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The Relevance of Primary Dealers for Public Bond Issues

Wolfgang Breuer
Abstract: We analyze the role of different kinds of primary and secondary market interventions for the government’s goal to maximize its revenues from public bond issuances. Some of these interventions can be thought of as characteristics of a "primary dealer system". After all, we see that a primary dealer system with a restricted number of participants may be useful in case of only restricted competition among sufficiently heterogeneous market makers. We further show that minimum secondary market turnover requirements for primary dealers with respect to bond sales seem to be in general more adequate than the definition of maximum bid-ask-spreads or minimum turnover requirements with respect to bond purchases. Moreover, official price management operations are not able to completely substitute for a system of primary dealers. Finally it should be noted that there is in general no reason for monetary compensations to primary dealers since they already possess some privileges with respect to public bond auctions.

Keywords: Public bonds, primary dealers, official market interventions, bond auctions, market makers

JEL classification: G18, H63
I. Introduction

The question how primary and secondary markets for governmental bonds should be organized in order to cover efficiently public deficits is of timeless importance. Recent developments in Germany as well as in other European countries show that in dynamic systems like real capital markets there is an everlasting need of trying to improve upon the conditions for raising capital by a country’s government.

In Germany, a major change took place at the beginning of 1998 when the former Federal Bond Consortium ("Bundesanleihekonsortium") was substituted by the Bund Issues Auction Group ("Bietergruppe Bundesemissionen").\(^1\) When founded in 1952 the Federal Bond Consortium was intended to work as an issuing syndicate for German governmental securities.\(^2\) In 1990 the offering technique for German public bonds was changed in such a way that from then on it became possible to place only a fraction of an issue by the Federal Bond Consortium, while the other portion was placed using a discriminatory auction.\(^3\) With respect to this change in the way how to issue governmental bonds the creation of the Bund Issues Auction Group in order to succeed the former Federal Bond Consortium seems quite straightforward as from the beginning of 1998 on German public bonds will only be tendered.\(^4\)

In connection with this major change regarding the primary market for German public bonds the question was raised how many banks should be allowed to participate in public bond tenders and what other rights and duties they should have. To be precise it has been discussed if there should be some kind of a "primary dealer system" for German government bonds.\(^5\)

In Section II. a brief characterization of primary dealer systems as well as an overview regarding the use of primary dealers by other industrial countries is given. Furthermore we present

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\(^1\) See for example Deutsche Bundesbank, 1997, Änderungen des Emissionsverfahrens bei Bundeswertpapieren.
\(^2\) See for example Deutsche Bundesbank, 1995, Der Markt für deutsche Bundeswertpapiere, 44-46.
\(^5\) See for example Bundesverband deutscher Banken, 1997, Der europäische Rentenmarkt in der Währungsunion, 10-11.
some arguments in favour of and against a primary dealer system which are usually stated by officials if asked to do so.

Since it is not easy to assess the possible advantages or disadvantages of a system of primary dealers, it is necessary to analyze those different aspects in a more formal framework in order to become more precise. Our initial setting is defined in Section III. where we analyze a simple deterministic model of public debt financing with perfect market maker competition. We consider both primary and secondary market transactions and start our analysis with the derivation of equilibrium outcomes if there are no official interventions. After this we analyze different ways how the government can influence primary and secondary market outcomes. These measures can be interpreted as components of a system of primary dealers or some kind of governmental "price management operations" on the secondary market. Especially we will ask which actions are best suited in order to increase the government’s revenues from issuing bonds. We will see that in case of perfect market maker competition there is only very little room for official interventions in order to increase revenues from government bond issues. In general, we need some kind of market imperfection if we want to rationalize official interventions. In Section IV. we therefore repeat our analysis presented in Section III. for the case of market makers being able to act as monopolists. This leads to a rationale for comprehensive official primary and secondary market interventions. Moreover these measures now may result in a restriction of the number of active bidders for public bonds on the primary market, so we may indeed get some kind of "primary dealer system". At first glimpse especially the possible usefulness of certain secondary market interventions may be somewhat surprising, since those official interventions will adversely affect market makers’ maximum profits from secondary market transactions and one might suspect these measures to lower market makers’ optimal bids for government bonds on the primary market. In fact, this does not need to be true. We therefore will elaborate on the relevant relationships in more detail. In Section V. we briefly discuss the consequences of uncertainty and informational asymmetries between the public debt manager on the one hand and market makers on the other hand with respect to the overall bond demand on the secondary market. Such modifications generally do not seem to affect our overall conclusions. The same holds true if we allow for oligopolistic competition among market makers on the secondary market. We discuss this scenario in Section V. in more detail. In Section VI. we will summarize our main results and draw some conclusions.
II. Primary dealer systems

A primary dealer system is just one way to organize primary and secondary markets for government bonds. Table 1 gives a brief overview over some selected industrial countries in which primary dealer systems are established. We see that primary dealer systems are established in quite a lot of countries, although especially in Germany and Japan there is no kind of primary dealership. In each of the countries of Table 1 primary dealers are a restricted number of institutions endowed with certain rights and obligations regarding the issuance and the trade of government bonds. This certainly is rather a vague characterization of a primary dealer system but in fact actual primary dealer systems are not such homogenous as to allow one simple general definition of primary dealership. For example Irish primary dealers generally are the only ones who are permitted to participate in government bond auctions\(^6\) while the British primary dealers’ privileges are restricted to the submission of greater volumes of non-competitive bids (i.e. bids without price announcements)\(^7\). While Spanish primary dealers must quote bid and ask prices on demand to clients paying attention to certain maximum spreads, there is no such obligation for Austrian primary dealers.\(^8\)

Despite this heterogeneity of real life primary dealers we certainly need some operational description to work with for the rest of this paper. In order to cover some very typical properties of primary dealers we will use the following characterization of a "full-fledged" primary dealership: Primary dealers are institutions which have the exclusive right to submit (competitive\(^9\)) bids in auctions for government bonds. Moreover, a primary dealer has the obligation to participate in a "substantial" way in these auctions, to quote two-way prices for government bonds on secondary markets with paying attention to maximum spread and/or minimum turnover requirements. Furthermore, primary dealers may have access to some other privileges such as special financing facilities which can be transformed into monetary equivalents. Finally the number of primary dealers typically is small compared to the number of all potential (here: competitive) bidders in public bond auctions. This means that a primary

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9 I.e. bids with a specific price announcement.
dealer system should coincide with a restriction of the number of potential (competitive) bidders.

<table>
<thead>
<tr>
<th>Country</th>
<th>Number of Primary Dealers</th>
<th>Year of Introduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Austria</td>
<td>25</td>
<td>1989</td>
</tr>
<tr>
<td>Belgium</td>
<td>12</td>
<td>1990</td>
</tr>
<tr>
<td>Canada</td>
<td>45</td>
<td>1940</td>
</tr>
<tr>
<td>Finland</td>
<td>5</td>
<td>1992</td>
</tr>
<tr>
<td>France</td>
<td>20</td>
<td>1987</td>
</tr>
<tr>
<td>Ireland</td>
<td>6</td>
<td>1995</td>
</tr>
<tr>
<td>Italy</td>
<td>28</td>
<td>1988</td>
</tr>
<tr>
<td>Norway</td>
<td>8</td>
<td>1995</td>
</tr>
<tr>
<td>Spain</td>
<td>15</td>
<td>1988</td>
</tr>
<tr>
<td>Sweden</td>
<td>9</td>
<td>1982</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>19</td>
<td>1986</td>
</tr>
<tr>
<td>United States of America</td>
<td>37</td>
<td>1960</td>
</tr>
</tbody>
</table>

Table 1: Primary dealer systems in selected industrial countries

When asked, officials usually state that one of the main reasons for implementing a primary dealer system was enhancing secondary market "liquidity" for government bonds. The main objection stated by opponents to a primary dealer system refers to restricted competition on the secondary market and a greater danger of collusion on the primary market. While the problem of enhancing collusion among bidders might be apparent, the other arguments, especially the possible advantages of a primary dealer system from the government’s point of view, remain quite unclear. What does it mean to increase market liquidity and why is the government interested in doing so? In this paper we therefore abstract from problems of

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10 See Appendix 1. In this appendix there is a brief overview with respect to primary dealer systems in the countries specified by Table 1 as well as a short characterization of public bond markets of countries without a primary dealer system.

11 So we were told, e.g., by M. Astberg, Debt Management Department, The Swedish National Debt Office, 08/14/1998.

12 This was pointed out, e.g., by J. Gyntelberg, Financial Markets Department, Danmarks Nationalbank, 08/10/1998, and H.-C. Kesselring, Schweizerische Nationalbank, August 1998.
collusion and focus mainly on the possible advantages of a primary dealer system as described by the definition previously given. To do so, it is first necessary to define an adequate model.

III. Primary dealership with perfect market maker competition

1. The model

We consider a public debt manager representing a government which wants to raise money by issuing some prespecified number $B$ of bonds at time 0. Any of these bonds will lead to payments amounting to unity at time 1. The government’s general objective is to maximize revenues from the sale of these bonds at time 0. Certainly, in reality public debt managers are concerned of more than just only one objective. But on the one hand the coverage of the government’s borrowing needs at minimum costs (which in general means to maximize sales revenues) seems to be the most important goal for public debt managers. In particular, in Germany such an objective can be derived from section 7 of the German federal budget rules (“Bundeshaushaltsordnung”). On the other hand, in many cases other objectives can be viewed as means to assure the realization of maximum revenues from issuing a given number of bonds. For example, enhancing secondary market liquidity can in general be thought of as a means to improve upon the issuance conditions for public debt.

As is common practice in many industrial countries, we assume that government bonds are issued by a discriminatory sealed-bid auction with a minimum price of $p_{\text{min}}^{\text{bid}} \geq 1$ per unit at time 0. So we may view the product $B \cdot p_{\text{min}}^{\text{bid}}$ as the government’s minimum financing goal. There are $n$ different banks which are allowed to submit competitive bids in this auction. Each bank submits only one bid which consists of a price and the number of bonds the bank wants to purchase at this price. So we allow for the possibility of limiting bids with respect to the number of bonds asked for.

Furthermore there are $q$ other investors who may submit non-competitive bids amounting to exactly $\beta$ units of government bonds each. Non-competitive bids will be served at the average accepted competitive bid. To simplify our analysis we assume that all investors who are

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13 For a comprehensive discussion of possible goals for public bond management see Bröker, G., 1993, Government Securities and Debt Management in the 1990s, 37-45.
allowed to place non-competitive bids will do so.\textsuperscript{14} We further postulate that any competitive bidder can at most get a fraction \( \alpha \in [1/n; 1] \) \((n \in \mathbb{N})\) of the \( R = B - q \cdot \beta \) remaining bonds.

After successful banks have received public bonds in turn for their payment all\textsuperscript{15} banks enter a secondary market as market makers for just one trading round. They have to announce bid and ask prices in advance and these prices have to be constant for the whole trading round. Furthermore, there are two types of other market participants, call them “liquidity traders”, who are willing to buy or sell government bonds. On the one hand there are the non-competitive bidders from the auction. They will be called type 1-individuals and they may act as buyers or as sellers of government bonds on the securities market. On the other hand there are other subjects who are not endowed with government bonds. We call this last group of agents type 2-individuals. They only can act as buyers of public bonds.

Consider first a buyer of public bonds. The gross demand \( D \) of such an individual is defined as the number of bonds this individual wants to hold after the trading round. This gross demand depends on the ask price \( p^{\text{ask}} \) the individual has to pay for additional bonds. To keep things simple we assume that for any individual buying bonds gross demand \( D \) can be described by the following equation:\textsuperscript{16}

\[
D(p^{\text{ask}}) = a - b \cdot p^{\text{ask}} \quad (a, b > 0).
\]

As we get the net demand \( d \) of any individual by subtracting his initial endowment (after the auction) \( w \) from \( D \), type 1- and type 2-individuals only differ with respect to their net demand. Note, that equation (1) is only relevant in case of \( D-w \geq 0 \). In a situation with \( D-w < 0 \) (which is only possible for type 1-individuals) there is no effective net demand for the security. But there still may be some net supply. For a type 1-individual who wants to sell bonds we first can determine his gross demand \( D \) in case of a market maker’s bid price \( p^{\text{bid}} \). The relevant formula is identical to (1) with the only difference that \( p^{\text{ask}} \) has to be substituted by \( p^{\text{bid}} \). A type 1-individual will only sell bonds if his gross demand calculated according to (1) with

\textsuperscript{14} The introduction of the possibility of non-competitive bids is only necessary to assure that there may be some supply of bonds by liquidity traders. For the rest of the paper we thus assume the amount \( q \cdot \beta \) to be exogenously given.

\textsuperscript{15} Even banks which did not get any public bonds on the primary market may act as market makers on the secondary market.

\textsuperscript{16} It is possible to extend the analysis to the case of more general demand functions, but this complication would not be honored by additional insights into our relevant questions. See also footnote 28.
\[ p = p^{(\text{bid})} \] is smaller than his initial endowment \( w \) with public bonds. In this case the difference \( w - D \) in principle describes his net supply of bonds to the market:

\[
\begin{align*}
\text{(2) } s(p^{(\text{bid})}) &= w - D(p^{(\text{bid})}) \\
&= w - a + b \cdot p^{(\text{bid})}.
\end{align*}
\]

Equation (2) holds true as long as \( s(p^{(\text{bid})}) \) from (2) does not exceed the individual’s endowment \( w \) with bonds. Otherwise the individual’s net supply of bonds is restricted to this initial endowment \( w \): \( s(p^{(\text{bid})}) \equiv w \).

All trading takes place during the trading round at time 0. We denote with \( q_i \) the number of agents of type \( i = 1, 2 \) entering the market during this trading round. Even before market makers bid for public bonds on the primary market they learn the numbers \( q_1 \) and \( q_2 \) of both types of liquidity traders. By announcing bid and ask prices on the secondary market they have to be aware of the problem to run out of bonds. So bid and ask prices have to be quoted in such a way as to assure any market maker’s possibility to deliver all bonds in demand. Hence, market makers’ inventories of bonds after the trading round have to be nonnegative.\(^{17}\)

Furthermore, there is some inter-dealer "clearing system": market makers are allowed to buy from or sell to other market makers at their announced prices where (net) delivery obligation for a market maker is restricted to the number of bonds which remain after he has served the whole demand from liquidity traders.\(^{18}\)

Market makers are only interested in maximizing their final wealth at time 1. We assume that they will submit bids on the primary market in order to get some government bonds as long as their overall resulting revenues are nonnegative. The same may hold true for their secondary market behavior: Market makers will be willing to quote prices with positive demand and/or supply if their revenues from secondary market trading are nonnegative. Monetary

\(^{17}\) The analysis could be extended in order to allow for uncertain numbers of liquidity traders of both types at the time market makers fix their quotes. We will briefly discuss this issue in Section V. In any case we assume that market makers’ monetary endowment is sufficient to pay for all bonds supplied to them.

endowments and bonds which are available to market makers after trading has taken place will be stored at an interest rate of zero until time 1.

2. Equilibrium without official interventions

The outcome of the trading on the secondary market for government bonds as well as the revenues from auctioning these bonds will depend on the specific assumptions regarding market characteristics as well as market participants’ behavior. We start our analysis by stating some assumptions which will define a simple reference model.

**Assumption 1:** There are no restrictions with respect to market makers’ maximum spreads and minimum secondary market turnovers. All market participants are able to learn all market makers’ quotations without incurring any information costs. If there are some market makers offering the same best ask or bid price then demand or supply from liquidity traders will be equally distributed among these market makers. There is no official authority selling bonds to or buying bonds from market makers on the secondary market and there is no obligation for competitive bidders to demand a certain minimum amount of bonds on the primary market.

Let us first consider secondary market transactions under these special assumptions. We call a market maker on the secondary market who is facing some positive demand for government securities by liquidity traders "active". Otherwise he may be called "inactive" or "passive". With this definition in mind it is easy to prove the following lemma.

**Lemma 1:** As long as there are \( n > 1 \) active market makers on the secondary market in equilibrium all of them will quote the same price \( p^* \geq 1 \) as ask and as bid price during the trading. Furthermore, there will be at most one equilibrium on the secondary market. If this equilibrium is characterized by \( p^* > 1 \) then all active market makers will totally run out of bonds at the end of the trading round. With \( I_{tot} \) being the overall inventory of all market makers, the resulting equilibrium price \( p^* > 1 \) can then be determined as

\[
p^* = a + \frac{I_{tot}}{b} + q_1 + q_2 \cdot \beta.
\]
Proof:

In any case liquidity traders will only choose to sell government bonds to a market maker with the maximum bid price. Let us call this bid price $p^{\text{bid}}(\ast)$. As long as $p^{\text{bid}}(\ast) > 1$, any market maker can raise his net revenues at time 1 by offering a slightly higher bid price than the maximum bid price of the other market makers. So we may conclude that „successful“ bid prices have to be greater than or equal to 1. This means that the only possibility to reach positive revenues from market making on the secondary market stems from selling bonds at an ask price greater than the according bid price to liquidity traders. Moreover, because of complete market transparency all active market makers will quote the same ask price $p^{\text{ask}}(\ast) \geq 1$. If this ask price would be smaller than a market maker’s corresponding bid price $p^{\text{bid}}$, then there would be an incentive for any other market maker to buy at $p^{\text{ask}}(\ast)$ from this competitor and simultaneously sell to him at $p^{\text{bid}} > p^{\text{ask}}(\ast)$ just realizing unbounded revenues. So, in equilibrium we must have any market maker’s ask price being not smaller than his corresponding bid price.

In any case active market makers are the only ones who are willing to buy bonds from type 1-individuals at a bid price greater than one. Therefore we may conclude that all active market makers are successful bidders with $p^{\text{bid}}(\ast) \geq 1$. Now assume active market makers are realizing $p^{\text{ask}}(\ast) > p^{\text{bid}}(\ast)$ so that they gain a revenue from market making greater than zero. In such a case any one of them as well as any inactive market maker has an incentive to offer a slightly better ask price $p^{\text{ask}}(\ast) - \varepsilon > p^{\text{bid}}(\ast)$ ($\varepsilon > 0$) so that he can attract all demand. Let us call such a market maker „deviant“.

If a deviant market maker’s inventory is sufficient to cover this additional demand he certainly will be able to raise his revenues by this behavior. Things are a little bit more complicated if the market maker’s endowment is not large enough to cover this demand. Then the market maker will on the one hand raise his bid price slightly up to $p^{\text{bid}}(\ast) + \delta$ with $p^{\text{ask}}(\ast) - \varepsilon > p^{\text{bid}}(\ast) + \delta$ ($\delta > 0$), so that he will also attract the whole supply of government bonds. If revenues for a market maker were positive for an ask price $p^{\text{ask}}(\ast)$ and a bid price $p^{\text{bid}}(\ast)$ then the deviant market maker will be able to choose $\varepsilon$ and $\delta$ in such a way that his overall revenues will be positive, too. This seems to be apparent for the case that his inventory after trading with type 1- and type 2-individuals is nonnegative. But indeed there may be the problem that the deviant
market maker runs out of bonds. To avoid such a situation the market maker can buy securities from other market makers at \( p^{(ask)} \). This is possible, since other market makers will face a demand for bonds by liquidity traders amounting to zero. For \( \varepsilon \) being sufficiently small the deviant market maker will suffer only an infinitesimal loss from this transaction, thus increasing his overall revenue to an amount beyond that which could be reached by the quotation of the original prices \( p^{(ask)} \) and \( p^{(bid)} \). This possibility to deviate will always exist as long as supply of government bonds from liquidity traders at \( p^{(bid)} \) has not reached its maximum value. This means that gross demand by a type 1-individual at \( p^{(bid)} \) has to be greater than zero. In fact, this condition certainly is always met because otherwise gross demand by a type 2-individual at \( p^{(ask)} \geq p^{(bid)} \) would also be at most zero which would imply that there could not be any active market maker for the ask price \( p^{(ask)} \).

So we may conclude that in equilibrium we must have \( p^* = p^{(ask)} = p^{(bid)} \geq 1 \). Certainly a situation with ask and bid price being equal to one may constitute an equilibrium as there is not any possibility to gain by deviation. For this equilibrium to emerge the initial overall inventory of all active market makers must at least amount to \( (a - b) \cdot q_2 - (\beta - a + b) \cdot q_1 \), because otherwise some market makers would run out of bonds.

If overall inventories of all market makers are smaller than this critical value, \( p = 1 \) will not describe an equilibrium, but there may be an equilibrium for \( p > 1 \). In such a situation all active market makers will completely sell their inventories at these prices because otherwise there would be an incentive for at least one of them to lower his ask price a bit in order to sell his excess inventory, closing any gap with respect to his supply capacities for public bonds by a suitable demand for bonds at other market makers. We thus may conclude that an equilibrium price \( p^* > 1 \) must obey the following equation:¹⁹

\[
(4) \quad I_{tot} + q_1 \cdot [\beta - a + b \cdot p^*] - q_2 \cdot [a - b \cdot p^*] = 0
\]

\[
\iff \quad p^* = \frac{a}{b} - \frac{I_{tot} + q_1 \cdot \beta}{b \cdot (q_1 + q_2)}.
\]

¹⁹ This equation holds true even if we have \( \beta - a + b \cdot p < 0 \). Notice further that the case \( \beta - a + b \cdot p > \beta \) is not possible as long as we assume that there is some positive demand for bonds at \( p \).
In general $I_{Tot}$ describes the total inventory of all active market makers. Since under the assumption of a strict inequality $p^* > 1$ any active market maker is realizing positive revenues during the trading period (compared to the holding of bonds until time 1), there is an incentive for any bank to be active. We thus may conclude that any market maker will quote the same identical bid and ask price according to (4) in equilibrium which implies that $I_{Tot}$ turns out to be the overall inventory of all market makers.20 Q.e.d.

As overall demand by liquidity traders can be reduced to any nonnegative level by increasing $p$, we can draw the following additional conclusion: There will always be a unique equilibrium on the secondary market with respect to $p$ for any values of $q_1$ and $q_2$.

After this brief characterization of secondary market occurrences we can now go on analyzing the properties of primary market equilibria. First of all it is easy to see that the following lemma holds true:

**Lemma 2**: In equilibrium all competitive bidders will submit the same (unlimited) bid $p^{(bid)*} \geq 1$, if we abstract from the possibility $p^{(bid)*} = p^{(bid)}_{\text{min}}$. If in contrast we have $p^{(bid)*} = p^{(bid)}_{\text{min}}$, then overall demand by competitive bidders for public bonds may be smaller than $R$.

**Proof:**
Suppose there are $n < n$ banks winning the tender by bidding $p^{(bid)*} > p^{(bid)}_{\text{min}}$, all realizing the same portion $1/n$ of $R \equiv B - q \cdot \beta$. In this case there are some banks getting nothing from the issuance. If one of these banks would also submit a bid $p^{(bid)*}$, then the inventory of any active market maker on the secondary market would be reduced in such a way that the resulting secondary market price $p^*$ would not be changed, since the overall initial inventory of all competitive bidders would remain constant: $(n + 1) \cdot R / (n + 1) = n \cdot R / n = R$.

20 Notice that there is no problem for any market maker to run out of bonds, as long as (4) holds because any bond shortage for some market makers will then be accompanied by an according surplus of bonds for other market makers which can just be equalized by inter-dealer trading.
If the original n "successful" bidders were able to reach a nonnegative revenue from participating in the auction this therefore must hold true for the case of n + 1 bidders because of the constant value for the equilibrium bid and ask price $p^*$. The situation just described therefore cannot be an equilibrium. Now suppose that total demand by all n banks submitting a bid $p^{(bid)*} > p^{(bid)}_{\text{min}}$ only amounts to a sum smaller than R. Such a situation could only be possible if there were not any one of the first n bidders rationed. But this implies that there would be an incentive for any of the first n banks to reduce its bid below $p^{(bid)*}$ in order to cover its demand for public bonds at a lower price. Such a situation therefore cannot describe an equilibrium, either. This proves the first part of Lemma 2.

Now suppose that we have $p^{(bid)*} = p^{(bid)}_{\text{min}}$, then it would not be possible to lower one's bid any more in order to get public bonds at smaller prices. Moreover, as long as there is not any rationing there will be no incentive to increase one's bid. Therefore in case of $p^{(bid)*} = p^{(bid)}_{\text{min}}$ not all of the R bonds intended to be sold to competitive bidders may be successfully issued. Q.e.d.

If market makers' total inventory after bidding is so large that we get $p^* = 1$ on the secondary market, then we certainly can only have $p^{(bid)*} = 1$ as equilibrium price on the primary market. So we may focus on values for R being small enough to imply $p^* > 1$ on the secondary market.

**Proposition 1:** Suppose $\alpha \geq 1/(n-1)$. Then for given values of B and R so that $p^* > 1$ results on the secondary market there will be a unique equilibrium on the primary market with $p^{(bid)*} = p^*$ as long as we have $p^* \geq p^{(bid)}_{\text{min}}$. A situation with $p^* < p^{(bid)}_{\text{min}}$ cannot describe an equilibrium.

**Proof:**
Assume that we have $p^* < p^{(bid)}_{\text{min}}$. In such a situation there is not any market maker willing to submit a bid on the primary market because resulting net revenues from this transaction will certainly be negative.
Now assume that we have \( p^* > p_{\text{bid}}^{\min} \). Because of \( p^* \geq p_{\text{bid}}^{*} \) we know from Lemma 2 that we can restrict our analysis to situations where all competitive bidders get the same number \( R / n \) of bonds as initial inventory for a certain bid \( p_{\text{bid}}^{*} \). For this initial inventory we know that there is just one possible secondary market equilibrium \( p^* \) for any values of \( q_1 \) and \( q_2 \). Moreover competitive bidders’ net revenues have to be nonnegative.

Suppose that revenues for bidders are indeed positive. For \( p^* > 1 \) this is only possible if we have \( p^* > p_{\text{bid}}^{*} \) as well. If any of these bidders would increase his bid a little bit above \( p_{\text{bid}}^{*} \), he could enlarge his share up to at most \( \alpha \) thereby becoming the market maker with the largest inventory. From Lemma 1 we know that this modified outcome of the auction would not affect equilibrium secondary market price \( p^* \). So this deviant market maker would certainly increase his net revenues. Therefore with \( p^* > p_{\text{min}}^{\text{bid}} \) an equilibrium on the primary market can only exist if we have \( p_{\text{bid}}^{*} = p^* \). In such a situation there will be no incentive for any bidder to increase one’s bid beyond \( p^* \). Moreover, reducing one’s bid below \( p_{\text{bid}}^{*} \) would lead to a zero portion because of other bidders unlimited bids. So we really get an equilibrium for \( p_{\text{bid}}^{*} = p^* \).

If we have \( p^* = p_{\text{bid}}^{\min} \), then we certainly must have \( p^* = p_{\text{bid}}^{*} \), too. Q.e.d.

Since the resulting secondary market price is monotone decreasing in \( I_{\text{Tot}} \), for any given parameter constellation there is at most one equilibrium price \( p^* \) on the primary and the secondary market. If the resulting secondary market price for \( I_{\text{Tot}} = R \) is not smaller than \( p_{\text{bid}}^{\min} \), this will be the unique equilibrium. If the opposite is true, then there can only be an equilibrium with \( p^* = p_{\text{bid}}^{\min} \) and overall bond demand by competitive bidders being smaller than \( R \).

If we had \( \alpha < 1 / (n - 1) \) then there would be no incentive for any competitive bidder to submit a bid greater than \( p_{\text{bid}}^{\min} \) on the primary market even in case of \( p^* > p_{\text{bid}}^{\min} \). Indeed, in such a case only for \( p^* = p_{\text{bid}}^{\min} \) there may be an equilibrium but in general there will be no equilibrium at all. Therefore we can conclude that the government should choose \( \alpha \geq 1 / (n - 1) \) if it wants to assure the issuance to be successful even under the assumption of exogenously fixed minimum bid price \( p_{\text{bid}}^{\min} \).
To complete our analysis let us briefly consider the case of just one bank participating at the auction as a competitive bidder. To keep consistency we must then have $\alpha = 1$. Since the bank in consideration is the only one to place a competitive bid, it will only (at most) submit the minimum bid price $p^{(bid)} = p^{(bid)}_{min} \geq 1$. Moreover, the overall situation on the secondary market here is similar to the case of monopolistic market makers which we will discuss in detail in Section IV.

Note that even for given minimum bid price $p^{(bid)}_{min}$ there is in general no need for more than just two market makers. Indeed the government will be indifferent between different numbers $n > 1$ of competitive bidders.

With these results we can now go on to ask if official interventions on the secondary and/or the primary market will help the government to raise more money. First we turn to secondary market interventions.

3. Primary dealership versus price management operations

a. Secondary market interventions

Establishing a primary dealer system would imply that the government could define some maximum bid-ask-spread for the secondary market as well as some minimum turnover per market maker. Since in the setting of this Section III. all market makers’ bid-ask-spreads are always down to zero there is not any possibility to intervene by defining maximum bid-ask-spreads.\footnote{We refrain from discussing the practically as well as theoretically unimportant case of \emph{negative} bid-ask-spreads.} The same is true for defining (absolute or relative) minimum turnovers although the derivation of this result is somewhat more complicated.
First of all note that we still must have \( p^{\text{ask}*} = p^{\text{bid}*} \), because otherwise there would be an incentive for any market maker to deviate by decreasing \( p^{\text{ask}*} \) and increasing \( p^{\text{bid}*} \) just a little bit. Since such a deviant market maker would attract all bond demand and supply on the secondary market he certainly would meet any minimum turnover requirement if this were true for the other market makers without his deviation. Furthermore bid and ask price must be specified in such a way as to assure that any of the active market makers will run out of bonds after trading because otherwise there would be an incentive to reduce one’s bond demand on the primary market.

Now suppose that there are \( n \) winning bidders submitting the same bid. Certainly, the bid \( p^{\text{bid}} \) submitted on the primary market will lead to net revenues by each winning bidder of just zero because otherwise any bidder would have an incentive to slightly increase his bid. The only reason why the other \( n - n \) banks will refrain from bidding would be that they could not meet turnover requirements on the secondary market in case of winning the bid because of their partitioned share of the issue being too small. As a consequence of minimum turnover requirements we thus may have a smaller number of active market makers on the secondary market but since their overall inventory remains unchanged so do the resulting bid and ask prices. After all, minimum turnover requirements may be useless in our model or just only lead to a smaller number of market makers on the secondary market without affecting any prices.

The same holds true if the government intends to reduce the number \( R \) of bonds to be sold to competitive bidders on the primary market just in order to sell the remaining bonds separately on the secondary market. We may call such a behavior “price management operation”. Since this measure would reduce the supply of bonds by market makers just in the same way as the supply of bonds by other market participants would be enlarged this procedure would neither influence primary nor secondary market equilibrium prices.

Let us summarize our findings in the following proposition.

**Proposition 2:** Let Assumption 1 hold. Then the government cannot influence its primary market revenues by formulating maximum bid-ask-spreads or minimum turnovers as well as by the sale of bonds on the secondary instead of on the primary market.
b. Primary market interventions

Since under the assumption of perfect market maker competition the government cannot influence its revenues on the primary market by secondary market interventions, we might conjecture that the same holds true for the definition of minimum bid prices for competitive bidders on the primary market. Somewhat surprisingly, this conjecture is wrong. If the public debt manager raises \( p_{\text{bid}}^{(\text{min})} \) to such a level that the resulting secondary market price for \( I_{\text{Tot}} = R \) does not describe an equilibrium any longer, then we still may get an equilibrium with \( p^* = p_{\text{bid}}^{(\text{bid})} \) where overall demand by market makers is smaller than \( R \). Indeed, if we want to maximize overall governmental revenues \( p^* \cdot (I_{\text{Tot}} + \beta \cdot q_1) \) we typically will be able to derive an inner solution because equation (4) just leads us to a quadratic objective function with respect to \( I_{\text{Tot}} \) (or \( p^* \)).\(^{22}\) We thus get in general binding minimum bid price requirements on the primary market if we allow for situations where not all original \( R \) bonds are sold to competitive bidders.

In contrast to minimum bid requirements the definition of some minimum number \( r \) of bonds which has to be demanded by any competitive bidder will not lead to higher revenues for the government. Such a requirement must only be taken into consideration in case of \( p^* = p_{\text{bid}}^{(\text{bid})} \). But under this condition any binding requirement with respect to market makers’ bond demand may only lead to negative overall revenues by some market makers. Since these market makers would not be willing to participate in the bond auction at all, the government will only reduce the number of bidders in equilibrium on the primary market by this kind of intervention, but it cannot increase its revenues from the auction. We therefore come immediately to the following proposition.

**Proposition 3:** Let Assumption 1 hold. Then the government can positively influence its primary market revenues by formulating minimum bid price requirements for the bond auction on the primary market, but not by defining some mandatory minimum bond demand \( r \) per competitive bidder on the primary market. If we assume a given number \( R \) of bonds shall be

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\(^{22}\) One might argue that bond demand by non-competitive bidders should be sensitive to variations of minimum bid prices on the primary market as well. This effect certainly makes increases in minimum bid price less attractive but does not change the general relationships, especially the possibility to derive an inner solution.
sold on the primary market to competitive bidders in any case, then the government is not able to influence its revenues by any kind of official intervention.

After all, in case of perfect market maker competition there is only little room for official interventions. If we view the number R of bonds to be sold to competitive bidders as exogenous, then indeed there are no possibilities for the government to positively influence its primary market revenues. Anyway, it is hardly possible to interpret the resulting equilibria as some kind of primary dealer system. Obviously, we need market imperfections in order to motivate a broader range of official interventions. In this context, some kind of monopolistic power for market makers on the secondary market may be helpful to explain the usefulness of governmental interventions.

IV. Primary dealership with monopolistic market makers

In sharp contrast to our model of Section III. we now state the following alternative assumption.

Assumption 2: Suppose that each bank has its own clientele and denote with \( q_{ij} \) the number of type i-individuals (i = 1, 2) approaching bank j during the trading round. Let each individual approaching bank j in order to buy bonds be characterized by a (gross) demand function

\[
D_j(p_{j,\text{ask}}) = a_j - b_j \cdot p_{j,\text{ask}}.
\]

Similarly type 1-individuals who want to sell bonds to bank j will have a net supply function \( s_j \) with

\[
s_j(p_{j,\text{bid}}) = w - D_j(p_{j,\text{bid}}) = w - a_j + b_j \cdot p_{j,\text{bid}}.
\]

Assumption 2 describes a situation where every market maker is a monopolist with his own customers. A market maker’s quotation may influence demand and supply by his clientele, but it will not lead to customer fluctuations among market makers. Such a situation may be the consequence of some extreme kind of informational asymmetry or personal preferences on the
side of the liquidity traders. Moreover, in order to stress the monopolistic character of each market maker, from now on we abstract from any kind of inter-dealer trading. 23

1. Equilibrium without official interventions

Let us start to analyze the consequences of Assumption 2 by looking at the market makers’ behavior on the secondary market. Each market maker j will define his ask price $p_j \text{ask}$ and his bid price $p_j \text{bid}$ just in order to maximize his gain from trading. For the rest of this paper we will focus on situations with $p_j \text{ask} \leq a_j / b_j$ and $p_j \text{bid} \geq (a_j - \beta) / b_j$, i.e. situations with non-negative demand for and supply of bonds with respect to any market maker j. Under these conditions we may formalize the market maker’s decision problem in the following way:

$$\max \{ p_j \text{ask} \cdot q_{2j} \cdot (a_j - b_j \cdot p_j \text{ask}) - (p_j \text{bid} - l) \cdot q_{1j} \cdot (\beta - a_j + b_j \cdot p_j \text{bid}) - I_j \cdot (p_j \text{bid} - l) \}$$

subject to

$$q_{1j} \cdot (\beta - a_j + b_j \cdot p_j \text{bid}) + I_j - q_{2j} \cdot (a_j - b_j \cdot p_j \text{ask}) \geq 0.$$

As long as we have a positive bond supply to the market maker and an optimal bid price $p_j \text{bid}* > 1$, constraint (8) will certainly be binding because otherwise the market maker could increase his overall revenues by lowering $p_j \text{bid}$. For situations with zero bond supply as well as situations with positive bond supply but $p_j \text{bid}* \leq 1$ the same holds true because in such situations the market maker in consideration would certainly have had an incentive to reduce his limit order in the auction. 24 Therefore in equilibrium any market maker’s j inventory constraint must be binding. Moreover we can conclude that the market maker will choose his initial inventory $I_j$ in such a way as to assure $p_j \text{ask}* \geq 1$. Otherwise it would be better for him to refrain from selling all his bonds to liquidity traders so that his inventory constraint for given $I_j$ would not be binding. This leads us to the following lemma.

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23 In what follows we will see that market makers’ inventory constraints in general will be binding. For such a situation the possibility of inter-dealer trading as defined in Section III. would be redundant, anyway.

24 We refrain from analyzing market makers that do not demand any public bonds on the primary market because such market makers will not contribute to government’s revenues from the issuance of bonds at all.
**Lemma 3:** Suppose that Assumption 2 holds. Then a market maker’s $j$ optimal price quotations are given by

$$p_{j}^{(ask)*} = a_j \frac{2 \cdot I_j + \beta \cdot q_{1j}}{b_j} < b_{j} \cdot (q_{1j} + q_{2j})$$

and

$$p_{j}^{(bid)*} = \frac{1}{b_{j}} \left\{ q_{2j} \left( \frac{2 \cdot I_j + \beta \cdot q_{1j}}{b_{j} \cdot (q_{1j} + q_{2j})} - I_j \right) \cdot \frac{1}{q_{1j}} - \beta + a_j \right\},$$

as long as we have $\beta \cdot q_{2j} \geq 2 \cdot I_j$.

**Proof:**

For (8) being binding we get:

$$p_{j}^{(bid)} = \frac{1}{b_{j}} \left\{ q_{2j} \left( a_j - b_{j} \cdot p_{j}^{(ask)} \right) - I_j \right\} \cdot \frac{1}{q_{1j}} - \beta + a_j \right\}.$$

After substituting this term for $p_{j}^{(bid)}$ in the market maker’s objective function and deriving this expression with respect to $p_{j}^{(ask)}$ we get (9) as a necessary and sufficient condition for the optimal market maker’s ask price. Inserting (9) in (11) gives us the corresponding expression for the optimal market maker’s bid price as described by (10). This expression is not smaller than $(a_j - \beta) / b_j$, if we have $\beta \cdot q_{2j} \geq 2 \cdot I_j$.

Notice further that we have

$$p_{j}^{(ask)*} - p_{j}^{(bid)*} = \frac{\beta}{2 \cdot b_{j}} > 0.$$

This implies that for $p_{j}^{(bid)} = p_{j}^{(bid)*}$ supply of bonds will not only be positive, but also smaller than $\beta$ per type 1-individual approaching market maker $j$. Q.e.d.

We may now use the results of Lemma 3 in order to describe the equilibrium on the primary market. First of all, we are able to determine a market maker’s optimal demand for bond inventory depending on his primary market bid price $p_{j}^{(bid)}$. 


Lemma 4: A market maker’s j optimal demand for public bonds on the primary market can be described by

$$I_j^* = \frac{(a_j - b_j \cdot p_j^{(bid)}) \cdot (q_{ij} + q_{2j}) - \beta \cdot q_{ij}}{2}.$$  

Proof:
In order to derive (13) we first have to insert (9) and (10) into (7). Setting the derivation of (7) with respect to $I_j$ equal to zero then gives us (13). Q.e.d.

From (13) we know that for any market maker the optimal bond demand on the primary market is a declining function of $p_j^{(bid)}$. With these results we are now able to consider competitive bidders’ behavior on the primary market.

Proposition 4: Let Assumption 2 hold true. Then there exists a unique equilibrium on the primary market if and only if all winning competitive bidders submit the same bid $p_j^{(bid)}$ and their overall demand for bonds amounts to at most R units.

Proof:
First assume that there are at least two different bids which are submitted by winning competitive bidders. The only reason for submitting the higher bid price would be to assure a higher portion of the issue. But this portion of the issue which can be reached by submitting the higher bid price could also be assured by a significant reduction of one’s bid price down to a level slightly above the second highest bid. In equilibrium we thus must have that all successful bidders submit the same bid $p_j^{(bid)}$. If any successful bidder is rationed at $p_j^{(bid)}$ there would be an incentive for him to increase his bid a little. So we may conclude that there is not any rationing at bid price $p_j^{(bid)}$ so that overall bond demand can at most amount to R units of bonds. But then in turn there may be an incentive to lower one’s bid because of this lack of rationing. To exclude the possibility of such a lower bid we must have $p_j^{(bid)} = p_{min}^{(bid)}$. Q.e.d.

As a consequence of their heterogeneity not all winning bidders will limit their bids to the same amount, but all bidders who demand a positive number of bonds will realize positive net revenues from the participation in the auction. Moreover, because of their monopolistic power
market makers can quote prices in such a way as to realize positive bid-ask spreads on the secondary market. These are relevant differences compared to the case of perfect competition. We therefore might expect that this may also be the case if we go on analyzing the government’s possibilities to influence primary and secondary market outcomes.

2. Primary dealership versus price management operations

a. Secondary market interventions

We first turn to the question how the government can influence market makers’ secondary market behavior. Such influence may prove useful for the government if it simultaneously increases market makers’ demand for public bonds on the primary market. We start our discussion with the introduction of maximum bid-ask-spreads for market makers. Such a requirement turns out to be an additional restriction for the market maker’s optimization problem with respect to his price quotations \( p_j^{\text{ask}} \) and \( p_j^{\text{bid}} \) on the secondary market. This problem was originally described by formulas (7) and (8) but now has to be extended to allow for

\[
(14) \quad p_j^{\text{ask}} - p_j^{\text{bid}} \leq d_1,
\]

where \( d_1 \) denotes the maximum bid-ask-spread for any market maker.

Certainly, we are only interested in cases where (14) is binding. Since we already know that (8) has to be binding, too, in equilibrium, we can derive a market maker’s ask and bid price quotations by simultaneously solving the two equations which are implied by (8) and (14). So we get

\[
\text{Lemma 5: Denote with } p_j^{\text{ask}*} \text{ market maker’s optimal ask price when there are no official interventions. In case of a binding maximum bid-ask-spread requirement market maker } j \text{ will choose an optimal ask price}
\]

\[
(15) \quad p_j^{\text{ask}(d_1^\star)} = p_j^{\text{ask}*} - \left( \frac{\beta - 2 \cdot b_j \cdot d_1}{2 \cdot b_j \cdot (q_{ij} + q_{ij}^*)} \right),
\]

with

\[
(16) \quad d_1 \leq \frac{\beta}{2 \cdot b_j}.
\]
Proof:

(15) follows immediately as the solution of the equation system described by (8) and (14). For \( d_1 = \beta/(2 \cdot b_j) \) we just get \( p_j^{(\text{ask})} = p_j^{(\text{ask})} \). The maximum bid-ask-spread requirement thus becomes binding for smaller values of \( d_1 \) than this critical value. Q.e.d.

Hence, we see that a market maker’s optimal ask price quotation is increasing in \( d_1 \). A binding maximum spread restriction will therefore result in lower market makers’ ask prices. It is easy to see that the opposite is true for a market maker’s bid price:

\[
(17) \quad p_j^{(\text{bid})} = p_j^{(\text{ask})} - d_1 = p_j^{(\text{bid})} + \frac{(\beta - 2 \cdot b_j \cdot d_1) \cdot q_{2j}}{2 \cdot b_j \cdot (q_{ij} + q_{2j})}.
\]

(15) and (17) can be inserted in (7) which leads to an objective function solely depending on the market maker’s inventory \( I_j \). Somewhat surprisingly, the derivation of this expression leads to the same formula for the optimal market maker’s bond demand as already developed in (13). This means for binding maximum spread requirements market makers will adjust their secondary market quotation in such a way as to keep the difference between demand for and supply of bonds which they are facing on the secondary market constant. This leads to the following lemma.

**Lemma 6:** In case of monopolistic market makers any kind of maximum spread requirements will not affect the resulting primary market equilibrium and thus neither the number of bonds sold nor their price which can be achieved via the bond auction by the government.

At first sight Lemma 6 does not seem to be very surprising. One might argue that any binding maximum spread requirement will only lead to reduced ask and increased bid prices in such a way that there is no additional demand by market makers for public bonds on the primary market. Therefore there seems to be no need for the market maker to change his demand on the primary market for given primary market bid price \( p_j^{(\text{bid})} \). Unfortunately, this argument holds true for *any* kind of official intervention even if this intervention influences primary market behavior. So we might suspect that our irrelevance result is a direct consequence of the assumption of linear demand and supply functions, and indeed in Appendix 2 of this paper it is shown that this suspicion does prove right. In general a variation of \( d_1 \) might influence a market maker’s optimal inventory \( I_j \) of public bonds but there appears to be no obvious
systematic relationship, so this kind of intervention does not seem to be very useful in case of monopolistic market makers.

Summarizing, if the public debt manager wants to raise government’s revenues on the primary market the definition of maximum bid-ask-spreads seems to be the wrong way, at least, if we consider the case of monopolistic market makers. This conclusion with respect to the ineffectiveness of official interventions does not apply any longer if we allow for the requirement of minimum turnover rates.

Let \( d_2 \) denote the minimum (absolute\(^{25} \)) turnover that a market maker is required to realize via bond sales on the secondary market. In this case the following inequality has to be added to the market maker’s decision problem described by (7) and (8):

\[
(18) \quad p_j^{(ask)} \cdot (a_j - b_j \cdot p_j^{(ask)}) \geq \frac{d_2}{q_{2j}}.
\]

We may interpret the fraction \( d_2/q_{2j} \) as the required minimum turnover per type 2-individual approaching market maker \( j \). In what follows we denote this fraction by \( d_{2j,r} \). Once again, we are only interested in such values of \( d_{2j,r} \) that (18) becomes binding. Since (18) does not depend on the market maker’s bid price we can immediately solve (18) with respect to \( p_j^{(ask)} \).

Neglecting negative solutions, this gives us

\[
(19) \quad p_j^{(ask),_{d_{2j,r}}} = \frac{a_j + \sqrt{a_j^2 - 4 \cdot b_j \cdot d_{2j,r}}}{2 \cdot b_j}.
\]

For (19) to be an admissible solution we must have that

\[
(20) \quad d_{2j,r} \leq \frac{a_j^2}{4 \cdot b_j}.
\]

This means that the minimum turnover requirement must not be so demanding that it cannot be reached by the market maker \( j \) at all.

\(^{25}\) Since in equilibrium total market makers’ turnovers from bond sales are given, absolute minimum turnover requirements can easily be transformed into relative ones. Our partial-equilibrium analysis therefore applies to this case, too. The disadvantage of relative minimum turnover requirements in case of monopolistic market makers is that relative requirements force market markers to form expectations with respect to other market makers’ behavior thus leading to the possibility of multiple equilibria. We therefore restrict our analysis from now on to absolute turnover requirements.
Since we already know that (8) has to be binding we get the relevant value of the market maker’s bid price by inserting (19) in (8):

\[ p_{j}^{\text{bid},d_{2j,r}} = \frac{2 \cdot (a_{j} - \beta) \cdot q_{1j} - 2 \cdot I_{j} + \left( a_{j} - \sqrt{a_{j}^{2} - 4 \cdot b_{j} \cdot d_{2j,r}} \right) \cdot q_{2j}}{2 \cdot b_{j} \cdot q_{1j}}. \]  

According to (19) and (21) an increase in \( d_{2j,r} \) leads to lower ask prices and higher bid prices. Market makers have to reduce their ask prices because their optimal price quotations are located in the region of elastic demand, so that an increase in turnover from bond sales is only possible by increasing the number of bonds sold to liquidity traders. In order to satisfy higher bond demand market makers will have to increase their bid prices, thus increasing the bond supply by liquidity traders.

With (19)\(^{26}\) and (21) inserted in (7) and forming the derivation with respect to \( I_{j} \) it is possible to determine a market maker’s optimal demand for public bonds on the primary market as a function of his primary market bid price. Recall that \( I_{j}^{*} \) is the optimal bond demand by market maker \( j \) on the primary market when there are no official interventions. So we get:

\[ I_{j}^{d_{2j,r}^{*}} = \frac{a_{j} \cdot (q_{1j} + q_{2j}) - (\beta + b_{j} \cdot p_{j}^{\text{bid}}) \cdot q_{1j} - \sqrt{a_{j}^{2} - 4 \cdot b_{j} \cdot d_{2j,r} \cdot q_{2j}}}{2} \]

\[ = I_{j}^{*} + \frac{(b_{j} \cdot p_{j}^{\text{bid}} - \sqrt{a_{j}^{2} - 4 \cdot b_{j} \cdot d_{2j,r}}) \cdot q_{2j}}{2}. \]

From (22) we see that there now may be some relevant implications of official secondary market interventions for bidders’ primary market behavior. In case of minimum turnover requirements \( d_{2} \) with\(^{27}\)

\[ \frac{a_{j}^{2}}{4 \cdot b_{j}} \geq d_{2j,r} \geq \frac{a_{j}^{2} - b_{j}^{2} \cdot p_{j}^{\text{bid},2}}{4 \cdot b_{j}}, \]

this requirement will lead to ceteris paribus higher bond demand by the market maker in consideration.\(^{28}\) The intuition for this result seems to be quite clear. In order to be able to

\( ^{26} \) Notice that the market maker’s optimal ask price does not depend on his bond demand on the primary market and thus is not relevant for the determination of the optimal value for \( I_{j} \).

\( ^{27} \) Moreover, we must allow for the fact that the supply of bonds to market maker \( j \) cannot be greater than \( \beta \cdot q_{1j} \). This condition will certainly be fulfilled with respect to (21) for \( \beta \) being high enough.

\( ^{28} \) It is not difficult to generalize this result to the case of arbitrary demand and supply functions. The procedure is the same as in Appendix 2. Besides the fact that (gross) bond demand by liquidity traders is falling in a market maker’s ask price we only need the assumptions that a market maker’s optimal ask price is falling in \( d_{2j,r} \) and that the Hessian matrix to the market maker’s optimization problem is negative definite.
realize a greater turnover from bond selling the market maker has to lower his ask price so that
he can sell a greater number of bonds. But this greater demand can only be satisfied if the
market maker’s bond inventory ceteris paribus is increased. Unfortunately, as was mentioned
earlier, exactly such an intuition could also be applied to the case of maximum bid-ask-spreads,
although the analytical results for this scenario are not in line with this intuition. So what is the
relevant difference between these two kinds of interventions? In the second case the market
maker is not forced to narrow the gap between ask and bid price in a certain way. This means
that for given (new) optimal ask price $p^{(ask),d_1,r,*}_j$ on the secondary market the market maker
can choose whether to increase (anticipatorily) his demand for bonds on the primary market or
on the secondary market (via an increase of $p^{(bid)}$). As stated above, the market maker will in
fact use this degree of freedom in order to strengthen his demand for public bonds on the
primary market. This mechanism in connection with an adequate definition of the minimum
primary market bid price $p^{(bid)}_{min}$ may give the public debt manager the opportunity to increase
government’s revenues from bond sales.

Our result of advantageous minimum turnover requirements for market makers’ bond sales will
even hold true if we assume that these requirements are only applicable to active bidders on the
primary market. This means that we may allow for a scenario where market makers that do not
bid on the primary market are not subject to minimum turnover requirements on the secondary
market. In this case any market maker has an additional outside option which might lead to a
maximal acceptable turnover requirement for him smaller than the one described by (23), but
nevertheless there will be room for increasing government’s revenues. Additionally, even in
this case there would not be any necessity to compensate regulated market makers because for
well-defined minimum turnover requirements voluntary acceptance of this regulation could be
assured. In this sense a primary dealer system may increase secondary market liquidity in a way
which is useful for the government. We will return to this issue in Section IV.2.b, but before
we have to analyze the case of minimum turnover requirements with respect to bond purchases
by market makers.

From the previous results it is easy to suspect that this second kind of minimum turnover
requirements will lead to lower market makers’ demand for public bonds on the primary
market. This is in fact the case, although binding minimum turnover requirements with respect
to bond purchases will induce market makers to lower their ask and increase their bid prices
just in a similar way as in case of binding minimum turnover requirements with respect to bond sales. This kind of intervention thus will not be interesting for a public debt manager trying to maximize government’s revenues from bond purchases. Our results therefore show that not all potential measures of enhancing secondary market liquidity might be advantageous for the government.

The last kind of intervention considered in this paper will be to sell (or buy) a number of $d_3$ bonds (per market maker) on the secondary instead of on the primary market by the public debt manager. As mentioned earlier, we may call an intervention of such a kind a "price management operation”. Let us assume that the number $d_3$ of bonds sold to ($d_3 > 0$) or bought ($d_3 < 0$) from market makers by the government is distributed among all $n$ active market makers according to their portion of type 1- or type 2-individuals approaching a certain market maker $j$, thus reducing or increasing the number of bonds sold on the primary market to competitive bidders by the same amount.

Such a kind of secondary market intervention is a possible way to deal with a situation where overall bond demand on the primary market is not in line with the official supply of $R$ bonds to competitive bidders. Especially, if overall demand is lower than $R$ the government could intend to sell some bonds on the secondary market. If overall demand exceeds $R$, then the government could on the one hand increase its bond supply on the primary market and on the other hand buy some bonds back on the secondary market. Any positive number $d_3$ of bonds sold on the secondary market by the public debt manager has the same effects as a variation of the total initial endowment $\beta q_1$ of type 1-individuals on the secondary market. From (13) we know that $d_3 > 0$ will reduce the sum of all optimal market maker’s bond inventories $I_j^*$ ($j = 1, ..., n$), but only to an amount of $d_3/2$. In an analogous way $d_3 < 0$ will imply the same consequences as an increase in the parameter $a_i$ of any type 2-individual’s net demand function by an amount $d_3 \cdot q_{2j} \sum_{i=1}^{n} q_{2i} (j = 1, ..., n)$. This in turn will ceteris paribus lead to an overall increase of market makers’ optimal bond inventories by $d_3/2$. Therefore it is in fact possible to adjust $d_3$ for given values $p_{\text{min}}^{(\text{bid})}$ and $R$ in such a way as to assure an equilibrium on the primary market where overall bond demand by competitive bidders is exactly identical to official bond supply. We may summarize our last findings in the following lemma.
Lemma 7: In case of monopolistic market makers binding minimum turnover requirements with respect to market makers’ secondary market bond sales will increase government’s revenues from public bond issuances, binding minimum turnover requirements with respect to bond purchases will reduce government’s revenues. Secondary market sales of any number $d_3$ of public bonds by the government will only lead to a reduction of $d_3/2$ in market makers’ optimal bond demand on the primary market. This also holds true for $d_3 < 0$.

b. Primary market interventions

As we have seen in Proposition 4 a necessary condition for an equilibrium on the primary bond market was that overall demand by market makers was at most equal to the governmental bond supply at $p^{(bid)}_{\min}$. In case of excess supply the public debt manager may lower $p^{(bid)}_{\min}$ in order to increase bond demand by competitive bidders. Accordingly, in case of excess demand the public bond manager is able to raise $p^{(bid)}_{\min}$ up until overall demand equals public bond supply. The situation is similar to the one in case of perfect competition. To identify relevant differences between a situation with monopolistic market makers and a situation with perfect market maker competition we now assume that it is optimal for the government to sell all $R$ bonds to competitive bidders. In such a situation we know from Proposition 3 that there would be no possibility to influence government revenues by official interventions in case of perfect market maker competition. This is no longer true for monopolistic market makers. Indeed there are several possibilities to increase the government’s revenues from the bond auction.

First of all the government could reduce its bond supply on the primary market by a certain amount $d_3$ just in order to sell these bonds on the secondary market. This transaction would reduce overall bond demand on the primary market by competitive bidders only by an amount $d_3/2$ thus leading to a situation with excess demand on the primary market which enables the public debt manager to an additional increase of $p^{(bid)}_{\min}$. Unfortunately, the secondary market sale of $d_3$ additional bonds will lower market makers’ bid prices on the secondary market. It therefore is not clear how the public debt manager should choose $d_3$ in order to maximize overall revenues from the sale of public bonds. But in general it is possible to get $d_3^* > 0$ as an optimal solution.
To keep our analysis simple let us for illustrative purposes assume that all n competitive bidders face the same demand and supply situation on the secondary market. In such a case the optimal amount $d_3$ of public bonds sold on the secondary instead of on the primary market can be determined by solving the following optimization problem:

\[
\max_{d_3, p_{\text{min}}} \quad \min_{d_3} \left( R - d_3 \right) \cdot p_{\text{bid}}^{(d_3)} + d_3 \cdot p_{\text{bid}}^{(d_3)*} \\
\text{s.t.} \quad R - d_3 = n \cdot I^*_j,
\]

where the identical optimal inventory $I^*_j$ for all market makers is determined by equation (13) and equations (9) and (10) hold as well. The first-order necessary conditions of (24) finally lead us to the following optimal value for $d_3$:

\[
(25) \quad d^*_3 = \frac{\left[ 4 \cdot R + \beta \cdot (2 \cdot q - q_2) \right] \cdot q_1}{2 \cdot (2 \cdot q_1 + q_2)}.
\]

(25) describes the optimal value for $d_3$ as long as $d^*_3$ is not negative or greater than $B \equiv R + \beta \cdot q$. This means that it will pay the government to sell some parts of its bonds on the secondary market as long as we have $4 \cdot R + \beta \cdot (2 \cdot q - q_2) > 0$. Certainly this condition may be fulfilled, but especially in situations with high values for $\beta$ and $q_2$ the government would refrain from this kind of market intervention.

In any case, the public debt manager can do even better by postulating some minimum turnover values for market makers’ sales on the secondary market. At least it would be useful to define a minimum turnover requirement which is equal to the minimum of market makers’ maximum acceptable possible sales turnovers. If the government increases its turnover requirements beyond this point, at least one market maker has to disappear as he is not willing (or able) any longer to fulfill turnover requirements. Such a harsh postulate may be sensible if there are enough market makers whose demand for public bonds on the primary market can be increased significantly by raising minimum turnover requirements. Certainly, minimum turnover requirements will work best if all market makers face identical demand and supply situations on the secondary market. If this is not the case then it seems to be helpful if there are some big bidders which are able to absorb the whole supply of public bonds for high minimum turnover requirements. In such a situation we get a natural restriction of the number of public bond

\[^{29}\text{It would be possible to allow for bidder heterogeneity without affecting the qualitative results but implying a much more complicated formula.}
\]

\[^{30}\text{It should be mentioned that for any value of } d_3 \text{ the net supply of bonds to market makers by the government always will be } R.\]
dealers, i.e. some kind of primary dealer system. So we are now able to summarize our results in the following proposition.

**Proposition 5:** Let Assumption 2 hold and assume that R bonds should be sold to market makers on the primary or the secondary market. In such a situation the government may use at least three kinds of market interventions in order to maximize revenues from its bond issuances. Firstly, it will define a minimum bid price $p_{\min}^{(bid)}$ so as to assure that total demand by competitive bidders equals the bond supply reserved for them. Secondly, the government will define some minimum turnover requirements for a market maker’s bond sales which in general will imply that some banks cannot act as market makers any longer. And thirdly, the government may sell a certain positive number $d_3$ of its bonds on the secondary instead of on the primary market. While the first two measures are always advantageous, there may be situations where the last intervention will not prove useful.\(^{31}\)

Up to now, we did not address the question if it may prove useful for the public debt manager to define a certain minimum number $r$ of government bonds which have to be bid for by any competitive bidder on the primary market. Indeed, such an obligation may now be an adequate means to increase official revenues on the primary market. To illustrate this case let us abstract from any other kind of intervention (besides the definition of a minimum bid price $p_{\min}^{(bid)}$ for competitive bidders on the primary market). It is now possible to derive a function $I_j^{(max)}(p_{\min}^{(bid)},.)$ which gives us ceteris paribus any market maker’s j maximum bond demand for given minimum bid price $p_{\min}^{(bid)}$ so that market maker j just remains willing to participate in the auction. It is easy to see that $I_j^{(max)}(p_{\min}^{(bid)},.)$ is monotone increasing if we increase the numbers $q_{1j}$ and $q_{2j}$ of type 1- and type 2-individuals approaching market maker j by the same factor $\gamma$, since if a market maker k with $\gamma_k = 1$ breaks even for $I_k^{(max)}(p_{\min}^{(bid)}, \gamma_k = 1)$ (compared to the case $I_j = 0$, i.e. no participation in the auction), then another market maker j with $\gamma_j > 1$ breaks even for $I_j^{(max)}(p_{\min}^{(bid)}, \gamma_j) = \gamma_j \cdot I_k^{(max)}(p_{\min}^{(bid)}, 1)$. In any case, the public debt manager could

---

\(31\)One might wonder if an incentive for choosing $d_3 > 0$ could arise solely as a consequence of the definition of minimum turnover requirements by the public debt manager. This indeed may be the case. But even in such situations where there are already minimum turnover requirements it might be that the government should refrain from $d_3 > 0$.  

29
certainly require market maker \( j \) to buy at most \( I_{j}^{(\text{max})} (p_{\text{min}}^{(\text{bid})}, \gamma_{j}) \) public bonds on the primary market. But now assume that the minimum number \( r \) of bonds which is to be bought by any competitive bidder on the primary market has to be the same for all market makers in consideration. A possible reason could be that the public debt manager may only know that there are some different types of competitive bidders facing different bond demand and supply (possibly characterized by different proportionality factors \( \gamma \)) on the secondary market but is not able to identify the demand and supply situation of any certain market maker. The motivation of introducing such a minimum requirement \( r \) by the public debt manager could now be that it enables him to raise \( p_{\text{min}}^{(\text{bid})} \) beyond the value which would otherwise arise because in case of \( n \) active market makers an obligation \( r \equiv R/n \) would assure the sale of the whole issuance.\(^{32}\) Thereby a market maker will be willing to accept the minimum requirement \( r \) even if he could raise his revenues by lowering his bid as long as his overall revenues will not become smaller than in case of \( I_{j} = 0 \).

It is clear that such a minimum requirement \( r \) will be most useful in order to increase the government’s revenues from the bond auction if all competitive bidders are facing similar demand and supply situations. In the extreme case of \( n \) identical competitive bidders the public debt manager will certainly use the minimum requirement \( r = R/n \) in order to raise the minimum bid price \( p_{\text{min}}^{(\text{bid})} \) beyond the value which could otherwise arise. Things are somewhat more difficult if there are different types of bidders facing different demand and supply situations. Assume that there are just two different types of competitive bidders, one of them being "big”, i.e. having a lot of customers in the sense described above, and the other one being "small”, i.e. possessing only a few potential customers. Let \( n_{1} \) be the number of big competitive bidders and \( n_{2} \) be the number of small competitive bidders with \( n = n_{1} + n_{2} \).

In such a situation for any minimum bid price and any minimum bond amount \( r \) the big market makers would be able to pay more for their respective shares \( r/R \) of the issuance than the small ones. This implies that the public debt manager could in general follow three basic strategies.

\(^{32}\) Once again we abstract from the possibility that non-competitive bidders’ primary market demand is declining when minimum bid prices for competitive bidders are increased. If this were the case, the public debt manager would have to increase \( R \) in order to compensate for non-competitive bidders’ reaction. The same would also hold true in problem (24).
Firstly, he could define such a minimum bid price and such a corresponding minimum bond amount \( r = R/n \) that only the big bidders would be willing to participate in the auction. If such an equilibrium is possible it is certainly the best solution for the government and it could be labeled as a "primary dealer system". If such an equilibrium cannot be established because the big bidders are not "big" enough, then the public debt manager may refrain from any minimum amount requirement \( r \) or set \( r \) in such a way that even the small market makers are willing to accept this requirement. In the latter case the minimum amount requirement would certainly not be binding for the big bidders. So we learn from this rather qualitative considerations that a primary dealer system with a small number of participants will be advantageous if there are quite heterogenous competitive bidders with the larger ones being so big that they are able to absorb the whole issuance.

We thus may conclude that now even for given numbers of bonds to be sold on the primary market there is much room for official market interventions. Moreover, these interventions lead to situations which are endowed with certain characteristics of primary dealer systems and they cannot be completely substituted by some kind of governmental "price management operations" on the secondary market. This last point is worth to be emphasized because with respect to German public bond issuances it is sometime argued that there is not any reason for a primary dealer system to enhance secondary market liquidity because of price management operations by the German Federal Reserve Bank ("Deutsche Bundesbank"). We need not to check if these price management operations are indeed effective, as in any case they are not suited to substitute for a primary dealer system.

V. Primary dealership with oligopolistic market makers

Certainly, our analysis is rather basic and there are many possible starting points to extend it. In particular, we may allow for asymmetric information between the government and the competitive bidders with respect to demand and supply situations on the bond market as well as for oligopolistic competition among market makers on the secondary market.

Asymmetric information between the government and competitive bidders would not change much because at least the market makers’ decision problems on the secondary market would be unaltered. The government now would have to distinguish between a number of possible
states of the world in order to determine the (state-dependent) consequences of its interventions. But this complication would not affect the general effectiveness of the government’s possible measures. Indeed, we further could introduce uncertainty for the market makers’ expectations with respect to future secondary market situations they might face. Under such circumstances risk-neutral market makers would try to maximize expected revenues subject to their inventory constraint which now has to be fulfilled for any possible future state of the world. Since this inventory constraint will be binding in at least one state of the world our conclusions may as well hold true under these modified assumptions.

Because of space constraints we therefore will now focus on the problem of oligopolistic competition. If we allow for oligopolistic competition among market makers we may distinguish between substitutive and complementary relationships. Certainly we would assume substitutability to be the normal case since liquidity traders might change their preferred market maker if other market makers’ ask prices are ceteris paribus reduced or corresponding bid prices are increased. But indeed it may also be possible that there are complementary relationships among market makers. To see this, suppose that liquidity traders costlessly only know average values of ask and bid prices on the secondary market but in order to learn ask and bid price of a certain market maker they have to approach him what might cause some kinds of transaction costs. If these transaction costs are so high that liquidity traders will restrict themselves to approach (possibly randomly) at most only just one market maker then a reduction of any market maker’s ask price will reduce the average level of ask prices on the secondary market and thus attract ceteris paribus more liquidity traders who are now willing to incur costs of contacting any specific market maker. The same may hold true for a ceteris paribus variation of market makers’ bid prices thus leading to some kind of complementary relationships.

Although complementary relationships thus seem to be possible, in what follows we will restrict ourselves to analyze solely the somewhat more plausible case of substitutive relationships among market makers.

Ceteris paribus an increase in any competitor’s ask price should lead to greater demand by liquidity traders approaching any market maker j. Accordingly, rising competitors’ bid prices on the bond market should lead to falling bond supply by type 1-individuals approaching
market maker \( j \) in consideration. In general we would assume that these variations in bond demand and supply with respect to a market maker \( j \) will be a consequence of a variation in the numbers \( q_{1j} \) and \( q_{3j} \) of liquidity traders approaching market maker \( j \). Unfortunately, allowing for such interdependences leads to rather complex formulas. To keep the analysis simple we therefore state the following assumption.

**Assumption 3**: Total bond demand a market maker \( j \) is confronted with on the secondary market for given ask prices \( p_{i}^{(\text{ask})} (i = 1, \ldots, n) \) shall be given by

\[
d_j(p_j^{(\text{ask})}, p_{j-1}^{(\text{ask})}, \ldots, p_j^{(\text{ask})}, \ldots, p_n^{(\text{ask})}) \cdot q_{2j} = (a_j - b_j \cdot p_j^{(\text{ask})}) \cdot q_{2j} + v_j \sum_{i=1, i \neq j}^{n} p_i^{(\text{ask})}
\]

\[
= \left( a_j - b_j \cdot p_j^{(\text{ask})} + b_j^+ \cdot \sum_{i=1, i \neq j}^{n} p_i^{(\text{ask})} \right) \cdot q_{2j},
\]

with \( v_j \equiv b_j^+ \cdot q_{2j} > 0 \).

Accordingly, liquidity traders` bond supply to market maker \( j \) can be described by

\[
s_j(p_j^{(\text{bid})}, p_{j-1}^{(\text{bid})}, \ldots, p_j^{(\text{bid})}, \ldots, p_n^{(\text{bid})}) \cdot q_{1j} = (\beta - a_j + b_j \cdot p_j^{(\text{bid})}) \cdot q_{1j} - v_j^+ \sum_{i=1, i \neq j}^{n} p_i^{(\text{bid})}
\]

\[
= \left( \beta - a_j + b_j \cdot p_j^{(\text{bid})} - b_j^+ \cdot \sum_{i=1, i \neq j}^{n} p_i^{(\text{bid})} \right) \cdot q_{1j},
\]

with \( v_j^+ \equiv b_j^+ \cdot q_{1j} > 0 \).

We thus assume that market maker \( j \) is facing some bond demand which does not depend on his ask price \( p_j^{(\text{ask})} \), but only on the other market makers` ask prices which all will influence this “basic” demand in the same way. Similarly, there exists some “basic” supply of bonds to market maker \( j \). For simplicity reasons we assume that the resulting equations for bond demand and supply remain quite “symmetric” even in the case of oligopolistic competition.
With these modifications it is easy to derive any market maker’s j optimal ask and bid price as well as the optimal amount of his inventory since his inventory constraint will keep binding for oligopolistic competition, too.

**Proposition 6**: Let Assumption 3 hold. Then for given competitors’ price quotations any market maker’s optimal behavior on the secondary market can be described by

\[
(28) \quad p_{j}^{(ask)*} = \frac{a_{j}}{b_{j}} + \frac{1}{2 \cdot b_{j}} \left[ q_{1j} \cdot \left( \sum_{i=1, i \neq j}^{n} p_{i}^{(ask)} + \sum_{i=1, i \neq j}^{n} p_{i}^{(bid)} \right) + 2 \cdot q_{2j} \cdot \sum_{i=1, i \neq j}^{n} p_{i}^{(ask)} \right] - 2 \cdot I_{j} - \beta \cdot q_{1j}
\]

and

\[
(29) \quad p_{j}^{(bid)*} = \frac{1}{b_{j}} \left[ \left( \sum_{i=1, i \neq j}^{n} p_{i}^{(ask)} - b_{j} \cdot p_{j}^{(ask)*} \right) - I_{j} \right] \cdot \frac{1}{q_{1j}} - \beta + a_{j} + b_{j}^{*} \sum_{i=1, i \neq j}^{n} p_{i}^{(bid)}
\]

as long as we have \( p_{j}^{(ask)*} \leq \frac{a_{j} + b_{j}^{*} \sum_{i=1, i \neq j}^{n} p_{i}^{(ask)}}{b_{j}} \)

as well as

\[
\left( a_{j} + b_{j}^{*} \sum_{i=1, i \neq j}^{n} p_{i}^{(bid)} \right) / b_{j} \geq p_{j}^{(bid)*} \text{ and } p_{j}^{(bid)*} \geq \frac{a_{j} + b_{j}^{*} \sum_{i=1, i \neq j}^{n} p_{i}^{(bid)} - \beta}{b_{j}}
\]

Moreover, any market maker’s j optimal bond demand on the primary market for given anticipated secondary market equilibrium is equal to

\[
(30) \quad I_{j}^{*} = \frac{(a_{j} \cdot b_{j} \cdot p_{min}^{(bid)}) \cdot (q_{1j} + q_{2j}) - \beta \cdot q_{1j} + b_{j}^{*} \cdot q_{2j} \cdot \sum_{i=1, i \neq j}^{n} p_{i}^{(ask)} + q_{1j} \cdot \sum_{i=1, i \neq j}^{n} p_{i}^{(bid)}}{2}
\]

Notice that in (30) we have used the fact that in equilibrium on the primary market we will have \( p_{j}^{(bid)*} = p_{min}^{(bid)} \).

(28) and (29) must hold true simultaneously for n market makers, so we generally have to solve a linear equations system with 2n equations. Since the general solution leads to rather lengthy formulas we refrain from explicitly solving this system and instead of this directly turn to an analysis of the consequences of official market interventions. As Assumption 3 does not
alter market makers’ general behavior on the primary market, we will lay our focus on secondary market interventions.

Once again, we start with the introduction of some maximum spread amount \( d_1 \). In analogy to equation (15) we may derive as market maker’s j optimal ask price the following formula as long as \( d_1 \) is not too high:

\[
\begin{align*}
\pi^{(\text{ask}),j}_{*} &= \frac{a_j}{b_j} - \frac{b \cdot d_1 \cdot q_{2j} - b_j^+ \cdot \left( q_{2j} \cdot \sum_{i=1, i \neq j}^{n} p_i^{(\text{ask})} + q_{1j} \cdot \sum_{i=1, i \neq j}^{n} p_i^{(\text{bid})} \right)}{b \cdot (q_{1j} + q_{2j})},
\end{align*}
\]

while his optimal bid price can be derived by inserting (31) into (29). Using these results it is not difficult to determine market maker’s j optimal demand for inventory \( I_j \). We finally get

\[
\begin{align*}
I_j^{d_1*} &= \frac{(a_j - b_j \cdot p_{\text{min}}^{(\text{bid})} \cdot (q_{1j} + q_{2j}) - b \cdot q_{1j} + b_j^+ \cdot \left( q_{2j} \cdot \sum_{i=1, i \neq j}^{n} p_i^{(\text{ask})} + q_{1j} \cdot \sum_{i=1, i \neq j}^{n} p_i^{(\text{bid})} \right))}{2}.
\end{align*}
\]

Not very surprisingly, market maker’s j optimal inventory does not depend directly on the maximum spread \( d_1 \). But since \( d_1 \) influences the competitors’ price quotations there may be an indirect influence from \( d_1 \) on \( I_j^{d_1*} \). In general we might conjecture that market makers’ ask prices are rising in \( d_1 \) and bid prices are falling. It thus remains unclear which of these two effects is generally dominating. In order to answer this question we must introduce some more very specific assumptions. To be precise, let us assume that all market makers are homogenous in such a way that they all face identical demand and supply situations on the secondary market. As a consequence of this assumption we get symmetric equilibria, i.e. equilibria with \( p^{(\text{ask}),j}_{*} = p^{(\text{ask}),1}_{*}, p^{(\text{bid}),j}_{*} = p^{(\text{bid}),1}_{*} \) for all \( j = 1, ..., n \). For this case, (29) and (31) reduce to just two linear equations which can easily be solved simultaneously. As their derivations with respect to \( d_1 \) (for given values \( I_j = I_j (j = 1, ..., n) \)) we finally obtain:

\[
\begin{align*}
\frac{\partial p^{(\text{ask}),j}_{*}}{\partial d_1} &= \frac{-q_1}{q_1 + q_2}, \\
\frac{\partial p^{(\text{bid}),j}_{*}}{\partial d_1} &= \frac{q_2}{q_1 + q_2}.
\end{align*}
\]

From (33) in connection with (32) we may actually derive
This leads us to the following lemma.

**Lemma 8:** Let Assumption 3 hold and assume all market makers to be identical. Then our irrelevance result from Lemma 6 with respect to maximum spread requirements still prevails although there is oligopolistic competition among market makers.

Lemma 8 seems to verify that any kind of maximum spread amount is not an adequate means to increase government’s revenues on the primary market.

We are thus able to analyze our second kind of secondary market interventions, i.e. some minimum turnover requirement with respect to market makers’ bond sales on the primary market. For \( d_{2j,r} \equiv d_2/q_{2j} \) being not too high we get the following two equations determining the optimal price quotation by any market maker \( j \):

\[
(35) \quad p^{\text{(ask)},d_{2j,r},*}_j = \frac{a_j + b_j^+ \sum_{i=1 \atop i \neq j}^n p_i^{\text{(ask)}} + \left( a_j + b_j^+ \sum_{i=1 \atop i \neq j}^n p_i^{\text{(ask)}} \right)^2 - 4 \cdot b_j \cdot d_{2j,r}}{2 \cdot b_j},
\]

and

\[
(36) \quad p^{\text{(bid)},d_{2j,r},*}_j = \frac{1}{2 \cdot b_j \cdot q_{1j}} \left[ a_j \cdot (2 \cdot q_{1j} + q_{2j}) - 2 \cdot I_j - 2 \cdot \beta \cdot q_{1j} + b_j^+ \cdot \left( 2 \cdot q_{1j} \cdot \sum_{i=1 \atop i \neq j}^n p_i^{\text{(bid)}} + q_{2j} \cdot \sum_{i=1 \atop i \neq j}^n p_i^{\text{(ask)}} \right) \\
- q_{2j} \cdot \left( a_j + b_j^+ \cdot \sum_{i=1 \atop i \neq j}^n p_i^{\text{ask}} \right)^2 - 4 \cdot b_j \cdot d_{2j,r} \right].
\]

With (35) and (36) it is easy to derive market maker’s \( j \) optimal bond demand on the primary market:
According to (37), a market maker’s j optimal bond demand is decreasing in his competitors’ ask prices and increasing in their bid prices and in \( d_{2j,r} \). Since (35) describes any market maker’s ask price as a positive function of all other ask prices and a negative function of \( d_{2j,r} \), an increase of \( d_{2j,r} \) is graphically equivalent to a downward shift of all market makers’ reaction "surfaces" thus leading to ceteris paribus lower equilibrium ask prices. In a similar way a rise in \( d_{2j,r} \) will ceteris paribus result in higher bid prices. Therefore we may immediately conclude that an increase in \( d_{2j,r} \) will raise any market maker’s j optimal bond demand on the primary market.

Once again we see that oligopolistic competition does not seem to change our main results as long as we have only substitutive relationships. Hence, we may suspect that this result carries over to the case where we consider minimum turnover requirements with respect to market makers’ bond purchases on the secondary market. Indeed things are a little bit more complicated so that only under the assumption of homogenous oligopolists we are able to easily verify our conjecture. In order to do so we have to resolve three equations for optimal values of \( p^{(ask)} \), \( p^{(bid)} \), and I simultaneously. It is then easy to show that the derivation of \( I^* \) with respect to the minimum turnover requirement is never equal to zero. Since we already know that this derivative is always greater than zero for \( b^* = 0 \) and moreover continuous in \( b^* \), we must have that this derivative is for any (admissible) value for \( b^* \) greater than zero. So we may state the following lemma.

**Lemma 9:** Let Assumption 3 hold. Then binding minimum turnover requirements with respect to secondary market sales will raise any market makers’ bond demand on the primary market. In case of homogenous oligopolists binding minimum turnover requirements with respect to

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33 Note that any market maker’s bid price according to (36) is monotone decreasing in competitors’ ask prices.
secondary market purchases will lead to lower optimal bond inventories for any market maker.

We thus are now left with exploring direct public bond sales or purchases on the secondary market as the last possible kind of official intervention which is subject of this paper. One more time the introduction of oligopolistic competition only leads to minor modifications of our results derived for the case of monopolistic market makers. Especially the sale of $d_{3,r} \cdot q_{ij}$ additional public bonds (with $d_{3,r} \equiv d_3 / \sum_{i=1}^{n} q_{ij}$ and $d_3 > 0$) on the secondary market to any market maker $j$ leads to the following optimal inventory demand by market maker $j$:

$$I_{j}^{d_{3,r} \ast} = \frac{1}{2} \left( (a - b \cdot p_{min}^{(bid)} \cdot (q_{ij} + q_{2j}) - \beta \cdot q_{ij} - d_{3,r} \cdot q_{ij} + b^{+} \cdot \left( q_{2j} \cdot \sum_{i=1}^{n} p_{i}^{(ask)} + q_{ij} \cdot \sum_{i=1}^{n} p_{i}^{(bid)} \right) \right).$$

(38) implies that overall demand for public bonds on the primary market by competitive bidders will be reduced by at least just one half of the sum $d_3$ of all public bonds directly placed on the secondary market by the government if market makers’ bid and ask prices on the whole are not rising with $d_3$. In particular, in case of homogenous competitors this result actually holds true, because we get after some calculations:

$$p_{(ask),d_{3,r} \ast}^{(bid)} = \frac{a + b \cdot p_{min}^{(bid)}}{2 \cdot b + b^{+} \cdot (n - 1)},$$

$$p_{(bid),d_{3,r} \ast}^{(bid)} = \frac{a + b \cdot p_{min}^{(bid)} - \beta - d_{3,r}}{2 \cdot b + b^{+} \cdot (n - 1)}.$$ 

Furthermore for this case of homogenous competitors it is easy to derive a formula which is analogue to (25) in case of monopolistic competition:

$$d_{3}^{\ast} = \frac{[4 \cdot b \cdot R + b \cdot \beta \cdot (2 \cdot q - q_{2}) - b^{+} \cdot (n - 1) \cdot (2 \cdot R + \beta \cdot q)] \cdot q_{1}}{2 \cdot [2 \cdot b \cdot q_{1} + b \cdot q_{2} - b^{+} \cdot (n - 1) \cdot q_{1}]}.$$

This last expression is monotone decreasing in $b^{+}$. This implies that increasing oligopolistic competition makes secondary market sale of government bonds by public debt managers less attractive.

**Proposition 7:** Let Assumption 3 hold. In such a situation the results of Proposition 5 will in general remain valid.
After this rough analysis of the case of oligopolistic competition we are now able to state our main results and their implications for the effectiveness of various official interventions on primary and/or secondary markets for government bonds.

VI. Conclusions

Our main results can be summarized as follows:

1) In general we have to distinguish between primary and secondary market interventions.
2) It does not matter whether official interventions may increase or decrease market makers’ profits as long as government’s overall revenues from bond sales on the primary (and secondary) market can be raised.
3) Official interventions will be more useful if there is only restricted competition among market makers.
4) In general there is no need to compensate market makers with respect to official interventions because they will be willing to participate as long as their overall revenues remain greater than in case of refraining from participation in the auction.
5) Minimum turnover requirements with respect to market makers’ bond sales seem to be a more adequate means than maximum spread definitions and cannot - in general - be substituted by direct official secondary market bond sales or purchases.
6) The definition of minimum bid prices and minimum bid amounts for competitive bidders on the primary market is in general useful in order to raise public revenues.
7) In case of strong heterogeneity among market makers secondary market minimum turnover requirements as well as primary market minimum bid amount postulates will lead to only a small number of participating market makers, i.e. some kind of primary dealership.

Very roughly, we thus can distinguish three different scenarios. The first one can be characterized by very intense competition. In such a case there is only few room to raise government’s revenues by official interventions. Secondly, there may be many homogenous oligopolistic or monopolistic market makers. Then we may recommend official interventions but the number of participating banks in equilibrium will be quite high. Finally, there may be a very heterogenous banking structure with a few players being able to totally absorb any official
bond issuance. In such a case some kind of "real" primary dealerships may emerge in equilibrium.

There is no doubt that the analysis presented in this paper is rather basic. For example we completely abstracted from any problems of collusion, just focusing on possible welfare increases by official market interventions. Moreover, in reality there exist many different kinds of primary dealer systems which are only partly similar to the one discussed in this paper. Especially, in many cases there is not an exclusive right for primary dealers to participate in government bond auctions as competitive bidders (any longer). Even if we allow for additional competitive bidders the introduction of primary and secondary market interventions for at least some of all bidders would be preferable from the government’s point of view. Certainly without some privileges these regulated bidders would not be willing to voluntarily accept government’s interventions. A welfare increase for the government therefore is in general only possible if there are additional revenues from the bond issuance in spite of "primary dealers'" compensations. This might be the case if we have some external effects (e.g. as a consequence of oligopolistic competition among market makers), but it would certainly be much easier for the government if compensation payments could be circumvented. In order to do so we just have to introduce some duties for all (competitive) bidders of an auction. This directly leads to the approach presented in this paper.

Certainly, the advantages and disadvantages of primary dealer systems need much more clarification in the future but maybe this paper has been able to increase the interest in this topic which - to our knowledge - so far has not been treated analytically.
Appendix 1:34

Countries with specialists in government securities - "primary dealers"

Austria
Belgium
Canada
Finland
France
Ireland
Italy
Norway
Spain
Sweden
United Kingdom
U.S.A.

Countries with no system of primary dealership

Denmark
Germany
Japan
The Netherlands
Switzerland

34 In general the following overview describes the situation as it was in 1998. As a consequence of the European Monetary Union we may expect that its members will express all monetary variables in Euro instead of their former domestic currencies. This change of their numéraires does not per se affect the following illustrations.
### Countries with specialists in government securities - "primary dealers"

<table>
<thead>
<tr>
<th>Country</th>
<th>Austria</th>
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</thead>
<tbody>
<tr>
<td>Year of introduction</td>
<td>1989</td>
</tr>
<tr>
<td>Number of PD and types of institutions</td>
<td>25 banks</td>
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**Main conditions of admission and special facilities/requirements**

Primary dealers require sufficient capitalization, branch locations, staff size and turnover of fixed income portfolios in Austrian Schillings and other relevant currencies. The final decision as to the eligibility for participation lies with the issuer in accordance with the Österreichische Kontrollbank.

**General duties/obligations**

- Primary dealers are obliged to submit bids at each auction with a minimum amount of 1/25\(^{th}\) of the issue amount (generally 1 divided by the number of auction participants).
- Formerly, there was a contractual market making obligation for primary dealers which has been abolished in the meantime. Nevertheless, primary dealers are still expected to make a market in government bonds.
- There are reporting requirements concerning the turnover figures in Austrian government bonds.

**Special rights/privileges/facilities**

- Primary dealers receive a fee of 10 basis points (in price) for their competitive allotments in the auctions.
- Primary dealers are the only ones participating in public bonds auctions.

**Form of the auction**

Multiple price auction

**Non-competitive bids and multiple bids**

Non-competitive bids are accepted; multiple bids are permitted.

**References:**

### Countries with specialists in government securities - "primary dealers"

<table>
<thead>
<tr>
<th>Country</th>
<th>Belgium</th>
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<tbody>
<tr>
<td>Year of introduction</td>
<td>1990</td>
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<tr>
<td>Number of PD and types of institutions</td>
<td>12 banks and securities firms</td>
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### Main conditions of admission and special facilities/requirements

Admission is decided by the Minister of Finance on the basis of performance and trading capacity.

### General duties/obligations

- To participate regularly in auctions for Treasury certificates and linear bonds on the primary market (0.5% of the volume attained by the primary dealers) in accordance with "Specifications" and a "Code of Good Conduct" defined by the Treasury.
- To ensure that fixed rates or prices are quoted at the request of clients and other primary dealers with the regard to Treasury securities (minimum quoting obligation of BEF 250 million for Treasury certificates and 100 million for linear bonds; existence of maximum range of quotations in basis points).
- To act as a market maker on the secondary market.
- To contribute towards promoting Treasury certificates and linear bonds with a view to ensuring that they are invested both in Belgium and abroad.

### Special rights/privileges/facilities

- Exclusive right to make non-competitive bids in auctions of T-bills and OLOs (Obligation linéaires).
- Privileged banking facilities at the National Bank of Belgium.
- Monthly consultations with the Belgian Treasury on the lines of linear bonds to be offered and on the states of the markets in general.

### Form of the auction

Multiple price auction.

### Non-competitive bids and multiple bids

Non-competitive bids are accepted and mainly reserved for primary dealers and for the National Bank of Belgium on behalf of foreign central banks and similar institutions; primary dealers are permitted to present a non-competitive bid (at the weighted average issues price) to the Treasury up to a maximum of 30% of the arithmetic average of their competitive participation; multiple bids are permitted.

### References:

Countries with specialists in government securities - "primary dealers"

Country: Canada

Year of introduction: 1940: "Primary Distributors" / 1954: "Jobbers" comprising a subset of the primary distributor group

Number of PD and types of institutions: 45 "Primary Distributors" comprising investment dealers and banks / 9 "Jobbers"

Main conditions of admission and special facilities/requirements
- To be incorporated in Canada
- Established record of dealing in fixed-income securities
- Membership in the Investment Dealers Association of Canada (for supervisory purposes)
- Minimum capital requirement (about Can-$ 50 million)

General duties/obligations
- Bid meaningfully (at least non-competitively) at all treasury bill and bond auctions
- Maintain active and continuous presence in secondary markets of all government securities
- Weekly reporting of fixed-income activity to the Bank of Canada

Special rights/privileges/facilities
- Special money market refinancing arrangements with the Bank of Canada
- Bank of Canada's market operations are conducted through "Jobbers".
- Only "Primary Distributors" are allowed to participate in public bond auctions.

Form of the auction
Multiple price auction

Non-competitive bids and multiple bids
Each "Primary Distributor" may submit one non-competitive bid up to a maximum of Can-$ 2 million (minimum of Can-$ 25 000 in multiples of Can-$ 5 000); multiple competitive bids are permitted (minimum of Can-$ 250 000 in multiples of Can-$ 50 000).

References:
Countries with specialists in government securities - "primary dealers"

Country: Finland
Year of introduction: 1992
Number of PD and types of institutions: 10 banks and broker firms

Main conditions of admission and special facilities/requirements
Commercial banks and securities broking firms authorized by the National Debt Office are first accepted for a trial period of six month

General duties/obligations
- To participate in bond auctions at a reasonable price with respect to the current market
- To participate actively and in accordance with good trading practice in secondary market trading in all benchmark bonds
- To give binding two-way price quotations for benchmark bonds to other primary dealers (FIM 10 million), at a margin of 5 yield bps (in practice 2 bps, as agreed upon among primary dealers) and to give binding bid quotations to the Bank of Finland, to the State Treasury and to customers outside the system
- To provide the Bank of Finland with information on transactions in government bonds

Special rights/privileges/facilities
- No pecuniary compensation, but the sole right to participate in the government auctions for serial bonds
- Right to participate in extra auction at fixed price on the day following the regular auction (30 % of their accepted amount)
- Benefits from the role as market maker in so far as banks and securities broking firms outside the system are only able to participate in the tenders through market makers. In this way information is released to the primary dealers which is of specific interest to them.

Form of the auction
Uniform price auction

Non-competitive bids and multiple bids
Non-competitive bids are not accepted; multiple bids are permitted.

Countries with specialists in government securities - "primary dealers"

Country: France  
Year of introduction: 1987  
Number of PD and types of institutions: 20 SVT (Spécialistes en Valeurs du Trésor)

Main conditions of admission and special facilities/requirements
- Authorized banks and securities firms with strong organizational qualifications and a minimum capital of FF 300 million  
- Admission is decided by the Minister of Economics, Finance and Industry.

General duties/obligations
- To bid for adequate amounts at auctions of new issues of government securities  
- To provide permanently for a sufficient liquidity in the secondary market which requires at least a participation quote of 2% of the total transaction volume in each of four product categories  
- To ensure that prices are quoted at the request of clients or other primary dealers (SVT)  
- To promote generally the sale of French government securities both domestically and internationally  
- Obligation to inform the Ministry of Economics, Finance and Industry regularly about market developments  
- To advise the government in its issue policy which requires extensive knowledge of investor needs

Special rights/privileges/facilities
- Limited Right to receive additional amounts of new issues at the average auction price in a non-competitive auction  
- Separation of coupons from government bonds and their renewed combination

Form of the auction
- Multiple price auction; Fungible Government Bonds: auctions at irregular intervals in ECU (since 1989); Treasury Notes: regular auctions since 1986

Non-competitive bids and multiple bids
- Non-competitive bids are not accepted, except for primary dealers after the auction, with limitations; multiple bids are permitted.

References:
Countries with specialists in government securities - "primary dealers"

Country: Ireland
Year of introduction: 1995
Number of PD and types of institutions: 6 banks and broker firms

Main conditions of admission and special facilities/requirements
- Excellence in management depth and experience, dealing capacity, marketing strategy, including demonstrated geographic distribution capacity
- Capital adequacy and the ability to support the National Treasury Management Agency (NTMA)
- Admission is decided by the National Treasury Management Agency in accordance with the central bank

General duties/obligations
- Obligation to quote, on demand to clients, firm bid and offer prices in each of a specified list of nine bonds in a specified minimum size and within a maximum specified spread
- A dedication of at least IRE 5 million capital exclusively to market making in Irish Government bonds on an on-going basis with a limit of IRE 8 million
- Expectation to be an active participant in the NTMA's annual funding program
- Commit oneself to making a market for a minimum period of three years

Special rights/privileges/facilities
- Exclusive access to the NTMA's fixed rate funding, which is executed by tap and auctions
- Bond-switching facilities in response to direct requests from primary dealers are offered.
- The NTMA makes, residually, repo and, if required, reverse repo facilities available to the six primary dealers.
- The NTMA quotes to primary dealers firm bid prices in the specified minimum size in each of the bonds in which they have price-making obligations.

Form of the auction
Multiple price auction

Non-competitive bids and multiple bids
Following the auction, a non-competitive auction, at the average auction price, confined to primary dealers is held; the amount of bonds available in the non-competitive auction is up to 20% of the amount sold in the competitive auction; multiple bids are permitted.

References:
Countries with specialists in government securities - "primary dealers"

Country: Italy  
Year of introduction: 1988 (revised in 1994)  
Number of PD and types of institutions: 14 "Specialists in Government Bonds", 28 "Primary Dealers" (including all "Specialists")

Main conditions of admission and special facilities/requirements
1) The selection procedure concerning "Specialists in Government Bonds" is based on two types of requirements: a) minimum conditions relating to net worth, volume of bonds auctioned and of trade in the secondary bond market; b) capacity to ensure appropriate liquidity conditions on the market at all times. The standards are based on a number of indicators including the size of the bid-ask spreads, continuity of trading and the number of bonds traded.
2) The "Primary Dealers" are subject to less stringent requirements than "Specialists"; they are not subject to requirements with regard to the primary market.

General duties/obligations
- To carry out the function of market makers in government securities
- "Specialists in Government Bonds" are subject to additional requirements in terms of subscription to government bonds at auctions and volume traded on the secondary market.

Special rights/privileges/facilities
"Specialists in Government Bonds" enjoy some privileges, such as the exclusive right to participate at supplementary auctions at fixed price; in addition they receive a guaranteed percentage of the amount offered in the supplementary issue. This amount depends on their own market share of the previous three auctions of the same security.

Form of the auction
- Since 1988, the allotment has been made for medium and long-term securities by uniform price auction subject to a minimum price; since 1992, the minimum price has been replaced by an excluding price mechanism. For Treasury Bills the allotment is made by competitive price auction.
- After each standard auction the Treasury issues a supplementary tranche of securities for medium and long-term securities (up to 10 % of the standard issue) which is allotted at the same stop-out price of the standard auction.

Non-competitive bids and multiple bids
Non-competitive bids are not accepted.

References:  
Countries with specialists in government securities - "primary dealers"

Country: Norway
Year of introduction: 1995
Number of PD and types of institutions: 8 banks and broker firms

Main conditions of admission and special facilities/requirements
- An equity capital requirement of NOK 50 million for guaranteeing the most active, professional and strongest operators in the market.
- There are no legal barriers preventing foreign banks and investment undertakings from participating in the primary dealer system, but the Norges Bank tries to stipulate their participation.
- The central bank places greatest emphasis on applicants’ participation in the secondary market for government securities; an annual review of the selected primary dealers with regard to their performances will take place.

General duties/obligations
- Binding bid and offer prices must be quoted every trading day for certain bonds. The binding prices shall apply to lots of NOK 10 million in each loan. The spread between bid and offer prices shall not exceed certain ranges; only under special circumstances they are allowed to double at maximum the spread for a short period.
- The primary dealer shall provide Norges Bank with information regarding market conditions as well as important changes and trends in the market.
- The primary dealer shall submit an annual report to Norges Bank on the general development of the company and immediately inform Norges Bank of conditions concerning the primary dealer's economic or financial position and any changes in the company's management, or corresponding conditions in other companies in the same group as the primary dealers, even including information about the technical or personnel situation.

Special rights/privileges/facilities
- Introduction of a borrowing facility for securities in Norges Bank under which each primary dealer can cumulatively borrow a maximum of NOK 100 million in each issue.
- Right to use the appellation "Primary Dealer", which is assumed to have an independent market value due to the fact that they acquire greater insight into the trading flows in the market.
- The central bank will only use primary dealers for any transactions in the secondary market for government bonds, with the exception of repurchase agreements in connection with monetary policy operations.

Form of the auction
Uniform price auction

Non-competitive bids and multiple bids
Non-competitive bids are not accepted; multiple bids are permitted.

References:
Countries with specialists in government securities - "primary dealers"

Country  Spain
Year of introduction  1988
Number of PD and types of institutions  15 "Creadores de Mercado"

Main conditions of admission and special facilities/requirements
Primary dealers have to be commercial banks, savings banks or securities houses. They have a direct dealing relationship with the Bank of Spain, and they have access to the inter-dealer brokers network.

General duties/obligations
- To participate regularly in the auctions of government securities
- To maintain an active and continuous two-way secondary market in government securities, with specified minimum and maximum spreads
- To help the Bank of Spain in the organization of the secondary market; some targets of trading and quotation must be covered
- To promote generally the sale of Spanish government securities both domestically and internationally

Special rights/privileges/facilities
- To receive information about the financing of the Treasury and the public debt policy
- To propose measures about the public debt market
- To participate, exclusively, in some kind of auctions called "segundas vueltas"

Form of the auction
Combination of uniform and multiple price auction

Non-competitive bids and multiple bids
Non-competitive bids are accepted.

Countries with specialists in government securities - "primary dealers"

Country         Sweden
Year of introduction  1982
Number of PD and types of institutions  9 banks and broker firms

Main conditions of admission and special facilities/requirements
- Authorization by the government (for banks) or by the financial Supervisory Authority (for broker firms)
- Experience in fixed income markets is one precondition for the authorization of broker firms.

General duties/obligations
- To bid in the auctions and to reach a market share of 2.5 per cent both in the auctions and in the secondary market for both bills and bonds; since September 1998, primary dealers are no longer obliged to quote two-way prices among themselves.
- To provide adequate market information within the financial community

Special rights/privileges/facilities
- Right to bid in the auctions for bills and bonds. For bonds there is a non-competitive facility, which allows the dealers to buy an additional 10 per cent of the sold volume two days later at the average price in the auction. Each dealer can buy a share of this 10 per cent based on his average participation in the last four auctions for bonds.

Form of the auction
- Multiple price auction

Non-competitive bids and multiple bids
Non-competitive bids are accepted within an individual limit of SKr 5 million; the average accepted auction price is applied; multiple competitive bids are permitted.

Countries with specialists in government securities - "primary dealers"

<table>
<thead>
<tr>
<th>Country</th>
<th>United Kingdom</th>
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<tbody>
<tr>
<td>Year of introduction</td>
<td>1986</td>
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<tr>
<td>Number of PD and types of institutions</td>
<td>19 Gilt Edged Market Makers (GEMMs)</td>
</tr>
</tbody>
</table>

**Main conditions of admission and special facilities/requirements**
- GEMMs have to be members of the Stock Exchange

**General duties/obligations**
- Significant participation in auctions and quotation of competitive bids, unless there are not any formal purchasing obligations
- To maintain continuous and effective two-way prices in all issues at which GEMMs stand committed to deal irrespective of trading conditions

**Special rights/privileges/facilities**
- Dealing relationship with the Bank of England
- Securities borrowing
- Short-term credit lines
- Access to inter-dealer brokers network

**Form of the auction**
- Discriminatory auction; they are conducted on a bid price basis, in which successful competitive bidders are allotted stocks at the prices which they bid; the Bank does not set a minimum price.

**Non-competitive bids and multiple bids**
- In the case of multiple price auctions, non-competitive bids (one bid per bidder) are accepted. GEMMs can each make a single non-competitive bid of up to 0.5% of the (nominal) amount on offer. Other bidders can make a non-competitive bid of up to £ 500 000 (nominal), subject to a minimum application of £ 1 000 (nominal) and a limit of one bid per applicant.

**References:**
Countries with specialists in government securities - "primary dealers"

<table>
<thead>
<tr>
<th>Country</th>
<th>U.S.A.</th>
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<tbody>
<tr>
<td>Year of introduction</td>
<td>1960</td>
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<tr>
<td>Number of PD and types of institutions</td>
<td>32 banks and broker firms (1 October 1998)</td>
</tr>
</tbody>
</table>

Main conditions of admission and special facilities/requirements
- Commercial banking organizations that are subject to official supervision by U.S. Federal supervisors or brokers/dealers registered with the SEC
- Certain minimum capital standards under the Basle Capital Accord must be met: commercial banking institutions must have at least US-$ 100 million of Tier I capital; registered brokers/dealers must have capital in excess of the SEC's or the Treasury's regulatory levels and have at least US-$ 50 million in regulatory capital.
- Strong commitment to continued participation as a market maker over long-term
- Management experience, a reasonable profitability record and good internal control systems
- Foreign-owned primary dealers are not designated when their home country does not provide the same competitive opportunities in underwriting and the distribution of government debt for U.S. companies as it does for domestic firms.

General duties/obligations
- To make reasonably good markets in their trading relationship with the Federal Reserve Bank of New York (FRBNY) in the full range of U.S. Government securities for a reasonably diverse group of customers (primary and secondary market)
- To participate meaningfully in Treasury auctions (relative to other auction participants)
- To provide the trading desk with market information and analysis that are helpful in the formulation and implementation of monetary policy
- No longer any duty to maintain a one percent share of the total customer activity reported by all primary dealers in aggregate

Special rights/privileges/facilities
- Exclusive trading relationship with the Federal Reserve Bank which allows for beneficial refinancing facilities

Form of the auction
- Single-price auctions are used for 2- and 5-year notes, multiple-price auctions are used for all other marketable securities.

Non-competitive bids and multiple bids
- Non-competitive bids are accepted up to an individual limit of US-$ 5 million at the weighted average price of accepted competitive bids.

References:
Countries with no system of primary dealership

Country: Denmark

Issuance procedure on the primary market
All government bonds and Treasury notes are issued on tap.
A fixed framework for the opening procedure was established in 1994.
All members of the Copenhagen Stock Exchange can buy government bonds and Treasury notes from Danmarks Nationalbank in the trading system of the Copenhagen Stock Exchange, which assures both the immediate distribution of market information concerning the issue of government securities and the equal access to buy the required amount of government securities for all securities dealers.
Sales on the opening day have similarities with an auction-style sale.

Market making in government securities
There are two market making arrangements for Danish government securities: the Voluntary Market-Making Agreement under the auspices of the Copenhagen Stock Exchange, and the Quote-on-Request Agreement under the auspices of the Danish Securities Dealers Association. At the start of 1998, 15 securities dealers participated in the Voluntary Market-Making Agreement. They are obliged to quote two-way prices with a maximum spread of DKK 0.10 per 100 for the only included security, the 10-year benchmark bond. All members of the Copenhagen Stock Exchange can join this agreement.
Participants of the Quote-on-Request Agreement, which covers all liquid government securities except for the 10-year benchmark bond, can freely choose the bonds in which they wish to participate. On request, they shall quote two-way prices with a maximum spread of 10 ticks to other participants for amounts of between DKK 10 and 100 million. The Quote-on-Request Agreement can be joined by members of the Danish Securities Dealers Association who are also members of the Copenhagen Stock Exchange. At the start of 1998, the 15 securities dealers participating in the Voluntary Market-Making Agreement also participated in the Quote-on-Request Agreement.

References:
Countries with no system of primary dealership

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<th>Country</th>
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<tbody>
<tr>
<td>Germany</td>
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Issuance procedure on the primary market

Until the end of 1997 Federal bonds were issued through the Federal Bond Consortium ("Bundesanleihekonsortium") using a combined syndicate and auction procedure whereas five-year special Federal bonds, Federal Treasury notes and Treasury discount papers were issued exclusively by auctions. All domestic credit institutions holding a giro account with a Land Central Bank were entitled to bid in this auction procedure.

Since January 1998 Federal bonds, parts of five-year special Federal bond issue amounts, Federal Treasury notes and Treasury discount papers have been issued uniformly by auction through a newly formed "Bund Issues Auction Group" ("Bietergruppe Bundesemissionen"; number of members in July 1998: 78). Resident credit institutions, securities trading houses and securities trading banks as well as German branches of foreign enterprises meeting certain standards of the German Banking Act ("Kreditwesengesetz") may become members provided they are authorized to engage in issuing business as required in the German Banking Act. A further obligation is to hold a Land Central Bank giro account and safe custody account or an account with the Deutsche Börse Clearing AG. Application for membership in the Auction Group may be made at any time. Moreover, membership of the auction group necessitates sufficient placing power for Federal securities. Seen from the present perspective, this requirement is met if the members have submitted successful bids for at least 0.05 % of the total issue amounts allocated at auctions in one calendar year. Auction group members are expected to participate in auctions as regularly as possible and with competitive bids. A ranking list of the auction group members by size of their shares in the issue volume allocated will be published annually.

The bids must be a par value of not less than DM 1 million or an integral multiple thereof and should quote the price, as a percentage of the par value, at which the bidders are prepared to buy the Federal securities offered. It is possible to make non-competitive bids and to submit several bids at different prices. The bids which are accepted by the issuer will be allotted at the price bid. Non-competitive bids are filled at the weighted average price of the competitive bids accepted.

A portion of the issue amount of all listed Federal securities is invariably set aside for market management operations. The Bundesbank gradually sells the amounts concerned in the market through the stock exchanges as part of its market management operations for Federal securities.

Market making in government securities

Federal bonds, five-year special Federal bonds and Treasury notes are traded actively on the German stock exchanges and over-the-counter in Germany and abroad. There are no obligations for the members of the "Bund Issues Auction Group" to be active in the secondary market.

Countries with no system of primary dealership

Country: Japan

Issuance procedure on the primary market

There are three main methods by which Japan's government issues bonds: underwriting by a syndicate, public offering via auctions and underwriting by the Ministry of Finance's Trust Fund Bureau.

1) The "syndicate system" is a means for the government to conclude a contract for the public offering or underwriting of government bonds with a group established for that purpose. The syndicate's function is then to provide a guarantee for the full amount of bonds they plan to underwrite. Currently, 10-year coupon-bearing government bonds and 5-year discount government bonds are allocated to syndicate members on a fixed-share basis. All the 1,822 syndicate members (April 1998), including banks, securities firms etc., have direct access to new issues and are responsible for bringing new bonds into the market. The syndicate underwriting system for 10-year coupon-bearing government bonds is partially conducted through a bidding system (60 % via auction, 40 % with fixed amount and price). For those members whose underwriting share among the syndicate members does not exceed 0.3 %, they can bid either through the competitive price auction system or the uniform price auction (the non-competitive bidding) system. Both are offered at the same time. Non-competitive biddings are executed at the average of the price paid in the corresponding competitive price auction. The maximum amount of bids for each member is 40 % of the issue amount multiplied by the underwriting share of each member.

2) Currently 20-, 6-, 4- and 2-year coupon-bearing government bonds and treasury bills are offered in public offering auctions, in which the Ministry of Finance sets issue amounts and terms based on the submissions of market participants. Syndicate members can bid either through the competitive price auction system or the uniform price auction (the non-competitive bidding) system. Both are offered at the same time. The maximum amount of bids for each member is 500 million yen. The offered amount of the uniform price auction (the non-competitive bidding) is 10 % of the total issuance. The required conditions for a bidder for a public offering are prescribed by a government ordinance and a list of the participants in public offering auctions is published by official gazettes.

3) The Ministry of Finance's Trust Fund Bureau has played an important role in absorbing a sizeable amount of the Japanese government bonds issued, supplementing the placement of bonds through the syndicate system. Underwriting by the Trust Fund Bureau represents a kind of intra-governmental financial transaction.

Market making in government securities

Transactions by securities dealers account for more than half of the transactions of Japanese government bonds in the secondary market. In general there are not any specific obligations for securities dealers to be engaged in the secondary market with the exception of a small group acting as Japanese government bond brokers in order to create a market among financial institutions.

Countries with no system of primary dealership

Country: The Netherlands

Issuance procedure on the primary market

Government loans can be issued in three ways:

1) The most common issue technique is the tap issue in which the Dutch State Treasury Agency announces the price at which it is willing to sell debt securities and the institutions advise their preferred amount to subscribe.

2) Alternatively the tender issue can be applied where eligible institutions are invited to advise the amount they wish to subscribe for and the price they are willing to pay. The Agency then determines the (uniform) issue price.

3) An additional issue instrument was created in 1994 with the "issue portfolio". The Dutch State Treasury Agency can include up to 20 % of a tap issue in the issue portfolio as a reconciling item (outside the budget). The debt securities in the portfolio are placed in the market generally during the weeks after the issue. The issue portfolio enables the government to remain active in the market for some time after an issue, making the success of the issue less dependent on the official subscription period. The securities are issued on the same settlement conditions as those applicable to transactions in the secondary market. In the uniform price auction, non-competitive bids are not accepted; multiple bids are permitted.

Market making in government securities

The trade in Dutch State Loans (DSLs) in the secondary market which has been split into a retail and a wholesale segment can take place either in direct dealing between buyer and seller or through the Inter Dealer Broker (IDB). The task of the IDB who acts purely as a broker and is not permitted to assume positions of its own, is always to find a counterpart.

The Dutch State Treasury Agency is not active in the secondary market in order to smoothen the prices/quotations. Operations in the secondary market are only related to early redemptions (redemptions made before the loan reaches maturity).

Countries with no system of primary dealership

Country: Switzerland

Issuance procedure on the primary market
- All types of investors are permitted to submit bids. Commercial banks, savings banks and securities broker-dealer firms collect bids from customers and receive a commission from the Federal Department of Finance of 0.5 per cent of the amounts involved.
- Uniform price auction
- Non-competitive bids are accepted within individual limits of SF 100,000.

Market making in government securities
The market is left to its own devices. There is no activity of the National Bank of Switzerland in the secondary market.

References:
Appendix 2:

In general we can assume that any market maker faces a certain net demand \( d(p_j^{\text{ask}}) \) per type 2-individual and a certain net supply \( s(p_j^{\text{bid}}) \equiv \beta - d(p_j^{\text{bid}}) \) per type 1-individual. Let us further assume that the first derivative of \( d(p_j) \) with respect to \( p_j \) is negative. Then any market maker’s objective is to find price quotations \( p_j^{\text{ask}} \), \( p_j^{\text{bid}} \) and an inventory \( I_j \) in such a way as to resolve the following optimization problem:

\[
(A1) \quad (p_j^{\text{ask}} - 1) \cdot d(p_j^{\text{ask}}) \cdot q_{2j} - (p_j^{\text{bid}} - 1) \cdot s(p_j^{\text{bid}}) \cdot q_{1j} - (p_j^{\text{bid}} - 1) \cdot I_j \rightarrow \max
\]

subject to

\[
(A2) \quad I. \quad p_j^{\text{ask}} - p_j^{\text{bid}} = d_1^*,
\]

II. \( q_{1j} \cdot s(p_j^{\text{bid}}) + I_j - q_{2j} \cdot d(p_j^{\text{ask}}) = 0. \)

The substitution of I. from (A2) into (A1) and the formation of a Lagrange function with \( \lambda \) being the corresponding Lagrange multiplier in order to allow for restriction II. lead to the following necessary conditions for market maker’s behavior:

\[
(A3) \quad \begin{align*}
I. \quad & [(p_j^{\text{ask}},d_1^* - 1) \cdot d'(p_j^{\text{ask}},d_1^*) + d(p_j^{\text{ask}},d_1^*)] \cdot q_{2j} - [(p_j^{\text{ask}},d_1^* - d_1) \cdot s'(p_j^{\text{ask}},d_1^* - d_1) - q_{1j} \cdot s'(p_j^{\text{ask}},d_1^* - d_1)] - q_{2j} \cdot d'(p_j^{\text{ask}},d_1^*) = 0, \\
II. \quad & -(p_j^{\text{bid}} - 1) - \lambda = 0, \\
III. \quad & -[q_{1j} \cdot s(p_j^{\text{ask}},d_1^* - d_1) + I_j^* - q_{2j} \cdot d(p_j^{\text{ask}},d_1^*)] = 0. 
\end{align*}
\]

By inserting II. from (A3) into I. we get an equation system for two variables \( p_j^{\text{ask}},d_1^* \) and \( I_j^* \):

\[
(A4) \quad \begin{align*}
A(p_j^{\text{ask}},d_1^*,I_j^*) &= [(p_j^{\text{ask}},d_1^* - 1) \cdot d'(p_j^{\text{ask}},d_1^*) + d(p_j^{\text{ask}},d_1^*)] \cdot q_{2j} \\
&- [(p_j^{\text{ask}},d_1^* - d_1 - 1) \cdot s'(p_j^{\text{ask}},d_1^* - d_1) + s(p_j^{\text{ask}},d_1^* - d_1)] \cdot q_{1j} \\
&+ (p_j^{\text{bid}} - 1) \cdot q_{1j} \cdot s'(p_j^{\text{ask}},d_1^* - d_1) - q_{2j} \cdot d'(p_j^{\text{ask}},d_1^*) = 0, \\
B(p_j^{\text{ask}},d_1^*,I_j^*) &= -[q_{1j} \cdot s(p_j^{\text{ask}},d_1^* - d_1) + I_j^* - q_{2j} \cdot d(p_j^{\text{ask}},d_1^*)] = 0.
\end{align*}
\]

Now assume that the second order sufficient conditions for a local maximum are satisfied. This implies that the Jacobian matrix belonging to the equation system (A4) with respect to \( p_j^{\text{ask}},d_1^* \) and \( I_j^* \) is negative definite. We can now apply the implicit function theorem in connection with Cramer’s rule in order to determine the derivative of \( I_j^* \) with respect to \( d_1 \). We get
Because of our assumptions the sign of \( \frac{dI_j^{d^*}}{dd_j} \) is the opposite of the sign of the determinant in the numerator of the fraction of the right-hand side of (A5). Let us call this numerator \( N(p_j^{(ask)}, I_j^{d^*}) \). Then we have:

\[
\text{(A6) } \det[N(p_j^{(ask)}, d_1^{d^*}, I_j^{d^*})] = \frac{\partial A(p_j^{(ask)}, d_1^{d^*}, I_j^{d^*})}{\partial p_j^{(ask)}, d_1^{d^*}} \cdot \frac{\partial B(p_j^{(ask)}, d_1^{d^*}, I_j^{d^*})}{\partial d_1^{d^*}} - \frac{\partial A(p_j^{(ask)}, d_1^{d^*}, I_j^{d^*})}{\partial d_1^{d^*}} \cdot \frac{\partial B(p_j^{(ask)}, d_1^{d^*}, I_j^{d^*})}{\partial p_j^{(ask)}, d_1^{d^*}}.
\]

From the second order sufficient conditions we now that \( \frac{\partial A}{\partial p_j^{(ask)}, d_1^{d^*}} \) is negative. Moreover we have \( \frac{\partial B}{\partial d_1} > 0 \), so that the product of these two derivatives becomes negative. For the other two derivatives we get:

\[
\text{(A7) } \frac{\partial A(p_j^{(ask)}, d_1^{d^*}, I_j^{d^*})}{\partial d_1} = q_{ij} \cdot [(p_j^{(ask)}, d_1^{d^*} - d_1 - p_j^{(bid)}) \cdot s''(p_j^{(ask)}, d_1^{d^*} - d_1) + 2 \cdot s'(p_j^{(ask)}, d_1^{d^*} - d_1)],
\]

\[
\frac{\partial B(p_j^{(ask)}, d_1^{d^*}, I_j^{d^*})}{\partial p_j^{(ask)}, d_1^{d^*}} = -[q_{ij} \cdot s'(p_j^{(ask)}, d_1^{d^*} - d_1) - q_{2j} \cdot d'(p_j^{(ask)}, d_1^{d^*})].
\]

While the second derivative of (A7) is negative, the sign of the first one is in general indeterminate. It seems plausible that this derivative is positive, but especially in this case we are not able to determine generally the sign of (A6). Now consider a special situation where we have \( d'(p_j^{(ask)}, d_1^{d^*}) = -s'(p_j^{(ask)}, d_1^{d^*} - d_1) \). We then could collect all terms in (A6) involving second order derivatives of \( d() \) and \( s() \), since from the discussion of linear functions we know that the other terms must sum up to zero. In case of \( s''(p_j^{(ask)}, d_1^{d^*} - d_1) = 0 \) and \( d''(p_j^{(ask)}, d_1^{d^*}) > 0 \) as well as \( p_j^{(ask), d_1^{d^*}} - p_j^{(bid)} > 0 \) we would get a positive sign for (A6) and a negative one for (A5). The opposite is ceteris paribus true for \( d''(p_j^{(ask), d_1^{d^*}}) < 0 \).
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