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## Income demand and labor supply: Robbins and Buchanan revisited

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INCOME DEMAND AND LABOR SUPPLY:  
ROBBINS AND BUCHANAN REVISITED

Gerhard Michael Ambrosi

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INCOME DEMAND AND LABOR SUPPLY:  
ROBBINS AND BUCHANAN REVISITED

Introduction

Several years ago J.M. Buchanan<sup>1)</sup> pleaded to return labor supply analysis to the "simple and essentially correct explanation by Lionel Robbins in 1930<sup>2)</sup>"(p.383) which rested on an income demand curve. In developing his argument, Buchanan resorted to a type of demand analysis which Friedman had claimed to be Marshallian.<sup>3)</sup> From reading Robbins's article it becomes quite clear, however, that he did not use the so-called "Marshallian" demand concept. Noticing this discrepancy one would not be astonished if Buchanan's reasoning were criticized by other economists and indeed it was. But when A. Leijonhufvud<sup>4)</sup> rebuked Buchanan's plea in the journal "History of Political Economy" he did not employ this argument of doctrinal history just mentioned. He rather used an analytical justification: "The alternative apparatus suggested by Buchanan should not be adopted. Since the conclusions it generates are essentially arbitrary, nothing can be learned from its use."(p.266). In replying<sup>5)</sup> Professor Buchanan said he did not

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- 1) James M. Buchanan "The Backbending Supply Curve of Labor: An Example of Doctrinal Retrogression?" History of Political Economy 3 (Fall 1971) 383-90
  - 2) Lionel Robbins "On the Elasticity of Demand for Income in Terms of Effort" Economica 10 (June 1930) 123-29
  - 3) Milton Friedman "The Marshallian Demand Curve" Journal of Political Economy (December 1949) 463-74
  - 4) Axel Leijonhufvud "The Backbending Supply Curve of Labor: Comment on Buchanan; with his Reply" History of Political Economy 5 (1973) 261-67
  - 5) "Reply by Professor Buchanan" History of Political Economy 5 (1973) 266-67

"argue for the use of any specific 'alternative apparatus'"(p.267). The main point of his paper according to him was rather that a supply curve for labor could be constructed which is "fully consistent with a theory of demand that is, itself, consistent in a full general equilibrium setting"(p.266f).

If we now examine the results of the debate between Leijonhufvud and Buchanan we will find that it was rather inconclusive on a number of points and so we propose to have a closer look at the following issues: Firstly, an evaluation of the merits of Robbins's analysis of income demand relative to the standard textbook treatment of labor supply as described by Buchanan is still missing. Secondly, up to now the question has not been answered satisfactorily, why Buchanan's application of Friedman's "Marshallian" demand concept turns out to be inconsistent. Finally we will deal with Buchanan's search for a supply curve for labor which is "fully consistent with a theory of demand that is, itself, consistent in a full general-equilibrium setting."

The Model of Household behavior: Robbins and the Standard Textbook

It is strange that neither Buchanan, favoring Robbins's income demand curve nor Leijonhufvud, favoring the labor supply curve, actually utilize the respective curves. Rather they rely on the indifference-curve-budget-constraint representation of the choice situation of the household. Thus there are in total three different ways of depicting the labor supply decisions of a household and it may be helpful to demonstrate at the outset of the discussion just how they are interrelated.

(p.25 below)

Consider figure one and regard point Q in the first quadrant. Real income in terms of goods(G) is measured on the horizontal axis while leisure(L) is measured along QO, QO being total time available. Assuming that the household has a well-ordered utility function  $U(G,L)$  we draw the indifference curves  $11'$  and  $22'$  through points  $P_1$  and  $P_2$ . Given certain value functions, e.g. those represented by the lines  $OG_1$  and  $OG_2$ ,  $P_1$  and  $P_2$  represent equilibrium allocations of goods and leisure for an utility maximizing household. It may be mentioned that the figure so far discussed corresponds to Buchanan's figure 1 (The Backbending... op.cit. p.388), in particular our point  $P_1$  corresponds to his  $E''$  and our  $P_2$  corresponds to his point E. The locus of all equilibrium points is Od and this curve is comparable to Leijonhufvud's "endowment-constant reciprocal demand curve"(op.cit.p.262).

We can easily construct a labor supply curve on the basis of the information contained in the first quadrant. There is a specific price system associated with the value function that corresponds to  $OG_1$ . It may be represented by a vector originating <sup>from O with</sup> (positive or negative) direction to  $P_1$ . Noticing that the real wage associated with  $P_1$ , and expressed by  $\tan \angle G_1 O Q$ , may also be expressed by  $\tan \angle R_1 O p_0$ , and that the real wage associated with  $P_2$  may be expressed as  $\tan \angle R_2 O p_0$ , we fix the price component of the real wage vector at  $p_0$ . Then we can express <sup>the</sup> change in the real wage that goes with a transition from  $P_1$  to  $P_2$  as a change from  $(w/p_0)_1$  to  $(w/p_0)_2$  along the horizontal axis. The amounts of time demanded by the household in  $P_1$  and  $P_2$  are  $QE_1$  and  $QE_2$ , respectively. These points form the coordinates for  $S_1$  and  $S_2$  in the second quadrant.  $S_1$  and  $S_2$  lie on  $ss'$ , the locus of real wage rates and leisure demanded which is generated while moving along  $Od$  in the first quadrant. The curve  $ss'$  may be considered as the labor supply curve if time is not measured from  $Q$  onwards, but rather starting from point  $O$ . It is readily seen that the curve may be backbending (as in the present case), an outcome which depends on the elasticity of substitution in consumption of leisure and goods.<sup>1)</sup>

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1) If the elasticity of substitution is constant, no backbending occurs. If it is equal to unity labor supply is inelastic to real wage changes. See G. Hanoch "The "Backward-Bending" Supply of Labor" Journal of Political Economy 73 (1965) 636-42

The discussion so far has generated what Buchanan calls "the standard textbook discussion of the backbending supply curve for labor". Proceeding in an exactly analogous way we may now derive a demand curve for real income in terms of goods in the fourth quadrant of figure 1. There the vectors  $O'P_1'$  and  $O'P_2''$  have the same direction as the vectors  $OP_1$  and  $OP_2$  in the first quadrant. The reciprocal of the real wage is given by  $\tan \angle P_1'O'I_1'$  and by  $\tan \angle P_2''O'I_2'$  and may be measured as  $(p/w_0)_1$  and  $(p/w_0)_2$  along the vertical axis in the fourth quadrant, if the wage component of the vector of relative prices is fixed at  $O'w_0$ . Combining these relative prices with the respective quantities demanded, i.e.  $I_1'$  and  $I_2'$ , the points  $P_1'$  and  $P_2'$  are described. The curve  $dd'$  is the locus of such points generated while moving for a corresponding stretch along  $Od$ .

It may be noted that the demand curve for real income in the fourth quadrant looks "normal" while the demand curve for leisure in the second quadrant does not. But what is abnormal in the present context is not the shape of one of the curves, rather it is the usage of the word "normal". The two curves are quite different and really not comparable to each other because they are derived for analytically quite distinct cases. An increase in the price of leisure ( $w$ ) from  $(w/p^0)_2$  to  $(w/p^0)_1$  causes the value line  $OG_2$  to pivot to  $OG_1$ . The attainable set of the household, formerly given by the triangle  $G_2OQ$ , has increased. On the other hand, in the case of an increase in the price of income ( $p$ )



from  $(p/w^0)_1$  to  $(p/w^0)_2$  the value line pivots from  $OG_1$  to  $OG_2$ , thereby decreasing the attainable set of the household. Normally, when a demand curve is derived, the increase in the price of the good in question results in a shrinking of the attainable set, as in the case of the income demand curve, and not in an enlargement of the set, as in the case of the leisure demand curve. Since it is - or should be - the norm in academic discussions not to compare uncomparables, a result obtained in an analytic setup where price increases increase the attainable set should not be compared to results obtained in a setup where price increases decrease the attainable set. Therefore no statements concerning "normalcy" should be made if it is observed that the curve in the second quadrant of figure one does not have the same characteristics as the one in the fourth quadrant.

Nevertheless, given certain choice theoretic assumptions the three curves under consideration -  $Od$ ,  $ss'$ , and  $dd'$  - convey the same information concerning labor supply. Each one will tell e.g. whether labor supply is backbending or not. Consider point  $P_2$  along  $Od$ . We know that

$$E_2P_2/E_2O = (w/p)_2 \text{ i.e. } E_2P_2(p/w)_2 = E_2O .$$

Expressed in words this last relationship tells us that the amount of goods demanded times the reciprocal of the real wage is equal to the nonleisure (i.e. working) time of the household. Thus in the fourth quadrant the area of rectangle  $O'I_2P_2'(p/w_0)_2$  which expresses just this product should measure the amount of work supplied in point  $P_2$ . A transition from  $P_2$  to  $P_1$  involves a reduction of work supplied from  $OE_2$  to  $OE_1$ . From this consideration we are led to expect that rectangle  $O'I_1P_1'(p/w_0)_1$  should be smaller than the previous one and indeed it is. In general it may be said that the supply of labor curve will have a negative elasticity if the demand for income curve has an elasticity less than one.

Alternatively to the derivation of labor supply from the income demand curve one may start out from the labor supply curve in the second quadrant and convey the corresponding income demand by a similar construction. Each curve may thus be drawn from the information contained in the other ( given the value of  $p_0$  or alternatively  $w_0$  ). Each representation of labor supply implies the other - the  $Od$ -curve, the  $ss'$ -curve, and the  $dd'$ -curve.

If we now look up the original article by Robbins (op.cit.) and compare his analysis to the one that we derived here from the "standard textbook" representation of labor supply we will see immediately that they are identical. The  $dd'$ -curve

in his first figure is quite similar to the one drawn here in the fourth quadrant and a comparison of the  $O_d$  curve in his second figure will reveal a correspondence to the one depicted in our first quadrant. Since the "standard textbook" model is a real income model we may safely equate our "Goods" with Robbins's "Quantity of Income" and since work effort is measured in our first quadrant along  $OQ$ , starting from  $O$ , we may call this axis also the "Quantity of Effort" axis as Robbins has done. Thus the "standard textbook" model of the first quadrant may be considered as identical to Robbins's second figure. In view of the fact that the  $dd'$ -curve is implied by the  $O_d$ -curve it seems somewhat misleading when Buchanan (The Backbending... op.cit. p.383) considers it as "perhaps unfortunate that Robbins paired a demand curve for income, his first diagram, with an offer curve of total effort in a second diagram..". From the discussion above it should be quite clear that the two curves in question are not "unfortunate pairs" but rather "patheno-genetic offsprings" the one implying the other.

Buchanan's Application of "Marshallian" Demand Concepts

In arguing against the standard analysis of labor supply Buchanan set out to demonstrate that the backward bending labor supply curve does not depend on income effects. This he showed by assuming that changes in real wages are compensated so that the household under consideration is always kept on the same level of utility. It may be noted as a historical aside that Friedman (op.cit.) claimed that such thought experiments are limiting cases of Marshallian demand analysis.

Turning to figure 1 in order to discuss this case we start from point  $P_2$  in the first quadrant and remember that this point is represented by  $P'_2$  in the fourth quadrant. Now the real wage is increased from the prevailing level  $(w/p)_2$  to  $(w/p)_1$ . Normally the household would move along  $Od$  to  $P_1$ , but he is kept from so doing by appropriate compensatory devices. Instead he moves along the original indifference curve to  $P_3$ . Marking the corresponding point in the fourth quadrant (which is characterized by the co-ordinates  $(p/w_0)_1$  and  $I'_3$ ) with  $P'_3$ , and repeating this thought experiment for several other real wage rates, we obtain the "Marshallian" income demand curve  $P'_2P'_3$ . Obviously this curve is less

elastic than the  $dd'$ -curve (in the comparable range) which already was inelastic enough to generate a backward bending labor supply curve. We are thus led to the conclusion that this income compensated thought experiment leads to (even stronger) backbending labor supply behavior. This conclusion is confirmed by noticing that the area of the rectangle  $O'I_2P'_2(p/w_0)_2$  which depicts the labor supply at the real wage  $(w/p)_2$  shrinks to the considerably smaller proportions of  $O'I_3P'_3(p/w_0)_1$  when the higher real wage  $(w/p)_1$  is offered. Buchanan can now claim he has proven that it is redundant to resort to income effects in order to explain backbending labor supply phenomena. These phenomena appear even if income effects are assumed away.

The challenge of Leijonhufvud (op.cit.) can be demonstrated in the present framework by pointing out that in the second quadrant the point which corresponds to  $P_3$  in the first quadrant is  $S_3$  and the stretch  $S_2S_3$  generated by similar changes in the real wage suggests not backward bending but forward falling labor supply. Thus for the same thought experiment - a transition from  $P_2$  to  $P_3$  in the first quadrant - we obtain two conflicting results: A prediction of an increase in labor supply if looking at  $S_2S_3$  in the second quadrant and a prediction of a decrease in labor supply if looking at the rectangles

associated with  $P_2'$  and  $P_3'$  in the fourth quadrant. Certainly a paradoxical conclusion to which Leijonhufvud has rightly called attention.

In explaining the paradox we will resort to the following equation:

$$pG = wE - pA$$

where  $G$  = goods,  $E$  = work effort,  $A$  = changes in assets measured in units of goods,  $p$  = price of goods,  $w$  = nominal wage.

In Robbins's and in standard analysis of labor supply,  $A$  is always considered to be zero. From  $pG = wE$  one then obtains the relation  $E = (p/w)G$ , the right-hand side of this expression being described by the rectangle formed by the axes and the co-ordinates of a point along  $dd'$ . In these cases we may look at the goods demanded by a household at a specific real wage and infer from this observation how much work he is prepared to supply. Or we may look at his supply of work and infer the demand for goods. But as soon as changes in assets are admitted this analysis breaks down. In this case it is impossible to tell e.g. how much work a household is prepared to supply by merely looking at his demand for goods. He may run down assets and consume more per time unit than he could finance by work effort alone. Or the household under consideration may have to pay a tax and will have to work more than one would expect if one only regarded the planned purchases of goods.

In all these cases where  $A$  is not zero the relationship  $E = (w/p)(G+A)$  holds.

In the context of figure 1 this means that in the standard case the value function is only permitted to pivot in the constant endowment point  $O$ . In the Buchanan case the value function changes as if the household in question were accumulating assets when the real wage is increased. For point  $P_3$ , the change in assets would be  $OT$  of goods. Considering this we will regard it in the present context as quite wrong to employ an analysis as if  $A$  were zero. If a situation characterized by  $P_3$  in the first quadrant is to be analyzed in the framework of the fourth quadrant, the work effort supplied is not given by  $(p/w)G = O'I_3^1P_3^1(p/w_0)_1$  but by  $(p/w)(G+A) = UI_3^1P_3^1V$ . Comparing now this area to the one associated with point  $P_2'$ , where  $A$  is zero, we will see that it is indeed larger. Therefore we will conclude that as the real wage is increased from  $(w/p)_2$  to  $(w/p)_1$  work effort will increase. Thus the analysis becomes consistent with the one in the second quadrant which suggested a forward falling labor supply curve. In a similar way the time used for asset formation must be taken into account when trying to infer the demand for goods from the labor supply curve in the second quadrant. If that is done it will become apparent that the three representations in the three quadrants are still consistent.

Of course it is quite immaterial whether the household "reilly" accumulates  $OT$  of assets or pays a tax of equal amount or is locked up for  $OT$  of time units during which he could have earned  $OT$  of goods - as long as nothing happens to the shape and position of the indifference curves. In each case the household is deprived of consuming a certain amount of goods and/or time "now", i.e. in the time period analyzed, and our point is that these deprivation effects must not be neglected.

The difference between the two representations of labor supply in the second and in the fourth quadrant may now be explained as a difference in the identification of the "deprivation effect": In the second quadrant it is represented inside the quadrant, e.g. at  $(w/p_0)_1$  it is  $(\frac{w}{p_0})_1 F' = OF$  whereas the "Marshallian" demand curve in the fourth quadrant relies on a representation outside that quadrant. At  $(p/w_0)_1$ , for example, it is the distance  $(p/w_0)_1 V = OT$  in the third quadrant. Only when these "deprivation effects" are not taken into account is Buchanan vulnerable to the charge of having presented an inconsistent analysis.



Labor Supply in a General Equilibrium Setting

As already quoted, Professor Buchanan considered as his main point that labor supply analysis should be consistent with a full general equilibrium setting. His emphasis was, as he says, upon "the development of an analytical framework that helps us to explain the workings of the whole market system and not its component parts" (Reply...op.cit. p.267). So far we have looked only at one of the component parts of the economic system, however. We have just analyzed the behavior of a single household. Even if we interpret the isoquants in question as representing community preferences, the analytical scope is nevertheless very limited. In the case of Buchanan's application of "Marshallian" demand concepts the focus is even narrower, since only movements along a single isoquant are analyzed. If we now turn again to Buchanan's article in order to look for the other component parts of the economic system, we will search in vain. No considerations concerning a "whole market system" can be found in that article. Buchanan's point with regard to labor supply in a general equilibrium setting may be considered as a well taken declaration of intention. But no such analysis can be found in that article.

We will now take up Buchanan's suggestion to analyze labor supply in an general equilibrium model. We may start out by remembering that the simplest representation of such a model was offered by T.C. Koopmans<sup>1)</sup>. This model is shown in the first quadrant of figure two. There, the set  $W$  is Koopmans's "production set" and  $X$  is his "consumption set". Along  $OQ$  work effort is measured and goods are measured along  $OI$ . The similarity of this model to the one in figure one will be obvious, the set  $X$  being another way of saying that the existence of indifference curves is assumed. The difference to the previous model consists in introducing the production set  $W$  into the analytical framework. The point of general equilibrium is  $P^*$ . It comes about when the Walrasian auctioneer calls out price  $(w/p)^*$ . Then the entrepreneurs have to choose a value function with the slope of line  $E^*I^*$ . They will settle for this line out of the infinitely many ones with identical slope, because the valuation on the production set is maximized for that line with the given slope which goes through  $P^*$ .

Fixing  $p_0$  along the vertical axis and  $w_0$  along the horizontal axis in the fourth quadrant, we may now proceed in an analogous way to the one taken in the construction of the  $ss'$  and  $dd'$ -curves in figure one. Given  $P^*$  in the first quadrant we obtain the co-ordinates  $E^*$  and  $(w/p_0)^*$  in the second quadrant, describing point  $M$ . In the fourth

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1) See the first essay in: Tjalling C. Koopmans Three Essays on the State of Economic Science New York 1957

quadrant point N corresponds to  $P^*$ .

Suppose the Walrasian auctioneer did not call out  $(w/p)^*$  but  $(w/p)_1$  which implies a valuation line of the slope of line  $L'1'$ . The entrepreneurs choose this particular line because it maximizes the value of  $W$  in  $P_2$ . Their option for point  $P_2$  means that they decide to demand  $OE_1$  of labor and to supply  $E_1P_2$  of goods, i.e. they want to realize point  $D_2$  in the second quadrant and point  $\Sigma_2$  in the fourth. The workers, however, having been notified that they have to consider valuation line  $L'1'$  as given want to be in point  $P'_2$  when their consumption set can be partitioned in the way depicted in figure 2. They want to supply not  $OE_1$  but  $OE'_1$  of labor and they want to buy not  $E_1P_2$  but  $E'_1P'_2$  of goods, i.e. households would rather realize point  $S_2$  in the second quadrant and point  $\Delta_2$  in the fourth. Thus we see that at the comparatively high real wage rate  $(w/p)_1$  there is an excess supply of labor of  $E_1E'_1 = D_2S_2$  and an excess demand for goods equal to  $I_1I'_1 = \Sigma_2\Delta_2$ .

The situation just described is a case of unemployment in the sense of Clower<sup>1)</sup>. Indeed the analytical situation described by our line  $L'1'$  is identical to the one described in figure one of Clower's article by his line  $L(p_f/p_g)$ . It will maybe remembered that Clower's main thesis in that connection was that equilibrium might not be attained if

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1) R.W. Clower "The Keynesian Counter-revolution: a Theoretical Appraisal" in: The Theory of Interest Rates, edited by F.H. Hahn and F. Brechling, International Economic Series, (Macmillan) pp. 103-25

certain dynamic conditions are given which were analyzed in some detail by Leijonhufvud<sup>1)</sup>, who, in so doing, relied on considerations of Alchian<sup>2)</sup>, who in turn regarded himself to "follow the lead of Hicks [3] and Hutt [4]" (op. cit. p. 117).

Finally, if the real wage is set very low by the auctioneer, the entrepreneurs might e.g. want to realize point  $P_3$  on the valuation line  $L_1$ , demanding  $OE_2$  of labor and supplying  $E_2P_3$  of goods. In that case they would want to realize point  $D_3$  in the second quadrant and point  $\Sigma_3$  in the fourth. The lines  $\delta\delta'$  and  $\sigma\sigma'$  are the labor demand and goods supply lines which may be generated for further thought experiments with the entrepreneurs in this fashion. Returning to valuation line  $L_1$  we note that while entrepreneurs want to be in point  $P_3$  households want to realize  $P'_3$ , supplying  $OE$  of labor and demanding  $E P'_3$  of goods. Thus they want to be in  $S_3$  on line  $ss'$  in the second quadrant and in  $\Delta_3$  along  $dd'$  in the fourth, these two curves being the labor supply and the goods demand curves, respectively, generated by further thought experiments along these lines.

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- 1) Axel Leijonhufvud On Keynesian Economics and the Economics of Keynes New York 1968
  - 2) Armen A. Alchian "Information Costs, Pricing, and Ressource Unemployment" in: Western Economic Journal 7 (1969) 109-28
  - 3) Alchian quotes J.R. Hicks The Theory of Wages (London, 1943), pp. 42-45
  - 4) W.H. Hutt The Theory of Idle Ressources (London, 1939)

It is highly doubtful, however, whether thought experiments along these lines are justified when the valuation line has a flatter slope than in general equilibrium, as is the case with valuation line  $L_1$ . The reasons are the following. Along  $L_1$  point  $P_3$  cannot be realized because entrepreneurs will not get the  $OE_2$  workers they need as factors of production, because for that kind of wages workers don't want to work that long hours in our model. On the other hand, the households's option,  $P'_3$  along  $L_1$ , cannot be realized either because it is not in the attainable set of the economy which is given by the intersection of the production set  $W$  and the consumption set  $X$ . Thus it is unimaginable that any transactions take place in a point along  $L_1$  - unless we introduce phenomena like forced labor. It will carry us too far, however, to speculate what exactly will happen in such a case. We will just warn the reader not to draw rash conclusions by having the supply and demand curves represented by broken lines where we are in the region of excess demand for labor and excess supply of goods. Opinion differs whether the broken curves are analytically relevant. Hirshleifer<sup>1)</sup>, who derives individual and market supply curves for two goods (pp.15 and 17) in a framework quite similar to the one here employed, sees no difficulty in an off-production-possibility-curve analysis. J.M.Keynes<sup>2)</sup>, however, obviously

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1) J. Hirshleifer Investment, Interest, and Capital  
Englewood Cliffs 1970

2) John Maynard Keynes The General Theory of Employment, Interest, and Money London 1936

thought that only points along  $OP^*$  <sup>1)</sup> but none beyond  $P^*$  or beyond the production possibility set are analytically relevant<sup>2)</sup>.

What can be learnt from this rather tentative discussion of labor supply in a general equilibrium setting? The lesson we want to draw is : a macroeconomic model like the one described by D. Patinkin<sup>3)</sup> can not be imagined as being pieced together from microeconomic elements. In the case of figure two the reason is not that the information contained in that figure is insufficient for microeconomic analysis but rather that such a procedure might lead to wrong results. Indeed it would be quite possible to construct a microeconomic model of a labor market on the basis of figure two. Noticing that the boundary of set  $W$  is nothing else than a production function, we could obtain the marginal productivity curve which is identical to the  $\delta\delta'$ -curve in the second quadrant. Then, employing the standard textbook analysis of labor supply, we might take  $O$  as the constant endowment point and derive a labor supply curve while pivoting the value line  $OG$  in  $O$ . We might, for simplicity, assume that in  $R$  and other relevant points the elasticity of substitution between leisure and goods is unity. Then we will obtain an offer curve like  $RR_1$  in the

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- 1) Take point  $P_2$  in figure 2, characterizing unemployment in the sense of Clower, and decrease the real wage. Then "... employment increases ... until a point  $[P^*]$  comes at which there is no surplus of labour available at the then existing real wage ...!" (op.cit.p.289)
- 2) "But after this point  $[P^*]$  ... wages and prices, and consequently profits also, should all rise in the same proportion ... , the 'real' position, including the volume of output  $[I^*]$  and employment  $[E^*]$  being left unchanged in all respects" (ibid.)
- 3) See Figure IX-7, p. 204 in: Don Patinkin Money, Interest, and Prices New York 1965 2nd ed.

first quadrant or , corresponding to it, a labor supply curve like  $R_1'R'$  in the second quadrant. It might well be that no equilibrium exists between the household and the firm under consideration. If e.g. the subsistence minimum is reached in R or R', respectively, the  $\delta\delta'$ -curve cannot be reached by the labor supply curve and we are led to predict that no equilibrium transactions are possible in the given analytical framework.

"If we transfer these concepts to the economy as a whole..." (Patinkin, op.cit. p.202), i.e. if we argue by analogy to microeconomics , as Patinkin has done , we might be led to the wrong result that the same situation will be observed in "the economy as a whole". Whereas if we kept partial analytical microeconomics apart from total analytical macroeconomics we would see immediately that equilibrium transactions might exist in the points  $P^*$ , M, and N.

It is not crucial, of course, that the existence of a subsistence minimum is assumed in order to show that arguing by analogy will lead to results which are inconsistent with general equilibrium analysis. If we assume that the microeconomic labor supply curve  $R_1'R'$  extends far enough to meet the microeconomic labor demand curve we might think that the equilibrium amount of labor services exchanged is  $OE_3$ . But in (total-analytical) fact it is  $OE^*$ .

In deriving the microeconomic labor supply curve we could have taken point  $l^*$  instead of point O as the constant endowment point and let the valuation line pivot in that point. Then we would have obtained the same equilibrium

point as in general equilibrium analysis - but a supply function quite different from  $ss'$ . This difference could be crucial for an analysis of static stability. But why should we let the valuation line always go through  $l^*$ ? Why not through  $l'$ ? Or through  $l$ ? Because only if we take  $l^*$  as constant endowment point do we obtain the correct general equilibrium point? But did we not set out with our analysis in order to find the equilibrium point? Now we have to know it before we can choose the correct constant endowment point.

Our problems originate from the fact that in the "Koopmans Economy" the "constant endowment point" is not constant. It must be imputed from the shape of the production set in conjunction with the price system called out by the auctioneer. One could save standard textbook labor supply analysis by adopting the analysis of long run productive equilibrium for the firm offered by Hirshleifer (op.cit.p.125).<sup>1)</sup> His zero-wealth-increment solution implies that the value function always pivots in the origin, thus relieving us of the problem which constant endowment point to choose. But in Hirshleifer's analysis the production set shifts with each new price vector, implying that factor demand is equal to (i) the demand from engaging in production plus (ii) whatever amount prospective

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1) This was pointed out to me by Professor Jaeger, Free University Berlin



producers want to offer in order to obtain the right to use the productive opportunities. Now this is a very interesting theory of factor markets but certainly it is not identical to the theory of standard textbook labor demand analysis. Thus we could save standard textbook labor supply analysis by giving up standard textbook labor demand analysis. Since this will not lessen our problems in defending "standard textbook analysis" in the context of macroeconomic labor market models it might now be time to admit that with analogy as analytical principle we have ended up in considerable confusion.<sup>1)</sup>

Of course the charge that unjustified analogy is used might also be fired against the Koopmans-method here employed. After all, basically it suggests that a decentralized perfect competition market economy may be regarded just like a super-schizophrenic Robinson who has three personalities at once: an auctioneer's, an entrepreneur's, and a household's. Nevertheless this method has the advantage of demonstrating the redistribution effects which occur when alternative price vectors are called out. These effects the pure partial analysis ignores when drawing its supply and demand curves. They might make considerable difference in labor market analysis, however.

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1) No wonder there has been prolonged reluctance to accept Patinkin's vision of the labor market. See in this connection but different context the "Comment" by A. Leijonhufvud in: History of Political Economy, Vol. 6. No.2 pp 164-170

We agree with Professor Buchanan on the necessity of non-partial-equilibrium labor market analysis in macro-economic contexts like the one of Patinkin (op.cit.). We do not agree, however, that "Marshallian" demand curves for income will bring us any further in that regard. Our thesis is : in a general equilibrium setting the valuation line must not move around a given isoquant when alternative price systems are called out (Buchanan-analysis) but around a given production possibility set (Koopmans-analysis) <sup>1)</sup> at least as long as there is unemployment in the sense of Clower.

### Conclusion

In this article it was shown that (i) Robbins's analysis of labor supply rests on the same choice theoretic paradigm as the standard textbook analysis, its crucial assumption being a fixed endowment of time and goods. (ii) Buchanan's analysis is of a different choice theoretic paradigm than Robbins's in so far as its results rest on the assumption of variable endowments of time and goods. If endowment changes are identified and explicitly taken account of, Buchanan's analysis is not inconsistent. Its conclusions are not arbitrary. Therefore Leijonhufvud's justification for dismissing his approach does not hold. (iii) If labor supply is to be analyzed geometrically in a general equilibrium

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1) M.J. Bailey "The Marshallian Demand Curve" in: The Journal of Political Economy , 62, 1954, 255-61 , also presents a graphical "general equilibrium model" (see his Figure 1). The crucial difference between his analysis and ours is that in his model entrepreneurs never maximize the value of the production set unless the value line prevails which assures general equilibrium whereas they always maximize it in our model as long as they move along the  $gg'$  and  $cc'$ -curves.

setting neither "standard textbook" labor supply curves nor "Marshallian" income demand curves should be employed. The Koopmans-model seems to be an appropriate geometrical framework in this case. Clower's illustration (op.cit.) of unemployment in a general equilibrium setting is identical to the one shown in this article. The Koopmans-model is seldom used in macroeconomic labor market analysis although it is analytically more satisfying than reasoning by analogy to "standard textbook" analysis.



