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The Market for Conservation and Other Hostages

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The Market for Conservation and Other Hostages

Abstract

A “conservation good” (such as a tropical forest) is owned by a seller who is tempted to consume (or cut), but a buyer benefits more from conservation. The seller does conserve if the buyer is expected to buy, but the buyer is unwilling to pay as long as the seller conserves. This contradiction implies that the market for conservation cannot be efficient and conservation is likely to fail. A leasing market is inefficient for similar reasons and dominates the sales market if and only if the conservation value is low, the consumption value high, and the buyer’s protection cost large. The theory explains why optimal conservation often fails and why conservation abroad is leased, while domestic conservation is bought.

JEL-Code: Q300, Q230, D780, D620, H870.

Keywords: conservation, deforestation, dynamic games, sales v rental markets.

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13.6.13
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1. Introduction

1.1. Conservation Goods

This paper introduces the notion of "conservation goods" and shows how they differ fundamentally from traditional goods in dynamic settings. Traditional goods are purchased by buyers who intend to consume the good: trade is typically predicted to take place immediately if the buyer's consumption value is larger than the seller's. For conservation goods, however, the buyer is satisfied with the status quo: he does not desire to consume the good, but only to prevent the seller from consuming it in the future.

Tropical forests are conservation goods - in the jargon of this paper. On the one hand, the South benefits from selling the timber and clearing the land for agriculture or oil extraction. On the other, the North prefers conservation in the South because the tropical forests are among the most biodiverse areas in the world, they are inhabited by indigenous people, and deforestation contributes to 11-17 percent of the world's carbon dioxide emissions, which cause global warming.¹

If the North's conservation value is larger than the South's value of logging, Coasian bargaining should ensure that the forest is preserved: the North should simply buy the forests from the South, or pay the current owners for conservation. The North has plenty of opportunities to do so, either individually or collectively through the World Bank or the United Nations. The recent emergence of REDD (Reducing Emissions from Deforestation and Forest Degradation) funds does provide such financial incentives to conserve.² However, REDD is a recent phenomenon and offered to a very limited extent. Given the Coasian view, it is puzzling why the North is not buying conservation from the South on a much larger scale.

Conservation goods are different from traditional goods, but they are not confined to rainforests. Engel et al. (2008) discuss many examples of payments for environmental/ecosystem services (PES). In the United States, The Nature Conservancy frequently uses land acquisition as a tool of its conservation effort. In fact, the conservation goods can be real captives or hostages,³ a piece of art, or historical ruins: as long as the good is conserved, the buyer may be in no hurry to pay. As an efficient climate policy, this author has argued elsewhere that a climate coalition could greatly benefit from purchasing and conserving fossil fuel deposits in nonparticipating countries (Harstad, 2012). The puzzle is, again, why such a market for conservation is not observed in reality.

In other situations, the market for conservation exists but the outcome is inefficient. To mention a recent example, villagers on the Solomon Islands had agreed with the Earth

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¹The estimates have varied within this interval since IPCC (2007). Negative externalities from forest loss and degradation cost between $2 trillion and $4.5 trillion a year according to The Economist (September 23, 2010, citing a UN-backed effort, The Economics of Ecosystems and Biodiversity, TEEB).

²There are several ways of defining REDD; see Karsenty (2008) on details or Parker et al. (2009) for a summary of the various proposals and the distinction between RED, REDD, and REDD+. The 2010 Cancun Agreements (UNFCCC, 2010) recognize the importance of reducing deforestation and forest degradation, but are quite imprecise regarding who should pay and how this should be implemented.

³The present model, predicting whether an exogenously given hostage will survive, contributes to the literature on hostage-taking (surveyed by Sandler and Arce, 2007). However, I ignore how the incentive to take hostages is affected by commitment (Selten, 1988), reputation, or uncertainty (Lapan and Sandler, 1988).
Island Institute to protect bottlenose dolphins in return for $2.4 million SBD (Solomon Island Dollars). After receiving little payment, however, the villagers retaliated by slaughtering as many as 900 dolphins.\(^4\) An ancient example is the nine books of Sibylline prophecy that were offered to the last King of Rome, Tarquinius Superbus. Books with prophecies were consulted in the stress of war, or in time of plague or famine, and the king was perhaps in no hurry to pay as long as these books would be available later. Consequently, the seller had to gradually burn six books before the king agreed to buy the remaining three.\(^5\)

This paper attempts to explain these puzzles and to investigate why and when the market for conservation is inefficient. In addition, I compare the sales market and the leasing market in order to detect the equilibrium and the best choice.

1.2. Overview and Outline

To formalize the market for conservation, I present a dynamic model with a seller (S), a buyer (B), and a good (e.g., the forest). In each period, B decides whether to buy. As long as B has not yet bought, S has the possibility of consuming - or "cutting." The game is a stopping game which ends after sale or consumption. The only novelty in the game is that B benefits if S conserves.

As in most games with an infinite time horizon, there are multiple subgame-perfect equilibria, and some of these are efficient. However, if we require the equilibrium to be either stationary, Markov-perfect, or renegotiation-proof, then the equilibrium is essentially unique. But there is no equilibrium in pure strategies. The buyer cannot buy with probability one, since S would then conserve and there would be no need to buy; a contradiction. The seller cannot cut with probability one, since B would then hurry to buy and S would conserve in the meantime; another contradiction. Instead, the unique equilibrium is in mixed strategies. B buys with a probability such that S is just indifferent when considering cutting, and S cuts with a probability such that B is indifferent when considering buying. S is more likely to cut if the conservation value is low and, perversely, B is more likely to buy if the value of cutting is high.

If S has the possibility of investing and increasing the conservation value, she would never make such an investment: even if the price would increase following a higher conservation value, S would not benefit since B would then be less likely to buy. On the other hand, S has a strong incentive to raise the value of cutting, since that makes B more likely to buy.

Since the sales market is inefficient, I also consider a rental market where B can temporarily pay S for conservation. For the same reasons as before, it cannot be an


\(^5\)According to the legend, the seller was a strange woman who appeared before the king. She asked for a steep price and the king declined. The woman asked again for the exact same price for six books after burning three of them. The king laughed at her, but after the woman burned another three books, he accepted the original price for the last three books (Ihne, 1871:74-75). I am thankful to Wiola Dziuda for suggesting this example.
equilibrium that B rents with a very high probability, since S would then always conserve, making it unnecessary for B to rent. The inefficiencies are thus similar to the sales market. By comparison, the model predicts the rental market to be both better and the equilibrium choice if and only if the conservation value is small relative to the consumption value, while B’s maintenance or protection cost is high relative to S’s protection cost. In other words, domestic conservation will be bought, while conservation across the border (which would require high protection costs) will be rented.

The main model is kept as simple as possible, but it can be extended a number of ways. The mixed-strategy equilibrium can be "purified" by relaxing the assumption that the good is indivisible, if just the equilibrium probabilities for cutting and buying are interpreted as the fractions that are cut and bought in each period. This way, the results describe the gradual decline of the forest. Other extensions are discussed in Section 4: While I first assume that there is a protection cost only for the buyer, the model can easily be reformulated to allow for such a cost for the seller as well. And while I first assume that time is discrete and a rental contract lasts exactly one period, the same results hold in a continuous-time model in which the rental agreement is of arbitrary length. The continuous-time model is particularly apt when allowing for multiple buyers. However, regardless of the number of buyers or sellers, the stable equilibria coincide with the one-buyer-one-seller situation investigated in Sections 2-3.

As far as this paper is concerned, it is irrelevant whether the price for conservation is exogenously given or an endogenous outcome of a bargaining game between the buyer and the seller. The intriguing results of the paper arise as long as (i) the price is larger than the seller’s benefit of cutting, and (ii) trading at this price is voluntary. If the price is endogenously given as the bargaining outcome between the parties, then assumption (i) can be satisfied or replaced by assuming the following game instead: First, B decides whether to contact or negotiate with S and, if B contacts S, then (i) both parties capture a positive fraction of the bargaining surplus, and (ii) if negotiations should fail, the default or threat point is that the seller cuts (in equilibrium, this is indeed a best response for S). If we relaxed assumption (ii), such that B could not hide from negotiating with S, then the parties would trivially come to an agreement immediately. Thus, assumptions (i)-(ii) are both strong and crucial.

The paper is organized as follows. After explaining my contribution to the literature, Section 2 presents and analyzes the sales market. Section 3 analyzes rental arrangements before the two markets are compared. Although the basic model is very simple, a large number of extensions are analyzed in Section 4: although the main results continue to hold, each extension brings new light to the market for conservation. The Appendix

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6If the threat point were instead that S would cut with the same probability as if B had never contacted S, then, under assumption (i'), one can show that the equilibrium conservation price would equal the consumption value and B would buy with probability one. The results are then trivial. But if we simultaneously relax assumption (i') by allowing S to make a take-it-or-leave-it offer to B, then there are multiple equilibria in mixed strategies: one for every price that is above the consumption value but below the conservation value. Otherwise, the results are identical to those below (for details, see the earlier working paper version, Harstad, 2011).

7In this case, S would have an incentive to announce a low price to attract B to the bargaining table; however, such an announcement would have no impact if S could renege once B were sitting at the bargaining table.
presents the proofs.

1.3. Contributions to the Literature

Earlier explanations of deforestation have pointed to corruption, electoral cycles, unclear property rights, multiple users and owners, multiple buyers, leakage, and the difficulties in monitoring and enforcing contracts.\(^8\) But even when we abstract from these obstacles, the current paper shows that inefficiencies continue to exist in the market for conservation, because they are fundamentally tied to the nature of the good.

The effects analyzed in this paper are not discussed elsewhere in the environmental economics literature, to the best of my knowledge. On the contrary, it is frequently argued that the expectation of a future environmental policy leads to less conservation (Kremer and Morcom, 2000) or more pollution today (the "green paradox"; Sinn 2008 and 2012). In this paper, however, the owner of a resource may prefer conservation if a future environmental policy is anticipated. The reason for the contrasting results is that in the present paper, the owner is (more than) fully compensated for conservation and thus benefits when the future policy arrives.

Coase (1960) argued that if property rights are well defined and there are no transaction costs, then the outcome is efficient and invariant to the initial allocation of rights. However, Coasian bargaining may break down if one assumes small transaction costs (Anderlini and Felli, 2006) or private information (Farrell, 1987). Dixit and Olson (2000) and Ellingsen and Paltseva (2012) have argued that when the agents are free to opt out of the negotiations, some of them may prefer to "stay home" if the others agents are, in any case, providing some (although inefficiently few) public goods. These assumptions are not necessary for the inefficiencies detected in this paper; instead, it is the nature of the good that leads to inefficiency, since the buyer prefers to buy later rather than sooner - as long as the seller does not consume the good in the meantime.

Theoretically, bargaining between a buyer and a seller tends to be efficient and without delay when information is complete.\(^9\) While the reasoning in this paper requires a dynamic framework, the model is different from both durable goods markets\(^10\) and classic war-of-

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\(^8\)See, for example, Alston and Andersson (2011) and Angelsen (2010), and the references therein. While many treat forests as a renewable resource (as Copeland and Taylor, 2009), I model the logging of (virgin) forests and their ecosystem values as being irreversible. For empirical studies of the determinants of deforestation, see Burgess et al. (2011), Damette and Delacote (2012), and, for an earlier overview, Angelsen and Kaimowitz (1999). Although there are often multiple users of the same forest, REDD-contracts may force them to act as one single seller (Phelps et al., 2010).

\(^9\)See the survey by Osborne and Rubinstein (1990). There are exceptions, however. Hendon and Tranæs (1991) study a setting with two heterogeneous buyers and show that there may be no stationary equilibrium in pure strategies: the seller must sometimes sell to the low-valuation buyer to raise the price paid by the high-valuation buyer. (Groes and Tranæs, 1999, find the same in a resale market.) These papers ignore mixed stationary strategies, matching is random and exogenous, and, most importantly, the good is consumed and not of the conservation-good type studied here.

\(^10\)As conjectured by Coase (1972) and shown by Bulow (1982), the seller of a durable good has an incentive to reduce the price over time. For this effect, it is essential that there is more than one buyer valuation. In this paper, there is only one buyer type and the price does not drop over time. In contrast to durable goods, a conservation good is something the buyer would prefer to buy later rather than sooner, as long as it continues to exist and the price remains the same. This preference is driving the inefficiency studied here.
attrition models.\textsuperscript{11} The closest theoretical literature is instead the relatively few papers on sales in the presence of externalities. To see this closeness, note that the game in this paper would remain essentially identical if, as an alternative to cutting the forest, the owner could sell the forest to a logger. Such a sale would then create a negative externality on the buyer interested in conservation.

Sales in the presence of externalities were first discussed by Katz and Shapiro (1986) and later analyzed by Jehiel et al. (1996), who let the seller commit to a sales mechanism. Jehiel and Moldovanu (1995a) allow for negotiations after the seller is randomly matched with one of several potential buyers. If the time horizon is finite, delay can occur if many periods remain before the deadline, whether the externality is positive or negative. With negative externalities, this delay is generated by a war-of-attrition game between potential "good" buyers who each hope the other good buyer(s) will purchase the good before the bad buyer does (causing negative externalities on the good ones). This story requires at least three buyers, in contrast to my model. Furthermore, trade will take place with certainty closer to the deadline.

If the buyers have bounded recall, Jehiel and Moldovanu (1995b) detect possible delay even when time is infinite. While that paper does not analyze equilibria in mixed strategies, it does detect conditions under which there is no equilibrium in pure stationary strategies.

However, if time is infinite and recall is not bounded, Björnerstedt and Westermark (2009) show that, in all these models, there cannot be any delay for sales under negative externalities when attention is restricted to stationary strategies. In other words, trade occurs as soon as the seller is matched with the "right" buyer.

This result is nonrobust, as the current paper shows. Formally, the main difference is that I endogenize matching between the buyer and the seller. Rather than imposing an exogenous matching, as in the literature just mentioned, I follow Diamond (1971) by letting the buyer choose whether to contact the seller. The nonrobustness is obviously a two-edged sword, implying that the delay emphasized in this paper would not survive if a buyer was always forced to meet with the seller.\textsuperscript{12}

\textsuperscript{11}War-of-attrition games were first studied by Maynard Smith (1974) in biological settings, but are often applied in economics. According to Tirole (1998:311), "the object of the fight is to induce the rival to give up. The winning animal keeps the prey; the winning firm obtains monopoly power. The loser is left wishing it had never entered the fight." Muthoo (1999:241) provides a similar definition. In this paper, in contrast, the buyer is perfectly happy with the status quo, and he does not hope that the seller will act. Once the buyer acts, he is also very happy that he did not give in earlier. It is only in the extension with multiple buyers that the game between the various buyers becomes somewhat similar to a war of attrition.

\textsuperscript{12}Jehiel and Moldovanu (1999) also endogenize matching when allowing for resale and externalities. When the seller can reach out to any (set of) buyer(s), they show that the identity of the final buyer is independent of the initial owner. This is in contrast to my paper, and the explanation is, once again, that in my model the buyer can decide whether to get in touch with the seller.

One generalization of the literature on sales is the set of papers studying contracting in the presence of externalities, where the principal can contract with multiple agents (Segal, 1999; Segal and Whinston, 2003; Genicot and Ray, 2006). A related (and overlapping) set of papers studies coalition formation when there are externalities; an interesting question is then whether the grand coalition will form in equilibrium when this would be socially efficient (see the survey by Ray, 2007, and the references therein). These generalizations, however, are somewhat orthogonal to this paper.
The choice between buying and renting is not analyzed by any of the papers mentioned above. The economics literature comparing sales and leasing focuses on rather different trade-offs. Bulow (1982) showed how leasing arrangements can solve the monopolist’s problem as illustrated by the Coase conjecture, since the produced quantity is then always returned to the seller and a later expansion of the quantity would thus reduce the price and harm the seller. Furthermore, if the seller possesses private information, then adverse selection, or the market of the lemons, can be avoided with leasing contracts (Hendel and Lizzeri, 2002; Johnson and Waldman, 2003). Johnson and Waldman (2010) compare this benefit with the moral hazard that occurs when the agent does not own the good himself (discussed by Henderson and Ioannides, 1983; Smith and Wakeman, 1985; Mann, 1992). If it is the buyer who possesses private information, then leasing today reveals a high willingness to pay and the price may thus increase tomorrow (Hart and Tirole, 1988). This "ratchet effect" creates inefficiencies that are avoided in the sales market. (For textbook discussions, see Tirole, 1998, or Bolton and Dewatripont, 2005.)

None of these benefits or drawbacks is present in this paper. Instead, my assumption that the initial owner may have a lower cost of protecting or maintaining the good is recognized by textbooks in finance: "under a full-service lease the lessor provides maintenance; in many cases he may be in a better position to provide such service" (Levy and Sarnat, 1994:662).

While the seller’s reduced maintenance cost suggests that a leasing arrangement may have an advantage, this paper uncovers a new drawback of leasing markets: since the market for conservation is not efficient, payoffs may be higher if the parties trade once and for all rather than continue forever in the inefficient market. Combined, this generates a new trade-off which suggests that goods with very high conservation values should be purchased, while leasing conservation is better for geographically remote areas, for which the buyer would face relatively high protection costs.

2. The Sales Market

2.1. Stage Game

There is one seller (S or "she"), one buyer (B or "he"), and one indivisible good initially owned by the seller. First, the buyer decides whether to buy the good. If he does, the game ends. If he does not buy, the seller decides whether to consume or conserve the good.

The game is quite standard, and its terminal payoffs are illustrated in Figure 2.1. If the buyer purchases the good, he enjoys the direct benefit $D$ although he must pay the price $P$, which in turn equals the seller’s payoff. For the results of this paper, it is irrelevant whether the price is "exogenously" given or an equilibrium outcome of the negotiations that may follow after B has contacted S (more on this below). If the buyer does not buy and the seller consumes the good, the seller enjoys the payoff $C > 0$. All parameters are common knowledge.

13 A few recent papers study how to design conservation agreement in the presence of moral hazard (Gjertsen et al., 2010), asymmetric information (Mason and Plantinga, 2013; Mason, 2013), and imperfect monitoring (Delacote and Simonet, 2013).
Figure 2.1: *If B does not buy, S decides whether to cut. The terminal nodes present the buyer’s payoff, then the seller’s payoff.*

The only novelty in this stage game is that the buyer enjoys some benefit from the existence of the good, whether or not it is purchased. This "existence" value, or perhaps "environmental" benefit, is represented by $E > 0$. Payoffs are normalized such that if B does not buy and S does not consume, both payoffs are zero. Thus, the existence value $E$ is experienced as a loss by B if and only if S consumes. A positive existence value is relevant for all examples mentioned in the Introduction. To fix ideas and point to an important example, I will refer to the good as a unit of forest and S’s consumption as cutting.

Quite generally, $D$ measures the difference in utility for B between owning and having the good conserved by S. For a traditional good, such as the purchase of a car, $D > 0$ and $E = 0$. But for a "pure" conservation good, where B enjoys the full value of the good whether B buys or S conserves, then $E > 0$ and $D = 0$. If there is a cost of maintaining or protecting the forest, then B may actually prefer that S own and conserve it: the maintenance cost can be measured by $-D$. Thus, $D < 0$ is quite reasonable when it comes to conservation goods. Section 3 allows B to pay S to conserve the good as a leasing arrangement (then, B saves the protection cost $-D$). The model can easily be reformulated to permit a maintenance cost also for the seller. In this section, $D$ plays a minor role and the reader is free to simplify by assuming that $D = 0$.

Note that the addition of $E$ does not alter the play of the static game: it is a unique best response for S to consume, given that she reaches her decision node, and (therefore) it is a unique best response for B to buy for every $P \in (C, D + E)$, which I assume for

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14 As explained in Section 4.2, $C$ can be interpreted as the sum of the value of cutting plus S’s present discounted cost of protecting the forest forever (a cost which is saved when S cuts), if just $-D$ measures B’s protection cost minus S’s protection cost. It has been documented that local authorities often have lower protection costs than those further away (Somanathan et al., 2009), implying $D < 0$ in this model.
now. To describe the equilibrium formally, let B’s strategy simply be the probability that he buys, $b \in [0, 1]$, while S’s strategy is the probability that she cuts, $c \in [0, 1]$, given that she reaches her decision node.

**Proposition 0.** *In the static version of the game, the unique subgame-perfect equilibrium is $b = c = 1$.\

I emphasize the case $P \in (C, D + E)$ to ensure that trade is mutually interesting and to avoid multiple best responses. The origin of the price $P$ is actually irrelevant for the results of this paper: it can be exogenously given or an endogenous outcome of a bargaining game between B and S. To see this, suppose B first decides whether to contact S. If he does, they negotiate the sales price $P$. When we let the bargaining outcome be characterized by the generalized (or "asymmetric") Nash bargaining solution and $\alpha \in (0, 1)$ is S’s relative bargaining power, while $1 - \alpha$ is B’s relative bargaining power, then the negotiated price is:  

$$P = (1 - \alpha) C + \alpha (D + E) \in (C, D + E).$$  

(2.1)

If the equilibrium price is instead coming from another process, one can let (2.1) define $\alpha$. Quite generally, $\alpha$ measures how close the price is to the buyer’s maximal willingness to pay rather than to the seller’s minimal requirement. In any case, we have the equivalence $\alpha \in (0, 1) \Leftrightarrow P \in (C, D + E)$.

Note that the motivation for trade depends on the parameters. If $P < D$, as for traditional goods, the buyer buys because the benefit $D$ is larger than the price. In contrast, if $D < P < D + E$, the buyer would prefer the status quo, and the purchase gives the buyer a negative payoff - but in equilibrium the buyer buys to prevent the seller from cutting. Thus, the good is a conservation good, or a "hostage": it is the threat of cutting which makes B pay. When the price is given by (2.1), the inequalities $D < P < D + E$ hold for every $\alpha \in (0, 1)$ if just $D < C < D + E$.

**Definition.** The good is a conservation good if $D < C < D + E$.

### 2.2. The Dynamic Sales Game and Equilibrium

Consider the stage game above and suppose that the game ends if the buyer buys or the seller cuts. If no such action is taken, the forest is conserved and the game continues to the next period with the identical stage game. The dynamic game is thus a quitting or stopping game which continues until one player deliberately stops.

As before, payoffs are normalized to be zero unless the game stops. Now, the existence value $E$, for example, should be interpreted as the present discounted cost of losing the forest’s conservation value forever. The common discount factor is $\delta \in (0, 1)$, so if the forest is cut at time $t$, the present-discounted value of this cost measured at time zero is

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15 In addition, all proofs in the Appendix allow for $P \in [C, D + E]$ (or $\alpha \in [0, 1]$, to use the notation described below). Other cases are trivial.

16 It is well-known that the generalized Nash bargaining solution "is identical to the (limiting) bargaining outcome that is generated by the basic alternating-offers model" (Muthoo, 1999, p. 52, drawing on Binmore, 1987). Any difference in discount rates would then influence the allocation of bargaining power.
\( \delta^t E \). Parameter \( C \) can be interpreted as the market value of the timber when the forest is cut, plus the present-discounted value of the agricultural crops that thereafter can be grown on the land. Parameter \( D \), or \(-D\), may be interpreted as the present discounted cost to B from protecting the forest forever after his purchase. The assumption that parameters are constant over time can be relaxed (see the next footnote).

A player’s strategy is a mapping from the set of histories to a probability for stopping the game when this player has the possibility to act. A player can act only if no player has already acted, so the set of relevant histories is simply summarized by time. There are many subgame-perfect equilibria in this game, and several refinements could be considered. It turns out that the set of renegotiation-proof equilibria, the set of Markov-perfect equilibria, and the set of stationary equilibria essentially coincide and permit a unique equilibrium. Section 4.3 explains this in detail; for now, I take the shortcut of simply restricting attention to stationary equilibria. After all, every pair of subgames starting at times \( t \) and \( t' \) are identical and there is no reason that the strategies should be contingent on the date. (If they were, the equilibrium would be neither Markov-perfect nor renegotiation-proof, as explained in Section 4.) With this refinement, an equilibrium is simply summarized by the stationary pair \((b, c)\).

If \( C > D + E \), no price can make trade mutually beneficial. If \( D > P \), as for traditional goods, it is easy to see that B prefers to buy immediately and with probability one, so \( b = 1 \) in equilibrium. Consider instead conservation goods satisfying \( D < C < D + E \), such that \( D < P \) for every \( \alpha \in (0, 1) \). For such conservation goods, the stopping game has a special feature: both players prefer that B, rather than S, stops the game. But while B most prefers the status quo, S ranks this option last. With these rankings of the outcomes, it turns out that the equilibrium must be in mixed strategies (and strictly so, if \( \delta \) is close to 1).

**Proposition 1.** The strategies \((b, c)\) constitute an equilibrium if and only if:

\[
\begin{align*}
(i) & \quad b = 1 \quad \text{and} \quad c = 1 \quad \text{if} \quad P \in \left( C, \frac{C}{\delta} \right); \\
(ii) & \quad b = 1 \quad \text{and} \quad c = \left[ \frac{(1-\delta)(P-D)}{E-\delta(P-D)}, 1 \right] \quad \text{if} \quad P = \frac{C}{\delta}; \\
(iii) & \quad b = \frac{C}{P-C} \left( \frac{1-\delta}{\delta} \right) \quad \text{and} \quad c = \left[ \frac{(1-\delta)(P-D)}{E-\delta(P-D)}, 1 \right] \quad \text{if} \quad P \in \left( \frac{C}{\delta}, D + E \right).
\end{align*}
\]

Figure 2.2 illustrates how \( b \) and \( c \) depend on the price of conservation. The proposition holds whether this price is exogenous or endogenous. In particular, we can assume that if B contacts S, then the parties negotiate the price. Under the (credible) threat that S will cut unless B buys, the generalized Nash bargaining solution predicts that the price is given by (2.1), above. A large \( P \) is thus driven by the seller’s large bargaining power index, \( \alpha \), or by the buyer’s high willingness to pay, \( D + E \).

**Corollary 1.** With the price given by (2.1), (2.2) becomes:

\[
\begin{align*}
(i) & \quad b = 1 \quad \text{and} \quad c = 1 \quad \text{if} \quad \alpha < \frac{1-\delta}{\delta} \frac{C}{D+E-C}; \\
(ii) & \quad b = 1 \quad \text{and} \quad c = \left[ \frac{(1-\delta)(\alpha+(1-\alpha)(C-D)/E)}{1-\delta(\alpha+(1-\alpha)(C-D)/E)}, 1 \right] \quad \text{if} \quad \alpha = \frac{1-\delta}{\delta} \frac{C}{D+E-C}; \\
(iii) & \quad b = \frac{C}{D+E-C} \left( \frac{1-\delta}{\alpha} \right) \quad \text{and} \quad c = \left[ \frac{(1-\delta)(\alpha+(1-\alpha)(C-D)/E)}{1-\delta(\alpha+(1-\alpha)(C-D)/E)}, 1 \right] \quad \text{if} \quad \alpha > \frac{1-\delta}{\delta} \frac{C}{D+E-C}.
\end{align*}
\]
Proposition 1 and its corollary have quite a few implications. Case (i) arises when \( P \) is small; this could be because \( \alpha \) or \( D + E \) is small relative to \( C \). Then, \( C > \delta P \) and \( S \) prefers cutting to selling in the next period, so \( B \) needs to buy with probability one to save the forest. However, if \( \alpha \) or \( D + E \) is larger, or if \( \delta \) approaches one, then we always have \( C < \delta P \) and thus case (iii). In this situation, there is no equilibrium in pure strategies: If \( B \) bought for sure, \( S \) would prefer to conserve rather than cut, implying that \( B \) would not need to buy - a contradiction. If \( B \) never bought, \( S \) would always cut, but then \( B \) would prefer to buy - another contradiction. The only equilibrium is in mixed strategies where \( B \) buys with a probability which is such that \( S \) is just indifferent between cutting and conserving, and \( S \) cuts with a probability which makes \( B \) just indifferent between buying and not buying.

This reasoning explains the comparative static illustrated by Figure 2.2. On the one hand, the buyer becomes less tempted to buy when \( P \) (or \( \alpha \)) is large, and \( S \) must thus cut with a higher probability to make \( B \) willing to buy. On the other hand, a high \( P \) (or \( \alpha \)) makes the seller more tempted to wait for a sale, and the buyer buys with a smaller chance in equilibrium, still without necessarily triggering cutting.

If the seller’s benefit of cutting, \( C \), is large, then she becomes tempted to cut and \( B \) must buy with a larger probability, according to the above results. It is thus more likely that the buyer will purchase a forest, with the intention of conserving it, if the forest is actually quite profitable to cut. If the conservation value \( E \) is large, \( B \) becomes more eager to buy and, for \( B \) to be still willing to wait, \( S \) must cut at a slower rate. Note that these claims are true whether \( P \) is constant or reacts to \( C \) and \( E \) as in (2.1), as confirmed by Proposition 1 and Corollary 1. It is also straightforward to let parameters vary over time.\(^{17}\)

\(^{17}\)If parameters \( C, D, E, \) and \( P \) varied over time, this should be reflected by using subscript \( t \) for period \( t \), for example. Then, \( S \) and \( B \) would be indifferent (and thus willing to randomize) at their decision nodes.
2.3. The Fate of the Forest

At every point in time, there are three possible outcomes. The forest may have been cut, it might have been bought, or the game is still in play. If the game has not already stopped, there is a chance that it will stop in the next period. Thus, the cumulated probability that the forest will have been cut before the end of period $T$ increases in $T$. Similarly, the cumulated probability that the forest will have been purchased before the end of period $T$ is also increasing in $T$. The sum of these cumulated probabilities approaches one as $T$ goes to infinity: eventually, the forest is either purchased or cut. These probabilities can be derived from Proposition 1 and they are illustrated in Figure 2.3.

Corollary 2. The conservation good exists with a probability that declines over time to
when, respectively:

$$b_{t+1} = \frac{C_t}{P_{t+1} - C_t} \left( \frac{1 - \delta}{\delta} \right)$$  \hspace{1cm}  

$$c_t = \frac{(1 - \delta)(P_t - D_t)}{E_t - \delta (P_t - D_t)}.$$  

The Nash bargaining solution for $P_t$ would be given by $P_t = (1 - \alpha) C_t + \alpha (D_t + E_t)$.

To see the following corollary, note that the probability that the forest has been bought before the end of period $T$ is:

$$b + (1 - b)(1 - c) b + \ldots [(1 - c)(1 - b)]^{T-1} b = \frac{b \left( 1 - \left[ (1 - b)(1 - c) \right]^T \right)}{1 - (1 - b)(1 - c)}.$$  

When we substitute for the equilibrium $b$ and $c$ and let $T \to \infty$, this becomes:

$$\frac{E}{(P - D) - \delta} \left[ \frac{E - \delta [(1 - \alpha)(C - D) + \alpha E]}{E + \delta \alpha (D + E - C) / C - 1} \right] \left[ (1 - \alpha)(C - D) + \alpha E \right].$$  

when we substitute for $P$. The probability decreases in $\delta$ because of B's vanishing first-mover advantage; it is thus more interesting to look at the limit when $\delta \to 1$, giving the corollary. Another way of seeing the corollary is by inspecting the equilibrium of the continuous-time model in Section 4.4.
the probability that it is eventually purchased. This probability is

\[
\frac{1}{1 + (1 - D/C) \alpha + (E/C) \alpha^2 / (1 - \alpha)}
\]

when \( \delta \to 1 \).

At the end of the day, it is more likely that the forest still exists if the good is close to being a traditional good - in the sense that the direct value \( D \) is large while the conservation value \( E \) is small (both relative to the consumption value). A large \( E \) means that the equilibrium price for conservation is high, which in turn implies both that the buyer is less likely to buy and that the seller is more likely to cut. The effect of the equilibrium price also explains why the good is less likely to be conserved if the seller’s bargaining power is high. If the seller has most of the bargaining power (\( \alpha \to 1 \)), the forest is eventually cut with probability one. If the seller’s bargaining power is arbitrarily small (\( \alpha \to 0 \)), however, the good is eventually bought and conserved with probability one.

2.4. Purification and Gradual Cutting

Many conservation goods, such as forests, are divisible. The owner may be able to cut any fraction she wishes, and the buyer can purchase less than the entire forest (although he may not want to, since conserving one part of the forest could lead to "leakage" and more deforestation in other parts).

If the good is divisible, then randomization is not necessary for the equilibrium described above. To see this, assume that \( C, D, E, \) and \( P \) are all measured per unit of the forest. These linearities require that both players be risk-neutral and that marginal benefits do not change as the forest shrinks. The linear payment is consistent with existing REDD contracts, which do specify payments that are linear in the deforestation reduction.\(^\text{19}\) When the good is divisible in this way, \( c \) can be interpreted as the fraction of the forest that is cut in each period or, more generally, the expected fraction that is cut. Likewise, \( b \) can be interpreted as the expected fraction that is purchased in each period.

**Corollary 3.** Suppose the good is divisible. The equilibria in Proposition 1 and Corollaries 1 and 2 survive if \( b \) and \( c \) are interpreted as the expected fractions that are bought and cut, respectively.

In fact, every Markov-perfect equilibrium must satisfy Proposition 1 and Corollary 1.\(^\text{20}\)

\(^{19}\)For example, Norway’s contract with Guyana specifies a payment of 5 USD per ton of CO2, multiplied with 100 ton CO2 per hectare of forest, in turn multiplied with an estimated (and partly negotiated) baseline level of deforestation minus the actual level of deforestation (http://www.regjeringen.no/en/dep/md).

\(^{20}\)To see this claim and Corollary 3, let \( x_t \) be the size of the forest at the start of period \( t \), while \( y_t \) is the size of the forest at the subsequent interim cutting stage. Given \( b \) and \( c \), we have \( E[y_{t+1}] = x_t (1 - b) \) and \( E[x_{t+1}] = y_t (1 - c) \). The buyer prefers buying now rather than in the next period if and only if:

\[
x_t (D - P) \geq x_t (-cE + (1 - c)(D - P)),
\]

while S prefers cutting to conserving if and only if:

\[
y_t C \geq \delta y_t P + \delta (1 - b) y_t C.
\]

Note that \( x_t \) and \( y_t \) drop out, leaving the proof of Proposition 1 unchanged.
Thus, when combined with Corollary 3, Corollary 2 and Figure 2.3 describe the gradual shrinkage of the forest rather than the probability that it still exists. The remaining size of the forest is predicted to approach a fraction of the original size. This fraction decreases in the seller’s bargaining power and the continuation value $E$ but increases in the direct value $D$ (both relative to the consumption value $C$), as discussed above.

### 2.5. Payoffs and Investments

At the start of the game, the equilibrium payoff for the buyer, $V_B$, is pinned down by the fact that buying is always a best response. Thus, $V_B = D - P$. For a traditional good such that $P < D$, or if $P < C/\delta$, we have learned that the buyer purchases with probability one, so that the seller’s equilibrium payoff at the start of the game is $V_S = P$. The sum of these payoffs is simply $D$, locally independent of $P$ or $\alpha$.

The more interesting situation with conservation goods arises when $C/\delta \leq P$. In equilibrium, the seller is indifferent between cutting and waiting for her discounted equilibrium payoff, $\delta V_S$. Thus,

\[
V_B = D - P = -(1 - \alpha)(C - D) - \alpha E; \quad \text{(2.3)}
\]

\[
V_S = \frac{C}{\delta}. \quad \text{(2.4)}
\]

Interestingly, the sum of payoffs declines in the equilibrium price - or in the seller’s bargaining power, $\alpha$. This is in contrast to the case for the traditional good. For conservation goods, a low price implies that the probability (or the level) of trade is larger, and so is efficiency.

Given these equilibrium payoffs, we can easily study the players’ incentives to influence any of the parameters in the model - if they could. Although such influence is not formally in the model, it follows straightforwardly that, for example, S has no incentive to raise B’s value of conservation at the beginning of the game. For a given $P$, this would make it more attractive for B to contact S unless, as will happen in equilibrium, S is less likely to cut. Thus, S’s payoff stays unchanged. Even when $P$ increases following a larger $E$ (as in (2.1)), S would not benefit since B buys less if $P$ is high (Proposition 1). A higher price is always associated with a corresponding decrease in $b$, which ensures that S’s payoff is not altered.

Also this result is in stark contrast to the traditional case: For traditional goods (when $P < D$, or if $P < C/\delta$), the buyer buys with probability one and S does have an incentive to raise both $D$ and $E$ since S can then expect a higher price (2.1). If $\alpha \in (0, 1)$, the buyer has some of the bargaining power and the price does not rise one-to-one when S enhances.

---

When the good is divisible, we may have other MPEs as well, if strategies can be conditioned on the fraction consumed so far. However, note that the amount of the good that is left is not "payoff-relevant" because $x_t$ and $y_t$ drop out when we compared payoffs above. Using the reasoning from Maskin and Tirole (2001), one may thus argue that the fraction that is so far cut is not payoff-relevant and that the MPEs should not be conditioned on it. With such reasoning, the Markov-perfect $b$ and $c$ are unique for $P \in (C, C/\delta) \cup (C/\delta, D + E)$, and they are in line with Proposition 1.

When the price is endogenous, then Corollary 1 holds if (i) the buyer first specifies the fraction of the forest he is thinking of buying before he negotiates the price of this fraction with the seller, and (ii) the threat point is that if this bargaining fails, S will immediately cut this part of the forest.
the value of the good: this is the familiar hold-up problem, which leads to investments which are suboptimally low but still positive. In contrast, for conservation goods (when \( P > \max \{ D, C/\delta \} \)), S has absolutely no incentive to raise B’s valuation, no matter how the bargaining power is allocated.

But consider instead the seller’s incentive to improve her outside option, i.e., the value of the timber or the land, \( C \). In reality, S can raise \( C \) by investing in roads and logging capacity or by negotiating market access with trading partners. From (2.4), we know that \( \partial V_S/\partial C = 1/\delta > 1 \). Thus, S’s incentive to raise the value of cutting, \( C \), is larger than it would have been if conservation had not been an issue (then, \( \partial V_S/\partial C = 1 \)). If B might buy, S has a smaller chance of being able to enjoy the value of cutting, \( C \). This effect ought to reduce S’s optimal investment in \( C \), particularly when \( P \) is given. However, if \( C \) increases marginally, B must buy with a larger probability. This effect is very beneficial for S and strongly motivates S to raise \( C \).

**Corollary 4.** Suppose \( C/\delta \leq P \). The seller has no incentive to increase the value of conservation, \( E \), but strong incentives to raise the value of cutting, \( C \).

The equilibrium payoffs (2.3) also allow us to discuss the buyer’s incentives. A boycott, for example, that reduces \( C \), may not necessarily benefit B. In fact, in isolation (for a fixed \( P \)), a lower \( C \) reduces S’s payoffs while B’s payoff is unaltered: the small \( C \) makes cutting less tempting for the seller, and the buyer can therefore buy with a smaller probability. The buyer can benefit from a reduced \( C \) only if that will reduce the negotiated price (2.1). But even in this situation, B’s increased benefit is \( \partial V_B/\partial (-C) = 1 - \alpha < 1 \), smaller than S’s loss, \( 1/\delta > 1 \). Hence, a boycott may improve B’s bargaining position when B purchases conservation, but B will subsequently buy with a smaller probability in equilibrium and the sum of payoffs declines.\(^{21}\)

3. The Rental Market

3.1. Stage Game

In a rental or leasing market for conservation, the buyer pays the current owner to conserve the good for a given length of time. There are several reasons to analyze this market: (i) the sales market, analyzed above, proved to be very inefficient; (ii) existing REDD-contracts specify payments conditional on annual avoided deforestation,\(^{22}\) and they are thus more similar to leasing arrangements than to sales; (iii) it may be very costly for a buyer to protect a forest that is geographically far away from illegal logging; and (iv) a buyer may fear that the seller will attempt to renationalize the forest after a sale.

\(^{21}\)The model can easily be reformulated to let S also enjoy some conservation value. If parameter \( E_S \) represents the seller’s present discounted value of conservation, she will enjoy this value unless the good is cut. As long as \( E_S < C \), we have \( b > 0 \) and the seller’s equilibrium payoff is \( E_S + (C - E_S)/\delta \), which is decreasing in \( E_S \). Intuitively, if \( E_S \) were increased, S would be less willing to cut and, to make her indifferent, B must be less likely to buy. This decrease in \( b \) harms S. Thus, even if S could invest in eco-tourism, for example, she would have no incentive to do this.

\(^{22}\)Norway’s REDD+ agreements with Brazil, Guyana, Indonesia, Tanzania and Mexico can be found online: http://www.regjeringen.no/en/dep/md/. The United Nations’ REDD program is described here: http://www.un-redd.org/.
Arguments (iii) and (iv) suggest that buying may be a costly conservation method. This cost can be captured by assuming that $D < 0$ in B’s payoff following a purchase. In fact, the best interpretation of $-D$ is that it represents the present-discounted cost for B when protecting the forest forever against illegal logging or S’s attempt to renationalize it. Naturally, this cost is not paid when B leases the forest, since S is then interested herself in protecting the forest so that it can also be rented in the future. The model can easily be reformulated to capture a protection cost for the seller as well (as explained in Section 4.2).

Figure 3.1 illustrates that the stage game is otherwise quite similar to the one for the sales market. At the beginning of a period, B first decides whether to contact S, anticipating that he then will be able to rent conservation at the (equilibrium or exogenous) price $p$. If B rents, the rental price $p$ is this period’s payoff to S. If B does not rent, S decides whether to cut or conserve. If S conserves, payoffs are zero in this period. Cutting ends the game and gives the payoff $C$ to S and the payoff $-E$ to B, just as before. The good is a conservation good if $C < E$.

The one-period version of the rental game is thus very similar to the one-period version of the sales game, and the two are identical if $D = 0$. As before, let $b \in [0, 1]$ denote the probability that B rents, while $c \in [0, 1]$ is the probability that S cuts at her decision node. For every price strictly between $C$ and $E$, the unique subgame-perfect equilibrium is $b = c = 1$. The sum of equilibrium payoffs is strictly higher in the rental market than in the sales market if $D < 0$, but strictly lower if $D > 0$.

### 3.2. The Dynamic Rental Game and Equilibrium

As justified above, one difference assumed here is that the buyer does not enjoy $D$ in the rental market. A second assumed difference is that a rental contract is temporary and
future contracts cannot be negotiated in advance. While Section 4.4 permits rental contracts of any length, this section simplifies by considering only one-period rental contracts. So, if B rents, S agrees to conserve and to protect the good for one period but thereafter, the players enter the next period with the identical stage game. Thus, only cutting ends the game. If one or both of these two assumptions are relaxed, the comparison between the two markets becomes trivial, as discussed at the end of this section.

Just as before, there are multiple subgame-perfect equilibria. The set of renegotiation-proof equilibria, the set of Markov-perfect equilibria, and the set of stationary equilibria essentially coincide and permit a unique equilibrium. Section 4.3 explains this in detail; this section is again taking the shortcut of simply focusing on the stationary equilibrium. After all, the subgames at the beginning of different periods are always identical as long as the forest still exists, and the date as well as the history are "payoff-irrelevant" (Section 4.3). The equilibrium strategies can thus again be summarized by the stationary pair \((b, c)\).

As mentioned, the rental price can, for present purposes, be exogenously given or endogenously determined as described in Section 4.1. In either case, the interesting and reasonable case is where the present-discounted cost of renting forever, \(p / (1 - \delta)\), lies between \(C\) and \(E\). If \(p / (1 - \delta) \leq C\), it is a best response for S to cut and not accept a leasing contract. If \(p / (1 - \delta) \geq E\), it is a best response for B to not rent.

Proposition 2. The strategies \((b, c)\) constitute an equilibrium if and only if:

\[
\begin{align*}
\text{(i)} & \quad b = 1 \quad \text{and} \quad c = 1 \quad \text{if} \quad \frac{p}{1 - \delta} \in (C, \frac{C}{\delta}) ; \\
\text{(ii)} & \quad b = 1 \quad \text{and} \quad c \in \left[ \frac{p(1 - \delta)}{E(1 - \delta) - \delta p}, 1 \right] \quad \text{if} \quad \frac{p}{1 - \delta} = \frac{C}{\delta} ; \\
\text{(iii)} & \quad b = \frac{C}{p} (\frac{1 - \delta}{\delta}) \quad \text{and} \quad c = \frac{p(1 - \delta)}{E(1 - \delta) - \delta p} \quad \text{if} \quad \frac{p}{1 - \delta} \in \left( \frac{C}{\delta}, E \right) .
\end{align*}
\]

The equilibrium is analogous to that of the sales market. If \(p < C (1 - \delta) / \delta\), the seller prefers cutting to renting forever after, and the buyer must and will rent immediately and always with probability one. In contrast, if \(p > C (1 - \delta) / \delta\), the seller prefers conservation if she expects that the buyer will rent; but if S conserves, B does not want to rent. So, if \(p > C (1 - \delta) / \delta\), the unique equilibrium is in mixed strategies. In each period, the buyer rents with a probability which is such that S is just indifferent between cutting or not, and S cuts at a rate which makes B just willing to rent in a given period.

The comparative static is similar to the sales market. For example, if the price for conservation is high, B is willing to rent only if S cuts at a faster rate (so \(c\) increases in \(p\)), while S is willing to cut only if B rents with a lower probability (so \(b\) declines in \(p\)).

\[23\]Both these differences between leasing and buying are well recognized and standard to assume; see the textbook by Levy and Sarnat (1994:657-76). The temporal aspect of the leasing contract is also reflected in the discussion of Tirole (1998:81-4).

\[24\]Note that there is no need to assume that S can precommit when promising to conserve the forest in cases (ii) and (iii). The seller is indifferent when considering protection of the good at the cutting stage, whether or not B has rented the good for this period. Thus, S does not need to commit and B’s payment does not need to be conditioned on actual conservation. However, if S were assumed to protect the good with the same probability whether or not the good would be rented, then there would be no value in a rental arrangement and B would never rent.
If \( b \) and \( c \) were drawn as functions of \( p \), the figure would be quite similar to Figure 2.2, above.

Corollaries 1-4 extend to the rental market. Consider first the incentives (Corollary 4): If \( S \) could invest in order to enhance \( B \)'s conservation value, \( E \), she would not lift a finger (in cases (ii)-(iii)). The seller’s equilibrium continuation value is given by \( C/\delta \), which is independent of \( E \) and even \( p \). This also implies, as in the sales market, that \( S \) has a stronger incentive to increase the value of cutting, \( C \), than she would have had if conservation were not an issue.

Corollary 3 also continues to hold. Just as for the sales market, \( b \) can be interpreted as the (expected) fraction of the remaining forest that is rented in a given period, while \( c \) can be interpreted as the (expected) fraction of the remaining forest, not already under contract, that is cut at the cutting stage. This way, the strategies above can be purified.

Corollary 2 is somewhat different for the rental market. As long as the forest stands, there is always a chance that \( B \) does not rent and that \( S \) cuts in case (iii). To illustrate this, the cumulated probability that the forest is cut, as a function of time, is illustrated in Figure 3.2: compare this to the analogous Figure 2.3 for the sales market. When the forest is divisible and can be gradually cut, the curve can be interpreted as the expected size of the remaining forest as a function of time.

Section 4.1 endogenizes the rental price and explains how Corollary 1 extends to the rental market.

### 3.3. Lease or Buy Conservation?

Despite the similarities, the rental market and the sales market are not equivalent. On the one hand, the rental market has the disadvantage that the game never ends before \( S \) cuts: this is illustrated by the difference between Figures 2.3 and 3.2. On the other, in the rental market, the seller is protecting the good and not the buyer. Thus, if \( D < 0 \), renting every period would be a first-best outcome, while a sale would generate inefficiency. In fact, a sales market exists (for some \( P \)) only if \( E > C + (-D) \), while the rental market
exists (for some $p$) whenever $E > C$.

The rental price can be exogenously given or an endogenous outcome of a bargaining game, as discussed earlier. To make positive predictions regarding the choice between sales and rental contracts, consider the sales market and the equilibrium sales price $P$, given by (2.1). If the buyer has contacted the seller, the two of them may consider a one-period rental contract as an alternative to a sale. It is reasonable that they will sign a rental contract if and only if there exists some rental price such that both $B$ and $S$ are better off by signing such a contract rather than signing the sales contract. The rental contract lasts one period only, and in the following period, either (i) $B$ may consider renting again at some (equilibrium) price, or (ii) $B$ and $S$ may revert to the equilibrium in the sales market. Regardless of whether the future is expected to be (i) or (ii), the following proposition describes the condition for when a rental contract exists which Pareto-dominates selling at the negotiation stage.

**Proposition 3.** Consider the equilibrium in the sales market. Once $B$ has contacted $S$, there exists a rental contract which both $B$ and $S$ strictly prefer to selling if and only if both $C < E$ and:

\[
-D > \max \left\{ 0, \frac{\delta P - C}{1 - \delta} \right\} \Leftrightarrow \quad (3.2)
\]

\[
-D > \max \left\{ 0, \frac{\delta \alpha E}{1 - \delta + \delta \alpha} - C \right\}. \quad (3.3)
\]

While a sale would eliminate the possibility of future cutting, renting has an advantage if it is costly for the buyer to protect the forest after a purchase, i.e., if $-D > 0$ is
sufficiently large. The model therefore predicts that conservation of areas far away will be leased, while local conservation, perhaps within the buyer’s own country, will be bought.

A sale is also more attractive if the seller’s benefit from cutting, $C$, is small, since $B$ is then less likely to show up (and rent) again, as explained above, which makes it beneficial to trade now to end the inefficient game.

Furthermore, note that a sale is more likely if the equilibrium sale price is large. If the seller’s bargaining power $\alpha$ is large, then $P$ is also large and $S$ will require a high rental price to be willing to lease rather than sell. But with such high conservation prices, the equilibrium rate of cutting is large. Rather than risk future cutting in the leasing market, $S$ and $B$ are better off trading once and for all. For the same reason, if the conservation value $E$ is large (so that $P$ would be large), we should expect to see sales rather than leasing arrangements.

Figure 3.3 illustrates the parameter-set under which ”rent” is preferred by $B$ and $S$. The rental market does not exist below the horizontal line in blue (there, $E < C$). If $E < C - D$, i.e., below the downward-sloping straight line, the sales market does not exist. Condition (3.3) is illustrated as the kinked downward-sloping line (in red). Thus, in the area ”rent” (or ”buy”), both markets exist but the rental (or sales) market is strictly better. In the area ”RENT” (or ”BUY”), only the rental (sales) market exists. 25

As a concluding remark, let us reconsider the two assumptions mentioned at the start of Section 3.2. If we relax the second assumption by allowing $B$ and $S$ to sign a rental agreement that lasts forever, then the only difference is that $B$ enjoys $D$ following a purchase but not when he is renting; as a result, buying is always preferred when $D > 0$ but never when $D < 0$. If we relax the first assumption instead such that $D$ (or the per-period equivalent $D(1 - \delta)$) is also enjoyed (or suffered) in a renting agreement, then renting will never take place, since a sale more effectively eliminates the future probability for an inefficient cutting. If both assumptions are relaxed, the two markets are equivalent.

4. Extensions and Robustness

The model above is deliberately kept as simple as possible. As a result, it can easily be extended in a number of directions. This section briefly describes some extensions which show that, although the results above continue to hold, each extension sheds new light on the market for conservation. The subsections do not build on each other, so each extension can be read isolated or separately.

4.1. Endogenous Rental Price

The results above hold whether the price for conservation is exogenous or endogenous. The sale price has already been endogenized by (2.1). Now, suppose that if $B$ contacts $S$, then the rental price is negotiated. Let the generalized Nash bargaining solution characterize

25Note that as $\delta \to 1$, the area where renting is preferable converges to the area ”RENT” where the sales market is nonexistent. The explanation is that as each period becomes very short, the cost of risking cutting in each of these periods becomes arbitrarily large. This suggests that one-period contracts are unreasonable when $\delta \to 1$, which motivates the extension to longer leasing arrangements, discussed below.
the outcome, where $\alpha \in [0,1]$ is, as before, the seller’s bargaining power index (i.e., the fraction of the bargaining surplus captured by the seller). There are at least two reasonable threat points: (a) as in the sales market, we may presume that if the parties cannot agree on a cooperative arrangement, then $S$ cuts. Alternatively, (b) if the parties cannot agree on a rental price, $B$ may instead buy at price (2.1). The equilibrium rental price is the same for both threat points, it turns out.\footnote{The intuition for this equivalence is that the surplus from selling, relative to cutting, is distributed by the same sharing rule ($\alpha$) as when sharing the surplus from renting.}

**Proposition 4.** Let the generalized Nash bargaining solution characterize the rental price where the threat point is that either (a) $S$ cuts or (b) $B$ buys at price (2.1). In either case,

$$\frac{p}{1-\delta} = \max \left\{ \frac{(1-\alpha)C + \alpha E}{\delta} \frac{\alpha}{\delta} \frac{E}{\delta} \right\}$$

$$= \left\{ \begin{array}{ll}
(1-\alpha)C + \alpha E & \text{if } \alpha < \left(\frac{1-\delta}{\delta}\right) \frac{C}{E-C} \\
\frac{\alpha}{1-\delta} \frac{E}{\delta} & \text{if } \alpha \geq \left(\frac{1-\delta}{\delta}\right) \frac{C}{E-C}
\end{array} \right\}.$$  

(4.1)

Just as in the sales market, the price for conservation in the rental market increases in $\alpha$ and $E$.

We can rewrite the first line in (4.1), case (i), as $C/\delta > p/(1-\delta)$, which implies that $c = b = 1$ according to Proposition 2. $B$ will then rent in every period and the present discounted value of these rents will be identical to (2.1) when $D = 0$. When $\delta$ is sufficiently close to 1, however, we enter the second line, case (ii).

In case (ii), the inequality can be rewritten as $C/\delta \leq p/(1-\delta)$, which implies that the seller is always indifferent at the cutting stage and her continuation value is the same ($C/\delta$), whether or not she promises to conserve in this period. Thus, her bargaining surplus is simply $p$, independent of $C$. This explains why $C$ does not influence $p$ in this case. A somewhat larger $C$ is reflected in the larger $b$, but not in a larger price.

By combining Propositions 2 and 4, we get the following corollary (analogous to Corollary 1 for the sales market):

**Corollary 5.** With the rental price given by (6.2), (3.1) becomes:

\begin{align*}
\text{(i)} & \quad b = 1 \quad \text{and} \quad c = 1 \quad \text{if} \quad \alpha < \left(\frac{1-\delta}{\delta}\right) \frac{C}{E-C}; \\
\text{(ii)} & \quad b = 1 \quad \text{and} \quad c \in [\alpha,1] \quad \text{if} \quad \alpha = \left(\frac{1-\delta}{\delta}\right) \frac{C}{E-C}; \\
\text{(iii)} & \quad b = \frac{C}{E} \left(1 + \frac{1/\delta - 1}{\alpha}\right) \quad \text{and} \quad c = \alpha \quad \text{if} \quad \alpha > \left(\frac{1-\delta}{\delta}\right) \frac{C}{E-C}.
\end{align*}

The comparative static for $b$ is just as in the sales market and Corollary 1: $b$ increases in $C$ but decreases in $E$, $\delta$, and $\alpha$. And, just as before, the larger the seller’s bargaining power $\alpha$, the faster the rate of cutting and the lower the probability for cooperation between the buyer and the seller. While $c$ increases in $\alpha$, as before, $c$ does not depend on any other parameter in the rental market. If $E$ were large, for example, the buyer would be more tempted to buy for a fixed $p$; this would require a smaller $c$, in line with (3.1). But when $p$ is linear in $E$, the buyer does not become more tempted to buy when $E$ is large, so that to keep $B$ indifferent, $c$ must stay invariant to $E$.\footnote{The intuition for this equivalence is that the surplus from selling, relative to cutting, is distributed by the same sharing rule ($\alpha$) as when sharing the surplus from renting.}
4.2. Protection Costs for Seller and Buyer

It is straightforward to introduce a maintenance or protection cost also for the seller. In fact, the analysis does not need to be modified at all, since $C$ can be interpreted as the value of cutting plus the seller’s saved protection cost, if just $D$ is interpreted as the difference between the players’ protection costs.

To be precise, let $G_B$ be B’s present discounted cost of forever maintaining or protecting the forest (against illegal logging, for example), while $G_S$ is the analogous cost for S. Denote by $\underline{C}$ the market value of the timber and/or the present discounted value of forever using the land for agriculture. When the status-quo payoffs are normalized to zero, the seller gains $\underline{C} + G_S$ at the moment she stops the game.

**Proposition 5.** Let $G_S$ be the protection cost for the seller and $G_B$ the protection cost for the buyer, while $\underline{C}$ is the seller’s value of cutting (in addition to the protection cost avoided). All the above results continue to hold with the interpretations:

\[
C \equiv \underline{C} + G_S \quad (4.2)
\]
\[
D \equiv G_S - G_B \quad (4.3)
\]

Note that it follows that $D < C$ (when $\underline{C} > 0$ and $G_B > 0$), as was frequently assumed above.

To see the proposition, normalize payoffs to zero in the status quo and let $P$ be the actual sales price. Relative to the status quo, S enjoys $G_S + \underline{C}$ when cutting and $P + G_S$ when selling, while B gets $-P - G_B$ when buying. These payoffs coincide with those illustrated in Figure 2.1 when we complement (4.2)-(4.3) with

\[
P \equiv P + G_S.
\]

When the price $P$ is negotiated, S requires at least $C$ (since she receives $G_S$ whether she sells or cuts) while B is willing to pay at most $E - G_B$. When we let the generalized Nash bargaining solution characterize the sales price $P$, where $\alpha$ is S’s share of the bargaining surplus, it follows that:

\[
P = (1 - \alpha) \underline{C} + \alpha (E - G_B) \Rightarrow
P = P + G_S = (1 - \alpha) \underline{C} + \alpha (E + D),
\]

exactly as before. Thus, for the sales market, all results continue to hold if $C$ is interpreted as the sum of $\underline{C}$ and $G_S$, while $D$ is interpreted as the difference in protection costs. If $G_S < G_B$, then $D < 0$.\(^{27}\)

The rental market is also unchanged. In particular, S gets $C \equiv \underline{C} + G_S$ when cutting and only $p$, say, when B rents for just one period. If $\underline{p}$ is the outcome of the generalized Nash bargaining solution, as discussed above, then one can show that $\underline{p} = p$, where $p$ is described by (6.2). In sum, the comparison between renting and selling is the same as in Proposition 3, although parameters $C$ and $D$ can, as explained, be interpreted more broadly.

\(^{27}\)Somanathan et al. (2009) document that local authorities manage forests at lower costs, and achieve more preservation, than do central authorities in the central Himalayas of India.
4.3. Equilibria, Refinements, and Renegotiation

This subsection is intended to describe more precisely the set of strategies, equilibria, and the reasons why the equilibria emphasized above are relevant. In short, I show that as long as we require the equilibria to be renegotiation-proof or, alternatively, Markov-perfect, then they must take the form described by the propositions above.

Consider first the sales market. In every period $t$, B first chooses an action $a_t^B \in \{0, 1\}$ and, if $a_t^B = 0$, then S chooses an action $a_t^S \in \{0, 1\}$. A history at the start of period $t$ is a sequence of actions $\{a_1^B, a_1^S, ..., a_{t-1}^B\}$ or $\{a_1^B, a_1^S, ..., a_{t-1}^B, a_t^S\}$. For each player, a strategy specifies a randomization over possible actions after each possible history. This game is a quitting game with alternating moves which stops as soon as one player has chosen action 1. Thus, an action is possible at time $t$ only after a history of zeros, so a player’s strategy at $t$ is necessarily conditioned on the unique history in which no player has stopped. Therefore, the strategies at the beginning of period $t$ is simply a pair $s_t = (b^t, c^t)$ where $b^t = (b_t, b_{t+1}, ...)$ and $c^t = (c_t, c_{t+1}, ...)$; each $b_\tau \in [0, 1]$ denotes B’s probability for choosing $a_\tau^B = 1$ while each $c_\tau \in [0, 1]$ denotes S’s probability for choosing $a_\tau^S = 1$ for every integer $\tau \geq t$. Similarly, $s^t_+ = (c^t, b^{t+1})$ is the strategies at the cutting stage in period $t$. The strategy pair $s^t$ is a Nash equilibrium if $b^t$ is a best response to $c^t$ and $c^t$ is a best response to $b^t$. Furthermore, $s^t$ is subgame-perfect if $s^\tau$ and $s^\tau_+$ are Nash equilibria for every $\tau \geq t$.

There are many subgame-perfect equilibria in this game. Some of them are in pure strategies and some always lead to conservation. As before, the price $P$ can be exogenous or endogenous. If $P < D$, then B’s unique best response would be $b_t = 1$. If $P < C/\delta$, then $c_t = 1$ and thus $b_t = 1$ are strictly best responses for every $t$, and thus a unique sub-game perfect equilibrium. We will therefore consider again the more interesting case where $P > C/\delta$ and $P > D$.

Example. If $P > C/\delta$, we can always find an integer $\Delta > 1$ such that:

$$\delta^{\Delta} P \leq C < \delta^{\Delta-1} P.$$ 

The first inequality means that S prefers cutting today to selling in $\Delta$ periods, while the second inequality means that S prefers selling in $\Delta - 1$ periods to cutting. The following is thus a subgame-perfect equilibrium in pure strategies: $b_t = c_t = 1$ when $t \in \{1, 1 + \Delta, 1 + 2\Delta, ..\}$, while $b_t = c_t = 0$ otherwise. The outcome is that B buys immediately, since S will otherwise cut immediately. If S deviates, B will not buy in the following period since B knows that it is thereafter in the interest of S to wait for trade in $\Delta - 1$ periods.

Unfortunately, the strategies in the Example are neither stationary nor Markov-perfect. Markov-perfect equilibria require that strategies be contingent only on "payoff-relevant" partitions of histories (Maskin and Tirole, 2001). Here, the only payoff-relevant aspect to any history is whether or not the game has terminated. The time itself is not payoff-relevant: if one player’s strategy is not contingent on time, then the other player cannot benefit from such contingency either. It follows that in the game of this paper,
the Markov-perfect equilibria must be in stationary strategies. Clearly, the strategies in the above Example are not stationary, so they cannot be Markov-perfect.\textsuperscript{30}

The equilibrium in the Example is not renegotiation-proof, either. At the interim stage in period 1 (i.e., S is ready to cut because B did not buy), then both players prefer to renegotiate or redefine the time (to period 2, for example). If both players can strictly benefit by simply coordinating on another pair of equilibrium strategies, then the initial equilibrium is clearly vulnerable. Thus, one may want to restrict attention to equilibria which are robust to such simple re-coordination - particularly when one is interested in the most efficient equilibria.

The rest of this subsection shows that by adhering to standard notions of renegotiation-proofness,\textsuperscript{31} the equilibrium must converge to those described by Proposition 1 (for the sales market) and Proposition 2 (for the rental market) after at most two periods. In other words, if one insists on renegotiation-proof equilibria and believes that the game started more than two periods ago, then the equilibrium must be unique and exactly as described in Sections 2-3.

As noted, there are two different types of subgames, depending on whether B or S is the next player to act. Two subgames are called identical if the next player to act is the same in both subgames.

**Definition.** A subgame-perfect equilibrium \((s^t \text{ or } s^t_+)\) is *weakly renegotiation-proof* if the continuation payoff profiles at any pair of identical subgames are not strictly ranked.

In other words, there is no time at which both players would strictly benefit from following the strategies specified for a different time (where the identity of the next mover is preserved). Let \(S^w\) denote the set of weakly renegotiation-proof equilibria where B is the next player to act. Note that \(S^w\) must be independent of time. The set \(S^w_+\) is defined analogously.

**Definition.** A subgame-perfect equilibrium \(s^t \in S^w\) or \(s^t_+ \in S^w_+\) is *strongly renegotiation proof* if no continuation payoff profile is strictly Pareto-dominated by the continuation payoff profile of another \(s^t' \in S^w\) or \(s^t_+ \in S^w_+\).

Let \(S^a \subseteq S^w\) and \(S^a_+ \subseteq S^w_+\) denote the set of strongly renegotiation-proof equilibria. It turns out that every equilibrium which is (weakly or strongly) renegotiation-proof coincides with the one described by Proposition 1, at least after the first two periods.

\textsuperscript{30}If \(b\) and \(c\) are interpreted as the fraction that is bought and cut, then the size of the remaining forest is a stock on which strategies could be conditioned on. However, if one player’s strategy is not contingent on the level of the remaining stock, then the other player cannot benefit from such a contingency, either. In this way, the size of the forest is not payoff-relevant, and the strategies in a Markov-perfect equilibrium should not be contingent on it (again following the logic of Maskin and Tirole, 2001).

\textsuperscript{31}There are several (different) definitions of renegotiation-proofness. Here I adopt the definitions for infinitely repeated games by Farrell and Maskin (1989), also presented in the textbook by Mailath and Samuelson (2006:134-8). Alternative but related definitions are provided by Pearce (1987), Bernheim and Ray (1989), Asheim (1991), and Abreu, Pearce, and Stachetti (1993). For definitions of renegotiation-proofness for finitely repeated games, see Benoit and Krishna (1993), Bernheim, Peleg and Whinston (1987), or Bergin and MacLeod (1993).
The proof in the Appendix explains and fully characterizes the set of all renegotiation-proof equilibria.

**Proposition 6.**
(i) Consider the sales market and suppose that \( P > C/\delta \Leftrightarrow \alpha > (1/\delta - 1) C/ (D + E - C) \). We then have \( S^w = S^s \) and \( S^s_+ = S^s_+ \). Furthermore, if \( s \in S^w = S^s \) or \( s_+ \in S^w_+ = S^s_+ \), then
\[
 b_t = b \quad \text{and} \quad c_t = c \quad \forall t > 2,
\]
where \( b \) and \( c \) are as described by Proposition 1.
(ii) Consider the rental market and suppose that \( p/ (1 - \delta) > C/\delta \Leftrightarrow \alpha > (1/\delta - 1) C/ (E - C) \). We then have \( S^w = S^s \) and \( S^s_+ = S^s_+ \). Furthermore, if \( s \in S^w = S^s \) or \( s_+ \in S^w_+ = S^s_+ \), then
\[
 b_t = b \quad \text{and} \quad c_t = c \quad \forall t > 2,
\]
where \( b \) and \( c \) are as described by Proposition 2.

Part (ii) says that the reasoning above also holds for the rental market. The main difference is that, in the rental market, the set of relevant histories at the start of period \( t \) must satisfy \( a^S_\tau = 0, \tau < t \), but not necessarily \( a^B_\tau = 0 \). Thus, the strategies can be contingent on whether (and when) \( B \) has rented in the past. Such contingencies, however, do not have much bite when the players can renegotiate: there can still be no pair of histories such that both players strictly benefit from following the strategies described for another possible subgame.

4.4. Multiperiod Leasing Contracts

This subsection permits rental contracts of any length. Let \( T \) be the (possibly infinite) upper boundary on the length of rental contracts. It is easy to show that, once \( B \) has contacted \( S \), they strictly prefer a rental contract of length \( T \) to shorter contracts of length \( t < T \). That is, \( B \) may pay \( S \) to conserve the forest for \( T \) periods in return for some rental price, \( p_T \).

With this extension, Proposition 2 and 3 can be generalized to the following.

**Proposition 7.** (i) For every \( p_T / (1 - \delta^T) \in [C, E] \), the equilibrium in the rental market must satisfy:
\[
 b = c = 1 \quad \text{if} \quad \frac{p_T}{1 - \delta^T} < \frac{C}{\delta};
\]
\[
 b = 1, c \in \left[ \frac{1 - \delta}{E (1 - \delta^T) / p_T - \delta}, 1 \right] \quad \text{if} \quad \frac{p_T}{1 - \delta^T} = \frac{C}{\delta};
\]
\[
 b = \frac{1 - \delta}{\delta p_T / C + \delta^T - \delta} \quad \text{and} \quad c = \frac{1 - \delta}{E (1 - \delta^T) / p_T - \delta} \quad \text{if} \quad \frac{p_T}{1 - \delta^T} \in \left( \frac{C}{\delta}, E \right);
\]
\[
 b \in \left[ 0, \frac{1 - \delta}{\delta p_T / C + \delta^T - \delta} \right] = C \quad \text{and} \quad c = 1 \quad \text{if} \quad \frac{p_T}{1 - \delta^T} = E.
\]
(ii) There exists a rental price $p_T$ which is strictly preferred to sale by both $B$ and $S$ if and only if:

\[-D > \max \left\{ 0, \frac{\alpha}{\delta^{-T} - 1 + \alpha} E - \frac{\delta^{-1} - 1 + \alpha}{\delta^{-T} - 1 + \alpha} C \right\} \tag{4.5}\]

The intuition is as before, but the effects of $T$ is new. A longer rental agreement implies that the inefficiency of renting, relative to buying, declines. As $T \to \infty$, (4.5) holds whenever $D < 0$. If $T = 1$, Propositions 2 and 3 follow as special cases.

We can also use this model to introduce continuous time. Suppose each "period" is of length $\Delta$ and $r$ is the discount rate, so $\delta = e^{-r\Delta}$ and $\delta^{T(\Delta)} = e^{-rT(\Delta)}$, where $T(\Delta)$ is the length of the rental contract in real time. If this length is kept fixed while $\Delta \to 0$ (i.e., if the sequence of moves become arbitrarily frequent although the length of the rental contract stays constant), condition (4.5) becomes

\[-\frac{D}{E - C} > \frac{\alpha}{e^{rT} - 1 + \alpha}.\]

This expression is confirmed in the continuous-time model in the next subsection. That subsection is also endogenizing the rental price for an arbitrary duration of the rental agreement.

4.5. Continuous Time

In this subsection, the results of Propositions 1-3 are restated for the case with continuous time. The common discount rate is $r$, while $\tilde{b}$ and $\tilde{c}$ denote the Poisson rates at which $B$ contacts $S$ and $S$ cuts, respectively, if the game has not yet ended. If the rental contract can be of any length up to $T$, where $T$ is an exogenous upper threshold (perhaps limited by political economy forces), then the equilibrium contract will always be of length $T$.

**Proposition 8.** Suppose time is continuous and a rental contract can be of length $T$.

(i) In the sales market, the unique equilibrium is in mixed strategies:

\[
\tilde{b} = r \frac{C}{P - C} = r \frac{P - D}{E - P + D} \tag{4.6}
\]

since the equilibrium sales price is, as before, $P = (1 - \alpha)C + \alpha(D + E)$.

(ii) In the rental market, the unique equilibrium is in mixed strategies:

\[
\tilde{b} = r \frac{C}{P - C(1 - e^{-rT})} = r \frac{E}{(E - C)(1 - e^{-rT})k} \quad \text{and} \quad \tilde{c} = r \frac{E}{(E - C)(1 - k)} \tag{4.7}
\]

since the equilibrium rental price (analogous to Proposition 4) is:

\[
\frac{p}{1 - e^{-rT}} = (1 - k)C + kE, \quad \text{where} \quad k \equiv \frac{\alpha}{1 - e^{-rT} + e^{-rT}\alpha}. \tag{4.8}
\]
(iii) Once $B$ has contacted $S$, there exists a rental contract which both $B$ and $S$ strictly prefer to selling if and only if:

$$\frac{-D}{E-C} > \frac{\alpha}{\alpha + e^{rT} - 1}.$$  \hspace{1cm} (4.9)

Part (i) is similar to Proposition 1, and in fact identical when the discount rate is $\delta = e^{-\Delta}$, $\Delta$ is the length of a period, $\tilde{b} = b/\Delta$, $\tilde{c} = c/\Delta$, and one takes the limit as $\Delta \to 0$. At every point in time, if the sales market has ended, then the good is conserved (and purchased) with probability

$$\frac{\tilde{b}}{\tilde{b} + \tilde{c}} = \frac{(1 - \alpha) C/\alpha}{\alpha E + (1 - \alpha) (C - D) + (1 - \alpha) C/\alpha},$$

which we can rewrite to confirm Corollary 2.

Part (ii) of Proposition 8 is identical to Proposition 2 if $T = \Delta$ and $\Delta \to 0$. Part (iii) is similar to Proposition 3, but the effect of $T$ is new. Remember that the disadvantage with a rental contract is that the players continue to randomize as soon as one rental contract has expired. If $B$ and $S$ can commit to a longer rental contract, then this disadvantage is somewhat mitigated, and a rental contract becomes more attractive compared to a sales contract. Thus, if $T$ is sufficiently large, (4.9) always holds unless $D \geq 0$.

Parts (i) and (iii) hold whether the rental price is exogenous or endogenous. If we let the rental price be characterized by the generalized Nash bargaining solution in which the threat point is either that (a) $S$ cuts or (b) $B$ buys at price $P$, then the rental price must be given by (4.8). Thus, the present-discounted cost of renting always is a weighted average between $C$ and $E$, intuitively increasing in the seller’s bargaining power, $\alpha$.

4.6. Multiple Buyers

The continuous time model can easily allow multiple buyers. To simplify, suppose there are $n$ identical potential buyers (heterogeneity is permitted below). Thus, every $i \in N \equiv \{1, ..., n\}$ receives the payoff $-E$ when $S$ cuts, the payoff $D - P$ if $i$ buys, and zero if $j \in N \setminus i$ buys or if $S$ conserves. In the rental market, the payoffs are analogous. As before, let $\tilde{b}$ represent the Poisson rate at which $S$ is contacted by some buyer. Thus, in a symmetric equilibrium, every $i$ contacts $S$ at the rate $\tilde{b}_i$ which satisfies

$$(1 - \tilde{b})^n \leftrightarrow \tilde{b}_i = 1 - \left(1 - \tilde{b}\right)^{1/n}.$$  \hspace{1cm} (4.10)

Perhaps surprisingly, most of the results continue to hold:

**Proposition 9.** Suppose there are $n$ identical potential buyers. In the symmetric equilibrium, Proposition 8 continues to hold with the following modifications:

(i) Cutting increases in $n$ in the sales market:

$$\tilde{c} = r \frac{1 + (1 - 1/n) C/(P - C)}{E/(P - D) - 1}.$$
(ii) Cutting also increases in \( n \) in the rental market:

\[
\tilde{c} = \frac{r + (1 - 1/n) \left( 1 - e^{-rt} \right) b}{E \left( 1 - e^{-rt} \right) / p - 1}.
\]

In comparison to Proposition 8, the result is disappointing. If more countries benefit from conservation, a planner would be more eager to conserve the forest but the outcome is the reverse. The rate at which some buyer (or a renter) contacts \( S \) is unchanged if \( n \) grows, but \( S \) cuts at a faster rate!

The intuition is the following. When \( n \) is large, every buyer \( i \) benefits since other buyers may contact \( S \) and pay for conservation. This reduces \( i \)'s willingness to contact \( S \) and, for \( i \) to still be willing to pay, \( S \) must cut at a faster rate.\(^{32}\)

Nevertheless, the similarities to the one-buyer case may be more surprising than the differences. First, \( \tilde{b} \) is independent of \( n \), given the price. The reason is that \( S \) is willing to randomize only if the rate at which some buyer will buy or rent, multiplied by the price, makes \( S \) indifferent. Second, in equilibrium, every buyer receives the payoff pinned down by the payoff he would receive if he were to contact \( S \) immediately and in isolation. Thus, in equilibrium the buyers do not gain from the presence of other buyers: the benefit that the other countries may pay for conservation cancels out with the cost of the faster cutting rate. For related reasons, the buy-versus-rental decision is also independent of \( n \): in both markets, the payoffs to \( i \in N \) as well as to \( S \) are unaffected by \( n \).

So far, this subsection has described the symmetric equilibrium in which all buyers might buy with the same chance. This equilibrium is not particularly stable, however: If the \( \tilde{b}_i \) increased marginally, the best response for \( j \neq i \) would be \( \tilde{b}_j = 0 \). This, in turn, would motivate \( i \) to raise \( \tilde{b}_i \) to \( \tilde{b} \) as described by Proposition 8. This iterative process of best replies can never converge to the equilibrium with multiple active buyers.

Suppose buyers are heterogenous, such that \( D_i \) and \( E_i \) are depending on the identity of the buyer \( i \in N \). If \( i \in N \) is the unique active buyer, the cutting rate is given by Proposition 8 where \( D \) and \( E \) should be replaced by \( D_i \) and \( E_i \). This implies that the cutting rate must depend on \( i \), so we should write it as \( \tilde{c}_i \). For another buyer \( j \in N \) to be willing to abstain from contacting the seller, it must be that \( j \) tolerates a higher rate of cutting than the active buyer \( i \) does, so \( \tilde{c}_j \geq \tilde{c}_i \). If also the prices are endogenous, as described above, then we arrive at the following result.

**Proposition 10.** Suppose buyers have heterogenous \( D_i < C \) and \( E_i > C - D_i \). (i) The single active buyer in the sales market has a high \( D_i \) and \( E_i \) and satisfies:

\[
i = \arg \min_{j \in N} \frac{E_j}{D_j + E_j - C}.
\]

\(^{32}\)The outcome is still worse if the aggregate conservation value \( E \) is held constant while \( n \) increases (i.e., if the buyers go from acting collectively to acting independently). Then, \( E = E/n \) and, for a given \( P \) or \( p \), \( S \) cuts even faster when \( n \) grows, since \( E \) also decreases. (However, when the equilibrium price decreases in \( E \), this effect is somewhat but not fully mitigated.) In this situation renting would be more likely as \( n \) grows, since Proposition 8(iii) states that renting is more likely when the buyer’s value is low.

Note that there is an analogy to the "Kitty Genovese" game (Osborne, 2003, Ch. 4.8), where the likelihood that someone calls the policy is decreasing in the number of observers to the crime. In that game, however, there is no player similar to the seller in the current game, and thus nothing reflects the increased cutting.
(ii) The single active renter in the rental market has the highest possible $E_i$, so $i = \arg\max_{j \in N} E_j$.

4.7. Multiple Sellers

Multiple sellers can also be introduced into the model. There are several ways of doing this. This subsection presents two variations which ensure that the results above continue to hold. Other variants are mentioned at the end.

(a) Independence. Suppose there are multiple sellers, $j \in M \equiv \{1, \ldots, m\}$, who each own a conservation good characterized by $C_j$, $D_j$, and $E_j$. Following the reasoning above (Section 2.2), assume that a buyer’s loss when multiple forests are cut is linearly separable. Thus, if a buyer purchases every forest in the set $M_B$ while every forest in the set $M_C$ is cut, where $M_B \cup M_C = M$, then the buyer’s payoff would be:

$$\sum_{j \in M_B} (D_j - P_j) - \sum_{j \in M_C} E_j.$$ 

In this situation, whether forest $j \in M$ is purchased, conserved, or cut does not influence any player’s payoff when he is considering risking the purchase, conservation, or cutting of another forest. It follows that the game between seller $j \in M$ and the buyer(s) is strategically independent of the outcome or the play with another seller. Thus, each of the games (one for each seller) can be analyzed in isolation, exactly as is done above.

(b) Satiation. An alternative assumption is to assume that a buyer experiences the same loss $E$ if and only if all forests are cut. The conservation of one of them suffices, so two or more forests will never be purchased. In this case, it is intuitive that as soon a buyer has contacted one of the sellers, the other sellers will find it optimal to cut immediately. The price which thereafter is negotiated between the contacted seller and the contacting buyer (or renter) is thus the same as described above.\footnote{If instead cutting takes some time, such that a buyer can contact a second seller after he fails to agree with the first, then the threat point is more favorable to the buyer and the Nash bargaining outcome predicts a lower price: $P = (1 - \alpha^2) C + \alpha^2 (D + E)$.}

Given this game, it is easy to verify that there cannot be any symmetric equilibrium in which either all sellers cut at the same time (since the buyer would then strictly benefit from immediately contacting one of the sellers), or in which multiple sellers $j \in M$ cut at interior rates $\tilde{c}_j \in (0, \infty)$, while the buyer will contact each of the sellers with the Poisson rate

$$b = \frac{r}{\alpha^2 (D + E - C)}.$$ 

Thus, small modifications in the multiple-seller model can have substantial consequences for the results, suggesting that further research is warranted.

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since the buyer(s) will then strictly benefit from not contacting the sellers until at most one forest remains. Hence, at most one seller can cut at an interior rate.

In sum, in every stationary equilibrium, all but one seller cut immediately, and the subgame between the remaining seller and the buyer(s) is exactly as described above. If the sellers are heterogeneous in that $C_j$ and $D_j$ are forest-specific, then a buyer is willing to defer buying one of the other forests only if that would have led to lower payoffs. When the prices are endogenous, as described above, it follows that the one remaining seller in the sales market must have the lowest possible $C_j - D_j$, since that generates the lowest price for conservation and thus the highest payoff to the buyer. In the rental market, the single remaining seller must have the lowest possible $C_j$, for the analogous reason.

**Proposition 11.** Suppose that there are multiple sellers and conservation goods.

(i) With Independence, Propositions 1-10 above describe the equilibrium play between the buyer(s) and each of the sellers.

(ii) With Satiation, in every stationary equilibrium all but one of the sellers cut immediately and the subsequent play between the remaining seller and the buyer(s) is described by Propositions 1-10. The identity of the single remaining seller must satisfy

$$j \in \arg \min_{l \in M} C_l - D_l$$

in the sales market, and

$$j \in \arg \min_{l \in M} C_l$$

in the rental market.

In the case of Independence, if there are multiple homogeneous buyers, then there can be different active buyers for each of the sellers. For each seller, stability requires that there is at most one active buyer, just as before. With heterogeneity, then for each forest $l \in M$, the single active buyer $i \in N$ is a buyer satisfying Proposition 10 where parameters $C$, $D_j$, and $E_j$ ought to be replaced by the seller-specific parameters $C^l$, $D_j^l$, and $E_j^l$. With this, Proposition 10 characterizes the matching between the set of sellers and the set of buyers.

**Corollary 6.** With Independence and heterogeneous buyers and sellers, for each seller $l \in M$, the single active buyer $i \in N$ satisfies

$$i = \arg \min_{j \in N} \frac{E_j^l}{D_j^l + E_j^l - C^l}$$

in the sales market, and $i = \arg \max_{j \in N} E_j^l$ in the rental market.

For example, if the only heterogeneity is in regard to the conservation values $E_j^l$, then Proposition 10 states that for every forest $l$, the single active buyer (or renter) is the buyer with the highest conservation value of this particular forest. If the only heterogeneity is in regard to the $-D_j$'s, instead, then for every forest, the active buyer is the one who can protect the forest at the lowest cost.
The analysis should not end here. Rather than the two extreme assumptions of Independence and Satiation, it would be both more reasonable and more general to let the buyers face conservation values that are arbitrary nonlinear functions of the stocks of conserved forests. This situation could create interesting strategic games between the different sellers as well as between the buyers - raising a host of new issues that deserve to be investigated in future research.

4.8. Other extensions

A large number of additional extensions are analyzed in the earlier working paper version (Harstad, 2011): It presents the equilibrium with multiple heterogenous active buyers; discusses "privatization" of the conservation good; and studies whether multiple buyers would benefit more from collective action rather than simple coordination. That version of the paper also allow for meeting costs, upper limits on the probabilities $b$ and $c$, and stochastic parameters as a way to purify the strategies. Most importantly, the earlier working paper presumes that after $B$ has agreed to negotiate, then $S$ makes a take-it-or-leave-it offer regarding the price. If $B$ rejects the offer, then $S$ cuts with the same probability as if $B$ had not contacted $S$ in the first place. This modification of the game implies that there are multiple equilibria: For every $P \in (C, D + E)$ in the sales market, there is an equilibrium where $S$ asks for the price $P$ and where the probabilities for buying and cutting (the $b$ and the $c$ associated with this $P$) is exactly as given by Proposition 1, above. In the rental market, an analogous claim holds. Thus, this complementary model confirms that it is irrelevant where the conservation price is actually coming from, as long as it might be larger than seller’s value of cutting.

5. Conclusions

This paper introduces the notion of conservation goods and shows that they are quite different from traditional goods. A buyer is satisfied with the status quo and is willing to buy only if the seller is likely to end conservation; but the seller conserves if she believes the buyer will pay. Reasonable equilibria are in mixed strategies, implying that conservation ends with a positive probability - or gradually at a positive rate.

The analysis uncovers a new trade-off between buying and leasing. On the one hand, rental markets are also characterized by mixed-strategy equilibria, so conservation inevitably ends, sooner or later, as long as the good is not purchased. This suggests that conservation is a more likely outcome in a sales market than in a rental market. On the other hand, renting implies that the seller has an incentive to protect or maintain the good, and this might be less expensive than if the buyer (which might be a foreign country) protects or maintains the good. By comparison, the results predict that domestic conservation will be bought, while conservation in other countries (where protection would be expensive) will be rented. This seems consistent with anecdotal evidence: REDD contracts are rental arrangements; national parks are not.

The model above is rather simple, but it has proven flexible enough to be extended in a number of directions. Nonetheless, many questions remain open. To isolate the key feature of conservation goods, I have abstracted from uncertainty, private information,
reputation-building, learning, moral hazard, and more complicated utility functions or bargaining procedures. These aspects should be examined in future research so that we can better understand the important and puzzling nature of conservation markets.

6. Appendix: Proofs

Proof of Proposition 1. At the start of each period, the continuation values for B and S depends on the equilibrium strategies:

\[
V_B(b, c) = b(D - P) + (1 - b) [-cE + (1 - c) \delta V_B(b, c)];
\]
\[
V_S(b, c) = bP + (1 - b) [cC + (1 - c) \delta V_S(b, c)].
\]

The game is a quitting game and B’s decision is whether to stop and get the payoff \(D - P\) or to continue. If continuation is a best response, it remains a best response in the subsequent periods and B’s payoff is:

\[
V_B(0, c) = -\frac{cE}{1 - \delta (1 - c)} \geq V_B(1, c) = D - P \text{ if } c \leq \frac{P - D}{E - \delta (P - D)} (1 - \delta).
\]

It follows that B’s best response is

\[
b = 0 \text{ if } c < \frac{P - D}{E - \delta (P - D)} (1 - \delta),
\]

\[
b \in [0, 1] \text{ if } c = \frac{P - D}{E - \delta (P - D)} (1 - \delta), \quad (6.1)
\]

\[
b = 1 \text{ if } c > \frac{P - D}{E - \delta (P - D)} (1 - \delta),
\]

where \((1 - \delta)(P - D) / [E - \delta (P - D)] \in (0, 1)\) if \(P > D\). If \(P < D\), as for traditional goods, then \(b = 1\) for every \(c \in [0, 1]\). If \(P = D\), the unique best response is \(b = 1\) if \(c > 0\), while any \(b \in [0, 1]\) is a best response if \(c = 0\).

S’s decision at the cutting stage is whether to cut to get \(C\) or to continue. If continuation is a best response, it remains a best response in the following periods and S’s payoff is:

\[
\delta V_S(b, 0) = \frac{\delta bP}{1 - \delta (1 - b)} \geq V_S(b, 1) = C \text{ if } b \geq \frac{C}{P - C} \frac{1 - \delta}{\delta}.
\]

It follows that S’s best response is

\[
c = 0 \text{ if } b > \frac{C}{P - C} \frac{1 - \delta}{\delta},
\]

\[
c \in [0, 1] \text{ if } b = \frac{C}{P - C} \frac{1 - \delta}{\delta},
\]

\[
c = 1 \text{ if } b < \frac{C}{P - C} \frac{1 - \delta}{\delta}.
\]
where \( C (1 - \delta) / \delta (P - C) \in (0, 1) \) if \( P > C / \delta \). So, if \( P > C / \delta \) and \( P > D \) (case (iii)), then the unique equilibrium is in mixed strategies. If \( P < C / \delta \) (case (i)), then \( c = 1 \) is the unique best response for every \( b \in [0, 1] \). If \( P = C / \delta \) (case (ii)), then \( c = 1 \) is the unique best response if \( b > 0 \), while any \( c \in [0, 1] \) is a best response if \( b = 1 \). Combined with (6.1), the equilibrium must satisfy:

\[
\begin{align*}
(a) & \quad b = \frac{C}{P-C} \left( \frac{1-\delta}{\delta} \right) \quad \text{and} \quad c = \left( \frac{1-\delta(P-D)}{E-\delta(P-D)} \right) \quad \text{if} \quad P \in \left( \max \{ C, D \}, D + E \right); \\
(b) & \quad b = 1 \quad \text{and} \quad c = 1 \quad \text{if} \quad P \in \left( \frac{C}{\delta}, D \right); \\
(c) & \quad b \in \left[ \frac{C}{P-C} \left( \frac{1-\delta}{\delta} \right), 1 \right] \quad \text{and} \quad c = 0 \quad \text{if} \quad P = D > \frac{C}{\delta}; \\
(d) & \quad b = 1 \quad \text{and} \quad c \in \left[ \frac{1-\delta(P-D)}{E-\delta(P-D)}, 1 \right] \quad \text{if} \quad P = \frac{C}{\delta}; \\
(e) & \quad b \in \left[ 0, \frac{C}{P-C} \left( \frac{1-\delta}{\delta} \right) \right] \quad \text{and} \quad c = 1 \quad \text{if} \quad P = D + E.
\end{align*}
\]

When attention is limited to \( P \in \left( \max \{ C, D \}, D + E \right) \), Proposition 1 follows. \( \text{QED} \)

**Proof of Proposition 2.** The proof is analogous to the proof of Proposition 1, and it follows as a special case from the proof of Proposition 7. The proof is thus omitted here, but available upon request. \( \text{QED} \)

**Proof of Proposition 3.** First, note that if \( E < (\leq) C \), there is no rental price that makes both players (strictly) better off than cutting, so the rental market does not exist. Thus, assume \( E > C \).

In the sales market, \( V_B = D - P \). In the rental market, \( V_B = -p / (1 - \delta) \) since renting forever in every period is a best response. If B expects to revert to the sales market in the following period, then \( V_B = -p + \delta (D - P) \). In both cases, B is indifferent between renting and buying if:

\[
P = \frac{P}{1-\delta} + D \iff p = (1 - \delta) (P - D). \tag{6.2}
\]

Once B has contacted S to buy, S expects the payoff \( P \). If S accepts a one-period rental agreement instead, her payoff is \( p + \delta V_S \). If the following periods are characterized by the sales market, then, if \( D < P \), \( V_S = \min \{ P, C/\delta \} \). If \( D > P \), then \( V_S = P \). So, there exists a \( p \) which makes both B and S strictly prefer a rental contract in the present period if and only if:

\[
\begin{align*}
(i) & \quad D > \frac{\delta P}{1-\delta} - \frac{C}{1-\delta} \quad \text{if} \quad P > C/\delta \quad \text{and} \quad P > D; \\
(ii) & \quad D > 0 \quad \text{if} \quad P \leq C/\delta \quad \text{or} \quad P < D.
\end{align*}
\]

If \( D > P \), both conditions fail and renting is never better than buying. If \( D = P \), then \( V_S = P \) if \( P \leq C/\delta \), while \( V_S \in [C/\delta, P] \) if \( C/\delta < P \); but regardless of \( V_S \in [C/\delta, P] \), there is no \( p \) such that both players can strictly gain by renting. When we combine (i), (ii) and (2.1), we can conclude that renting is better if and only if (3.2) holds.

If B and S instead anticipate that the following periods will also be characterized by the rental market with some equilibrium price \( \bar{p} \), then \( V_S = \min \{ \bar{p} / (1 - \delta), C/\delta \} \). In this
situation, there exists a \( p \) which makes both \( B \) and \( S \) strictly prefer renting today rather than trading at price \( P \) if and only if:

\[
(1 - \delta) (P - D) + \delta V_S > P \Rightarrow
\]

\[
-D > \frac{\delta P}{1 - \delta} - \frac{C}{1 - \delta} \quad \text{if} \quad \bar{p} / (1 - \delta) > C / \delta;
\]

\[
-D > \frac{\delta P}{1 - \delta} - \frac{\delta \bar{p}}{(1 - \delta)^2} \quad \text{if} \quad \bar{p} / (1 - \delta) \leq C / \delta \Rightarrow
\]

\[
-D > \frac{\delta P}{1 - \delta} - \frac{\delta}{1 - \delta} \min \left\{ \frac{\bar{p}}{1 - \delta}, \frac{C}{\delta} \right\}. \quad (6.3)
\]

These conditions are more likely to hold for some \( \bar{p} \) if this \( \bar{p} \) is large, but the largest \( \bar{p} \) which \( B \) is willing to accept is given above by (6.2). Thus, condition (6.4) becomes:

\[
-D > \max \left\{ \frac{\delta P}{1 - \delta} - \frac{\delta (P - D)}{1 - \delta}, \frac{\delta P - C}{1 - \delta} \right\} \Rightarrow (3.2).
\]

When we substitute for the price from (2.1) and rearrange, we get (3.3). QED

**Proof of Proposition 4.** If \( B \) rents at \( p \) this period, anticipating the equilibrium rental price \( \bar{p} \) in the following periods, then \( B \)'s payoff is:

\[
-p - \frac{\delta \bar{p}}{1 - \delta},
\]

while, from Proposition 2, \( S \)'s payoff is

\[
p + \delta V_S = p + \min \left\{ C, \frac{\delta \bar{p}}{1 - \delta} \right\}.
\]

(a) Consider the threat point "cut," where \( S \) immediately cuts if the negotiations on the rental price fail. Relative to this, \( B \)'s surplus when renting at \( p \) is

\[
\Delta_{rent}^B = E - p - \frac{\delta \bar{p}}{1 - \delta},
\]

while \( S \)'s bargaining surplus is:

\[
\Delta_{rent}^S = p + \min \left\{ C, \frac{\delta \bar{p}}{1 - \delta} \right\} - C.
\]

The generalized Nash bargaining solution requires that

\[
\Delta_{rent}^S = \alpha \left( \Delta_{rent}^S + \Delta_{rent}^B \right)
\]

\[
= \alpha \left( \min \left\{ C, \frac{\delta \bar{p}}{1 - \delta} \right\} - \frac{\delta \bar{p}}{1 - \delta} + E - C \right) \Rightarrow
\]

\[
p = (1 - \alpha) C + \alpha E - (1 - \alpha) \min \left\{ C, \frac{\delta \bar{p}}{1 - \delta} \right\} - \alpha \frac{\delta \bar{p}}{1 - \delta}. \quad (6.5)
\]
In equilibrium, \( p = \bar{p} \). If \( C \leq \frac{\delta p}{1-\delta} \), (6.5) becomes:

\[
p = (1 - \alpha) C + \alpha E - (1 - \alpha) C - \alpha \frac{\delta p}{1-\delta} \Rightarrow \frac{p}{1-\delta} = \frac{\alpha}{1-\delta + \delta \alpha} E.
\]

The condition \( C \leq \frac{\delta p}{1-\delta} \) does indeed hold if:

\[
C \leq \frac{\delta \alpha}{1-\delta + \delta \alpha} E \Leftrightarrow \alpha \geq \frac{C}{E-C} \frac{1-\delta}{\delta}.
\]

If instead \( C > \frac{\delta p}{1-\delta} \), then (6.5) becomes (when \( p = \bar{p} \)):

\[
p = (1 - \alpha) C + \alpha E - (1 - \alpha) \frac{\delta p}{1-\delta} - \alpha \frac{\delta p}{1-\delta} \Rightarrow \frac{p}{1-\delta} = (1 - \alpha) C + \alpha E.
\]

The condition \( C > \frac{\delta p}{1-\delta} \) does indeed hold if

\[
C > \delta (1 - \alpha) C + \delta \alpha E \Rightarrow C > \frac{\delta \alpha}{1-\delta + \delta \alpha} E. \quad (6.6)
\]

So, if \( C \) is larger than this threshold, then \( S \) would rather cut than wait for a future rental agreement, and \( B \) needs to rent in every period with certainty. Bargaining failure leads to cutting so \( p \) will reflect the level of both \( C \) and \( E \). However, as \( \delta \to E \), (6.6) requires that \( C > E \), which is never satisfied for conservation goods in the rental market. In summary,

\[
\frac{p}{1-\delta} = \max \left\{ (1 - \alpha) C + \alpha E, \frac{\alpha}{1-\delta + \delta \alpha} E \right\} \Leftrightarrow (6.2).
\]

(b) Consider the threat point "sale," where \( B \) buys at \( P \) if the negotiations on the rental price fail. Relative to this, \( B \)'s surplus when renting at \( p \) is

\[
\Delta_B^{sale} = -p - \frac{\delta p}{1-\delta} - D + P,
\]

while \( S \)'s bargaining surplus is:

\[
\Delta_S^{sale} = p + \min \left\{ C, \frac{\delta p}{1-\delta} \right\} - P.
\]

The generalized Nash bargaining solution requires

\[
\Delta_S^{sale} = \alpha (\Delta_S^{sale} + \Delta_B^{sale}) = \alpha \left( \min \left\{ C, \frac{\delta p}{1-\delta} \right\} - \frac{\delta p}{1-\delta} - D \right) \Rightarrow (6.5).
\]

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The rest of the proof follows the same steps as after (6.5) in part (a). QED

**Proof of Proposition 5.** The proof is in the text.

**Proof of Proposition 6.** (i) First, I prove that (4.4) must hold for $s \in S^w$. Let $U_B(t)$ and $U_S(t)$ be the interim continuation values for $B$ and $S$ just before the cutting stage in period $t$. I allow $B$ and $S$ to renegotiate (also) at the interim stage. Let $b$ and $c$ be defined by (2.2) in Proposition 1.

**Lemma 1.** If $s \in S^w$, then for every $t > 1$, (a) $b_t \leq b$ and (b) $c_t \leq c$.

**Proof.** The proof is by contradiction.

(a) Suppose $s \in S^w$ and that for some $t > 1$, $b_t > b$. This can be optimal only if $-(P-D) \geq U_B(t)$ and $c_t \geq c$, which implies that $U_S(t) = C$. At $t-1$, $S$’s unique best response is $c_{t-1} = 0$, since then $U_S(t-1) = \delta [b_t P + (1 - b_t) U_S(t)] > \delta [b P + (1 - b) C] = C = U_S(t)$. Also, note that $U_B(t-1) = -\delta (P-D) > -(P-D) \geq U_B(t)$. In sum, $U_B(t-1) > U_B(t)$ and $U_S(t-1) > U_S(t)$, so at interim period $t-1$ both $B$ and $S$ strictly prefer at interim period $t$ to renegotiate to the equilibrium.

(b) If for some $t > 1$, $c_t > c$, then $B$’s response is $b_t = 1$. This contradicts (a). QED

**Lemma 2.** If $s \in S^w$, then for every $t > 2$, (a) $b_t \geq b$ and (b) $c_t \geq c$.

**Proof.** (a) Suppose $b_t < b$ for $t > 2$. Then, we must have $c_{t-1} = 1$ at $t > 1$, which contradicts Lemma 1(b).

(b) Suppose $c_t < c$ for $t > 1$. Then, $b_t = 0$, which contradicts part (a). QED

Second, we can use Lemma 1 and Lemma 2 to construct the set $S^w$. In particular, $s \in S^w$ if and only if one of these cases describes the equilibrium:

1. $b_t = b$ and $c_t = c \forall t \geq 1$.
2. $b_1 \in [0, 1]$, $c_1 = c$, $b_t = b$, and $c_t = c \forall t > 1$.
3. $b_1 = 0$, $c_1 < c$, $b_t = b$, and $c_t = c \forall t > 1$.
4. $b_1 = 1$, $c_1 > c$, $b_t = b$, and $c_t = c \forall t > 1$.
5. $b_1 = 1$, $c_1 = 1$, $b_2 < b$, $c_2 = c$, $b_t = b$, and $c_t = c \forall t > 2$.
6. $b_1 = 1$, $c_1 = 1$, $b_2 = 0$, $c_2 < c$, $b_t = b$, and $c_t = c \forall t > 2$.

By comparison, it is straightforward to verify that there is no $s \in S^w$ and time such that both players strictly benefit from switching to another $s' \in S^w$. This holds also if the players can switch to another weakly renegotiation-proof equilibrium at the interim stage (when $S$ is the next player to act). To see this, simply note that at the interim stage, the set of weakly renegotiation-proof equilibria $S^w_+$ consists of just the set of cases (1)-(6) but modified so that the first entry involving $b_1$ is omitted from each case. It follows that if $s \in S^w$, then $s \in S^s$. Since $S^s \subseteq S^w$, it follows that $S^s = S^w$. Analogously, $S^s_+ = S^w_+$. This completes the proof.

(ii) The proof for the rental market follows the same lines except that in the proof of Lemma 1(a), $(P-D)$ should be replaced by $p/(1 - \delta)$. QED

**Proof of Proposition 7.** (i) The seller prefers to cut if:

$$C \geq b(\delta p_r + \delta^r C) + (1 - b) \delta C \Rightarrow$$

$$b \leq \frac{1 - \delta}{\delta p_r + \delta^r C - \delta C} C.$$
The buyer prefers to rent rather than do nothing if:

\[
\frac{p_r}{1 - \delta^r} \geq cE + (1 - c) \frac{\delta p_r}{1 - \delta^r} \Rightarrow \\
c \geq \frac{1 - \delta}{1 - \delta^r} \frac{p_r}{E - \delta p_r/(1 - \delta^r)} = \frac{p_r (1 - \delta)}{E (1 - \delta^r) - \delta p_r}.
\]

Part (i) follows straightforwardly.

(ii) For every \( p_r < E \), B’s payoff is pinned down by the fact that contracting with S in every period is a best response. Once B has contacted S, B would prefer a rental agreement if:

\[
\frac{p_r}{1 - \delta^r} \leq P - D.
\]

The seller, at this stage, strictly prefers a rental agreement if:

\[
P < p_r + \delta^r \min \left\{ \frac{p_r}{1 - \delta^r}; \frac{C}{\delta} \right\},
\]

since, for every \( p_r/(1 - \delta^r) \geq C \), the seller’s continuation value in the rental market would be

\[
\min \left\{ \frac{p_r}{1 - \delta^r}; \frac{C}{\delta} \right\}.
\]

So, for the highest rental price which B is willing to accept, S strictly prefers a rental contract if:

\[
P < (P - D) (1 - \delta^r) + \delta^r \min \left\{ P - D; \frac{C}{\delta} \right\} \Rightarrow \\
D < \min \left\{ 0, \frac{\delta^r}{1 - \delta^r} \left( \frac{C}{\delta} - P \right) \right\} \Rightarrow \\
D < \min \left\{ 0, \frac{\delta^r}{1 - \delta^r} \left( \frac{C}{\delta} - (1 - \alpha) C - \alpha (D + E) \right) \right\} \Rightarrow (4.5).
\]

QED

Proof of Proposition 8. The proposition follows from Proposition 9 when setting \( n = 1 \).

Proof of Proposition 9. (i) The sales market: The aggregate \( b = \sum_{i \in N} b_i \) that makes S willing to randomize is given by:

\[
C = \int_0^\infty Pbe^{-t(r+b)}dt = \frac{bP}{r+b} \Rightarrow \\
b = \frac{rC}{P-C}.
\]

(6.7)

For \( i \in N \), the rate at which someone else buys is \( b_{-i} \equiv b - b_i \). Buyer \( i \) is willing to randomize when:

\[
P - D = \int_0^\infty (cE + b_{-i} \cdot 0) e^{-t(r+b_{-i}+c)}dt = \frac{cE}{c + b_{-i} + r} \Rightarrow \\
c = \frac{(P - D) (b_{-i} + r)}{D + E - P} = \frac{(P - D) (b (n-1) / n + r)}{D + E - P},
\]

(6.8)
where the equality $b_i = b/n$ is used since every $b_{-i}$ must be the same in order for (6.8) to hold for all $i \in N$.

(ii) The rental market: If $S$ is willing to mix, then $V_S = C$ and:

$$C = \int_{0}^{\infty} (p + Ce^{-rT}) be^{-t(r+b)}dt = \frac{b(p + Ce^{-rT})}{r + b} \Rightarrow$$

$$b = \frac{rC}{p - C(1 - e^{-rT})}.$$  

If buyer $i \in N$ is willing to rent and to pay $p$ at frequency $T$, then $V_i = -p/ (1 - e^{-rT})$.

If $i$ is also willing to wait, then the following also hold:

$$V_i = -\int_{0}^{\infty} (E + b_{-i} \left[-e^{-rT}V_i\right]) e^{-t(r+b_{-i}+c)}dt$$

$$= -\frac{cE - b_{-i}e^{-rTV_i}}{r + b_{-i} + c} \Rightarrow$$

$$c = -\frac{(b_{-i} + r) V_i + b_{-i}e^{-rTV_i}}{E + V_i} = \frac{(r + b_{-i} (1 - e^{-rT})) p/ (1 - e^{-rT})}{E - p/ (1 - e^{-rT})}$$

$$= \frac{[r/ (1 - e^{-rT}) + b (1 - 1/n)] p}{E - p/ (1 - e^{-rT})}.$$  

The endogenous rental price. If $B$ rents at $p$ this period, anticipating the equilibrium rental price $\bar{p}$ in the future, then $B$’s payoff is:

$$-p = \frac{e^{-rT} \bar{p}}{1 - e^{-rT}};$$

while $S$’ payoff is

$$p + e^{-rT}V_S = p + Ce^{-rT}.$$

(a) Consider the threat point "cut", where $S$ immediately cuts if the negotiations on the rental price fails. Relative to this, $B$’s surplus when renting at $p$ is

$$\Delta_{B}^{rent} = E - p - \frac{e^{-rTp}}{1 - e^{-rT}},$$

while $S$ bargaining surplus is:

$$\Delta_{S}^{rent} = p + Ce^{-rT} - C.$$  

The generalized Nash bargaining solution requires

$$\Delta_{S}^{rent} = \alpha \left( \Delta_{S}^{rent} + \Delta_{B}^{rent} \right)$$

$$= \alpha \left( E - \frac{e^{-rTp}}{1 - e^{-rT}} - C (1 - e^{-rT}) \right) \Rightarrow$$

$$p = \alpha E - \alpha \frac{e^{-rTp}}{1 - e^{-rT}} + (1 - \alpha) C (1 - e^{-rT}). \quad (6.9)$$

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In equilibrium, $p = \bar{p}$, so (6.9) becomes:

$$
p (1 - e^{-rT} (1 - \alpha)) = \alpha (1 - e^{-rT}) E + (1 - \alpha) C (1 - e^{-rT})^2 \Rightarrow
\frac{p}{1 - e^{-rT}} = \frac{\alpha E + (1 - \alpha) (1 - e^{-rT}) C}{1 - e^{-rT} + e^{-rT} \alpha}.
$$

(b) With the threat point "sale", where B buys at $P$ if the negotiations on the rental price fails, then the same analysis as in (a) leads to the same $p$ for the same reason as in the proof of Proposition 4.

(iii) By comparison: Since the buyer is willing to immediately buy in the sales market, and to always rent in the rental market, his payoffs in the two markets are identical if:

$$
\frac{p}{1 - e^{-rT}} = P - D. \quad (6.10)
$$

Once B has contacted S, S strictly prefers the rental contract if and only if:

$$
P < p + e^{-rT} V_S = p + e^{-rT} C. \quad (6.11)
$$

Thus, for the most expensive rental contract that B would be willing to accept (i.e., ensuring that (6.10) holds), S strictly prefers the rental contract (the inequality (6.11) is satisfied) if:

$$
P < (1 - e^{-rT}) (P - D) + e^{-rT} C \Rightarrow
Pe^{-rT} < - (1 - e^{-rT}) D + e^{-rT} C \Rightarrow
P - C < -(e^{-rT} - 1) D \Rightarrow (4.9). \quad (6.12)
$$

Substituting for $P$, we get (4.10). QED

Proofs of Propositions 10-11 follow from the reasoning in the text.

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