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320
SOME THEORETICAL ISSUES IN KEYNESIAN

STABILIZATION POLICY,

100⁸
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Contents

1. Introduction
2. An Outline of the Keynesian Paradigm: IS-LM and p-Y Analysis
3. Extension to a Two-Sector Model: Relative Prices and Related Issues
4. Problems Associated with the Government Budget Restraint
5. The Liquidity Preference Relation and the Effectiveness of Monetary Policy
6. Supplement: The Role of Money Supply Analysis in Keynesian Economics
7. List of Symbols

1. Introduction

The standard Keynesian position is expressed by an array of income-expenditure models which all share some common characteristic features:

(1) The crucial linkage transmitting monetary impulses on the pace of economic activity is based on the positions and slope properties of two semi-reduced form equations, the IS-LM curves. Apart from the minor role of price and interest induced wealth effects in the expenditure function, the standard model "connects" the monetary sector with the real sector by a single variable, the long term interest rate. This interest rate linkage is referred to as "the borrowing cost conception of the transmission mechanism". It is argued that borrowing costs are a critical magnitude relative to total investment costs so that a change in the long term interest rate, brought about for instance by a change in the money supply, will have a decisive influence on total expenditure for new capital. But this argument is based on a priori reasoning, for it is exactly the denial of an empirically strong linkage which reduces the model to a simple multiplier approach.

(2) The standard Keynesian model includes several assets, but introduces only two different yields, the rate of return on money, which is set equal to zero, and the long term interest rate. Real capital, government bonds, bank loans and other private debts are thus regarded as perfect substitutes.¹⁾ Relying on "a two-asset

1) See James Tobin, "Money, Capital and Other Stores of Value," American Economic Review, Vol. 51, May 1961, pp. 26-37.

model" is the second common denominator of Keynesian Economics.²⁾

Our following discussion is centered on these two building blocks. In section 2 we review different versions of the Keynesian paradigm in the context of a one-sector model and state some of the major policy implications. We will show that under certain model assumptions the market interest rate is a very unreliable indicator of the expansiveness and contractiveness of monetary policy.

In section 3 we will extend the basic model by including independent demand and supply functions for both consumption and investment goods. The interplay of the relative output prices modifies and changes some of the major results, especially those bearing on the effects of fiscal policy operations and the indicator problem. We should note that the inclusion of relative output prices is quite consistent with the Keynesian view of the transmission mechanism, because it is still the relative borrowing cost theory which specifies the structural conditions for the effectiveness of monetary policy operations.

2) If we follow Axel Leijonhufvud's interpretation in his book "On Keynesian Economics and the Economics of Keynes, New York 1968," we have to accept his main thesis that Keynesian economics are quite distinct from Keynes' own theory. Keynes' own efforts were always directed towards a reformulation of inherited price theory which failed its major test during the Great Depression. The usual Keynesian theory, however, generally deemphasises the role of relative prices both in the specification of the mechanism transmitting monetary impulses and in the explanation of inflexibilities and rigidities in the price system.

More recently some economists have argued that the Keynesian paradigm has not treated the bond-finance and new money-finance cases of government deficits symmetrically which has caused incorrect implications regarding the multiplier effects of government spending. In section 4 we try to demonstrate that this position is mistaken. The government budget restraint can be included into the basic system without changing any of the derived conclusions.

The standard system cannot separate the different effects of changes in the money supply brought about by pure wealth increases and open-market operations. In section 5 we demonstrate that an appropriate reformulation of the liquidity preference relation allows a more subtle differentiation of various monetary policy operations.

Section 6 supplements the discussion by the inclusion of the money supply process.

2. An Outline of the Keynesian Paradigm: IS-LM and p-Y Analysis

The basic Keynesian model is usually stated as follows: ¹⁾

$$(1.1) \quad Y = C + I + \frac{\bar{G}}{p} \quad \text{output market equilibrium}$$

$$(1.2) \quad C = C\left(Y, \frac{\bar{T}}{p}\right) \quad \text{consumption function}$$
$$0 < C_1 < 1; 0 > C_2 > -1$$

$$(1.3) \quad I = I\left(i, \frac{w}{p}\right) \quad \text{investment function}$$
$$I_1 < 0; I_2 < 0$$

Under special model assumptions we can write:

$$(1.3a) \quad I = I(i)$$

$$I' < 0$$

$$(1.4a) \quad \frac{\bar{M}}{p} = m^*\left(Y, \frac{\bar{T}}{p}, i\right) \quad \text{money market equilibrium}$$
$$m_1^* > 0; m_2^* < 0; m_3^* < 0$$

$$(1.4) \quad \frac{\bar{M}}{p} = m(Y, i)$$

$$m_1 > 0; m_2 < 0$$

$$(1.5) \quad Y = Y(L; \bar{K}) \quad \text{aggregate production function}$$
$$Y' > 0; Y'' < 0$$

$$(1.6) \quad Y'(L) = \frac{w}{p} \quad \text{demand for labor function}$$

$$(1.7) \quad L = L\left(\frac{w}{p}\right) \quad \text{supply of labor function}$$
$$L' > 0$$

To separate the endogenous variables from the exogeneous and predetermined variables, we add to the latter variables a bar.

We may exclude the government budget constraint for reasons which will be discussed in section 4 of our paper. The general

1) Compare for instance Martin J. Baily, National Income and the Price Level: A Study in Macrotheory, New York 1962.

structure of the model is too familiar to enter into the particulars, but some minor points should be mentioned. (1) We note that the money demand function (equation (1.4a)) includes real tax payments as an argument. This contradicts the assumption that the fiscal parameters are only associated with the IS curve. The reasoning for the inclusion of the tax payments into the money demand function is the following: If we interpret real income and real tax payments as proxies for private real wealth, which is not included directly, then a change in \bar{T} reflects a change in the private sector's wealth position and will certainly affect the demand for money. But if we interpret real income only as summarizing the factors determining the transactions volume, the exclusion of real tax payments as an argument is justified (equation (1.4)). This is the standard procedure which we will follow. Because our system includes a government sector, there are some difficulties in the interpretation of the demand for money function. We can accept the equation as an approximation to a more precisely specified formulation. (2) Differing from the usual procedure we have included the real wage rate as an additional argument into the investment function. This follows directly from the transformation of the marginal efficiency of capital calculus into a function describing the demand for new capital. Only under special circumstances, which will be stated below, an omission of this variable is justified. (3) We ignore some implicit finance assumption of the Keynesian model which are crucial for a correct interpretation of the liquidity preference function and the government budget restraint. These problems will be discussed in later sections. (4) Our treatment of tax payments is admittedly crude. We know that income taxes affect the marginal choice between income and leisure and by this affecting the supply of labor.

Excise taxes will affect the supply behavior of producers, *s.s.o.*

To simplify the analysis we have ignored a more detailed specification. We have included both government expenditures and tax payments as exogenous variables. But this procedure is not a restriction of the scope of our analysis, because we can always define a mathematical operation taking care of the built-in-stabilization case related to tax payments.

If we fix the price level and interpret the output market equilibrium condition (equation (1.1)) as an aggregate supply function we can condense equations (1.1)- (1.4) into two semi-reduced forms, the IS and LM curves:

$$(1.8) \quad i = f\left(Y, \frac{\bar{T}}{\bar{p}}, \frac{\bar{G}}{\bar{p}}\right) \quad \text{IS curve}$$

$$f_1 < 0; \quad f_2 < 0; \quad f_3 > 0$$

$$(1.9a) \quad i = g^*\left(Y, \frac{\bar{T}}{\bar{p}}, \frac{\bar{M}}{\bar{p}}\right) \quad \text{LM curve}$$

$$g_1^* > 0; \quad g_2^* < 0; \quad g_3^* < 0$$

But according to the standard procedure we exclude $\frac{\bar{T}}{\bar{p}}$ as an argument and write

$$(1.9) \quad i = g\left(Y, \frac{\bar{M}}{\bar{p}}\right) \quad \text{LM curve}$$

$$g_1 > 0; \quad g_2 < 0$$

A decrease of \bar{p} in equation (1.8) corresponds to a weighted increase in both \bar{T} and \bar{G} . Because these changes are opposite, the sign of a variation of \bar{p} depends on the difference of the levels of \bar{T} and \bar{G} .

The effect or total impulse of a policy measure is defined as the induced change of the equilibrium values of the endogenous variables as a consequence of a change of one or more of the

exogenous policy variables. A derivative involving system parameters has therefore to be understood as a total derivative.

To correspond to the meaning of the definition of the transmission mechanism in the usual sense, these total derivatives should be expressed in terms of the levels and partial derivatives of the variables as specified by the original structural equations of the complete system:

(1.10) - (1.12)

$$\frac{\partial Y}{\partial \bar{M}} = \frac{-I'}{\Delta \bar{p}} > 0; \quad \frac{\partial Y}{\partial \bar{T}} = \frac{-m_2 C_2}{\Delta \bar{p}} < 0; \quad \frac{\partial Y}{\partial \bar{G}} = \frac{-m_2}{\Delta \bar{p}} > 0$$

(1.13) - (1.15)

$$\frac{\partial i}{\partial \bar{M}} = \frac{C_1 - 1}{\Delta \bar{p}} < 0; \quad \frac{\partial i}{\partial \bar{T}} = \frac{C_2 m_1}{\Delta \bar{p}} < 0; \quad \frac{\partial i}{\partial \bar{G}} = \frac{m_1}{\Delta \bar{p}} > 0$$

The Jacobian determinant of the system is

$$\Delta = (-m_2 (1 - C_1) - m_1 I') > 0$$

From these total derivatives we can infer the necessary and sufficient conditions for an effective transmission of monetary and fiscal impulses. In the literature we often find reference to elasticity conditions of the IS and LM curve. This view is mistaken, because there is no way to transform our results into elasticity conditions.

Given these results it is easy to explain, why Keynesians advocate a fiscal policy mix which allegedly works more reliably and more directly.

Ever since the Oxford survey of the late thirties Keynesians question the responsiveness of investment with respect to a change in the long term interest, the "channel" through which monetary policy works, and this responsiveness determines the total impulse of a variation of the money supply on gross

national product. The crucial magnitude determining the effectiveness of fiscal policy is m_2 , a "well-behaved" magnitude.²⁾

But it is not at all clear what is meant, if fiscal policy is referred to as affecting directly the crucial variables determining output and employment (or even the price level) opposite to the more indirect effect of monetary policy. In the case of tax policy, both fiscal and monetary policy stand at par, because the total rate of aggregate absorption is affected only "indirectly" through a change of gross national product components (i.e. C resp. I). In the case of a change of government expenditure one may reason that such a policy will affect directly a gross national product component. But this reasoning is misleading, because the government does not buy part of "gross national product" but only certain good categories. This will change the relative price structure of both inputs and outputs. If we leave our one-good economy, the argument appears less appealing. This is demonstrated in section 3. In the following discussion we will refer to equations (1.8) and (1.9) as version I of the basic Keynesian model.

2) Given the money stock, the "crowding out effect" on private investment expenditures can be at most equal to the change in public expenditure, a situation unlikely to occur.

Figure 1 summarizes once more the essential features:

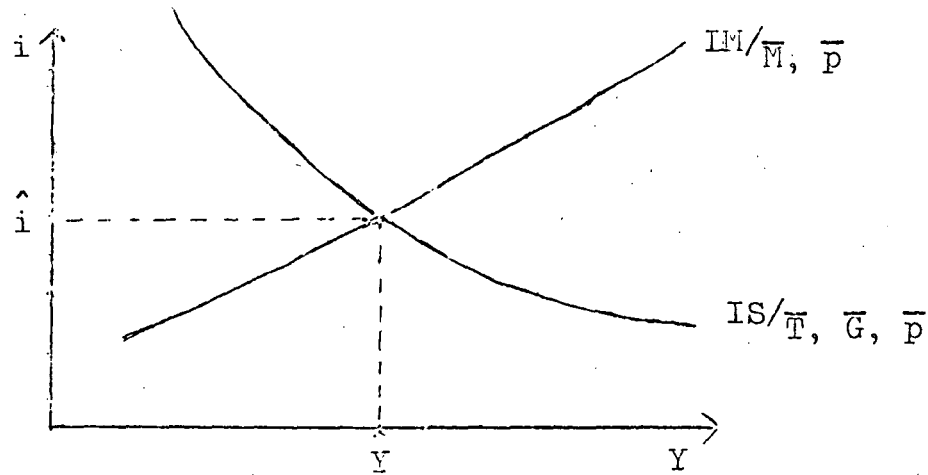


figure 1

Presented with varying assumptions about rigid prices and wages, money illusion, liquidity traps, fixed input-output coefficients, supplemented by an elasticity pessimism, more or less based on ideological preconceptions rather than on empirical evidence, version I could be varied in such a way as to explain much of the academic efforts of more orthodox Keynesians. As a policy model it is the popular frame used to rationalize programs, conceptions and activities of public policy formation during the last decades. As a natural by-product of this development the LM part of the frame become either excluded or ranked as of minor importance.

According to Leijonhufvud the majority school of the Keynesian tradition encompasses at least two major factions:³⁾ the "Revolutionary Orthodoxy", which we have briefly characterized in the last paragraph, and the "Neoclassical Resurgence" or the "Keynesian Counterrevolution".

³⁾ Axel Leijonhufvud, *On Keynesian Economics and the Economics of Keynes*, op. cit. pp. 6-8.

This latter group which is mainly interested in the "theoretical issues raised by the "Keynesian Revolution" - the imposition of their own vision on Keynes' program notwithstanding - regard the institutional implementation of the Keynesian model as artificial restrictions, completely uninteresting from an analytical point of view. Their basic model can be derived as follows: We solve equations (1.5) - (1.7) for real income and the real wage rate. This operation determines real income in the IS-LM frame. The IS curve can be used to solve for the interest rate and the LM curve gives us the equilibrium value of the price level. It may seem that these solution steps contradict the "Keynesian tradition", emphasizing the fact that interest rates are approximately determined on the money market. This view is mistaken, for, if the model is complex enough, we can always solve it in such a way to make it "appear" like this, second and more fundamentally, it is useless and misleading to classify theories according to the formal procedure which provides for the equilibrium values (or according to the equation which is missing or temporarily excluded).⁴⁾

$$(1.16) \quad i = f\left(Y^f, \frac{\bar{T}}{p}, \frac{\bar{G}}{p}\right) \quad \text{IS curve}$$
$$f_1 < 0; \quad f_2 < 0; \quad f_3 > 0$$

4) The last statement paraphrases Milton Friedman's procedure. See his "A Theoretical Framework for Monetary Analysis," Journal of Political Economy, April 1970. A similar approach is taken by David Fand, "Keynesian Monetary Theories, Stabilization Policy, and the Recent Inflation," Journal of Money, Credit and Banking, Vol. 1, 1969, pp. 556 - 587.

$$(1.17) \quad i = g(Y^f, \frac{\bar{M}}{p}) \quad \text{LM curve}$$

$$g_1 > 0; \quad g_2 < 0$$

This model, we refer to it as version II, is illustrated in figure 2.

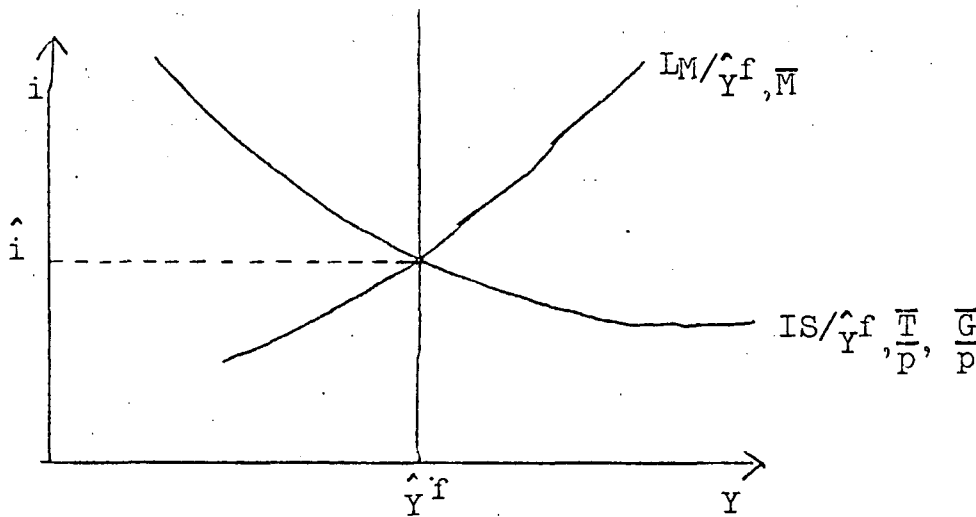


figure 2

There is probably no other macroeconomic model than the simple full employment Keynesian model which is manipulated more often by students of economics. But it nevertheless seems that important implications, especially those bearing on the controversy between fiscalists and monetarists,⁵⁾ were overlooked. If we introduce real tax payments, real government expenditures and the nominal money supply as policy parameters, the results are:

(1.18) - (1.19) ⁶⁾

$$\frac{\partial i}{\partial Y} = \frac{(1-C_1) \bar{M}}{\Delta p^2} > 0; \quad \frac{\partial p}{\partial Y} = \frac{-I' m_1 - m_2(1-C_1)}{\Delta} < 0$$

5) This should by no means suggest that this current controversy can be settled in the context of the simple Keynesian model. Both factions use completely different "visions" about the question, how the economy operates at large.

6) Given an unchanged investment function this effect has to be related to changes in the supply of labor function.

(1.20) - (1.21)

$$\frac{\delta i}{\delta \bar{P}} = \frac{-C_2 \bar{M}}{\Delta p^2} < 0; \quad \frac{\delta p}{\delta \bar{P}} = \frac{m_2 C_2}{\Delta} < 0$$

(1.22) - (1.23)

$$\frac{\delta i}{\delta \bar{G}} = \frac{-\bar{M}}{\Delta p^2} > 0; \quad \frac{\delta p}{\delta \bar{G}} = \frac{m_2}{\Delta} > 0$$

(1.24) - (1.25)

$$\frac{\delta i}{\delta \bar{M}} = 0; \quad \frac{\delta p}{\delta \bar{M}} = \frac{I'}{\Delta p} = \frac{p}{\bar{M}}$$

where Δ is defined as

$$I' \frac{M}{p^2} < 0$$

If, however, we consider nominal tax payments, nominal government expenditures and the nominal money supply as policy instruments, we get the following results:

(1.28) - (1.29)

$$\frac{\delta i}{\delta Y} = \frac{(1-C_1) \bar{M}}{\Delta^* p^2} - m_1 \frac{\bar{C}_2 \bar{I} + \bar{G}}{\Delta^* p^2} < 0; \quad \frac{\delta p}{\delta Y} = \frac{-I' m_1 - m_2 (1-C_1)}{\Delta^*} < 0$$

(1.28) - (1.29)

$$\frac{\delta i}{\delta \bar{T}} = \frac{-C_2 \bar{M}}{\Delta^* p^3} < 0; \quad \frac{\delta p}{\delta \bar{T}} = \frac{m_2 C_2}{\Delta^* p} < 0$$

(1.30) - (1.31)

$$\frac{\delta i}{\delta \bar{G}} = \frac{-\bar{M}}{\Delta^* p^3} > 0; \quad \frac{\delta p}{\delta \bar{G}} = \frac{m_2}{\Delta^* p} > 0$$

(1.32) - (1.33)

$$\frac{\delta i}{\delta \bar{M}} = \frac{C_2 \bar{T} + \bar{G}}{\Delta^* p^3} < 0; \quad \frac{\delta p}{\delta \bar{M}} = \frac{I'}{\Delta^* p} > 0$$

The signs are derived under the condition that

$$\Delta^* = \frac{I'M}{p^2} + m_2 \frac{C_2\bar{T} + \bar{G}}{p^2} < 0 \text{ and } C_2\bar{T} < G$$

This implies that the budget is either balanced, mildly contractive or on a deficit basis.⁷⁾

If, on the other hand, the budget is overly restrictive Δ^* can change its sign which leads to a corresponding change in the signs of the total derivatives in (1.26 - 1.33). Equations (1.24) and (1.25) expel the familiar results that money is "neutral", if the budget position is fixed in real terms. A change in the money supply leads to a proportional change in the price level without affecting the long term interest rate. The most interesting case is a change of the money supply under conditions of "money illusion" in the government's taxing and spending activities: If the budget is slightly contractive, balanced or expansive, a change in the money supply leads to a decrease in the long term interest rate and to a less than proportional fall in the price level. The total impulse, and this is the most important result, depends on the current budget situation. The position of the budget preconditions the effectiveness of monetary policy (equations (1.32) - (1.33)).

The result is consistently overlooked in the current debate between the fiscalists and monetarists. It would be worthwhile to refor-

7) In order to avoid possible misunderstandings, we should state that an unbalance in the government budget is always offset by an appropriate change in the government debt outstanding and not by issuing new money. This corresponds to the implicitly stated finance assumption of the Keynesian model (compare section 4 of this paper). To incorporate the money finance case of the government deficit requires a reinterpretation of our basic model.

ulate both the research strategies and the empirical results at hand given these somewhat modified aspects. It should be noted that the same situation prevails, if the economy operates under less than full employment conditions.

Judged from our derivatives we can derive that the interest rate behavior is a much less reliable indicator of the thrust of monetary policy than for instance the money stock. Given our model it is less probable that the interest rate has a high degree of systematic association with nominal income, our target variable, than the money stock. This follows from the fact that it is more probable that the price level is positively related to the behavior of the money stock with the consequence of a positive association with nominal income. Similar results can be derived for situations of less than full employment. This result clearly modifies and corrects the findings of J. Pierce⁸⁾ who argued that under the static Keynesian frame both the interest rate and the money stock are equivalent measures indicating the expansiveness or restrictiveness of monetary policy. (i.e. variations in \bar{M}).

To conclude our discussion of the basic Keynesian model we introduce version III, a version which is used in more sophisticated Keynesian stability discussions. We derive this version by solving equations (1.1) - (1.4) of our basic model for an aggregate demand function, which expresses the price level as a function of real income and the policy parameters, and by solving the aggregate production function and the demand for labor function for an

8) James Pierce, "Some Rules for the Conduct of Monetary Policy," in Controlling Monetary Aggregates, The Federal Reserve Bank of Boston, Boston 1969, p. 134.

aggregate supply curve, given an institutionally fixed money wage. This is appropriate, if we only consider situations at less than full employment. If we reach the full employment level, we have to change the procedure, for now the supply of labor curve becomes operative. The aggregate supply curve will bend backward at the full employment level.⁹⁾ Analyzing a model like this we have to make sure that an overall equilibrium is defined, a case which cannot be taken for granted as the figure 3 illustrates.

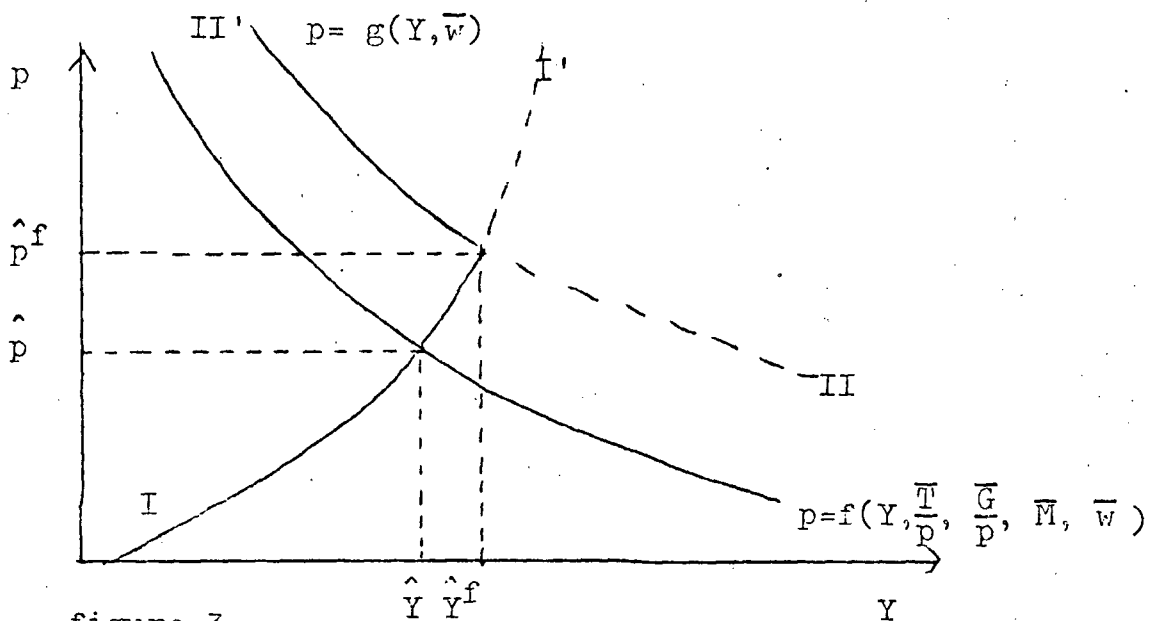


figure 3

The two supply curves I I' and II II' correspond to the two possible cases of a substitution of either the demand for labor function or the supply of labor function over the full range of possible variations of the real wage rate.

If we liberate the labor market from its institutional restraints, the edge of the aggregate supply curve will move downward until

9) The backward bending part of the aggregate supply is only drawn for illustrative purposes. This part corresponds to a complicated dynamic process which would eventually change the investment function too. For some of the implications see Don Patinkin, Money, Interest, and Prices, New York 1965, pp. 313-324.

the full employment price level is reached.

Because we already discussed the workings of the model under conditions of full employment and because we will display the under-employment situation with the aid of a two sector model, we can omit a further discussion and state only the signs of the derivatives. To simplify the computation we assume that the budget position is fixed in real terms:

$$(1.36) \quad p = f(Y, \frac{\bar{T}}{p}, \frac{\bar{G}}{p}, \bar{M}, \bar{w}) \quad \text{aggregate demand curve}$$

$$f_1 < 0; f_2 < 0; f_3 > 0; f_4 > 0; f_5 < 0$$

$$(1.35) \quad p = g(Y, \bar{w}) \quad \text{aggregate supply curve}$$

$$g_1 > 0; g_2 > 0$$

We note that for the first time the real wage rate as an argument of the investment function becomes operative. Under version I of our model both the nominal wage rate and the price level were arbitrarily fixed and under version II the real wage rate was uniquely determined on the labor market, so that in both cases this argument dropped out.

The economically appealing slope properties of the aggregate demand curve in figure 3 and the unconditioned sign specifications of the derivatives hide a fundamental ambiguity: the stated results are only valid, if we introduce an a priori unjustified order restriction for the denominator of f_1 in equation (1.34), which is

more than puzzling: $I_1 \frac{\bar{M}}{p^2}$ must be absolutely greater than $m_2 I_2 \frac{\bar{w}}{p^2}$.

Given this order condition we can summarize the impulse factors as follows:

(1.36) - (1.39)

$$\frac{\delta Y}{\delta T} < 0; \frac{\delta T}{\delta G} > 0; \frac{\delta Y}{\delta M} > 0; \frac{\delta Y}{\delta W} > 0$$

(1.40) - (1.43)

$$\frac{\delta p}{\delta T} < 0; \frac{\delta p}{\delta G} > 0; \frac{\delta p}{\delta H} < 0; \frac{\delta p}{\delta W} > 0$$

The basic ambiguities about the effects of different policy operations will arise, as demonstrated in the discussion of version II, if we fix the government budget structure in nominal terms.

3. Extension to a Two-Sector Model: Relative Prices and Related issues

Keynes' own efforts were always directed towards a disaggregation of total "output" into two broad good categories: investment goods and consumption goods. This is clearly demonstrated in his discussion of the "marginal efficiency of investment" schedule which includes explicitly both the demand for investment goods and the supply for investment goods as distinct from the corresponding demand and supply for consumption goods. This procedure conserves a minimum price theoretical foundation of aggregate demand and supply analysis and focuses directly on the supply conditions of two industries which could for instance render more precise the analytical basis of the much debated key-industry conception of

modern Keynesian stabilization policy. A theoretical advantage of the explicit introduction of the demand and supply conditions of investment goods is to give a more rigorous definition of the marginal efficiency of capital versus marginal efficiency of investment conceptions which are often introduced in a very confusing way into the one-good world of modern textbook Keynesianism. ¹⁾

A simple two-sector model of employment, income and the price level could be introduced as follows:

(2.1)	$I = I^c + I^i + \frac{\bar{I}^g}{q}$	market equilibrium for investment goods
(2.2)	$I^c = I^c(i, q, p, w)$	demand function for I-goods of the C-goods industry
	$I_1^c < 0; I_2^c < 0; I_3^c > 0; I_4^c < 0$	
(2.3)	$I^i = I^i(i, q, w)$	demand function for I-goods of the I-goods industry
	$I_1^i < 0; I_2^i > 0; I_3^i < 0$	
(2.4)	$C = C^p + \frac{\bar{C}^g}{p}$	market equilibrium for consumption goods
(2.5)	$C^p = C^p\left(\frac{pC + qI}{p}, \frac{\bar{T}}{p}\right)$	consumption function
	$0 < C_1 < 1; -1 < C_2 < 0$	
	$Y = \frac{pC + qI}{p}$	real gross national product in terms of consumption goods

1) For a formal development compare Björn Thalberg, A Keynesian Model Extended by Explicit Demand and Supply Functions for Investment Goods, Uppsala 1962; and Peer M. Wijkman, "The Marginal Efficiency of Capital and Investment: A Didactic Exercise," Swedish Journal of Economics, Vol. 63, 1965, pp. 263-277. Wijkman's didactic exercise gives an excellent presentation of the two conceptions, but contains at the same time some confusing mathematical errors. Thalberg's book was very useful to derive the implications concerning the foundation of the investment demand of the two industries.

$$(2.6a) \quad \frac{\bar{M}}{P} = m^* \left(\frac{pC + qI}{P}, i \right) \text{ money market equilibrium}$$

$$m_1^* > 0; \quad m_2^* < 0$$

We assume that m^* is linear homogenous in "real income" and write:

$$(2.6) \quad \frac{\bar{M}}{P} = m(i) \frac{pC + qI}{P} \quad \text{money market equilibrium}$$

$$\frac{\bar{M}}{pC + qI} = m(i)$$

$$m < 0$$

$$(2.7) \quad I = I(L^i); \quad K^i \quad \text{production function for I-goods}$$

$$I' > 0; \quad I'' < 0$$

$$(2.8) \quad C = C(L^c) \quad \text{production function for C-goods}$$

$$C' > 0; \quad C'' < 0$$

$$(2.9) \quad I'(L^i) = \frac{W}{q} \quad \text{demand for labor function of the I-goods industry}$$

$$(2.10) \quad C'(L^c) = \frac{W}{P} \quad \text{demand for labor function of the C-goods industry}$$

$$(2.11) \quad L = L\left(\frac{W}{P}\right) \quad \text{supply of labor function}$$

$$(2.12) \quad L = L^i + L^c \quad \text{definition of employment}$$

Analogously to our discussion of the last section we present the two-sector model in three versions. If we arbitrarily fix the nominal wage, the price level of both investment and consumption goods and interpret equations (2.1) and (2.4) as supply functions, equations (2.1) - (2.6) define a set which is economically equivalent to our previously derived equations defining the IS-LM curves.

We defined real gross national income as: $Y = \frac{pC + qI}{p}$

Because p and q are fixed, this term is a linear homogeneous function of $\frac{pC + qI}{pC^* + qI^*}$, where C^* and I^* are base quantities.

This defines a true quantity index, from which follows that we can apply the Hicks Composite-Good Theorem²⁾ and write:

$$(2.13) \quad i = f \left[Y(p,q), \bar{w}, \frac{\bar{I}^g}{q}, \frac{\bar{C}^g}{p} \right] \quad \text{IS curve}$$

$$(2.14) \quad i = g \left[Y(p,q), \bar{M} \right] \quad \text{LM curve}$$

All the mathematical derivations of the last section applied to the basic model could be restated without any change of the main consequences.

This is not true for version II of our extended model which corresponds to the full employment solution of our basic model. There is no unique real wage-employment combination describing a full employment solution. Changes in the relative output prices have feedbacks both on the relative shares of the labor force employed in the two sectors and on total employment. This forces us to work through the complete model. There is no formal equivalent to the IS-LM frame under full employment conditions.

If we differentiate our extended model totally with respect to all policy parameters, the row by row result can be stated in the form of a labeled coefficients matrix from which all our further conclusions can be derived:

2) See for instance Paul A. Samuelson, *Foundations of Economic Analysis*, Cambridge, Mass. 1947, pp. 160-163.

The labeled coefficients matrix can be written more compactly in the partitioned forms

$$(2.15)-(2.16) \quad \left(\begin{array}{c|c} \underline{O} & \underline{V} \\ \hline \underline{F} & \underline{A} \end{array} \right) \quad \text{resp.} \quad \left(\begin{array}{c|c} \underline{O} & \underline{V}^* \\ \hline \underline{F} & \underline{A} \end{array} \right)$$

A and A* contain the elements of the Jacobian determinants of our two-sector model augmented by 4 columns defining the coefficients of the policy parameters. We may either define the fiscal policy instruments in nominal terms or in real terms with the corresponding change in the columns 7 and 8. This defines A and A*. The meaning of the other matrices is self-evident.

It should be recalled that we exclude all effects of the so-called built-in-stabilization of the government budget. This procedure is more flexible, because we can always include these effects by defining an appropriate mathematical restriction on the impulse equations (f.i. equations (1.11), (1.12), (1.14), and (1.15)). Only in case of estimation and forecasting we have to formulate our model in terms of various directly controllable parameters, for instance in the case of fiscal policy in terms of various tax parameters.

The above stated matrices refer to what we introduce as version II of our extended model which defines the "full employment" case. If we exclude row number 8, which is associated with equation (2.11), the supply of labor function, and if we switch the signs of the elements of column 9, an operation which excludes the nominal wage as an endogenous variable, we get a system equivalent to version III of our basic model. The associated labeled coefficients matrix is defined as follows:

$$(2.17)-(2.18) \quad \left[\begin{array}{c|c} \underline{0} & \underline{V} \\ \hline \underline{\mu}' & \underline{A}' \end{array} \right] \text{ resp. } \left[\begin{array}{c|c} \underline{0} & \underline{V}^* \\ \hline \underline{\mu}' & \underline{A}'^* \end{array} \right]$$

We know from our discussion of the basic model that an unrestricted labor market guarantees a unique full employment equilibrium value for real income. Fiscal and monetary policy can only affect the structure of the aggregate absorption rate but not the total. As mentioned, these results do not hold any more in the context of a two-sector model, for both nominal and "real" income can be affected by various fiscal and monetary policy measures. We have to include the complete model structure and work through all equations to derive the desired policy informations. The structure of version III of our extended model is a little bit easier to handle. In addition to this, it is certainly a more often used paradigm for shaping overall stabilization policy than the full employment model which is more relevant under growth aspects. We therefore postpone a discussion of some of the implications of version II and start with a more extensive description of version III.

The signs of the Jacobian determinants associated with \underline{A}' and \underline{A}'^* are negative under reasonably stated economic order conditions. This is especially true for the determinant associated with \underline{A}'^* . We know from our previous discussion that in the case of budget situation, fixed in nominal terms, an identification of the signs becomes more difficult.

We first present the results of version III under the condition of a government budget fixed in real terms:

$$(2.19) \quad \frac{\delta q}{\delta \bar{M}} = \frac{1}{\Delta A'^*} (I_1^c + I_1^i) (-p) I' [(1 - C_1^p) C'C' - pC']$$

$$C_1^p \frac{\partial I}{\partial p^2}] > 0$$

$$(2.20) \quad \frac{\delta p}{\delta \bar{M}} = \frac{1}{\Delta A'^*} (I_1^c + I_1^i) pC'' (-C_1^p \frac{q}{p} I'I' + C_1^p \frac{q}{p} II'') > 0$$

$$(2.21) \quad \frac{\delta i}{\delta \bar{M}} = \frac{1}{\Delta A'^*} pC'' \{ pI'' [(-I_2^c + I_2^i) C_1^p \frac{qI}{p^2} + C_1^p \frac{I}{p} (-I_3^c)] - I' [I'C_1^p \frac{q}{p} I + C_1^p \frac{q}{p} I' (-I_3^c)] \} + C'(1 - C_1^p) C'[I'I' - pI''(-I_2^c - I_2^i)] \stackrel{<}{=} 0$$

$$(2.22) \quad \frac{\delta I}{\delta \bar{M}} = \frac{1}{\Delta A'^*} (I_1^c + I_1^i) I'I' [(1 - C_1^p) C'C' - C_1^p \frac{q}{p^2} I] > 0$$

$$(2.23) \quad \frac{\delta C}{\delta \bar{M}} = \frac{1}{\Delta A'^*} (-I_1^c - I_1^i) C'C' (-C_1^p \frac{q}{p} I'I' + C_1^p pI'') > 0$$

The signs of a total impulse of a change of \bar{M} on I and C must follow from the signs of the same monetary impulse on q and p . Given the nominal wage rate q , I and N^i resp. p , C and N^c move always in the same direction. The sign of $\frac{\delta i}{\delta \bar{M}}$ is indeterminate because both the money stock and nominal income increase, but their quotient, which determines the interest rate, is unknown. This once more confirms our previously stated result: the interest rate is a very unreliable indicator for the expansiveness or contractiveness of monetary policy. In our simple model this was due to the fact that the government budget was fixed in nominal terms here it is the consequence of the interaction of relative prices of consumption and investment goods. There is no "money illusion" on the sides of private and government agencies which produces this result, and it is reasonable to expect that in the case of a budget structure fixed in nominal terms this basic ambiguity will increase. This result cannot be overstated. It is a much

debated problem in recent stabilization policy. We could derive a much stronger critical position using the Keynesian model than f.i. the monetarists who argue that it is the divergence of real and nominal interest rates which make the market interest rate a very unreliable indicator. These divergences will only reinforce the uncertainty of a policy based on the interest rate as a monetary indicator.

Given real taxes and real government expenditures for consumption goods, an increase in \bar{M} is always associated with an increase in nominal income deflated by the consumption goods price index. An increase in the money stock is associated with an overall increase in economic activity, measured either in terms of the output rate of our two industries, in terms of utilization rate of the labor force or in terms of "real" income.

The inclusion of relative output prices has certainly broadened the transmission mechanism, but it is still the borrowing conception on which the transmission mechanism is based and which determined the effectiveness of monetary policy. If the marginal response coefficients of investment demand with respect to changes in the interest rate (I_1^c and I_1^i) are zero in both industries a change in the money stock will neither affect prices nor quantities of the two gross national product components. The new stock will be absorbed into the portfolios at a lower interest rate.

Recently, Arthur Okun has advanced the so-called key-industry conception of monetary policy. This reasoning is completely based on the borrowing cost conception of the transmission mechanism. This reasoning does not deny an effectiveness of monetary policy, for instance by operating with extreme interest elasticities for the expenditure functions, but argues that monetary policy will affect mainly and directly certain key-industries selling products

for which the demand is strongly dependent on both the level and the variations of the interest rate. And because it happens that these industries have not only key positions in a hierarchy defined by this interest criterion but also in a hierarchy of socially desirable good categories, monetary policy is said to have an unwarranted allocative effect. The induced accelerative and multiplicative effects of change in the output of these key-industries are negligible and therefore not worth the price paid in the first instance. If the key-industry conception makes sense at all it should be advanced in a multi-sector model and not in a one-sector model. In a multi-sector model the conception of the key-industry is untenable, even by relying on the Keynesian paradigm. But a final refutation of the underlying hypothesis could only be advanced in the context of a Keynesian model containing at least three industries: a consumption goods industry and two independent investment goods industries. The interplay of only one investment goods industry with one consumption goods industry does not produce the adequate test environment, because the nominal value-added of the investment goods industry enters into the consumption function and is connected with the consumption goods industry by the multiplier process.

In the context of a three-sector model the hypothesis could be tested by ascertaining a significant change in the distribution of the output pattern of the two investment goods industries following a change in monetary policy, i.e. a change in \bar{M} . To test this implication we extended our model by including a second investment good industry. This modified model contains six independent investment functions describing the demand of three industries for two products. We state the results of our findings without proof: By assigning high values to the marginal interest response

coefficients to the demand functions for the output of an arbitrarily selected investment good industry and assigning lower values or zero values to the demand functions for the output of the other industry, we constructed the appropriate test conditions. But we could not discover any systematic reallocation of the output pattern of the two industries as predicted by the hypothesis, an observation which should be counted as strong negative evidence. The connection of all industries by a complex relative price mechanism precludes an a priori conjecture about the role of a special industry, even if we make strong assumptions about the position in the spectrum of one relative price, i.e. the interest rate.

A test of the key-industry hypothesis in the context of two sector-model is not completely consistent with the hypothesis, but nevertheless we could use our conclusions, stated in equations (2.22) and (2.23), to derive some weaker indirect evidence with regard to the conjecture: The only industry selling products for which the demand is depending on the interest rate is the investment goods industry. We assign to this industry the role of the key-industry. We should now expect that a change of the money supply will lead to a significant reallocation of the output structure. But the outputs of both industries increase. Both I_1^C and I_1^i appear in equations (2.22) and (2.23) as a sum multiplying a string of partial derivatives mainly specifying productivity relationships and relative prices. From this we can conclude that we have to reject the basic assumption of the key-industry conception, the conjecture that the position of an industry in a spectrum of marginal interest response coefficients is a meaningful starting point. It is interesting to note once more that the basis for our rejection of the key-industry conception is not derived from the

analytics of a different economic vision but from the very Keynesian model which was used to rationalize this speculative conjecture.

There is a striking asymmetry between the results of a change in government expenditure for investment goods and a change in government expenditure for consumption goods. The direction of the first change can be uniquely inferred from the structure. The effects of the second policy operation is completely indetermined.

$$(2.24) \quad \frac{\delta q}{\frac{\delta I^G}{p}} = \frac{1}{\Delta A'^*} (-m'(pC + qI) qI'') [(1-c_1^p)C'C' - c_1^p \frac{qI}{p^2} pC''] > 0$$

$$(2.25) \quad \frac{\delta p}{\frac{\delta I^G}{p}} = \frac{1}{\Delta A'^*} m'(pC + qI) pI'' (-c_1^p \frac{q}{p} I'I' + qI'' c_1^p \frac{I}{p}) > 0$$

$$(2.26) \quad \frac{\delta i}{\frac{\delta I^G}{p}} = \frac{1}{\Delta A'^*} qI'' [(1-c_1^p)C'(-L(i)I)C' + m(i) pC'(-c_1^p \frac{I}{p})C' + pC''(c_1^p \frac{I}{p} m(i)C + c_1^p \frac{qI}{p^2} m(i)I)] + I'(-c_1^p)I' [(-m(i)p)C'C' + pC''m(i)C] + m(i) pI'[(1-c_1^p)C'C' - pC''c_1^p \frac{qI}{p^2}] > 0$$

$$(2.27) \quad \frac{\delta I}{\frac{\delta I^G}{p}} = \frac{1}{\Delta A'^*} \{-m'(pC + qI)I'[(1-c_1^p)C'C' - c_1^p \frac{qI}{p^2} pC'']\} > 0$$

$$(2.28) \quad \frac{\delta C}{\frac{\delta I^G}{p}} = \frac{1}{\Delta A'^*} [-m'(pC + qI)C'C'(-c_1^p \frac{q}{p} I'I' + c_1^p \frac{I}{p} qI'')] > 0$$

Because real taxes and real government expenditures for C-goods are kept constant, we can infer from the last equation that real national income will rise too.

For a change of government expenditures for real consumption goods the results are

$$(2.29) \quad \frac{\delta q}{\frac{\delta U^G}{p}} = \frac{1}{\Delta A'^*} qI'' [(-I_1^c - I_1^i)(-m(i) pC'C' + pm(i)CC'') + m'(pC + qI)(I_1^c pC'')] < 0$$

$$(2.30) \frac{\frac{\delta p}{\delta C^G}}{p} = \frac{1}{\Delta A^{**}} (-p) C'' [(-I_1^c - I_1^i)(-m(i) q I' I' + q I'' m(i) I) + m'(pC + qI)(I' I' + (I_2^c - I_1^c) q I'')] \stackrel{<}{\geq} 0$$

$$(2.31) \frac{\frac{\delta I}{\delta C^G}}{p} = \frac{1}{\Delta A^{**}} (-I' I') [(-I_1^c - I_1^i)(-m(i) p C' C' + m(i) C p C'') + m'(pC + qI) I_3^c p C''] \stackrel{<}{\geq} 0$$

$$(2.32) \frac{\frac{\delta i}{\delta C^G}}{p} = \frac{1}{\Delta A^{**}} (-1) \{ I' I' (-L(i) p C' C' + m(i) C p I'') - m(i) q I' I' (-I_3^c) p I'' + q I'' [-m(i) p C' (-I_2^c - I_2^i) C'] + p I'' L(I_1^c - I_1^i) m(i) C - m(i) I I_3^c \} \stackrel{<}{\geq} 0$$

$$(2.33) \frac{\frac{\delta C}{\delta C^G}}{p} = \frac{1}{\Delta A^{**}} C' C' \{ [(-I_1^c - I_1^i) (-m(i) q I' I' + m(i) I q I'')] + m'(pC + qI) [I' I' + (I_2^c + I_2^i) q I''] \} \stackrel{<}{\geq} 0$$

If we multiply each equation by C_2^p , a negative magnitude, we get the results for a change in real taxes. The effects of this policy operation is of course indeterminate too.

We could, of course, specify some order conditions to infer some more information. But most of these conditions cannot be justified by a priori reasoning alone. From this we have to conclude that real taxes and real government expenditure for consumption goods are very unreliable policy instruments.

One of the important conjectures of the monetarist position is that monetary has predominantly aggregative consequences, whereas fiscal policy has more allocative effects. The refutation of the key-industry conception strengthened the monetarist position with regard to the aggregative effect of monetary policy, but

the same reasoning weakens the second part of the argument: The linkage of our two industries by the relative price structure does not justify an a priori conjecture that a change in government expenditure for investment goods will lead to a decisive rearrangement of the output structure with little effect on the total component.

Until now the discussion was based on the assumption that the items in the government budget are fixed in real terms. We know from our previous discussion that most of our policy measures become unreliable, if we introduce "money illusion" into the government budget. This has an obvious consequence stated often only programmatically: fiscal and monetary policy should be coordinated. This is certainly not the case, if for instance monetary policy is expansive, measured by the increase in the money stock, and fiscal policy at the same time restrictive, measured by the real expenditure for the outputs of the two industries. This conclusion is, of course, stated in the frame of the Keynesian paradigm and therefore valid, only if the model is the correct guidance to the solution of given policy problems.

If the government reacts passively to the operation of the central monetary authorities, i.e. \bar{T}^e , \bar{C}^e , \bar{T} are constant, the results of a change of the money stock are as follows:

$$(2.34) - (2.38) \quad \frac{\delta q}{\delta \bar{M}} > 0; \quad \frac{\delta p}{\delta \bar{M}} > 0; \quad \frac{\delta i}{\delta \bar{M}} \leq 0; \quad \frac{\delta I}{\delta \bar{M}} > 0;$$

$$\frac{\delta C}{\delta \bar{M}} > 0; \quad \text{if } c_1^p \frac{q}{p^2} I + c_2^p \frac{T}{p^2} + \frac{\bar{G}}{p^2} > 0$$

These results are easily derived by an appropriate substitution of two different elements from our basic labeled coefficients matrix into the solution set described by equations (2.19)-(2.23). We state once more that the position of the budget determines the effectiveness of monetary policy.

The signs of a change in government expenditure are the same as in the case of a budget position fixed in real terms with the exception of an undetermined effect on the real interest rate:

$$(2.39) - (2.43) \quad \frac{\delta q}{\delta \overline{I}^g} > 0; \quad \frac{\delta p}{\delta \overline{I}^g} > 0; \quad \frac{\delta i}{\delta \overline{I}^g} < 0; \quad \frac{\delta C}{\delta \overline{I}^g} > 0; \quad \frac{\delta I}{\delta \overline{I}^g} > 0;$$

As before, all the signs of the effects of a change in government nominal taxes and nominal expenditure for consumption goods are indeterminate.

We conclude our discussion of the two-sector model with a short discussion of version II which corresponds to the full employment case of our basic model. If the government budget position is fixed in real terms, we can derive the classical neutrality proposition: An increase in the money supply will lead to a proportional increase in all nominal variables leaving constant all real magnitudes included the real interest rate. If, however, the government does not offset its budget position after a change in relative and absolute prices, the central monetary authority can affect both the structure and the magnitude of real output. There is no unique full employment solution defined. But an exact specification of monetary policy under these assumptions is difficult to derive. This is intuitively plausible: The complete structure determines an other relative price, the nominal wage rate, which was previously fixed. This complicates the price mechanism and introduces new interpretation problems.

4. Problems Associated with the Government Budget Restraint

More recently some economists¹⁾ have argued that the traditional IS-IM analysis has not treated the bond-finance and new money-finance cases of government deficits symmetrically, which has led to incorrect conclusions regarding the multiplier effects of government spending.

This issues need some clarification, because our discussion was presented under the explicit assumption that an exclusion of the government budget restraint is justified.

We will show that a correctly interpreted Keynesian model allows a logically consistent incorporation of the government budget restraint. By this we mean that the inclusion of the government budget restraint is not a specification problem but an interpretation problem. All the familiar results are valid, if we correctly interpretate the implicitly stated finance assumptions of the standard model.²⁾

We will try to derive our proposition from a simple variant of our previously stated basic model. We will use version I, because it allows a simple exposition of the basic ideas:³⁾

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- 1) For example William L. Silber, "Fiscal Policy in IS-IM Analysis: A Correction," Journal of Money, Credit and Banking, Vol. II, 1970, pp. 461-472. For similar views compare the references cited in Silber's article.
 - 2) The standard reference in this context is of course L.A. Metzler, "Wealth, Saving, and the Rate of Interest," Journal of Political Economy, Vol. 59, 1951, pp.93-110.
 - 3) Some conceptual tools for the following procedure are developed in Jack Hishleifer, Investment, Interest, and Capital, Englewood Cliffs, N.J., 1970, pp.153-194.

$$(4.1) \frac{W_0}{\bar{p}} = \frac{\lambda Y}{i} + \frac{\bar{N}_0}{\bar{p}i} + \frac{\bar{M}_0}{\bar{p}} - \frac{\bar{T}}{\bar{p}i}$$

wealth at the beginning of the "period"⁴⁾

$$(4.2) Y = C\left(\frac{W_0}{\bar{p}}, Y + \frac{\bar{N}_0}{\bar{p}} - \frac{\bar{T}}{\bar{p}}; \lambda\right) + I\left(i, \frac{W}{\bar{p}}\right) + \alpha + \frac{\bar{G}}{\bar{p}}$$

commodity-market equilibrium

$$(4.3) \frac{\bar{M}_1}{\bar{p}} = m\left(\frac{W_0}{\bar{p}}, Y, i; \lambda\right)$$

money-market equilibrium

$$(4.4) \frac{\bar{G}}{\bar{p}} + \frac{\bar{N}_0}{\bar{p}} - \frac{\bar{T}}{\bar{p}} = \frac{\bar{M}_1}{\bar{p}} - \frac{\bar{M}_0}{\bar{p}} + \frac{N_1 - \bar{N}_0}{\bar{p}i}$$

government budget restraint.

$$(4.5) \frac{W_1}{\bar{p}} = \frac{\lambda Y}{i} + \frac{N_1}{\bar{p}i} + I + \alpha + \frac{\bar{M}_1}{\bar{p}} - \frac{\bar{T}}{\bar{p}i}$$

wealth at the end of the "period"

Given the values of the policy parameters \bar{G} , \bar{M}_0 , \bar{M}_1 , \bar{N}_0 , and \bar{T} , or more simply their real values, the five equations are sufficient to determine the endogenous variables $\frac{N_1}{\bar{p}}$, i , $\frac{W_0}{\bar{p}}$, $\frac{W_1}{\bar{p}}$, and Y .

Because we are not directly interested in the magnitude of W_1 we omit equation (4.5) which imposes no further restriction on the remaining structure.

Equation (4.1) states the implicit finance assumption of the Keynesian model: Gross national product is multiplied by the fraction of nonhuman income to total income and capitalized at the current market interest rate. The operation defines the market

4) We use the term "period" as a theoretical term analogously to the Hicksian "week" as applied for instance in Don Patinkin, Money, Interest, and Prices, op.cit.passim. Our analysis thus describes a one-period equilibrium.

value of the capital stock $\frac{\lambda Y}{i}$. This procedure is admittedly crude and ignores some important difficulties, but it is a good first approximation. All investment projects are financed by issuing common stock or interest bearing debt. Because we exclude human capital from our wealth restraint, it is safe to assume that only a part of the current and future tax liabilities will be capitalized. To the degree wealth owners capitalize these tax obligations, the government debt will be offset. To this degree the tax liabilities are treated as negative bonds. Equation (4.2) describes the familiar commodity-market equilibrium condition and equation (4.3) the corresponding money-market condition. Because any increase in the money supply has to be absorbed into the portfolios of the wealth owners at the end of the "period", the demand has to absorb the supply magnitude \bar{M}_1 which includes the money-finance part of the government deficit. Equation (4.4) specifies the outlay and receipt components of the government budget. The budget deficit must be covered either by printing new money or issuing new interest bearing debt. An analogous statement holds for the case of a current surplus. It will be noted that the coupon payments for the increased debt will be due for the first time during the next "period". If we hold \bar{M}_1 and \bar{T} constant, a change in government expenditure for final output will be matched by an equivalent change in the value of government bonds outstanding at the end of the "period". Because our representative wealth owner regards all debt instruments as homogeneous goods, it will be indifferent between an increase in its wealth brought about by an increase in the number of government bonds and an equivalent increase in privately generated wealth, i.e. private investment. The new issued debt will be expected to pay the current market rate beginning with the next "period".

Before we analyze the implications of the model we should state the basic similarity between this model and our basic Keynesian model. First we singled out a parameter α denoting autonomous real investment. This does not change the basic structure, but does allow a more convenient analysis of shifts of private investment decisions. If we assume that the government deficit is always financed by issuing new interest bearing debt and a surplus budget will always result in a repayment of a corresponding amount of interest bearing debt, the money supply, \bar{M}_0 , at the beginning of the "period" is equal to the supply of the end of the "period", \bar{M}_1 .

Under this assumption equation (4.4) could be cancelled. Neglecting all wealth effects eliminates equation (4.1) and the corresponding wealth arguments in the behavior functions. These operations reduce the model to the familiar IS-IM frame, which is defined by equations (4.2) and (4.3). These equations restate equations (1.8) and (1.9) defined in section 1.

Our previous exclusion of the government budget restraint is justified, if and only if the following results can be derived

$$(4.6) \quad \frac{\delta i}{\delta \bar{G}} = \frac{\delta i}{\delta \alpha} = - \frac{1}{\Delta i} (m_2 + m_1 \frac{\lambda}{i}) > 0$$

$$(4.7) \quad \frac{\delta Y}{\delta \bar{G}} = \frac{\delta Y}{\delta \alpha} = \frac{1}{\Delta i} (m_3 - m_1 \frac{\lambda Y + \frac{N_0}{p} \frac{\delta \bar{M}_1}{p}}{i^2}) > 0$$

These equalities hold, as we can verify by inspection of the system (4.1) - (4.4). Δ is the Jacobian determinant of the system. This magnitude has a negative sign.

If we adjust for the interest induced wealth effect and introduce wealth as a constant, we get the familiar textbook results. We have therefore to reject the proposition that in the case of a bond-financed government deficit the standard procedure fails to account properly for the wealth effect.

The arguments of those economists who charge the Keynesian IS-LM analysis for a failure to incorporate the proper wealth effects of a bond-financed government deficit can be formalized in terms of our model:

$$(4.8) \quad di = \frac{\delta i}{\delta \frac{\bar{G}}{p}} d \frac{\bar{G}}{p} + \frac{\delta i}{\delta \frac{\bar{N}_0}{p}} d \frac{\bar{N}_0}{p}$$

$$(4.9) \quad dY = \frac{\delta Y}{\delta \frac{\bar{G}}{p}} d \frac{\bar{G}}{p} + \frac{\delta Y}{\delta \frac{\bar{N}_0}{p}} d \frac{\bar{N}_0}{p}$$

where we have to adjust $d \frac{\bar{N}_0}{p}$ in such a way that $d \frac{\bar{N}_0}{p} = i d \frac{\bar{G}}{p}$.

This allows a transformation of the two differentials into derivatives. Because $\frac{\delta i}{\delta \frac{\bar{N}_0}{p}}$ is definitely positive, this procedure

leads to an overestimation of di in the first equation. The sign of $\frac{\delta Y}{\delta \frac{\bar{N}_0}{p}}$ is indetermined so that the sign of the total effect of

a bond-financed increase in government expenditure on gross national product is indetermined. This is exactly the allegedly correct result stated by some of the critics.⁵⁾ We recall that in our analysis the sign of this critical magnitude was uniquely determined. The indeterminacy is the consequence of including a

5) See f.i. William L. Silber, op.cit.

kind of a windfall gain or gift into the budget restriction, i.e. the unjustified variation of $\frac{N_0}{p}$, or otherwise stated, the consequence of a failure to separate balance sheet items from the flow concepts of the income statement. The critical magnitude $i \frac{d\bar{G}}{p}$ is nothing more than a perpetual yield corresponding to a part of the private saving decisions.

Another problem associated with the unjustified modification should be noted: This operation completely ignores the financing of the coupon payments of the increased government debt, a problem which we will discuss in a moment.

If we finance the government deficit by printing new money, our model yields the following results:

$$(4.10a) \quad di = \frac{\delta i}{\frac{\delta \bar{G}}{p}} d\frac{\bar{G}}{p} + \frac{\delta i}{\frac{\delta \bar{M}_1}{p}} d\frac{\bar{M}_1}{p}$$

$$(4.11) \quad dY = \frac{\delta Y}{\frac{\delta \bar{G}}{p}} d\frac{\bar{G}}{p} + \frac{\delta Y}{\frac{\delta \bar{M}_1}{p}} d\frac{\bar{M}_1}{p}$$

where we have to restrict $d\frac{\bar{M}_1}{p}$ in such a way that

$$d\frac{\bar{M}_1}{p} = - \left(\frac{\frac{\delta \frac{N_1}{p}}{\frac{\delta \bar{G}}{p}}}{\frac{\delta \frac{N_1}{p}}{\frac{\delta \bar{M}_1}{p}}} \right) d\frac{\bar{G}}{p}$$

The compensation is such that the number of government debt instruments at the end of the period will remain unchanged. di in the first equation is indeterminate, dY in the second equation is definitely positive.

Our results correspond with what we expect from Keynesian analysis: An increase in government expenditure shifts the IS curve to the right, the following rightward shift of the LM curve reinforces the income rising effect, but offsets the increase of the interest rate by weakening the crowding-out-effects of the increased government expenditures.

In terms of our model the critics of the Keynesian approach would proceed as follows:

$$(4.12) \quad di \frac{\delta i}{\delta \frac{\bar{G}}{p}} d\frac{\bar{G}}{p} + \frac{\delta i}{\delta \frac{\bar{M}_0}{p}} d\frac{\bar{M}_0}{p} + \frac{\delta i}{\delta \frac{\bar{M}_1}{p}} d\frac{\bar{M}_1}{p}$$

$$(4.13) \quad dY = \frac{\delta Y}{\delta \frac{\bar{G}}{p}} d\frac{\bar{G}}{p} + \frac{\delta Y}{\delta \frac{\bar{M}_0}{p}} d\frac{\bar{M}_0}{p}$$

where the restriction of $d\bar{M}_0$ and $d\bar{M}_1$ are as follows:

$$d\frac{\bar{M}_0}{p} = d\frac{\bar{G}}{p}$$

$$d\bar{M}_1 = \left(\begin{array}{cc} \frac{\delta \frac{N_1}{p}}{\delta \frac{\bar{G}}{p}} & \frac{\delta \frac{N_1}{p}}{\delta \frac{\bar{M}_0}{p}} \end{array} \right) \cdot \frac{-d\frac{\bar{G}}{p}}{\frac{\delta \frac{N_1}{p}}{\delta \frac{\bar{M}_1}{p}}}$$

This compensation leaves the number of debt instruments at the end of the "period", N_1 , constant, but includes a wealth effect by the change of \bar{M}_0 at the beginning of the "period". Both in the case of a bond-financed deficit and a money-financed deficit an unjustified gift to the private wealth owners is included, a procedure which fails to specify the proper wealth restriction.

Until now we bypassed the problem of the financing of the coupon payments. Coupon payments may be financed either by raising taxes, by issuing new debt or by printing new money. All three cases can be analyzed in the context of our model.

Because the interest payments for the increased government debt are first due during the next "period", we can subtract the current tax payments. This net magnitude was included in our previous discussions (sections 2 and 3). But this procedure is only correct, if we assume that the economic agents do not anticipate now either future changes in tax obligations, future changes of the money supply or changes of the stock of government debt outstanding, which are the three modes to finance the changed coupon payments during the next "periods". The general omission of wealth effects in the standard paradigm justified this assumption.

The following discussion outlines the general problem: Suppose that an increase of the government debt at the end of the period, $\frac{N_1 + \bar{N}_0}{\bar{p}_i}$, will lead to an increase in taxes in the next "period" to finance the increased coupon obligations due during the next "period": $\frac{i(N_1 - \bar{N}_0)}{\bar{p}_i}$. Suppose further that the community correctly anticipates this increase and capitalizes these tax obligations at the current market rate.⁶⁾

$$\frac{d\frac{\bar{T}}{\bar{p}}}{i(1+i)}$$

The structure of our model does not allow to compensate for this effect by changing $\frac{\bar{T}}{\bar{p}}$ directly, because \bar{T} is included as the current tax liability both in the computation of disposable income and current government receipts. But we can incorporate this

⁶⁾ The correct discount factor is not i but $i(1+i)$, because the first increase in taxes is due during the next "period", but the loss is accounted for at the beginning of the current "period".

effect by an appropriate variation of s .

$$(4.14) \quad di = \frac{\delta i}{\delta \bar{G}} \frac{d\bar{G}}{p} + \frac{\delta i}{\delta s} ds$$

$$(4.15) \quad dY = \frac{\delta Y}{\delta \bar{G}} \frac{d\bar{G}}{p} + \frac{\delta Y}{\delta s} ds$$

where $ds = \frac{i}{\bar{T}(1+i)} d \frac{\bar{G}}{p}$

If the increased government debt leads to increased future tax obligations and if these future tax obligations are completely capitalized, we should substitute these two equations for the previously stated ones (equations (4.8) and (4.9)). If private wealth owners capitalize future liabilities only incompletely, it is of course possible to adjust accordingly.

Our little model is very flexible with regard to different policy operations. Open market operations, for instance, could be described either by analyzing equal but opposite changes in $\frac{\bar{M}_0}{p}$ and $\frac{\bar{N}_0}{pi}$ - with the appropriate adjustments in \bar{M}_1 and N_1 - or by analysing a pure change in \bar{M}_1 over and above the appropriations necessary for the financing of the budget deficit. The second operation is mathematically much easier to handle, because it does not presuppose any change in the fiscal budget items. The disadvantage is that we have to disentangle the money-finance case of the government deficit from the open-market operation part, in order to assess the correct effects of fiscal and monetary operations. The first case corresponds to the usual procedure proposed in the literature⁷⁾ and can be directly related to our

7) See f.i. Don Patinkin, Money, Interest, and Prices, op.cit., pp. 507-509.

previous discussions.⁸⁾ A detailed analysis of the proposal will be given in section 6.

A pure increase in the money supply, equivalent to a once for all change in the wealth position of the private sector, can be analysed by computing the effects of a proportional change in both \bar{M}_0 and \bar{M}_1 on the endogenous variables. This operation is equivalent to our previous discussion of a change of the money supply under various model assumptions.

We know that monetary and fiscal policy often move in the same direction, which makes it difficult to separate the effects of a ceteris-paribus-fiscal action from a so-called mutatis-mutandis-effect. The first effect is computed on the assumption that the money stock is constant, the second effect presumes a permissive behavior of the central monetary authorities to hold the interest rate constant, to eliminate possible crowding-out effects by an opposite change of private investment expenditure. Our little model allows a first approximation to a more precise definition of these various fiscal and monetary operations.

8) Relying on the concept of the Hicksian week, the first operation would be defined as being carried out at the beginning of the "week", given of course the equilibrium values of all endogenous variables, and the second operation would be conceptually related to the end of the "week".

5. The Liquidity Preference Relation and the Effectiveness of Monetary Policy

The general omission of wealth effects in the standard paradigm does not allow a systematic assessment of different operations leading to an increase in the money supply, i.e. increases brought about by fiscal deficits, open-market-operation, or pure wealth changes. In the last section, we have shown that we can separate the pure increase in the money supply from an increase brought about by fiscal deficits. In this section we try to demonstrate that a careful reformulation of the liquidity preference relation allows a more subtle differentiation of various monetary operations.

To illustrate this we use a variant of version I of our basic model. To this structure we add a wealth restriction

$$(5.1) \quad \frac{W_o}{\bar{p}} = \frac{\lambda Y}{1} + \frac{\bar{N}_o}{\bar{p}i} + \frac{\bar{M}_o}{\bar{p}} - \frac{s\bar{T}}{\bar{p}i}$$

From this equation we can derive a definition of real income

$$(5.2) \quad Y = \frac{iW_o - \bar{N}_o - i\bar{M}_o + s\bar{T}}{\bar{p}}$$

We substitute this definition into the familiar commodity-market equilibrium condition.

(5.3)

$$\frac{iW_o - \bar{N}_o - i\bar{M}_o}{\bar{p}^\lambda} = C\left(\frac{iW_o - \bar{N}_o - i\bar{M}_o + s\bar{T}}{\bar{p}^\lambda} + \frac{\bar{N}_o}{\bar{p}} - \frac{\bar{T}}{\bar{p}}\right) + I(i) + \frac{\bar{G}}{\bar{p}}$$

This equation restates the overall wealth position of the community as a function of the interest rate, the money stock and the number of bonds outstanding at the beginning of the "period", taxes due for the "period" and the volume of government expenditures. The flow magnitudes real income, real consumption and real

investment are implicitly included.

$$(5.4) \quad \frac{W_0}{p} = \varphi\left(i; \frac{\bar{M}_0}{p}, \frac{\bar{N}_0}{p}, \frac{\bar{T}}{p}, \frac{\bar{G}}{p}, \lambda, s\right)$$

$$\varphi_1 < 0; \varphi_2 = 1; \varphi_3 > 0; \varphi_4 < 0; \varphi_\lambda > 0; \varphi_s < 0$$

To analyse the effects of various monetary policies we add our liquidity preference function which is modified by the inclusion of real wealth and a vector R^e describing the individual distribution of future interest rates, or otherwise stated: The information about the current rate and the expected individual future rates allows to derive the distribution of the individual expected holding period yields. In the following discussion we introduce this vector as a parameter. It should be noted that the following exposition does not correctly describe the Keynesian approach to portfolio behavior.¹⁾ Our presentation is a generalisation describing a much broader class of possible portfolio behavior.

$$(5.5) \quad \frac{\bar{M}_0}{p} = \frac{\bar{M}_1}{p} = m \left(\frac{W_0}{p}, \frac{iW_0 - \bar{N}_0 - i\bar{M}_0 + s\bar{T}}{p^\lambda}, i; R^e \right)$$

To simplify this analysis, we assume that a government deficit or surplus is always covered by an increase or decrease of the government bonds outstanding at the end of the "period" and that wealth holders do not anticipate changes in tax obligations related to the financing of the coupon payments.

Given the overall budget restriction, we derive the implicitly defined demand function for real securities, i.e. privately issued

1) For a detailed analysis of the micro- and macro-implications of the Keynesian approach to portfolio behavior compare Hans G. Monissen, "Analyse der Keynesianischen Liquiditätspräferenzfunktion," Kredit und Kapital, 4. Jahrgang, 1971, Heft 1.

stocks and government bonds, remembering the assumption that all private investment is financed by issuing common stock.²⁾ It is obvious that a more sophisticated analysis would separate the common stock market or the market for real capital from the market for long term fixed interest bearing loans, which means that we have to determine the supply price of capital (or the required rate on real capital) in addition to the interest rate on long term government bonds. But this modification leads us into the realms of both the monetarists and the proponents of the "New View".

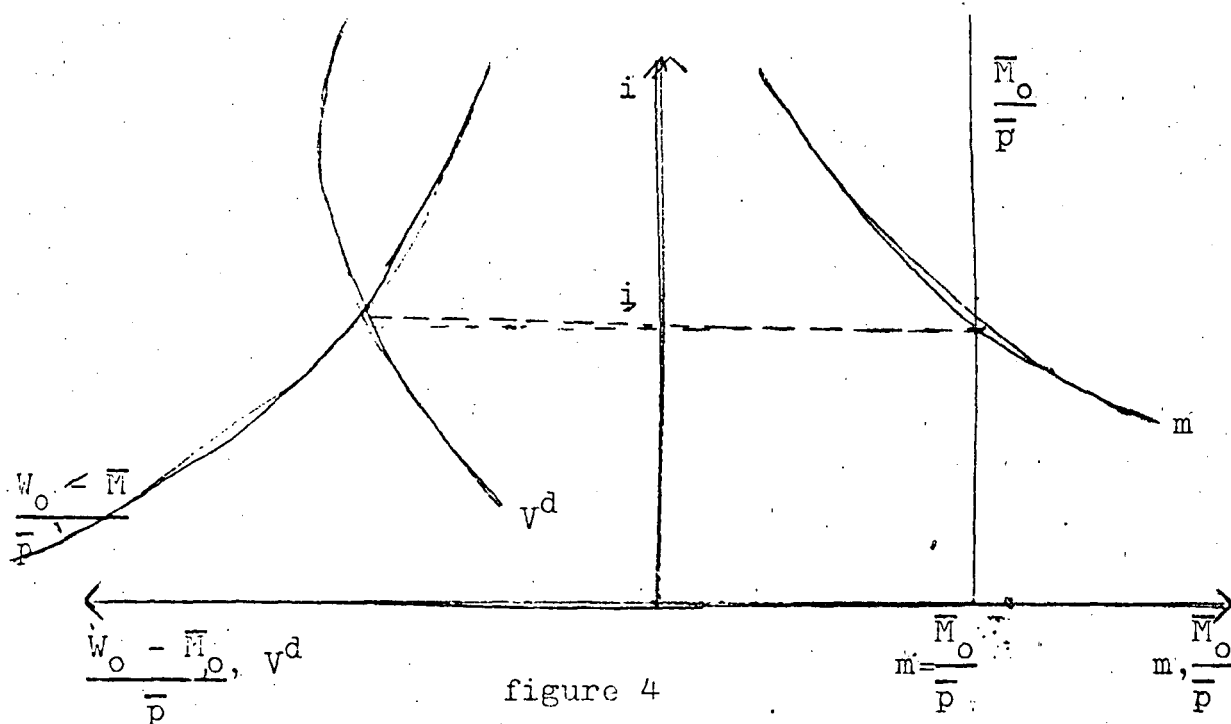
The stock demand for interest bearing claims can be derived from the overall budget restriction and the demand for money function.

$$(5.6) \quad V^d = \frac{W_0}{\bar{p}} - m \left(\frac{W_0}{\bar{p}}, \frac{iW_0 - \bar{N}_0 - i\bar{M}_0 + s\bar{T}}{\bar{p}^\lambda}, i; R^e \right)$$

It is interesting to note that the following two cases lead to equivalent results: We can either derive the demand for securities from the current wealth position or from the wealth position at the beginning of the next "period". It is simpler to formulate a demand function derived from the current wealth restriction.

Figure 4 illustrates the model under the conditions stated.

2) Under certainty our model is compatible with a much broader class of finance assumptions, see Merton H. Miller and Franco Modigliani, "Dividend Policy, Growth and the Valuation of Shares," in Stephen H. Archer and Charles A. d'Ambrosio, eds., *The Theory of Business Finance, A Book of Readings*, New York 1967, pp. 339-366.



Total real wealth is measured by the horizontal difference between the curves $\frac{W_0 - M_0}{\bar{p}}$ and $\frac{W_0 - \bar{M}_0}{\bar{p}}$. This additive connection of the money stock follows from our previously stated derivative ∂_2 . The curve $\frac{W_0 - M_0}{\bar{p}}$ includes the equilibrium condition of the commodity-market and therefore the changes in real income as a consequence of a variation of real investment which depends on the interest rate. Because the commodity-market is included by its equilibrium condition, we know from Walras' law that the equilibrium condition of either one of the remaining markets, i.e. the money and bond markets, does not impose any restriction on the total model structure.

It should be noted that if we exclude the commodity market or introduce investment as a constant, i.e. the interest responsiveness of investment is zero, our model reduces to a simple portfolio model.

For our analytical discussion we will use the model in the following version:

$$(5.7) \quad \frac{\bar{M}_0}{p} = \frac{\bar{M}_1}{p} = m\left(\frac{W_0}{p}, \frac{iW_0 - \bar{N}_0 - i\bar{M}_0 + \bar{T}}{p^\lambda}, i\right)$$

$$(5.8) \quad \frac{W_0}{p} = \psi\left(i, \frac{\bar{M}_0}{p}, \frac{\bar{N}_0}{p}, \frac{\bar{T}}{p}, \frac{\bar{G}}{p}\right)$$

A change of the interest rate leads to an opposite change of the demand for money. The demand for money will change as the result of a wealth or income effect and a substitution effect which work in the same direction. Because the change in the interest rate will lead to an opposite change in real wealth, the sign of a change in the interest rate on the demand for securities is indeterminate.

$$(5.9) \quad \frac{\delta m}{\delta i} = m_1 \psi_1 + \frac{m_2}{\lambda} \left(i\psi_1 + \frac{W_0}{p} - \frac{\bar{M}_0}{p}\right) + m_3 < 0$$

$$(5.10) \quad \frac{\delta V^d}{\delta i} = \frac{\delta \frac{W_0}{p}}{\delta i} - \frac{\delta m}{\delta i} >$$

To derive a set of mutatis-mutandis-relationships expressing various monetary and fiscal operations, we first differentiate the model totally with respect to the exogeneous variables.

$$(5.11) \quad -\left(m_1 + \frac{n_2 i}{\lambda}\right) dW_0 - \left(\frac{n_2}{\lambda} \frac{W_0}{p} - \frac{n_2}{\lambda} \frac{\bar{M}_0}{p} + n_3\right) di \\ = -\left(1 + \frac{n_2 i}{\lambda}\right) d\frac{\bar{M}_0}{p} - \frac{n_2}{\lambda} d\frac{\bar{N}_0}{p} + \frac{n_2}{\lambda} d\frac{\bar{T}}{p}$$

$$(5.12) \quad dW_0 \quad -\psi_1 di \\ = \psi_2 d\frac{\bar{M}_0}{p} + \psi_3 d\frac{\bar{N}_0}{p} + \psi_4 d\frac{\bar{T}}{p} + \psi_5 d\frac{\bar{G}}{p}$$

Using our previously stated derivatives, we see that Δ , the determinant of the coefficients on the left side of equations (5.11) and (5.12), is negative.

$$\Delta = \psi_1 m_1 + m_3 < 0$$

$$(5.13) \frac{\frac{\delta \frac{W_0}{\bar{p}}}{\bar{M}_0}}{\frac{\delta \frac{W_0}{\bar{p}}}{\bar{p}}} = \frac{1}{\Delta} (\psi_1 + m_3) > 0$$

$$(5.14) \frac{\frac{\delta i}{\bar{M}_0}}{\frac{\delta \frac{W_0}{\bar{p}}}{\bar{p}}} = \frac{1}{\Delta} (-m_1 + 1) < 0, \text{ for } m_1 < 1$$

$$(5.15) \frac{\frac{\delta \frac{W_0}{\bar{p}}}{\bar{N}_0}}{\frac{\delta \frac{W_0}{\bar{p}}}{\bar{p}}} = \frac{1}{\Delta} \left\{ \psi_1 \frac{m_2}{\lambda} + \psi_3 \left(\frac{m_2}{\lambda} \frac{W_0}{\bar{p}} - \frac{m_2}{\lambda} \frac{\bar{M}_0}{\bar{p}} + m_3 \right) \right\} \begin{matrix} < \\ > \end{matrix} 0$$

$$(5.16) \frac{\frac{\delta i}{\bar{N}_0}}{\frac{\delta \frac{W_0}{\bar{p}}}{\bar{p}}} = \frac{1}{\Delta} \left\{ -\psi_3 m_1 - \frac{c'}{1-c'} \frac{\lambda}{i} \frac{m_2^i}{\lambda} \right\} > 0$$

$$(5.17) \frac{\frac{\delta \frac{W_0}{\bar{p}}}{\bar{G}}}{\frac{\delta \frac{W_0}{\bar{p}}}{\bar{p}}} = \frac{1}{\Delta} \left\{ \psi_5 \left(\frac{m_2}{\lambda} \frac{W_0}{\bar{p}} - \frac{m_2}{\lambda} \frac{\bar{M}_0}{\bar{p}} + m_3 \right) \right\} \begin{matrix} < \\ > \end{matrix} 0$$

$$(5.18) \frac{\frac{\delta i}{\bar{G}}}{\frac{\delta \frac{W_0}{\bar{p}}}{\bar{p}}} = \frac{1}{\Delta} \left\{ -\psi_5 \left(m_1 + \frac{m_2^i}{\lambda} \right) \right\} > 0$$

$$(5.19) \frac{\frac{\delta \frac{W_0}{\bar{p}}}{\bar{M}_1}}{\frac{\delta \frac{W_0}{\bar{p}}}{\bar{p}}} = \frac{1}{\Delta_1} \psi_1 > 0$$

$$(5.20) \frac{\frac{\delta i}{\bar{M}_1}}{\frac{\delta \frac{W_0}{\bar{p}}}{\bar{p}}} = \frac{1}{\Delta} < 0$$

The sign of the effects of a variation of \bar{N}_0/\bar{p} and \bar{G}/\bar{p} on real wealth is indeterminate, a consequence of the inclusion of the investment function into the wealth restraint. It should be noted, that if we exclude the commodity-market from our analysis, a change

of \bar{M}_0/\bar{p} will always lead to a change of W_0/\bar{p} in the same direction, which means that the increase in the interest rate does not offset the initial change in real wealth brought about by an increase number of government bonds outstanding. Every increase in \bar{M}_0/\bar{p} shifts the money supply curve to the right. From equation (5.14) we derive that a change in the interest rate depends on m_1 , the marginal propensity to hold money out of a given stock of real wealth. We assume that m_1 is greater than zero and less than one. This information allows to derive our first mutatis-mutandis-relationship describing the reaction of the interest rate as a consequence of a change in the money stock (i.e. a change in real wealth).

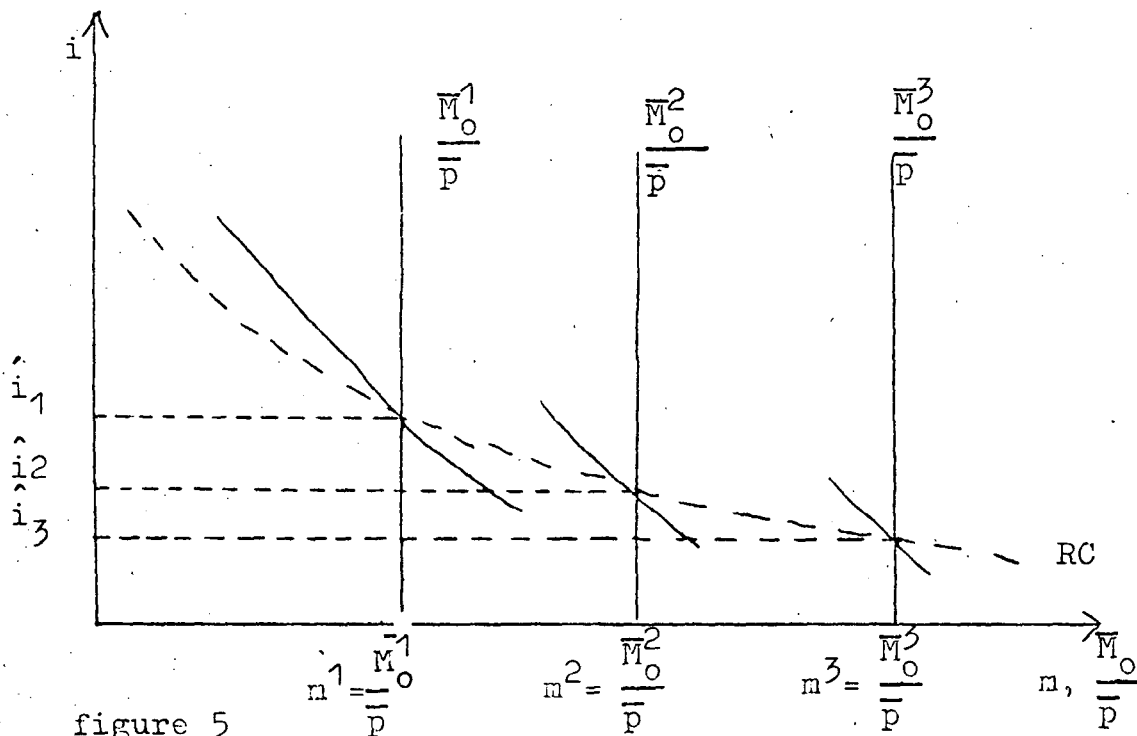


figure 5

If we connect the equilibrium points we derive the reaction curve RC. It should be noted that the resulting curve is a market equilibrium curve and should not be confused with the underlying money demand function. If m_1 is equal to one, the RC is horizontal. This situation could be formally interpreted as the liquidity trap case. But if we assume that the community is inhabited by

bulls and bears, $m_1 = 1$ implies that the increase of the money supply is completely distributed to the latter class, a generally unjustified assumption. Only if a certain minimum interest rate is reached, at which nobody is willing to hold interest bearing claims, the total increase in the money supply leads to an equivalent increase in the demand for money. But as long as the number of interest bearing claims is greater than zero, this minimum rate cannot be reached, or otherwise stated, as long as the capital stock is positive, the current market interest rate is always greater than the minimum rate, at which everybody would convert interest bearing claims into money.

We recall from the last section that an uncompensated change of \bar{M}_1/\bar{p} , \bar{M}_0/\bar{p} kept constant, corresponds to a change of the financing of the government budget. A proportional change of \bar{M}_0/\bar{p} , which in equation (5.7) was set equal to \bar{M}_1/\bar{p} , corresponds to an initial increase in real wealth. We derive the following order condition:

$$(5.21) \quad \frac{\frac{\delta \frac{W_0}{\bar{p}}}{\bar{M}_0/\bar{p}}}{\frac{\delta \frac{W_0}{\bar{p}}}{\bar{p}}} > \frac{\frac{\delta \frac{W_0}{\bar{p}}}{\bar{p}}}{\frac{\delta \frac{W_0}{\bar{p}}}{\bar{M}_1/\bar{p}}}$$

This result is intuitively plausible, because the change of $\frac{\bar{M}_0}{\bar{p}}$ corresponds to a gift to the community, but the change of $\frac{\bar{M}_1}{\bar{p}}$ describes only a change in the composition of the portfolio. From this follows that

$$(5.22) \quad \frac{\frac{\delta i}{\bar{M}_0/\bar{p}}}{\frac{\delta \frac{W_0}{\bar{p}}}{\bar{p}}} > \frac{\delta i}{\frac{\delta \frac{W_0}{\bar{p}}}{\bar{p}}}$$

The two demand curves in figure 4 depict the conditions for discrete open-market operations. We may shift the two supply curves along the demand curves and reach any projected target

interest rate. From the construction follows that in any point the excess demand for money is equal to the excess demand for securities. Fixing any interest rate the monetary authority buys (sells) the negative (positive) excess demand for securities by issuing (withdrawing) the equivalent positive (negative) excess demand for money. Technically we analyse open-market operations as follows:

$$(5.23) \quad d \frac{W_o}{p} = \frac{\frac{\delta W_o}{p}}{\frac{\delta \bar{M}_o}{p}} d \frac{\bar{M}_o}{p} + \frac{\frac{\delta W_o}{p}}{\frac{\delta \bar{N}_o}{p}} d \frac{\bar{N}_o}{p}$$

$$(5.24) \quad di = \frac{\frac{\delta i}{\frac{\delta \bar{M}_o}{p}}}{\frac{\delta \bar{M}_o}{p}} d \frac{\bar{M}_o}{p} + \frac{\frac{\delta i}{\frac{\delta \bar{N}_o}{p}}}{\frac{\delta \bar{N}_o}{p}} d \frac{\bar{N}_o}{p}$$

where we have to use the following restriction

$$d \frac{\bar{M}_o}{p} i + d \frac{\bar{N}_o}{p} = 0$$

The sign of the first expression is indeterminate, the sign of the second expression is negative, if we use the stated restriction and increase the money supply, i.e. we analyse open-market purchases. An open-market sale leads to an analogous increase in the market interest rate.

As a consequence of the interest-induced wealth effect in the demand for money function this curve will rotate in the new equilibrium position. The original curve and the new curve have one common point, the new equilibrium point. But the curve is generally flatter, if the monetary authority has engaged in open-market sales, and generally steeper, if the new equilibrium is the consequence of an open-market purchase. From this we derive the interesting result that discrete open-market operations are

irreversible. Starting from a low interest rate to a higher one with a consequent reversal to the beginning rate will enrich the community. The opposite operation will lead to an impoverishment.

The process is reversible, if we use open-market operation continuously. The resulting interest reaction curve is an envelope to the array of money supply curves which we can construct in any point on a continuously covered interest rate spectrum.

6. Supplement: The Role of Money Supply Analysis in Keynesian Economics

Until now we have omitted any discussion of the process underlying the supply of money. But this omission only reflects the attitude of Keynesian thinking, namely the refusal to accept relative price theory as an analytical frame for analyzing economic processes. When Harry G. Johnson commented in his survey of Monetary Theory published in 1962 that money supply theory had been thoroughly neglected in monetary analysis, the criticism was certainly addressed to mainstream Keynesianism and the associated income-expenditure approach. The more recent interest in money supply analysis is connected with two different, but definitely Non-Keynesian groupings, the monetarists and the proponents of the "New View".

If Keynesians refer to the money supply theory then either assume that the money stock is given, i.e. could be absolutely controlled by the monetary authorities, or, after a very sketchy discussion of bank money multiplier, they offer the other extrem, writing the supply of money as a function of the interest rate, completed by further arguments as f.i. the level of free reserves.

Our following discussion tries to illustrate the following points: The basic Keynesian portfolio assumption, offering only a choice between long-term interest bearing claims and money, if combined with the Keynesian transmission mechanism, reduces the money supply analysis to a mathematical exercise. The Keynesian approach does not ascribe an independent role to the credit market in addition to other markets for loanable funds. This leads to the consequence that the IS-LM frame is still the best summary description of the Keynesian system. Our analysis could be used for a thorough investigation of some of the recently formulated large scale econometric models which are moulded according to the standard frame: Relying on the borrowing conception of the transmission mechanism but blowing up the monetary subsystems. To state our results a little bit dramatically we vary the Brunner-Meltzer money supply theory in such a way as to derive an ultra-Keynesian portfolio model. We start with the following balance sheets:

Banking Sector

$\frac{R}{P}$	total commercial banks reserves	$\frac{D^s}{P}$	demand deposit of the public
	$\frac{R}{P} = \frac{R_c}{P} + \frac{R_r}{P}$	$\frac{D_g^s}{P}$	demand deposit of the government
	excess reserves + required reserves	$\frac{D^t}{P}$	time deposits
$\frac{E}{P}$	earning assets net of capital accounts	$\frac{A}{P}$	discounts and advances
		$\frac{W}{P} = 0$	

Private Non-Banking Sector

$(\frac{\lambda Y}{i} + \frac{\bar{N}_o}{pi} - \frac{E}{p})$	$\frac{s\bar{T}}{pi}$	discounted value of the tax obligations
net holdings of long term interest bearing claims	$\frac{W_o}{p}$	net wealth of the private sector
$\frac{M_c}{p}$		currency held by the public
$\frac{D^s}{p}$		demand deposits of the public
$\frac{D^t}{p}$		time deposit

Consolidated Balance Sheet of the Private Sector

(Gurley-Shaw-Case)

$\frac{\lambda Y}{i}$	market value of the real capital stock	$\frac{D^s}{p}$	demand deposits of the government
$\frac{\bar{N}_o}{pi}$	government securities outstanding	$\frac{A}{p}$	discounts and advances
$\frac{M_c}{p}$	currency held by the public	$\frac{s\bar{T}}{pi}$	discounted value of the future tax obligations
$\frac{R}{p}$	total commercial-bank reserves	$\frac{W_o}{p}$	net wealth of the private sector

Consolidated Balance Sheet of the Private Sector

(Posek-Saving-Case)

$\frac{\lambda Y}{i}$	market value of the real capital stock	$\frac{A_p}{pi}$	market value of discounts and advances
$\frac{\bar{N}_o}{pi}$	government securities outstanding	$\frac{s\bar{T}}{pi}$	discounted value of the future tax obligations
$\frac{M_c}{p}$	currency held by the public		

$\frac{D_p^S}{p} + \frac{D_g^S}{p}$	total demand deposits	$\frac{R}{p} + \frac{i^t D^t}{pi}$	discounted values of the debt components of demand and time deposits
$\frac{D^t}{p}$	time deposits		
$\frac{R}{p}$	total commercial bank reserves	$\frac{W_o^*}{p}$	net wealth of the private sector

From the last consolidated statement we could derive the corresponding balance sheet of the banking sector under Pesek-Saving assumptions. The following discussion is based on the Gurley-Shaw assumptions, but all results can be restated in terms of the Pesek-Saving procedure.

From the first consolidated balance sheet of the private sector we derive the overall budget restriction:

$$(7.1) \quad \frac{W_o}{p} = \frac{\lambda Y}{i} + \frac{N_o}{pi} + \frac{M_c}{p} + \frac{R}{p} - \frac{D_g^S}{p} - \frac{A}{p} - \frac{sT}{pi}$$

Using the definition of the adjusted base the definition simplifies to

$$(7.2) \quad \frac{W_o}{p} = \frac{\lambda Y}{i} + \frac{N_o}{pi} + \frac{B_o^a}{p} - \frac{D_g^S}{p} - \frac{sT}{pi}$$

If we disregard the demand deposits of the government, we can interpret the adjusted monetary base as the money stock according to our previous discussions in which we abstracted from the existence of banks.

We have two interesting options: First we could explain the joint demand of both commercial banks and the non-bank public for the autonomously given stock $\frac{B_o^c}{p}$. This would preserve a complete formal analogy to the analysis of our last section. This means that, if we define the money supply as the adjusted monetary base, all our previously stated results remain valid. There is nothing

wrong with such an interpretation in the Keynesian world. This formulation can be directly related to the discussion of the real-balance effect which is based on this narrower wealth definition.

The other option is to use a money conception in the usual definition. We will show that the first option can be easily derived from this second option.

In addition to our interest rate on all kinds of long term claims i , we introduced the interest rate on time deposits. We can get rid of this rate by a typical Keynesian device, namely postulating a price-setting function, a functional relationship between the two rates:

$$(7.3) \quad i^t = i^t(i), \quad i^t > 0$$

Following the Keynesian tradition to partition all financial and real assets into two broad classes, it is more appropriate to use a money demand function in a broader sense, including time deposits.

$$(7.4) \quad \frac{M}{P} = m\left(\frac{W_0}{P}, Y, i, i^t(i); R^e\right) = \frac{D_p^s}{P} + \frac{M_c}{P} + \frac{D^t}{P}$$

The bank behavior is completely described by the following system of eleven equations determining eleven endogenous variables.

$$(7.5) \quad \frac{B}{P} = \frac{R}{P} + \frac{M_c}{P} \quad \text{uses of the monetary base}$$

$$(7.6) \quad \frac{B^a}{P} = \frac{B}{P} + \frac{A}{P} \quad \text{the adjusted monetary base}$$

$$(7.7) \quad \frac{M}{P} = \frac{D_p^s}{P} + \frac{M_c}{P} + \frac{D^t}{P} \quad \text{the money supply}$$

$$(7.8) \quad \frac{E}{P} = \frac{D_p^s}{P} + \frac{D^t}{P} + \frac{D_c^s}{P} + \frac{A}{P} - \frac{R}{P} \quad \text{earning assets of commercial banks}$$

$$(7.9) \quad \frac{R}{P} = \frac{R_r}{P} + \frac{R_e}{P} \quad \text{allocation of total reserves}$$

$$(7.10) \quad \frac{R_r}{P} = \bar{r}^d \left(\frac{D_p}{P} + \frac{D_c^s}{P} \right) + \bar{r}^t \frac{D^t}{P} \quad \text{required reserves}$$

$$(7.11) \quad \frac{R_e}{p} = c[i, i^t(i), \bar{p}, \bar{r}^d, \bar{r}^t] \left(\frac{D_p^s}{p} + \frac{\bar{D}_G^s}{p} + \frac{D^t}{p} \right) \text{ excess reserves}$$

$$e_1 < 0, e_2^? < 0, e_3^? > 0, e_4^? < 0, e_5^? < 0$$

$$(7.12) \quad \frac{A}{p} = b[i, i^t(i), \bar{p}, \bar{r}^d, \bar{r}^t] \left(\frac{D_p^s}{p} + \frac{D_G^s}{p} + \frac{T}{p} \right) \text{ commercial bank borrowings}$$

$$b_1 > 0, b_2^? > 0, b_3 < 0, b_4^? > 0, b_5^? > 0$$

$$(7.13) \quad \frac{M_c}{p} = k \left(\frac{W_0}{p}, \frac{T}{p} \right) \frac{D_p^s}{p} \text{ allocation ratio}$$

$$k_1^? < 0, k_2^? < 0$$

$$(7.14) \quad \frac{D^t}{p} = t \left(\frac{W_0}{p}, Y, i, i^t(i); R^e \right) \frac{D_p^s}{p} \text{ allocation ratio}$$

$$t_1 > 0, t_2^? > 0, t_3 < 0, t_4 > 0$$

$$(7.15) \quad \frac{D_F^s}{p} = d \frac{D_p^s}{p} \text{ government behavior}$$

If we fix i, p, Y in addition to the policy variables $\bar{B}^a, \bar{r}^d, \bar{r}^t, \bar{p}, \bar{d}$, we are left with eleven linear equations in eleven endogenous variables:

$$\frac{B}{p}, \frac{R}{p}, \frac{M_c}{p}, \frac{A}{p}, \frac{M}{p}, \frac{D_p^s}{p}, \frac{D^t}{p}, \frac{E}{p}, \frac{D_G^s}{p}, \frac{R_r}{p}, \frac{R_e}{p}$$

From these equations we derive the money multiplier defined as follows:

$$(7.16) \quad \frac{M}{p} = \frac{1 + t + k}{e(1+t+d) + \bar{r}^d (1+d) + t \bar{r}^t - b(1+t+d) + k} \frac{\bar{B}^a}{p}$$

using the definition

$$r = \frac{1 + d}{1+t+d} \bar{r}^d + \frac{t}{1+t+d} \bar{r}^t + e$$

we arrive at the basic Brunner-Meltzer equation

$$(7.17) \quad \frac{M}{p} = \frac{1 + t + k}{(r-b)(1+t+d) + k} \frac{\bar{B}^a}{p}, \text{ for } r > b$$

We once more recall that we use the money definition in the broader sense including time deposits.

To derive a demand function for adjusted base money, we need some further relationships:

(7.18) - (7.19)

$$\frac{A}{P} = \frac{b(1+t+d)}{(r-b)(1+t+d) + k} \frac{\bar{B}^a}{P} ; \frac{D_n^s}{P} + \frac{D^t}{P} = \frac{1+t}{(r-b)(1+t+d) + k} \frac{B^a}{P}$$

(7.20) - (7.21)

$$\frac{R}{P} = \frac{(1+d)(\bar{r}^d+e) + t(\bar{r}^t+e)}{(r-b)(1+t+d) + k} \frac{\bar{B}^a}{P} ; \frac{D_g^s}{P} = \frac{d}{(r-b)(1+t+d) + k} \frac{\bar{B}^a}{P}$$

As a policy model it is very convenient to express the endogenous variables as functions of the adjusted monetary base, a variable under direct control of the monetary authorities. But as a theoretical model this device is less convenient, because we have to decide, how this adjustment of the monetary base should be carried out. This problem could be solved in various ways: We could offset any change in A by an appropriate change of the parameters \bar{r}^d , \bar{r}^t or \bar{B} or combinations of these parameters. Or we could formulate our policy operations in terms of both the monetary base and the adjusted monetary base using appropriate restrictions mathematically defined similarly to the ones used in our previous discussion. But for our purposes we can solve this problem simply by ignoring the possibility of commercial bank borrowings with the central monetary authority.

After this somewhat heroic assumption, we can proceed by a restatement our basic equations left:

(7.22)

$$\frac{W_o}{P} = \frac{\lambda Y}{i} + \frac{N_o}{P_i} + \frac{B_o^a}{P} - \frac{d}{(r-b)(1+t+d) + k} \frac{B_o^a}{P} - \frac{s\bar{T}}{P_i} \quad \text{wealth definition}$$

$$(7.23) \quad Y = C\left(Y + \frac{\bar{N}_0}{p} - \frac{\bar{T}}{p}\right) + I(i) + \frac{\bar{G}}{p} \quad \text{commodity market equilibrium}$$

(7.24)

$$\frac{\bar{B}_0^a}{p} = \frac{(1+d)(\bar{r}^d + e) + (t(\bar{r}^t + e))}{(r-b)(1+t+d) + k} \frac{\bar{B}_0^a}{p} + m\left(\frac{W_0}{p}, Y, i, i^t(i); R^e\right) - \frac{1+t}{(r-b)(1+t+d) + k} \frac{\bar{B}_0^a}{p} \quad \text{market equilibrium for adjusted base money}$$

resp.

(7.25)

$$\frac{1+t+k}{(r-b)(1+t+d) + k} \frac{\bar{B}_0^a}{p} = m\left(\frac{W_0}{p}, Y, i, i^t(i); R^e\right) \quad \text{money market equilibrium}$$

If we either use the market equilibrium condition for adjusted base money or the money market equilibrium condition for money, we are left with three equations in three unknown variables, $\frac{W_0}{p}$, Y and i . To complete the multiplier expressions we have to substitute the above defined quotient functions.

Because the formal analysis is completely analogous to our previous derivations there is no reason to go into the technical details. Because the IS curve can be derived in the familiar way, there is no change of the structure of the underlying transmission mechanism. We could use either the equilibrium condition for adjusted base money or for money to define in the broader sense to drive a LM curve. On the other side we could solve the first two equations for real income and define a modified wealth restriction. In this case the diagrammatical device developed in the last section could be used to focus on the money market. Once more we could rely on either of the remaining equations to solve for the interest rate.

We should note that the demand and supply functions for earning assets are residuals provided for by Walras' law. Neither equation imposes any restriction on the remaining equation system. Consistent with the Keynesian approach we introduced the market equivalents for real capital $\frac{\lambda Y}{i}$, government bonds $\frac{N_0}{p_i}$, the present value of future tax payments $\frac{sT}{p}$ and bank earning assets E as homogeneous goods, representatives of the class of long term interest bearing claims. This is certainly an ultra-Keynesian approach which gives some hints how to reformulate and reconstruct the Keynesian model, namely by incorporating both an independent market for real capital and an independent market for bank credit. The relative prices determined on these markets will enter all the expenditure functions which really modifies and extends the narrow Keynesian conception of the transmission mechanism.

List of Symbols

- A discounts and advances in nominal terms
- B monetary base in nominal terms: $R + M_c$
- B^a adjusted monetary base in nominal terms: $B - A$
- C real expenditure: for final output of consumers, resp. output of consumption goods in the two-sector model
- C^g nominal expenditures for consumption goods of the government
- C^d real expenditure for consumption goods of consumers
- D_g^s demand deposits of the government in nominal terms
- D_p^s demand deposits of the public in nominal terms
- D^t time deposits in nominal terms
- G nominal expenditures for final output of the government
- I real expenditures for final output of investors, resp. output of investment goods in the two-sector model
- I^c real expenditure: for investment goods of the consumption goods industry
- I^g nominal expenditure: for investment goods of the government
- I^i real expenditure: for investment goods of the investment goods industry
- i market interest rate: nominal rate equal to real rate
- i^t interest rate on time deposits
- earning assets assets of commercial banks net of capital accounts in nominal terms
- K total physical capital stock
- K^c capital stock of the consumption goods industry
- K^i capital stock of the investment goods industry
- L total labor input
- L^c labor input in the consumption goods industry
- L^i labor input in the investment goods industry
- M nominal money stock
- m_c currency held by the public in nominal terms
- N number of standard bonds promising to pay a dollar per period

p absolute price level, resp. price of consumption goods in the two-sector model

q price of investment goods

R total commercial bank reserves in nominal terms

R^e vector describing the individual expected interest rates

R^e excess reserves in nominal terms

R^r required reserves in nominal terms

r^d reserve - requirement ratio for demand deposits

r^t reserve-requirement ratio for time-deposits

T nominal tax payments

V^d real net demand for interest bearing claims

W total, nominal wealth of the private sector

w nominal wage rate

Y real gross national product

α autonomous investment

λ distribution parameter

ρ discount rate

ξ tax capitalization parameter