

Deissenberg, Christophe

Working Paper

Optimal control of linear econometric models with intermittent controls

Diskussionsbeiträge - Serie A, No. 127

Provided in Cooperation with:

Department of Economics, University of Konstanz

Suggested Citation: Deissenberg, Christophe (1979) : Optimal control of linear econometric models with intermittent controls, Diskussionsbeiträge - Serie A, No. 127, Universität Konstanz, Fakultät für Wirtschaftswissenschaften und Statistik, Konstanz

This Version is available at:

<https://hdl.handle.net/10419/78114>

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.

FAKULTÄT FÜR
WIRTSCHAFTSWISSENSCHAFTEN UND STATISTIK
UNIVERSITÄT KONSTANZ

OPTIMAL CONTROL OF LINEAR ECONOMETRIC
MODELS WITH INTERMITTENT CONTROLS

Christophe | Deissenberg

Serie A - Nr. 127

DISKUSSIONSBEITRÄGE

D-7750 Konstanz
Postfach 7733

Serie A

Volkswirtschaftliche Beiträge

Nr. 127

OPTIMAL CONTROL OF LINEAR ECONOMETRIC
MODELS WITH INTERMITTENT CONTROLS

Christophe | Deissenberg

Serie A - Nr. 127

June 1979

A05467 70
Kf Wirtschaft
Kf Kval

Optimal Control of Linear Econometric Models
with Intermittent Controls

Christophe Deissenberg

June 1979

Abstract

Dynamic Programming is used to derive the optimal feedback solution to the minimization of a quadratic welfare loss-functional subject to a linear econometric model, when the value of some instrument variables can not be optimized in every model period, but only in single ones. In this way, the relative inertia of fiscal policy-making, as compared to monetary policy-making, can e.g. be taken into account. Analytical expressions are derived for the optimal feedback rules and for the minimum expected losses, and iterative schemes are proposed for their numerical computation. It is suggested that a numerical analysis of the economic gain to be realized by making more frequent adjustment of fiscal policy variables than is actually the case could yield valuable information for policy-makers.

I. INTRODUCTION

In recent years, several authors have addressed problems in macroeconomic policy-making with the help of linear-quadratic control methods, see e.g. Pindyck [1973], Chow [1975]. Specifically, these authors consider the linear or linearized econometric model in state-variable form

$$(1a) \quad x_t = A_t x_{t-1} + B_t u_t + D_t z_t + \varepsilon_t, \quad t = 1, 2, \dots,$$

$$(1b) \quad x_0 = \hat{x},$$

where x_t is a $(n \times 1)$ vector describing the state of the economy in the period t , \hat{x} being the given initial state; u_t is a $(m \times 1)$ vector of instruments or control variables whose value is set by the policy-maker; z_t is a $(p \times 1)$ vector of exogenous or predetermined variables; ε_t is a $(n \times 1)$ vector of stochastic disturbances, which are normal-distributed with mean 0 and serially uncorrelated; the parameter matrices A_t , B_t , and D_t are exactly known.

For every initial state \hat{x} , (1a) describes the possible behavior of the economy over time as a function of the instrument variables u_t , of the uncontrollable events z_t and of the random phenomena ε_t . Because of the disturbance ε_t this behavior is only probabilistically known. The time-dependency of the matrices A , B , and D expresses eventual changes in the economy's structure over time.

The afore mentioned authors also assume an objective function in the form of a quadratic welfare loss-functional

$$(2) \quad W = E \left[\sum_{t=1}^N \frac{1}{2} (x_t - \bar{x}_t)^T K_t (x_t - \bar{x}_t) \right] \rightarrow \min$$

where $E[.]$ is the expectation operator, T denotes transposition, \bar{x}_t is an exogenously specified target vector of desired values for the state in t and K_t is a given non-negative definite weighting matrix. In general it is appropriate to define (1a) so that x_t includes u_t as subvector, thus making u_t an argument of W without complicating the notation unnecessarily. (The weighting of u_t in W can express the technical or political costs of using macroeconomic instruments. It may also be used to keep the fluctuation of u_t within reasonable bounds without explicitly introducing restrictions on these fluctuations.)

The optimal policy problem is to determine the instrument sequence u_1^*, \dots, u_N^* that minimizes the welfare loss (2) subject to the economic constraints (1a) and (1b). It can be shown--see e.g. Chow [1975]--that the optimal control in t , u_t^* , is a deterministic, linear function of the current state x_{t-1}

$$(3) \quad u_t^* = G_t x_{t-1} + g_t, \quad t = 1, \dots, N,$$

(linear feedback rule), with

$$(4a) \quad G_t = -(B_t^T H_t B_t)^{-1} B_t^T H_t A_t,$$

$$(4b) \quad g_t = -(B_t^T H_t B_t)^{-1} B_t^T (H_t D_t z_t - h_t),$$

$$(5a) \quad H_{t-1} = K_{t-1} + (A_t + B_t G_t)^T H_t (A_t + B_t G_t), \quad H_N = K_N,$$

$$(5b) \quad h_{t-1} = K_{t-1} \bar{x}_{t-1} + (A_t + B_t G_t)^T (h_t - H_t D_t z_t), \quad h_N = K_N \bar{x}_N,$$

The system (4) - (5) can easily be solved numerically. A feasible computational scheme is to use alternatively (4) and (5) for $t = N, N-1, \dots, 1$ together with the initial condition $H_N = K_N$, $h_N = K_N \bar{x}_N$ to determine backward in time the matrices and vectors $G_N, g_N; H_{N-1}, h_{N-1}; G_{N-1}, g_{N-1}; \dots; G_1, g_1$. Note that the feedback matrices G_t and forcing vectors g_t are independent of the values taken by x and u over $t = 0, \dots, N$ so that they can be predetermined before even x_0 is known. Given G_t and g_t , the optimal control u_t^* is determined by (3) as a function of the current state of the economy, x_{t-1} .

The above solution (3) - (5) assumes that the value of every component of u , i.e., of every instrument variable, can be optimized by the policy-maker in every period t , $t = 1, \dots, N$. In actual policy making, however, this may indeed not be true. Consider, for example, the case when the econometric model is a quarterly model, and the instrument vector includes both fiscal policy and monetary policy variables. On the one hand, monetary policy variables can usually be adjusted more or less continuously in time. Therefore, it will in general be justified to assume that the components of u corresponding to the monetary policy variables can be optimized in every period t . On the other hand, for technical, political, or institutional reasons, many important fiscal policy variables like tax rates etc. are normally kept constant over the whole fiscal year. In our example, this means that the corresponding components of u can only be optimized every four periods, in $t_1, t_1+4, t_1+8, \dots, t_1+4n, \dots$. Between t_1+4n and $t_1+4(n+1)$ they remain "frozen" at the value they were given in t_1+4n .

In this note we extend the basic problem (1) - (2) in order to be able to take into consideration such instrument variables v which can not be optimized in every period t but only in single periods $t_0, t_1, \dots, t_i, \dots$ with $t_{i+1} - t_i > 1$ for at least one i . The variables v will be called

intermittent control variables; the variables u , which can be optimized in every period, permanent control variables. (Our introductory example was, of course, unnecessarily restrictive in considering a situation where the interval between successive adjustments of the intermittent controls is a constant number of periods. The above problem formulation allows for variable intervals between optimizations of v .) We derive analytical expressions for the optimal feedback rules with intermittent controls and for the minimum expected welfare loss, and propose iterative schemes for the numerical computation of these expressions. The social costs caused by the impossibility of adjusting the instruments v in other periods than in $t_0, t_1, \text{etc.}$, can thus be determined numerically.

II. OPTIMAL POLICY MAKING WITH INTERMITTENT CONTROLS

In order to take into consideration intermittent controls, the problem

(1) - (2) is extended to

$$(6) \quad J = E \left[\sum_{t=1}^N \frac{1}{2} (x_t - \bar{x}_t)^T K_t (x_t - \bar{x}_t) \right] \rightarrow \min_{(u_1^T, \dots, u_N^T, v_{t_0}^T, \dots, v_{t_K}^T)},$$

subject to

$$(7a) \quad x_t = A_t x_{t-1} + B_t u_t + C_t v_t + D_t z_t + \varepsilon_t,$$

$$(7b) \quad x_0 = \hat{x},$$

$$(8) \quad v_t = v_{t_i}, \quad t_i \leq t < t_{i+1}, \quad t, t_i \in \{0(+1)N\}.$$

where v_t are the intermittent and u_t the permanent controls. Here also

(7a) can be defined so that x_t includes u_t and v_t as subvectors. The solution of this problem is now derived using Dynamic Programming, see Bellman [1957], Chow [1975].

According to Bellman's Principle of Optimality the optimal control for the period t , y_t^* , can be obtained as the solution of the one-period optimization problem

$$(9) \quad V_t = E \left[\frac{1}{2} (x_t - \bar{x}_t)^T K_t (x_t - \bar{x}_t) + V_{t+1}^* \mid \sigma_t \right] \rightarrow \min_{y_t}$$

where $V_{t+1}^* = \min V_{t+1}$, σ_t describes the process behavior up to time t , and

$$(10) \quad y_t^T := \begin{cases} (u_t^T, v_t^T) & \text{for } t = t_i, \quad i = 0, 1, \dots, K, \\ (u_t^T) & \text{for } t \neq t_i. \end{cases}$$

That is, V_t is to be minimized with respect to u_t and v_t whenever $t = t_i$, and with respect to u_t alone whenever $t \neq t_i$. Thus $y_t^{*T} = (u_t^{*T}, v_t^{*T})$ for $t = t_i$, and $y_t^{*T} = (u_t^{*T})$ for $t_i < t < t_{i+1}$. In this last case the "v-part" of the control is a priori given with the value $v_{t_i}^*$. Note that by (7a) and the assumption, ε_t is independent from σ_t , it is possible to replace σ_t in (9) with x_{t-1} without changing the optimization's result. This suggests that here also the optimal control in t will be independent of x_τ , $\tau \neq t-1$.

Let's assume that the functional equation V_t is of the form

$$(11) \quad V_t = E \left[\frac{1}{2} x_t^T H_t x_t + x_t^T Q_t v_t + \frac{1}{2} v_t^T P_t v_t - x_t^T h_t - v_t^T p_t + c_t / \sigma_t \right],$$

with H_t and P_t non-negative definite. This is obviously true for

$t = N$ with

$$(12) \quad H_N := K_N, \quad P_N = 0, \quad Q_N = 0, \quad h_N := K_N \bar{x}_N, \quad p_N = 0, \quad c_N = \frac{1-T}{2} K_N \bar{x}_N.$$

For simplicity's sake let's use the notation Φ_t to designate the set of matrices and vectors $(H_t, Q_t, P_t, h_t, p_t, c_t)$. We will show that expression (11) is reproducible for $t-1$ when it holds for t -- i.e., that it holds for all $t \leq N$. While doing so, we will derive the optimal feedback rule for determining y_t^* and provide computationally feasible schemes to obtain Φ_t for $t = \tau-1$ from its value for $t = \tau$.

Using the model (7a) for x_t in (11) and taking expectations, we obtain V_t as a function of x_{t-1} and y_t alone,

$$(13) \quad \begin{aligned} V_t = & \frac{1}{2} (A_t x_{t-1} + B_t u_t + C_t v_t + D_t z_t)^T H_t (A_t x_{t-1} + B_t u_t + C_t v_t + D_t z_t) + \\ & + (A_t x_{t-1} + B_t u_t + C_t v_t + D_t z_t)^T Q_t v_t - \\ & - (A_t x_{t-1} + B_t u_t + C_t v_t + D_t z_t)^T h_t + \\ & + \frac{1}{2} v_t^T P_t v_t - v_t^T p_t + c_t + E \left[\frac{1}{2} \varepsilon_t^T H_t \varepsilon_t \right]. \end{aligned}$$

A case differentiation is now to be made:

Case a: $t_i < t < t_{i+1}$. Then $v_t = v_{t_i}$ and (13) is to be minimized with respect to u_t alone. The necessary (and, as will be shown later, sufficient) conditions for a minimum are

$$(14) \quad \frac{\partial V_t}{\partial u_t} = B_t^T H_t (A_t x_{t-1} + B_t u_t + C_t v_t + D_t z_t) + B_t^T Q_t v_t - B_t^T h_t = 0.$$

The solution of (14) yields the optimal decision for the period t

$$(15) \quad u_t^* = u_t^*(x_{t-1}, v_{t_i}) = G_t x_{t-1} + M_t v_{t_i} + g_t ,$$

with

$$(16a) \quad G_t = -(B_t^T H_t B_t)^{-1} B_t^T H_t A_t ,$$

$$(16b) \quad M_t = -(B_t^T H_t B_t)^{-1} B_t^T (H_t C_t + Q_t) ,$$

$$(16c) \quad g_t = -(B_t^T H_t B_t)^{-1} B_t^T (H_t D_t z_t - h_t) .$$

Equation (15) is the optimal feedback rule for $t \neq t_i$. Like its equivalent (3) for the basic problem (1) - (2), it is linear. The system (16) plays a similar role to (4) in the basic problem.

Substituting (15) in (11) for u_t , we obtain the minimum expected welfare loss at period t , conditional on the economy's behavior before t , i.e., equivalently on x_{t-1} :

$$(17) \quad V_t^* = \frac{1}{2} (A_t x_{t-1} + B_t G_t x_{t-1} + B_t g_t + B_t M_t v_t + C_t v_t + D_t z_t)^T H_t \cdot \\ \cdot (A_t x_{t-1} + B_t G_t x_{t-1} + B_t g_t + B_t M_t v_t + C_t v_t + D_t z_t) + \\ + (A_t x_{t-1} + B_t G_t x_{t-1} + B_t g_t + B_t M_t v_t + C_t v_t + D_t z_t)^T \cdot (Q_t v_t - h_t) + \\ + \frac{1}{2} v_t^T P_t v_t - v_t^T p_t + c_t + E \left| \frac{1}{2} \varepsilon_t^T H_t \varepsilon_t \right| .$$

Setting this last expression for $t = t+1$ in (9) one sees after some simplifications that V_{t-1} can be written in the form (11) with

$$\begin{aligned}
(18a) \quad H_{t-1} &= K_{t-1} + (A_t + B_t G_t)^T H_t (A_t + B_t G_t), & H_N &= K_N, \\
(18b) \quad Q_{t-1} &= (A_t + B_t G_t)^T H_t (B_t M_t + C_t) + (A_t + B_t G_t)^T Q_t, & Q_N &= 0, \\
(18c) \quad P_{t-1} &= P_t + (B_t M_t)^T H_t (B_t M_t) + (B_t M_t + C_t)^T Q_t, & P_N &= 0, \\
(18d) \quad h_{t-1} &= K_{t-1} \bar{x}_{t-1} + \frac{1}{2} (A_t + B_t G_t)^T (h_t - H_t D_t z_t), & h_N &= K_N \bar{x}_N, \\
(18e) \quad p_{t-1} &= (B_t g_t + D_t z_t)^T Q_t - p_t, & p_N &= 0, \\
(18f) \quad c_{t-1} &= \frac{1}{2} \bar{x}_{t-1}^T K_{t-1} \bar{x}_{t-1} + c_t + E \left| \frac{1}{2} \varepsilon_t^T H_t \varepsilon_t \right| - \\
&\quad - \frac{1}{2} (B_t g_t + D_t z_t)^T h_t, & c_N &= \frac{1}{2} \bar{x}_N^T K_N \bar{x}_N,
\end{aligned}$$

this last system of equations being equivalent to (5).

The numerical solution of (15) - (16), (17) and (18) is straightforward. Given Φ_t , one computes G_t , M_t and g_t by (16). This yields (15) and, given x_{t-1} , u_t^* as well as, by (17), v_t^* . The values obtained for the matrices G_t , M_t and g_t , used in (18), lead to Φ_{t-1} . Thus we have shown how to derive (11) for $t = \tau-1$ from (11) for $t = \tau$ and how to derive the optimal feedback rule at period t , u_t^* , and the expected minimum cumulated loss over $\tau = t, \dots, N$, v_t^* , when $t \neq t_i$.

Case b: $t = t_i$. In this case V_t is to be minimized with respect to $(u_t^T, v_t^T) := y_t^T$. Otherwise the same derivations as in case a apply. We have

$$(20) \quad \frac{\partial V_t}{\partial y_t} = R_t^T H_t (A_t x_{t-1} + R_t y_t + D_t z_t) + S_t y_t - R_t h_t - s_t + (A_t x_{t-1} + D_t z_t)^T W$$

with

$$(21) \quad R_t := (B_t, C_t) \quad , \quad W_t := (0, Q_t) \quad , \quad s_t := \begin{bmatrix} 0 \\ P_t \end{bmatrix} \quad ,$$

$$s_t := \begin{bmatrix} 0 & 2B_t^T Q_t \\ 0 & P_t + 2C_t^T Q_t \end{bmatrix} \quad .$$

It follows for y_t^*

$$(22) \quad \begin{bmatrix} u_t \\ v_t \end{bmatrix}^* = G_t x_{t-1} + g_t$$

with

$$(23a) \quad G_t = -(R_t^T H_t R_t + S_t)^{-1} (R_t^T H_t + W_t^T) A_t \quad ,$$

$$(23b) \quad g_t = -(R_t^T H_t R_t + S_t)^{-1} R_t^T (H_t D_t z_t - h_t) + W_t^T D_t z_t - s_t \quad ,$$

the linear feedback rule (22) being equivalent to (3), and (23) to (4). v_t^* is accordingly given by

$$(24) \quad \begin{aligned} v_t^* = & \frac{1}{2} (A_t x_{t-1} + R_t G_t x_{t-1} + R_t g_t + D_t z_t)^T H_t (A_t x_{t-1} + R_t G_t x_{t-1} + R_t g_t + D_t z_t) + \\ & + (A_t x_{t-1} + R_t G_t x_{t-1} + R_t g_t + D_t z_t)^T [\tilde{W}_t (R_t G_t x_{t-1} + R_t g_t) - b_t] + \\ & + \frac{1}{2} (R_t G_t x_{t-1} + R_t g_t)^T [\tilde{P}_t (R_t G_t x_{t-1} + R_t g_t) - s_t] + \\ & + c_t + E \left[\frac{1}{2} \varepsilon_t^T H_t \varepsilon_t \right] \end{aligned}$$

with

$$(25) \quad \tilde{W}_t = \begin{bmatrix} 0 & 0 \\ 0 & Q_t \end{bmatrix} \quad , \quad \tilde{P}_t = \begin{bmatrix} 0 & 0 \\ 0 & P_t \end{bmatrix} \quad .$$

Accordingly V_{t-1} can be expressed in the form (11) with

$$(25a) \quad H_{t-1} = K_{t-1} + (A_t + R_t G_t)^T H_t (A_t + R_t G_t) + G_t^T S_t G_t + 2A_t^T W_t G_t, \quad ,$$

$$(25b) \quad h_{t-1} = K_{t-1} \bar{x}_{t-1} - (A_t + R_t G_t)^T H_t (R_t g_t + D_t z_t) + (R_t G_t)^T h_t + \\ + A_t^T (h_t - W_t g_t) + G_t^T (s_t - S_t g_t - W_t^T D_t z_t) \quad ,$$

$$(25c) \quad c_{t-1} = \frac{1}{2} \bar{x}_{t-1}^T K_{t-1} \bar{x}_{t-1} + E \left[\frac{1}{2} \epsilon_t^T H_t \epsilon_t \right] + c_t + \frac{1}{2} g_t^T S_t g_t + \\ + \frac{1}{2} (D_t z_t + R_t g_t)^T H_t (D_t z_t + R_t g_t) + (R_t g_t)^T (H_t D_t z_t + H_t) \quad ,$$

$$(25d) \quad P_{t-1} = 0 \quad , \quad Q_{t-1} = 0 \quad , \quad p_{t-1} = 0 \quad ,$$

these last equations playing a similar role as (5) in the basic problem. The numerical solution of (22) - (23), (24) and (25), which leads to a) the optimal feedback rule at period t , b) u_t^* , c) V_t^* , and d) (11) for $t = \tau_i - 1$ when $t = \tau_i$, is again straightforward. It will not be explicitly discussed here.

We have now solved the whole optimization problem. The initial values of the different matrices and vectors needed to construct V_t for $t = N$, i.e. Φ_N , are given by (12). Based on these initial values, the values of the system Φ_t for $t < N$ and the optimal feedback rules for $t \leq N$ can be computed backward in time according to the formulas presented above. The major difference with the solution of the basic problem is that here the computation follows a "two regimes" scheme, the computational rules being different for $t = t_i$ and $t \neq t_i$. The total expected minimum loss over $t = 1, \dots, N$ is, of course, given by V_1^* .

In the above derivation, we did not check the second order conditions for a minimum of V_t . In fact, these conditions are always satisfied: The matrices H_t and P_t , which define the quadratic part in V_t , are

non-negative definite for all t -- for $t = N$ by definition and, as can easily be recognized, for $t < N$ by construction. The possibility of multiple minima, however, remains open. In our derivations we implicitly assumed that $(B_t^T H_t B_t)$ and $(R_t^T H_t R_t + S)$ are invertible, see (4), (16), and (23). This is obviously a sufficient condition for the uniqueness of y_t^* . It is also necessary: When these matrices are singular, the optimal solution is no longer unique. However, optimal solutions always exist, one of which can be obtained by replacing the inverse in (4), (16), or (23) by the corresponding Moore-Penrose generalized inverse. See e.g. Garbade [1976] for details.

Note that $v_{t_i}^*$ is both a function of the random variable x_{t_i-1} and an argument of the optimal feedback rule in t , $t_i < t < t_i+1$. Therefore, contrary to the basic case, the optimal feedback rule in t , $t_i < t < t_i+1$ can not be completely fixed a priori. Although it can be partially pre-computed conditional on x_{t-1} , its final value can only be determined after x_{t_i-1} is realized. The optimal feedback rules in $t \in \{t_i\}$, on the other hand, can here also be computed a priori. Of course, the solution given in this note assures that v_{t_i} is chosen in such a way as to be optimal with respect to both its functions as a control in t , $t_i \leq t < t_i+1$, and as an argument of the feedback rule in t , $t_i < t < t_i+1$, i.e. in fact optimal with respect to the whole future behavior of the economy and the assumed decision-making mechanism. However, consider two problems with intermittent controls which are identical except for the fact that in problem 2 the intermittent controls can not only be optimized in the same periods t_i as in problem 1, but also in supplementary periods t_j . Assuming that the controls v_t are not redundant or degenerate, the total expected minimum welfare loss over $t = 1, \dots, N$, expressed by V_1^* , will be smaller for problem 2 than for problem 1 because of the relaxation of the constraints on v_t in the former problem in comparison to the latter. (For a discussion of necessary and sufficient conditions for

the non-degeneracy of controls see e.g. Garbade [1976], Preston [1977]). The extra expected loss in problem 1 can conceptually be traced to two sources. In problem 1 the instruments v_t can not be adjusted as often as in problem 2 to control the expected behavior of the economy. Furthermore, they do not, as often as in problem 2, take into account the deviations of the actual state of the economy from its expected value. In problem 1, the permanent controls v_t are thus more heavily used to both control the dynamics and counteract the random disturbances of the system.

From a policy-making point of view, it should be interesting to know the reduction in the value of the welfare loss-function that can be attained by making possible an optimization of the intermittent controls in supplementary periods t_j . This reduction can be interpreted as an estimate of the social costs caused e.g. by the "inertia" of fiscal policy or, equivalently, as an estimate of the increase in economic performance which could be attained by "speeding up" the fiscal policy-making process. (Of course, any such interpretation would be subject to serious qualifications, which can not be discussed within the scope of this note.) At least for the general case there is no useful analytical expression for this reduction of the welfare loss. However, it is easy enough to compute it numerically as the difference between V_1^* when v_t is optimized only for $t \in \{t_i\}$ (problem 1) and V_1^* when v_t is optimized for $t \in \{t_i\} \cup \{t_j\}$ (problem 2). An interesting limit case is when one allows v_t to be optimized in every t . Very preliminary numerical experiments using a small ad hoc model with the permanent controls discount rate and the intermittent controls taxes and government spending show that, for "sensible" values of K and \bar{x} , the expected minimum welfare loss with quarterly optimization of the fiscal policy instruments can be less than one fifth the loss with yearly optimization of the same instruments. This

relatively large reduction in welfare loss is of course partly a result of the quadratic form of W , which overproportionally penalizes increasing deviations from the desired values \bar{x}_t . Therefore even relatively small absolute improvements can yield large reductions in the expected loss.

For simplicity's sake we considered in this note only one vector of intermittent controls. However, the model can readily be expanded to cover the case of different vectors v_t, y_t, z_t, \dots of intermittent controls which can be optimized respectively only in $t \in \{t_i\}$, $t \in \{t_j\}$, etc. In practical applications the number of such vectors should increase with smaller lengths of the model's time period.

I. CONCLUSIONS

In recent years, much effort has been devoted to the development of macro-econometric models with short time periods--quarterly, monthly, or even weekly models. Are these models optimized to help determine the "best" course of economic policy, it is in general no longer realistic as with models with longer periods to assume that all instrument variables can be optimally adjusted in every period. Particularly certain fiscal policy variables necessarily remain fixed over several model periods. In this note, we have shown how to take into consideration such "intermittent controls" within the usual linear-quadratic framework. The numerical solution of the extended problem is similar to and only slightly more complicated than the solution of the basic linear-quadratic problem with only permanent controls. We expect that the optimization of econometric models taking into consideration the intermittence of certain controls in real life will not only result in more justified recommendations to policy-makers, but could also shed light on the economic costs of the existing institutional and technical constraints on frequent readjustments of some instrument variables.

REFERENCES

- Bellman, R. E. [1957], Dynamic Programming (Princeton, New Jersey: Princeton University Press).
- Chow, G. [1975], Analysis and Control of Dynamic Economic Systems (New York-London-Sydney-Toronto: John Wiley & Sons, Inc.).
- Garbade, K. [1976], "On the Existence of Uniqueness of Solutions to Multi-Period Linear Quadratic Optimal Control Problems," in International Economic Review, 17, 3, 719-731.
- Pindyck, R. [1973], Optimal Planning for Economic Stabilization (Amsterdam: North-Holland Publishing Company).
- Preston, J. [1977], "Existence, Uniqueness, and Stability of Linear Optimal Stabilization Policies," in Pitchford and Turnovsky (Eds): Applications of Control Theory to Economic Analysis, (Amsterdam-New York-Oxford: North-Holland Publishing Company), 255-292.