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Credit Ceilings, Asset Reserves and the Control of the Money Supply

by

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DISKUSSIONSBEITRÄGE

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Credit Ceilings, Asset Reserves and the Control of the Money Supply

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Diskussionsbeiträge des Fachbereichs Wirtschaftswissenschaften Nr. 53
In Germany the recent discussion on the reform of monetary policy has not yet come to an end, even though the chances for legislative innovations have greatly diminished. It has become obvious that the German government is still attempting to combine such a reform with restrictions of the autonomy of the German Bundesbank. I do not intend to give a survey of this discussion. Instead, I try to provide some further results concerning credit ceilings and asset reserves, both being among proposed instruments of monetary policy so far unavailable in Germany.

The analysis is built on the non-linear money-supply hypothesis developed by K. BRUNNER and A. H. MELTZER. In this hypothesis both the money and the bank credit supply are conceived to be the algebraic product of a multiplier and the monetary base. The great bulk of my paper is an outlay of the steps made in order to incorporate several variants of credit ceilings and, alternatively, of asset reserves into the BRUNNER-MELTZER model.

It is important to note that credit ceilings and asset reserves of the types discussed below do not relate to the monetary base (both adjusted and unadjusted); they only affect the multipliers. Therefore, assuming that the controllability of the monetary base is not altered by introducing or operating on either credit ceilings and asset reserves, prospective changes in the effectiveness of monetary policy may be judged on the basis of (prospective) changes in the controllability of the multipliers.
Now, the controllability of multipliers is uniquely related to the elasticities of the multipliers with respect to the various parameters forming these multipliers, and changes in the elasticities reflect changes in controllability. By an analysis of changes in elasticities, it will be possible to show that both credit ceilings and asset reserves weaken the potency of monetary policy in controlling the money supply, the result being slightly less unfavourable if bank credits are the target of monetary policy. A more detailed summary statement of results is given at the end of part A and B.

Part A deals with credit ceilings. In section I of that part I restate some elements of the BRUNNER-MELTZER model, which I need in section II, where various forms of ceilings (loan, investment and general earning asset ceilings) are cast in model form. In section III "constrained" multipliers are derived. Elasticities of multipliers and their reaction to ceilings changes are dealt with in section IV. Section V contains summary statements of the results for credit ceilings.

In Part B, section I four models of assets reserves are introduced. In section II of that part multipliers are developed, some properties of which are stated in section III. Elasticities of the multipliers and their reaction to asset reserve requirement changes are presented in section IV, while section V gives a summary statement of the results for asset reserves.
Part A

I.

I start out with a budget restraint, i.e. a consolidated balance sheet of the commercial banking sector as expressed by equation (1)

\[(1) \quad L + I + R = D + T + S + B_0.\]

Total assets are allocated between loans (\(L\)), investments (\(I\)) and reserves (\(R\)). Total liabilities consist of demand deposits (\(D\)), time deposits (\(T\)), savings deposits (\(S\)) and borrowings from the central bank (\(B_0\)). Dividing both sides of the balance sheet by the sum of all deposits, \((D + T + S)\), gives an equation in ratios:

\[(2) \quad 1 + i + r_1 = 1 + b\]

The loan ratio \((l)\), the investment ratio \((i)\), the total reserve ratio \((r_1)\) and the borrowing ratio \((b)\) describe the behaviour of the banking sector.

The ratios are behavioural parameters. They are functions and depend on an array of determinants, the complete statement of which is not necessary for my purposes. Yet, it will be illustrative to draw attention to some of the determinants and to give a more detailed description of the reserve ratio.

In equations (3), (4) and (5)

\[(3) \quad 1 = 1 (i_G, i_L, \rho_0 \ldots )\]
\[
(4) \quad i = i (i_G, i_L, \rho_o, \ldots)
\]
\[
(5) \quad b = b (i_G, i_L, \rho_o, \ldots)
\]

\(i_G\) designates the average yield on government securities, \(i_L\) denotes an index of loan rates, and \(\rho_o\) is the discount rate.

The total reserve ratio equals the sum of the required reserve ratio \(r^r\) and the excess reserve ratio \(r^e\). The excess reserve ratio is a true behavioural parameter of the banking sector as illustrated by equation (6):

\[
(6) \quad r^e = r^e (i_G, \rho_o, i_L, \ldots)
\]

The required reserve ratio is given by equation (7):

\[
(7) \quad r^r = \frac{1 \cdot r^d + t \cdot r^t + s \cdot r^s}{1 + t + s}
\]

\(r^d, r^t\) and \(r^s\) represent reserve requirements as determined by the central bank respectively for demand deposits, time deposits and savings deposits. \(t\) and \(s\) - not in their role as indeces - are symbols for time and savings deposits scaled by demand deposits, thus representing part of the allocative behaviour of the nonbank public as described by equation (8) and (9):

\[
(8) \quad t = t (i_T, i_S, i_G, i_L, \ldots)
\]
\[
(9) \quad s = s (i_S, i_T, i_G, i_L, \ldots)
\]
By virtue of (6), the total reserve ratio may be considered to be a behavioural function of the banking sector, too:

\[ r_1 = r^F + r^e = r_1 (r^d, r^t, r^s, i_G, i_L, i_T, i_S, \rho_O, \ldots) \]

There is no point in further detailing these behavioural functions, since in the following I shall usually omit all the arguments of the behavioural parameters. For the same reason it may be justified to introduce without specifying details the currency ratio as an additional behavioural parameter of the public, which describes the allocation of its money holdings between currency and demand deposits.

Some additional terminology will be required in what follows:

- \( K = L + I = \) earning assets
- \( e = l + i = \) earning asset ratio
- \( r_2 = r_1 - b = \) adjusted reserve ratio

II.

It is my aim to analyze a variety of ceilings, i.e. ceilings on the total of banks' earning assets as well as on single components of them. For reasons of symmetry it is sufficient to concentrate on loan ceilings and on ceilings for total earning assets. Since in the following the word loan may be replaced by the word investment without affecting any of the formal results, general statements and conclusions of our analysis, a separate analysis of investment ceilings is not necessary. As it turns out, even ceilings
on total earning assets do not require separate treatment. My technical handling of the loan ceiling case will be general enough to include ceilings on total earning assets as a special case of loan ceilings.

Formally, I shall represent a loan ceiling by the parameter $\rho_1$ restricting the desired loan ratio (1) to a constrained loan ratio ($l^c$) as shown by equation (11) and (12).

\begin{align}
(11) \quad l^c &= \rho_1 \cdot l \quad \text{(constrained loan behaviour)} \\
(12) \quad \rho_1 &= \bar{\rho}_1 \quad 0 \leq \rho_1 \leq 1 \quad \text{(autonomous restriction)}
\end{align}

Since the budget constraint (1) or (2) is assumed to be always effective, a restriction on loans implies restrictions on the other behavioural parameters of the banking sector. In a general fashion these "induced" restrictions are represented by equations (13), (14) and (15),

\begin{align}
(13) \quad \rho_1 l + r^x + \rho_2 (i + r^e) &= 1 + \rho_3 b \\
(14) \quad \rho_2 &= \rho_2 (\rho_1, \ldots) \quad \text{(induced restriction)} \\
(15) \quad \rho_3 &= \rho_3 (\rho_1, \ldots) \quad \text{(induced restriction)}
\end{align}

where $\rho_2$ and $\rho_3$ represent parameters constraining respectively $(i + r^e)$ and $b$. $\rho_2$ and $\rho_3$ are functions of the "autonomous" constraint $\rho_1$:
Equation (13) reduces to equation (2) in case $p_1 = 1$. That is to say:

\begin{equation}
\rho_2 = \rho_3 = 1 \text{ if and only if } \rho_1 = 1
\end{equation}

If $p_1 < 1$, then there are two different budget constraints holding simultaneously for the commercial banking sector: a "free" budget constraint as given by equation (2) and a "restricted" budget constraint as given by equation (13). The two constraints are noncontradictory for all values of $p_1$, and they are identical for $p_1 = 1$.

Before turning to the specification of functions (14) and (15), I shall complete the formal representation of constrained behaviour

\begin{equation}
b^c = \rho_3 b \quad \text{(constrained borrowing behaviour)}
\end{equation}

\begin{equation}
i^c = \rho_2 i \quad \text{(constrained investment behaviour)}
\end{equation}

\begin{equation}
r_{e,c} = \rho_2 r^e \quad \text{(constrained excess reserve behaviour)}
\end{equation}

As soon as (15) is specified, one may substitute for $\rho_3$ in (13). From that, (14) may be derived by way of isolating $\rho_2$. In order to specify (15), I assume that the borrowing ratio (5) may be described as a sum of two components:

\begin{equation}
b = b_1 + b_1 \quad \text{(total borrowing behaviour)}
\end{equation}

\begin{equation}
b_1 = b_1 (i_L, \rho_o, \ldots) \quad \text{(borrowing for loans behaviour)}
\end{equation}

\begin{equation}
b_1 = b_1 (i_G, \rho_o, \ldots) \quad \text{(borrowing for investments behaviour)}
\end{equation}
Thus, I associate each component of the earning asset ratio with a component of the borrowing ratio and vice versa. And each component of the borrowing ratio is considered to be a function and is assumed to be independent of the interest rates ruling for the "other" component of the earning asset ratio.

I further assume that loan restrictions affect the borrowing behaviour only by a parallel revision of its borrowing-for-loan component. Thus, the constrained borrowing ratio may be written as:

\[ b^c = \rho_1 b_1 + b_1 \]

from which by virtue of (17) it immediately follows that

\[ \rho_3 = \frac{1}{b} \left( \rho_1 b_1 + b_1 \right) \]

\[ \rho_3 = \rho_1 \beta + (1 - \beta) \]

\[ = 1 - (1 - \rho_1) \beta \]

where

\[ 0 \leq \beta \equiv \frac{b_1}{b} \leq 1 \]

\[ 0 \leq (1 - \beta) \equiv \frac{b_1}{b} \leq 1 \]

\[ \beta = \beta (i_L, i_G, \rho_o, \ldots) \]

As shown by (25), \( \rho_3 \) may be interpreted as a weighted average of the restrictions placed on components of the earning asset ratio.
The restriction on loans is captured by \( \rho_1 \), the (assumed lack of) restrictions on investments is represented by the coefficient of 
\((1 - \beta)\), which is equal to 1, and the weights are given by equations 
(26) - (28). Generally, \( \rho_3 \) is a function as characterized by equation 
(29), (30) and (31).

\[
(29) \quad \rho_3 = \rho_3 (\rho_1, i_L, i_G, \rho_o, \ldots).
\]

This function may be formulated more explicitly by means of equation 
(25) and (28). From (25) it also follows that

\[
(30) \quad \rho_3 < 1 \text{ if and only if } \rho_1 < 1
\]

provided \( \rho_1 > 0 \) and \( \beta > 0 \). In addition, if \( \beta > 0 \), it is obvious that

\[
(31) \quad \frac{\delta \rho_3}{\delta \rho_1} > 0
\]

In the "special" case of a general nondifferentiated earning asset 
ceiling the restrictions are of equal size for all components of 
the earning asset ratio. Therefore \( \rho_3 \) is a weighted average of 
identical restrictions:

\[
(32) \quad \rho_3 = \rho_1 \beta + \rho_1 (1 - \beta)
\]

and (29) degenerates to

\[
(32 \text{a}) \quad \rho_3 = \rho_1.
\]
Thus, as far as $p_3$ is concerned, the case of a general earning asset ceiling is also contained in (25). Formally, this general ceiling merely requires the insertion of identity (33) into (25):

\[(33) \quad \beta = 1.\]

(33) is a degenerate form of function (28).

Finally, the function $p_2$ may be derived from (13) and (25).

\[(34) \quad p_2 = \rho_1 + \frac{1}{1 + \rho_2} \left[ 1 + b (1 - \beta) - r \right] (1 - \rho_1)\]

(34) is a specification of (14).
The following assumption will be used throughout the discussion of loan ceilings:

\[(35) \quad (1 - b\beta) > 0 \quad \text{for} \ 0 < \beta < 1\]

This assumption merely claims that loans are higher than borrowings for loans. It is a correct assumption for Germany, and it is very likely to hold for many countries.

Now, it is easy to derive from (2), (20), (26), (27) and (35) that

\[(36) \quad \rho_2 > 1 \quad \text{if and only if} \quad \rho_1 < 1\]

regardless of the size of $\beta$. In addition, it follows that

\[(37) \quad \frac{\partial \rho_2}{\partial \rho_1} < 0\]

As far as $\rho_2$ is concerned, the case of a general earning asset ceiling is also contained in (34). Formally, this general ceiling merely requires the use of both (33) and (38) in order

\[(38) \quad i = 0\]

to substitute for $\beta$ and $i$ in (34).

In the case of a general earning asset ceiling, the analogue of assumption (35) is, of course:

\[(39) \quad (1 + i - b) > 0\]
(39) may formally be derived from (35) by adding (38) to (35) and by taking \( \beta \) as given in (33).

Under this rather weak assumption, both (36) and (37) hold for the case of general earning asset ceiling, too.

III.

By the definition of the monetary base from the users' side I have

\[ (40) \quad B = C^P + R^r + R^e \]

where \( B \) stands for monetary base, \( C^P \) is a symbol for currency (of the public) in circulation outside commercial banks, \( R^r \) designates required reserves and \( R^e \) denotes excess reserves.

The concept of the adjusted base will be used, likewise, and may be represented by

\[ (41) \quad B^a = B - B_o \]

where \( B^a \) designates the adjusted base, and \( B_o \) stands for borrowings of the commercial banking sector from the central bank.

To determine both the money supply \( (M) \) as defined by

\[ (42) \quad M = C^P + D \]

and the supply of credits or the demand for earning assets by banks, \( (L + I) \), the base is transformed by suitable multipliers incorporating
behavioural parameters of the commercial banking sector and the nonbank public. The money multipliers for the different base concepts are:

\[ m_1 = \frac{1 + k}{(r^r + r^e)(1 + t + s) + k} \quad \text{(unadj. base model)} \]

\[ m_2 = \frac{1 + k}{(r^r + r^e - b)(1 + t + s) + k} \quad \text{(adj. base model)} \]

The credit or earning asset multipliers are:

\[ a_1 = \frac{(1 - r^r - r^e + b)(1 + t + s)}{(r^r + r^e)(1 + t + s) + k} \quad \text{(unadj. base model)} \]

\[ a_2 = \frac{(1 - r^r - r^e + b)(1 + t + s)}{(r^r + r^e - b)(1 + t + s) + k} \quad \text{(adj. base model)} \]

A proper treatment of the effects of loan and credit ceilings requires the derivation of suitably constrained multipliers. This goal can easily be reached by replacing the unconstrained behavioural parameters of the multiplier expressions by their constrained (c) counterparts.

Introducing the following notation for constrained reserve ratios

\[ r^c = (r^r + r^e, c) = (r^r + \rho_2 r^e) \]

\[ r^c_2 = (r^r + r^e, c - b^c) = (r^r + \rho_2 r^e - \rho_3 b) \]

I can write the constrained multipliers as

\[ m^c_1 = \frac{1 + k}{r^c_1 (1 + t + s) + k} \quad \text{(unadj. base model)} \]

\[ a^c_1 = \frac{(1 - r^c_1)(1 + t + s)}{r^c_1 (1 + t + s) + k} \]
Exploiting the definitions of $\rho_2$ and $\rho_3$, as given by (25) and (34), I find

\begin{align}
(53) \quad r_1^c &= (r^r + r^e) \tilde{\rho}_1 + \frac{r^r_i + \left[1 + b(1 - \beta)\right] r^e}{(i + r^e)} (1 - \tilde{\rho}_1) \\
(54) \quad r_2^c &= (r^r + r^e - b) \tilde{\rho}_1 + \frac{r^r - b(1 - \beta) i + r^e}{(i + r^e)} (1 - \tilde{\rho}_1).
\end{align}

(53) and (54) are more explicit statements of (47) and (48).

They show that the constrained reserve ratio may be interpreted as a weighted average of two different reserve ratios, the weights being $\rho_1$ and $(1 - \rho_1)$. The first of the two ratios is the one prevailing if credit ceilings are absent. I shall call it the unconstrained reserve ratio. The other ratio which I shall call a forced reserve ratio is itself a weighted average of two different reserve ratios. The first of the two ratios forming the forced ratio is related to investments, the second one is related to excess reserves and the weights are given by the relative size of unconstrained investments and excess reserves.

The forced reserve ratio may further be described as follows. If loans are constrained, then the commercial banks are partly forced to allocate deposits and borrowings either to investments or to excess reserves. Each allocation absorbs or requires additional reserve holdings. In the unadjusted base model, forced allocation
to investments absorbs required reserves to an extent expressed by the ratio \( r_r^r \), while forced allocation to excess reserves absorbs borrowed and unborrowed reserves to an extent expressed as a ratio by the sum of (the ratios) \( b \cdot (1 - \beta) \) and 1.

Similarly, in the adjusted base model, forced allocation to investments requires unborrowed reserves to an extent indicated by the ratio \( r_r^r - b \cdot (1 - \beta) \), while forced allocation to excess reserves is absorbing unborrowed reserves to an extent indicated by the ratio 1.

The case of general earning asset ceilings does not require a separate treatment. Constrained reserve ratios for this case merely require the use of (33) and (38) in order to substitute for \( \beta \) and \( i \) in (53) and (54). The interpretation of constrained ratios in this case is analogue to the one given for loan ceilings.

Recalling (30) and (36), it is easy to derive from (47) and (48) that

\[
\begin{align*}
\begin{cases}
r_1^c > r_1 \\
r_2^c > r_2 = (r_1 - b)
\end{cases}
\end{align*}
\text{if and only if } \rho_1 < 1,
\]

regardless of the size of \( \beta \).

In addition, it may be shown that

\[
- \frac{\partial r_i^c}{\partial \rho_1} > 0 \quad i = 1, 2
\]

Since the "forced" reserve ratio is larger than the unconstrained reserve ratio, their average, the constrained reserve ratio, turns
out to be larger than the unconstrained reserve ratio. Therefore, the constrained reserve ratio really is an augmented reserve ratio. Lowering the loan or total earning asset ceiling increases the constrained reserve ratio.

From (55) it follows at once that:

\[
\begin{align*}
m_i^c & < m_i \\
\rho_i^c & < \rho_i \\
& \text{if and only if } \quad \rho_i^c < 1
\end{align*}
\]

regardless of the size of \( \beta \).

In addition, from (56)* we have

\[
\frac{\partial m_i^c}{\partial \rho_1} < 0 \quad \frac{\partial \rho_i^c}{\partial \rho_1} < 0
\]

Thus, both the money and the credit multiplier is reduced by lowering an effective loan ceiling or by lowering an effective total earning asset ceiling. This result is independent of the base version used.

Since both types of ceilings primarily affect the total reserve ratios, it is obvious that equally contractive effects can be reached by an appropriate change in reserve requirements. Consequently, a central bank in possession of reserve requirements as a policy instrument does not increase its aggregate effectiveness by introducing loan or general credit ceilings as an additional policy instrument.

Clearly, loan ceilings or ceilings on other components of earning assets in contrast to ceilings on total earning assets also have allocative effects on the asset supply side and consequently on the structure of interest rates. But these effects are not at stake in my present analysis concentrating on stabilization issues. Thus,
I find, that for the sole purpose of an effective stabilization policy, there is no point in a central bank lobbying for loan or credit ceilings as a policy instrument in addition to the already available reserve requirements instrument.

IV.

My next step is to present elasticities of the multipliers incorporating loan and earning asset ceilings. There are three tables to begin with. The first table defines the elasticities of the money multiplier and the asset multiplier both for the unadjusted \( i = 1 \) and the adjusted \( i = 2 \) base model \(^3\). Generally, these elasticities are differences of respective elasticities for the numerator \( Z \) and the denominator \( N \) of a multiplier.

For numerator and denominator elasticities are given in tables 2 and 3, where use is made of the following notation:

\[
Z = (1 - r^C_2)(1 + t + s) \\
N_i = r^C_i (1 + t + s) + k \\
 m^C_i = \frac{1 + k}{N_i} \\
 a^C_i = \frac{Z}{N_i}
\]

and the fact that:

\[
Z = (1 + t + s) + k - N_2.
\]
Now, there are two further tables. They contain the main results of the first part of this paper. Table 4 gives signs and values of the money multiplier elasticities and states their reaction to a change in credit ceilings. In table 5 the same is given for the credit multiplier elasticities.

The effects of credit ceilings on the elasticities depend on the parameter with respect to which an elasticity is taken. Yet, it is obvious that the effects may be classified according to whether the parameter is controlled by the central bank authorities, as is true for \( r^1, r^d, r^t, r^s \) and \( \rho_1 \), or whether it is controlled by the bank and non-bank public, as is true for \( k, t, s, i, b, 1 - \beta \) and \( r^e \).

Introducing or lowering existing ceilings (reducing \( \rho_1 \) below 1 or further below 1), increases the elasticities of the money multiplier with respect to parameters not under control of the authorities; however, this does not hold for \( k, b \) and \( r^e \). For \( k \) there may be an increase or a decrease, while for \( b \) (adj. base model) there is a decrease in the elasticity. The elasticity of the money multiplier with respect to \( r^e \) decreases if ceilings are placed on total earning assets, while it may increase or decrease if only loans are restricted by ceilings. The elasticities of the money multiplier with respect to those parameters that are under (direct) control of the authorities are reduced without exception, if credit ceilings are introduced or lowered.

The reactions of the credit multiplier differ from those of the money multiplier. It is only for ceilings on total earning assets as opposed to loan ceilings that I can give definite statements about the reaction of the credit multiplier elasticities. In that
case, the reduction of the elasticities is paramount with exceptions relating to parameter b (unadj.). Ceilings on total earning assets thus reduce the sensitivity of the credit multiplier with respect to parameters controlled by private banks and the nonbanks public, as well as with respect to parameters controlled by monetary authorities.

V.

The main results of part A of this paper dealing with two models of credit ceilings may be summarized by the following arguments against credit ceilings:

1. Credit ceilings are not necessary for an effective control of the aggregate supply of money and credit, in particular if the instrument of minimum required reserves is available.

2. Applying credit ceilings in addition to minimum required reserves reduces the effectiveness of control of the money supply by minimum required reserves of a given amount.

3. Applying credit ceilings (in addition to minimum required reserves) predominantly increases the effects on the money multiplier of given relative changes in the behaviour of the (private) nonbank public and thus weakens the potency of monetary policy.

4. For given relative changes in the behaviour of commercial banks this predominance of increases in elasticities does not necessarily prevail. Whether there exists a balance between increases and decreases or a dominance of increases over decreases depends on the precise type of credit ceiling model considered.
With respect to the goal of an effective control of the money supply by central bank authorities, these results clearly point towards a dominance of undesirable consequences of loan or general earning asset ceilings 4).

As far as ceilings on total earning assets are concerned and not just loan ceilings, the evaluation slightly changes in favor of ceilings, if the policy goal consists of an effective control of the credit supply instead of the money supply. Thus, to some degree the final evaluation of ceilings on total earning assets depends on the attachment to one of two alternative targets of monetary policy: money stock or volume of bank credits.
Part B

I.

Part B deals with requirements of asset reserves, in addition to requirements of liability reserves.

I shall distinguish five broad categories of assets:

a) Loans (L)

b) investments (I)

c) total earning assets \( K = L + I \)

d) "unborrowed" total earning assets \( K - B_0 \)

e) "borrowed" total earning assets \( B_0 \)

For reasons of symmetry, I may again neglect either of the first two categories. I choose to avoid the longer word "investments", but (again) it is understood that in the following the word "loan" may be replaced by the word "investment" without affecting the formal results, general statements and conclusions of my analysis of loan reserves.

I shall distinguish four types or models of asset reserves, according to the asset category in proportion to which asset reserves are required. Formally, the chosen types are characterized by the following equations:

\[
\begin{align*}
(1) & \quad R_{a1}^A = r_A L \\
(2) & \quad R_{a2}^A = r_A K \\
(3) & \quad R_{a3}^A = r_A (K - B_0) \\
(4) & \quad R_{a4}^A = r_A B_0
\end{align*}
\]

(model 1 = m 1)  
(model 2 = m 2)  
(model 3 = m 3)  
(model 4 = m 4)
In order to derive multipliers, it is appropriate to restate the users' side of the monetary base \( B \) in each of the four cases:

\[
B_j = B^1 + R^a_j \quad \text{(model } j, \ j = 1, \ldots, 4) \tag{5}
\]

\( B^1 \) is equal to the sum of currency and all reserves except asset reserves, i.e.

\[
B^1 = k \cdot D + (r^r + r^e) (1 + t + s) D, \tag{6}
\]

and \( R^a_j \) equals asset reserves of type \( j \).

By a similar restatement of the consolidated balance sheet of the banking sector, I now have

\[
L + I + R^r + R^e + R^a_j = (D + T + S) + B_j \quad j = 1, 2, \ldots, 4 \tag{7}
\]

Alternative rearrangements of the balance sheet equation

\[
L + R^a = (D + T + S) + (B_O - I - R^r - R^e) = (1 + r_A) L \quad \text{(m 1)} \tag{8}
\]

\[
K + R^a_2 = (D + T + S) + (B_O - R^r - R^e) = (1 + r_A) K \quad \text{(m 2)} \tag{9}
\]

\[
(K - B_O) + R^a_3 = (D + T + S) - (R^r + R^e) = (1 + r_A) (K - B_O) \quad \text{(m 3)} \tag{10}
\]

\[
B_O + R^a_4 = (1 + r_A) B_O \quad \text{(m 4)} \tag{11}
\]

and appropriate scaling procedures give expressions for \( R^a_j \).
These expressions may be used to fully specify $B_j$ in terms of demand deposits ($D$) and behavioural parameters. The details of this specification need not be stated here explicitly.

II.

My next step is to redefine the total reserve ratio for both the adj. and the unadj. base version of each of the four models. Scaling all reserves to total deposits, $(1 + t + s) D$, the total reserve ratio may be represented by

$$r^a = r_1 + r^a$$

where

$$r_1 = r^r + r^e$$

Here asset reserves are captured by the ratio $r^a$ which should be sharply distinguished from the ratio $r_A$, the latter being associated with different scales. Index 1 still refers to the unadjusted base version, while (below) index 2 is again chosen to indicate the adjusted base version:

$$r^a_2 = r^a_1 - b.$$
In table 6 reserve ratios are defined in columns 1 to 4, each column relating to an equally numbered model.

Using these definitions for $r^a_i$ ($i = 1, 2$), numerators ($Z$), denominators ($N_i$), and multipliers may be represented in a general fashion covering all four models by:

$$Z = (1 - r^a_2) (1 + t + s)$$

$$N_i = r^a_i (1 + t + s) + k$$

$$m^a_i = \frac{1 + k}{N_i}$$

$$a^a_i = \frac{Z}{N_i}$$

where again

$$Z = (1 + t + s) + k - N_2$$

**III.**

From table 6 it may be seen that

$$r^a_i = r^a_i(r_A), \quad \frac{\partial r^a_i}{\partial r_A} > 0. \quad i = 1, 2$$

Therefore, both the money and the credit multiplier is reduced by introducing or increasing an effective rate of asset reserves ($r_A$). This result is of course independent on the base version used. Thus, the restrictive effect of asset reserves on the supply of money and
credit is obvious and may be stated as follows:

\[ m_i^a < m_i \]

(22) \[ i = 1, 2 \quad \text{if and only if} \quad r_A > 0 \]

\[ a_i^a < a_i \]

\[ \frac{\partial m_i^a}{\partial r_A} < 0 \quad \frac{\partial a_i^a}{\partial r_A} < 0 \quad i = 1, 2 \]

\[ m_i \text{ and } a_i \text{ have been defined in (43) - (46) of part A.} \]

Since all four types of asset reserves primarily affect the total reserve ratio, the same contractive effects can be reached by an appropriate change in existing liability reserve requirements instead of introducing or increasing existing asset reserve requirements.

Table 1 and 2 are also applicable in the case of asset reserves, if, in the \( x \)-set, \( \rho_i \) is replaced by \( r_A \), while \( (1 - \beta) \) is eliminated from that set. In addition, elasticities of the multipliers may again be derived from elasticities of the denominators \( (N_i) \). These latter elasticities are given in table 7.

IV.

The main results of part B are contained in tables 8 and 9. Table 8 gives signs and values of the money multiplier elasticities and states their reaction to a change in asset reserve requirements.

In table 9 the same is given for the credit multiplier elasticities. The effects of asset reserves on the elasticities depend on the parameter with respect to which the elasticity is taken. Again, it
is meaningful to distinguish between two classes of parameters. The one class contains parameters under control of the authorities, such as $r^R, r^d, r^t, r^s$ and $r_A$. The other class contains parameters not under control of the authorities, but under control of the bank and nonbank public, such as $k, t, s, i, b$ and $r^e$.

In each of the four asset reserve models and, with a few exceptions, introducing or increasing asset reserve requirements increases the elasticities of the money multiplier with respect to parameters not controlled by the authorities. The exceptions relate to $k, b$ and $i$. While a decrease is possible for $k$, the elasticities with respect to $i$ simply cannot change in three of the four models. A less weak exception is $b$ in the adjusted base version with a decrease of the elasticity, while the elasticity with respect to $b$ in the unadjusted base version of model 3, again, simply cannot change.

With one exception the elasticities of the money multiplier with respect to parameters controlled by the authorities are reduced in each of the four models, if asset reserve requirements are introduced or increased. The exception is: for $r_A$ this does not happen before $r_A$ has become large.

In general, it is not possible to indicate unequivocally the response of credit multiplier elasticities to an increase in asset reserve requirements. But it may be seen from table 9, that the elasticities with respect to $k$ definitely decreases while the elasticities with respect to $t$ and $s$ definitely decrease, only if the ratios of liability reserves required for time and savings deposits do not differ. The elasticity with respect to the investment ratio $i$ increases in model 1.
(asset reserves on loans), while this elasticity cannot change in all other models. The elasticity with respect to \(b\) in the unadjusted base version of model 3 cannot change either. The elasticity of the credit multiplier with respect to asset reserve requirements will also increase, if these requirements are sufficiently small. The reaction is undetermined for large \(r_A\) except for model 4, where the reaction definitely remains positive.

Thus, I find that in those cases where definite statements about reactions of credit multiplier elasticities can be made, except with respect to \(i\) and \(b\) unadj. (model 3), an increase in asset reserve requirements reduces the sensitivity of the credit multiplier with respect to parameters controlled by the bank and nonbank public, while it increases the sensitivity of the credit multiplier with respect to the asset reserve parameter controlled by the authorities.

V.

The main results of part B of this paper dealing with four models of asset reserves may be summarized by the following arguments against asset reserve requirements:

1. Asset reserves are not necessary for an effective control of the aggregate supply of money and credit, particularly if the instrument of liability reserves is available.

2. Applying asset reserves in addition to liability reserves reduces the effectiveness of control of the money supply by liability reserves of a given amount.
3. Applying asset reserves in addition to liability reserves predominantly increases the effects on the money multiplier of given relative changes in the behaviour of the (private) nonbank public and thus weakens the potency of monetary policy.

4. For given relative changes in the behaviour of commercial banks this predominance of increases in elasticities does not necessarily prevail. Whether there exists a balance between increases and decreases or a dominance of increases over decreases depends on the precise type of asset reserve model considered.

With respect to the goal of an effective control of the money supply by central bank authorities, these results clearly point towards a dominance of undesirable consequences of asset reserve requirements.

If the policy goal consists of an effective control of the credit supply instead of the money supply, this evaluation of asset reserves slightly changes. Thus, as with credit ceilings, to some degree the final evaluation of asset reserves depends on the attachment to one of two alternative targets of monetary policy: money stock or volume of credits.

Conclusion

Obviously, after an appropriate exchange of names for the policy instruments under discussion, the summary statements given at the end of part B appear as mere repetition of the statements given at the end of part A. Thus, asset reserves and credit ceilings are both found to be not only unnecessary, but even undesirable for the purpose of an effective control of the money supply.
Footnotes


2) More precisely I associate each component of the earning asset ratio with a separate component of the reserve holdings and each component of the latter with a separate component of the borrowing ratio. Thus, one might distinguish between an unborrowed reserve ratio for loans and another one for investments.

3) It is clear that the reaction both of the supply of money and credit to a credit ceiling is independent of the model version (adj. or unadj.) chosen for analysis. Thus, if we find multiplier elasticities to be unequal between the two model versions

\[ -\varepsilon(m^C_1, \rho_1) > -\varepsilon(m^C_2, \rho_1) \]
\[ -\varepsilon(a^C_1, \rho_1) > -\varepsilon(a^C_2, \rho_1) \]

we should not be surprised, as there exists a compensating inequality of base elasticities:

\[ \varepsilon(B^C, \rho_1) = 0 \]
\[ -\varepsilon(B^C, \rho_1) < 0 \]

4) Rough calculations show that this judgement needs not to be changed if differences in the variability of parameters are taken into account and if changes of elasticities are weighted accordingly.

5) See footnote 4.
### Table 1

**Elasticities of the money multiplier** \( \epsilon (m_i, x) \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( \epsilon (m_i, x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k )</td>
<td>[ \frac{k}{1+k} - \epsilon(N_i, k) = \frac{k}{1+k} - \frac{k}{N_i} ]</td>
</tr>
<tr>
<td>all ( x ) (except ( k ))</td>
<td>[ - \epsilon(N_i, x) ]</td>
</tr>
</tbody>
</table>

**Elasticities of the credit multiplier**

<table>
<thead>
<tr>
<th>( x )</th>
<th>( \epsilon (a_i, x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k )</td>
<td>[ - \epsilon(N_i, k) ]</td>
</tr>
<tr>
<td>( t, s, k )</td>
<td>[ \frac{x}{\bar{Z}} - \frac{N_2}{\bar{Z}} \cdot \epsilon(N_2, x) - \epsilon(N_i, x) ]</td>
</tr>
<tr>
<td>all ( x ) (except ( t, s, k ))</td>
<td>[ - \frac{N_2}{\bar{Z}} \cdot \epsilon(N_2, x) - \epsilon(N_i, x) ]</td>
</tr>
</tbody>
</table>

1. **Legend for**
   - \( i \) as an index:
     - \( i = 1 \): unadjusted base model
     - \( i = 2 \): adjusted base model

2. \( x \in \{ k, t, s, i, b, (1-\beta), r^e, r^r, r^d, r^t, r^s, \rho \} \)
Table 2

Elasticities of the numerator of the credit multiplier

<table>
<thead>
<tr>
<th>$x$</th>
<th>$\varepsilon(Z,x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k$</td>
<td>$\frac{k}{Z} - \frac{N_2}{Z} \varepsilon(N_2, k) = 0$</td>
</tr>
<tr>
<td>$t, s$</td>
<td>$\frac{x}{Z} - \frac{N_2}{Z} \varepsilon(N_2, x)$</td>
</tr>
<tr>
<td>all $x$ except $k, t, s$</td>
<td>$-\frac{N_2}{Z} \varepsilon(N_2, x)$</td>
</tr>
</tbody>
</table>

$x \in \{k, t, s, i, b, (1-\beta), r^e, r^r, r^d, r^t, r^s, \rho_1\}$
### Table 3: Credit ceilings

Elasticities of the denominator of both the money and the credit multiplier

\[ x \in \{ N_i, x \} \]

<table>
<thead>
<tr>
<th>( k )</th>
<th>[ \frac{k}{N_i} ]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t )</td>
<td>[ \frac{t (r^t - r^r)}{N_i} \cdot \frac{(i + \rho_t r^e)}{(i + r^e)} + \frac{t (N_i - k)}{(1 + t + s)N_i} ]</td>
</tr>
<tr>
<td>( s )</td>
<td>[ \frac{s(r^s - r^r)}{N_i} \cdot \frac{(i + \rho_s r^e)}{(i + r^e)} + \frac{s(N_i - k)}{(1 + t + s)N_i} ]</td>
</tr>
<tr>
<td>( i )</td>
<td>[ -i \frac{N_i}{(1 + t + s) \cdot \frac{r^e}{(i + r^e)} \cdot (1 - \rho_i)} ]</td>
</tr>
</tbody>
</table>

\([\text{unadj. base model}]

\[ b \cdot \frac{(1 + t + s)}{N_1} \cdot \frac{r^e (1 - \rho)}{(i + r^e)} \cdot (1 - \rho) \]

\([\text{adj. base model}]

\[ -b \cdot \frac{(1 + t + s)}{N_2} \cdot \left[ \rho_t + (1 - \rho) \cdot \frac{i (1 - \rho)}{i + r^e} \right] \]

\([1 - \beta]\)

\([\text{unadj. base model}]

\[ (1 - \beta) \cdot \frac{(1 + t + s)}{N_1} \cdot \frac{r^e b}{(i + r^e)} \cdot (1 - \rho) \]

\([\text{adj. base model}]

\[ -(1 - \beta) \cdot \frac{(1 + t + s)}{N_2} \cdot \frac{ib}{(i + r^e)} \cdot (1 - \rho) \]

\(r^e\)

\[ \frac{r^e (1 + t + s)}{N_i} \cdot \left\{ \rho_t + i \cdot \frac{[1 + b(1 - \beta) - r^r]}{(i + r^e) \cdot (1 - \rho)} \right\} \]

\(r^r\)

\[ \frac{r^r (1 + t + s)}{N_i} \cdot \frac{(i + \rho_t r^e)}{(i + r^e)} \]

\(r^d\)

\[ \frac{r^d}{N_i} \cdot \frac{(i + \rho_t r^e)}{(i + r^e)} \]

\(r^t\)

\[ \frac{r^t}{N_i} \cdot \frac{(i + \rho_t r^e)}{(i + r^e)} \]

\(r^s\)

\[ \frac{r^s}{N_i} \cdot \frac{(i + \rho_t r^e)}{(i + r^e)} \]

- cont'd -
Table 3 continued

<table>
<thead>
<tr>
<th></th>
<th>( \hat{\varphi}_1 ) ( \frac{(1 + t + s)}{N_1} ) ( \left{ \left( r^r + r^e \right) - \frac{r^i + [1 + b(1 - \beta)]r^e}{(1 + re)} \right} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>adj. base model</td>
<td>( \hat{\varphi}_1 ) ( \frac{(1 + t + s)}{N_2} ) ( \left{ \left( r^r + r^e - b \right) - \frac{[r^r - b(1 - \beta)]i + r^e}{(1 + re)} \right} )</td>
</tr>
</tbody>
</table>

Legend for \( i \) as an index:

\( i = 1 \): unadjusted base model

\( i = 2 \): adjusted base model
### Table 4

Money multiplier elasticities and their reaction to a reduced ceiling on loans or total earning assets

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x )</td>
<td>adjusted or unadj. base model</td>
<td>sign or value of: ( \epsilon(m^{C}, x) ) for ( \rho_{1} \leq 1 )</td>
<td>( - \frac{\partial \epsilon}{\partial \rho_{1}} \left</td>
</tr>
<tr>
<td>( k )</td>
<td>either</td>
<td>? for Germany: negative if ( \rho_{1} ) is large enough, will be positive for small ( \rho_{1} ) and ceilings on total earning assets (( i=0, \beta=1 )), may be positive for small ( \rho_{1} ) and loan ceilings.</td>
<td>? decrease as long as ( \epsilon(\cdot) ) is negative, increase otherwise</td>
</tr>
<tr>
<td>( t )</td>
<td>either</td>
<td>- generally if ( r_{t} \leq r_{F}^{F} ) as for Germany</td>
<td>+ generally increase if negative and if ( r_{t} \leq r_{F}^{F} ); for Germany increase even though ( r_{t} &gt; r_{F}^{F} )</td>
</tr>
<tr>
<td>( s )</td>
<td>either</td>
<td>- generally if ( r_{S} \leq r_{F}^{F} ); for Germany even though ( r_{S} \leq r_{F}^{F} )</td>
<td>+ generally increase if negative and if ( r_{S} \leq r_{F}^{F} ) as for Germany</td>
</tr>
<tr>
<td>( i )</td>
<td>either</td>
<td>+ generally; zero for ( \rho_{1} = 1 )</td>
<td>+ increase (generally)</td>
</tr>
<tr>
<td>( b )</td>
<td>unadj. ( (i=1) )</td>
<td>- generally; zero for ( \rho_{1} = 1 )</td>
<td>+ increase (generally)</td>
</tr>
<tr>
<td>( b )</td>
<td>adj. ( (i=2) )</td>
<td>+ generally</td>
<td>- decrease (generally)</td>
</tr>
<tr>
<td>( 1-\beta )</td>
<td>unadj.</td>
<td>- generally; zero for ( \rho_{1} = 1 )</td>
<td>+ increase (generally)</td>
</tr>
<tr>
<td>( 1-\beta )</td>
<td>adj.</td>
<td>+ generally; zero for ( \rho_{1} = 1 )</td>
<td>+ increase (generally)</td>
</tr>
</tbody>
</table>

- cont'd -
<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r^e$</td>
<td>either</td>
<td>- generally</td>
<td>? indeterminate for loan ceilings; - decrease for ceilings on total learning assets (i=0, B=1)</td>
<td></td>
</tr>
<tr>
<td>$r^r$</td>
<td>either</td>
<td>- generally</td>
<td>- decrease (generally)</td>
<td></td>
</tr>
<tr>
<td>$r^d$</td>
<td>either</td>
<td>- generally</td>
<td>- decrease (generally)</td>
<td></td>
</tr>
<tr>
<td>$r^t$</td>
<td>either</td>
<td>- generally</td>
<td>- decrease (generally)</td>
<td></td>
</tr>
<tr>
<td>$r^s$</td>
<td>either</td>
<td>- generally</td>
<td>- decrease (generally)</td>
<td></td>
</tr>
<tr>
<td>$\rho_1$</td>
<td>unadj.</td>
<td>+ generally</td>
<td>- decrease (generally)</td>
<td></td>
</tr>
<tr>
<td>$\rho_2$</td>
<td>adj.</td>
<td>+ generally</td>
<td>- decrease (generally)</td>
<td></td>
</tr>
</tbody>
</table>

i in column 1 stands for the investment ratio; in the remaining columns i is an index.

The rows i, b(unadj.) and (1-B) (both adj. and unadj.) only hold for the special case of a loan ceiling. In the case of a ceiling on total earning assets, i.e. the case formally represented by i = 0 and B = 1, the respective elasticities are zero and do not respond to a change in $\rho_1$. 
Credit multiplier elasticities and their reaction to a reduced ceiling on loans or total earning assets.

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x)</td>
<td>adjusted or unadj. base model</td>
<td>sign or value of: (\varepsilon(a_1^c, x)) for (p_1^t = 1)</td>
<td>(- \frac{2</td>
</tr>
<tr>
<td>(k)</td>
<td>either</td>
<td>- generally</td>
<td>- decrease generally</td>
</tr>
<tr>
<td>(t)</td>
<td>either</td>
<td>+ generally if (r^t \leq r^r); true for Germany if (p_1 = 1) even though (r^t &gt; r^r)</td>
<td>? indeterminate; decrease if (- rt = r^r); decrease for Germany if ceiling on total earning assets ((i = 0, B = 1))</td>
</tr>
<tr>
<td>(s)</td>
<td>either</td>
<td>+ generally if (r^s \leq r^r) as for Germany</td>
<td>? indeterminate; decrease if (- rs = r^r); decrease for Germany if ceiling on total earning assets ((i = 0, B = 1))</td>
</tr>
<tr>
<td>(i)</td>
<td>either</td>
<td>+ generally; zero for (p_1 = 1)</td>
<td>+ increase (generally)</td>
</tr>
<tr>
<td>(b)</td>
<td>unadj. ((i = 1))</td>
<td>+ very likely (^1); true for (p_1 = 1); true for (i = 0, B = 1)</td>
<td>? indeterminate for loan ceilings; no change for ceilings on total earning assets ((i = 0, B = 1))</td>
</tr>
<tr>
<td>(b)</td>
<td>adj. ((i = 2))</td>
<td>+ generally</td>
<td>? indeterminate for loan ceilings; decrease for ceilings on total earning assets ((i = 0, B = 1))</td>
</tr>
<tr>
<td>(1 - \beta)</td>
<td>unadj.</td>
<td>+ very likely (^2); zero for (p_1 = 1)</td>
<td>+ increase if (\varepsilon()) is positive</td>
</tr>
</tbody>
</table>

1) positive (negative) if \(i - a_1^c r^e\) is positive (sufficiently negative) 
2) positive (negative) if and only if \((i - a_1^c r^e)\) is positive (negative)

- cont'd -
### Table 5 continued

<table>
<thead>
<tr>
<th></th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 - β</td>
<td>adj.</td>
<td>+ generally; zero for $\rho_1 = 1$</td>
<td>+ increase (generally)</td>
</tr>
<tr>
<td>$r^e$</td>
<td>either</td>
<td>- generally</td>
<td></td>
</tr>
<tr>
<td>$r^r$</td>
<td>either</td>
<td>- generally</td>
<td></td>
</tr>
<tr>
<td>$r^d$</td>
<td>either</td>
<td>- generally</td>
<td></td>
</tr>
<tr>
<td>$r_t$</td>
<td>either</td>
<td>- generally</td>
<td></td>
</tr>
<tr>
<td>$r_s$</td>
<td>either</td>
<td>- generally</td>
<td></td>
</tr>
<tr>
<td>$\rho_1$</td>
<td>unadj.</td>
<td>+ generally</td>
<td></td>
</tr>
<tr>
<td>$\rho_1$</td>
<td>adj.</td>
<td>+ generally</td>
<td></td>
</tr>
</tbody>
</table>

$i$ in column 1 stands for the investment ratio, in the remaining columns $i$ is an index.

The rows for $i$ and $(1-\beta)$ (both adj. and unadj.) only hold for the special case of a loan ceiling. In the case of a ceiling on total earning assets, i.e. the case formally represented by $i = 0$ and $\beta = 1$, the respective elasticities are zero and do not respond to a change in $\rho_1$. In this case $\epsilon (a^0_1, b)$ is also irresponsible to a change in $\rho_1$. 

indeterminate for loan ceilings; decrease for ceilings on total earning assets ($i=0, \beta=1$)
## Table 6

### Asset reserves

Definition of reserve ratios scaled to total deposits

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>$(1+b)-(r^r+r^e+i)$</td>
<td>$(1+b)-(r^r+r^e)$</td>
<td>$(1 - r^r - r^e)$</td>
<td></td>
</tr>
<tr>
<td>$r^a$</td>
<td></td>
<td>$\mu \frac{r_A}{1 + r_A}$</td>
<td></td>
<td>$r_A - b$</td>
</tr>
<tr>
<td>$r_1^a$</td>
<td>un-adj. base model</td>
<td>$r^r + r^e + \mu \frac{r_A}{1 + r_A}$</td>
<td></td>
<td>$r^r + r^e + r_A - b$</td>
</tr>
<tr>
<td>$r_2^a$</td>
<td>adj. base model</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 7

Asset reserves
Elasticities of the denominator of both the money and the credit multiplier

\( e (N_i, x) \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k )</td>
<td>( \frac{k}{N_i} )</td>
<td>( \frac{t(r^t - r^r)}{N_i (1 + r_A)} ) + ( \frac{t(N_i - k)}{(1 + t + s)N_i} )</td>
<td>( \frac{t(r^t - r^r)}{N_i} ) + ( \frac{t(N_i - k)}{(1 + t + s)N_i} )</td>
<td></td>
</tr>
<tr>
<td>( t )</td>
<td>( \frac{s(r^s - r^r)}{N_i (1 + r_A)} ) + ( \frac{s(N_i - k)}{(1 + t + s)N_i} )</td>
<td>( \frac{s(r^s - r^r)}{N_i} ) + ( \frac{s(N_i - k)}{(1 + t + s)N_i} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( b )</td>
<td>( \frac{b(1 + t + s) r_A}{N_i (1 + r_A)} )</td>
<td>0</td>
<td>( \frac{b(1 + t + s) r_A}{N_i} )</td>
<td></td>
</tr>
<tr>
<td>( \text{unadj. base model} )</td>
<td>( \frac{-b(1 + t + s)}{N_2 (1 + r_A)} )</td>
<td>( \frac{-b(1 + t + s)}{N_2} )</td>
<td>( \frac{-b(1 + t + s) (1 - r_A)}{N_2} )</td>
<td></td>
</tr>
<tr>
<td>( i )</td>
<td>( \frac{-i(1 + t + s) r_A}{N_i (1 + r_A)} )</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>( r^e )</td>
<td>( \frac{r^e (1 + t + s)}{N_i (1 + r_A)} )</td>
<td>( \frac{r^e (1 + t + s)}{N_i} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( r^r )</td>
<td>( \frac{r^r (1 + t + s)}{N_i (1 + r_A)} )</td>
<td>( \frac{r^r (1 + t + s)}{N_i} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( r^d )</td>
<td>( \frac{r^d}{N_i (1 + r_A)} )</td>
<td>( \frac{r^d}{N_i} )</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

- cont'd -
Table 7 continued

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_t$</td>
<td>$\frac{r_t t}{N_i (1 + r_A)}$</td>
<td></td>
<td>$\frac{r_t t}{N_i}$</td>
<td></td>
</tr>
<tr>
<td>$r^s$</td>
<td>$\frac{r^s s}{N_i (1 + r_A)}$</td>
<td></td>
<td>$\frac{r^s s}{N_i}$</td>
<td></td>
</tr>
<tr>
<td>$r_A$</td>
<td>$\frac{^\mu (1 + t + s) r_A}{N_i (1 + r_A)^2}$</td>
<td></td>
<td>$\frac{b (1 + t + s) r_A}{N_i}$</td>
<td></td>
</tr>
</tbody>
</table>

$\mu$ is defined in table 6

i in column 1 is the investment ratio, in all other columns i is an index

i = 1: unadj. base model
i = 2: adj. base model
Table 8

Money multiplier elasticities and their reaction
to an increase in required asset reserves

| x  | adj. or unadj. base model | sign or value of \( e(m^a_i,x) \) for \( r_A > 0 \) | \( \frac{\delta |e(m^a_i,x)|}{\delta r_A} \) = change of due to an increase of \( r_A \) |
|-----|--------------------------|---------------------------------|----------------------------------------------------------------------|
|     |                          | \( m_1 \) \( m_2 \) \( m_3 \) \( m_4 \) | \( m_1 \) \( m_2 \) \( m_3 \) \( m_4 \) |
|      |                         | for Germany (without \( m^1 \)) | decrease as long as \( e( \) is negative, increase otherwise |
|      |                         | generally if \( r^t > r^r \)    | generally increase if negative and if \( r^t \leq r^r \); for Germany increase even though \( r^t > r^r \) |
| \( t \) | either                  | - - - -                         | + + + +                                                                   |
|      |                         | generally if \( r^t > r^r \)    | generally increase if negative and if \( r^t \leq r^r \); for Germany increase even though \( r^t > r^r \) |
| \( s \) | either                  | - - - -                         | + + + +                                                                   |
|      |                         | generally if \( r^t > r^r \), for Germany even though \( r^s < r^r \) | generally increase if negative and if \( r^s \leq r^r \) as for Germany |
| \( i \) | either \( r_A > 0 \)    | + + + +                         | + no change                                                               |
|      | \( r_A = 0 \)           | 0 0 0 0                         | + no change                                                               |
| \( b \) | unadj. \( r_A > 0 \)    | - - 0 -                         | + + no change +                                                           |
|      | \( r_A = 0 \)           | 0 0 0 0                         | + + no change +                                                           |
| \( b \) | adj.                    | + + + +                         | - - - -                                                                   |
| \( r_A \) | either \( r_A > 0 \)    | - - - -                         | - - - -                                                                   |
|      | \( r_A = 0 \)           | 0 0 0 0                         | (±) (±) (±) +                                                            |

- cont’d -
legend:

1. +  positive if sign (or value)  
   increase if change (or reaction)

2. -  negative if sign (or value)  
   decrease if change (or reaction)

3. (+) positive or increase for small $r_A$  
   negative or decrease for large $r_A$

4. (−) negative or decrease for small $r_A$  
   positive or increase for large $r_A$
Table 9
Credit multiplier elasticities and their reaction to an increase in required asset reserves

| x   | adj.or unadj.base model | sign or value of $\varepsilon(a_1^a, x)$ for $r_A > 0$ | $\frac{\delta |\varepsilon(a_1^a, x)|}{\delta r_A}$ due to an increase of $r_A$ |
|-----|-------------------------|---------------------------------|--------------------------------------------------|
|     |                         | m1    | m2    | m3    | m4    | m1    | m2    | m3    | m4    | m1    | m2    | m3    | m4    |
| k   | either                  | -     | -     | -     | -     | -     | -     | -     | -     | -     | -     | -     | -     |
|     |                         | generally if $r_t \leq r_r$; true for Germany for $r_A = 0$ even though $r_t > r_s$ | generally indeterminate; decrease if $r_t = r_s$ |
|     |                         | generally if $r_s \leq r_r$ as for Germany | generally indeterminate; decrease if $r_t = r_s$ |
| i   | either $r_A > 0$        | +     | 0     | 0     | 0     | +     | no change |
|     | $r_A = 0$               | 0     | 0     | 0     | 0     | +     | no change |
| b   | unadj.                  | (?^)  | (?^)  | 0     | (?^)  | ?     | ?     | ?     | no change |
| b   | adj.                    | +     | +     | +     | +     | ?     | ?     | ?     | ?     |
| r_A | either $r_A > 0$        | -     | -     | -     | -     | (?^)  | (?^)  | (?^)  | (+)   |
|     | $r_A = 0$               | 0     | 0     | 0     | 0     | (?^)  | (?^)  | (?^)  | (+)   |

Legend:
1. + : positive if sign (or value), increase if change (or reaction)
2. - : negative if sign, decrease if change
3. ? : indeterminate
4. (?^) : positive or increase for small $r_A$
   indeterminate for large $r_A$