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## Article

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# Optimal monitoring to implement clean technologies when pollution is random

Inés Macho-Stadler · David Pérez-Castrillo

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**Abstract** We consider a model of polluting firms subject to tax on emissions, monitoring, and penalties in case of underreporting and which face a choice between a more expensive clean and a less expensive dirty technology. Moreover, emissions are subject to random events. We show that the optimal monitoring is a cut-off policy, where all reports below a threshold are inspected with the same probability, while reports above the threshold are not monitored. If the adoption of the technology is firms' private information, too few firms will adopt the clean technology under the optimal monitoring policy. However, when the environmental agency can check the technology adopted by the firms, the optimal policy may induce overswitching or underswitching to the clean technology.

**Keywords** Production technology · Random emissions · Environmental taxes · Optimal monitoring policy

**JEL Classification** K32 · K42 · D82

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## 1 Introduction

Pollution prevention and clean technologies have come to the forefront in reducing and controlling the environmental effects created by firms. Environmental Agencies (EAs) face the important challenge of encouraging the adoption of such measures and compelling compliance with environmental laws and regulations. For this aim, they often design a deterrence policy based on inspections. This paper contributes to the literature that analyzes the optimal inspection policy taking into account firms' strategic behavior (see [Cohen 1999](#), and [Sandmo 2000](#), for two extensive reviews of the literature). We build and analyze a model where firms choose a production technology which, together with some random event, determines the final emission level. That is, we explicitly take into account the random nature of pollution and its effects on the optimal inspection policy.

We consider the coexistence of two alternative technologies: a clean technology and a dirty technology. A “clean technology” is a manufacturing process or product technology that reduces pollution or waste energy use, or material use in comparison with the “dirty technology”. That is, expected level of emissions when production is carried out with the clean technology is lower than if the firm uses the dirty technology. For both technologies, the realized emission level is random and it is privately observed by the firm. Indeed, although firms can limit emissions of pollutants by deciding the production technology, by adjusting the mix of outputs and inputs, and through the use of abating technologies and management practices, this control is often not precise. Many factors such as weather, equipment failures, and human error may cause realized emissions to differ from intended emissions. Also, input relative price changes may affect the level of polluting input used.

In our framework, the environmental regulation is based on taxes over reported emissions, monitoring, and penalties over unreported emissions. Firms report their emission level and pay the taxes associated to them. The true emission level can only be observed (and made verifiable) by the EA after an inspection.

We consider situations where the EA wants to induce firms to make a major discrete investment to help the environment. Firms face different costs to adopt the clean technology, which usually cannot be observed by the EA. In this paper, we study how to provide incentives for the adoption of the clean technology at the lowest costs and which firms should be encouraged to do so. In particular, we analyze the optimal monitoring strategy when the EA takes into account the random nature of pollution: bad luck may cause a high level of emissions even when the firm adopts the clean technology while good luck may diminish emission level of a firm that uses the dirty technology.

Before analyzing the two main scenarios, for expositional purposes, we study an instrumental framework where we assume that the EA faces a single firm and knows the firm's cost of adopting the technologies but the technology chosen is not verifiable. We show that the optimal inspection policy that induces a firm to adopt the clean technology is a cut-off strategy where all the reports under the cut-off are inspected with the same probability and reports over this cut-off are not audited.

In the first main scenario that we consider, the EA faces a population of firms that differ in the cost of adopting the clean technology and where both the technology

adopted by the firms and their costs are non-verifiable. The optimal monitoring is the cut-off policy that would be designed for the “marginal” firm as if its emissions distribution was an average between the clean and the dirty technology. Second, we analyze situations where the technology adopted by each firm is observable, although the cost encountered by the firm is not. In this case, only those firms producing with the dirty technology will be inspected through the cut-off rule corresponding to the “marginal” firm.

In all the scenarios, the policy calls for a monitoring policy that is extensive rather than intensive: it is optimal to monitor with the same probability a large range of reports. This makes the firm pay higher expected taxes when emissions are high, which gives it stronger incentives to adopt the clean technology.

In all cases, firms with low adoption costs will be induced to switch to the clean technology while high-cost firms will keep the dirty one. We compare the conditions under which firms are pushed to adopt the clean technology with the case where the EA has all the information (first-best). When the technology adopted is private information for the firms, the optimal monitoring policy induces too few firms to choose the clean technology as compared to the first best since giving the firm incentives to adopt is costly. In contrast, when the cost is firms’ private information, while the technology adopted is verifiable, the EA may want to push firms to adopt the clean technology too often to save monitoring costs as firms with the clean technology will not be monitored.

Several papers have considered that pollution emissions frequently produce stochastic environmental damages.<sup>1</sup> But they have studied different aspects from our paper. Some authors have analyzed the advantages and disadvantages of introducing self-reporting (whereas in our paper is assumed to be in place) on the emission level in situations where emissions are random. In particular, [Innes \(1999\)](#) analyzes a model where there are ex post benefits of cleaning-up if an environmental accident (high level of pollution) occurs. In his model, firms choose the level of care (that can be interpreted as the choice of a technology), and this care affects the probability of an accident. Innes shows that when there is no self-reporting a firm will engage in clean-up only if audited, while the firm always cleans-up when self-reporting is in place. [Malik \(1993\)](#) compares the case with and without self-reporting in a situation where collecting penalties and taxes is costly and the monitoring technology is imperfect (including both types I and II of errors). In this framework, self-reporting does not necessarily reduce regulation costs because of costly sanctioning (see also [Kaplow and Shavell 1994](#); [Livernois and McKenna 1999](#)). [Hamilton and Requate \(2006\)](#) analyze the choice between emission caps and environmental quality standards when emissions are random. They show that when firms invest in abatement equipment, an emission standard induces over-investment relative to the socially optimal resource allocation, while under-investment tends to occur under an ambient environmental policy ([Requate and Unold 2003](#); [Bontems and Bourgeon 2005](#), study the incentives

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<sup>1</sup> For example, the damage from a given amount of effluent released in a river depends on features which vary temporally, such as seasonal fluctuations in water volume, temperature and turbidity. The effect of airborne emissions on air quality depends on prevailing atmospheric conditions, such as thermal structure, circulation, pressure, and humidity.

provided through different policy instruments in models where emissions are deterministic).

The model analyzed in this paper also contrasts with most of the models that study the optimal inspection policy, since they assume that the firm decides directly its (non-random) emission level. For example, [Harford \(1978, 1987\)](#) analyzes the optimal inspection frequency, considering that the solution is interior, and obtains that the probability of monitoring influences the firm's report but not its actual emissions. In [Macho-Stadler and Pérez-Castrillo \(2006\)](#), we show that when emissions are deterministic (the firm fully controls them), the EA optimal strategy induces a corner solution, in the sense that there are always firms that do not comply with the environmental objective and others that do comply but all of them evade the environmental taxes. In addition, in the papers considering deterministic emissions the optimal inspection policy does not depend on the report. In contrast, in this paper where emissions depend on the decision of the firm and some random events, first, firm's reported emissions varies from firm to firm as a function of the true emission level, and second, the optimal monitoring does depend on the firm's report and is tougher for low reports. This contingent policy allows to give the firm the right incentives to choose the clean technology, as low emissions appear less often than when the dirty technology is adopted. Concerning the optimality of the use of environmental taxes, [Macho-Stadler \(2008\)](#) shows that, when the compliance issue is taken into account, it is less costly to achieve any level of compliance through taxes than using standards or tradable permits.

In terms of methodology, analyzing the audit policy to induce compliance with the environmental policy is close to the literature on optimal auditing in tax evasion (some related papers are [Reinganum and Wilde 1985](#); [Scotchmer 1986](#); [Sánchez and Sobel 1993](#); [Macho-Stadler and Pérez-Castrillo 1997](#)). However, in this literature the taxpayer is assumed to have no choice other than reporting an income level. In contrast, the problem addressed in the present paper is more complex and has not been considered before. In our model, the agent decides first on the technology of production and second on the report about emissions once the (random) emission level is realized. Therefore, our model combines two informational dimensions: a moral hazard problem with respect to the choice of the production technology and an adverse selection problem concerning the report of emissions.

Some previous papers have analyzed how the regulatory regime via emissions taxes or standards may affect firms' adoption of emissions abatement technology (see, for example, [Downing and White 1986](#); [Milliman and Prince 1989](#); [Gersbach and Requate 2004](#); [Tarui and Polasky 2005](#)). Our paper is complementary to these contributions as we show how the monitoring policy, in environments where emissions cannot be identified without inspection, can be designed to optimize firms' adoption at the lowest cost. Finally, in a context with environmental standards, [Arguedas \(2005\)](#) analyzes the eventual adoption by a firm of a cleaner technology in exchange for reductions of the fines for non-compliance.

The paper is organized as follows. In Sect. 2, we introduce the model and analyze a firm's report given its technology and the inspection policy. Section 3 introduces the scenario with two different technologies. In Sect. 4, we analyze the instrumental framework where there is a single firm and both the firm and the EA know the cost of adopting the clean technology. We characterize the policy that the EA puts in place if

it wants to induce this firm to adopt the clean technology. Sections 5 and 6 constitute the central part of the paper. We characterize the optimal monitoring policy for the EA that faces a set of firms when their adoption cost is not observable assuming either that the technology adopted is non verifiable (Sect. 5), or that it is verifiable (Sect. 6). In Sect. 7, we conclude. All proofs are in the Appendix.

## 2 The Firm’s report under emission taxes

We model situations where a firm’s emissions are random, but the likelihood of high or low level of emissions depends upon the firm’s choice of technology. A firm’s level of emissions (or damages)  $e$  is distributed in the interval  $[e, \bar{e}]$  according to the distribution function  $F(e; E)$ , where  $E$  denotes the production technology chosen by the firm. We assume that  $F(\cdot; E)$  is continuously differentiable and that  $f(e; E) = \partial F(e; E)/\partial e > 0$  on  $[e, \bar{e}]$ . The cost of the technology  $E$  is sunk.

We assume that emissions are taxed according to a linear schedule, with marginal tax rate  $t$ . However, emission levels are private information of the firm and they can only be assessed if the firm is monitored by the EA. The firm is asked to send a report  $z \in [e, \bar{e}]$  on its emissions, once they are realized. It may choose a report  $z$  that does not coincide with the true emission level  $e$ .

The EA monitors the firm to control its emissions. We denote by  $\alpha(z)$  the probability that the EA will audit the emissions of the firm when it reports a level of emissions  $z$ . The strategy  $\alpha(\cdot)$  followed by the EA is decided previous to the choice of the technology  $E$ , that is, we assume that the EA is able to commit to its monitoring strategy. The EA’s monitoring cost is linear in the number of audits and, without loss of generality, we normalize the cost of monitoring one firm to 1. If the firm is monitored and its level of emissions is found to be higher than its report, then a penalty is imposed to the firm. If the firm overreports its pollution and it is monitored, then it gets the excess tax paid back and there is no bonus for overreporting. For simplicity, we assume that the penalty is linear in the underreported emissions. We also assume that the marginal penalty rate, denoted  $\theta$ , is exogenous. Parameter  $\theta$  includes the taxes due to the EA, hence  $\theta > t$ .

The firm’s expected costs when the emissions are  $e$ , the report is  $z$  and the monitoring strategy is  $\alpha(\cdot)$  are:

$$\begin{aligned} tz + \alpha(z)\theta[e - z] & \text{ if } z \leq e, \\ tz + \alpha(z)t[e - z] & \text{ if } z > e. \end{aligned}$$

The timing of the decisions is as follows. First, the EA decides on the monitoring strategy  $\alpha(\cdot)$ . Second, the firm chooses the technology  $E$  at a certain cost. Emissions are realized according to the density function  $f(e; E)$ . Third, after having observed the realized emissions  $e$ , the firm decides on the report  $z$  and pays the taxes  $tz$ . The firm is monitored with probability  $\alpha(z)$ . If it is audited and it has underreported, then the firm pays the penalty  $\theta[e - z]$  (if it has overreported, then it gets  $t[e - z]$  back).

The firm chooses  $z$  to minimize its costs, as a function of the realized emissions  $e$ . That is, at the last stage, the firm chooses  $z(e)$ . We denote  $c(e; \alpha(\cdot))$  firm’s expected costs when its emission level is  $e$  and it makes the report that minimizes its costs.

We now study firm's behavior concerning its report  $z$  and firm's expected cost given its technology  $E$  and the inspection policy  $\alpha(\cdot)$ . The analysis in this section is closely related to the one developed by [Scotchmer \(1986\)](#) and [Sánchez and Sobel \(1993\)](#).

**Lemma 1** *A firm whose emission level is  $e$ :*

- (i) *never reports more than its emissions:  $z \leq e$ ;*
- (ii) *never reports  $z < e$  if  $\alpha(z) > t/\theta$ ;*
- (iii) *reports honestly, i.e.,  $z = e$ , only if  $\alpha(z) \geq t/\theta$  for all  $z \in [\underline{e}, e]$ .*

The intuition behind Lemma 1 is the following. Given the tax rate  $t$  and the penalty rate  $\theta$ , a monitoring probability of  $t/\theta$  is enough to spur honest behavior. Therefore, a firm never submits a report  $z$  lower than its real emission  $e$  if reporting  $z$  leads to inspection with a probability higher than  $t/\theta$ . On the other hand, the firm will not report honestly if it can submit a report  $z < e$  that is monitored with a probability lower than  $t/\theta$ .

According to Lemma 1, the EA will not have incentives to inspect any report with a probability higher than  $t/\theta$ , since monitoring is costly. Therefore,  $t/\theta$  is an upper bound for the optimal monitoring probability.

Proposition 1 states the main consequences of the application of a monitoring policy  $\alpha(\cdot)$  when the firm reacts optimally to this policy.

**Proposition 1** *Once the EA has announced the monitoring policy  $\alpha(\cdot)$ , if the firm chooses a report  $z(e)$  that minimizes its costs when the emission level is  $e$ , then:*

$$\alpha(z(e)) \text{ is nonincreasing in } e, \quad (1)$$

and

$$c(e; \alpha(\cdot)) = c(\underline{e}; \alpha(\cdot)) + \theta \int_{\underline{e}}^e \alpha(z(x)) dx. \quad (2)$$

Moreover, if (1) and (2) hold, then  $z(e)$  minimizes firm's expected costs over the set of all possible equilibrium reports, i.e.,  $\{z|z = z(e^o) \text{ for some } e^o \in [\underline{e}, \bar{e}]\}$  when the emission level is  $e$ .

We now explain the main insights of Proposition 1, which is a classic result in continuous-type adverse selection models. For any given report, the penalty that the firm pays if it is caught underreporting increases with its realized pollution level. Therefore, the higher the emission level, the more incentives the firm has to chose reports with low monitoring probability. That is, incentive compatibility constraints explain that  $\alpha(z(e))$  is nonincreasing in  $e$ . As to the expected costs, Equation (2) states that the cost borne by the firm when its emissions are  $e$  is the integral of the monitoring probability of every level below  $e$ . This equation is also explained by the firm's possibility of underreporting. By inspecting with probability  $\alpha(z(x))$ , the EA makes the firm pay an expected penalty of  $\theta\alpha(z(x))$  when its emission level is  $x$ . But this similarly affects the firm's expected costs when it emissions are higher than  $x$ , since  $z(x)$  is always a

possible report for this firm. Hence, Equation (2) provides the expected cost borne by the firm when its emission level is  $e$ .

Note that, although the tax rate  $t$  does not explicitly appear in Equation (2), it plays a role as it sets the upper bound for the probability  $\alpha(\cdot)$ . The rate  $t$  is only important for those emission levels for which the firm reports honestly. For example, if the report  $z(e)$  is such that  $\alpha(z(e)) = t/\theta$  for all  $e \leq \hat{e}$  and  $\alpha(z(e)) < t/\theta$  otherwise, then we can write

$$c(e; \alpha(\cdot)) = c(\underline{e}; \alpha(\cdot)) + t [\hat{e} - \underline{e}] + \theta \int_{\hat{e}}^e \alpha(z(x)) dx.$$

We can use Proposition 1 to compute firm’s expected costs  $C(E; \alpha(\cdot))$  of using the technology  $E$ :

**Proposition 2** *Given the monitoring policy  $\alpha(\cdot)$ , if the report strategy  $z(\cdot)$  minimizes firm’s costs for all emission levels, then:*

$$C(E; \alpha(\cdot)) = c(\underline{e}; \alpha(\cdot)) + \theta \int_{\underline{e}}^{\bar{e}} \alpha(z(e)) [1 - F(e; E)] de. \tag{3}$$

In this section, we have developed the analysis for an exogenous monitoring policy. In the next sections, we characterize the optimal monitoring policy from the EA’s point of view.

### 3 Two production technologies

We analyze a situation where two production technologies are possible:  $E^D$  and  $E^C$ . Technology  $E^C$  is a cleaner but also more expensive technology than  $E^D$  (subscript  $C$  stands for “clean” and  $D$  for “dirty”).<sup>2</sup> We assume that the firm is initially producing according to  $E^D$  and we denote by  $\Delta$  the cost of switching from the dirty technology to the clean one. We can also see our model as a situation where the firm is not using any of the two technologies and, assuming that both of them give positive profits (anticipating the monitoring activities of the EA), it has to chose one of them. In this case,  $\Delta$  is interpreted as the difference in costs of the technologies, i.e., the cost to adopt the former instead of the later.<sup>3</sup>

Given the policy announced by the Government and the EA involving taxes over reported emissions, monitoring, and penalties over unreported emissions, the firm will

<sup>2</sup> We will discuss about sufficient conditions for  $E^C$  to be preferable from a social point of view in the welfare analysis developed in Sect. 5, which will also apply to Sect. 6.

<sup>3</sup> In our framework, the emissions from both technologies are equally difficult to inspect. Some authors have analyzed technologies that can affect the observability of firms’ emissions. Heyes (1993) considers a model where firms may invest in decreasing “inspectability”. Millock et al. (2002) studies a choice of technology that affects the verifiability of emission: adopting the technology allows nonpoint sources to become point sources.



choose the clean technology if and only if its total expected costs are lower than using the dirty technology, that is, if  $C(E^C; \alpha(\cdot)) + \Delta \leq C(E^D; \alpha(\cdot))$ . This inequality can be written as the following incentive constraint:

$$\Delta \leq \theta \int_{\underline{e}}^{\bar{e}} \alpha(z(e)) [F(e; E^C) - F(e; E^D)] de. \tag{4}$$

It might be the case that the firm chooses technology  $E^D$  for any possible monitoring strategy. Indeed, if the difference in cost  $\Delta$  is very large, the firm may prefer paying all the expected taxes corresponding to the emissions induced by  $E^D$  rather than adopting the clean technology. In what follows, we will assume that the set of functions  $\alpha(\cdot)$  that lead the firm to choose  $E^C$  is not empty, which is equivalent to state that the toughest policy leads the firm to use the clean technology, that is Equation 4 holds (with strict inequality) for  $\alpha(z) = t/\theta$  for all  $z$ .<sup>4</sup>

**Assumption 1**  $\Delta < t \int_{\underline{e}}^{\bar{e}} [F(e; E^C) - F(e; E^D)] de$ .

Although part of the analysis of the optimal policy can be developed without any additional assumption concerning the distribution functions  $F(e; E^C)$  and  $F(e; E^D)$ , the complete characterization of the policies requires further assumptions. In particular, we will assume that the density functions  $f(e; E^C)$  and  $f(e; E^D)$  are linear. Also, to help notation, we will normalize  $[\underline{e}, \bar{e}] = [0, 1]$ .

**Assumption 2**  $f(e; E^C) = a + 2[1 - a]e$ ,  $f(e; E^D) = b + 2[1 - b]e$ , for all  $e \in [0, 1]$ , where  $a, b \in (0, 2)$ , and  $a > b$ .

Note that the property  $F(1; E^C) = F(1; E^D) = 1$  characterizes the slope of the linear functions  $f(1; E^C)$  and  $f(1; E^D)$ , once we choose the independent terms  $a$  and  $b$ . Moreover, the idea that  $E^C$  is a cleaner technology than  $E^D$  is reflected in the inequality  $a > b$ . Also note that although Assumption 2 is restrictive, it allows the flexibility of dealing with distribution functions  $F(e; E^C)$  and  $F(e; E^D)$  that may be linear ( $a = 1$  or  $b = 1$ ) concave ( $a > 1$  or  $b > 1$ ), or convex ( $a < 1$  or  $b < 1$ ). On the other hand, it is a strong assumption that is helpful to identify a simple monitoring policy. After Proposition 3, we state reasonable hypotheses that allow reaching similar conclusions to those obtained under Assumption 2, at least when the cost of adopting the clean technology is low.

The monitoring policy decided by the EA strongly influences the choice between  $E^C$  and  $E^D$ . In the remaining of the paper, we look for the optimal monitoring policy in three scenarios.

<sup>4</sup> Assumption 1 can also be written as  $\int_{\underline{e}}^{\bar{e}} tef(e; E^C)de + \Delta < \int_{\underline{e}}^{\bar{e}} tef(e; E^D)de$ , which means that the clean technology is cheaper than the dirty one if the firm always complies with environmental taxes.

#### 4 A reference framework: optimal monitoring when the cost $\Delta$ is public information

In this section, we characterize the optimal monitoring policy if the EA wants a firm to adopt technology  $E^C$ . We consider a situation where the EA knows the cost  $\Delta$ , but is uninformed about the technology that the firm adopts and about the realized emission level. This informational set-up is not realistic in most scenarios since the precise cost for a firm to adopt a technology is better known by the firm. However, it is extremely useful as a reference since the analysis will help us to understand the effects at work and to better present the results in the other two cases, where  $\Delta$  is assumed to be firm’s private information (studied in Sects. 5, 6). The optimization problem of the EA, that minimizes monitoring costs, is program [P] below. The first three constraints take into account that the firm will chose its report strategically, while the last constraint states that the firm should have incentives to adopt the clean technology:

$$\begin{aligned}
 & \underset{(\alpha(z))_{z \in [\underline{e}, \bar{e}]}}{\text{Min}} \int_{\underline{e}}^{\bar{e}} \alpha(z(e)) dF(e; E^C) \\
 & \text{s.t.: } \alpha(z(e)) \text{ is nonincreasing in } e \\
 & \alpha(z(e)) \in [0, t/\theta] \text{ for all } e \in [\underline{e}, \bar{e}] \\
 & z(e) \text{ minimizes } c(e, z; \alpha(\cdot)) \text{ for all } e \in [\underline{e}, \bar{e}] \\
 & \Delta \leq \theta \int_{\underline{e}}^{\bar{e}} \alpha(z(e)) [F(e; E^C) - F(e; E^D)] de.
 \end{aligned}$$

Let us remark that program [P] is different from (and more complex than) the programs that describe adverse selection problems analyzed in the previous literature, as [Sánchez and Sobel \(1993\)](#).<sup>5</sup> Program [P] includes the incentive constraint (4), which was not present in previous contributions, and which adds a new dimension of asymmetric information (a moral hazard problem) to the existing adverse selection problem. It leads to some additional difficulties in the search for the optimal monitoring policy.

We show in the Appendix that the optimal monitoring policy is always very simple: for any distribution function  $F(\cdot)$ , there exists a solution  $\alpha(\cdot)$  to [P] that takes on at most one value different from 0 and  $t/\theta$ .

Under Assumption 2, the optimal policy is even simpler as it only takes two values. To state this Proposition, let us define the function  $h(e)$  as follows:

$$h(e) \equiv f(e; E^C) - \frac{F(e; E^C) - F(e; E^D)}{\int_{\underline{e}}^e [F(x; E^C) - F(x; E^D)] dx} F(e; E^C).$$

Under Assumption 2,  $h(e)$  is first negative and then positive. We denote by  $e^*$  the cut-off level such that  $h(e) < 0$  if  $e < e^*$  and  $h(e) > 0$  if  $e > e^*$ , that is,  $e^*$  is defined by  $h(e^*) = 0$ . Note that the cut-off level  $e^*$  only depends on the shape of the

<sup>5</sup> However, the techniques developed in [Sánchez and Sobel \(1991\)](#) will help us solve [P].

distribution functions, and in particular is independent of the cost  $\Delta$ . We also define  $\widehat{\gamma} < t/\theta$ ,  $z^* < e^*$ , and  $\widehat{e} \geq e^*$  as follows:

$$\begin{aligned} \widehat{\gamma}\theta \int_{\underline{e}}^{e^*} [F(e; E^C) - F(e; E^D)] de &= \Delta, \\ z^* = \underline{e} + \frac{\Delta}{t} \frac{(e^* - \underline{e})}{\int_{\underline{e}}^{e^*} [F(e; E^C) - F(e; E^D)] de}, \\ t \int_{\underline{e}}^{\widehat{e}} [F(e; E^C) - F(e; E^D)] de &= \Delta. \end{aligned}$$

Proposition 3 characterizes the optimal monitoring policy as a function of the report,  $\alpha(z)$ , as well as firm’s reporting behavior given the optimal monitoring policy,  $z(e)$ .

**Proposition 3** *Suppose Assumptions 1 and 2 hold. We distinguish two regions:*

(a) *If  $\Delta < t \int_{\underline{e}}^{e^*} [F(e; E^C) - F(e; E^D)] de$ , then the following policy  $\alpha^*(z)$  is optimal:*

$$\begin{aligned} \alpha^*(z) &= \widehat{\gamma} \text{ for all } z \in [\underline{e}, z^*], \\ \alpha^*(z) &= 0 \text{ for all } z \in [z^*, \bar{e}]. \end{aligned}$$

*Facing the monitoring policy  $\alpha^*(z)$ , the firm’s reporting strategy is the following:*

$$\begin{aligned} z(e) &= \underline{e} \text{ for all } e \in [\underline{e}, e^*), \\ z(e) &= z^* \text{ for all } e \in [e^*, \bar{e}]. \end{aligned}$$

(b) *If  $\Delta \geq t \int_{\underline{e}}^{e^*} [F(e; E^C) - F(e; E^D)] de$ , then the following policy  $\alpha^*(z)$  is optimal:*

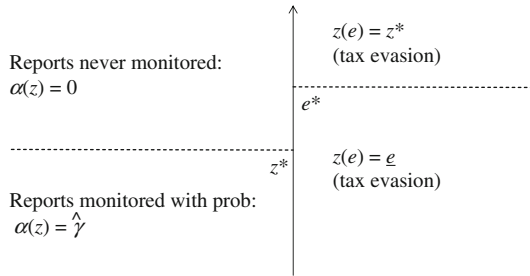
$$\begin{aligned} \alpha^*(z) &= t/\theta \text{ for all } z \in [\underline{e}, \widehat{e}), \\ \alpha^*(z) &= 0 \text{ for all } z \in [\widehat{e}, \bar{e}]. \end{aligned}$$

*Facing the monitoring policy  $\alpha^*(z)$ , the firm’s reporting strategy is the following:*

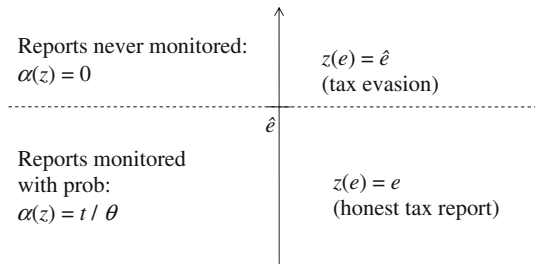
$$\begin{aligned} z(e) &= e \text{ for all } e \in [\underline{e}, \widehat{e}), \\ z(e) &= \widehat{e} \text{ for all } e \in [\widehat{e}, \bar{e}]. \end{aligned}$$

The optimal monitoring policy is very simple. We highlight its main characteristics. First, for any cost  $\Delta$ , the EA will always monitor, at least, the reports corresponding to all the emission levels lower than the cut-off value  $e^*$ . Note that the cut-off  $e^*$  is usually high; under Assumption 2 and for the intermediate case  $a = 1$ , we

**Fig. 1** Monitoring and reporting behavior when switching costs are low



**Fig. 2** Monitoring and reporting behavior when switching costs are high



have  $e^* = 3/4$ .<sup>6</sup> Second, the probability of monitoring is the same for all the reports subject to audit. Third, as long as the incentive problem is not very acute, in the sense that adopting the clean technology is not very costly, the EA will only monitor when the realized emission level is lower than  $e^*$  (for that purpose, it will only monitor when the reported emission level is lower than  $z^*$ ). The probability of audit  $\hat{\gamma}$  is increasing with  $\Delta$  until it reaches the maximum value  $t/\theta$ . This defines the borderline between cases (a) and (b). From then on, the EA audits additional reports. That is why, when the incentive problem is very severe, the monitoring probability is the highest possible, among the sensible ones, (i.e.,  $\alpha = t/\theta$ ) for all the reports subject to audit.

Given the optimal monitoring policy, there is always under-reporting when the switching cost is not too high (Region (a)). Indeed, the firm makes the lowest report  $\underline{e}$  if its emissions are under  $e^*$ ; otherwise, it pays the taxes corresponding to the report  $z^*$  to make sure that it will not be monitored. In Region (b), there is truthful reporting for lower emission levels and under-reporting for higher levels. The optimal monitoring policy and firm’s reporting behavior are depicted in Figs. 1 and 2.

In Fig. 1 (the interpretation of Fig. 2 is similar) the left-hand side represents the EA’s optimal policy as a function of the firm’s report (which is measured in the vertical axe), while the right-hand side represents the firm’s reporting behavior as a function of the realized emission (also measured in the vertical axe). We can see in this figure

<sup>6</sup> It can be shown that  $e^* = \frac{-2a + \sqrt{4a^2 + 6a(1-a)}}{2(1-a)} \in (0, 1)$  when  $a \neq 1$ , and that  $e^*$  is an increasing function of  $a$ .

that the cut-off level of emissions  $e^*$  from which on the firm starts reporting positive emissions is strictly higher than the report  $z^*$  that provides immunity.

We now explain the intuitions behind Proposition 3. The EA's objective is to dissuade the firm from using the dirty technology at the lowest (monitoring) cost. To “convince” the firm, the EA must choose a monitoring strategy that makes the firm bear high expected environmental costs (also taking into account the penalties) if it uses the dirty technology, and low expected costs if it produces according to the clean one.

A dirty technology produces high emission levels more often than a clean technology. For the case of linear density functions over the interval  $[0, 1]$  (Assumption 2), the dirty technology has higher density for  $e \in (1/2, 1]$  and lower density for  $e \in [0, 1/2)$ . Therefore, in terms of dissuasion, the EA would find it beneficial to make the firm pay as much as possible when realized emissions are high and as little as possible when realized emissions are low. However, the EA does not observe the realized emission level, it only receives the firm's report.

According to Equation (2) in Proposition 1, the cost borne by the firm when the emission level is  $e^o$  is the integral of the monitoring probability of every level below  $e^o$ . That is, increasing the probability of monitoring the report corresponding to a level  $e$  affects in the same way the cost suffered for every emission level higher than  $e$ . Hence, monitoring the report corresponding to a high emission level, say  $e' > 1/2$ , provides incentives to adopt the clean technology, as it affects the cost borne for every realized emission  $e \geq e'$ . On the other hand, monitoring the report corresponding to a low emission level, say  $e'' < 1/2$ , provides mixed incentives since it affects the cost associated to both high (every  $e > 1/2$ ) and low (every  $e \in [e', 1/2)$ ) emission levels.

The difficulty is that, from Equation (1) in Proposition 1, the EA is constrained to use a monitoring probability nonincreasing in the emission level. That is, if the EA wants to monitor the (firm's optimal) report corresponding to a certain level of emissions  $e^o$ , then it is forced to monitor the reports corresponding to all the levels  $e < e^o$  with, at least, the same frequency.

To understand how the EA solves the previous trade-off, consider that  $\Delta$  is small so that inducing the firm to switch to the clean technology is easy (Region (a) in Proposition 3). Could it make sense for the EA to monitor only the reports corresponding to low emission levels? The answer is no. The EA does better monitoring reports chosen by a larger range of emission levels (including levels higher than  $1/2$ ) with lower probability. The cost paid in case of high emission level will be the same, while the cost borne in case of lower emission levels will be lower, which gives the firm more incentives to adopt the clean technology. Is it optimal for the EA to set a full flat policy (i.e.,  $e^* = \bar{e}$ )? The answer to this question is also negative because monitoring the report corresponding to emission levels very close to  $\bar{e}$  only affects the payment of a very small interval of emissions.

In the case where the density function  $f(e; E^C)$  is uniform, i.e.,  $a = 1$ , the trade-off leads to a flat policy where all the reports corresponding to  $e < 3/4 = e^*$  are monitored with the same probability. When  $f(e; E^C)$  is not uniform, the argument is more complex, as switching monitoring probabilities from one level to the other has consequences in terms of monitoring costs. This is why when the density function

$f(e; E^C)$  is decreasing, it is optimal to state an even flatter technology ( $e^* > 3/4$ ), while the opposite happens when  $f(e; E^C)$  is increasing.<sup>7</sup>

We now discuss on the generality of the results with respect to the shape of the distribution functions, that is, when Assumption 2 is dropped and general distribution functions are considered. First, as already said, the solution always takes on at most one value different from 0 and  $t/\theta$ . Second, monitoring every single emission with some probability is never optimal. Third, the property that the monitoring policy is flat for quite a wide range of emissions can be stated under reasonable hypotheses. For example, assume that  $F(e; E^C) > F(e; E^D)$  for all  $e \in (e, \bar{e})$ ,  $F(e; E^C) - F(e; E^D)$  is first increasing and then decreasing in  $e$ , and  $F(e; E^C)$  is concave in  $e$ . Under this sufficient condition, we can prove that there exists a cut-off value  $z^\#$  (that lies in the region of emissions where  $F(e; E^C) - F(e; E^D)$  is decreasing) such that  $\alpha(z)$  is constant for all  $z < z^\#$ . In particular, only the reports  $z < z^\#$  are monitored (with a low probability) when the cost of adopting the clean technology is low. On the other hand, it seems more difficult to propose general sufficient conditions to establish the precise form of the optimal monitoring strategy for intermediate reports, which is relevant when the cost of adopting the clean technology is high.

Next, Corollary 1 states the monitoring cost  $ECost(\Delta)$  of the implementation of the clean technology as a function of the parameters of the model.

**Corollary 1** *Suppose Assumptions 1 and 2 hold.*

(I) *Expected monitoring costs  $ECost$  are the following:*

(Ia) *If  $\Delta < t \int_{\underline{e}}^{e^*} [F(e; E^C) - F(e; E^D)]de$ , then:*

$$ECost(\Delta) = \frac{\Delta}{\theta} \frac{F(e^*; E^C)}{\int_{\underline{e}}^{e^*} [F(e; E^C) - F(e; E^D)]de}.$$

(Ib) *If  $\Delta \geq t \int_{\underline{e}}^{e^*} [F(e; E^C) - F(e; E^D)]de$ , then:*

$$ECost(\Delta) = \frac{t}{\theta} F(\hat{e}; E^C).$$

(II) *Expected monitoring costs are increasing in the difference  $\Delta$  and they are decreasing with the penalty rate  $\theta$ ; they are higher the less clean is technology*

<sup>7</sup> It is worth comparing our context with situations in which the objective of the agency is to raise the largest amount of taxes, for a given technology. In these situations, the agency is much less interested in focusing on high-emission levels. For example, in the tax evasion literature it is assumed that the distribution of income is given (i.e., there is no “choice of technology to earn income”) and the objective of the enforcement agency is to maximize the collected revenues (taxes plus penalties). In this case, the optimal policy consists in auditing all the taxpayers reporting incomes lower than a certain cut-off income with a probability high enough so that those reports will happen to be truthful, while the taxpayers earning higher incomes will report the cut-off income and will not be subject to audit. The main intuition for this result is similar to the one we have provided in the main text: putting pressure over the report corresponding to an emission level increases the revenue collected from every higher level. That is, it is beneficial to concentrate the monitoring in the lowest levels of income (with the maximum probability  $t/\theta$ ).

$E^C$  and the less dirty is technology  $E^D$ . Finally, expected costs are increasing in the ratio  $t/\theta$  in Region (b).

First, the higher the cost  $\Delta$  for the firm to switch to the clean technology, the higher the monitoring cost required to give it incentives to adopt  $E^C$ . Second, a higher penalty rate  $\theta$  makes it easier to “convince” the firm, hence it decreases the EA’s cost. Third, the larger (in terms of expected emissions) the difference between the two technologies, the more the EA’s monitoring can target the dirty technology, which also decreases monitoring costs. Finally, if the tax rate  $t$  increases, the EA must increase the monitoring probability if it wants the firm to be honest when the level of pollution is low (which is the optimal policy in Region (b)). Therefore, the monitoring costs increase with  $t$ . That is, a tough policy in terms of penalty rate and (in Region (b)) a soft policy in terms of tax rate help in keeping low monitoring costs.<sup>8</sup>

### 5 Optimal policy when both the technology adopted and the cost $\Delta$ are not observable by the EA

We address the EA’s optimal policy when both the cost  $\Delta$  of adopting the clean technology and the technology used by a firm are firms’ private information. We model this situation as follows. The EA faces a set of firms characterized by the cost parameter  $\Delta$ . Each firm knows its parameter  $\Delta$ . The EA does not know the particular cost  $\Delta$  of a firm, but it knows that  $\Delta$  is distributed in the family of firms according to the density function  $g(\Delta)$  over the interval  $[0, \bar{\Delta}]$ ; we denote by  $G(\Delta)$  the distribution function of  $\Delta$ .<sup>9</sup>

The environment considered in this section corresponds to situations where the precise “technology” used by the firm is difficult to verify. Firms decide whether to implement rigorous protocols or organizational modes that help decreasing the expected emissions. They also choose whether to employ good (and expensive) human resources and whether to adopt the best practices and know how to improve the functioning of existing equipment in order to reduce risks. In other words, “clean” or “dirty” refer to the decisions that firms take with respect to the maintenance and improvement of the environmental performance of existing technology or to avoiding mistakes. A firm uses a clean technology when it devotes (monetary and human) resources to the good functioning of its equipment, while a firm produces according to a dirty technology when it does not put in place the means to ensure the correct running of the equipment, thus leading to higher expected level of emissions.

The EA cares about expected environmental damage, hence its concern is whether the firms choose the clean or the dirty technology. It weights the benefits of the expected reduction of the damage against the monitoring costs and the firms’ cost to implement the clean technology. The EA’s policy is anonymous, i.e., every type of firm is subject

<sup>8</sup> Note that if the penalty rate is proportional to the tax rate, say  $\theta = (1 + \pi)t$ , then an increase in  $t$  decreases expected costs in region (a) and an increase in  $\pi$  decreases expected costs in both regions.

<sup>9</sup> The analysis developed in this and next section can also be interpreted as the study of the optimal monitoring policy when the EA monitors only one firm whose parameter  $\Delta$  is unknown and distributed according to the function  $G(\Delta)$ . Propositions and corollaries have an immediate interpretation in this alternative context.

to the same monitoring policy. Inspection of the incentive compatibility constraint (4) makes it clear that incentives to switch to the clean technology are strictly decreasing with the switching cost. That is, for a given monitoring policy, if a firm with parameter  $\Delta$  adopts  $E^C$ , a firm with parameter  $\Delta' < \Delta$  will also adopt it. Therefore, any policy  $\alpha(\cdot)$  will induce a firm to adopt  $E^C$  if its parameter lies in an interval  $[0, \Delta^n]$ , for some  $\Delta^n \in [0, \bar{\Delta}]$ .<sup>10</sup>

Next proposition characterizes the policy that minimizes monitoring costs when the EA wants all firms with  $\Delta$  in the interval  $[0, \Delta^n]$  to switch to  $E^C$ . The policy is qualitative the same as the one stated in Proposition 3, although the cut-off levels are different. The precise value for the parameters  $e^n, z^n, \hat{e}^n$ , and  $\hat{\gamma}^n$  that appear in Proposition 4 are given in the Appendix. They do not correspond to the optimal cut-off levels whenever the EA would like to give incentives to switch technology to a firm with parameter  $\Delta^n$ . That is, the homogeneous monitoring policy does not coincide with the optimal policy for the “marginal firm”  $\Delta^n$ . It is the optimal policy for a “fictitious” firm with the following characteristics: it has adoption costs of  $\Delta^n$ ; its incentives to switch are given by the difference between the distribution functions  $F(e; E^C)$  and  $F(e; E^D)$ ; however, its actual emissions are given by the (average) distribution function  $G(\Delta^n)F(e; E^C) + [1 - G(\Delta^n)]F(e; E^D)$ .

**Proposition 4** *Suppose the firms’ cost parameter  $\Delta$  is distributed according to  $G(\Delta)$ , it is firm’s private information, the EA cannot observe the technology choice, and Assumptions 1 and 2 hold. Then, when the EA wants firms with  $\Delta \in [0, \Delta^n]$  to adopt  $E^C$ :*

(a) *If  $\Delta^n < t \int_{\underline{e}}^{e^n} [F(e; E^C) - F(e; E^D)]de$ , then the following policy  $\alpha^n(z)$  is optimal:*

$$\begin{aligned} \alpha^n(z) &= \hat{\gamma}^n \text{ for all } z \in [\underline{e}, z^n], \\ \alpha^n(z) &= 0 \text{ for all } z \in [z^n, \bar{e}]. \end{aligned}$$

(b) *If  $\Delta^n \geq t \int_{\underline{e}}^{e^n} [F(e; E^C) - F(e; E^D)]de$ , then the following policy  $\alpha^n(z)$  is optimal:*

$$\begin{aligned} \alpha^n(z) &= t/\theta \text{ for all } z \in [\underline{e}, \hat{e}^n], \\ \alpha^n(z) &= 0 \text{ for all } z \in [\hat{e}^n, \bar{e}]. \end{aligned}$$

The policy  $\alpha^n(z)$  stated in Proposition 4 requires monitoring all reports below a cut-off value ( $e^n$  or  $\hat{e}^n$  depending on the region) with the same probability, that is, a large range of (low) reports are monitored with a uniform probability, while high reports are never monitored. The discussion after Proposition 3 provides the main intuitions behind the optimality of the policy proposed in Proposition 4.

Once we have identified the optimal monitoring policy if the EA wants that firms with  $\Delta \in [0, \Delta^n]$  adopt  $E^C$ , we discuss the choice of  $\Delta^n$ , that is, the decision concerning the set of firms that will be induced to switch to the clean technology. The optimal

<sup>10</sup> The letter  $n$  in  $\Delta^n$  stands for (technology adoption) non verifiable. In next section, the adoption is supposed verifiable and we will use  $\Delta^v$ .



policy minimizes the expected total costs: environmental damage plus monitoring and abatement costs. We elaborate on the three parts of the EA’s expected costs.

Abatement costs are easy to treat. They are incurred by the firms, they are increasing in  $\Delta^n$  and they amount to:

$$\int_0^{\Delta^n} \Delta g(\Delta) d\Delta.$$

The expected monitoring costs  $ECost^n([0, \Delta^n])$  of the policy  $\alpha^n(z)$  when the adoption of the technology is not observable also depend on the interval  $[0, \Delta^n]$ . Using the envelop theorem in program  $[P^M]$  in the proof of Proposition 4, we can deduce that:<sup>11</sup>

$$\frac{\partial ECost^n([0, \Delta^n])}{\partial \Delta^n} = \gamma g(\Delta^n) \left[ F(e_2; E^C) - F(e_2; E^D) \right] + \frac{[G(\Delta^n)f(e_2; E^C) + [1 - G(\Delta^n)]f(e_2; E^D)]}{[F(e_2; E^C) - F(e_2; E^D)]\theta}, \quad (5)$$

where  $e_2 = e^n$  and  $\gamma = \widehat{\gamma}^n$  in Region (a) and  $e_2 = \widehat{e}^n$  and  $\gamma = t/\theta$  in Region (b). An increase in the cut-off level  $\Delta^n$  has two effects on the monitoring costs. First, to induce firms with a higher switching cost to adopt  $E^C$ , a higher monitoring probability is necessary. This affects a firm independently on its type and is reflected in the second term in the right-hand side of (5). Second, there are types of firms that were keeping  $E^D$  before the increase in the cut-off and are adopting  $E^C$  after the change. Firms using  $E^C$  are monitored more often (although their expected payment is lower) than if they keep  $E^D$  (this is due to the property that the monitoring probability should be non-decreasing in realized emission, see Proposition 1). Both effects go in the same direction: inducing more firms to adopt  $E^C$  increases the monitoring costs.

Finally, expected environmental damage is a function of the expected emission level of the technology and it may also depend on its variance as the damage is often considered a convex function of the emission level. In our model, the critical decision in terms of environmental damage is which technology is adopted by the firms. Hence, we denote  $D(G(\Delta^n))$  the expected damage due to emissions when there are  $G(\Delta^n)$  firms producing with the clean technology and  $1 - G(\Delta^n)$  firms using  $E^D$ . The function  $D(G(\Delta^n))$  is assumed strictly decreasing in  $G(\Delta^n)$ , i.e., in  $\Delta^n$ . In environments with many firms producing with a similar technology and whose level of actual emission is not correlated,  $D(G(\Delta^n))$  is decreasing if  $E^C$  induces a lower expected emission level than  $E^D$ , i.e.,  $\int_e^{\bar{e}} edF(e; E^C) < \int_e^{\bar{e}} edF(e; E^D)$ . If the number of firms is small or, more importantly, there is strong correlation between firms’ actual emissions, we need further assumptions that make sure that a potential higher volatility of  $E^C$  does not translate into higher expected damage. A sufficient assumption to ensure

<sup>11</sup> The optimal solution of program  $[P^M]$  always involves  $e_1 = \underline{e}$ .

that  $D(G(\Delta^n))$  is decreasing is:<sup>12</sup>  $\int_{e^o}^{\bar{e}} edF(e; E^C) < \int_{e^o}^{\bar{e}} edF(e; E^D)$  for all  $e^o < \bar{e}$  (although weaker assumptions can also be sufficient).

Therefore, the optimal policy minimizes:

$$D(G(\Delta^n)) + ECost^n([0, \Delta^n]) + \int_0^{\Delta^n} \Delta g(\Delta)d\Delta.$$

We compare the chosen level  $\Delta^n$  with the first-best, denoted  $\Delta^*$ , which is the optimal level if a firm’s cost  $\Delta$  was public information and the firm’s technology verifiable. In the first-best, the Government (or the EA) would weight benefits of adopting technology  $E^C$  due to the reduction in the damage against costs of adoption,  $\Delta$ . Given that  $ECost^n([0, \Delta^n])$  is increasing in  $\Delta^n$ , it is immediate that the optimal decision will push too few firms to acquire the clean technology compared with the first-best situation, that is,  $\Delta^n < \Delta^*$ . Hence, the expected level of damage will be higher than the first-best level of damage:

**Corollary 2** *Suppose the cost parameter  $\Delta$  and the technology adopted are the firms’ private information. Then, the optimal monitoring policy induces firms to adopt technology  $E^C$  for an interval of parameters  $[0, \Delta^{n*}]$  that is smaller than the first-best interval  $[0, \Delta^*]$ .*

**6 Optimal monitoring when the technology adopted by the firms is verifiable but the cost  $\Delta$  is not**

In this section, we study the environments where the EA can easily verify the technology adopted by each firm. However, it does not know the adoption costs  $\Delta$ . The framework considered in this section is a plausible one when the technologies represent different types of physical capital. It may then be easy to check whether a firm has indeed adopted a given emissions-reducing technology, but it is not easy for the EA to assess each firm’s cost of the adoption. We can have in mind a set of firms or industries that rely on internal combustion engines for production. Some of these industries may be transportation, some manufacturing, some gas, diesel, coal. All emit carbon. All can purchase an abatement technology, but the cost of the technology is unknown by the EA and can differ across industries.

Given that the EA is not concerned about the environmental taxes raised, the optimal policy in this case involves not monitoring at all a firm that decides to switch to  $E^C$ . Therefore, a firm can “buy” immunity from environmental taxes by adopting the clean technology. This is the first characteristic of the optimal policy. Second, for similar reasons as in the previous section, for any given monitoring policy (that will only be applied to the firm if it keeps  $E^D$ ) the firm adopts  $E^C$  if its parameter  $\Delta$  lies in an interval  $[0, \Delta^v]$ .

<sup>12</sup> The density functions satisfying Assumption 2 have this property.

What is the optimal monitoring policy for the firm when it adopts  $E^D$ ? It needs to give incentives for the firm to switch to  $E^C$  even when its costs are  $\Delta^v$  and the distribution of emissions of those firms that are monitored is  $F(e; E^D)$ . Therefore:

**Proposition 5** *Suppose the firms’ cost parameter  $\Delta$  is distributed according to  $G(\Delta)$ , it is firms’ private information, the EA can observe the technology choice, and Assumptions 1 and 2 hold. Then, the optimal policy when the EA wants that firms with  $\Delta \in [0, \Delta^v]$  adopt  $E^C$  is:*

- (i) *A firm that adopts  $E^C$  is not monitored.*
- (ii) *A firm that adopts  $E^D$  is audited according to the policy found in Proposition 3 for a firm with adoption costs equal to  $\Delta^v$ .*

From Proposition 5, we see that the monitoring policy will only be applied to firms that use  $E^D$ , which happens when their parameter lie in the interval  $(\Delta^v, \bar{\Delta}]$ . Moreover, the policy applied is the one that would be optimal if the EA would face a firm with “known” adoption cost of  $\Delta^v$ . As we already described in Sect. 4, the optimal monitoring policy is a simple cut-off policy: reports lower than a certain threshold are inspected with a constant probability while any firm can avoid inspection by reporting that threshold emission level.

The expected monitoring costs of the policy in this case  $ECost^v([0, \Delta^v])$  are:

$$\begin{aligned}
 ECost^v([0, \Delta^v]) &= [1 - G(\Delta^v)] \int_{\underline{e}}^{e^*} \widehat{\gamma} f(e; E^D) de \\
 &= [1 - G(\Delta^v)] \frac{\Delta^v F(e^*; E^D)}{\theta \int_{\underline{e}}^{e^*} [F(e; E^C) - F(e; E^D)] de},
 \end{aligned}$$

when the parameters lie in Region (a) of Proposition 3, i.e., when  $\Delta$  is low. In Region (b):

$$ECost^v([0, \Delta^v]) = [1 - G(\Delta^v)] \frac{tF(\widehat{e}; E^D)}{\theta}.$$

Consider Region (a) (the qualitative properties in Region (b) are similar). It is immediate that:

$$\frac{\partial ECost^v([0, \Delta^v])}{\partial \Delta^v} = [[1 - G(\Delta^v)] - g(\Delta^v)\Delta^v] \frac{F(e^*; E^D)}{\theta \int_{\underline{e}}^{e^*} [F(e; E^C) - F(e; E^D)] de}.$$

As it was the case in the previous section, an increase in  $\Delta^v$  has two effects on the monitoring costs. On the one hand, for firms with a higher switching cost to adopt  $E^C$ , the monitoring probability must increase to “convince” those firms to adopt the clean technology. On the other hand, the population of firms that are monitored is smaller, as more firms switch to  $E^C$ . That is, there is an effect (the positive term in the previous

equation) that makes the monitoring cost increase, while another effect (the negative term) goes in the sense of decreasing monitoring costs.

Corollary 3 highlights the main implication of the previous discussion: there are environments where there is too much adoption of clean technology compared with the first-best situation.

**Corollary 3** *Suppose the cost parameter  $\Delta$  is the firm's own private information and that the adoption of the technology is verifiable. Then, the optimal monitoring policy induces the firm to adopt technology  $E^C$  for an interval of parameters  $[0, \Delta^{v*}]$  that may be larger or shorter than the first-best interval  $[0, \Delta^*]$ .*

Typically, we should expect too much adoption of the clean technology, as compared to the first best situation, precisely in those environments where the first best requires adoption for a large range of parameters ( $\Delta^*$  is high), while the informational asymmetry should cause too little adoption when  $\Delta^*$  is low. For example, if  $g(\Delta)$  is uniform, then too many firms switch to the clean technology when  $\Delta^* > \bar{\Delta}/2$ .<sup>13</sup> The reason for this result is the following. On the one hand, if the EA decides to increase adoption from  $\Delta^*$  to  $\Delta^* + d\Delta$ , it saves on monitoring costs because a total of  $g(\Delta^*)d\Delta$  firms switch to  $E^C$  and they do not need to be monitored any longer. On the other hand, the increase in the cost due to using a marginally tougher monitoring depends on the amount of firms that still chose  $E^D$ , which is equal to  $(1 - G(\Delta^*))$ . Therefore, the larger  $\Delta^*$  the more likely it is that it pays the EA to (marginally) induce more firm in the population to switch to  $E^C$ .

For example, consider firms' decision whether to adopt renewable energy processes (burning biomass) instead of processes based on fossil energy. The adoption of either process is easy to check, while the actual extra cost due to switching to renewable energy use may be difficult to assess by the EA. To give the firms incentives to adopt clean processes, the EA will monitor the pollution of fossil energy plants. Will the optimal monitoring policy lead to too many or too few renewable plants? On the one hand, the cost of the monitoring should imply a lower-than-optimal "firms' effort", that is, too little adoption of the clean plants. However, on the other hand, monitoring is only applied to those firms that still use fossil energy. This gives the EA an extra motivation to monitor, as tougher monitoring makes the number of monitored firms decrease. As the previous corollary shows, the optimal policy may imply overswitching or underswitching to renewable energy processes.

## 7 Conclusion

We have considered a situation where the environmental policy is based on taxes over reported emissions, monitoring, and penalties. We have assumed that the EA faces a population of firms. Firms' emissions depend on a decision (adopting the clean or the dirty technology) and random events. In addition each firm has private information concerning its realized emission level. We analyze the optimal monitoring strategy

<sup>13</sup> In this discussion, we are implicitly assuming that the second-order condition for the EA's objective function with respect to  $\Delta$  is concave, which happens, for example, if  $B(\cdot)$  is concave enough.

for the regulator and which firms in the population are induced to adopt the clean technology.

The added value of our paper lies in the characterization of this monitoring policy when the firms cannot fully control their emissions, they just decide its distribution. This random characteristic is not present in previous papers considering optimal auditing and we show that, in contrast with deterministic models, in our framework the firm's tax return depends on its emissions and the optimal inspection depends on the firm's report. We have developed the analysis in two different scenarios depending on whether the technology adopted by the firm is verifiable or not. In both cases, the optimal policy is a cut-off policy, where all reports below a threshold are inspected with the same probability, while reports above the threshold are not monitored. The main difference between the two scenarios is that, when the EA can check the technology adopted by the firms, it will only monitor those that keep the dirty technology (the others will not pay taxes even if bad luck makes them have high emissions), while all firms need to be subject to monitoring when the technology is not verifiable. It is intuitive that the EA achieves a higher welfare in the first scenario than in the second. Therefore, it may have incentives to promote the use of technologies and management protocols that are easier to verify.

We have also shown that if the adoption of the technology is firms' private information, too few firms will adopt the clean technology under the optimal monitoring policy. However, when the EA can check the technology adopted by the firms, the optimal policy may induce overswitching or underswitching to the clean technology.

In this paper, we have assumed that the environmental policy is based on taxes over reported emissions, monitoring, and penalties. We have not considered the possibility that the Government or the EA might give a firm a subsidy if it switches to the clean technology, or that it imposes a fixed penalty to firms keeping the dirty technology. When the technology adopted by the firm is not verifiable (i.e., only the firm knows the expected level of pollution of the technologies), the previous policies based on subsidies or penalties cannot be implemented, as they require the EA to be able to check whether a change to a clean technology has taken place. On the other hand, when the EA can easily check whether a firm has adopted a more environmentally friendly technology (or whether it is using the technology trying to minimize emissions), a fixed reward or penalty can be optimal. Therefore, our analysis applies to those situations where, due to political, technical, or moral hazard constraints, a policy based on fixed subsidies or penalties is not possible.

Let us note also that we have focused the analysis in those environments where the main decision concerning emissions is made *ex-ante*: the firm chooses the production technology, hires well-trained (or badly-trained) human resources, adopts (or not) the best practices and know how, or decides whether to implement costly protocols and organizational modes. We have not allowed the firms the possibility of adjusting their decisions once production is in place. In those industries where this adjustment is easy and cheap, a cut-off policy might not provide the right incentives to abate emissions when bad luck pushes actual emissions to a high level. Including the possibility of costly adjustment is an interesting and complex extension of our model that is beyond the scope of the present paper.

As to further policy implications of our analysis, many experts insist on the importance of the adoption (and the development) of environmental friendly technologies in reducing carbon emissions. The deployment of such energy-efficient and low-carbon technologies is costly and, in order to induce firms to adopt them, firms need to receive incentives. In our model, these incentives come from making the use of the dirty technology costly, via taxes or permits on emissions, but also from the design of the correct enforcement policy. We provide some insight on the optimal monitoring strategy depending on whether the adoption of the clean technology is public or private information when firms do not have perfect control on emissions. Finally, some analysts signal that among the weaknesses of the Kyoto Protocol there is the fact that the agreement did not provide the countries sufficient incentives for compliance. To give some advise for international agreements such as the Kyoto and the Post-Kyoto Protocols one has to take into account that national governments maintain sovereignty and the design of enforcement policies is in the hands of national or even regional institutions. To study the incentives and the enforcement of international agreements in such a hierarchy is an interesting avenue for future research.

### Appendix

*Proof of Lemma 1* First, reporting more than the true emissions is never optimal, since the expected payment is always higher. Second, if  $e > z$  and  $\alpha(z) > t/\theta$ , then  $tz + \alpha(z)\theta[e - z] > tz + t[e - z]$ , which is the payment the firm would make if it would report  $e$ . Therefore, reporting  $z$  is not optimal. Finally, by similar reasons, reporting  $e$  is not optimal when  $\alpha(z) < t/\theta$  for some  $z \in [e, e]$ .  $\square$

*Proof of Proposition 1* Consider two emission levels  $e_1$  and  $e_2$  with  $e_1 > e_2$  and the optimal reports corresponding to these levels,  $z(e_1)$  and  $z(e_2)$ . Given that the firm prefers reporting  $z(e_1)$  than  $z(e_2)$  when the emission level is  $e_1$ , and viceversa, we have:

$$c(e_1; \alpha(\cdot)) = tz(e_1) + \alpha(z(e_1))\theta[e_1 - z(e_1)] \leq tz(e_2) + \alpha(z(e_2))\theta[e_1 - z(e_2)],$$

$$c(e_2; \alpha(\cdot)) = tz(e_2) + \alpha(z(e_2))\theta[e_2 - z(e_2)] \leq tz(e_1) + \alpha(z(e_1))\theta[e_2 - z(e_1)].$$

These equations imply:

$$\alpha(z(e_1))\theta[e_1 - e_2] \leq c(e_1; \alpha(\cdot)) - c(e_2; \alpha(\cdot)) \leq \alpha(z(e_2))\theta[e_1 - e_2]. \tag{6}$$

First, since  $e_1 - e_2 > 0$ , (6) requires that  $\alpha(z(e_1)) \leq \alpha(z(e_2))$ , i.e.,  $\alpha(z(e))$  is nonincreasing in  $e$ . Second,  $\alpha(z(e))$  nonincreasing and (6) imply that  $c(e; \alpha(\cdot))$  is differentiable in  $e$  almost everywhere, with  $\frac{dc(e; \alpha(\cdot))}{de} = \alpha(z(e))\theta$  almost everywhere. Equation (2) immediately follows.

Finally, assume (1) and (2) hold. Then, a firm with emission level  $e$  reporting  $z(e^o)$  has an expected cost of:

$$\begin{aligned} tz(e^o) + \alpha(z(e^o))\theta[e - z(e^o)] &= c(e^o; \alpha(\cdot)) + \alpha(z(e^o))\theta[e - e^o] \\ &= c(e; \alpha(\cdot)) + \theta \int_e^{e^o} \alpha(z(x))dx + \alpha(z(e^o))\theta[e - e^o] \\ &= c(e; \alpha(\cdot)) + \theta \int_e^{e^o} [\alpha(z(x)) - \alpha(z(e^o))] dx. \end{aligned}$$

Given (1),  $\int_e^{e^o} [\alpha(z(x)) - \alpha(z(e^o))]dx \geq 0$ .

Therefore,  $z(e)$  is optimal in  $\{z|z = z(e^o) \text{ for some } e^o \in [\underline{e}, \bar{e}]\}$ . □

*Proof of Proposition 2* According to Equation (2):

$$C(E; \alpha(\cdot)) = \int_{\underline{e}}^{\bar{e}} c(e; \alpha(\cdot))dF(e; E) = c(\underline{e}; \alpha(\cdot)) + \int_{\underline{e}}^{\bar{e}} \left[ \theta \int_{\underline{e}}^e \alpha(z(x))dx \right] dF(e; E).$$

Integrating by parts, we obtain:

$$\begin{aligned} \int_{\underline{e}}^{\bar{e}} \left[ \int_{\underline{e}}^e \alpha(z(x))dx \right] dF(e; E) &= \left[ \left[ \int_{\underline{e}}^e \alpha(z(x))dx \right] F(e; E) \right]_{e=\underline{e}}^{e=\bar{e}} \\ &\quad - \int_{\underline{e}}^{\bar{e}} \alpha(z(e))F(e; E)de \\ &= \int_{\underline{e}}^{\bar{e}} \alpha(z(x))dx - \int_{\underline{e}}^{\bar{e}} \alpha(z(e))F(e; E)de. \end{aligned}$$

Equation (3) immediately follows. □

*Proof of Proposition 3* We can simplify program [P] as follows. We do not take into account the constraint that  $z(e)$  minimizes  $c(e, z; E^C; \alpha(\cdot))$ , and we denote the function  $\alpha(z(e))$  as  $\beta(e)$ . Once we identify  $\beta(e)$ , we will use Proposition 1 to decompose the function  $\beta(e)$  into the optimal monitoring function  $\alpha(z)$  and the report function  $z(e)$ . The optimal  $\beta(\cdot)$  solves the following program, that we will denote [P']:

$$\begin{aligned}
 & \underset{(\beta(e))_{e \in [\underline{e}, \bar{e}]}}{\text{Min}} \int_{\underline{e}}^{\bar{e}} \beta(e) dF(e; E^C) \\
 & \text{s.t.: } \beta(e) \text{ is nonincreasing in } e \\
 & \beta(e) \in [0, t/\theta] \text{ for all } e \in [\underline{e}, \bar{e}] \\
 & \Delta \leq \theta \int_{\underline{e}}^{\bar{e}} \beta(e) [F(e; E^C) - F(e; E^D)] de.
 \end{aligned}$$

Consider a solution  $\beta^*(\cdot)$  to program  $[P']$  and  $B^*$  the optimal budget. We claim that  $\beta^*(\cdot)$  is also the solution to the program  $[P'']$  below:

$$\begin{aligned}
 & \underset{(\beta(e))_{e \in [\underline{e}, \bar{e}]}}{\text{Max}} \int_{\underline{e}}^{\bar{e}} \beta(e) [F(e; E^C) - F(e; E^D)] de \\
 & \text{s.t.: } \beta(e) \text{ is nonincreasing in } e \\
 & \beta(e) \in [0, t/\theta] \text{ for all } e \in [\underline{e}, \bar{e}] \\
 & \int_{\underline{e}}^{\bar{e}} \beta(e) dF(e; E^C) \leq B^*.
 \end{aligned}$$

Indeed, if a function  $\beta'(\cdot)$  would exist involving a higher value for the solution,  $\beta^*(\cdot)$  would not be the solution to  $[P']$ : the EA could use  $\beta'(\cdot)$  that coincides with  $\beta^*(\cdot)$  until the lowest emission level  $e^o$  that satisfies

$$\Delta = \theta \int_{\underline{e}}^{e^o} \beta'(e) [F(e; E^C) - F(e; E^D)] de$$

and  $\beta''(e) = 0$  for all  $e > e^o$ . This policy would be cheaper than  $\beta^*(\cdot)$ , hence it would cost less than  $B^*$ , which is not possible.

We can now use known results (see, for example, Step 4 in the proof of Proposition 1 in Sánchez and Sobel 1991) to state that there exists a solution to  $[P'']$  that takes on at most one value different from 0 and  $t/\theta$ . That is, there exist  $\gamma \in (0, t/\theta)$ ,  $e_1$  and  $e_2$ , with  $\underline{e} \leq e_1 \leq e_2 \leq \bar{e}$ , such that the optimal function  $\beta(e)$  has the following shape:  $\beta(e) = t/\theta$  for all  $e \in [\underline{e}, e_1]$ ,  $\beta(e) = \gamma$  for all  $e \in (e_1, e_2)$  and  $\beta(e) = 0$  for all  $e \in [e_2, \bar{e}]$ . Therefore, we can rewrite  $[P']$  as  $[P''']$ :

$$\begin{aligned}
 & \underset{(\gamma, e_1, e_2)}{\text{Min}} \left\{ \frac{t}{\theta} F(e_1; E^C) + \gamma [F(e_2; E^C) - F(e_1; E^C)] \right\} \\
 \text{s.t.: } & \frac{\Delta}{\theta} = \frac{t}{\theta} \int_{\underline{e}}^{e_1} [F(e; E^C) - F(e; E^D)] de + \gamma \int_{e_1}^{e_2} [F(e; E^C) - F(e; E^D)] de.
 \end{aligned} \tag{7}$$

To find the solution of  $[P''']$ , we start by proving some claims.



*Claim 1* We can restrict attention to policies where  $e_2 < \bar{e}$ .

To prove Claim 1, consider the set of policies characterized by  $(e_1, e_2, \gamma)$ , with  $e_1 < \bar{e}$ . We do the analysis fixing the level of  $e_1$ . The parameter  $\gamma$  is given by (7), that is,

$$\gamma = \frac{1}{\int_{e_1}^{e_2} [F(e; E^C) - F(e; E^D)] de} \left[ \frac{\Delta}{\theta} - \frac{t}{\theta} \int_{\underline{e}}^{e_1} [F(e; E^C) - F(e; E^D)] de \right].$$

Therefore, the cost of the policy as a function of  $e_2$  is given by the function  $m(e_2)$ :

$$m(e_2) \equiv \frac{t}{\theta} F(e_1; E^C) + A \frac{F(e_2; E^C) - F(e_1; E^C)}{\int_{e_1}^{e_2} [F(e; E^C) - F(e; E^D)] de},$$

where  $A$  is a positive constant that does not depend on  $e_2$  (it is the second factor in the expression for  $\gamma$ ).  $m'(e_2 = \bar{e})$  is proportional to  $f(e_2; E^C) \int_{e_1}^{e_2} [F(e; E^C) - F(e; E^D)] de$ . Hence,  $m'(e_2 = \bar{e}) > 0$  given Assumption 2. This implies that, at the optimum, it is always the case that the cost is minimized for a value of  $e_2$  lower than  $\bar{e}$ .

*Claim 2* A policy such that  $e_1 = e_2 < e^*$  is not optimal.

We consider the policies of the form  $\beta(e) = \gamma$  for all  $e \in [\underline{e}, \tilde{e}]$  and  $\beta(e) = 0$  for all  $e \in [\tilde{e}, \bar{e}]$ , for which (7) holds. In this class of policies, we consider a marginal change in  $\tilde{e}$ , accompanied by the corresponding change in  $\gamma$  so that (7) still holds, i.e.,

$$\frac{\partial \gamma}{\partial \tilde{e}} = - \frac{\gamma [F(\tilde{e}; E^C) - F(\tilde{e}; E^D)]}{\int_{\underline{e}}^{\tilde{e}} [F(e; E^C) - F(e; E^D)] de}.$$

The cost of any policy in this class is  $\gamma F(\tilde{e}; E^C)$ . Hence, the change in cost due to the proposed marginal change is  $F(\tilde{e}; E^C) \partial \gamma + \gamma f(\tilde{e}; E^C) \partial \tilde{e} = h(\tilde{e}) \gamma \partial \tilde{e}$ . By Assumption 2,  $h(\tilde{e}) < 0$  given that  $\tilde{e} < e^*$ . Therefore, a marginal increase in  $\tilde{e}$  would reduce the cost. Consequently, a policy with  $\gamma = t/\theta$  (i.e.,  $e_1 = e_2$ ) cannot be optimal since there is room to increase  $\tilde{e}$  and decrease  $\gamma$  in a profitable way, which proves Claim 2.

*Claim 3* A policy such that  $e_1 < e_2$  is not optimal when  $e_1 < e^*$ .

We follow a similar strategy of proof as in Claim 2. Consider the class of policies of the form  $\beta(e) = \gamma'$  for all  $e \in [\underline{e}, e_1]$ ,  $\beta(e) = \gamma$  for all  $e \in [e_1, e_2]$ , and  $\beta(e) = 0$  for all  $e \in [e_2, \bar{e}]$ , with  $\gamma' > \gamma$ , for which Equation (7) holds (where we substitute  $t/\theta$  by  $\gamma'$ ). We want to show that  $\gamma' = t/\theta$  cannot be optimal within this class of policies (hence, it cannot be optimal in general). A marginal change in  $e_1$  accompanied by the corresponding change in  $\gamma'$  so that Equation (7) holds, must satisfy:

$$\frac{\partial \gamma'}{\partial e_1} = - \frac{(\gamma' - \gamma) [F(e_1; E^C) - F(e_1; E^D)]}{\int_{\underline{e}}^{e_1} [F(e; E^C) - F(e; E^D)] de}.$$

Given that the cost of the policy is  $\gamma'F(e_1; E^C) + \gamma[F(e_2; E^C) - F(e_1; E^C)]$ , the proposed marginal change in  $e_1$  will result in a change in costs of  $h(e_1)(\gamma' - \gamma)\partial e_1$ .

By the same reasons as in Claim 2, a marginal increase in  $e_1$  would decrease the costs whenever  $e_1 < e^*$  and  $\gamma' > \gamma$ . In particular, the policy where  $\gamma' = t/\theta$  cannot be optimal, since there is room to decrease  $\gamma'$  and increase  $e_1$ , which lowers the cost of the monitoring.

*Claim 4* A policy such that  $e_1 = e_2 > e^*$  and  $\gamma < t/\theta$  is not optimal.

The proof is similar to the proof of Claim 2. The difference is that now  $h(\bar{e})$  is positive since  $\bar{e} > e^*$ . Therefore, decreasing  $\bar{e}$  and increasing  $\gamma$  (when this change is possible, i.e., when  $\gamma < t/\theta$ ) decreases the costs of the policy.

*Claim 5* A policy such that  $e_1 < e_2$  is not optimal when  $e_1 \geq e^*$ .

To prove this Claim, we consider Program  $[P'']$ . By contradiction, suppose that the optimal  $e_1$  is an interior solution (we already now that  $e_2 < \bar{e}$ ). Denoting  $\lambda \geq 0$  the Lagrange multiplier of (7) in  $[P'']$ , the first order conditions of the Lagrange function with respect to  $e_1$  and  $e_2$  must hold:

$$\frac{\partial \mathcal{L}}{\partial e_1} = \left[ \frac{t}{\theta} - \gamma \right] \left[ f(e_1; E^C) - \lambda \left[ F(e_1; E^C) - F(e_1; E^D) \right] \right] = 0, \tag{8}$$

$$\frac{\partial \mathcal{L}}{\partial e_2} = \gamma \left[ f(e_2; E^C) - \lambda \left[ F(e_2; E^C) - F(e_2; E^D) \right] \right] = 0. \tag{9}$$

Given  $\gamma > 0$  and  $\gamma < t/\theta$ , from (8) and (9), it follows that:

$$\frac{f(e_1; E^C)}{F(e_1; E^C) - F(e_1; E^D)} = \frac{f(e_2; E^C)}{F(e_2; E^C) - F(e_2; E^D)}. \tag{10}$$

Under Assumption 2, Equation (10) is written as:

$$\frac{a + 2[1 - a]e_1}{[a - b][e_1 - e_1^2]} = \frac{a + 2[1 - a]e_2}{[a - b][e_2 - e_2^2]},$$

i.e.,  $[a + 2[1 - a]e_1]e_2^2 - [a + 2[1 - a]e_2]e_1^2 + a[e_1 - e_1^2] = 0$ . Easy calculations show that, when  $e_1 \geq e^*$  the previous equality does not have any solution (in  $e_2$ ) in the interval  $(e_1, 1]$ .

We now complete the characterization of the solution of  $[P''']$ . Claims 3 and 5 allow to state that the optimal policy has only two regions. Hence, it has the following form:  $\beta(e) = \widehat{\gamma}$  for all  $e \in [\underline{e}, \widehat{e}]$  and  $\beta(e) = 0$  for all  $e \in [\widehat{e}, \bar{e}]$ , where, given Claims 1 and 2,  $\widehat{e} \in [e^*, \bar{e}]$ . Finally, Claim 4 leaves as the unique candidate the following policy:

- (a) If  $\Delta < t \int_{\underline{e}}^{e^*} [F(e; E^C) - F(e; E^D)]de$ , then  $\beta(e) = \widehat{\gamma}$  for all  $e \in [\underline{e}, e^*]$  and  $\beta(e) = 0$  for all  $e \in [e^*, \bar{e}]$ , where  $\widehat{\gamma} < t/\theta$  is defined as in the Proposition.
- (b) If  $\Delta \geq t \int_{\underline{e}}^{e^*} [F(e; E^C) - F(e; E^D)]de$ , then  $\beta(e) = t/\theta$  for all  $e \in [\underline{e}, \widehat{e}]$  and  $\beta(e) = 0$  for all  $e \in [\widehat{e}, \bar{e}]$ , where  $\widehat{e} \geq e^*$  is defined as in the Proposition.

Finally, we transform the function  $\beta(e)$  into  $\alpha(z)$  and  $z(e)$ .

- (a) We first prove that, given the function  $\alpha^*(z)$  introduced in the proposition,  $z(e)$  is the optimal firms' strategy. It is easy to check that  $\widehat{\gamma} < t/\theta$  implies that firms either will report  $z = \underline{e}$  or  $z = z^*$ , any other possible report is dominated. The expected costs of a firm with emission level  $e$  are lower reporting  $\underline{e}$  than  $z^*$  if:

$$t\underline{e} + \widehat{\gamma}\theta [e - \underline{e}] < tz^* = t\underline{e} + \frac{\Delta (e^* - \underline{e})}{\int_{\underline{e}}^{e^*} [F(e; E^C) - F(e; E^D)] de},$$

i.e., given the characterization of  $\widehat{\gamma}$ ,

$$\frac{\Delta [e - \underline{e}]}{\int_{\underline{e}}^{e^*} [F(e; E^C) - F(e; E^D)] de} < \frac{\Delta (e^* - \underline{e})}{\int_{\underline{e}}^{e^*} [F(e; E^C) - F(e; E^D)] de},$$

or  $e < e^*$ .

Since  $z(e)$  is optimal for the firms given  $\alpha^*(z)$ , the policy  $\alpha^*(z)$  achieves the policy  $\beta(e)$  we found before, hence, it is optimal under Assumptions 1 and 2.

- (b) In this case, it is immediate to check that firms' strategy is optimal given  $\alpha^*(z)$  and that the policy  $\alpha^*(z)$  is then optimal. □

*Proof of Corollary 1* The proof follows easily from Proposition 3. □

*Proof of Proposition 4* Given  $\Delta^n$ , the EA solves the following program:

$$\begin{aligned} & \underset{(\beta(e))_{e \in [\underline{e}, \bar{e}]}}{\text{Min}} \quad B \\ & \text{s.t.: } \beta(e) \text{ is nonincreasing in } e \\ & \beta(e) \in [0, t/\theta] \text{ for all } e \in [\underline{e}, \bar{e}] \\ & G(\Delta^n) \int_{\underline{e}}^{\bar{e}} \beta(e) dF(e; E^C) + [1 - G(\Delta^n)] \int_{\underline{e}}^{\bar{e}} \beta(e) dF(e; E^D) \leq B \\ & \Delta^n = \theta \int_{\underline{e}}^{\bar{e}} \beta(e) [F(e; E^C) - F(e; E^D)] de. \end{aligned}$$

Following the same steps as in Proposition 3, there exists a solution to the previous program that takes on at most one value  $\gamma$  different from 0 and  $t/\theta$ . Also, the policy minimizing monitoring costs must solve program  $[P^M]$  below:

$$\begin{aligned} & \underset{(\gamma, e_1, e_2)}{\text{Min}} \quad \left\{ \frac{t}{\theta} F(e_1; E^M) + \gamma [F(e_2; E^M) - F(e_1; E^M)] \right\} \\ \text{s.t.: } & \frac{\Delta^n}{\theta} = \frac{t}{\theta} \int_{\underline{e}}^{e_1} [F(e; E^C) - F(e; E^D)] de + \gamma \int_{e_1}^{e_2} [F(e; E^C) - F(e; E^D)] de. \end{aligned} \tag{11}$$

where we have denoted  $F(e; E^M) \equiv G(\Delta^n)F(e; E^C) + [1 - G(\Delta^n)]F(e; E^D)$ . We note that the distribution function  $F(e; E^M)$  is the cumulative distribution function of a linear density function  $f(e; E^M) = a^n + 2[1 - a^n]e$ , where  $a^n = G(\Delta^n)a + [1 - G(\Delta^n)]b$ . We denote

$$h^n(e) \equiv f(e; E^M) - \frac{F(e; E^C) - F(e; E^D)}{\int_{\underline{e}}^e [F(x; E^C) - F(x; E^D)] dx} F(e; E^M).$$

Under Assumption 2,  $h^n(e)$  is first negative and then positive. We denote by  $e^n$  the cut-off level such that  $h^n(e^n) = 0$ . It is easily checked that  $e^n < e^*$ .

From now on, we can follow the same steps as in Claims 1 to 5 in the proof of Proposition 3, where we have to consider  $\Delta^n$  instead of  $\Delta$ ,  $e^n$  instead of  $e^*$ , and  $h^n(\cdot)$  instead of  $h(\cdot)$ . The claims lead to the following unique candidate policy:

(a) If  $\Delta^n < t \int_{\underline{e}}^{e^n} [F(e; E^C) - F(e; E^D)] de$ , then:

$$\begin{aligned} \beta^n(e) &= \widehat{\gamma}^n \quad \text{for all } e \in [\underline{e}, e^n], \\ \beta^n(e) &= 0 \quad \text{for all } e \in [e^n, \bar{e}], \text{ with} \\ \widehat{\gamma}^n \theta \int_{\underline{e}}^{e^n} [F(e; E^C) - F(e; E^D)] de &= \Delta^n. \end{aligned}$$

(b) If  $\Delta^n \geq t \int_{\underline{e}}^{e^n} [F(e; E^C) - F(e; E^D)] de$ , then:

$$\begin{aligned} \beta^n(e) &= t/\theta \quad \text{for all } e \in [\underline{e}, \widehat{e}^n], \\ \beta^n(e) &= 0 \quad \text{for all } e \in [\widehat{e}^n, \bar{e}], \\ \text{with } t \int_{\underline{e}}^{\widehat{e}^n} [F(e; E^C) - F(e; E^D)] de &= \Delta^n. \end{aligned}$$

Given the previous function  $\beta^n(e)$ , we follow the same steps as in the proof of Proposition 3 to show that the function  $\alpha^n(z)$  corresponds to  $\beta^n(e)$ . The cut-off value  $z^n$  that appears in the Proposition corresponds to the report made by a firm whose realized emission is  $e^n$  and is indifferent between reporting 0 (and being monitored with probability  $\widehat{\gamma}^n$ ) and reporting  $z^n$  and avoiding monitoring. That is,  $z^n$  is characterized by  $t\underline{e} + \widehat{\gamma}^n \theta [e^n - \underline{e}] = tz^n$ . □

*Proof of Proposition 5* It follows from Proposition 3. □

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