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**Article**

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SERIEs - Journal of the Spanish Economic Association

**Provided in Cooperation with:**

Spanish Economic Association

*Suggested Citation:* Cuevas, Ángel; Quilis, Enrique M. (2012) : A factor analysis for the Spanish economy, SERIEs - Journal of the Spanish Economic Association, ISSN 1869-4195, Springer, Heidelberg, Vol. 3, Iss. 3, pp. 311-338,  
<https://doi.org/10.1007/s13209-011-0060-9>

This Version is available at:

<https://hdl.handle.net/10419/77743>

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## A factor analysis for the Spanish economy

Ángel Cuevas · Enrique M. Quilis

Received: 7 July 2010 / Accepted: 16 April 2011 / Published online: 10 May 2011  
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**Abstract** We present a medium-scale dynamic factor model to estimate and forecast the rate of growth of the Spanish economy in the very short term. The intermediate size of the model overcomes the serious specification problems associated with large-scale models and the implicit loss of information of small-scale models. The estimated common factor is used to forecast the gross domestic product by means of a transfer function model. Likewise, the model solves the operational and informational limits posed by the presence of an unbalanced panel of indicators and generates multivariate forecasts of the basic indicators.

**Keywords** Dynamic factor model · Short-term economic analysis · Spanish economy · Kalman filter · Transfer function · Temporal disaggregation · Forecasting · Nowcasting

**JEL Classification** C22 · C53 · C82 · E27 · E32

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We thank A. Abad, J. Bógalo, M. Camacho, N. Carrasco, C. Cuerpo, R. Doménech, A. Estrada, L. González-Calbet, G. Pérez-Quirós, J.M. Ramos and A. Sanmartín for their valuable input at different stages of the project, and referees for comments that greatly improved the article. The views expressed in this paper are those of the authors and not necessarily those of the Spanish Ministry of Industry, Tourism and Trade or the Spanish Ministry of Economy and Finance.

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## 1 Introduction

Business cycle analysis has been spurred by the severity of the recent downturn of the world economy. The assessment of economic policies does require timely and precise information about general macroeconomic conditions. In this vein, the use of standard measures of aggregate economic activity based on the Quarterly National Accounts (QNA) imposes a delay in the decision-making process that may hamper its effectiveness.

In order to alleviate the information constraints imposed by these standard measures, we design a coincident indicator to estimate the state of the business cycle in the very short term on a real-time basis using dynamic factor analysis.

This attempt has some precedents, starting with [Stock and Watson \(1991, 2002\)](#) which may be considered modern descendants of the seminal work on cyclical indicators of [Burns and Mitchell \(1946\)](#). Factor models have become one of the most widely used techniques in applied econometric analysis because they provide a parsimonious way to parameterize dynamic models for vector time series, see [Bai and Ng \(2008\)](#) and [Stock and Watson \(2010\)](#) for a comprehensive review. The relationship of factor models with other multivariate techniques is analyzed in [Galeano and Peña \(2000\)](#), [Peña and Poncela \(2006b\)](#) and [Stock and Watson \(2005\)](#). Since the pioneering work of [Sargent and Sims \(1977\)](#) and [Geweke \(1977\)](#), these models have been used for macroeconomic analysis and forecasting. More recently, both central banks and academic institutions have created all sorts of real time indicators and disseminated them through their websites. These estimates and forecasts influence policy-makers and shape up public opinion. Notable examples are the indicator designed by [Aruoba et al. \(2009\)](#), published in real time by the Federal Reserve Bank of Dallas; [Chauvet \(1998\)](#), both for the United States economy (US) and for Brazil; [Giannone et al. \(2008\)](#) and [Evans et al. \(2002\)](#) also for the US economy; [Angelini et al. \(2008\)](#) for the Eurozone and [Camacho and Pérez-Quirós \(2009a,b\)](#) for both the Eurozone and the Spanish economy.

Applications related to finance are also numerous, many of them linked to risk management and term structure modeling, see [Chamberlain \(1983\)](#), [Chamberlain and Rothschild \(1983\)](#), [Litterman and Scheinkman \(1988\)](#), [Knez et al. \(1994\)](#), [Bechikh \(1998\)](#), [Reimers and Zerbs \(1999\)](#), [Ang and Piazzesi \(2003\)](#) and [Christensen et al. \(2009\)](#), among others. Factor models have been used to assess economic policies, see [Bernanke et al. \(2005\)](#), [Boivin and Giannoni \(2006\)](#), and [Forni and Gambetti \(2010\)](#); estimation and inferential issues are analyzed in [Peña and Box \(1987\)](#), [Watson and Engle \(1983\)](#), [Watson and Kraft \(1984\)](#), [Stock and Watson \(1988\)](#), [Escribano and Peña \(1994\)](#), [Peña and Poncela \(2004\)](#), [Peña and Poncela \(2006a\)](#). Factor models in the frequency-domain are described in [Priestley et al. \(1974\)](#), [Geweke \(1977\)](#), [Sargent and Sims \(1977\)](#), [Geweke and Singleton \(1981\)](#), [Singleton \(1983\)](#) and [Forni et al. \(2000\)](#), [Forni et al. \(2005\)](#), among others.

All the preceding models are designed either as small-scale or large-scale. Both methodologies present important shortcomings. On the one hand, small-scale models are relatively exposed to idiosyncratic shocks and suffer an implicit loss of information. On the other hand, the estimation of large-scale models by quasi-maximum likelihood methods, akin to those used in our model, can violate the weak cross-correlation assumption needed to ensure the consistency of their estimators. By contrast, our

model has an intermediate size that provides a natural hedge against the pitfalls of both small-scale and large-scale models.

The debate concerning the forecasting performance of small-scale models versus large-scale models is still an open issue. Our main contribution to the literature is twofold. First, we increase the number of indicators in a controlled way, fulfilling the assumption of weak cross-correlation among the idiosyncratic components which ensures the consistency of the estimators. Second, our model combines dynamic factor analysis with transfer function modeling, instead of ad hoc bridge equations.

The common factor underlying the observed indicators is estimated by means of the Kalman filter, after a suitable reparameterization of the model in state space form. In this way, we solve simultaneously the problem posed by the presence of an unbalanced panel (i.e., indicators with non-overlapping samples) and the generation of forecasts for individual indicators using a multivariate approach.

It must be emphasized that these predictions of the individual indicators are made in an explicit multivariate setting, avoiding the overparameterization and overfitting risks posed by other approaches (e.g. VAR models). Therefore, when making individual forecasts, the model makes an efficient use of the information contained in related indicators.

Moreover, transfer function models provide a simple and quantitatively consistent relationship between the common factor and the macroeconomic aggregates, GDP in particular. This linkage allows us to compile a contemporaneous estimate of GDP on a real-time basis. These models also provide confidence intervals for the GDP estimates, quantifying the uncertainty that surrounds them. It is important to note that the specification of the transfer function is checked with the results of a bivariate vector autoregressive and moving average (VARMA) model. The VARMA model provides an additional and rigorous foundation for the transfer function and prevents data mining.

This two-step approach (common factor estimation and transfer function) effectively disentangles the uncertainty due to the real-time estimation of actual business cycle conditions using monthly indicators from the uncertainty due to the relationship between GDP and monthly short-term indicators. This separation hedges us from idiosyncratic GDP changes that may distort the historical relationship between monthly indicators and quarterly macroeconomic aggregates measured by the QNA.

Additionally, the fact that the GDP compilation features (chain-linking, benchmarking, seasonal adjustment and balancing) are so different from the usual short-term indicators compilation practices, suggests the use of a two-step approach such as the one used in this work. These features, individually considered, are absent in the compilation of most short-term economic indicators and their concurrent use is completely missing. Hence, from the compilation perspective, GDP is a very special type of economic statistics, see [INE \(2002\)](#) and [Abad et al. \(2009\)](#).

Additionally, GDP is a synthetic statistic, the result of combining short-term indicators (monthly and quarterly data) with structural sources (annual data provided by the National Accounts and the Input-Output tables, see [INE 1993](#)). Thus, GDP is functionally equivalent to a common factor although not compiled using factor models. As a result, a one-step approach that considers GDP and short-term indicators in a unique framework may overweight the former due to its synthetic (or “artificial”) nature, rather than on a genuine communality derived from common economic fundamentals.

This methodology is applied to a broad set of monthly indicators of the Spanish economy, whose selection took into account their economic significance, their temporal and statistical coverage, and an appropriate degree of sources diversification. The size of the model (31 indicators) allows a feasible computerized processing and reduces the risks implied by idiosyncratic shocks affecting the estimation and forecasting of the common factor as well as its link to the quarterly GDP.

It should be also mentioned that a natural extension of the model would be its integration with a Markov switching model in the line of [Cancelo \(2005\)](#) and [Camacho et al. \(2010\)](#). These nonlinear integrated models allow simultaneous calculation of probabilities of recession while dealing with some specificity of common factor models (mixing frequencies, data revisions and ragged edges). However, in the context of the size of this model is computationally more complex, while the integrated models may be more sensitive in their results to changes in information, especially in the delimitation of the states.

The document is organized as follows. The second section outlines the econometric methodology, detailing the nature of the dynamic factor model, its estimation by means of the Kalman filter and its relationship with macroeconomic variables using transfer function models. The third section presents the basic short-term indicators and their preliminary statistical treatment. The empirical results appear in section four. Finally, a set of appendices describes the technical details of the model, in order to ensure the self-contained nature of the text.

## 2 Econometric approach

The starting point of our modeling approach is a dynamic one-factor model that captures in a parsimonious way the dynamic interactions of a set of monthly economic indicators. The common factor of the system is estimated by means of the Kalman filter, after casting the factor model in state space form. On the basis of this factor we design a synthetic index that is related to quarterly aggregate output through a transfer function model. The entire procedure has been adapted to operate with unbalanced data panels, in order to forecast both indicators as well as macroeconomic aggregates in real time (nowcasting).

### 2.1 Dynamic factor model

Dynamic factor analysis is based on the assumption that a small number of latent variables generate the observed time series through a stochastically perturbed linear structure. Thus, the pattern of observed co-movements is decomposed into two parts: communality (variation due to a small number of common factors) and idiosyncratic effects (specific elements of each series, uncorrelated along the cross-section dimension).

In this paper we assume that the observed, stationary growth signals of  $k$  monthly indicators are generated by a factor model:

$$z_{i,t} = \lambda_i f_t + u_{i,t} \quad (2.1)$$

Being:

- $z_{i,t}$ :  $i$ th indicator growth signal at time  $t$ .
- $\lambda_i$ :  $i$ th indicator loading on common factor.
- $f_t$ : common factor at time  $t$ .
- $u_{i,t}$ : specific or idiosyncratic component of  $i$ th indicator at time  $t$ .

The loadings  $\lambda_i$  measure the sensitivity of the growth signal of each indicator for changes in the factor.

Equation (2.1) considers only static (i.e., contemporaneous) interactions among the observed indicators through its common dependence on a latent factor. The model must be expanded in order to adapt it to a time series framework, thereby adding a dynamic specification for the common factor and the idiosyncratic elements. A finite autoregression of order  $p$ , AR( $p$ ), provides a sufficiently general representation for dynamics of the common factor:

$$\begin{aligned} (1 - \phi_1 B - \phi_2 B^2 - \phi_3 B^3 - \dots - \phi_p B^p) f_t &= a_t \\ a_t &\sim iid N(0, 1) \end{aligned} \tag{2.2}$$

In (2.2)  $B$  is the backward operator  $Bf_t = f_{t-1}$  and the variance of the innovation has been normalized. Depending on the characteristic roots of  $\phi_p(B)$  the model may exhibit a wide variety of dynamic behaviors. Determining the order  $p$  of the model is made taking into account the empirical dynamics of the static factor, according to standard order selection criteria. As will be seen below in the section on empirical results, the most appropriate order is  $p = 4$ .

We consider an AR(1) specification for the dynamics of the specific elements, allowing for some degree of persistence:

$$\begin{aligned} (1 - \psi_i B) u_{i,t} &= b_{i,t} \quad |\psi_i| < 1 \\ b_{i,t} &\sim iid N(0, v_i) \end{aligned} \tag{2.3}$$

Finally, we assume that all the innovations of the system are orthogonal:

$$\begin{aligned} E(a_t b_{i,s}) &= 0 \quad \forall i, t, s \\ E(b_{i,t} b_{j,s}) &= 0 \quad \forall i, j, t, s \end{aligned} \tag{2.4}$$

### 2.2 State-space representation and Kalman filter

Model (2.1)–(2.4) attempts to represent the static as well as the dynamic features of the data. We estimate the common and idiosyncratic factors using the Kalman filter, after a suitable reparameterization of the model in state-space form. This reparameterization requires the introduction of a state vector that encompasses all the required information needed to project future paths of the observed variables from their past realizations. In our case, this vector is:

$$X_t = (f_t \quad f_{t-1} \quad f_{t-2} \quad f_{t-3} \quad u_{1,t} \quad \dots \quad u_{k,t})' \tag{2.5}$$

The corresponding measurement equation is:

$$Z_t = (L \quad 0_{k \times 3} \quad I_k)X_t = HX_t \tag{2.6}$$

where  $L = \{\lambda_i | i = 1 \dots k\}$  represents the loading matrix. This equation allows us to derive the observed indicators from the (unobservable) state vector.

The transition equation completes the system and characterizes its dynamics:

$$X_t = GX_{t-1} + V_t \tag{2.7}$$

where  $G$  is a square matrix with dimension  $k + 4$ :

$$G = \begin{bmatrix} \phi_1 & \phi_2 & \phi_3 & \phi_4 & 0 & \dots & 0 \\ 1 & 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 1 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & \psi_1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \dots & \psi_k \end{bmatrix} \tag{2.8}$$

The innovations vector  $V_t$  is:

$$V_t = (a_t \quad 0 \quad 0 \quad 0 \quad b_{1,t} \quad \dots \quad b_{k,t})' \tag{2.9}$$

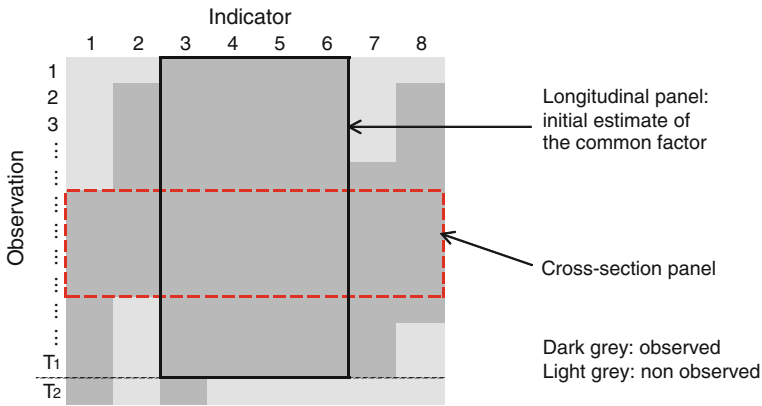
$V_t$  evolves as a Gaussian white noise with diagonal variance-covariance matrix as follows:

$$Q = E[V_t V_t'] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & v_1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \dots & v_k \end{bmatrix} \tag{2.10}$$

We assume that the time index  $t$  goes from 1 to  $T$ . The application of the Kalman filter requires  $\Theta = [H, G, Q]$  to be known. Since the model is not small-scale, full-system maximum likelihood estimates for  $\Theta$  are not feasible. Our solution was to derive them from the static version of the model estimated using bootstrap methods, see Appendix A for details. The Kalman filter is explained in O’Connell (1984) and Kim and Nelson (1999), among others.

### 2.3 Dealing with an unbalanced data panel

One of the major operational problems faced while analyzing multiple time series is the incomplete nature of the available information. In general, the availability of different



**Fig. 1** Unbalanced panel data

indicators is not homogeneous, which leads to the generation of a non-overlapping sample.

One way to deal with unbalanced panels consists in working only with complete panels, in the time dimension or in the cross-section dimension. As shown in Fig. 1, in the first case we may discard a large number of the relevant indicators, with likely adverse effects on factor estimation accuracy and forecasting performance. In the second case, the number of observations may be too small when some series have a short span, making the forecasting or backcasting horizon too long.

Given these drawbacks we propose a way to utilize all available information, both on the cross-section dimension and on the time dimension. The method, which is partially based on [Stock and Watson \(2002\)](#) and [Doz et al. \(2006\)](#), relies on an iterative process with the following steps:

1. Estimation of a static factor model by principal components using the longitudinal panel data. Obviously, the use of this panel involves a loss of information that will be compensated in the following stages.
2. The indicators that have been excluded from the longitudinal panel are individually regressed (by ordinary least squares, OLS) on the common factor. The estimated parameters are then used to calculate the missing data in these series from  $t = 1$  to  $t = T_1$ .
3. A new factor is calculated from the statically balanced panel, from  $t = 1$  to  $t = T_1$ , using the same procedure as in step 1. Hence, new parameters  $\Theta = [H, G, Q]$  are available.
4. Using the new parameters  $\Theta$  we apply the Kalman filter from  $t = 1$  to  $t = T_1$  to estimate the common factor. This factor is in turn projected to  $t = T_2$ .
5. With the estimated common factor derived from step 4 as a regressor, we rebalance again the panel using the same procedure used in step 2. Steps 2–5 are iterated until convergence is achieved. The convergence criterion states that the change of the likelihood function should not trespass a given threshold.

The initial longitudinal panel should be wide enough to be representative, easing the usual trade-off between temporal coverage and cross-section coverage. After several



tests, we selected January 1990 as the starting point of the panel data, providing a sensible balance in the above mentioned trade-off.

## 2.4 Linkage with macroeconomic variables via transfer function modeling

One of the main goals of the model consists in designing a connection between high-frequency indicators and the key variables that shape the macro scenario. In order to do it in a simple and efficient way, a transfer function model emerges as the ideal solution, providing real-time estimates of quarterly GDP using monthly indicators. Once we have completed the estimation process of the dynamic factor model, taken into account the basic nature of the indicators as (standardized) period on period rates of growth, we can follow [Mariano and Murasawa \(2003\)](#) and represent the factor at the quarterly frequency combining the monthly observations according to:

$$f_T = \left( \frac{1}{3} + \frac{2}{3}B + B^2 + \frac{2}{3}B^3 + \frac{1}{3}B^4 \right) f_t \quad (2.11)$$

where  $f_t$  represents the monthly dynamic common factor and  $f_T$  is its temporally aggregated (quarterly) counterpart with time indexes related by  $T = 3t$ . Hence, quarter  $T$  comprises months  $t$ ,  $t - 1$  and  $t - 2$ .

We consider that the dynamic relationship at the quarterly frequency between the common factor and the GDP may be articulated using a linear transfer function:

$$y_T = c + V(B)f_T + n_T \quad (2.12)$$

where:

- $y_T$  is the GDP, quarter on quarter rate of growth.
- $f_T$  is the dynamic common factor, temporally aggregated according to [Mariano–Murasawa](#).
- $n_T$  is a stochastic disturbance that obeys a stationary and invertible ARMA(p,q) model.

The intercept  $c$  represents the non-stochastic component of  $y_T$  and  $V(B)$  is the filter that passes on the information contained in  $f_T$  to contemporaneous and future values of  $y_T$ .

In order to specify the impulse-response  $V(B)$  in a parsimonious way we follow [Box and Jenkins \(1976\)](#) and represent it in a rational form. Hence, the model (2.12) becomes:

$$y_T = c + \frac{\omega_s(B)B^b}{\delta_r(B)} f_T + \frac{\theta_q(B)}{\phi_p(B)} u_T \quad (2.13)$$

where  $u_T \sim iid N(0, v_u)$  and  $\delta_r(B)$ ,  $\omega_s(B)$ ,  $\theta_p(B)$  and  $\theta_q(B)$  are polynomials on the backward operator  $B$  with orders  $r$ ,  $s$ ,  $p$  and  $q$ , respectively. We assume that all of them have their roots outside of the unit circle. The term  $b \geq 0$  is the pure delay of the transfer function.

We arrive at the final form for (2.13) following the adaptive methodology of Box–Jenkins, refined and tailored to the transfer function case by Liu and Hanssens (1982), Hanssens and Liu (1983) and Tsay and Wu (2003), among others. In particular, tentative identification of the orders  $b$ ,  $r$  and  $s$  of the (rational) impulse response is performed using the corner method (Beguin et al. 1980) as implemented by Liu (2005). The orders  $p$  and  $q$  of the model for the perturbation are determined using the the so-called *Smallest Canonical Analysis* (SCAN), see Tsay and Tiao (1985). This methodology provides a statistically well-rooted method to determine the dynamic form of the relationship between  $y_T$  and  $f_T$ , avoiding ad hoc data mining and other pitfalls of the standard bridge equation approach.

### 3 Data

This section details the indicators that have been selected for model estimation and the preliminary processing that they have gone through.

#### 3.1 Selection of indicators

Given the objective of the model and the econometric methodology at hand, we have made a relatively wide selection of monthly indicators. The selection process was carried out under the premise that indicators should be available timely and should provide a synthetic measure of the growth rate of the Spanish economy, being selected at their more aggregated level.<sup>1</sup> Additionally, they should have a correlation with GDP growth  $>0.4$  in absolute value. The 31 selected economic indicators, listed in Table 1, can be divided into 5 large blocks.

The first set includes information related to the domestic production. Among them we include the traditional series that are used to capture the evolution of economic activity, such as apparent consumption of cement, energy consumption or the industrial production index.

In the second block we have considered those economic variables related to the external sector, such as exports and imports of goods and services suitably deflated.

The third block consists of “soft” or qualitative indicators, where the economic sentiment indicator plays an important role due to their prompt availability. The financial variables are represented by (deflated) credit to firms and households.

Finally, the number of social security contributors, the number of registered contracts and the number of employed provided by the Labor Force Survey (LFS),<sup>2</sup> stands for the aggregate evolution of the Spanish labor market.

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<sup>1</sup> The initial set on which the selection has been made is available in the *Synthesis of Economic Indicators* published by the Ministry of Economy and Finance.

<sup>2</sup> The data provided by the LFS are the only ones compiled on a quarterly basis. In order to preserve the monthly nature of the data set, we have used temporal disaggregation techniques to derive consistent monthly figures, see Boot et al. (1967). The transformation has been applied to the seasonally adjusted levels.

Table 1 List of indicators

	Code	Source	Start date	Unit	Release date
<i>Domestic production</i>					
Total air traffic	AER	State Agency for Air Navigation	1990 01	Passengers	t + 12 days
Apparent consumption of cement	CEMN	Cement Partnership	1990 01	Thousand tons	t + 22 days
Consumer goods availabilities	DISPOCONS	GDMA	2000 01	Volume index	t + 50 days
Capital goods availabilities	DISPOEQ	GDMA	2000 01	Volume index	t + 50 days
Electric power consumption	ELE	Spanish Electricity Network	1990 01	Million kW/h	t + 1 day
Entry of tourists	ENT	Institute of Tourism Studies	1995 01	Thousand people	t + 22 days
Consumption of gasoline and diesel	GASOL	Petroleum Products Corporation	1990 01	Thousand of metric tons	t + 30 days
Industrial turnover index deflated by IPRI	ICNI	National Statistical Institute	2002 01	Deflated value index	t + 47 days
Services turnover index deflated by CPI of services	ICNSS	National Statistical Institute	2002 01	Deflated value index	t + 47 days
Industrial order books index deflated by IPRI	IEPI	National Statistical Institute	2002 01	Deflated value index	t + 47 days
Industrial production index	IPI	National Statistical Institute	1990 01	Volume index	t + 35 days
Construction industry production index	IPIC	Eurostat	1990 01	Deflated value index	t + 47 days
Retail trade index	IVCM	National Statistical Institute	1995 01	Deflated value index	t + 27 days
Sea goods transport	MARM	Ministry of Public Works	1990 01	Thousand Mt	t + 40 days
Car registrations	MATT	General Directorate of Traffic	1990 01	Units	t + 1 day
Truck and cargo van registrations	MATVC	General Directorate of Traffic	1990 01	Units	t + 1 day
Overnight stays in hotels	PERNO	National Statistical Institute	1999 01	Units	t + 23 days
Total gross salaries	RBT	Tax State Agency	1995 01	Deflated value index	t + 35 days

Table 1 continued

	Code	Source	Start date	Unit	Release date
Railway goods transport	REM	Spanish Railways	1990 01	Thousand passengers/km	t + 29 days
Large companies sales total	VEG	Tax State Agency	1995 01	Deflated value index	t + 35 days
Road passenger transport	VICAR	National Statistical Institute	1996 01	Units	t + 39 days
Periodified number of housing starts	VIVPER	Ministry of Public Works	1990 01	Units	t + 1 day
<i>External sector</i>					
Exports of goods deflated by UVI	EXBQ	Tax State Agency/GDMA	1990 01	Deflated value index	t + 50 days
Exports of services deflated by CPI of services	EXBS	Bank of Spain	1990 01	Deflated value index	t + 60 days
Imports of goods deflated by UVI	IMPB	Tax State Agency/GDMA	1990 01	Deflated value index	t + 50 days
Imports of services deflated by euro zone CPI	IMPS	Tax State Agency/Eurostat	1996 01	Deflated value index	t + 60 days
<i>Opinion</i>					
Economic sentiment indicator: SPAIN	ISE	European Commission	1990 01	Index 1990–2008 = 100	t – 1 day
<i>Financial variables</i>					
Credit to companies and families deflated by CPI	FIN	Bank of Spain/National Statistical Institute	1995 01	Deflated value index	t + 35 days
<i>Labour market</i>					
Social security contributors	AFI	Ministry of Labour	1990 01	Thousand people	t + 2 days
Registered contracts	CONTRA	Ministry of Labour	1990 01	Units	t + 2 days
Employed LFS	OCU	National Statistical Institute	1990 01	Units	t + 30 days

All indicators are freely available at <http://serviciosweb.meh.es/apps/igpe/default.aspx>  
 GDMA General Directorate for Macroeconomic Analysis

**Table 2** Allocation of indicators to macroeconomic aggregates and sectors

Indicator	Consumption	Investment	Exports	Imports	Industry	Construction	Services	Labor market
AER	X						X	
AFI								X
CEMN						X		
CONTRA								X
DISPOCONS	X							
DISPOEQ		X						
ELE	X				X	X	X	
ENT			X				X	
EXBQ			X					
EXSQ			X					
FIN	X	X						
GASOL	X						X	
ICNI					X			
ISE	X							
ICNSS							X	
IEPI					X			
IMPB				X				
IMPS				X				
IPI					X			
IPIC						X		
IVCM							X	
MARM							X	
MATT	X							
MATVC		X						
RBT	X							X
REM							X	
OCU								X
PERNO			X				X	
VICAR							X	
VIVPER		X				X		
VGE					X	X	X	

Another reason for the choice of these variables is to consider all the indicators used in the compilation of the QNA and its main output, GDP. See [Álvarez \(1989\)](#), [Martínez and Melis \(1989\)](#), [INE \(1993, 1994\)](#) and [Álvarez \(2005\)](#). To achieve this goal we attempt to cover in a sensible manner all the operations involved in the GDP compilation, both from the point of view of supply and demand:

As shown in [Table 2](#), we want that the main macroeconomic aggregates and sectors are adequately represented in the factor model. Such representation is strengthened

diversifying the information sources, to the extent feasible by available economic short-term statistics.

### 3.2 Preliminary processing

As already mentioned, the objective of the model is to provide a synthetic measure of the rate of growth of the economy. This goal requires identifying a reliable signal of growth to be fitted by the factor model. In practice, the identification of this signal requires applying a filter to the series that removes their secular trend from the observed data. A detailed analysis of the different measures of economic growth can be found in [Melis \(1991\)](#) and [Espasa and Cancelo \(1993\)](#).

In order to emphasize the short-term information contained in the indicators, we have chosen the regular first difference of the log time series to perform such decomposition. The high-pass nature of this filter ensures an adequate estimation of the short-term variation, ruling out at the same time the trend component.

For this filtering not to be distorted by the presence of seasonal and calendar factors, they have been removed by means of seasonal adjustment and time series techniques ([Gómez and Maravall 1996](#); [Caporello and Maravall 2004](#)). These transformations are also necessary in order to set a linkage, via Eq. (2.11), with the GDP growth, as both are corrected by the same factors (seasonal and calendar factors).

In the specific case of “soft” series, typically measured as balances of qualitative responses, the log transformation is not applied. Naturally, in all cases, the process of seasonal and calendar adjustment applies only if such effects are significant.<sup>3</sup> To facilitate the process of estimation and interpretation of the factor model, the filtered series are standardized:

$$\tilde{z}_{i,t}^{st} = \frac{Z_{i,t} - \mu_i}{\sigma_i} \quad (3.1)$$

Being  $\mu_i$  and  $\sigma_i$  the mean and standard deviation of the indicators in the selected sample. Thus, all series contained in the system are expressed in the same units of measurement.

## 4 Empirical results

The eigenvalues of the indicators correlation matrix across its cross-section dimension indicates the dominance of its maximum over the remaining eigenvalues (Fig 2).

A similar picture emerges from the scree plot of the same eigenvalues computed using the 31 indicators at the same time (Fig 3).

Both results suggest that a one-factor model may be a sensible model for the joint behavior of the 31 indicators.

<sup>3</sup> Consumer Goods Availabilities (DISPOCONS), Capital Goods Availabilities (DISPOEQ) and the Economic Sentiment Indicator (ISE) are not adjusted from seasonal and calendar effects because they have already been processed by our data provider, <http://serviciosweb.meh.es/apps/dgpe/default.aspx>.

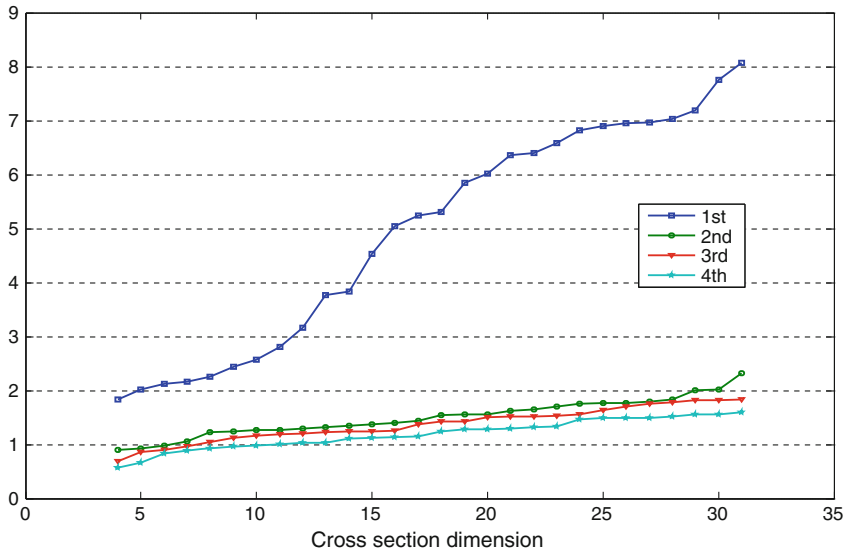


Fig. 2 Correlation matrix eigenvalues across its cross-section

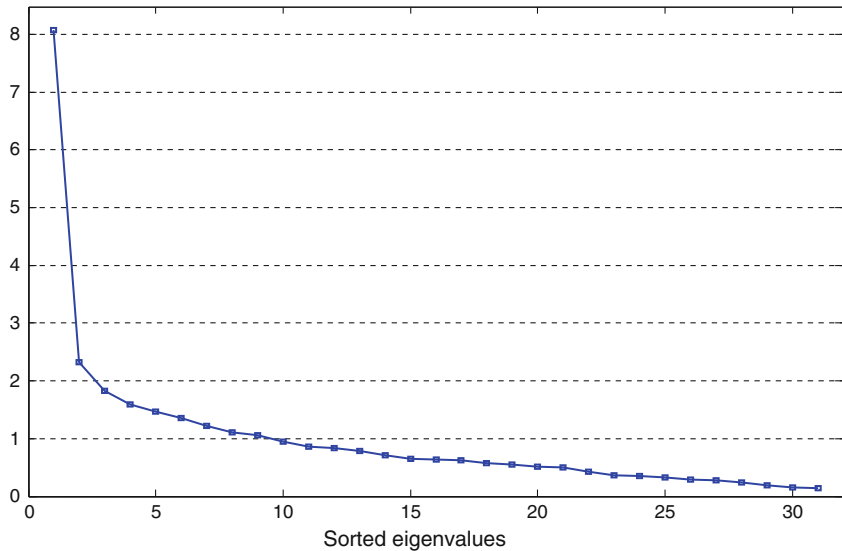


Fig. 3 Correlation matrix eigenvalues: scree plot

The loading vector is estimated by means of principal components factor analysis combined with resampling techniques, suitably adapted to the time series context by Politis and Romano (1994). Estimation is based on 10,000 bootstrap replicates. The resampling procedure uses the stationary bootstrap with an expected size block of 41 months. This method provides a measure of the precision of point estimates and does not require any assumption concerning the distributional features of the data.

**Table 3** Loading vector: bootstrap estimates and communalities

	Loadings		Communalities
	Estimate	Standard error	
ICNSS	0.88	0.07	0.78
ICNI	0.85	0.06	0.71
VGE	0.76	0.07	0.59
IPI	0.74	0.06	0.55
IEPI	0.74	0.08	0.53
AFI	0.73	0.11	0.58
CEMN	0.64	0.06	0.41
IVCM	0.60	0.07	0.36
GASOL	0.54	0.05	0.28
OCU	0.54	0.15	0.35
MATVC	0.51	0.05	0.26
DISPOCONS	0.50	0.09	0.24
MATT	0.47	0.06	0.22
CONTRA	0.47	0.06	0.22
FIN	0.46	0.10	0.22
IPIC	0.44	0.09	0.23
EXBQ	0.44	0.05	0.18
IMPB	0.43	0.06	0.18
ENT	0.40	0.06	0.15
VIVPER	0.40	0.16	0.21
DISPOEQ	0.36	0.08	0.12
AER	0.35	0.07	0.13
ISE	0.31	0.06	0.10
EXSQ	0.30	0.06	0.09
RBT	0.30	0.11	0.10
VICAR	0.26	0.04	0.06
IMPS	0.25	0.06	0.01
REM	0.22	0.13	0.06
ELE	0.20	0.06	0.04
MARM	0.19	0.05	0.03
PERNO	0.15	0.12	0.02

See Appendix A for details. Table 3 shows the results of estimating Eq. (2.1), sorted from highest to lowest loading. This table also includes the mean<sup>4</sup> communalities (defined as the ratio between the observed variance of each indicator and the variance explained by the factor model).

To set the lag order of the factor AR model we have used the Akaike information criterion (AIC), the Bayesian information criterion (BIC) and the partial autocorrelation

<sup>4</sup> The mean is computed averaging over all the resamples.



**Table 4** Common factor: AR(4) estimates

	$\phi_1$	$\phi_2$	$\phi_3$	$\phi_4$
Estimate	0.03	0.28	0.31	0.14
Standard error	0.06	0.06	0.06	0.07

function (PACF) applied to the static common factor. The results of these statistics suggest  $p = 3$  or  $p = 4$  as the appropriate order of the model. We have chosen  $p = 4$  in order to fully represent the systematic dynamics of the factor.<sup>5</sup>

Using the previous results and the corresponding static factor we estimate the parameters of Eq. (2.2) by ordinary least squares, obtaining Common factor: AR(4) estimates (Table 4).

Using the same estimation procedure applied to Eq. (2.3) we get Idiosyncratic factors: AR(1) estimates (Table 5).

The dynamic common factor is estimated using the Kalman filter and its quarterly counterpart, temporally aggregated using the Mariano–Murasawa formula. It shows a remarkable conformity with GDP growth, as may be appreciated in the next graph:<sup>6</sup>

Figure 4 shows their notable similitude, quantified by a high correlation (0.8) especially if one takes into account the presence of an important irregular component in both series.

The cross correlation function also shows a high degree of conformity between the common factor of the system and the GDP. The function has a maximum at lag zero, confirming the coincident nature of the factor with respect to GDP. Moreover, its asymmetric shape points to a tendency of the factor to lead GDP. This feature is very convenient for nowcasting and short-term forecasting (Table 6).

Following the methodology described in Liu (2005), the orders finally selected for the transfer function are:  $b = 0$ ,  $s = r = 1$  and  $p = q = 0$ . The formal expression is:

$$y_T = c + \frac{(\omega_0 + \omega_1 B)}{(1 - \delta B)} f_T + u_T \quad (4.1)$$

Moreover, a separate multivariate analysis, based on the estimation of an autoregressive and vector moving average (VARMA) model, clearly ascertains a unidirectional Granger-causality that goes from factor to GDP and not vice versa. This lack of feedback justifies the use of a transfer function. Furthermore, this analysis suggests a tentative similar model:  $b = 0$ ,  $r = s = 1$  and  $p = q = 1$ . It was found that the modeling of the disturbance may ultimately be simplified, obtaining  $p = q = 0$ . See Appendix B for additional details on the VARMA analysis.

Table 7 displays the estimation of the transfer function model by exact maximum likelihood:

<sup>5</sup> The detailed results are available upon request.

<sup>6</sup> The dynamic common factor has been scaled according to the affine transformation  $\alpha + \beta f_T$ , being  $\alpha$  and  $\beta$  the mean and standard deviation of GDP growth, respectively. This transformation enhances the comparability of both time series and preserves the directional pattern of the factor.

**Table 5** Idiosyncratic factors: AR(1) estimates

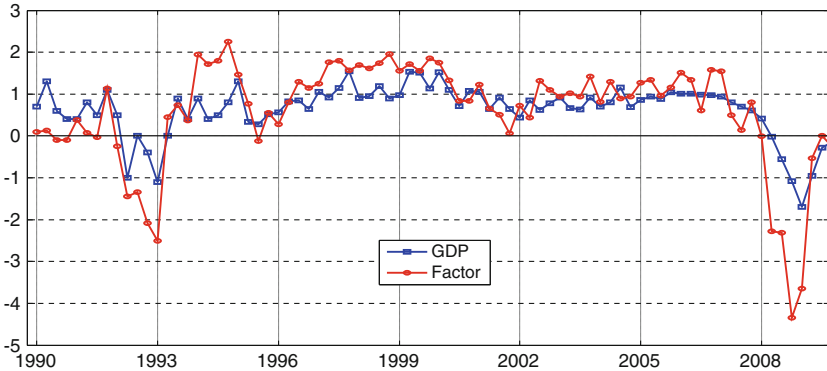
	$\psi_i$		$\sigma$
	Estimate	Standard error	
ICNSS	0.01	0.07	0.17
ICNI	-0.21	0.06	0.23
VGE	-0.33	0.06	0.36
IPI	-0.15	0.06	0.41
IEPI	-0.42	0.06	0.37
AFI	0.14	0.07	0.42
CEMN	-0.31	0.06	0.52
IVCM	0.06	0.07	0.67
GASOL	-0.34	0.06	0.65
OCU	0.59	0.05	0.42
MATVC	-0.37	0.06	0.66
DISPOCONS	-0.26	0.06	0.72
MATT	0.01	0.07	0.79
CONTRA	-0.22	0.06	0.74
FIN	0.48	0.06	0.62
IPIC	0.46	0.06	0.62
EXBQ	-0.38	0.06	0.70
IMPB	-0.55	0.05	0.58
ENT	-0.40	0.06	0.73
VIVPER	0.64	0.05	0.48
DISPOEQ	-0.51	0.06	0.66
AER	-0.27	0.06	0.82
ISE	-0.15	0.06	0.88
EXSQ	-0.37	0.06	0.79
RBT	-0.30	0.06	0.83
VICAR	-0.22	0.06	0.90
IMPS	-0.46	0.06	0.74
REM	-0.54	0.05	0.67
ELE	-0.50	0.06	0.72
MARM	-0.52	0.06	0.69
PERNO	-0.35	0.06	0.86

The ordering is the same as in Table 3

The dynamics implied by the estimated transfer function reveals the high degree of persistence of the GDP ( $\delta = 0.84$ ). The  $\omega(B)$  operator plays also an important role since, due to its low long-run gain ( $\omega(1) = 0.07$ ), compensates the inertia of GDP and links its forecasts more closely to those of the factor.

Following Tsay and Tiao (1985) we have performed a canonical analysis of the residuals (the so-called *Smallest Canonical Analysis*, SCAN). The results do not show any major inadequacy, in line with the autocorrelation function.

In order to check the robustness of the transfer function, we estimate an expanded version of (4.1). The augmented model is:



**Fig. 4** Dynamic common factor (scaled) and GDP growth

**Table 6** Common factor and GDP: cross correlation function

Lag	-5	-4	-3	-2	-1	0	1	2	3	4	5
	0.24	0.40	0.60	0.68	0.83	0.84	0.62	0.46	0.34	0.12	0.02

Negative (positive) lags indicate that the factor is leading (lagging) GDP

**Table 7** Transfer function estimates

	C	$\omega_0$	$\omega_1$	$\delta$	$\sigma$
Estimate	0.56	0.21	-0.14	0.84	0.25
Standard error	0.03	0.02	0.03	0.05	

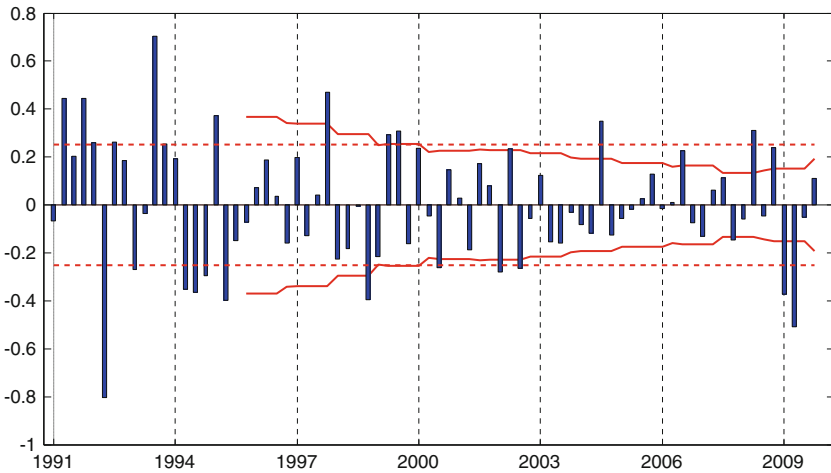
**Table 8** Extended transfer function estimates

	$\omega_2$	$\delta_2$	$\phi$	$\theta$
Estimate	-0.06	0.63	0.08	-0.03
Standard error	0.03	0.05	0.11	0.11
$\sigma$	0.24	0.27	0.27	0.26

$$y_T = c + \frac{(\omega_0 + \omega_1 B + \omega_2 B^2)}{(1 - \delta_1 B - \delta_2 B^2)} f_T + \frac{(1 - \theta B)}{(1 - \phi B)} u_T \tag{4.2}$$

Following [Box and Jenkins \(1976\)](#), the additional parameters are included one by one, in order to isolate as best as possible its individual contribution. [Table 8](#) presents the results:

In all the cases the additional parameter is not significant and/or it does not improve the fit of the model. As an additional check of the robustness of the model, we estimate the parameters of the transfer function (4.1) recursively from 2006:Q1 onwards.



**Fig. 5** Transfer function: residuals. *Horizontal lines* are  $\pm\sigma$  interval. The *dotted line* estimates  $\sigma$  using the full sample and the *solid line* estimates  $\sigma$  using a 5-year rolling window

In general, the recursive estimates remain confined in the intervals centered around the last estimate.<sup>7</sup>

There is some evidence of changing volatility reflected in the kurtosis (3.62), in the autocorrelation of the squared residuals (systematically positive) and in the variability of the variance of the residuals, as shown in Fig. 5.

However, this evidence is not strong enough to reject the gaussianity assumption using the Jarque–Bera test<sup>8</sup> but deserves additional analysis using more sophisticated methods in future research (e.g., stochastic volatility models). This issue is important because, as shown in Fig. 5, it may reflect changes in the size of the shocks affecting GDP.

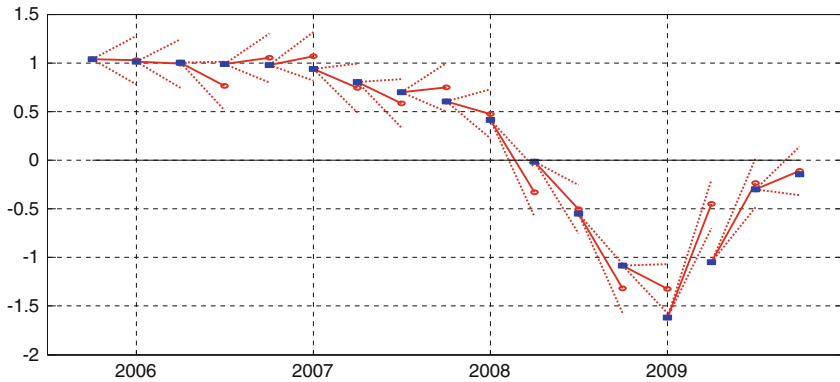
In order to evaluate the forecasting performance of the model we have done several backtesting exercises. In all cases, the model has proved its usefulness as a tool for short-term economic analysis and the assessment of the growth pattern. As an example, Fig. 6 shows the good tracking properties of the model during the previous 4 years.

We have compared the predictions made by the transfer function with those that have been generated by three standard univariate models used in forecasting GDP growth: a random walk, I(1); a first-order autoregressive and moving average, ARMA(1,1); and a fourth-order autoregression, AR(4). The first one represents a “no change” assumption, the second one a univariate transposition of the VARMA(1,1) model and the third one considers only pure AR representations.<sup>9</sup>

<sup>7</sup> The graph of the recursive estimates are available upon request.

<sup>8</sup> The test value, 1.38, generates a p value of 0.41.

<sup>9</sup> The order of the AR model,  $p = 4$ , has been determined using the Bayesian Information Criterion (BIC). The Akaike’s Information Criterion (AIC) suggested a much less parsimonious model ( $p = 6$ ). Anyway, its forecasting performance is much similar to the AR(4) model.



**Fig. 6** Backtesting 2006–2009. One-step ahead forecasts ( $\pm$ SE). *Solid squares* are actual GDP and *circles* are recursive one-step ahead forecasts. *Dotted lines* are  $\pm\sigma$  confidence intervals

**Table 9** Forecasting performance, 2006:Q1–2009:Q4

	I(1)	ARMA(1,1)	AR(4)	Transfer function
RMSE	0.36	0.40	0.37	0.21
MAE	0.26	0.29	0.25	0.15
RMSE: DM	0.01	0.02	0.07	
MAE: DM	0.02	0.02	0.10	

DM test is reported using the p value of the null hypothesis of forecasting performance equivalence

Table 9 shows alternative measures of the forecasting performance of the models during the span 2006:Q1–2009:Q4: root of mean squared errors (RMSE) and mean of absolute errors (MAE), both considering one-step ahead forecasts. This time span has been chosen to take into account both a period of high growth and a period of sharp and deep contraction of aggregate output. The table also includes the Diebold–Mariano 1995 test to check the equivalence of the forecasting performance of the alternative models.

The competitive edge of the transfer function model relies on its efficient use of monthly information combined with a proper dynamic specification, leading to better outcomes than its peers. The DM test presents the AR(4) model as the most close competitor of the transfer function, although the significance level is still quite small.

In order to complete the evaluation of the forecasting performance, it has been carried out a real time estimation exercise for the last four quarters (2009:Q3–2010:Q2). Figure 7 plots the evolution of the real-time forecast of GDP in such quarters on a daily basis, including its  $\pm\sigma$  confidence interval.

Observing graphs of Fig. 7, we can see how the model reacts to the coming out of data updates. This process reduces the amplitude of the confidence interval, as the cross-sectional estimates are replaced by actual data. Initially, when only “soft” indicators are available, the uncertainty associated with the estimate is greater. When

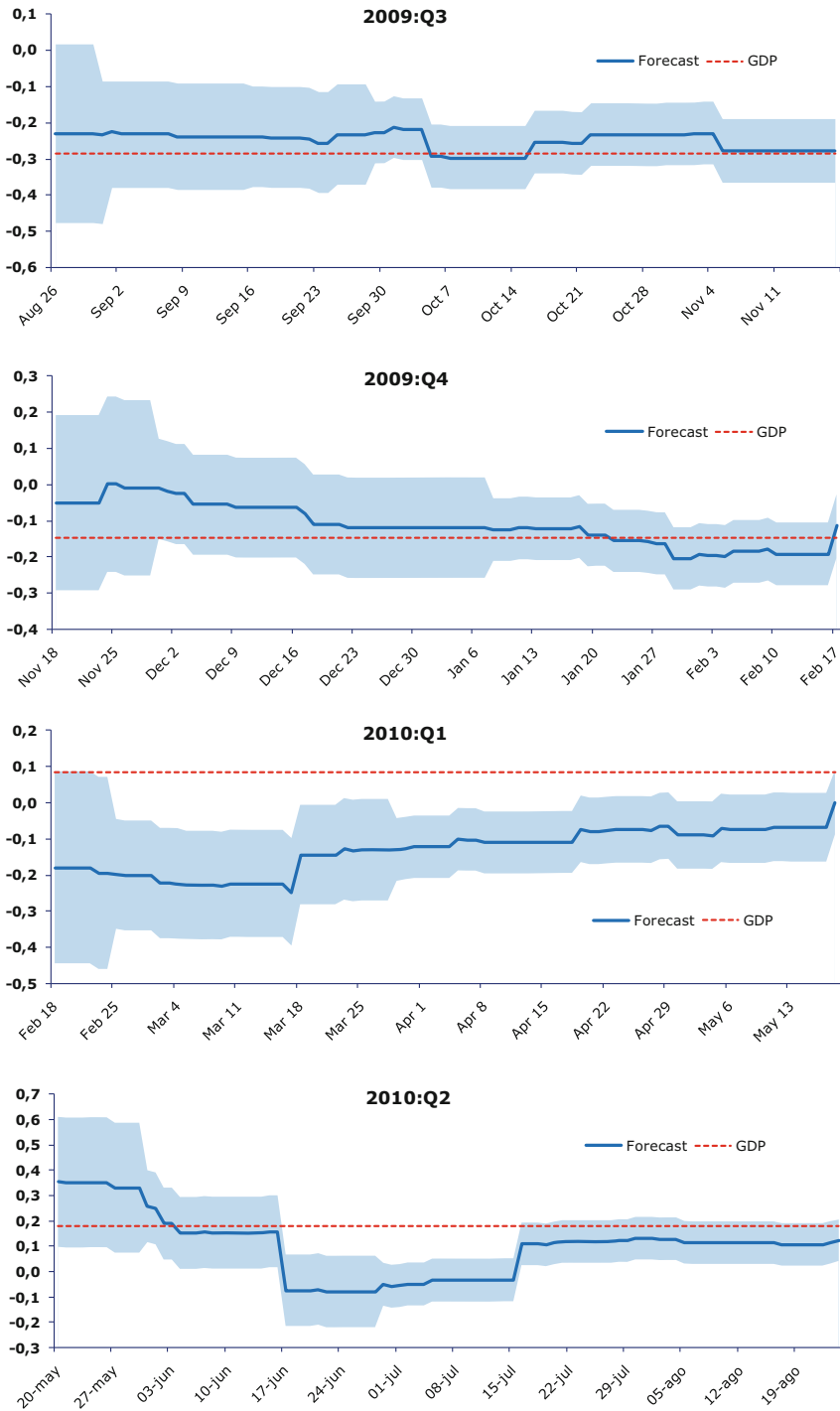


Fig. 7 GDP growth rate Real-time forecasts

**Table 10** Factor models for the Spanish economy

Model	Number of indicators	Preprocessing	First observation	Factor estimation	GDP forecasting
S-STING	11	Seasonal and calendar adjustment, seasonal differences and levels	1990:01	Maximum likelihood + Kalman filtering	Internal
MICA	12	Seasonal and calendar adjustment, seasonal differences and levels	1980:01	Maximum likelihood + Kalman filtering	Internal
FASE	31	Seasonal and calendar adjustment, seasonal differences and levels	1990:01	Static factor analysis + Kalman filtering	External

“hard” information arrives (industrial production index, large companies sales, etc.), the estimate becomes more accurate.

Figure 7 shows that these forecasts were in close agreement with the GDP flash release disseminated by the National Statistical Institute and the subsequent final figure (second estimate).<sup>10</sup> May be noted that, in most cases, the final data published has fallen within the confidence intervals associated with the estimation.

Finally, it is worth noting the work carried out in the same line for [Camacho and Pérez-Quirós \(2009a,b\)](#) and [Camacho and Doménech \(2010\)](#) who also estimate dynamic factor models for the Spanish economy,<sup>11</sup> providing GDP forecasts and synthetic measures of the state of the economy.<sup>12</sup> Table 10 compares the different characteristics of the models.

As can be seen, the first of the differential characteristics of the models is their size. Both the S-Sting and MICA took place in a small-scale size, easing their maximum likelihood estimation. The second feature is the different preliminary treatment of the indicators, sharing all of them the seasonal and calendar adjustment. Finally, apart from the greater sample period covered by the MICA model, our model does not include the GDP as an indicator to estimate the factor, since, as mentioned earlier in the article, the GDP is already synthetic statistic, the result of combining short-term indicators.

<sup>10</sup> The GDP flash estimate is released about six weeks after the end of the quarter. The second estimate, incorporating the complete GDP breakdown, is released just 1 week after the flash (except in August that left nearly 2 weeks to incorporate the structural information of Annual National Accounts).

<sup>11</sup> They are also known by their acronyms: S-Sting (Spain-Short Term Indicator for Growth) and MICA (*Modelo de Indicadores Coincidentes y Adelantados*, Model of Coincident and Leading Indicators).

<sup>12</sup> Using an affine methodology, [Camacho and Pérez-Quirós \(2009a,b\)](#) estimate and analyze a dynamic factor model for the Eurozone. [Camacho et al. \(2010\)](#) expand the model to incorporate non-linearities (via Markov-switching) in the behavior of the common factor. In the same vein, [Cancelo \(2005\)](#) estimates a dynamic factor model with Markov-switching features to analyze the GDPs of the Eurozone countries.

## 5 Conclusions

In this paper we have designed a real-time, coincident indicator of the Spanish business cycle. It has a straightforward interpretation as the dynamic common factor of a set of representative short-term monthly economic indicators. This synthetic indicator also plays a critical role in GDP forecasting, by means of a suitable dynamic projection based on transfer function modeling.

The model differs from others proposed in the literature due to its medium-scale. This feature provides a certain advantage over small-scale models due to its higher information content and, at the same time, avoids the technical problems concerning the consistency of the estimators that hamper large-scale models. Moreover, its two-step approach strengthens the operative characteristics of the model, providing a hedge from changes in the relationship between indicators and macroeconomic aggregates.

This work could be extended in several directions. The incorporation of leading indicators would enrich the dynamic structure of the model. Another possibility is to apply this methodology to other macroeconomic aggregates, being the demand-side components of GDP prime candidates. Anyway, since the model is eminently empirical, its use in a production mode will determine the way forward, including changes in the list of indicators and refinements of the estimation procedures.

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## Appendix A: Preliminary static estimation

By rewriting (2.1) in matrix form, we obtain:

$$Z_t = Lf_t + U_t \tag{A.1}$$

Being  $Z_t$ :  $k \times 1$ ,  $L$ :  $k \times 1$ ,  $f_t$ :  $1 \times 1$  and  $U_t$ :  $k \times 1$ .

The normalized eigenvector associated with the largest eigenvalue of the correlation matrix of  $Z$ , provides an estimate of the loading matrix  $L$ :

$$\hat{L} = \sqrt{\lambda_1} e(\lambda_1) \tag{A.2}$$

The variance–covariance matrix of the specific factors is then estimated as a residual:

$$\hat{\Psi} = \text{diag}(R - \hat{L}\hat{L}') \tag{A.3}$$

In order to obtain estimates of  $L$  and  $\Psi$  with appropriate standard errors, we apply (A.2) and (A.3) to the resampled time series. Resampling is performed using the bootstrap technique suggested by Politis and Romano (1994), in which the resampling is applied with reposition to blocks of varying size. The block size is selected each time according to a predefined probability distribution. In our application we have used



the geometric distribution with an expected block size of 41 months.<sup>13</sup> The results are robust with respect to alternative mean block size. The estimation is repeated 10,000 times and the corresponding averages and standard deviations provide the estimates for  $L$  and  $\Psi$ .

The stationary bootstrap provides more robust results than other resampling methods, notably those procedures based on the use of fixed size blocks, e.g. [Künsch \(1989\)](#). In fact, the former may be considered as a weighted average over block size of the latter, generating a smoothed version of it.

With the resulting point estimates of  $L$  and  $\Psi$  we transform the original factor model into one akin to a multivariate regression model. Hence, an initial estimate can be obtained using generalized least squares (GLS):

$$\hat{F} = [\hat{L}'\hat{\Psi}^{-1}\hat{L}]^{-1}[\hat{L}'\hat{\Psi}^{-1}Z] = \Theta(\hat{L}, \hat{\Psi})Z = \Theta Z \quad (\text{A.4})$$

A complete analysis of these issues can be found in [Mardia et al. \(1979\)](#).

## Appendix B: Varma analysis

In this section we estimate a vector autoregressive moving-average (VARMA) model to summarize the econometric relationship between the dynamic common factor ( $f_t$ ) and the GDP quarter on quarter rate of growth ( $y_t$ ), see [Tiao and Box \(1981\)](#), [Lütkepohl \(1991\)](#), [Reinsel \(1993\)](#) and [Tiao \(2001\)](#), for an in-depth analysis of such models.

Consider a  $k$ -dimensional vector,  $Z_t$ , which evolves following a VARMA( $p,q$ ) model, which can be expressed by the following equation:

$$\Phi_p(B)Z_t = c + \Theta_q(B)U_t \quad (\text{B.1})$$

Being  $\Phi_p(B)$  and  $\Theta_q(B)$  polynomial matrix operators of degree  $p$  and  $q$ , respectively. Furthermore, the vector  $U_t$  can be characterized by the following distributional properties:

$$U_t : k \times 1 \sim N(0, \Sigma) \quad (\text{B.2})$$

Being  $\Sigma$ , in general, a non-diagonal matrix. Additionally, it is assumed that all the roots of the determinantal polynomials  $|\Phi_p(B)|$  and  $|\Theta_q(B)|$  lie either on or outside the unit circle.

The canonical correlation analysis of Tsay–Tiao suggests that a low-order VARMA(1,1) provide a reasonable fit to the data. This model serves as a benchmark to check the adequacy of several specifications concerning the direction of (Granger) causality. The results are summarized in [Table 11](#).

The results strongly support the hypothesis that the dynamic common factor is an input in the determination of GDP and that the use of transfer function is well grounded.

<sup>13</sup> Following [Camacho et al. \(2005\)](#) in their implementation of stationary bootstrap for business cycle analysis.

**Table 11** VARMA(1,1): Granger-causality analysis

Hypothesis	Log likelihood	$\Delta$
Feedback	22.75	–
Factor causes GDP	22.98	0.23
GDP causes factor	–2.24	–25.00
Decoupling	–7.22	–29.98

**Table 12** VARMA model:  $(I - \Phi B)Z_t = c + (I - \Theta B)U_t$  constrained maximum likelihood estimation

	Estimate		Standard error		Eigenvalues
C	0.08		0.02		
	0		–		
$\Phi$	0.85	–0.02	0.03	0.03	0.85
	0	0.66	–	0.10	0.66
$\Theta$	1.00	–0.32	0.05	0.04	1.00
	0	–0.37	–	0.12	–0.37
$\Sigma \Gamma$	0.09	0.65			
	0.23	1.43			
Log likelihood	23.02				

0 and – mean restricted parameters.  $\Gamma$  is the correlation matrix linked to  $\Sigma$

**Table 13** VARMA model SCAN analysis of the residuals

q = >	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0
p	4	0	0	0	0	0	0	0	0
	5	0	0	0	0	0	0	0	0
	6	0	0	0	0	0	0	0	0
	7	0	0	0	0	0	0	0	0
	8	0	0	0	0	0	0	0	0

The estimation of the constrained<sup>14</sup> VARMA(1,1) model by exact maximum likelihood yields the following results (Table 12).

The residuals obtained from the VARMA model do not show any major inadequacy, as may be seen from their corresponding SCAN table (Table 13).

To further analyze the underlying structure of the VARMA model we perform a canonical analysis, following Box and Tiao (1977). The results are summarized in Table 14.

<sup>14</sup> The constraint  $c_2 = 0$  is considered in addition to the ones defined in the second row of Table 12.

**Table 14** VARMA model:

$$(I - \Phi B)Z_t = c + (I - \Theta B)U_t$$

Box–Tiao canonical analysis

Eigenvalues	0.78	0.27
Eigenvectors	1.00	0.94
	0.01	−0.34

The main results may be summarized as follows:

- There is a remarkable degree of persistence in the bivariate system, as denoted by the maximum eigenvalues of both  $\Phi$  (0.85) and the Box–Tiao canonical analysis (0.78). The behavior of the GDP explains most of this feature.
- Adding up to the dynamic (unidirectional) interactions, there is a significant degree of contemporaneous association between both series (0.65). This fact justifies the use of the common factor to nowcast GDP on a real-time basis.
- The system is non-invertible, due to the GDP intrinsic dynamics ( $\Theta_{1,1} = 1$ ). This fact may be the result of the seasonal adjustment procedure, see [Maravall \(1993\)](#).
- The Box–Tiao canonical analysis identifies a stable contemporaneous, positive relationship between GDP and the common factor. Deviations from this “equilibrium” feature revert to the mean at a high speed.

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