of credit.
Footnotes


3) It is clear that the reaction both of the supply of money and credit to a credit ceiling is independent of the model version (adj. or unadj.) chosen for analysis. Thus, if we find multiplier elasticities to be unequal between the two model versions

\[- (m_1^c, \rho_1) > (m_2^c, \rho_1)\]

\[- (a_1^c, \rho_1) > (a_2^c, \rho_1)\]

we should not be surprised, as there exists a compensating inequality of base elasticities:

\[(B^a, \rho_1) = 0\]

\[(B, \rho_1) > 0\]

4) This judgement of the German situation is based on rough calculations that take into account differences in the variability of parameters and weight changes of elasticities accordingly.
### Table 1

**Elasticities of the money multiplier**  \( = \varepsilon \left( m_i^c, x \right) \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( \varepsilon \left( m_i^c, x \right) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k )</td>
<td>( \frac{k}{1+k} \cdot \varepsilon \left( N_i, k \right) = \frac{k}{1+k} - \frac{k}{N_i} )</td>
</tr>
<tr>
<td>all ( x ) (except ( k ))</td>
<td>( - \varepsilon \left( N_i, x \right) )</td>
</tr>
</tbody>
</table>

**Elasticities of the credit multiplier**

<table>
<thead>
<tr>
<th>( x )</th>
<th>( \varepsilon \left( a_i^c, x \right) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k )</td>
<td>( - \varepsilon \left( N_i, k \right) )</td>
</tr>
<tr>
<td>( t, s )</td>
<td>( \frac{Z_1}{Z} - \frac{N_2}{Z} \cdot \varepsilon \left( N_2, x \right) - \varepsilon \left( N_1, x \right) )</td>
</tr>
<tr>
<td>all ( x ) (except ( t, s, k ))</td>
<td>( - \frac{N_2}{Z} \cdot \varepsilon \left( N_2, x \right) - \varepsilon \left( N_1, x \right) )</td>
</tr>
</tbody>
</table>

1. **Legend for**  
i as an index:  
i = 1 : unadjusted base model  
i = 2 : adjusted base model

2. \( x \in \{ k, t, s, i, b, (1-\beta), r^e, r^r, r^d, r^t, r^s, \rho \} \)
Table 2

Elasticities of the numerator of the credit multiplier

<table>
<thead>
<tr>
<th>$x$</th>
<th>$\varepsilon(\frac{Z}{x})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k$</td>
<td>$k \frac{N_2}{Z} - \frac{N_2}{Z} \varepsilon(N_2, k) = 0$</td>
</tr>
<tr>
<td>$t, s$</td>
<td>$x \frac{N_2}{Z} - \frac{N_2}{Z} \varepsilon(N_2, x)$</td>
</tr>
<tr>
<td>all $x$ (except $k, t, s$)</td>
<td>$-\frac{N_2}{Z} \varepsilon(N_2, x)$</td>
</tr>
</tbody>
</table>

$x \in \{k, t, s, i, b, (1-\beta), r^e, r^r, r^d, r^t, r^s, \rho\}$
### Table 3

Credit ceilings

Elasticities of the denominator of both the money and the credit multiplier

<table>
<thead>
<tr>
<th>x</th>
<th>( s(N_i, x) )</th>
</tr>
</thead>
</table>

| k | \( \frac{k}{N_i} \) |
| t | \( \frac{t(r_t - r^r)}{N_i} \cdot \frac{(i + \bar{r}_1 r^e)}{(1 + re)} + \frac{t(N_i - k)}{(1 + t + s N_i)} \) |
| s | \( \frac{s(r_s - r^r)}{N_i} \cdot \frac{(i + \bar{r}_1 r^e)}{(1 + re)} + \frac{s(N_i - k)}{(1 + t + s) N_i} \) |
| i | \( -i \frac{(1 + t + s)}{N_i} \cdot \frac{r^e[1 + b(1 - \beta - r^r)]}{(1 + r^e)^2} \cdot (1 - \bar{r}_1) \) |
| b | \( \frac{b(1 + t + s)}{N_1} \cdot \frac{r^e(1 - \beta)}{(1 + r^e)} \cdot (1 - \bar{r}_1) \) |
| \( (1 - \beta) \) | \( \frac{(1 + t + s)}{N_1} \cdot \frac{r^e \cdot b}{(1 + r^e)} \cdot (1 - \bar{r}_1) \) |
| \( (1 - \beta) \) | \( \frac{(1 + t + s)}{N_2} \cdot \frac{ib}{(1 + re)} \cdot (1 - \bar{r}_1) \) |
| \( r^e \) | \( \frac{r^e(1 + t + s)}{N_i} \cdot \frac{\bar{r}_1 + i \left[ 1 + b(1 - \beta) - r^r \right]}{(1 + r^e)^2} \cdot (1 - \bar{r}_1) \) |
| \( r^r \) | \( \frac{r^r(1 + t + s)}{N_i} \cdot \frac{(i + \bar{r}_1 r^e)}{(1 + re)} \) |
| \( r^d \) | \( \frac{r^d}{N_i} \cdot \frac{(i + \bar{r}_1 r^e)}{(1 + re)} \) |
| \( r_t \) | \( \frac{r^t \cdot t}{N_i} \cdot \frac{(i + \bar{r}_1 r^e)}{(1 + re)} \) |
| \( r^s \) | \( \frac{r^s \cdot s}{N_i} \cdot \frac{(i + \bar{r}_1 r^e)}{(1 + re)} \) |

- cont'd -
Table 3 continued

\begin{align*}
\text{unadj. base model} & \quad \tilde{\rho}_1 \left( \frac{1 + t + s}{N_1} \right) \left\{ \left( r^r + r^e \right) - \frac{r^i}{1 + r^e} \left[ 1 + b(1-b) \right] \right\} \\
\text{adj. base model} & \quad \tilde{\rho}_1 \left( \frac{1 + t + s}{N_2} \right) \left\{ \left( r^r + r^e - b \right) - \frac{[r^r - b(1-b)]i + r^e}{1 + r^e} \right\}
\end{align*}

legend for \( i \) as an index:

\begin{itemize}
\item \( i = 1 \): unadjusted base model
\item \( i = 2 \): adjusted base model
\end{itemize}
Table 4

Money multiplier elasticities and their reaction to a reduced ceiling on loans or total earning assets

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>adjusted or unadj. base model</td>
<td>sign of value of: $\epsilon(m_i^j, x)$ for $P_1 \leq 1$</td>
<td>$\frac{\partial</td>
</tr>
<tr>
<td>k</td>
<td>either</td>
<td>- negative if and only if $1 &gt; r_i^C (1 + t + s)$; for Germany: negative if $P_1$ is large enough, will be positive for small $P_1$ and ceilings on total earning assets ($i=0, \beta=1$), may be positive for small $P_1$ and loan ceilings.</td>
<td>? decrease as long as $\epsilon(.)$ is negative, increase otherwise</td>
</tr>
<tr>
<td>t</td>
<td>either</td>
<td>- negative if $r_t^r \geq r_r^r$ as for Germany; negative in the unadj. base model.</td>
<td>+ generally increase if negative and if $r_t^r \leq r_r^r$; for Germany increase even though $r_t^r &gt; r_r^r$</td>
</tr>
<tr>
<td>s</td>
<td>either</td>
<td>- negative if $r_s^r \geq r_r^r$; for Germany even though $r_s^r &lt; r_r^r$; negative in the unadjusted base model.</td>
<td>+ generally increase if negative and if $r_s^r \leq r_r^r$ as for Germany</td>
</tr>
<tr>
<td>i</td>
<td>either</td>
<td>+ positive; zero for $P_1 = 1$</td>
<td>+ increase</td>
</tr>
<tr>
<td>b</td>
<td>unadj. ($i=1$)</td>
<td>- negative for $P_1 &lt; 1$; zero for $P_1 = 1$</td>
<td>+ increase</td>
</tr>
<tr>
<td>b</td>
<td>adj. ($i=2$)</td>
<td>+ positive</td>
<td>- decrease</td>
</tr>
<tr>
<td>1-\beta</td>
<td>unadj.</td>
<td>- negative for $P_1 &lt; 1$; zero for $P_1 = 1$</td>
<td>+ increase</td>
</tr>
<tr>
<td>1-\beta</td>
<td>adj.</td>
<td>+ positive for $P_1 &lt; 1$; zero for $P_1 = 1$</td>
<td>+ increase</td>
</tr>
</tbody>
</table>

- cont'd -
<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>either</td>
<td>- negative</td>
<td>? indeterminate for loan ceilings; - decrease for ceilings on total earning assets ($i = 0, \beta = 1$)</td>
</tr>
<tr>
<td>$r^e$</td>
<td>either</td>
<td>- negative</td>
<td>- decrease</td>
</tr>
<tr>
<td>$r^d$</td>
<td>either</td>
<td>- negative</td>
<td>- decrease</td>
</tr>
<tr>
<td>$r^t$</td>
<td>either</td>
<td>- negative</td>
<td>- decrease</td>
</tr>
<tr>
<td>$r^s$</td>
<td>either</td>
<td>- negative</td>
<td>- decrease</td>
</tr>
<tr>
<td>$p_1$</td>
<td>unadj.</td>
<td>+ positive</td>
<td>- decrease</td>
</tr>
<tr>
<td>$p_1$</td>
<td>adj.</td>
<td>+ positive</td>
<td>- decrease</td>
</tr>
</tbody>
</table>

$i$ in column 1 stands for the investment ratio; in the remaining columns $i$ is an index.

The rows $i$, $b$(unadj.) and $(1 - \beta)$ (both adj. and unadj.) only hold for the special case of a loan ceiling. In the case of a ceiling on total earning assets, i.e., the case formally represented by $i = 0$ and $\beta = 1$, the respective elasticities are zero and do not respond to a change in $p_1$. 
Credit multiplier elasticities and their reaction to a reduced ceiling on loans or total earning assets.

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>adjusted or unadjusted base model</td>
<td>sign or value of: $\varepsilon(a_1^c, x)$ for $\rho \leq 1$</td>
<td>$\frac{\partial</td>
</tr>
<tr>
<td>k</td>
<td>either</td>
<td>- negative</td>
<td>- decrease</td>
</tr>
<tr>
<td>t</td>
<td>either</td>
<td>+ positive if $r^t \geq r^r$; true for Germany if $\rho_1 = 1$ even though $r^r &gt; r^t$</td>
<td>? indeterminate; decrease if $r^t = r^r$; decrease for Germany if ceiling on total earning assets ($i=0, \beta=1$)</td>
</tr>
<tr>
<td>s</td>
<td>either</td>
<td>+ positive if $r^s \leq r^r$ as for Germany</td>
<td>? indeterminate; decrease - if $r^s = r^r$; decrease for Germany if ceiling on total earning assets ($i=0, \beta=1$)</td>
</tr>
<tr>
<td>i</td>
<td>either</td>
<td>+ positive for $\rho &lt; 1$; zero for $\rho_1 = 1$</td>
<td>+ increase</td>
</tr>
<tr>
<td>b</td>
<td>unadj. ($i=1$)</td>
<td>+ very likely 1); true for $\rho_1 = 1$; true for $i = 0$, $\beta = 1$</td>
<td>? indeterminate for loan ceilings; no change for ceilings on total earning assets ($i=0, \beta=1$)</td>
</tr>
<tr>
<td>b</td>
<td>adj. ($i=2$)</td>
<td>+ positive</td>
<td>? indeterminate for loan ceilings; decrease for ceilings on total earning assets ($i=0, \beta=1$)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>unadj.</td>
<td>+ very likely 2); zero for $\rho_1 = 1$</td>
<td>+ increase if $\varepsilon(\cdot)$ is positive</td>
</tr>
</tbody>
</table>

1) positive (negative) if $i - a_1^c r^e$ is positive (sufficiently negative)
2) positive (negative) if and only if $(i - a_1^c r^e)$ is positive (negative)

---

cont'd
Table 5: continued

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 - β:</td>
<td>adj.</td>
<td>+ positive for $\rho_1 &lt; 1$; zero for $\rho_1 = 1$</td>
<td>+ increase</td>
</tr>
<tr>
<td>$r^e$</td>
<td>either</td>
<td>- negative</td>
<td></td>
</tr>
<tr>
<td>$r^r$</td>
<td>either</td>
<td>- negative</td>
<td>indeterminate</td>
</tr>
<tr>
<td>$r^d$</td>
<td>either</td>
<td>- negative</td>
<td>for loan ceilings; decrease for</td>
</tr>
<tr>
<td>$r^t$</td>
<td>either</td>
<td>- negative</td>
<td>ceilings on total earning assets (i=0, β=1)</td>
</tr>
<tr>
<td>$r^s$</td>
<td>either</td>
<td>- negative</td>
<td></td>
</tr>
<tr>
<td>$\rho_1$</td>
<td>unadj.</td>
<td>+ positive</td>
<td></td>
</tr>
<tr>
<td>$\rho_{\text{adj.}}$</td>
<td>adj.</td>
<td>+ positive</td>
<td></td>
</tr>
</tbody>
</table>

i in column 1 stands for the investment ratio, in the remaining columns i is an index.

The rows for i and (1-β) (both adj. and unadj.) only hold for the special case of a loan ceiling. In the case of a ceiling on total earning assets, i.e. the case formally represented by i = 0 and β = 1, the respective elasticities are zero and do not respond to a change in $\rho_1$. In this case $\varepsilon(a^c_1, b)$ is also irresponsive to a change in $\rho_1$. 