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# Structural change and the Kaldor facts in a growth model with relative price and non-Gorman preferences

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# Structural change and the Kaldor facts in a growth model with relative price effects and non-Gorman preferences

Timo Boppart

January 2011

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# Structural change and the Kaldor facts in a growth model with relative price effects and non-Gorman preferences*

Timo Boppart†

January 14, 2011

**Abstract**

Growth of per-capita income is associated with (i) significant shifts in the sectoral economic structure, (ii) systematic changes in relative prices and (iii) the Kaldor facts. Moreover, (iv) cross-sectional data shows systematic expenditure structure difference between rich and poor households. Ngai and Pissarides (2006) and Acemoglu and Guerrieri (2008) are consistent with observation (i)-(iii) but abstract form non-homotheticities of preferences. However, they cannot replicate the structural change between the U.S. goods and service sector quantitatively. This paper presents a growth model, which reconciles both forces of structural change - relative price and income effects - with balanced growth on the aggregate. The theory is simple and parsimonious and contains an analytical solution. The model can replicate shape and magnitude of the nonbalanced sectoral facts as well as the balanced nature of growth on the aggregate. In a structural estimation, the model's functional form is exploited to obtain estimates for the relative importance of income and price effects as determinants of the structural change.

Keywords: Structural change, relative price effect, non-Gorman preferences, Kaldor facts.
JEL classification: O14, O30, O41, D90.

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# 1 Introduction

It is a well documented empirical fact that economic growth is associated with significant shifts in the sectoral output, employment and consumption structure (see e.g. Kuznets (1957) and Kongsamut, Rebelo and Xie (2001)). This phenomenon is summarized under the term "structural change". As an example, figure 1 shows the relative decline of the goods sector (or the rise of the service sector) in the U.S. after World War II. The evolution of the logarithmized expenditure share devoted to goods is well approximated by a linear downward sloping trend.[1] The slope of this linear fit is $-0.0103$, which suggests that the expenditure share devoted to goods decreases (on average) at a constant annualized rate of one percent.

The nonbalanced nature of growth is displayed in prices too. Figure 2 plots the evolution of the logarithmized relative consumer price between goods and services. With some exceptions, as the first and second oil crisis in 1973 and 1979, the series is fairly good approximated by a linear downward sloping curve (see dashed line). The estimated slope coefficient of the fitted line is $-0.0160$, which suggests that the relative price of goods has on average been decreasing at a constant annualized rate of -1.6 percent.

Beyond the nonbalanced characteristics at the sectoral level, aggregate variables present a balanced picture of growth. Actually, the post-war U.S. often serves as a prime example of balanced growth on the aggregate. Balanced growth is best summarized by the Kaldor facts. These stylized facts state that the growth rate of real per-capita output, the real interest rate, the capital-output ratio and the labor income share are constant over time (see Kaldor (1961)). As a consequence, comprehensive models of structural change should also replicate the Kaldor facts.

To the best of my knowledge there exist two papers, which reconcile structural change, relative prices dynamics and the Kaldor facts in a growth model with endogenous savings: Ngai and Pissarides (2006) and Acemoglu and Guerrieri (2008).[2]

[1] Personal consumption expenditures account for about 70% of total output. In output, the same structural change can be observed. See Buera and Kaboski (2009b) who emphasize, that the rise of the service economy in terms of value added shares has in the U.S. mainly been driven by consumption.

[2] Changes in relative prices affect the expenditure structure whenever the elasticity of substitution across sector is unequal to unity. This mechanism of structural change goes back to Baumol (1967), who emphasizes total factor productivity (TFP) growth differences as a source of relative

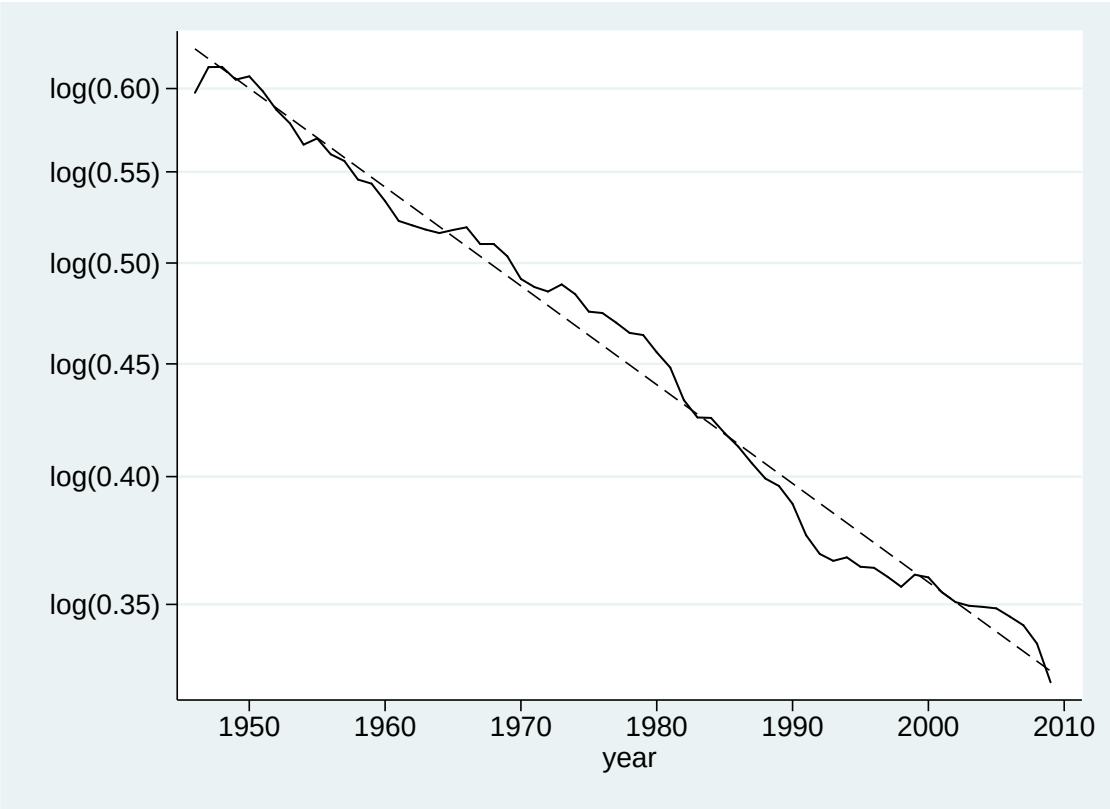


Figure 1: **Logarithm of expenditure share of goods.**

**Notes:** The figure plots the logarithm of the share of personal consumption expenditures devoted to goods in the U.S.. The dashed line represents a linear fit. The slope coefficient and its standard error are $-0.0103$ and $0.00015$, respectively. The simple regression attains an $R^2$ of 0.986. In levels the expenditure share of goods declined from 0.60 in 1946 to 0.32 in 2009. Source: BEA, NIPA table 1.1.5.

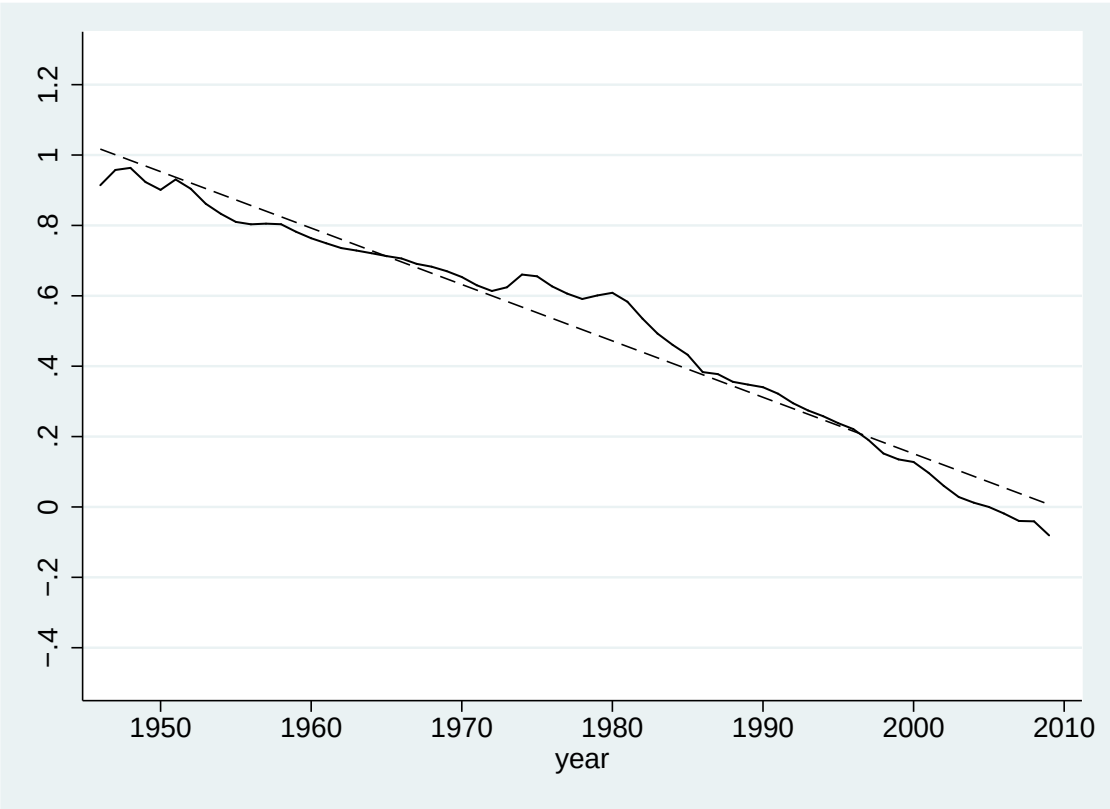


Figure 2: **Logarithm of the relative price between goods and services.**

**Notes:** The figure plots the logarithm of the relative consumer price between goods and services. The dashed line represents a linear fit. The slope coefficient and its standard error are $-0.016$ and $0.00038$, respectively. The simple regression attains an $R^2$ of 0.966. Source: BEA, NIPA table 1.1.4.

Both theoretical models feature a constant elasticity of substitution across sectors. However, in the U.S., the relative expenditure share of goods has declined at a faster rate than the relative price of goods. Hence, with relative price effects alone, theories with a constant elasticity of substitution cannot replicate the observed structural change.[3]

Acemoglu and Guerrieri (2008) emphasize that income effects are an "undoubtly important" determinant of structural change. Nevertheless, both Ngai and Pissarides (2006) and Acemoglu and Guerrieri (2008) abstract from non-homotheticity of preferences.[4] Empirically, there is clear evidence for an income effect. Figure 3 plots the logarithmized expenditure shares devoted to goods for the different pre-tax income quintiles. Rich households exhibit a significantly lower expenditure share of goods then poor households.[5] Moreover, all income quintiles display the same downward sloping trend as the aggregate data (see dashed line).

With non-unitary expenditure elasticities of demand, increases in real per-capita expenditure levels (due to growth) affect the sectoral expenditure shares.[6] Kongsamut, Rebelo and Xie (2001) and Foellmi and Zweimueller (2008) reconcile non-homothetic preferences and the Kaldor facts in an otherwise standard growth model with intertemporal optimization. However, in order to obtain balanced aggregate growth,

price changes. In Acemoglu and Guerrieri (2008), capital deepening and sectoral factor intensity differences is another driver of the relative price dynamic.

[3]This has already been pointed out by Buera and Kaboski (2009a).

[4]Acemoglu and Guerrieri (2008) conclude: "It would be particularly useful to combine the mechanism proposed in this paper with nonhomothetic preferences and estimate a structural version of the model with multiple sectors using data from the U.S. or the OECD."(Acemoglu and Guerrieri (2008), p. 493).

[5]An exception is the first income quintile, especially in the eighties, which can be explained by systematic differences in the household composition (see the regressions in section 3 which include additional controls).

[6]This mechanism of structural change is consistent with Engel's law, which is regarded as one of the most robust empirical regularities in economics (see Engel (1857), Houthakker (1957), Houthakker and Taylor (1970) and Browning (2008)). As a consequence, many models of structural change rely on income effects. See e.g. Matsuyama (1992), Echevarria (1997), Laitner (2000), Kongsamut, Rebelo and Xie (2001), Caselli and Coleman (2001) and Gollin, Parente and Rogerson (2002) which use quasi-homothetic intratemporal preferences or Falkinger (1990), Falkinger (1994), Zweimueller (2000), Matsuyama (2002), Foellmi and Zweimueller (2008) and Buera and Kaboski (2009b), which generate non-homotheticity by a hierarchy of needs.

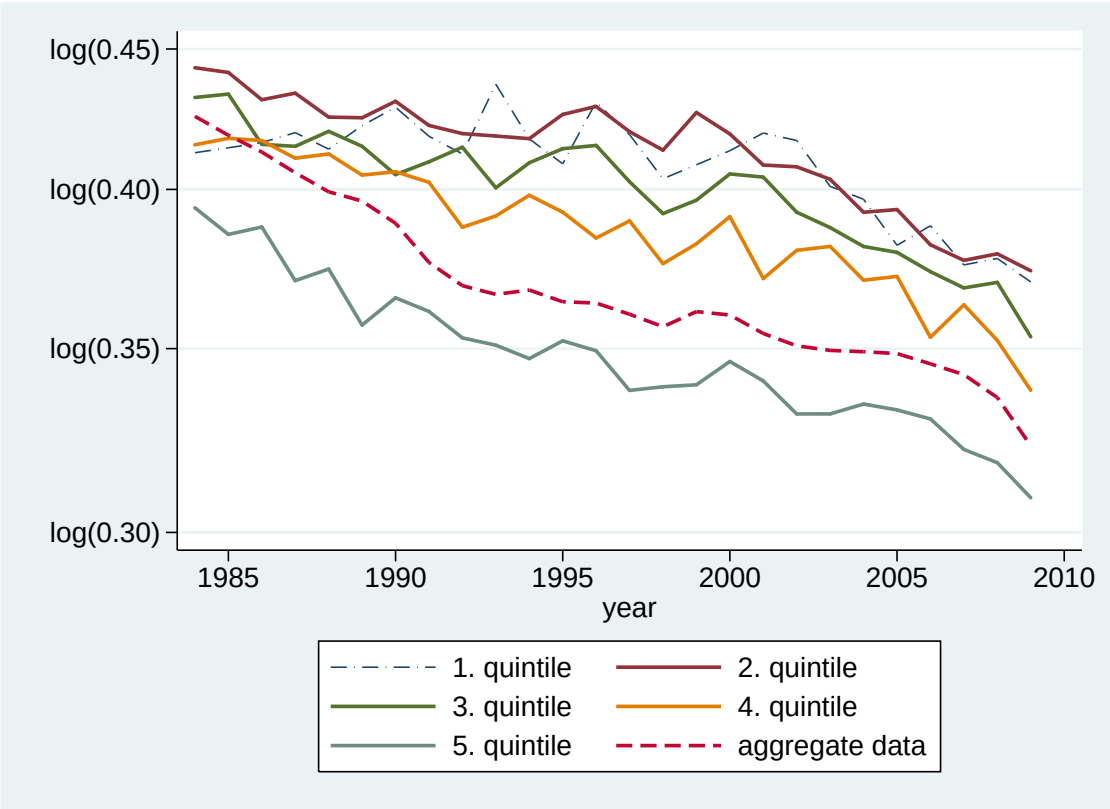


Figure 3: **Cross-sectional variation in logarithmized expenditure shares**

**Notes:** The figure plots the logarithm of the expenditure share devoted to goods for each pre-tax income quintile of the U.S.. The following expenditure categories are considered as services: food away from home; shelter; utilities, fuels, and public services; personal services; postage and stationery; other apparel products and services; other vehicle expenses; public transportation; health insurance; medical services; fees and admissions; other entertainment supplies, equipment, and services; personal care products and services; education; cash contributions; personal insurance and pensions. The remaining categories are considered as goods. Source: Consumer Expenditure Survey. The red dashed line is the same aggregate series as in figure 1.

both theories have to exclude relative price effects.[7] Hence, as pointed out by Buera and Kaboski (2009a), none of the existing growth models with endogenous savings and balanced growth, allows us to discuss both forces of structural change - relative price and income effects.

The contributions of this paper are as follows: First, it presents a neoclassical growth theory with intertemporal optimization, which reconciles the Kaldor facts with structural change simultaneously determined by relative price and income effects. Second, it shows that the theory can replicate within a unified framework the shape and magnitude of structural change and relative price dynamic identified in figure 1 and 2. Moreover, the model is consistent with cross-sectional expenditure structure differences and the parallel evolution of logarithmized expenditure shares of different income groups, depicted in figure 3. Finally, a structural estimation allows us to decompose the structural change into an income and substitution effect.[8]

The paper consists of four sections: Section 2 presents the theoretical growth model. In section 3 an estimation of the relative importance of income and substitution effects as determinants of structural change is carried out. Finally, section 4 concludes.

# 2 Theoretical model

There is a unit interval of households indexed by $i \in [0, 1]$. Each household consists of $N(t)$ identical members, where $N(t)$ grows at an exogenous rate $n \geq 0$. $N(0)$ is normalized to one, so we have $N(t) = \exp[nt]$. Each member of household $i$ is endowed with $l_i \in (\bar{l}, \infty)$, $\bar{l} > 0$, units of labor and $a_i(0) \in [0, \infty)$ units of initial wealth. These per-capita factor endowments can differ across households. Labor is supplied inelastically at every instant of time. Consequently, the aggregate labor

[7] In Kongsamut, Rebelo and Xie (2001) consistency with the Kaldor facts relies on a widely criticized knife-edge condition, which ties together preference and technology parameters and implies constant relative prices. Foellmi and Zweimueller (2008) have to assume that technological differences (which translate into a relative price dynamic) are uncorrelated with the hierarchical position of a good (and its sectoral classification). As figure 2 shows, stationarity of the relative good price is not supported by the data.

[8] See also the recent empirical works by Buera and Kaboski (2009a) and Herrendorf, Rogerson and Valentinyi (2009), which estimate for the U.S. the relative contribution of income and substitution effects to the structural change. In contrast to these two papers, the structural estimation of this work is based on a general equilibrium model which is consistent with the Kaldor facts.

supply $L(t) \equiv N(t) \int_0^1 l_i \, di$, grows at constant rate $n$.

## 2.1 Preferences

All households have the following additively separable representation of intertemporal preferences

$$\mathcal{U}_i(0) = \int_0^\infty \exp\left[-(\rho - n)t\right] V\left(P_1(t), P_2(t), e_i(t)\right) dt, \tag{1}$$

where $\rho \in (n, \infty)$ is the rate of time preference and $V\left(P_1(t), P_2(t), e_i(t)\right)$ is an indirect instantaneous utility function of each household member. This instantaneous utility function is specified over the prices of the two consumption goods, $P_1(t)$ and $P_2(t)$, and the nominal per-capita expenditure level of household $i$, $e_i(t)$. Henceforth, the first consumption good is called "good" whereas the second consumption good is "service". The indirect instantaneous utility function takes the following form

$$V\left(P_1(t), P_2(t), e_i(t)\right) = \frac{1}{\epsilon}\left[\frac{e_i(t)}{P_2(t)}\right]^\epsilon - \frac{\beta}{\gamma}\left[\frac{P_1(t)}{P_2(t)}\right]^\gamma - \frac{1}{\epsilon} + \frac{\beta}{\gamma}, \tag{2}$$

where $0 \leq \epsilon \leq \gamma < 1$ and $\beta, \gamma > 0$.[9] This intratemporal utility function falls into the class of "price independent generalized linearity" (PIGL) preferences defined by Muellbauer (1975) and Muellbauer (1976). PIGL preferences are more general than Gorman preferences. Nevertheless, PIGL preferences avoid an aggregation problem. Aggregate expenditure shares coincide with those of a household with a representative expenditure level (the representative household in Muellbauer's sense). Moreover, PIGL preferences ensure that this representative expenditure level is independent of prices. Because Engel curves are patently non-linear, PIGL preferences have explicitly an empirical justification and were widely used in expenditure system estimations (see e.g. the "Quadratic Expenditure System" (QES) by Howe, Pollak

[9]For $\epsilon = 0$ we get the limit case with $V(\cdot) = \log\left[\frac{e_i(t)}{P_2(t)}\right] - \frac{\beta}{\gamma}\left[\frac{P_1(t)}{P_2(t)}\right]^\gamma + \frac{\beta}{\gamma}$ and with $\gamma = \epsilon = 0$ we would obtain Cobb-Douglas preferences with $V(\cdot) = \log\left[\frac{e_i(t)}{P_1(t)^\beta P_2(t)^{1-\beta}}\right]$. As another special case, with $\beta = 0$, we would have only one consumption sector and CRRA preferences. But clearly, with only one consumption sector or with constant expenditure shares (i.e. Cobb-Douglas preferences), structural change cannot be discussed. Therefore, these two cases are excluded by the parametric restriction $\beta, \gamma > 0$. Nevertheless, it is remarkable that these two prevalent cases are special instances of (2).

and Wales (1979) or the "Almost Ideal Demand System" (AIDS) by Deaton and Muellbauer (1980)).[10]
Lemma 1 shows that function (2) satisfies the standard properties of a utility function.

**Lemma 1.** *Function* (2),

*(i) is a valid indirect utility specification that represents a preference relation defined over goods and services if and only if*

$$e_i(t)^{\epsilon} \geq \left[\frac{1-\epsilon}{1-\gamma}\right] \beta P_1(t)^{\gamma} P_2(t)^{\epsilon-\gamma}, \tag{3}$$

*(ii) is increasing and strictly concave in $e_i(t)$.*

*Proof.* See appendix A. □

Henceforth, I assume that condition (3) is fulfilled. Later, two conditions in terms of exogenous parameters are stated, which jointly ensure condition (3) for all individuals, at each date. Strict concavity of the intratemporal utility function will be a necessary condition for intertemporal optimization, which will be addressed below. The characteristics of the intratemporal preferences are best discussed in terms of the associated expenditure system. We have the following lemma.

**Lemma 2.** *At each point in time, intratemporal preferences imply the following expenditure system*

$$x_1^i(t) = \beta \frac{e_i(t)}{P_1(t)} \left[\frac{P_2(t)}{e_i(t)}\right]^{\epsilon} \left[\frac{P_1(t)}{P_2(t)}\right]^{\gamma}, \tag{4}$$

[10]The most general indirect form of PIGL preferences can be written as (see Muellbauer (1976))

$$V(P,e) = \left[\frac{e}{b(P)}\right]^{\vartheta} - \left[\frac{a(P)}{b(P)}\right]^{\vartheta},$$

where $\vartheta > 0$. $e$ is the expenditure level, $P$ is the price vector and $a(P)$ and $b(P)$ are linearly homogeneous functions. For a discussion of PIGL preferences in neoclassical growth theory see Boppart (2011). The functional form (2) is chosen, because it can jointly explain the constancy of the growth rates of the expenditure share devoted to goods (see figure 1), the relative price (see figure 2) and the expenditure level (one of the Kaldor facts). Moreover, the model is parsimonious. The two parameters, $\epsilon$ and $\gamma$, pin down separately, the expenditure elasticity of demand of goods and the asymptotic elasticity of substitution across sectors.

*and*

$$x_2^i(t) = \frac{e_i(t)}{P_2(t)} \left[ 1 - \beta \left[ \frac{P_2(t)}{e_i(t)} \right]^{\epsilon} \left[ \frac{P_1(t)}{P_2(t)} \right]^{\gamma} \right], \tag{5}$$

*where $x_j^i(t)$, $j = 1, 2$, is household $i$'s per-capita consumption of goods/services at date $t$.*

*Proof.* The derivation of the demand system is just an application of Roy's identity. □

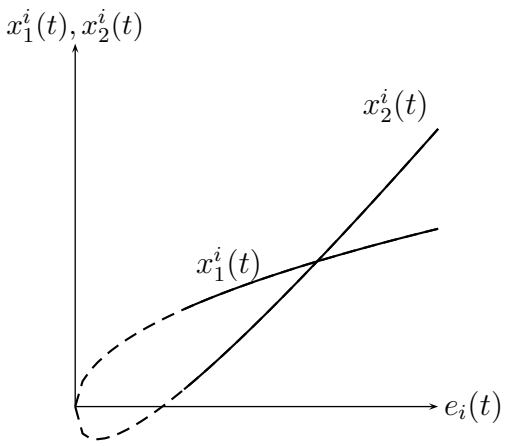


Figure 4: Engel curves

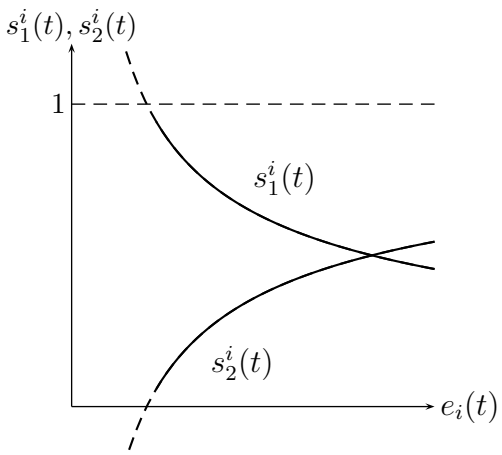


Figure 5: Expenditure shares

**Notes:** As indicated by the dashed sections, preferences are only well defined, if condition (3) holds (i.e. $e_i(t)$ exceeds a certain threshold).

The expenditure system reveals, that the demand for goods, $x_1^i(t)$, is an exponential function of order $1-\epsilon$ of the per-capita expenditure level. Moreover, the expenditure shares devoted to the two consumption sectors, $s_j^i(t)$; $j = 1, 2$, can be expressed as

$$s_1^i(t) = \beta \left[ \frac{P_2(t)}{e_i(t)} \right]^{\epsilon} \left[ \frac{P_1(t)}{P_2(t)} \right]^{\gamma} \text{ and } s_2^i(t) = 1 - \beta \left[ \frac{P_2(t)}{e_i(t)} \right]^{\epsilon} \left[ \frac{P_1(t)}{P_2(t)} \right]^{\gamma}. \tag{6}$$

For $\epsilon > 0$, figure 4 and 5 plot the Engel curves and the sectoral expenditure shares as a function of the per-capita expenditure level. In general, as the non-linear Engel curves reveal, preferences are non-homothetic and even do not fall into the Gorman class.

The elasticity of substitution across sectors and the expenditure elasticities of demand control the magnitude and direction of the income and substitution effects on expenditure shares. Growing real per-capita expenditure levels imply - according to

the income effect - an increasing expenditure share of the sector, whose expenditure elasticity of demand strictly exceeds unity. Besides, if the elasticity of substitution is strictly less than unity, according to the substitution effect, the sector which experiences a relative price increase, gains in terms of expenditure shares. The next lemma characterizes these two elasticities.

**Lemma 3.** *The intratemporal preferences,* (2)*, imply that*

*(i) the elasticity of substitution between goods and services,*

$$\sigma_i(t) = 1 - \gamma - \frac{\beta \left[\frac{P_1(t)}{P_2(t)}\right]^{\gamma}}{\left[\frac{e(\cdot)}{P_2(t)}\right]^{\epsilon} - \beta \left[\frac{P_1(t)}{P_2(t)}\right]^{\gamma}} \left[\gamma - \epsilon\right], \tag{7}$$

*is strictly less than unity (for all households at each date).*

*(ii) with $\epsilon > 0$, the expenditure elasticity of demand is positive, but strictly smaller than one for goods and larger than one for services.*

*(iii) with $\epsilon = 0$ we have homothetic preferences (expenditure elasticities of both sectors are equal to unity).*

*Proof.* See appendix A. □

Several things are worth noting: First, the restrictions on the preference parameters $\epsilon$ and $\gamma$ are such that the elasticity of substitution is strictly less than unity.[11] In the literature there seems to be a consensus that this is the empirically relevant case.[12] This notion is also confirmed by the structural estimation of section 3.
Second, in general, the elasticity of substitution varies over time and across households. Nevertheless, there is a special case with $\gamma = \epsilon$, in which the elasticity of substitution is constant for all households at each date.

[11]The utility function (3) could also generate cases where the elasticity of substitution is strictly larger than one. But these cases where excluded right away by the restriction $0 \leq \epsilon \leq \gamma < 1$.

[12]Baumol, Blackman and Wolff (1985) document a structural change toward the slower growing sector, which is in line with an elasticity of substitution smaller than one. Buera and Kaboski (2009a) calibrate their model with an elasticity of substitution equal to 0.5. See also Ngai and Pissarides (2006) and Acemoglu and Guerrieri (2008) which both emphasize the case where the inter-sectoral elasticity of substitution is less than one (in their calibration Acemoglu and Guerrieri (2008) use an elasticity of substitution equal to 0.76).

Third, with $\epsilon = 0$, we have homothetic preferences and consequently no income effect on expenditure shares. In contrast, as long as $\epsilon > 0$, goods are necessities with an expenditure elasticity of demand strictly smaller than one.[13]
Next, we turn to the household's intertemporal optimization problem. Households maximize (1) with respect to $\{e_i(t), a_i(t)\}_{t=0}^{\infty}$, subject to the budget constraint

$$\dot{a}_i(t) = [r(t) - n]\, a_i(t) + w(t) l_i - e_i(t), \tag{8}$$

and a standard transversality condition, which can be expressed as

$$\lim_{t\to\infty} e_i(t)^{\epsilon-1} P_2(t)^{-\epsilon} a_i(t) \exp\left[-(\rho - n)t\right] = 0. \tag{9}$$

$r(t)$ and $w(t)$ is the (nominal) interest and wage rate, respectively, and $a_i(t)$ denotes the per-capita wealth of household $i$ at date $t$. $a_i(0)$ is exogenously given. The result of intertemporal household optimization is summarized in the next lemma.

**Lemma 4.** *Intertemporal optimization yields the Euler equation*

$$(1-\epsilon) g_{e_i}(t) + \epsilon g_{P_2}(t) = r(t) - \rho, \tag{10}$$

*where $g_{e_i}(t)$ is the growth rate of per-capita consumption expenditures of household $i$ and $g_{P_2}(t)$ is the growth rate of the price of sector 2 at date $t$.*

*Proof.* See appendix A. □

The Euler equation takes the same functional form as in the standard one-sector growth model with CRRA preferences.[14] Additionally, since $g_{e_i}(t)$ is the only term that involves a household index $i$, the Euler equation implies that the growth rate of the per-capita expenditure levels is the same for all households at a given point in time, or formally,

$$g_{e_i}(t) = g_e(t), \ \forall i. \tag{11}$$

[13]Since this is the empirically relevant case (see figure 3), the opposite case, where services are necessities, is excluded by assuming $\epsilon \geq 0$.

[14]Without loss of generality, we could chose $P_2(t)$ as a numéraire. Then, the Euler equation would read as in the standard model: The intertemporal substitution elasticity of total consumption expenditures, $(1-\epsilon)$, times the per-capita growth rate is equal to the interest rate, minus the rate of time preference. However, in this model the intertemporal substitution elasticity is directly connected to the degree of non-homotheticity of intratemporal preferences, $\epsilon$.

Together with the desirable aggregation properties specific to all PIGL preferences, the feature, that all expenditure levels grow pari passu, simplifies the equilibrium analysis dramatically. Let us define $E(t)$ as the aggregate consumption expenditures and $X_j(t)$ as the aggregate demand for consumption $j = 1, 2$ at date $t$ (i.e. $E(t) \equiv N(t) \int_0^1 e_i(t)di$ and $X_j(t) \equiv N(t) \int_0^1 x_j^i(t)di$, $j = 1, 2$). Then, consumer behavior is summarized by the following proposition.

**Proposition 1.** *Under consumer optimization,*

*(i) the intertemporal behavior of the demand side is fully characterized by the following Euler equation, budget constraints and transversality conditions:*

$$(1-\epsilon)\left[g_E(t) - n\right] + \epsilon g_{P_2}(t) = r(t) - \rho, \ \forall t, \tag{12}$$

*where $g_E(t)$ is the growth rate of $E(t)$,*

$$\dot{a}_i(t) = \left[r(t) - n\right] a_i(t) + w(t) l_i - e_i(0) \exp\left[\int_0^t g_E(\varsigma) - n \, d\varsigma\right], \ \forall i, t, \tag{13}$$

*and*

$$\lim_{t\to\infty} a_i(t) \exp\left[-\int_0^t r(\varsigma) - n \, d\varsigma\right] = 0, \ \forall i, \tag{14}$$

*where $a_i(0)$, $\forall i$, is exogenously given.*

*(ii) the aggregate expenditure share devoted to goods, $S_1(t) \equiv \frac{P_1(t)X_1(t)}{E(t)}$, is given by*

$$S_1(t) = \beta \left[\frac{P_2(t)}{\frac{E(t)}{N(t)}}\right]^{\epsilon} \left[\frac{P_1(t)}{P_2(t)}\right]^{\gamma} \phi, \tag{15}$$

*where $\phi \equiv \int_0^1 \left[\frac{e_i(0)N(0)}{E(0)}\right]^{1-\epsilon} di$ is a scale invariant measurement of inequality of per-capita consumption expenditures across households. Furthermore, we have*

$$E(t) = P_1(t)X_1(t) + P_2(t)X_2(t). \tag{16}$$

*(iii) a household with $e_i(t) = \frac{E(t)}{N(t)}\phi^{-\frac{1}{\epsilon}} \equiv e^{RA}(t)$ is the representative agent in Muellbauer's sense.*[15]

*Proof.* See appendix A. □

[15] With $\epsilon = 0$, we have - according to Muellbauer's definition - the limit case $e^{RA}(t) = \frac{E(t)}{N(t)}$.

This proposition fully characterizes the demand side of this economy. Given a path of production factor, good and service prices, $\{r(t), w(t), P_1(t), P_2(t)\}_{t=0}^{\infty}$, equation (12) - (16) define the equilibrium evolution of the level and structure of aggregate consumption expenditures. Since in general, the intratemporal preferences do not fall into the Gorman class, a representative agent in the narrower sense does not apply and the distribution of per-capita expenditure levels matters. Nevertheless, the tractability of the specified preferences allows us to write the aggregate demand of goods and services as a function of just two terms: the aggregate expenditure level, $E(t)$, and a summary statistic of the distribution of per-capita expenditure levels at date $t = 0$, denoted by $\phi$. This is the outcome of two special properties:
First, the fact that preferences are part of the "generalized linearity" class, allows for a representative agent in Muellbauer's sense (see Muellbauer (1975) and Muellbauer (1976)). A household with the representative expenditure level, $e^{RA}(t)$, exhibits the same expenditure shares as the aggregate economy. Moreover, since preferences are even part of the PIGL class, the representative expenditure level is independent of prices. Consequently, aggregate demand can be expressed as a function of $E(t)$ and the scale invariant inequality measure of per-capita expenditure levels at date $t$, $\phi(t) = \int_0^1 \left[\frac{e_i(t)N(t)}{E(t)}\right]^{1-\epsilon} di$.
The second property is that intertemporal optimization implies for all households the same per-capita expenditure growth rate at any given point in time (see (11)). Then, $\phi(t)$ is constant over time and can therefore be expressed as a function of the $e_i(0)$ distribution.[16] This tractability allows me to solve the model analytically, despite household heterogeneity, non-Gorman intratemporal preferences and intertemporal optimization.[17]
To close the model, i.e. in order to determine the equilibrium path of production factor, good and service prices, the production side of the economy remains to be

[16] With $\epsilon > 0$, a high dispersion of per-capita expenditure levels is associated with a low value of $\phi$. In the homothetic case, we have a representative agent economy (in the narrower sense), where inequality does not matter (i.e. $\phi = 1$).

[17] As in models with 0/1 preferences (see e.g. Foellmi and Zweimueller (2006), Foellmi, Wuergler and Zweimueller (2009) and Wuergler (2010)), this paper presents a dynamic general equilibrium model with intertemporal optimization, where inequality affects the sectoral demand structure. But in contrast to 0/1 preferences, this model focuses on the intensive margin of consumption. Moreover, the model allows us to study any - possibly continuous - income distribution which is consistent with condition (3).

specified.

## 2.2 Production

There are three output goods: the output of the two consumption sectors $Y_1(t)$ and $Y_2(t)$ and an "investment good", $Y_3(t)$, which can be transformed one-to-one into capital, $K(t)$. Capital depreciates at constant rate $\delta \geq 0$. This implies for the law of motion of capital

$$\dot{K}(t) = X_3(t) - \delta K(t), \tag{17}$$

where $X_3(t)$ are aggregate gross investments (in terms of investment goods) at date $t$. The consumption sectors produce under perfect competition according to the following technologies

$$Y_j(t) = \exp\left[g_j t\right] L_j(t)^{\alpha} K_j(t)^{1-\alpha}, \; j = 1, 2, \tag{18}$$

where $L_j(t)$ and $K_j(t)$ denote labor and capital, respectively, allocated to sector $j$ at date $t$. Both production factors are fully mobile across sectors. $\alpha \in (0, 1)$ is the output elasticity of labor, which is identical across sectors.[18] Total factor productivity (TFP) expands at a constant, exogenous, sector-specific rate $g_j \geq 0$.[19] The investment good is produced by a linear technology

$$Y_3(t) = AK_3(t), \tag{19}$$

with $A > \delta$.[20] The market of investment goods is competitive, too. Henceforth, I normalize the price of the investment good at each date to one, i.e. $P_3(t) = 1$, $\forall t$.

[18](18) represents aggregate production functions, comprising total factor inputs, which come from direct production as well as indirect sources (production of intermediates). Valentinyi and Herrendorf (2008) estimate sectoral labor income shares with respect to final output/consumption rather than value added. The estimates for manufacturing, services, overall consumption and total output are all between 0.65 and 0.67. Hence, for final consumption, on the level of aggregation considered in this paper, the assumption of identical output elasticities of labor seems to be a good approximation. Nevertheless, for the sake of completeness, appendix D illustrates the equilibrium dynamic, when there are sectoral factor intensity differences.

[19]Appendix E shows how these sector specific TFP growth rates can be endogenized.

[20]The AK specification is not crucial. With $Y_3(t) = \exp\left[g_3 t\right] L_3(t)^{\kappa} K_3(t)^{1-\kappa}$, $g_3 \geq 0$, $\kappa \in (0, 1]$, the model would be consistent with a globally saddle-path stable steady state, displaying exactly the same properties as the equilibrium with the linear technology.

The production side of this economy is similar to the one in Rebelo (1991).[21] $K(t)$ is a "core" capital good, whose production does not involve nonreproducible factors. This makes endogenous growth feasible. But as long as $g_j \neq 0$, for some $j = 1, 2$, the economy also consists of an exogenous driver of growth.

## 2.3 Equilibrium

### 2.3.1 Definition

In this economy, an equilibrium is defined as follows:

**Definition 1.** *A dynamic competitive equilibrium is a time path of households' per-capita expenditure levels, wealth stocks and consumption quantities* $\left\{e_i(t), a_i(t), x_j^i(t)\right\}_{t=0}^{\infty}$, $j = 1, 2$, $\forall i$; *an evolution of prices, wage, interest and rental rate,* $\{P_j(t), w(t), r(t), R(t)\}_{t=0}^{\infty}$, $j = 1, 2$ *and a time path of factor allocations* $\{L_1(t), L_2(t), K_1(t), K_2(t), K_3(t)\}_{t=0}^{\infty}$, *which is consistent with household and firm optimization, perfect competition, resource constraints and market clearing conditions.*

### 2.3.2 Resource constraints and market clearing conditions

In equilibrium, capital and labor markets have to clear, i.e.

$$L(t) = L_1(t) + L_2(t), \text{ and } K(t) = K_1(t) + K_2(t) + K_3(t), \ \forall t. \tag{20}$$

Market clearing in goods, service and investment goods markets requires

$$Y_j(t) = X_j(t), \ j = 1, 2, 3, \ \forall t. \tag{21}$$

Since the price of the investment good is chosen as a numéraire, asset market clearing implies

$$N(t) \int_0^1 a_i(t) di = K(t), \ \forall t. \tag{22}$$

Finally, the market rate of return of capital has to equalize the rental rate net of depreciations, i.e. $r(t) = R(t) - \delta$, $\forall t$.

[21] With $\beta = 0$ and $g_2 = 0$ the model would coincide with the one by Rebelo (1991).

### 2.3.3 Equilibrium dynamic

Under the choice of numéraire, perfect competition, resource constraints and the market clearing conditions, the equilibrium in production is characterized by the following lemma.

**Lemma 5.** *Firm optimization implies at each date $t$,*

$$r(t) = A - \delta, \tag{23}$$

$$w(t) = A\frac{\alpha}{1-\alpha}\frac{K_1(t)+K_2(t)}{L(t)},\ j = 1,2, \tag{24}$$

$$P_j(t) = \exp\left[-g_j t\right]\left[\frac{A}{1-\alpha}\right]\left[\frac{K_1(t)+K_2(t)}{L(t)}\right]^{\alpha},\ j = 1,2, \tag{25}$$

$$Y_j(t) = \exp\left[g_j t\right]\left[\frac{L(t)}{K_1(t)+K_2(t)}\right]^{\alpha} K_j(t),\ j = 1,2, \tag{26}$$

*and*

$$\frac{K_1(t)}{L_1(t)} = \frac{K_2(t)}{L_2(t)} = \frac{K_1(t)+K_2(t)}{L(t)}. \tag{27}$$

*Proof.* See appendix A. □

The dynamic competitive equilibrium is fully characterized by equations (12)-(17) and (19)-(26). The endogenous variables are: $X_j(t)$ and $Y_j(t)$, $j = 1,2,3$; $a_i(t)$, $\forall i$; $E(t)$, $P_j(t)$, $j = 1,2$; $w(t)$, $r(t)$, $L_j(t)$, $j = 1,2$; $K(t)$ and $K_j(t)$, $j = 1,2,3$. $a_i(0)$, $\forall i$, are exogenously given.
When we solve for the dynamic competitive equilibrium, we obtain the following proposition.

**Proposition 2.** *Suppose we have*

$$A - \delta - \rho + \epsilon g_2 > 0, \tag{28}$$

$$\rho > (1-\alpha)\epsilon\left[A-\delta\right] + n + \epsilon g_2, \tag{29}$$

$$\alpha^{\epsilon}\bar{l}^{\epsilon} \geq \frac{1-\epsilon}{1-\gamma}\beta\left[\frac{L(0)}{K(0)}\frac{A\left(1-(1-\alpha)\epsilon\right)}{n+\rho-\epsilon g_2-\epsilon(1-\alpha)\left(A-\delta-n\right)}\right]^{\epsilon(1-\alpha)}, \tag{30}$$

*and*

$$\gamma\left[g_2 - g_1\right] - \epsilon\left[\frac{g_2 + (1-\alpha)\left[A-\delta-\rho\right]}{1-(1-\alpha)\epsilon}\right] \leq 0. \tag{31}$$

*Then, there exists a unique dynamic competitive equilibrium path along which*

*(i) per-capita consumption expenditures, wages, aggregate capital and capital allocated to the consumption sectors grow at constant rates*

$$g_E^* - n = g_w^* = \frac{A - \delta - \rho + \epsilon g_2}{1 - (1-\alpha)\epsilon} > 0, \tag{32}$$

$$g_K^* = g_{K_1+K_2}^* = g_E^*. \tag{33}$$

*The saving rate is constant and the real, investment good denominated interest rate is given by $A - \delta$. The prices of goods and services change at constant rates*

$$g_{P_j}^* = -g_j + \alpha \left[g_E^* - n\right], \; j = 1, 2. \tag{34}$$

*(ii) the expenditure share devoted to goods changes at constant rate*

$$g_{S_1}^* = -\gamma \left[g_1 - g_2\right] - \epsilon \left[g_2 + (1-\alpha)\left[g_E^* - n\right]\right] \leq 0. \tag{35}$$

*Capital and labor allocated to the goods sector grow at constant rates*

$$g_{K_1}^* = g_K^* + g_{S_1}^* \leq g_K^* \leq g_{K_2}^*(t), \text{ and } g_{L_1}^* = n + g_{S_1}^* \leq n \leq g_{L_2}^*(t), \; \forall t. \tag{36}$$

*The relative price between consumption goods and services changes at constant rate*

$$g_{P_1}^* - g_{P_2}^* = g_2 - g_1. \tag{37}$$

*Proof.* See appendix A. □

Part (i) of proposition 2 illustrates that on the aggregate the model features the standard properties of neoclassical growth theory. The per-capita growth rate is increasing in the marginal product of capital, $A$, and decreasing in the rate of time preference, $\rho$, and the depreciation rate, $\delta$. Furthermore, the Kaldor facts hold. Total labor income, $w(t)L(t)$, and the total capital income net of depreciation, $rK(t)$, grow at the same constant rate $g_E^*$ as aggregate output. Thus, the per-capita output growth rate, the capital-output ratio, the saving rate and the labor income share are constant.[22] Furthermore, the real, investment good denominated interest rate is equal to $A - \delta$. Since all consumption prices change at constant rates (see (34)),

[22]The capital-output ratio and the saving rate read $\frac{K(t)}{R(t)K(t)+w(t)L(t)} = \frac{1-\alpha}{A-\alpha\left[g_K^*+\delta\right]}$ and $\frac{R(t)K_3(t)}{R(t)K(t)+w(t)L(t)} = \frac{(1-\alpha)\left[g_K^*+\delta\right]}{A-\alpha\left[g_K^*+\delta\right]}$, respectively.

any price index with constant sectoral weights grows at a constant rate too. Hence, deflated by a price index with constant weights, the real per-capita growth and real interest rate would be constant. (For instance, the real, in goods or services denominated interest rate is time invariant.) But in an economy with structural change, the sectoral weights of an appropriate price index should be adjusted over time. This would yield a non-constant growth rate of the price index and a time varying real interest rate. But typically, changes in the growth rate of the price index due to weight adjustments are very small (see Ngai and Pissarides (2004)). The model predicts, that measured by the true cost of living price index of the representative household, the real interest rate in 2009 is 0.005 higher than its asymptotic value.[23] The model exhibits no transitional dynamic and can be solved analytically.[24] Without exogenous TFP growth (i.e. with $g_1 = g_2 = 0$), the aggregate behavior would be the same as in Rebelo (1991). However, the intertemporal substitution elasticity of expenditure, $\frac{1}{1-\epsilon}$, is tied together with the expenditure elasticity of demand for goods, $\epsilon$.[25]

Noteworthy, although preferences are non-Gorman and inequality matters, the Kaldor facts hold irrespective of the distribution of expenditure level. This holds true since the marginal propensity to save out of capital income is the same at all wealth levels (and the marginal propensity to save out of labor income is zero for all households). An unforeseen shock on the wealth distribution would change the demand structure, but not the aggregate saving rate. Consequently, capital accumulation and growth would not be affected.

Part (ii) of proposition 2 focuses on the non-balanced features of the model. Although the Kaldor facts hold, the aggregate expenditure share devoted to goods as well as the relative price between goods and services change over time. The functional forms that the simple model imposes are striking. The model predicts that

[23]The growth rate of the partial true cost of living price index of household $i$ is defined as $g_P^{TCL}(t) = g_{P_2}(t) + s_1^i(t)\left[g_{P_1}(t) - g_{P_2}(t)\right]$ (see Pollak (1975)). In the data, relative price growth rate is -1.6 percent and in 2009 the aggregate expenditure share of goods was 0.32, whereas its asymptotic value is zero.

[24]This is due to the $AK$ specification of the production function of investment goods. With a decreasing marginal product of capital, transitional dynamics would arise (see footnote 20).

[25]With $\epsilon = 0$, this interdependence reflects the result obtained by Ngai and Pissarides (2006): If preferences are homothetic, reconciliation of structural change with the Kaldor facts requires that intertemporal substitution elasticity of expenditures is equal to unity.

both the expenditure share of goods and the relative price of goods decrease at constant rates. Remarkably, this is consistent with the functional form of the stylized facts discovered in figure 1 and 2.

The shift in the aggregate demand structure transforms to the production side (see (36)). Capital allocated to the goods sector grows at a lower rate than the aggregate capital stock, which itself grows at a lower rate than capital allocated to the service sector. In contrast to $g^*_{K_1}$ and $g^*_K$, $g^*_{K_2}(t)$ expands at a time varying rate. The same applies to the allocation of labor. If $n$ is small relative to $g^*_{S_1}$, the absolute quantity of labor allocated to the goods sector can even decrease. Nevertheless, consumption of both goods and services increases steadily - even in per-capita terms. Thus, the goods sector declines only in relative and not in absolute terms.

The required parametric restrictions (28)-(31) are harmless. Reconciliation of the non-balanced features of growth with the Kaldor facts does not depend on any knife-edge condition. (28) ensures positive capital accumulation and growth in per-capita terms.[26] Condition (29) is necessary and sufficient for the transversality condition to hold. Furthermore, it is also sufficient to ensure finite utility. Condition (30) makes sure that condition (3) is met for all households at $t = 0$. Moreover, together with condition (31), it ensures condition (3) along the whole equilibrium path.

In general, the structural change is driven by both income and substitution effects. With $\epsilon > 0$ services are luxuries. Hence, due to per-capita growth, the expenditure share devoted to services tends to increase. In addition, if the relative price changes (i.e. $g_1 \neq g_2$), there is a substitution effect. Since the elasticity of substitution between the two consumption sectors is strictly less than one, the expenditure share of the sector with the higher TFP growth rate tends to decrease. The magnitude of the income and substitution effects is controlled by the exogenous preference parameters $\gamma$ and $\epsilon$. With $\epsilon = 0$ we have homothetic preferences and changes in expenditure shares are completely determined by the substitution effect. With $g_1 = g_2$ the relative price does not change and the entire structural change is driven by an income effect. In general, income and relative price effects can go in opposite directions. If,

[26]The analysis is constraint on equilibria with positive per-capita growth and capital deepening (which is the empirically relevant case). As long as $A - \rho + \epsilon g_2 - (1-\alpha)\epsilon\delta > 0$ the model would be consistent with positive gross investments (but possibly negative net ones). With $A - \rho + \epsilon g_2 - (1-\alpha)\epsilon\delta \leq 0$ we would obtain a corner solution with no investments. As a consequence, aggregate capital would decline at constant rate $-\delta$.

by sheer coincidence $-\gamma(g_1 - g_2) = \epsilon\,[g_2 + (1-\alpha)\,[g_E^* - n]]$, the two effects cancel each other so that there would be no structural change.[27]
In the next proposition the income and substitution components of structural change are analyzed in more detail.

**Proposition 3.** *Along the equilibrium path,*

*(i) for all households, the expenditure share devoted to goods changes at a constant rate* $g_{S_1}^* \leq 0$.

*(ii) according to the substitution effect, a decrease of the relative price of goods by one percent, decreases the expenditure share devoted to goods of household* $i$ *by* $-\gamma + \epsilon s_1^i(t) \leq 0$ *percents.*

*(iii) for all households, according to the income effect, an increase of the (instantaneous) utility level by one percent, decreases the expenditure share devoted to goods by* $\epsilon$ *percents.*

*Proof.* See appendix A. □

The model predicts that not only the aggregate, but also all individual expenditure shares of goods decrease at the identical, constant rate $g_{S_1}^*$. This is consistent with the linear and parallel decline of the logarithmized expenditure shares of different income quintiles (see figure 3). However, as part (i) and (ii) of proposition 3 show, if $\epsilon > 0$, the division of this change in expenditure shares into an income and substitution effect differs across households. For richer households (with a lower $s_1^i(t)$), the substitution effect is relatively more important. Consequently, as all households get richer, the relative importance of the income effect as a determinant of the aggregate expenditure share dynamics decreases. Since preferences allow for a representative agent in Muellbauer's sense, the substitution effect of the aggregate economy is the same as the substitution effect for the representative agent. Hence, a one percent decline in the relative price of goods decreases (according to the substitution effect) the aggregate expenditure share of goods by $-\gamma + \epsilon S_1(t) \leq 0$ percents.

[27] A trivial case, where this condition is fulfilled arises if neither an income nor a substitution effect exists. This occurs with homothetic preferences and a constant relative price ($\epsilon = g_1 - g_2 = 0$) or with Cobb-Douglas preferences ($\epsilon = \gamma = 0$).

It is insightful to take a closer look at the equilibrium toward which the economy converges, as time goes to infinity. To do so, we define:

**Definition 2.** *The asymptotic equilibrium is the dynamic competitive equilibrium path toward which the economy tends as time goes to infinity.*

Then, we have the following proposition (asymptotic equilibrium values are denoted by a superscript $A$).

**Proposition 4.** *Suppose now, condition* (31) *holds with strict inequality. Then, in the asymptotic equilibrium,*

*(i) the expenditure share devoted to goods is equal to zero, i.e.* $S_1^A = 0$.

*(ii) the expenditure elasticity of demand is* $1 - \epsilon$ *for goods and unity for services.*

*(iii) the elasticity of substitution between goods and services,* $\sigma_i^A$*, is equal to* $1 - \gamma$ *for all households* $i$.

*Proof.* See appendix A. □

Part (i) of proposition 4 shows that the service sector is the asymptotically dominant consumption sector. The existence of an asymptotically dominant sector is a common feature of the models by Ngai and Pissarides (2006), Foellmi and Zweimueller (2008) and Acemoglu and Guerrieri (2008). The asymptotic dominance of the service sector is not a fact of a trivial disappearance of the goods sector. In absolute terms, the asymptotically consumed quantity of goods goes to infinity - even in per-capita terms.

Part (ii) and (iii) of proposition 4 illustrate how parsimonious the model is. The expenditure elasticity of demand and the elasticity of substitution across sectors control size and magnitude of relative price and income effects on $S_1$. The model has exactly two exogenous parameters, $\epsilon$ and $\gamma$, which control separately the asymptotic values of these two elasticities. With $\epsilon = 0$ the asymptotic equilibrium is similar to the one by Ngai and Pissarides (2006) and Acemoglu and Guerrieri (2008). There is no income effect and the elasticity of substitution across sectors is constant. With $g_1 = g_2$, there is no relative price effect and the asymptotic equilibrium resembles the one by Foellmi and Zweimueller (2008). But in contrast to Foellmi and Zweimueller (2008), where the expenditure elasticity of demand of the asymptotically dominated

sectors converge to zero, it is asymptotically positive in this model. In general, with $\epsilon \neq 0$ and $g_1 \neq g_2$, both income and relative price effects are even asymptotically present.[28]

So far, it has been shown that the model is consistent with a unique dynamic competitive equilibrium path, along which the Kaldor facts hold and changes in expenditure shares and relative prices occur. Furthermore, the functional form of these nonbalanced features is consistent with the dynamics observed in the U.S. data. But whether the model can quantitatively replicate the size of structural change identified in figure 1 is another question. This will be assessed in the next section.

# 3 Structural estimation and a numerical example

## 3.1 Quantitative replication of structural change in the U.S. economy

The dynamic of structural change is described by (35). This equation relates the growth rate of the expenditure share of goods, $g^*_{S_1}$, to both the growth rate of the relative price of goods and the growth rate of per-capita expenditures in terms of services. In the U.S. data, the trend in the relative price of goods is well captured by a decline at a constant annualized rate of -0.016 (see figure 2). Furthermore, per-capita expenditures in terms of services grow on average at an annualized rate of 0.008.[29] The relative importance of the income and substitution effects is controlled by the two preference parameters $\epsilon$ and $\gamma$. In section 2 we presumed the parametric restriction

$$0 \leq \epsilon \leq \gamma < 1. \tag{38}$$

[28]This is an important difference to theories where the income effect relies on quasi-homothetic preferences. As Buera and Kaboski (2009a) show with their calibration: "The model fails to match the sharper increase in services and decline in manufacturing after 1960. [...] Explaining this would require a large, delayed income effect toward services. This is not possible with the Stone-Geary preferences, where the endowments and subsistence requirements are most important at low levels of income." (See Buera and Kaboski (2009a), p. 473-474.)

[29]The average growth rate of aggregate expenditures in terms of services is well approximated by a constant rate increase of 2.8 percents (see figure 7 in the appendix B). To match the average employment growth in the data, $N(t)$ is assumed to grow at constant rate $n = 0.02$. This yields for the growth rate of per-capita expenditures in terms of services $g^*_E - n - g^*_{P_2} = 0.008$.

Suppose we calibrate the model in a way, that the observed per-capita growth and relative price trend are matched. Then, the predicted growth rate of the expenditure share of goods is given by $g^*_{S_1} = -0.016\gamma - 0.008\epsilon$. In the data we observe $g^*_{S_1} = -0.01$ (see section 1). Given the model replicates the observed per-capita growth and price dynamic, we can then ask the question whether there exist $\epsilon$ and $\gamma$ combinations fulfilling (38), which generate the observed magnitude of structural change. The answer to this question is an unambiguous yes. Figure 6 plots all $\gamma$ and $\epsilon$ combinations for which the model predicts a $g^*_{S_1}$ of $-0.75$, $-1.00$ and $-1.25$ percents, respectively. Parameter combinations that violate restriction (38) are represented by the gray area. We clearly see, that if $\gamma$ and $\epsilon$ is in the range of $0.42-0.63$

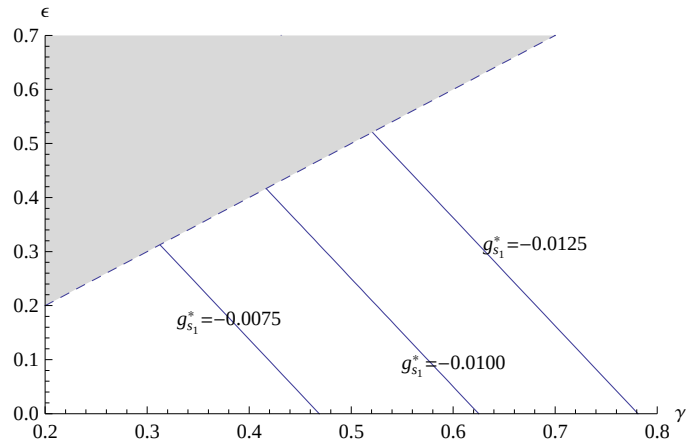


Figure 6: **$\gamma$ and $\epsilon$ combinations perfectly matching the structural change**

**Notes:** Given the observed per-capita growth and price dynamic, the solid lines plots all $\gamma$ and $\epsilon$ combination which are consistent with a $g^*_{S_1}$ of $-0.75$, $-1.00$ and $-1.25$ percent, respectively. The gray area represents $\gamma$ and $\epsilon$ combinations which violate the restriction $0 \leq \epsilon \leq \gamma$.

and $0-0.42$, respectively, (38) holds and the magnitude of structural change is about the same as in the U.S. data. An additional question is whether such parameter values are economically reasonable. According to proposition 4, $1-\gamma$ can be interpreted as the asymptotic elasticity of substitution across sectors, whereas $1-\epsilon$ is the expenditure elasticity of demand for goods. Consequently, a joint replication of the observed magnitude of structural change, per-capita growth and relative price dynamics requires an elasticity of substitution that converges (from below) to $0.37-0.58$ and an expenditure elasticity of demand for goods between 0.58 and

1. These are both reasonable ranges.[30] Therefore, we conclude that the model is able to replicate quantitatively the magnitude of per-capita growth as well as the nonbalanced price and expenditure features observed at sectoral level.

## 3.2 Structural estimation of $\epsilon$

As figure 6 illustrates, the effort to generate the same structural change as in the data per se is uninformative about the size of the income and substitution effect. $g^*_{S_1} = 0.01$ is consistent with both a very large or inexistent income effect ($\epsilon = \gamma = 0.42$ compared to $\epsilon = 0$ and $\gamma = 0.63$). This subsection aims to provide evidence on the magnitude of the income and relative price effects as drivers of structural change. To do so, I exploit cross-sectional expenditure variations detected in figure 3 to estimate $\epsilon$. Suppose we have different income groups $h = 1, 2, ..., H$. Then, the model implies that the expenditure share devoted to goods of income group $h$ is given by

$$S_1^h(t) = \beta \left[\frac{P_2(t)}{\bar{e}_h(t)}\right]^{\epsilon} \left[\frac{P_1(t)}{P_2(t)}\right]^{\gamma} \phi_h(t),$$

where $\bar{e}_h(t)$ is the average nominal per-capita expenditure level of income group $h$ and $\phi_h(t)$ is the expenditure inequality within group $h$. If within group expenditure levels are relatively homogeneous (i.e. if the income bins are relatively narrow), $\phi_h(t)$ is near unity for all income groups. Hence, we have $\log\left[S_1^h(t)\right] \approx b(t) - \epsilon \log[\bar{e}_h(t)]$, where $b(t) \equiv \log\left[\beta P_2(t)^{\epsilon-\gamma} P_1(t)^{\gamma}\right]$. Then, in order to identify $\epsilon$ by the cross-sectional variation, the logarithmized expenditure share of goods of each income group $h$ and date $t$ is regressed on a time dummy and the logarithmized group-specific per-capita expenditure level. For thirteen income groups and seven years (2003-2009), the results are summarized in column (1) of table 1. The estimate for $\epsilon$ is equal to 0.18 and significantly different from zero. Notably, this simple regression explains over 83 percent of the observed variation in logarithmized group-specific expenditure shares. Nevertheless, there are other household characteristics (as dependency-ratio, gender or race), which potentially affect the expenditure structure. Since these household characteristics are typically correlated with pre-tax income, the estimator of column

[30] In their calibration, Buera and Kaboski (2009a) and Acemoglu and Guerrieri (2008) use for the elasticity of substitution similar values. Furthermore, for different expenditure categories, expenditure elasticities of demand between 0.5 and 1 are often estimated (see e.g. Houthakker and Taylor (1970)).

(1) may be biased. Consequently, we control in columns (2)-(4) for additional household characteristics. With these additional controls, the estimate for $\epsilon$ increases to $0.27 - 0.28$. Two further robustness checks are performed in the last two columns of table 1. First, in column (4), the years 1984-2002, for which we observe only the lower eight income groups (up to 70'000 US$), are added. Second, in column (5) we drop the highest, unbounded income group, for which within group inequality may matter most. For both robustness checks, the estimated $\epsilon$ is about 0.23 and significantly different from zero.[31] Overall, there is clear evidence for the income effect. Reasonable values of $\epsilon$ range between 0.2 and 0.3.

Suppose $\epsilon$ is equal to 0.25. Then, to be consistent with a decline of the goods' expenditure share at a rate of -1 percent and the observed growth and price dynamic, requires $\gamma = 0.5$. With these parameter values, in 1946, 56% of the observed structural change is attributed to a relative price effect, whereas the remaining 44% are attributed to the income effect.[32] In 2009, the corresponding numbers are 67% and 33%, respectively. Furthermore, the model predicts that the relative contribution of the substitution effect will asymptotically converge to 80%.

So far, we just presumed that the model replicates the per-capita growth rate and price dynamic observed in the data. Matching the observed structural change and cross-sectional expenditure share differences pins down the preference parameters $\epsilon$ and $\gamma$. Only $\epsilon$ enters the equations which determine the aggregate behavior. Hence, for any parameter values of $\alpha$, $\delta$, $n$ and $A$, the exogenous TFP growth rates $g_1$ and $g_2$ can be adjusted such that the per-capita growth rate and relative price dynamic is matched. Finally, the rate of time preference is set in such a way that the Euler equation clears. Appendix C provides a full calibration of the model, which replicates the per-capita growth, price dynamic, structural change and labor income share in the U.S. economy. This emphasizes again the paper's main contribution: Providing a theory, which quantitatively replicates the structural change and relative price dynamic in the U.S. within a framework featuring balanced growth on the aggregate.

[31] For income quintiles instead of the 13 income groups analogous regressions with the sample 1984-2009 give for $\epsilon$ statistically highly significant estimates between 0.19 and 0.32 (not reported).

[32] In 1946, the goods sector accounted for 60% of total personal consumption expenditures. Then, the change in expenditure share attributed to the substitution effect is equal to an annualized rate of $(-0.5 + 0.25 \cdot 0.6) \cdot 1.6 = 0.56$ percents (see proposition 3). The expenditure share of goods has declined to 32% in 2009.

Dependent variable: Log expenditure share devoted to goods

| | (1) | (2) | (3) | (4) | (5) | (6) |
|---|---|---|---|---|---|---|
| Log per-capita exp. | $-0.183^{***}$ | $-0.269^{***}$ | $-0.274^{***}$ | $-0.277^{***}$ | $-0.230^{***}$ | $-0.235^{***}$ |
| | (0.009) | (0.019) | (0.021) | (0.023) | (0.020) | (0.037) |
| Size | | $0.060^{***}$ | | $0.046^{*}$ | 0.010 | 0.030 |
| | | (0.012) | | (0.024) | (0.016) | (0.026) |
| Children share | | | 0.243 | 0.144 | 0.089 | 0.152 |
| | | | (0.169) | (0.177) | (0.115) | (0.176) |
| Percent Black | | | $-0.006^{***}$ | $-0.003$ | $-0.002$ | $-0.003$ |
| | | | (0.002) | (0.002) | (0.002) | (0.002) |
| Elderly share | | | | 0.007 | $0.173^{***}$ | 0.027 |
| | | | | (0.097) | (0.048) | (0.095) |
| Percent Male | | | | $-0.001$ | $0.002^{***}$ | $-0.001$ |
| | | | | (0.001) | (0.000) | (0.001) |
| Obs. | 91 | 91 | 91 | 91 | 235 | 84 |
| $R^2$ | 0.838 | 0.874 | 0.874 | 0.880 | 0.759 | 0.798 |
| Time fixed effects | yes | yes | yes | yes | yes | yes |
| Considered years | 2003-2009 | 2003-2009 | 2003-2009 | 2003-2009 | 1984-2009 | 2003-2009 |

Table 1: Cross-sectional estimation of $\epsilon$

**Notes:** Standard errors in parenthesis. *** significant at 1 percent, ** significant at 5 percent, * significant at 10 percent. "Size" is the consumption unit size. "Children share" and "Elderly share" measures the household share with age $< 18$ and $\geq 65$, respectively. "Percent Male" and "Percent Black" refers to the corresponding percentages of reference persons. The 13 pre-tax income groups are: -4'999$, 5'000-9'999$, 10'000-14'999$, 15'000-19'999$ 20'000-29'999$, 30'000-39'999$, 40'000-49'999$, 50'000-69'000$, 70'000-79'000$, 80'000-99'000$, 100'000-119'000$, 120'000-149'000$ and 150'000$-. For the years 1984-2002 only the first 8 income classes are available.

# 4 Conclusion

This paper presented a parsimonious growth theory, which is consistent with structural change, relative price dynamics and the Kaldor facts. The model allows us to analyze both explanations of structural change - income and substitution effects - simultaneously. To the best of my knowledge, such a theory did not exist yet. The theory can quantitatively replicate the structural change, expenditure growth and relative price dynamic observed in the data. Moreover, it is consistent with cross-sectional differences in the expenditure structure.

The theory has been motivated and applied to the structural change between goods and services in the U.S. after World War II. But more generally, the theoretical model offers a simple framework in which structural change can be discussed. Consequently, the theory may prove to be useful also in other contexts, where changes in the sectoral economic structure are of relevance.

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# Appendix A: Proofs of lemmas and propositions

## Proof of lemma 1

**Proof of part (i):** (2) corresponds to the expenditure function

$$e(P_1(t), P_2(t), V_i(t)) = \left[\epsilon \left[V_i(t) + \frac{\beta}{\gamma}\left[\frac{P_1(t)}{P_2(t)}\right]^{\gamma} + \frac{1}{\epsilon} - \frac{\beta}{\gamma}\right]\right]^{\frac{1}{\epsilon}} P_2(t). \tag{39}$$

According to proposition 3.E.2 of Mas-Colell, Whinston and Green (1995), for an expenditure function to represent a locally non-satiated preference relation it has to be (a) homogeneous of degree one in prices, (b) strictly increasing in $V_i(t)$ and non-decreasing in all prices, (c) concave in prices and (d) continuous in prices and $V_i(t)$. It is readily to see, that (a) and (d) are fulfilled. So I start proving (b). If we take the first derivative with respect to $V_i(t)$ and use (39), we get $\frac{\partial e(\cdot)}{\partial V_i(t)} = \left[\frac{e(\cdot)}{P_2(t)}\right]^{1-\epsilon} P_2(t) > 0$. For the derivative with respect to prices we obtain (using (39) again) $\frac{\partial e(\cdot)}{\partial P_1(t)} = \beta\left[\frac{e(\cdot)}{P_2(t)}\right]^{1-\epsilon}\left[\frac{P_2(t)}{P_1(t)}\right]^{1-\gamma} > 0$ and $\frac{\partial e(\cdot)}{\partial P_2(t)} = \left[\frac{e(\cdot)}{P_2(t)}\right]^{1-\epsilon}\left[\left[\frac{e(\cdot)}{P_2(t)}\right]^{\epsilon} - \beta\left[\frac{P_1(t)}{P_2(t)}\right]^{\gamma}\right]$, which is positive as long as (3) holds (note that $\gamma \geq \epsilon$). This proves (b). To prove (c) we show that the Hessian reads

$$H = \Xi \begin{pmatrix} \frac{P_2(t)}{P_1(t)} & -1 \\ -1 & \frac{P_1(t)}{P_2(t)} \end{pmatrix},$$

where $\Xi = \beta\left[\frac{e(\cdot)}{P_2(t)}\right]^{1-2\epsilon} P_1(t)^{\gamma-1} P_2(t)^{-\gamma}\left[\beta(1-\epsilon)\left[\frac{P_1(t)}{P_2(t)}\right]^{\gamma} - (1-\gamma)\left[\frac{e(\cdot)}{P_2(t)}\right]^{\epsilon}\right]$. The eigenvalues are 0 and $\Xi\left[\frac{P_2(t)}{P_1(t)} + \frac{P_1(t)}{P_2(t)}\right]$. So both eigenvalues are less or equal to zero if $(1-\gamma)\left[\frac{e(\cdot)}{P_2(t)}\right]^{\epsilon} \geq \beta(1-\epsilon)\left[\frac{P_1(t)}{P_2(t)}\right]^{\gamma}$, which is guaranteed by (3). This proves (c) and completes the proof of part (i).

□

**Proof of part (ii):** $V(\cdot)$ is clearly increasing in $e_i(t)$ (see proof of part (i)). For the second derivative we get $\frac{\partial^2 V_i(t)}{\partial e_i(t)^2} = -(1-\epsilon)e_i(t)^{\epsilon-2}P_2(t)^{-\epsilon} < 0$.

□

## Proof of lemma 3

**Proof of part (i):** The Allen-Uzawa formula for the elasticity of substitution reads $\sigma_i(t) = \frac{\partial x_1^{i,H}(t)}{\partial P_2(t)} \frac{e_i(t)}{x_1^{i,H}(t) x_2^{i,H}(t)}$, where $x_j^{i,H}(t)$ is the Hicksian per-capita demand of household $i$ for sector $j = 1, 2$. Plugging in the expressions, simplifying and substituting (2) by $V_i(t)$, we obtain (7). With $\gamma > 0$ and (3), $\sigma_i(t)$ is clearly strictly smaller than one since $\gamma \geq \epsilon$. This proves part (i). For $\gamma = \epsilon$ the elasticity of substitution simplifies to $1 - \gamma$.

□

**Proof of part (ii):** The expenditure elasticity of demand for goods is $1 - \epsilon$, which is strictly less than one if and only if $\epsilon > 0$. Then, Engel aggregation implies that the expenditure elasticity of services is larger than one.

□

## Proof of lemma 4

The current value Hamiltonian of the household's intertemporal optimization is given by

$$\mathcal{H} = V(P_1(t), P_2(t), e_i(t)) + \lambda_i(t) \left[ a_i(t) \left[ r(t) - n \right] + w(t) l_i - e_i(t) \right].$$

We can then derive the first-order conditions

$$\dot{\lambda}_i(t) = \lambda_i(t) \left[ \rho - r(t) \right] \text{ and } e_i(t)^{\epsilon - 1} P_2(t)^{-\epsilon} = \lambda_i(t). \tag{40}$$

The transversality condition can be written as

$$\lim_{t \to \infty} \lambda_i(t) a_i(t) \exp\left[ -(\rho - n) t \right] = 0. \tag{41}$$

Finally, the first-order conditions, (40), simplify to (10).

□

## Proof of proposition 1

**Proof of part (i):** (11) and the definition of $E(t)$ yield $g_{e_i}(t) = g_E(t) - n$. Using this in (10) gives (12). Substituting $e_i(t)$ in (8) by $e_i(0)\exp\left[\int_0^t g_E(\varsigma) - n\, d\varsigma\right]$ gives (13). Using $\frac{\dot{\lambda}_i(t)}{\lambda_i(t)} = \rho - r(t)$ (see (40)) in (41) and ignoring the positive constant $\lambda_i(0)$ give (14).

□

**Proof of part (ii):** Aggregation of individual demands gives

$$X_1(t) = \beta P_1(t)^{-1} P_2(t)^{\epsilon} \left[\frac{P_1(t)}{P_2(t)}\right]^{\gamma} \left[\frac{E(t)}{N(t)}\right]^{-\epsilon} E(t)\phi(t), \tag{42}$$

$$X_2(t) = \frac{E(t)}{P_2(t)} - \beta P_2(t)^{\epsilon-1} \left[\frac{P_1(t)}{P_2(t)}\right]^{\gamma} \left[\frac{E(t)}{N(t)}\right]^{-\epsilon} E(t)\phi(t), \tag{43}$$

where $\phi(t) \equiv \int_0^1 \left[\frac{e_i(t)N(t)}{E(t)}\right]^{1-\epsilon} di$. Since $\phi(t)$ is a scale invariant measurement of inequality of per-capita consumption expenditures across households, (11) implies $\phi(t) = \phi(0) \equiv \phi$, $\forall t$. Then, (42) and (43) imply (15) and (16).

□

**Proof of part (iii):** (6) and (15) show that a household exhibits the same expenditure share as the aggregate economy if and only if $e_i(t) = \frac{E(t)}{N(t)}\phi^{-\frac{1}{\epsilon}}$.

□

## Proof of lemma 5

Optimization implies that the marginal rate of technical substitution is equal to the relative factor price, i.e.

$$\frac{w(t)}{R(t)} = \frac{\alpha}{1-\alpha}\frac{K_j(t)}{L_j(t)},\ j = 1, 2. \tag{44}$$

Hence, we have $\frac{K_1(t)}{L_1(t)} = \frac{K_2(t)}{L_2(t)}$. With (20), this gives (27). Next, the rental rate has to equalize the valued marginal products across all sectors. This yields

$$R(t) = A = (1-\alpha)\left[\frac{L(t)}{K_1(t) + K_2(t)}\right]^{\alpha} P_j(t)\exp\left[g_j t\right],\ j = 1, 2,$$

where (27) has been used. Solving for $P_j(t)$ gives (25). Using $R(t) = A$ and (27) in (44) yields (24). Finally, with (27), the production functions can be rewritten as (26).

□

## Proof of proposition 2

**Proof of part (i):** First, we show that there exists a unique equilibrium in which $g_e(t)$ grows at a constant rate. (16), (21), (25) and (26) imply $E(t) = \frac{A}{1-\alpha}\left[K_1(t) + K_2(t)\right]$. Hence, we have $g_E(t) = g_e(t) + n = g_{K_1+K_2}(t)$. Using this in (25) yields (34). Plugging (23) and (34) into (12) we get $\left[1 - (1-\alpha)\epsilon\right] g_e(t) = A - \delta - \rho + \epsilon g_2$. This proves that we have $g_e(t) = g_e^*$, $\forall t$, in equilibrium. Next, we show that - given $g_e(t) = g_e^*$ - the transversality condition holds if and only if per-capita wealth grows at rate $g_e^*$ too. With (23), the transversality condition, (14), can be rewritten as

$$\lim_{t\to\infty} a_i(t) \exp\left[-(A - n - \delta)t\right] = 0, \ \forall i. \tag{45}$$

(24), $g_E^* = g_{K_1+K_2}^*$ and $g_E(t) = g_e^* + n$ yield $g_w = g_e^*$. Then, with (23), the flow budget constraint, (13), simplifies to $\dot{a}_i(t) = \left[A - \delta - n\right] a_i(t) - \left[e_i(0) - w(0)l_i\right] \exp\left[g_e^* t\right]$. This linear differential equation has the following solution (see e.g. Acemoglu (2009), Section B.4)

$$a_i(t) = \mathcal{A}_i \exp\left[(A - \delta - n)\, t\right] + \frac{e_i(0) - w(0)l_i}{A - \delta - n - g_e^*} \exp\left[g_e^* t\right], \tag{46}$$

where $\mathcal{A}_i$ is a constant which is to be determined. Using this expression in (45) we get

$$\lim_{t\to\infty} \mathcal{A}_i + \frac{e_i(0) - w(0)l_i}{A - \delta - n - g_e^*} \exp\left[-\left(A - \delta - n - g_e^*\right) t\right] = 0.$$

Then, the transversality condition is fulfilled if and only if $\mathcal{A}_i = 0$ (note that (28) ensures that $A - \delta - n - g_e^* > 0$). $\mathcal{A}_i = 0$ implies that $a_i(t)$ grows at constant rate $g_e^*$. Since this is the case for all households $i \in [0, 1]$, this proves uniqueness of the equilibrium path with $g_E^* = g_K^*$.

Next, we show that (30) and (31) jointly ensure condition (3) for all individuals at each date. The poorest household has no wealth, i.e. $a_i(0) = 0$, and a labor

endowment of $\bar{l}$. Individuals with no wealth consume the entire income (see (46)). Hence, the poorest household exhibits $e_i(t) = w(t)\bar{l}$, $\forall t$. Then, in the view of (25), at $t = 0$, condition (3) can be rewritten as

$$w(0)^{\epsilon}\bar{l}^{\epsilon} \geq \beta \left[\frac{1-\epsilon}{1-\gamma}\right] \left[\frac{A}{1-\alpha}\right]^{\epsilon} \left[\frac{K_1(0)+K_2(0)}{L(0)}\right]^{\alpha\epsilon}. \tag{47}$$

Note that (17) and (20) yield $\frac{K_1(t)+K_2(t)}{K(t)} = \frac{A-\delta-g_K^*}{A}$ and we have $\frac{K_1(0)+K_2(0)}{L(0)} = \frac{w(0)}{A}\frac{1-\alpha}{\alpha}$ (see (24)). Then, (47) can be written as

$$\alpha^{\epsilon}\bar{l}^{\epsilon} \geq \beta \left[\frac{1-\epsilon}{1-\gamma}\right] \left[\frac{L(0)}{K(0)}\frac{A}{A-\delta-g_K^*}\right]^{\epsilon(1-\alpha)}.$$

Plugging in the expression for $g_K^*$, we see that this condition coincides with (30). The nominal expenditure levels and all prices grow in equilibrium at constant rates. Hence, given condition (3) holds at date $t = 0$, it also holds for $t > 0$ if $\epsilon(g_E^* - n) \geq \gamma g_{P_1}^* + (\epsilon - \gamma)g_{P_2}^*$. This is guaranteed by condition (31) and completes the proof of part (i).

□

**Proof of part (ii):** According to (15), $S_1(t)$ changes at rate $g_{S_1}(t) = -\epsilon\left[g_e(t) - g_{P_2}(t)\right] - \gamma\left[g_{P_2}(t) - g_{P_1}(t)\right]$. In equilibrium, this expression reduces to (35). The goods market clearing condition can be rewritten in growth rates as $g_{S_1}(t) = g_{P_1}(t) + g_{X_1}(t) - g_E^* \leq 0$. With (34), $g_{X_1}(t) = g_1 + \alpha g_{L_1}(t) + (1-\alpha)g_{K_1}(t)$ and $g_{K_1}(t) - g_{L_1} = g_K^* - n$ (see (27)) this implies (36). Finally, (34) follows immediately from (37).

□

## Proof of proposition 3

**Proof of part (i):** The individual expenditure shares are given by (6). Then, since $g_{e_i} = g_E^* - n$, $\forall i$, the individual expenditure shares devoted to goods grow at the same rate as their aggregate counterpart.

□

**Proof of part (ii):** In terms of prices and attained utility level, $V_i(t)$ the expenditure share of goods of household $i$ is given by (see (39) and (6))

$$s_1^i(t) = \beta \left[\epsilon \left[V_i(t) + \frac{\beta}{\gamma}\left[\frac{P_1(t)}{P_2(t)}\right]^{\gamma} + \frac{1}{\epsilon} - \frac{\beta}{\gamma}\right]\right]^{-1} \left[\frac{P_1(t)}{P_2(t)}\right]^{\gamma}. \tag{48}$$

In view of (48), the elasticity of $s_1^i(t)$ with respect to $\frac{P_1(t)}{P_2(t)}$ reads $-\gamma+\epsilon\beta\left[\frac{P_2}{e_i(t)}\right]^{\epsilon}\left[\frac{P_1(t)}{P_2(t)}\right]^{\gamma}$, which can be rewritten as $-\gamma+\epsilon s_1^i(t)$. This expression is non-positive since $s_1^i(t) \leq 1$ and $\gamma \geq \epsilon$.

□

**Proof of part (iii):** For the elasticity of $s_1^i(t)$ with respect to $V_i(t)$ we obtain $-\epsilon$.

□

## Proof of proposition 4

Because (31) holds with strict inequality, the expenditure share devoted to goods decreases at a constant rate and reaches asymptotically a value zero. This proves part (i). The expenditure elasticity of demand for goods is $1-\epsilon$ for all households at each date (see proof of part (ii) of lemma 3). Since we have growth in per-capita terms (see (32)), the expenditure level of all household goes to infinity. Then, the expenditure elasticity of demand for services reaches asymptotically unity and the elasticity of substitution reduces to $1-\gamma$ (see (7)). This proves part (ii) and (iii).

□

# Appendix B

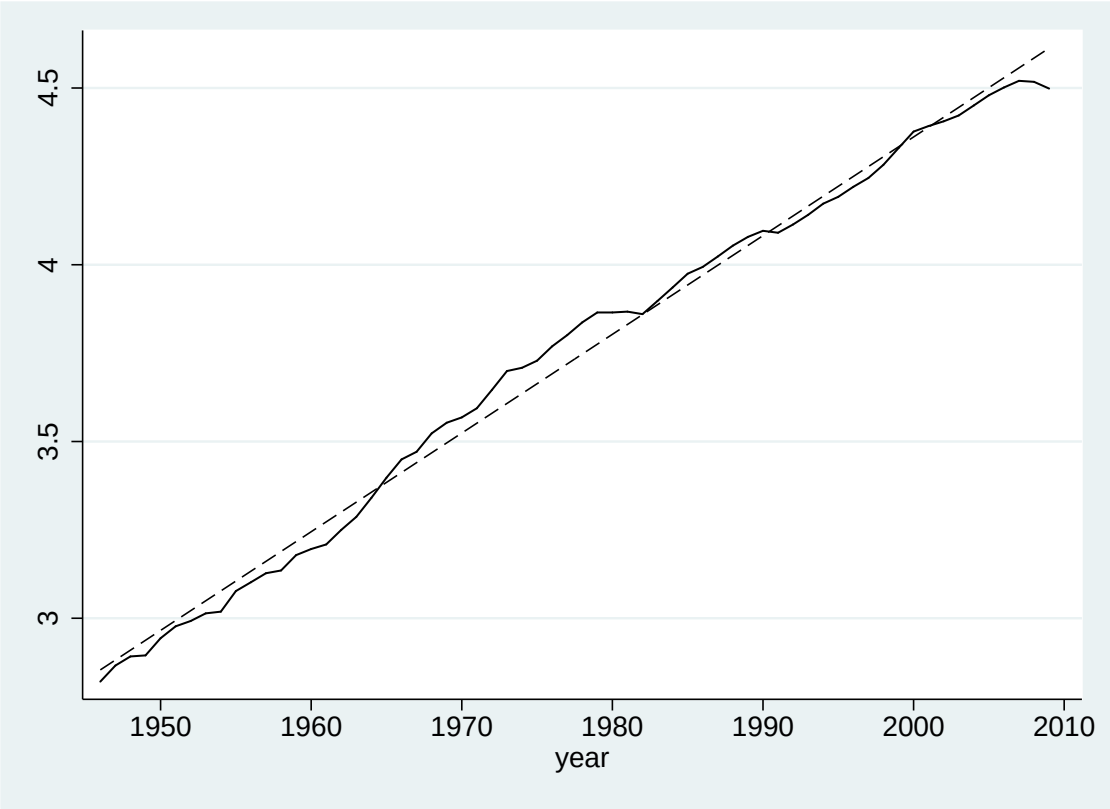


Figure 7: **Logarithmized aggregate expenditures in terms of services**

**Notes:** The figure plots the logarithmized aggregate expenditures in terms of services and a linear fit. The slope of the linear fit is equal to 0.028. Source: BEA, NIPA tables 1.1.4 and 1.1.5.

# Appendix C: Calibration

The theoretical model of section 2 has 10 parameters: the preference parameters, $\rho$, $\beta$, $\gamma$ and $\epsilon$, the technology parameters $\alpha$, $\delta$, $g_1$, $g_2$, and $A$ and the demographical parameter $n$. Motivated by the estimates of table 1, we chose $\epsilon = 0.25$ and $\gamma = 0.5$. To match the average employment growth, we set $n = 0.02$. For simplicity, I assume zero depreciations. The real (in investment goods denominated) interest rate is set to 10%, i.e. $A = 0.1$. We chose $\rho$, $g_1$ and $g_2$ such that the model replicates the output growth, relative price dynamics and the aggregate labor income share, $\frac{\alpha\left[A-g_E^*-\delta\right]}{A-\alpha\left[g_E^*+\delta\right]}$,

| parameter | value | target | data |
|---|---|---|---|
| population growth, $n$ | 0.0200 | annual labor force growth | 0.020 |
| output elasticity of labor, $\alpha$ | 0.7748 | labor income share | 0.670 |
| TFP goods sector, $g_1$ | 0.0193 | price dynamic of goods, $g^*_{P_1}$ | $-0.003$ |
| TFP service sector, $g_2$ | 0.0033 | price dynamic of services, $g^*_{P_2}$ | 0.013 |
| rate of time preference, $\rho$ | 0.1008 | output growth, $g_E$ | 0.041 |

Table 2: Calibration of the numerical example

**Notes:** The real interest rate, $A - \delta$, is set to 10%, whereas the depreciation rate, $\delta$, is assumed to be zero. For $\gamma$ and $\epsilon$ values of 0.5, and 0.25 are chosen. These values are according to the estimations of table 1 reasonable and match - for the given output growth and price dynamic - the observed structural change.

of the U.S. economy. The corresponding values are $g^*_E = 0.041$, $g^*_{P_1} = -0.003$, $g^*_{P_2} = 0.013$ and 0.67 for the labor income share. To match the labor income share we need $\alpha = 0.7748$. Then, according to (34) we have $g_1 = 0.0193$ and $g_2 = 0.0033$. Finally, the Euler equation, (12), gives $\rho = 0.1008$. The chosen parameter values are summarized in table 2. With these parameter values, the model perfectly replicates the structural change and price trend observed in figure 1 and 2 (see dashed lines). The expenditure share and the relative price of goods decrease at constant rates $g^*_{S_1} = -0.01$ and $g^*_{P_1} - g^*_{P_2} = -0.016$, respectively. The preference parameter $\beta$ can be chosen such that the level of expenditure shares is met. On the aggregate, standard properties of balanced growth are matched. The real interest rate is constant and the real per-capita growth rate in terms of investment goods is, as in the data, 2.1%. The capital-output ratio, the saving rate and labor income share are constant at 3.3, 0.135 and 0.67, respectively (see footnote 22).

# Appendix D: Equilibrium dynamic with factor intensity differences

Suppose the technologies are instead of (18)

$$Y_j(t) = \frac{\exp[g_j t]}{\alpha_j^{\alpha_j}(1-\alpha_j)^{1-\alpha_j}} L_j(t)^{\alpha_j} K_j(t)^{1-\alpha_j},\ j = 1, 2,$$

with $\alpha_j \in (0,1)$, $j = 1, 2$ and $\alpha_1 \neq \alpha_2$. We assume that the relative factor endowments are the same for all households, i.e. $\frac{a_i(0)}{l_i} = \frac{K(0)}{L(0)}$, $\forall i$. Under this assumption, changes in the expenditure shares - which will now affect the relative factor reward $\frac{R(t)}{w(t)}$ - do not affect the inequality measurement, $\phi$.[33] Finally, we assume that condition (3) holds with strict inequality.[34]

With factor intensity differences lemma 5 will not hold anymore. Together with zero profits and market clearing, firm's cost minimization yields $w(t)L_1(t) = \alpha_1 S_1(t)E(t)$ and $w(t)L_2(t) = \alpha_2 [1 - S_1(t)] E(t)$. Combining these expressions with the labor market clearing condition gives

$$w(t) = \frac{E(t)}{L(t)} [\alpha_2 + S_1(t) [\alpha_1 - \alpha_2]]. \tag{49}$$

The AK technology of the investment goods sector is unchanged. Consequently, we still have $r = R - \delta = A - \delta$. Equilibrium prices are given by

$$P_j(t) = \exp[-g_j t]\, w(t)^{\alpha_j} A^{1-\alpha_j},\ j = 1, 2. \tag{50}$$

Combining (15) with (49), (50) and the definition of $L(t)$ we obtain

$$S_1(t) = \tilde{\beta} \left[\frac{E(t)}{L(t)}\right]^{\alpha_1\gamma-(\gamma-\epsilon)\alpha_2-\epsilon} L(0)^{-\epsilon} [\alpha_2 + S_1(t)(\alpha_1 - \alpha_2)]^{\alpha_1\gamma-(\gamma-\epsilon)\alpha_2}, \tag{51}$$

where $\tilde{\beta} \equiv \beta\phi A^{\epsilon+\alpha_2(\gamma-\epsilon)-\alpha_1\gamma} \exp[((\gamma-\epsilon)g_2 - \gamma g_1)t]$. Differentiating (51) with respect to time gives

$$\frac{\dot{S}_1(t)}{S_1(t)} = \hat{\gamma}\left[\frac{\dot{E}(t)}{E(t)} - n\right] + (\gamma-\epsilon)g_2 - \gamma g_1 + [\alpha_1\gamma - (\gamma-\epsilon)\alpha_2] \frac{\dot{S}_1(t)[\alpha_1 - \alpha_2]}{\alpha_2 + S_1(t)[\alpha_1 - \alpha_2]}, \tag{52}$$

[33] Without this assumption, the joint $l_i$ and $a_i(0)$ distribution would have to be specified and potentially multiple equilibria arise.

[34] This assumption shortens the subsequent proofs, since a separate discussion of the case in which - by coincidence - $S_1(0) = 1$, can be avoided.

where $\hat{\gamma} \equiv \alpha_1\gamma - (\gamma - \epsilon)\alpha_2 - \epsilon$. With (49) and (50) the Euler equation (12) can be written as

$$[1 - \epsilon(1 - \alpha_2)] \left[\frac{\dot{E}(t)}{E(t)} - n\right] = A - \delta - \rho + \epsilon g_2 - \frac{\epsilon\alpha_2\dot{S}_1(t)\,[\alpha_1 - \alpha_2]}{\alpha_2 + S_1(t)\,[\alpha_1 - \alpha_2]}. \tag{53}$$

Finally, the law of motion of the capital stock is given by $\frac{\dot{K}(t)}{K(t)} = A - \delta + \frac{w(t)L(t)}{K(t)} - \frac{E(t)}{K(t)}$. With (49) this can be written as

$$\frac{\dot{K}(t)}{K(t)} = A - \delta - \frac{E(t)}{K(t)}\left[1 - \alpha_2 - S_1(t)(\alpha_1 - \alpha_2)\right]. \tag{54}$$

Equations (52), (53), (54) and the transversality condition define the evolution of $S_1(t)$, $E(t)$ and $K(t)$. $K(0)$ is exogenously given. The non-predetermined $E(0)$ implicitly pins down $S_1(0)$ according to (51).[35]

A constant growth path (CGP) is defined according to Acemoglu and Guerrieri (2008) as an equilibrium growth path along which expenditures grow at a constant rate. We have the following proposition:

**Proposition 5.** *Suppose we have*

$$(\gamma - \epsilon)g_2 - \gamma g_1 + [\alpha_1\gamma - (\gamma - \epsilon)\alpha_2 - \epsilon]\,\frac{A - \delta - \rho + \epsilon g_2}{1 - (1 - \alpha_2)\epsilon} < 0, \tag{55}$$

*and let us denote asymptotic values by a superscript $A$ (i.e. $z^A = \lim_{t\to\infty} z(t)$, for $z = S_1, g_E, g_K, g_w, g_{S_1}$). Then, there exists a globally saddle-path stable CGP with*

$$S_1^A = 0,$$

$$g_E^A - n = g_K^A - n = g_w^A = \frac{A - \delta - \rho + \epsilon g_2}{1 - (1 - \alpha_2)\epsilon},$$

$$g_{S_1}^A = (\gamma - \epsilon)g_2 - \gamma g_1 + [\alpha_1\gamma - (\gamma - \epsilon)\alpha_2 - \epsilon]\left[g_E^A - n\right] < 0.$$

[35] For any given $E(t)$, exactly one unique $S_1(t) \in (0, 1)$ fulfills (51). To see this, note that at $S_1(t) = 0$, the left-hand side (LHS) of (51) is zero, whereas the right-hand side (RHS) is $\in (0, 1)$ (the upper bound is ensured by the strict inequality of condition (3)). On the contrary, with $S_1(t) = 1$ we have LHS $= 1 >$ RHS. Hence, since both are continuous functions, there is at least one intersection between 0 and 1. Finally, since the LHS is linear and the RHS is either concave or convex on the entire domain we have at most two intersections. Hence, LHS and RHS cross exactly once between 0 and 1.

*Proof.* With the expressions for $g_E^A$ and $g_{S_1}^A$, (52) and (53) can be rewritten as

$$\frac{\dot{S}_1(t)}{S_1(t)} = \left[\frac{\dot{E}(t)}{E(t)} - g_E^A\right]\hat{\gamma} + [\alpha_1\gamma - (\gamma-\epsilon)\alpha_2]\frac{\dot{S}_1(t)\,[\alpha_1-\alpha_2]}{\alpha_2 + S_1(t)\,[\alpha_1-\alpha_2]} + g_{S_1}^A,$$

and

$$\frac{\dot{E}(t)}{E(t)} - g_E^A = -\frac{\epsilon\alpha_2\dot{S}_1(t)\,[\alpha_1-\alpha_2]}{[1-\epsilon(1-\alpha_2)]\,[\alpha_2 + S_1(t)\,[\alpha_1-\alpha_2]]}. \tag{56}$$

Solving these two equations for $\dot{S}_1(t)$ gives

$$\dot{S}_1(t) = \frac{g_{S_1}^A S_1(t)\left[S_1(t) + \frac{\alpha_2}{\alpha_1-\alpha_2}\right]}{\frac{\alpha_2}{\alpha_1-\alpha_2} + S_1(t)\left[1-\alpha_1\gamma + (\gamma-\epsilon)\alpha_2 + \alpha_2\epsilon\frac{\hat{\gamma}}{q}\right]},$$

where $q \equiv 1-\epsilon(1-\alpha_2)$. Hence, $\dot{S}_1(t)$ is zero if and only if $S_1(t) = 0$ (note that $S_1(t) \in [0,1]$). The equilibrium with $\dot{S}_1(t) = S_1(t) = 0$ is stable since

$$\left.\frac{\partial\dot{S}_1(t)}{\partial S_1(t)}\right|_{S_1(t)=0} = g_{S_1}^A < 0.$$

Hence, no matter where we start, $S_1(t)$ will always converge to $S_1^A = 0$ and consequently $\frac{\dot{E}(t)}{E(t)}$ will converge to $g_E^A$ (see (56)). Hence, asymptotically we have $\frac{\dot{K}(t)}{K(t)} = A-\delta-\frac{E(t)}{K(t)}\,[1-\alpha_2]$ and $E(t)$ grows at a constant rate. This is exactly the same structure as in the equilibrium of the main text. Then, by the identical argument as in the proof of proposition 2, the transversality condition is violated unless $\frac{\dot{K}(t)}{K(t)} = \frac{\dot{E}(t)}{E(t)}$. □

The CGP is very similar to the one in Acemoglu and Guerrieri (2008). But, the important difference to Acemoglu and Guerrieri (2008) is that this model features an income effect (as long as $\epsilon > 0$). In contrast to the asymptotic equilibrium in the main text (see proposition 4), the structural change is now also governed by the sectoral differences in the output elasticities of labor. The intuition is the same as in Acemoglu and Guerrieri (2008): We have capital deepening, whereby the relative factor price of labor, $\frac{w(t)}{R(t)}$, increases over time. This increases the relative price of the sector, which is more labor intensive. Finally, according to the substitution effect, this relative price drift affects the structural change. Condition (55) ensures $g_{S_1}^A < 0$ and guarantees global stability.

# Appendix E: Endogenizing $g_1$ and $g_2$

Suppose that, instead of (18), the production functions read

$$Y_1(t) = \tilde{\Delta}\chi_1^c(t)^{1-\Delta}\chi_1^a(t)^{\Delta}, \text{ and } Y_2(t) = \tilde{\Delta}\chi_2^c(t)^{1-\Delta}\chi_2^b(t)^{\Delta},$$

with $\tilde{\Delta} \equiv \frac{1}{\Delta^{\Delta}(1-\Delta)^{1-\Delta}}$. $\chi^a(t)$, $\chi^b(t)$ and $\chi^c(t)$ are three different intermediate inputs. $\chi^a(t)$, $\chi^b(t)$ are sector specific inputs, whereas $\chi^c(t)$ is a "general" input, which can be used in both sectors. $\Delta \in (0,1)$ is a measure of differences in production processes between goods and services.[36] Intermediate inputs are CES aggregators of different input-specific sets of available machines, $m_{\omega^l}(t)$,

$$\chi^l(t) = \left[\int_0^{M_l(t)} m_{\omega^l}(t)^{\frac{\nu-1}{\nu}}\right]^{\frac{\nu}{\nu-1}} d\omega^l, \; l = a, c,$$

and

$$\chi^b(t) = M_b(t)^{-\frac{1}{\nu-1}}\left[\int_0^{M_b(t)} m_{\omega^b}(t)^{\frac{\nu-1}{\nu}}\right]^{\frac{\nu}{\nu-1}} d\omega^b,$$

where $\nu > 1$. The measures of available machine varieties $M_l(t)$ are time varying. The technology of service specific intermediates, $\chi^b(t)$, does not allow for productivity gains due to specialization. Production of intermediates as well as goods and services is competitive. Market clearing implies $\chi_1^a(t) = \chi^a(t)$, $\chi_1^b(t) = \chi^b(t)$ and $\chi_1^c(t)+\chi_2^c(t) = \chi^c(t)$. Each machine type $\omega^l$ suitable in production of input $l = a, b, c$ is produced by a monopolist according to the following production function

$$m_{\omega^l}(t) = \frac{\nu}{(\nu-1)\alpha^{\alpha}(1-\alpha)^{1-\alpha}} L_{\omega^l}(t)^{\alpha} K_{\omega^l}(t)^{1-\alpha}, \; \forall \omega^a, \omega^b, \omega^c,$$

where $\alpha \in (0,1)$. $L_{\omega^l}(t)$ and $K_{\omega^l}(t)$ denote labor and capital, respectively, used in firm $\omega^l$ at date $t$. To simplify the expressions, we set henceforth $\nu = 2$. Monopolistically competitive machine producers face an iso-elastic demand and maximize their profits taking the wage and rental rate as given. Hence, all machine producers set their price equal to $w(t)^{\alpha}R(t)^{1-\alpha}$. The prices of intermediate inputs $l = a, b, c$ are then given by $p_l(t) = \frac{w(t)^{\alpha}R(t)^{1-\alpha}}{M_l(t)}$, $l = a, c$ and $p_b(t) = w(t)^{\alpha}R(t)^{1-\alpha}$. Then, under perfect competition, we have

$$P_1(t) = w(t)^{\alpha}R(t)^{1-\alpha}M_a(t)^{-\Delta}M_c(t)^{-(1-\Delta)}, \text{ and } P_2(t) = P_1(t)M_a(t)^{\Delta}. \tag{57}$$

[36] With $\Delta \to 0$ the production processes are identical.

A fraction $\Delta S_1(t)$ of total expenditures is spent on type $a$ machines. Then, because of symmetry and constancy of the markup, profits per firm are given by

$$\pi_{\omega^a}(t) = \frac{\Delta S_1(t)E(t)}{2M_a(t)}, \ \forall \omega^a. \tag{58}$$

The market size of all type $b$ and $c$ firms is $\Delta\left[1 - S_1(t)\right]E(t)$ and $(1-\Delta)E(t)$, respectively. Consequently, we have

$$\pi_{\omega^c}(t) = \frac{(1-\Delta)E(t)}{2M_c(t)}, \ \forall \omega^c, \text{ and } \pi_{\omega^b}(t) = \frac{\Delta\left[1 - S_1(t)\right]E(t)}{2M_b(t)}, \ \forall \omega^b. \tag{59}$$

Suppose a blueprint of a machine variety suitable in production of input $l = a, b, c$ can be invented according to a Cobb-Douglas production functions defined over the factor inputs labor and capital. Then, the innovation possibilities frontiers can be written as follows

$$\dot{M}_l(t) = \frac{1}{f_l \varkappa^{\varkappa}(1-\varkappa)^{1-\varkappa}} L_R^l(t)^{\varkappa} K_R^l(t)^{1-\varkappa}, \ l = a, b,$$

and

$$\dot{M}_c(t) = \frac{1}{f_c \vartheta^{\vartheta}(1-\vartheta)^{1-\vartheta}} L_R^c(t)^{\vartheta} K_R^c(t)^{1-\vartheta},$$

where $L_R^l(t)$ and $K_R^l(t)$ is labor and capital, respectively used for R&D directed to the intermediate sector $l$. $\vartheta \in (0,1)$ may differ from $\varkappa \in (0,1)$. $f_a$, $f_b$ and $f_c$ are positive constants. At date $t$ the value of a firm that produces machine $\omega^l$ is given by

$$v_{\omega^l}(t) = \int_t^{\infty} \pi_{\omega^l}(\tau) \exp\left[-\int_t^{\tau} r(\varsigma) d\varsigma\right] d\tau.$$

Henceforth, we consider an equilibrium with positive R&D investments in all sectors (i.e. firm values equalize R&D costs of a new blueprint). Moreover, let us focus on the constant growth path (CGP) of this economy, which is defined as an equilibrium path along which sectoral TFP growth rates (i.e. $g_{M_a}$ and $g_{M_c}$) are constant and a constant fraction of total labor and capital is devoted to R&D.

Along a CGP, the system of equilibrium conditions is similar to the equilibrium in the main text. Consequently, we have $g_E^* = g_K^* = g_w^* + n$ and $R(t) = A$ (using 2). With positive R&D investments we then have (where we used $g_w^* = g_E^* - n$)

$$v_{\omega^l}(t) = \frac{\pi_{\omega^l}(t)}{r - \varkappa(g_E^* - n)} = f_l w(t)^{\varkappa} A^{1-\varkappa}, \ l = a, b, \tag{60}$$

and

$$v_{\omega^c}(t) = \frac{\pi_{\omega^c}(t)}{r - \vartheta(g_E^* - n)} = f_c w(t)^{\vartheta} A^{1-\vartheta}.$$

Plugging in the expressions for the profits and differentiate both sides of these equations with respect to time yields

$$\varkappa(g_E^* - n) = g_{S_1}^* + g_E^* - g_{M_a}^*, \text{ and } \vartheta(g_E^* - n) = g_E^* - g_{M_c}^*. \tag{61}$$

The intuition is as follows: Along the CGP, total profits of the type $a$ machine sector grow at constant rate $g_{S_1}^* + g_E^*$ and market entry cost grow at rate $\varkappa(g_E^* - n)$. Hence in order to be consistent with zero ex ante profits, the growth rate of the number of firms has to fill the gap. Since the growth rate of the sector specific market depends on the pace of structural change, the growth rate $g_{M_a}^*$ is a function of $g_{S_1}^*$. Similarly, the total market size of type $c$ machines grows at rate $g_E^*$, whereas entry cost grow at rate $\vartheta(g_E^* - n)$. Hence, for zero ex ante profits the number of firms has to grow at the rate $g_E^* - \vartheta(g_E^* - n)$.
Then, according to (57), the sectoral prices grow along the CGP at the constant rates $g_{P_1}^* = \alpha(g_E^* - n) - \Delta g_{M_a}^* - (1-\Delta)g_{M_c}^*$ and $g_{P_2}^* = \alpha(g_E^* - n) - (1-\Delta)g_{M_c}^*$. With these price evolutions we obtain for the dynamic of $S_1$ (see (35))

$$g_{S_1}^* = -\gamma\Delta g_{M_a}^* - \epsilon\left[(1-\Delta)g_{M_c}^* + (1-\alpha)\left[g_E^* - n\right]\right]. \tag{62}$$

Finally, the Euler equation reads

$$(1 - (1-\alpha)\epsilon)\left[g_E^* - n\right] - \epsilon(1-\Delta)g_{M_c}^* = A - \delta - \rho. \tag{63}$$

We have the following proposition.

**Proposition 6.** *There exists a unique CGP with*

$$g_E^* = \frac{A - \delta - \rho + n\left[\epsilon(1-\Delta)\vartheta - \epsilon(1-\alpha) + 1\right]}{1 - \epsilon\left[(1-\alpha) + (1-\Delta)(1-\vartheta)\right]},$$

$$g_{M_c}^* = \frac{(1-\vartheta)(A - \delta - \rho) + n\left[1 - \epsilon(1-\alpha)\right]}{1 - \epsilon\left[(1-\alpha) + (1-\Delta)(1-\vartheta)\right]},$$

$$g_{M_a}^* = \frac{g_E^*(1 - \epsilon(1-\Delta)) + (g_E^* - n)(1 - \varkappa + \epsilon(\vartheta(1-\Delta) - (1-\alpha))}{1 + \Delta\gamma}.$$

*Proof.* Equations (61), (62) and (63) jointly define $g_E^*$, $g_{M_a}^*$, $g_{M_c}^*$ and $g_{S_1}^*$. Finally, to proof that this equilibrium path is a CGP, we have to show that a constant fraction of total resources is devoted to R&D. R&D investments of sector $c$ are given by $g_{M_c}^* M_c(t) w(t)^{\vartheta} R(t)^{1-\vartheta}$ which grows at rate $g_E^*$. For the type $a$ and $b$ machine sector we have (see (60) and (58))

$$f_a M_a(t) + f_b M_b(t) = \frac{\pi_{\omega^a}(t) M_a(t) + \pi_{\omega^b}(t) M_b(t)}{w(t)^{\varkappa} R(t)^{1-\varkappa}(r - \varkappa(g_E^* - n))} = \frac{\Delta E(t)}{2 w(t)^{\varkappa} R(t)^{1-\varkappa}(r - \varkappa(g_E^* - n))}.$$

Hence, R&D investments in type $a$ and $b$ machines are given by

$$\left[ f_a \dot{M}_a(t) + f_b \dot{M}_b(t) \right] w(t)^{\varkappa} R(t)^{1-\varkappa} = \frac{\Delta(g_E^* - \varkappa(g_E^* - n)) E(t)}{2(r - \varkappa(g_E^* - n))},$$

which grows at rate $g_E^*$ too. □

This proposition illustrates, that we can endogenize the sectoral TFP growth rates. The CGP is identical to the equilibrium in the main text with $g_1 = \Delta g_{M_a}^* + (1-\Delta) g_{M_c}^*$ and $g_2 = (1-\Delta) g_{M_c}^*$. The endogenization resembles the growth model without scale effects by Jones (1995). Nevertheless, notice the following two differences: First, since we have a multiple sector model with structural change, the sector specific TFP growth rate of goods depends also on the pace of structural change. Second, since the model contains capital accumulation as another source of endogenous growth, even without population growth (i.e. $n = 0$), we will have R&D investments along the CGP.