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The road not taken: competition and the R&D portfolio

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Abstract

When firms decide to invest in R&D, they have to choose not only the amount of resources to invest, but also which research projects to develop. This paper investigates the market portfolio of research projects. Contrary to most of the literature, which focuses only on the level of investment in innovation, this model captures both the variety of research projects undertaken and the amount of duplication of research. A characterization of the equilibrium market portfolio is provided. It is shown that an increase in the number of firms increases the variety of developed projects and increases the amount of duplication of research. An increase in the intensity of competition among firms leads to an increase in the variety of developed projects and a decrease in the amount of duplication of research. A characterization of the socially optimal portfolio is provided. It is shown under which conditions the market invests suboptimally in the variety and duplication of research projects. Market underinvestment in the variety of R&D projects is demonstrated for a large class of product market models.

Keywords: Innovation, competition, R&D portfolio, market structure

JEL Classification: L13, L22, O31

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1 Introduction

In 1998, the U.S. Department of Justice blocked the proposed merger of Lockheed Martin and Northrop Grumman, the largest proposed merger in the U.S. at the time. The merger would have reduced the number of firms supplying aircraft and electronic systems to the Department of Defense from three (including Boeing) to only two. According to Robinson (1999) and Rubinfeld and Hoven (2001), one of the main reasons why the Department of Justice, supported by the Department of Defense, opposed the merger was the concern that the merger would have had negative effects on innovation. However, the issue was not so much with the amount of funds invested in innovation, the bulk of which comes from the Department of Defense anyway.[1] Rather, the principal concern was that reducing the number of firms in the industry would reduce the diversity of approaches to innovation.

Existing literature on competition and innovation, in the long tradition of Schumpeter and Arrow, focuses for the most part on the level of investment in innovation. One of the main messages of this paper is that studying solely the level of investment in innovation is insufficient. What matters is not only the level of investment, but also the composition of the research portfolios of firms in the market. The level of investment in innovation does not determine the variety of approaches to innovation nor the amount of duplicative research that the firms undertake. However, both of these issues are relevant for society and for the firms which are pursuing the innovation.

This paper develops a model in which the questions of variety and duplication of research can be analyzed explicitly. There are N symmetric firms competing in a market. In the first stage, the firms can invest in innovation. There is a set of heterogeneous research projects and firms simultaneously choose the subset they wish to develop. The innovation is assumed to be drastic[2] and the discovery procedure is stochastic. All approaches are ex ante equally likely to be successful, but ex post only one approach will be successful. The approaches differ only in the cost needed to pursue them. There are no spillovers or patents. Each firm which invested in the successful approach receives the innovation while each firm that did not invest in the successful project receives nothing from its research. In the second stage, the firms compete on the product market either with or without the innovation.

Since all approaches are ex ante equally likely to be successful, the firms have an incentive to develop only the cheapest projects. However, the number of firms developing any given project also determines the number of firms which will compete on the product

[1] See Rubinfeld and Hoven (2001), p. 88.

[2] Innovation is drastic if whenever at least one firm innovates, firms without the innovation cannot compete. This is a standard assumption in the literature, introduced by Arrow (1962) see also Gilbert (2006). This assumption is not needed for the characterization of the equilibrium and is relaxed in Section 7.

market with the new technology. Thus, when choosing which projects to develop the firms face a trade off — cheaper approaches cost less to develop but will in equilibrium attract more competitors. From society's point of view, higher variety of research projects being developed is desirable because it increases the probability that the innovation will be discovered. On the other hand, more duplication of research projects is also desirable because it implies stronger product market competition ex post and lower deadweight loss. The market R&D portfolio is a function which captures how many firms are investing in each of the possible projects, and the variety of approaches is the fraction of projects which are developed by at least one firm.

I show that an equilibrium of the investment game always exists and that it is in general not unique. However, each equilibrium induces the same market R&D portfolio and furthermore each outcome of the investment game which induces the equilibrium market R&D portfolio is itself an equilibrium. Thus, the equilibrium market R&D portfolio is uniquely determined in equilibrium. Since the R&D portfolio captures both the variety of approaches to innovation and the extent of duplication of each research approach it is the main object of analysis in the paper. I provide a simple characterization of the equilibrium R&D portfolio and show that it follows a step function — with more expensive approaches being developed by fewer firms.

The characterization of the R&D portfolio is then used to derive comparative statics. I show that a decrease in the number of firms weakly decreases the variety of approaches to innovation and also weakly decreases the amount of duplication. Hence, a merger leads to a weakly decreasing variety of approaches to innovation. A policy implication drawn from this analysis is that the competition authorities should take into account the negative effects of a merger on the variety of approaches to innovation, in part giving theoretical foundation to the concern expressed in the Lockheed-Northrop case.

Next, I consider the effects of a change in the intensity of competition between firms, while keeping the number of firms constant. In the tradition of papers such as Schmidt (1997) and Schmutzler (2010), I define an increase in the intensity of competition as any exogenous change which decreases firm profits. An increase in the intensity of competition is shown to increase the variety of approaches to innovation and to decrease the amount of duplication in equilibrium. Thus, an increase in the intensity of product market competition leads to more specialized R&D portfolios. This illustrates why an increase in the intensity of competition can both increase and decrease the amount of resources invested in R&D — if the reduction in duplication of research efforts is greater than the increase in variety of research efforts, the total amount invested in R&D will decrease. If the opposite is true, the total amount invested in R&D will increase.

I provide a characterization of the socially optimal R&D portfolio and compare it with

the market R&D portfolio. I derive the condition under which the market investment in the variety of research approaches is optimal, too low or too high. Similarly, I derive the condition under which the market duplication of research approaches is optimal, too low or too high. I show that in a large class of homogeneous goods models, the market will always underinvest in the variety of approaches to process innovation. This result implies that if a government want to give R&D subsidies, these subsidies should be targeted at research approaches with high development costs and high potential payoffs.

The main body of the paper assumes that the innovation is drastic, firms are symmetric, have unlimited budgets and use only pure strategies. I consider the effects of relaxing these assumptions in turn and show that the equilibrium structure is in general robust. In particular, the assumption that the innovation is drastic is not needed at all for the characterization of equilibrium. It is only the comparative statics that become more complex if the innovation is non-drastic.

The outline of the paper is as follows. In Section 2, a brief overview of the related literature is provided. Section 3 describes the model. The equilibrium is characterized in Section 4. Comparative statics are analyzed in Section 5. The socially optimal portfolio and its relation to the market portfolio are analyzed in Section 6. In Section 7, I relax a number of assumptions made in the main body of the paper. Section 8 concludes. All mathematical proofs are relegated to the appendix.

2 Related literature

This paper contributes to the literature on the relationship between market structure and the incentives of the firms to invest in innovation. The focus of this literature is on the *amount of resources* that firms invest in R&D. Depending on the specifics of the model used, the literature finds that the competition in the marketplace can increase, decrease or have non-monotone effects on the amount invested in R&D. For surveys of this vast literature see Gilbert (2006), Sena (2004) and van Cayseele (1998). Vives (2008) and Schmutzler (2010) provide comprehensive studies for a range of market competition models and demand structures. Important contributions to this literature have been made from the endogenous growth literature, particularly from the models of *step-by-step innovations* (see for example Aghion et al. (2001) and Aghion et al. (2005)). This paper contributes to this literature by providing a model where not only the amount invested in innovation matters but also the composition of projects in which the firms invest. This approach allows us to consider how the variety and duplication of approaches to research change as the market structure changes, and how it relates to the socially optimal amount of variety and duplication of research, which is not possible with models which focus only

on the amount of investment.

While most papers in the theoretical literature on competition and innovation focus on the level of investment in innovation, several papers have considered the question how competing firms choose some aspect of the research strategy. Bhattacharya and Mookherjee (1986) and Klette and de Meza (1986) consider a model where undertaking research is like drawing a random variable. The maximum realization of the random variables determines the winner of the race (winner takes all) but also both private and social payoff. Firms choose a parameter of the density function, which determines the variance and in some scenarios the correlation of the research output. This parameter is interpreted as a research strategy of the firm. Dasgupta and Maskin (1987) consider a similar model. Results obtained by these models depend on the assumptions made about the distribution of research outcomes, but in a large class of cases, firms undertake excessive risk (because firms care who wins the race, while society only cares about the best research output; however see also Cabral (1994) and Kwon (2010) who find that the market is biased against risky research). At the same time, if reducing correlation is costly, firms will choose research strategies that are too correlated, since firms will not internalize the benefit low correlation confers to its opponent when its own research output is low. In this setting, the firms can choose only one research project (i.e., each firm chooses one parameter of the density function), hence these models cannot examine the variety of research projects. At the same time, in these papers the choice of correlation of the outcomes is interpreted as a measure of duplication. In this paper, duplication of research is literal — the firms can choose to pursue the same project.

Chatterjee and Evans (2004) present a dynamic model where two firms are searching for an innovation in a model of "buried treasure" (see also Fershtman and Rubinstein (1997)). There are two possible research projects and only one can yield the innovation, with the winner take all feature. However, developing the "right" project yields the innovation with some exogenously given probability. They find that the amount of correlation between research of the two firms can be too high or too low depending on the nature of asymmetry between the two research paths. Akcigit and Liu (2013) also consider a setup with two firms and two possible avenues for research, one is more profitable (in expectation) but may result in a dead-end and another which always yields a less profitable innovation if it is researched long enough. As opposed to Chatterjee and Evans (2004) they assume that firms cannot observe which research path their competitor is pursuing and they find that firms duplicate dead-end research and at the same time leave the risky research path too early. In contrast to this paper, this strand of literature assumes that firms can research only one project at a time, so the question of the choice of variety of research projects and the amount of duplicative research does not arise. This is, however,

the main focus of the present paper.

Most closely related to the present paper is the literature on multiproject innovation, which has been studied by Sah and Stiglitz (1987) and in the related work by Reynolds and Isaac (1992) and Farrell et al. (2003). Sah and Stiglitz (1987) assume that all projects are identical. The probability of success of any individual project depends only on the effort invested in this project and is independent of anything that might be happening with other projects. Using this setting and the Bertrand model of product market, Sah and Stiglitz (1987) show that the number of projects is invariant to the number of firms in the market, a result they refer to as the "strong invariance result." Reynolds and Isaac (1992) and Farrell et al. (2003) explore this setting further and show that the invariance result is sensitive to type of product market competition. In particular, they show that the invariance result does not hold under Cournot competition.

The main difference between this paper and the literature in tradition of Sah and Stiglitz (1987) is that here projects are assumed to be heterogeneous and that more than one firm can invest in the same project. Hence, firms need to decide which projects to develop and have to do so in a strategic manner, keeping in mind which projects their competitors are developing. In this paper, R&D portfolio is the main object of interest, while in the Sah and Stiglitz (1987) tradition it does not appear at all. There, projects are identical and it is immaterial which projects firms or their opponents develop. Thus, the model of Sah and Stiglitz (1987) does not capture the effects of variety of projects or the duplication of projects which is the main focus of analysis here.

3 A model of stochastic multiproject innovation

There are N symmetric firms[3] that compete in the pre-innovation market and that can invest in innovation. There is a continuum of research projects Ω, but only one project $\hat{j} \in \Omega$ leads to the innovation.[4] We can normalize the set of possible projects to the unit interval, that is $\Omega = [0, 1)$. I assume that all projects from the set $[0, 1)$ are successful with equal probability. Furthermore, each project has a fixed cost of development. Investing less than this cost means that firm will fail to develop the project and investing more will not improve the probability of the project being successful. In essence, the innovation mechanism is a lottery — developing different projects is akin to buying lottery tickets, the more lottery tickets you have the higher the probability you will win, but offering to pay more for a ticket will not increase its chances of winning. This fixed cost, fixed probability mechanism is similar to the one developed in Quirmbach (1993), the difference

[3] Asymmetric firms are studied in Section 7.3.

[4] The stochastic mechanism used to model innovation is adapted from Acemoglu and Zilibotti (1997).

being that here firms can invest in multiple projects.

The projects are assumed to differ in terms of the investment cost needed to develop them. Denote the cost of developing project $j \in [0, 1)$ as $C(j)$. Suppose that projects are ordered so that if $j > j'$ then $C(j) > C(j')$. Then we can view C as a function such that $C : [0, 1) \to \mathbb{R}^+$. I assume that C is continuous, differentiable and strictly increasing. The fact that the function C is increasing is simply a matter of ordering the projects j in the right way, strictness is assumed so that marginal reasoning will yield unique results. Continuity is assumed to make the problem more tractable. Furthermore, assume $\lim_{j\to 1} C(j) = \infty$. Since rewards from innovation are finite, this assumption ensures that firms will not want to invest in all possible projects. No exogenous restrictions are placed on the research budgets of firms, except in Section 7.5, which studies the consequences of limited research budgets and costly financing of research.

There are two possible levels of technology — old and new. The new technology is available only to the firms which invested in the successful innovation project, while the old technology is available to all firms. Let $n \leq N$ be the number of firms which developed the new technology. Denote with $R(n, N)$ the payoff of a firm with the new technology, where n is the number of firms with the new technology and N is the total number of firms. Analogously, denote with $r(n, N)$ the profits of a firm with the old technology. The difference between process and product innovations is not explicitly modelled. As long as the product market payoffs can be expressed in terms of the reward functions, the present model can be used to study both types of innovation.

Next, I list all assumptions that will be used in the paper. However, note that only Assumption 1 is used throughout, while the others are used for particular results.

Assumption 1 (Non-increasing reward to subsequent innovators)**.**
For all $n \in \{1, \ldots, N-1\}$ *it holds:*

$$R(n, N) - r(n-1, N) \geq R(n+1, N) - r(n, N).$$

This assumption implies that the gain from innovation does not increase as the number of innovators increases. It captures the intuition that a firm prefers that its competitors do not innovate. Thus innovations are strategic substitutes. While intuitive, this assumption needs to be checked for each model of product market competition. The consequences of relaxing this assumption will be considered in Section 7.2.

For some results, I assume that the innovation is drastic, in the sense that if there is at least one firm which has successfully developed the innovation, all firms which do not have the innovation cannot compete. That is, the laggards receive a payoff of zero and do not exert competitive pressure on the firms which has successfully innovated. For

process innovations, this implies that the price of a monopolist with the innovation is below the marginal cost of any firm without the innovation. For product innovation this implies that the old product is made obsolete and it cannot be sold on the market. This assumption will be relaxed in a Section 7.1, where the equilibrium will be characterized for non-drastic innovations. In the notation used here we have:

Assumption 2 (Drastic innovation)**.**
For all n, N and N' such that $1 \leq n \leq N \leq N'$ it holds: (i) $r(n, N) = 0$ and (ii) $R(n, N) = R(n, N')$.

Expression (i) ensures that laggards have zero profits while (ii) ensures that laggards do not exert competitive pressure on the innovators. Under Assumption 2, $R(n, N)$ is constant for any N, so from now on just $R(n)$ will be used to indicate the payoff of an innovator when there are n innovators. Furthermore, if Assumption 2 holds then Assumption 1 simplifies to the following two conditions: $R(n) \geq R(n+1)$ for all $n \geq 1$ and $R(1) - r(0, N) \geq R(2)$. The first expression states that the payoff per innovator weakly decreases as the number of innovators increases. The second expression states that the incentives of a prospective monopolist are greater than those of a single innovator when two firms innovate.

Assumption 3. *For every N it holds: $r(0, N) \geq r(0, N+1)$.*

This assumption states that as the number of firms which are active in the pre-innovation market increases, the profits of each individual firm do not increase. The intuition is simple: the additional firm will either not be competitive and have no effect on the profits of other firms, or it will put competitive pressure on other firms and decrease their profits, but it cannot increase their profits.

Next, denote with $W(n)$ the social welfare when there are n firms which have successfully innovated. That is, $W(0)$ denotes the welfare without the innovation, $W(1)$ denotes the welfare when there is only one firm with the innovation, and so on.

Assumption 4 (Non-increasing welfare returns)**.**
For every $n \in \{1, \ldots, N-1\}$ it holds $W(n) - W(n-1) \geq W(n+1) - W(n)$.

Each firm is assumed to be risk neutral and to maximize its expected profits. Profit maximization requires that firms either invest zero in a project or exactly the amount that is required to open the project. If a firm has invested a positive amount but less than $C(j)$, the project j would not be open and the investment would be wasted. Alternatively, if the firm invested more than $C(j)$, it would receive neither higher profits from project j nor would the probability of project j being successful increase. Thus, we can identify the strategy of a firm simply by the set of the projects in which it invests. Denote the

strategy of a firm i with $I_i \subseteq \Omega = [0,1)$ and call it the investment plan of firm i. In principle, I_i could be any measurable subset of the unit interval. To simplify exposition, assume that the set I_i consists only of a countable number of intervals, each closed from below and open from above.[5] Formally, the strategy space of firm i is the set $\mathcal{I}_i$ where:

$$\mathcal{I}_i := \{I_i \subset [0,1) : I_i = \cup_{k=1}^{\infty}[a_k, b_k) \text{ and } 0 \leq a_k < b_k < 1 \text{ for all } k\}.$$

In particular, note that this assumption ensures that the investment plan will not contain any isolated zero-mass points.

Let $I = [I_1, \ldots, I_N]$ be a vector of investment plans of all N firms. Define the function indicating the number of firms investing in a project, given a vector of investment plans I, as $n(j, I) : [0,1) \to \mathbb{N}^0$ as:

$$n(j, I) = \sum_{i=1}^{N} \mathbf{1}\Big(j \in I_i\Big),$$

where $\mathbf{1}(\cdot)$ is the indicator function.

Let $I_i^c := [0,1) \setminus I_i$. The expected profit of a firm i is then

$$\pi_i(I) = -\int_{I_i} C(j)dj + \int_{I_i} R(n(j,I))dj + \int_{I_i^c} r(n(j,I), N)dj. \tag{1}$$

The first part of the equation above represents the investment costs of firm i, the second part gives the expected profits from the new technology, while the third part gives the expected profits from the old technology. By Assumption 2, $r(n(j,I), N) = 0$ whenever $n(j,I) > 0$. However, it will be positive whenever $n(j,I) = 0$, which will occur in equilibrium with positive probability.[6]

When $N = 1$, that is, when there is a monopolist in the market, the above becomes a pure maximization problem. When there are more firms in the market we have to consider the effects of strategic interaction among firms. Specifically, $n(j,I)$ depends on the actions of other firms and thus the expected profit of one firm depends on the actions of other firms.

Finally, assume that investment in innovation is profitable. That is $R(1) - r(0,N) > C(0)$. This assumption guarantees positive investments in the equilibrium. If this assump-

[5] Since adding or removing zero-mass points does not change the payoff of any of the firms, allowing I_i to be general would mean that all statements regarding the properties of the equilibrium would have to be qualified by "almost everywhere". This assumption is purely of technical nature and does not affect the mechanics of the model. More importantly, note that for any measurable investment plan I_i which does not satisfy the assumption above, there always exists plan I_i' which does satisfy the assumption and only differs from I_i by zero-mass points, hence delivers the same payoff to all firms.

[6] Furthermore, the magnitude of $r(0,N)$ will determine the strength of the Arrow replacement effect, which is crucial for the equilibrium variety of research projects.

tion was not met, even the monopolist's return on the investment in the cheapest project would not justify its cost. Since $C(j)$ is strictly increasing and rewards are non-increasing in n, then no project could be profitable. Thus, if this assumption failed there would be a simple equilibrium in which firms did not invest at all.

4 Equilibrium

The vector of investment plans summarizes all decisions of all firms that are relevant for this problem. A vector of investment plans I^* is an equilibrium if no firm can increase its expected profit by unilaterally choosing an alternative investment plan I_i'. That is I^* is an equilibrium if, for any firm i, there does not exist an investment plan I_i' such that $\pi_i(I_i', I_{-i}^*) > \pi_i(I^*)$.

Proposition 1 (Existence, non-uniqueness and equivalence of equilibria)**.**
Suppose that Assumption 1 holds. Then:

1. *An equilibrium always exists.*
2. *If I^* is an equilibrium and $0 < n(j, I^*) < N$ for some $j \in [0, 1)$, then infinitely many equilibria exist.*
3. *If there are multiple equilibria they all result in the same market portfolio of research projects. That is, if I_1^* and I_2^* are equilibrium investment plans, then $n(j, I_1^*) = n(j, I_2^*)$ for all $j \in [0, 1)$. Furthermore, if I_1^* is an equilibrium then any investment plan I_3^* such that $n(j, I_1^*) = n(j, I_3^*)$ for all $j \in [0, 1)$ is also an equilibrium.*

While an equilibrium in pure actions[7] will always exist, typically there will also exist infinitely many equilibria. The proof of statement 2 in Proposition 1 (see the Appendix A.1) reveals the nature of the multiplicity. In equilibrium, identities of firms investing in any given project are in general not determined, only the number of firms investing is determined. Only when either all firms invest in a project or no firm invests in a project, we can infer the behavior of individual firms. Thus, when $0 < n(j, I^*) < N$ for some $j \in [0, 1)$, there are projects for which the identities of firms investing are not determined and as there is an infinite number of ways to assign investments to firms, there must be infinitely many equilibria.

Statement 3 of Proposition 1 clarifies this point further. It states that every equilibrium induces the same market portfolio of research projects — that is in every equilibrium the set of developed projects will be the same and the number of firms investing in each

[7]Symmetric mixed strategy equilibria with two firms are characterized in Section 7.4.

project will be the same. Thus, while there is a multiplicity of equilibria, the equilibrium market portfolio is unique. Since firms are identical, welfare does not depend on the identity of firms doing research. From the social welfare perspective, any two equilibria are equivalent.

Furthermore, not only do all equilibria induce the same market portfolio of research projects, but any investment that induces the equilibrium portfolio is itself an equilibrium. The intuition for this result is straightforward — the profitability of any research project depends only on the cost of the project and the number of competitors who are investing in the same project. In particular, it does not depend on any other investment that the firm or its competitors may be making. Hence, if in an equilibrium all profitable investments are exhausted and no unprofitable investments are made, then any other investment plan that prescribes the same investment portfolio in the same manner exhausts all profitable investments and has no superfluous investments.

Statement 3 of Proposition 1 implies that if I^* is an equilibrium then the function $n(j, I^*)$ fully characterizes the equilibrium portfolio of research projects. Since $n(j, \cdot)$ is the same for any equilibrium, we can denote the function characterizing the equilibrium portfolio of research projects as $n^*(j)$. Using the equilibrium constructed in the proof of statement one of Proposition 1 and applying Assumption 2 yields the following result.

Proposition 2 (Characterization of equilibrium portfolio)**.**
Suppose that Assumptions 1 and 2 hold. Denote with m the maximum number of firms investing in any project:

$$m = \max_{\{1,\dots,N\}} n$$
$$s.t.\ R(n) - r(n-1, N) - C(0) > 0$$

and with α_k for $k \in \{1, 2, \dots, m\}$ the most expensive project in which k firms can profitably invest. That is:

$$\begin{aligned} R(1) - r(0, N) - C(\alpha_1) &= 0 \\ R(2) - C(\alpha_2) &= 0 \\ &\vdots \\ R(m) - C(\alpha_m) &= 0. \end{aligned}$$

Let $\alpha_{m+1} = 0$ and $\alpha_0 = 1$. Then the equilibrium portfolio $n^*(j)$ is given by

$$n^*(j) = k \quad \text{if} \quad j \in [\alpha_{k+1}, \alpha_k).$$

An illustration of the equilibrium market portfolio for $N = 3$ and a process innovation

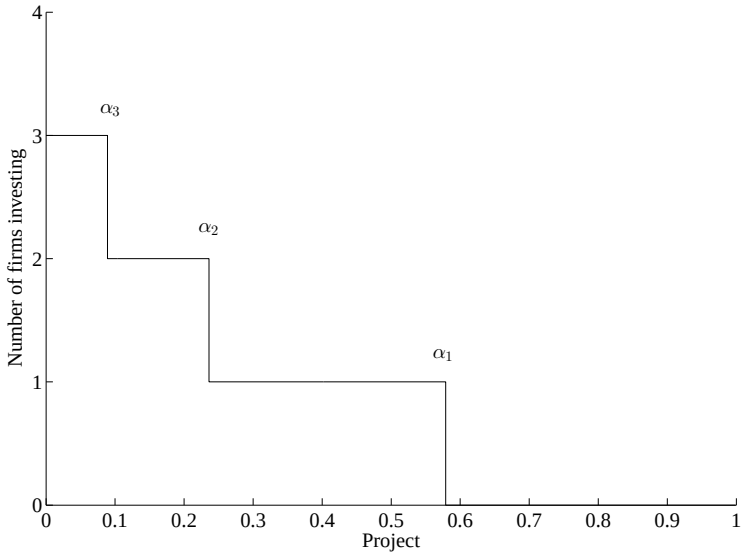

Figure 1: Equilibrium market portfolio.

in a Cournot market (which is an example from Section 5.3) is provided in Figure 1. Here, $m = 3$ represents the maximum number of firms that can profitably invest in any project. Since project 0 is by assumption the cheapest to develop, then m firms will invest in this project. Each point α_k is constructed so that, at the margin, if k firms invested the profit from investment would be zero. Since $C(j)$ is assumed to be strictly increasing, then at any point $j > \alpha_k$ strictly fewer than k firms can profitably invest. Since rewards are finite and costs to innovation approach infinity as $j \to 1$, values $\alpha_1, \alpha_2, \ldots, \alpha_m$ always exist. Furthermore, since $C(j)$ is increasing and by Assumption 1 the rewards to innovation are non-increasing it is easy to see that $\alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_m$.[8] From this observation it follows directly that the function $n^*(j)$ is weakly decreasing.

The set of all projects the market invests in is $[0, \alpha_1)$. Thus, I will refer to α_1 as the *variety of research projects* undertaken. I will say that the variety of research projects increases if α_1 increases. The probability that the market develops an innovation is equal to α_1. Hence an increase in the variety of research projects implies an increase in the probability that the market will develop an innovation. The function $n^*(j)$ captures the number of firms investing in any given project j in equilibrium. Hence, I will refer to the number $n^*(j)$ as the market *amount of duplication* of project j.

[8]Note that the inequality is weak (since the inequality in Assumption 1 is weak), so that it might happen for some $k \leq m$ that $\alpha_k = \alpha_{k+1}$. In this case, define $[\alpha_{k+1}, \alpha_k) = \emptyset$. Thus there will be no project that exactly k firms will develop.

5 Comparative statics

In this section, I will study how the market portfolio of research projects changes as the market structure changes. In particular, I will look at how a change in the number of active firms in the market and the intensity of competition among them affects the market portfolio of research projects. As can be seen from Proposition 2, the equilibrium portfolio is characterized by the maximum number of firms m investing in any project and the k-firm frontiers α_k, for $k \in \{1, \ldots, m\}$. I will analyze how a change in N and a change in the intensity of competition affect these variables.

5.1 Change in the number of firms

Consider first the case where the number of active firms in the market changes, while all other characteristics of the market remain the same.

Proposition 3 (Increase in the number of firms)**.**
Suppose that Assumptions 1, 2, and 3 hold. Let the number of firms in the pre-innovation market increase from N to N' so that the equilibrium investment plan changes from I to I'. In equilibrium, the variety of projects developed and the probability of developing an innovation weakly increases, that is $\alpha_1 \leq \alpha_1'$. The maximum number of firms investing also increases, that is $m \leq m'$.

The increase in the variety of developed projects is driven solely by the Arrow replacement effect.[9] In this setting, the firm investing near α_1 replaces $r(0, N)$ with $R(1)$. As $r(0, N) \leq r(0, N + 1)$, the Arrow replacement effect is weaker when there are $N + 1$ firms in the market. Consequently, firms attempt to escape the competition by investing in more expensive research projects than before and the variety of developed projects increases. This is equivalent to saying that the probability of discovering an innovation increases.

One implication of this result is that a merger in an imperfectly competitive industry will lead to a loss of variety of approaches to innovation that will be undertaken in post-merger (see Figure 2). Thus, competition authorities should take this loss of variety of approaches to innovation into account when reviewing merger cases, especially if innovation is important in the industry, as it was in the proposed Lockheed-Northrop merger.[10] Since the loss of the variety of approaches to innovation is driven by the Arrow replacement effect, the magnitude of the loss of variety will be proportional to the increase in

[9] If a firm innovates, it replaces its pre-innovation without the innovation with the post-innovation profits. Thus, the larger the pre-innovation profits of firms, the weaker its incentive to innovate. This is known as the Arrow replacement effect. See (Arrow, 1962, p. 620) and (Gilbert, 2006, p. 165).

[10] See Robinson (1999) and Rubinfeld and Hoven (2001).

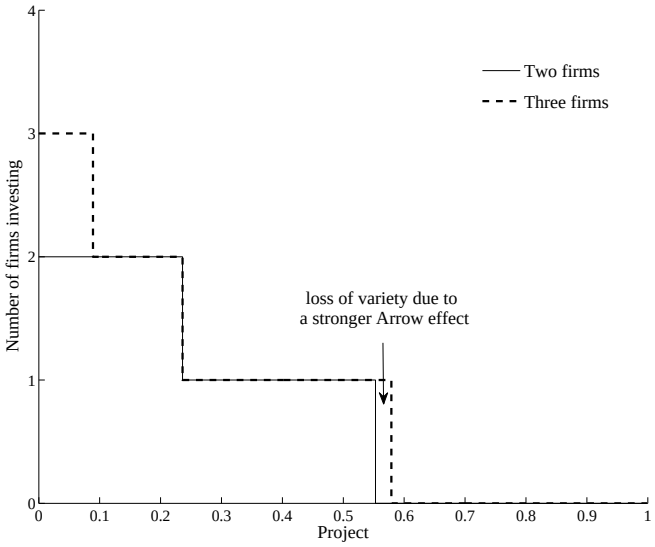

Figure 2: Merger in a 3-firm Cournot market.

profits (in the market without the innovation) due to the merger.

An increase in the number of active firms weakens the Arrow replacement effect both in this model and in the usual level-of-investment models. The difference arises in the effect on the ex post profits of firms. Here the firms are free to choose in which projects to invest. The number of firms investing in any given project, and hence the ex post number of competitors, is endogenously determined. In this setting, an additional firm ex ante does not translate into more competitors ex post. Thus the Schumpeter effect[11] in this model does not change as the number of ex ante active firms changes. This leads to the clear effect of an increase in the number of firms on the variety of projects developed, as only one firm will invest in the most expensive projects.

Consider in this context the invariance result of Sah and Stiglitz (1987), which states that the number of research projects is invariant to the number of firms in the market. The invariance can only hold if $r(0, N) = r(0, N + 1)$, that is, only if the Arrow replacement effect is constant. Clearly this will hold under homogeneous goods Bertrand competition as Sah and Stiglitz (1987) have originally assumed, since $r(0, N) = 0$ for any $N \geq 2$. Conversely, it will not hold (in general) under Cournot competition as $r(0, N)$ will be decreasing in N, which is in line with the results derived in Reynolds and Isaac (1992) and Farrell et al. (2003).

[11] Following Vives (2008), denote the reduction in innovation incentives due to lower ex post payoffs as the Schumpeter effect.

5.2 Change in the intensity of competition

The competitive structure of the market is not only determined by the number of firms which are active in the market, but also by the intensity of competition among firms. Suppose that there are two sets of reward functions $\{R, r\}$ and $\{R', r'\}$ such that $R(n) > R'(n)$ for every $n > 1$, $R(1) = R'(1)$ and $r(0, N) > r'(0, N)$. Then we can interpret the move from $\{R, r\}$ to $\{R', r'\}$ as an increase in the intensity of competition. Most standard examples of an increase in the intensity of competition correspond to this definition. In particular, in Section 5.3 I will consider a move from Cournot to Bertrand type of competition, but models of differentiated Cournot/Bertrand also correspond to this definition. The next result considers the effect of an increase in the intensity of competition on the market R&D portfolio.

Proposition 4 (Increase in the intensity of competition)**.**
Suppose that Assumptions 1 and 2 hold. Let $N \geq 2$ and suppose the intensity of competition increases so that the equilibrium investment plan changes from I to I'. Then the variety of research projects undertaken and the probability of discovering the innovation increase. That is $\alpha_1 < \alpha_1'$. The amount of duplication of research decreases. That is for each j such that $n(j, I) \geq 2$ we have $n(j, I) \geq n(j, I')$ with $n(j, I) > n(j, I')$ for at least some projects.

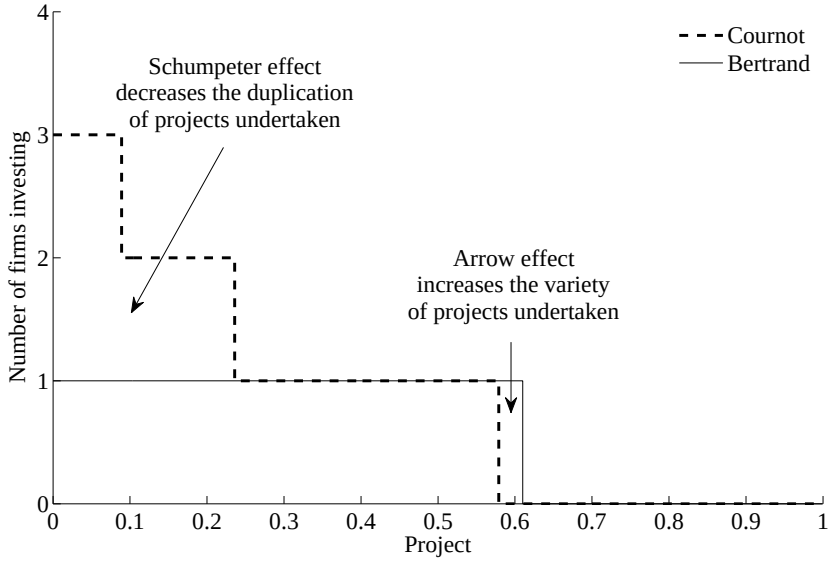

Figure 3: An illustration of an increase in intensity of competition.

An increase in the intensity of competition decreases the profits firms receive if no firm successfully innovates, thereby weakening the Arrow replacement effect and leading to an increase in the variety of developed research projects. On the other hand, it also

decreases payoffs to firms if there are multiple innovators, leading to a (weakly) decreasing number of firms investing in duplicative research projects. An increase in the intensity of competition "flattens out" the equilibrium research portfolio, reducing the duplication of costs (see Figure 3). However, it is not clear that an increase in the intensity of competition will lead to higher social welfare. On the one hand, duplication of costs is reduced and the variety of research projects is increased. However, less duplication of costs also implies fewer firms (though competing more vigorously!) in the product market leading to a possible efficiency loss. Which effect prevails will depend on the exact specification of the product market competition and the demand function.

An increase in the number of firms is sometimes used as an increase in the intensity of competition. The preceding results highlight the difference between an increase in the number of firms and an increase in the intensity of competition as defined here. An increase in the intensity of competition reduces firm profits whenever there are multiple firms competing. That is, it reduces firm profits both ex ante and ex post. The number of firms, due to the endogeneity of the ex post market structure, affects firm profits only ex ante. Thus, the Schumpeter effect is present only in the case of an increase in the intensity of competition and not in the case of an increase in the number of firms.

5.3 Example: Process innovation in a Cournot market

As an illustrative example, consider a simple Cournot model with homogeneous products, linear costs and linear demand. Suppose that there are three firms facing inverse demand of the form

$$P(q_1, q_2, q_3) = 1 - (q_1 + q_2 + q_3)$$

where q_i is the quantity supplied by the firm i. Denote with $\bar{c}$ the marginal cost of production with the old technology and with $\underline{c}$ the marginal cost of production with the new technology, where $\underline{c} \leq \bar{c} \leq 1$. That is, firms have the possibility to develop a process innovation which reduces their production cost from $\bar{c}$ to $\underline{c}$. The innovation is drastic if

$$\bar{c} \geq \frac{1 + \underline{c}}{2}, \tag{2}$$

where the right hand side of the inequality is the price which would be obtained if there was a monopolist with marginal cost $\underline{c}$ in the market. Suppose that the costs of research are given by

$$C(j) = b\sqrt{\frac{j}{1-j}}, \quad j \in [0, 1),$$

where $b > 0$ is a slope parameter. Observe that this choice of cost function implies $C(0) = 0$ so that $m = N$, that is at least some of the innovation projects are developed by all the firms in the market.

Using standard methods, the profits in Cournot markets with n firms and marginal costs c are given by $\Pi(n, c) = (1-c)^2/(n+1)^2$. From this equation it is possible to derive the ex post payoffs:

$$\begin{aligned} r(0,3) =& \frac{(1-\bar{c})^2}{16}, & R(1) =& \frac{(1-\underline{c})^2}{4}, \\ R(2) =& \frac{(1-\underline{c})^2}{9}, & R(3) =& \frac{(1-\underline{c})^2}{16}. \end{aligned}$$

In order to be able to apply Proposition 2, we have to check if Assumptions 1 and 2 hold. Assumption 2 holds whenever Equation (2) is satisfied. In addition, Assumption 1 holds whenever $\underline{c} \leq \bar{c} \leq 1$, which is assumed.[12] Hence, Proposition 2 can be used to characterize the equilibrium R&D portfolio.

Applying Proposition 2 yields the following k-firm frontiers for $k \in \{1, 2, 3\}$:

$$\begin{aligned} \alpha_3 &= \frac{R(3)^2}{b^2 + R(3)^2}, \\ \alpha_2 &= \frac{R(2)^2}{b^2 + R(2)^2}, \\ \alpha_1 &= \frac{(R(1) - r(0,3))^2}{b^2 + (R(1) - r(0,3))^2}. \end{aligned}$$

All projects in the interval $[0, \alpha_3)$ are developed by all three firms while the projects in the interval $[\alpha_3, \alpha_2)$ are developed by two firms. Projects in the interval $[\alpha_2, \alpha_1)$ are developed by just one firm while the projects in the interval $[\alpha_1, 1)$ are not developed at all. Thus, if the successful project is from the interval $[0, \alpha_1)$, the market will successfully develop the innovation and all firms which invested in the successful project will compete with the production costs $\underline{c}$. However, if the successful project is from the interval $[\alpha_1, 1)$ the market will not develop the innovation and all firms will compete with the production costs $\bar{c}$.

[12] If Assumption 2 is satisfied then the sufficient condition for Assumption 1 to hold is

$$\frac{(1-\underline{c})^2}{4} - \frac{(1-\bar{c})^2}{16} \geq \frac{(1-\underline{c})^2}{9}.$$

If $\bar{c} = 1$, the inequality is satisfied. If $\bar{c} < 1$, the expression simplifies to

$$\left(\frac{1-\underline{c}}{1-\bar{c}}\right)^2 \geq \frac{9}{20},$$

which is always satisfied since the left-hand expression is always greater than 1.

Figure 1 (in Section 4) illustrates the equilibrium market portfolio in the case where $b = 0.05$, $\bar{c} = 3/4$ and $\underline{c} = 1/2$.

Merger of two firms

Suppose now that two of the three firms merge, leaving everything else unchanged. That is, suppose that the merger affects only the number of firms which are active in the market. Denote with $\{r', R'\}$ payoffs after the merger and with $\{r, R\}$ payoffs without the merger. Clearly, $\{r, R\}$ are the same as before. The new payoff functions are given by:

$$r'(0,2) = \frac{(1-\bar{c})^2}{9},$$
$$R'(1) = \frac{(1-\underline{c})^2}{4},$$
$$R'(2) = \frac{(1-\underline{c})^2}{9}.$$

It is immediately clear that $r'(0,2) > r(0,3)$ while $R'(1) = R(1)$ and $R'(2) = R(2)$. The intuition behind this is that the merger increases profits in the market when all firms are active, because there are fewer competitors, hence $r'(0,2) > r(0,3)$. However, due to the drastic nature of innovation, post-innovation profits only depend on the number of firms which successfully innovated, hence $R'(1) = R(1)$ and $R'(2) = R(2)$. Since after the merger there are only two firms in the market, the maximum number of firms investing in any project is at most 2.

Applying Proposition 2 yields $m' = 2 < m = 3$, $\alpha'_2 = \alpha_2$ but $\alpha'_1 < \alpha_1$. This is in line with results derived in Proposition 3. Figure 2 (in Section 5.1) graphically illustrates the change in the market portfolio of research projects after the merger.

From Cournot to Bertrand competition

Consider again the scenario with three firms and suppose that the type of competition changes from Cournot to Bertrand. This change can be interpreted as an increase in the intensity of competition among the firms. How will the market portfolio of research projects change? Applying Proposition 4, the variety of research projects developed will increase while the duplication of research projects will decrease.

From above we know that with three firms engaged in a Cournot competition, the market portfolio will be characterized by the maximum number of firms investing m and the firm-frontiers α_3, α_2, and α_1. The equilibrium values of the market under Bertrand competition are denoted with a prime. When there are multiple symmetric firms competing in a homogeneous goods Bertrand market, in equilibrium firms set prices equal to marginal cost of production and earn zero profits. Hence, the payoff functions will be

$r'(0,3) = 0$ and $R'(2) = R'(3) = 0$. The monopolist earns the same profits in both cases, that is $R'(1) = R(1) = (1 - \underline{c})^2/4$.

Since Assumptions 1 and 2 clearly hold, Proposition 2 can be applied. It immediately follows that $m' = 1$, hence $n(j) < n'(j)$ for all $j < \alpha_2$. This drastic change in the amount of duplication is due to the fact that firms make no profits if there is a competitor, so firms choose to do no duplication at all. Simple calculations show that $\alpha_1' > \alpha_1$. Figure 3 (in section 5.2) illustrates the change in the market portfolio of research projects due to the change of competition from Cournot to Bertrand.

6 Optimal portfolio

There are several reasons to suspect that a market R&D portfolio will not be optimal. For example, when the innovator cannot appropriate the entire surplus because a part of the surplus is captured by the consumers, the incentive to innovate may be too low. On the other hand, if innovation enables firms to become more competitive in the market and steal business from their competitors, the incentive to innovate may be too high.[13] However, as this paper argues, looking solely at the levels of investment in innovation is misleading. Rather, the question to be posed is whether the market invests in the optimal variety of projects and whether it optimally duplicates projects. That is, the question is how the market R&D portfolio compares to the socially optimal portfolio.

The approach here is to ask what is the R&D portfolio that maximizes the expected social welfare. Firms are assumed to behave as before, that is to maximize profits. In particular, firms do not share the results of research, so some duplication of research will be optimal, as duplication of the successful project implies higher product market efficiency ex post.

Denote with $W(n)$ social welfare generated by the product market if there are n firms with the new technology, for every $n \leq N$. That is, $W(0)$ is welfare if no firm has successfully innovated and $W(N)$ is welfare if all firms have the new technology. We can decompose welfare into consumer surplus and producer surplus, that is $W(n) = CS(n) + nR(n)$ if $n \geq 1$ and $W(0) = CS(0) + Nr(0, N)$.

Analogously to Proposition 2, the optimal portfolio is characterized:

Proposition 5 (Characterization of the optimal portfolio)**.**
Suppose that Assumption 4 holds. Denote with m^o the optimal number of firms developing the least expensive project:

$$m^o = \max_{\{1,\ldots,N\}} n$$

[13]For an example of the under- and over-investment in innovation due to the two effects outlined here see Bester and Petrakis (1993).

$$s.t.\ W(n) - W(n-1) - C(0) > 0$$

and with α_k^o *for* $k \in \{1, 2, \ldots, m\}$ *the most expensive project in which at most* k *firms can optimally invest. That is:*

$$\begin{aligned} W(1) - W(0) - C(\alpha_1^o) &= \\ W(2) - W(1) - C(\alpha_2^o) &= \\ &\vdots \\ W(m^o) - W(m^o - 1) - C(\alpha_m^o) &= 0. \end{aligned}$$

Let $\alpha_{m+1}^0 = 0$ *and* $\alpha_0^o = 1$*. Then the optimal portfolio* $n^o(j)$ *is given by*

$$n^o(j) = k \quad \text{if} \quad j \in [\alpha_{k+1}^o, \alpha_k^o).$$

6.1 Market investment in variety

It is now possible to directly compare the market R&D portfolio with the optimal portfolio. In this way it is possible to identify if and how the market portfolio differs from the optimum and to suggest a way in which a policy intervention can improve the market outcome.

The net externality from investing in marginal variety (a research project that is not developed by any other firm) is given by:

$$\sigma = -(N-1)r(0,N) + [CS(1) - CS(0)]\,.$$

The expression outside of the bracket captures the negative externality imposed on the competitors of the firms making the marginal investment. They lose the profits they would obtain if no firm invested in the marginal project and the marginal project turned out to be successful. The expression inside the bracket captures the positive externality imposed on the customers — who receive the surplus associated with one firm innovating as opposed to the surplus associated without innovation. Corollary 1 states that the optimality of investment variety depends on the sign of the net externality imposed by the marginal variety.

Corollary 1 (Market investment in variety)**.**
Suppose that assumptions 1, 2 and 4 hold. Then the market will underinvest in the variety of R&D projects if and only if $\sigma > 0$*. The market will invest in the optimal variety of R&D projects if and only if* $\sigma = 0$*. The market will overinvest in the variety of R&D projects if and only if* $\sigma < 0$*.*

In principle the sign of σ should be checked for each model. However, as will be shown later, in a large class of homogeneous goods models the assumption that the innovation is drastic implies that $CS(1) \geq W(0) \geq CS(0) + (N-1)r(0,N)$. Thus, the market will in this case underinvest in the variety of R&D projects. The intuition for this is as follows. A process innovation is drastic if the monopolist's price is below the marginal cost of production without the innovation. Hence, consumer surplus with a monopolist (i.e., $CS(1)$), which is equal to the difference between the reservation price and the price paid, is greater than total welfare without the innovation (i.e., $W(0)$), which is equal to the difference between the reservation price and the cost of production.

To illustrate the market underinvestment in variety, consider a simple homogeneous product market similar to the one analysed in Mankiw and Whinston (1986). Suppose that the inverse market demand function is given by $P(Q)$, where Q is the aggregate output in the market and $P'(Q) < 0$ for all Q. There are N symmetric firms, each of which possesses a technology given by the cost function $\bar{c}(q)$, where $\bar{c}(0) = 0$, $\bar{c}'(\cdot) \geq 0$ for all $q \geq 0$. Firms can invest in R&D to develop a drastic process innovation, in which case their technology is given by the cost function $\underline{c}(q)$, where $\underline{c}(0) = 0$, $\underline{c}'(\cdot) \geq 0$ for all $q \geq 0$. A process innovation is drastic if a monopolist facing the cost function $\underline{c}(q)$ chooses a price which is below the marginal cost of production of a firm with the old technology. Formally, an innovating monopolist would choose a quantity q_1 such that $P(q_1) < \bar{c}'(0)$.

Proposition 6 (Underinvestment in homogeneous product markets)**.**
Suppose that Assumptions 1, 2 and 4 hold. Then a homogeneous product market with a potential drastic process innovation always underinvests in the variety of R&D projects.

As the case of homogeneous product market illustrates, a decentralized market will tend to underinvest in drastic innovations. It should be noted that the critical assumption in this example is not the type of the product market competition. Rather, the assumption that the innovation is drastic drives the result, in line with the observation that the market underinvestment will be greater for higher quality innovations.

Proposition 6 offers insights relevant to research policy. Suppose that society cannot affect the market structure or the behaviour of firms in the market but can offer subsidies for research. The market will tend to underinvest in the variety of drastic innovation by failing to develop high-cost projects which should optimally be developed. Thus, the research subsidies should be directed toward research projects with (1) high costs; and (2) and high potential payoffs.

6.2 Market investment in duplication

Typically, the market R&D portfolio will involve some duplication of research projects. As this is duplication of identical projects, it does not increase the probability that an innovation will be discovered. However, duplication is not entirely wasteful either. If multiple firms develop the same project and this project turns to be the successful one, then there will be more competitors on the product market. So for the cost of duplicating research the society receives the (weakly) higher product market efficiency. The efficient duplication of R&D projects is captured by the optimal portfolio.

In equilibrium, a firm duplicating a research project imposes both negative externalities on its competitors (in the form of business stealing effect) and positive externalities on the consumers (in the form of the efficiency effect). Define the net externalities effect of the k-th duplication as:

$$\delta(k) = \Big[(k-1)\big(R(k) - R(k-1)\big)\Big] + \Big[CS(k) - CS(k-1)\Big].$$

The first bracket captures the negative externalities generated by the investment of the k-th innovator, which are the reduction of profits of $k-1$ firms from $R(k-1)$ to $R(k)$. The second bracket captures the positive externalities which accrue to the consumers, and which are captured by the difference between $CS(k)$ and $CS(k-1)$, the consumer surplus when there are k competitors and $k-1$ competitors on the product market, respectively.

Corollary 2 (Market investment in duplication)**.**
Suppose that Assumptions 1, 2 and 4 hold. Denote with m the maximum number of firms investing in the market equilibrium and with m^o the maximal number of firms investing in the optimal equilibrium. For $2 \leq k \leq \min\{m, m^o\}$, denote with α_k the k-firm frontiers in the market portfolio and with α_k^o the k-firm frontiers in the optimal portfolio.

If $\delta(k) < 0$ then $\alpha_k^o < \alpha_k$ and the market overinvests in duplication of all projects $j \in (\alpha_k^o, \alpha_k)$. If $\delta(k) > 0$ then $\alpha_k < \alpha_k^o$ and the market underinvests in duplication of all projects $j \in (\alpha_k, \alpha_k^o)$. If $\delta(k) = 0$ then $\alpha_k^o = \alpha_k$ and the market optimally invests in duplication of all projects in the neighbourhood of α_k.

If $m \geq m^o$ and $\delta(k) \geq 0$ for all $k \in \{2, \ldots, m^o\}$, then the market (weakly) overinvests in duplication of all R&D projects. Conversely, if $m \leq m^o$ and $\delta(k) \leq 0$ for all $k \in \{2, \ldots, m\}$, then the market (weakly) underinvests in duplication of all R&D projects. If the net externalities are negative ($\delta(k) < 0$), then it would be optimal to reduce the equilibrium number of firms investing in projects (α_k^o, α_k) from k to $k-1$. If the externalities are positive then the number of firms should be increased in the interval (α_k, α_k^o).

From the perspective of a fixed project j, the question of whether the amount of duplication is optimal or not is essentially equivalent to the question whether the free entry in an industry with fixed costs is optimal or not. Here, the question is of an entry in a 'potential' industry, fixed costs are the cost of developing this specific project $C(j)$, and the number of firms that can enter is limited by the number of firms which are active in the pre-innovation market. Keeping in mind the upper bound on the number of firms imposed by N, the results derived in Mankiw and Whinston (1986) apply in this setting as well. For the homogeneous product market and ignoring the integer constraint, Mankiw and Whinston find that the free-entry equilibrium number of firms is not less than the socially optimal number of firms (i.e., there is no underinvestment in duplication in our terminology), and furthermore if the equilibrium price is above the marginal costs, then the equilibrium number of firms is strictly greater than the optimal number (i.e. there is overinvestment in duplication).[14] That is, Mankiw and Whinston identify conditions under which an industry equilibrium would tend toward excessive entry. In the context of the present model, this implies that there should be a tendency toward overinvestment in duplication of R&D projects. Taking into account the integer constraint weakens this result somewhat — Mankiw and Whinston establish that the free-entry equilibrium number of firms is not lower than the optimal number of firms less one.[15] In the notation of this paper, that would be $n^*(j) \geq n^o(j) - 1$ for appropriate project j. This suggests that while there might be underinvestment in the duplication of R&D projects, it will be bounded from below.

6.3 Market and optimal portfolios in a Cournot model

Consider again the Cournot example from Section 5.3. Social welfare generated in this product market by firms supplying total quantity Q is given by:

$$W_Q = \int_0^Q P(s)ds - Qc = \int_0^Q (1-s)ds - Qc = Q\left(1 - \frac{Q}{2} - c\right), \tag{3}$$

where c is the constant marginal cost of production. Using standard results, the total quantity supplied in a Cournot market with n firms is given by

$$Q(n,c) = \frac{n(1-c)}{n+1}.$$

Assumptions 1 and 2 hold and simple calculations show that Assumption 4 holds as well. Hence Propositions 5 and 6 can be applied. Proposition 6 immediately informs us

[14] Proposition 1 in Mankiw and Whinston (1986).
[15] Proposition 2 in Mankiw and Whinston (1986).

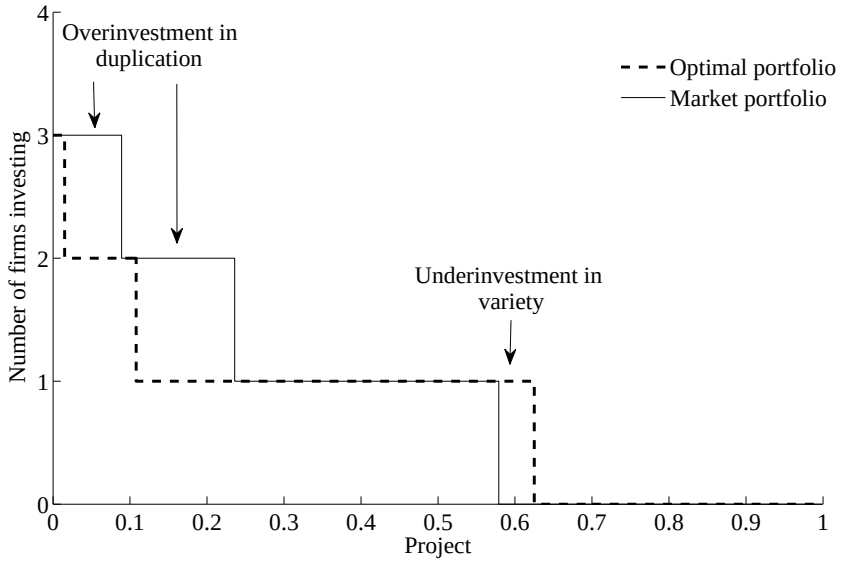

Figure 4: Optimal and market portfolios of research projects.

that there will be underinvestment in the variety of research projects. Figure 4 illustrates the difference between the optimal and the market portfolio in this market.

7 Extensions and robustness checks

This section relaxes several assumptions made in the model. For simplicity, in all following sections except 7.1, I will assume that there are only two firms in the market.

7.1 Non-drastic innovations

The assumption that innovation is drastic significantly simplifies the analysis, as it allows us to ignore all firms which have failed to innovate whenever at least one firm has innovated. However, there are many innovations which are incremental and which give only a slight advantage to the innovating firm over its rivals. This section relaxes Assumption 2 and provides a more general characterization of the market equilibrium portfolio.

First observe that Proposition 1 does not rely on Assumption 2. Hence, an equilibrium of the investment game exists and except in trivial cases an infinite number of equilibria exists. However, the equilibrium market portfolio is unique and any investment plan that generates the equilibrium market portfolio is itself an equilibrium of the investment game. The next result characterizes the equilibrium market portfolio.

Proposition 2a (Characterization of equilibrium portfolio)**.**
Suppose that Assumption 1 holds. Denote with m the maximum number of firms investing

in any project:

$$m = \max_{\{1,\ldots,N\}} n$$
$$s.t.\ R(n,N) - r(n-1,N) - C(0) > 0$$

and with α_k for $k \in \{1,2,\ldots,m\}$ the most expensive project in which k firms can profitably invest. That is:

$$\begin{aligned} R(1,N) - r(0,N) - C(\alpha_1) &= \\ R(2,N) - r(1,N) - C(\alpha_2) &= \\ &\vdots \\ R(m,N) - r(m-1,N) - C(\alpha_m) &= 0. \end{aligned}$$

Let $\alpha_{m+1} = 0$ and $\alpha_0 = 1$. Then the equilibrium portfolio $n^*(j)$ is given by

$$n^*(j) = k \quad \text{if} \quad j \in [\alpha_{k+1}, \alpha_k).$$

The basic form of the equilibrium portfolio is the same as in the case with the drastic innovation — it is still a step function with a declining number of firms investing as projects become more expensive. There are two differences however. First, the payoffs with the innovation $R(\cdot, N)$ are now functions of N, because the firms without the innovation can put competitive pressure on the firms with the innovation. Second, firms without the innovation can now obtain positive profits, which decreases the incentive to duplicate research.

As a consequence, the comparative statics results become ambiguous if Assumption 2 does not hold. Consider an increase in the intensity of competition, so that the payoff functions strictly decrease from $\{r, R\}$ to $\{r', R'\}$. If Assumption 2 holds, then Proposition 4 holds and the increase in the intensity of competition will result in an increase in the variety of projects undertaken. If Assumption 2 does not hold, the variety of projects undertaken will increase only if

$$r(0,N) - r'(0,N) > R(1,N) - R'(1,N),$$

or, in words, only if competition reduces profits without the innovation more than it does for the single innovator. An analogous condition is required for any other n-firm frontier as well as for the increase in the number of firms.

Figure 5 illustrates the Cournot duopoly example from Section 5.1 with drastic and with non-drastic innovation (i.e., the marginal cost of production after the innovation was increased by a small amount so that the example no longer satisfies Assumption 2). Figure

5 illustrates how the essential structure of the model does not depend on Assumption 2.

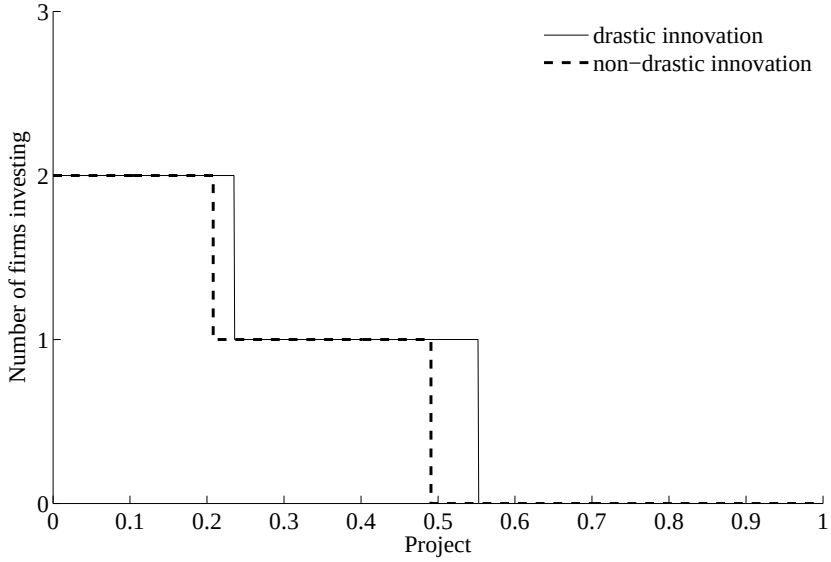

Figure 5: Drastic versus non-drastic innovation.

7.2 Innovations as strategic complements

Consider a case with two firms in the industry and relax Assumption 1. Assumption 1 will not hold if innovations are sufficiently strong complements, for example in the case of research spillovers. If Assumption 1 does not hold then it must be true that:

$$R(2,2) - r(1,2) > R(1,2) - r(0,2).$$

Assumption 2 is immaterial for the following discussion and it is not assumed to hold. Analogous to the use of k-firm frontiers before, let:

$$\alpha_1 = C^{-1}\big(R(1,2) - r(0,2)\big),$$
$$\alpha_2 = C^{-1}\big(R(2,2) - r(1,2)\big),$$

where $C^{-1}(\cdot)$ is the inverse of the function $C(\cdot)$. Since $C(\cdot)$ is a strictly increasing function, we have $\alpha_2 > \alpha_1$. This introduces ambiguity in the number of firms that will, in equilibrium, invest in the interval $[\alpha_1, \alpha_2)$. A single firm cannot profitably invest in any project in this interval, while two firms can. Hence, in equilibrium, it must hold that in any project in this interval, either no firm invests or both do. In the interval $[0, \alpha_1)$ both firms will invest, while in the interval $[\alpha_2, 1)$ neither firm will invest. If Assumption 1 does not hold, there will be an infinity of equilibrium market portfolios. In this sense,

Assumption 1 is essential for the model.

As a somewhat stylized example, motivated by the idea of absorptive capacity, consider the Cournot duopoly from Section 5.1, and suppose that there are spillovers from research and that a firm can implement the spillovers only if it has itself made the innovation. That is, let the marginal costs of production without innovation be 0.75, with innovation when there is only a single innovator 0.5, and with innovation when there are two innovators 0.25. In this specific example, this results in payoff functions which violate Assumption 1. Figure 6 illustrates the equilibrium market portfolio of research projects.

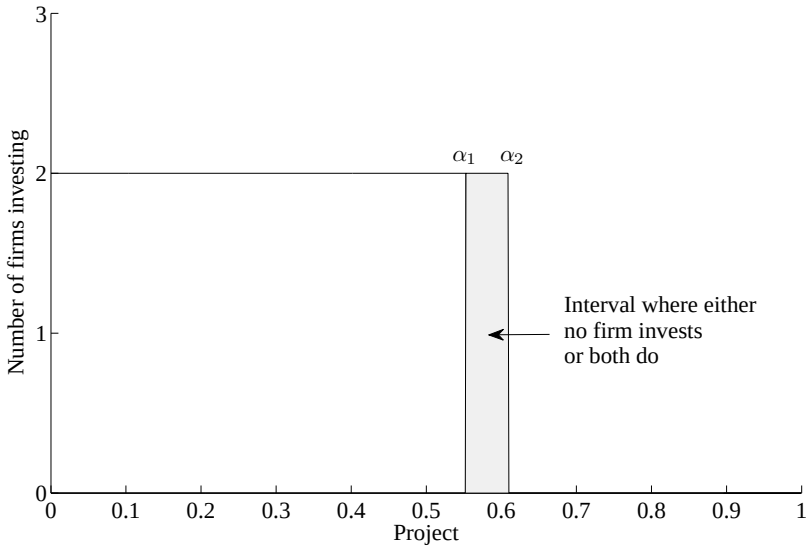

Figure 6: Violating Assumption 1.

7.3 Asymmetric firms

Suppose that two firms are producing homogeneous goods with different technologies, so that one firm has lower marginal costs of production than the other. Call the more efficient firm the leader and denote its marginal production cost with $\bar{c}_{lead}$. Call the less efficient firm the laggard and denote its marginal production cost with $\bar{c}_{lag}$. Suppose that the firms are symmetric in all other aspects and furthermore suppose that firms can invest in the development of a new production technology which would lower the production costs of whichever firm develops it to $\underline{c}$, such that $\bar{c}_{lag} > \bar{c}_{lead} > \underline{c}$. Suppose that Assumptions 1 and 2 hold.

First observe that if neither firm develops the innovation, firms will continue competing with the old technology and the leader's profits $r_{lead}(0, 2)$ will be greater than the laggard's profits $r_{lag}(0, 2)$. However, since the innovation is drastic, the profits post-innovation will be the same for both firms $R_{lead}(1) = R_{lag}(1)$ and $R_{lead}(2) = R_{lag}(2)$. To simplify

exposition, assume $C(0) < R(2)$. Analogously to before, denote the k-firm frontiers as

$$\begin{aligned}\alpha_{1,lag} &= C^{-1}\big(R(1) - r_{lag}(0,2)\big),\\ \alpha_{1,lead} &= C^{-1}\big(R(1) - r_{lead}(0,2)\big),\\ \alpha_2 &= C^{-1}\big(R(2)\big),\end{aligned}$$

where $\alpha_{1,lag}$ is the most expensive project in which the laggard would invest and $\alpha_{1,lead}$ is the most expensive project in which the leader would invest. It is straightforward to see that $\alpha_2 \leq \alpha_{1,lead} < \alpha_{1,lag}$. In equilibrium, both firms will invest in the interval $[0, \alpha_2)$, for any project in the interval $[\alpha_2, \alpha_{1,lead})$ either the leader or the laggard will invest (but only one will), and only the laggard will invest in the interval $[\alpha_{1,lead}, \alpha_{1,lag})$, while no firm will invest in the interval $[\alpha_{1,lag}, 1)$. Hence this model predicts that the laggard firms will be more likely to invest in the most expensive projects. Furthermore, if one is willing to assume that where both firms can invest they do so symmetrically, the laggard will be more likely to develop drastic innovations.

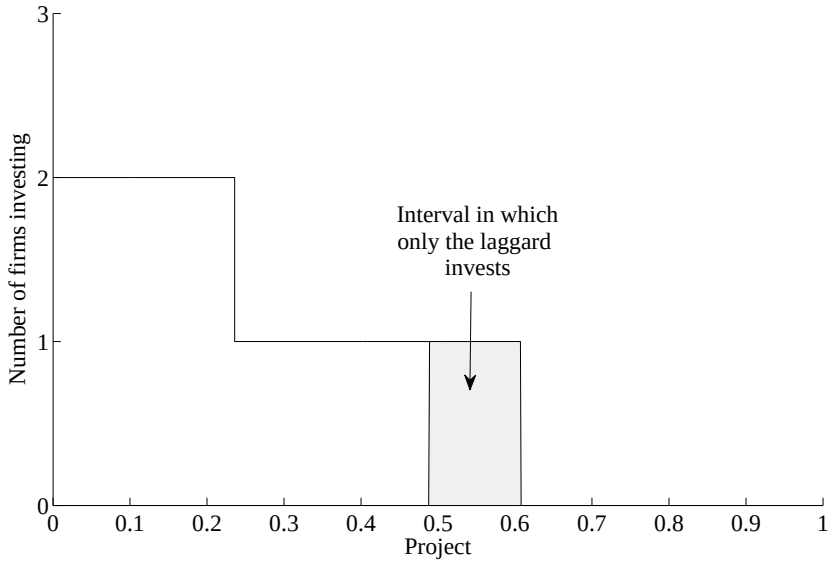

Figure 7: Asymetric firms.

As an illustrative example, consider again the Cournot duopoly from Section 5.3 and suppose that the pre-innovation costs of production of the leader are 0.75 while the pre-innovation costs of production of the laggard are 0.85. Leave the post-innovation costs of production unchanged. Figure 7 illustrates the equilibrium market portfolio of research projects.

7.4 Mixed strategies

Consider now the original setting, but suppose that firms are using mixed strategies. As a simplifying assumption, I will consider only the following pure strategy space

$$\mathcal{I}^m = \{0\} \cup \Big\{[0, j) : j \in (0, 1)\Big\}$$

and I will look only at symmetric mixed strategy equilibria (SMSE). Since now the pure strategy of a firm is restricted to choosing an interval $[0, j)$, it can be identified with the upper bound of the interval j. Denote with $f_i(j)$ the density that the firm i chooses the interval $[0, j)$ and with $F_i(j)$ the related cumulative distribution function.

Proposition 7 (Characterization of SMSE)**.** *Suppose $N = 2$ and the Assumptions 1 and 2 hold. Then a mixed strategy equilibrium is characterized by the cumulative distribution function:*

$$F(j) = \begin{cases} 0 & \text{if} \quad \dfrac{C(j) - R(2)}{R(1) - r(0,2) - R(2)} < 0 \\ \dfrac{C(j) - R(2)}{R(1) - r(0,2) - R(2)} & \text{if} \quad \dfrac{C(j) - R(2)}{R(1) - r(0,2) - R(2)} \in [0, 1] \\ 1 & \text{if} \quad \dfrac{C(j) - R(2)}{R(1) - r(0,2) - R(2)} > 1 \end{cases}$$

for $j \in [0, 1)$.

Suppose that $R(2) - C(0) > 0$. In pure actions, by Proposition 2 it holds: $m = 2$, $C(\alpha_1) = R(1) - r(0, N)$ and $C(\alpha_2) = R(2)$. Thus, both firms will invest in the interval $[0, \alpha_1)$, only one firm will invest in the interval $[\alpha_1, \alpha_2)$ and no firm will invest in $[\alpha_2, 0)$. Now consider SMSE. By Proposition 7, for $j \in [0, \alpha_1)$ it holds $F(j) = 0$, thus both firms invest in this interval with probability 1. For $j \in (\alpha_1, \alpha_2)$ it holds $0 < F(j) < 1$, thus firms invest with some probability less then one. If $j \in [\alpha_2, 0)$, then $F(j) = 1$, so that firms do not invest in this interval. Similar results hold if $R(2) - C(0) \leq 0$. Thus, the basic structure of the model is the same in both pure and mixed strategy equilibria. In particular the k-firm frontiers are the same. Furthermore, comparative statics results regarding variety of projects undertaken remain qualitatively the same, as anything that affects the one-firm frontier has qualitatively the same effect both in pure action and in mixed strategy equilibria. Figure 8 illustrates the difference between the (expected) equilibrium market portfolios for the Cournot duopoly example from Section 5.1. The mixed strategy equilibrium is "smoother" than the pure strategy equilibrium. The reason for this is that the integer problem is not present in the mixed strategy setting. In pure strategy equilibrium, some projects have higher expected profits than others (i.e. project

$\alpha_2+\epsilon$ is more profitable than $\alpha_1-\epsilon$ for some small positive ϵ). In mixed strategy equilibria, all projects in the interval where the mixing occurs have the same expected profits.

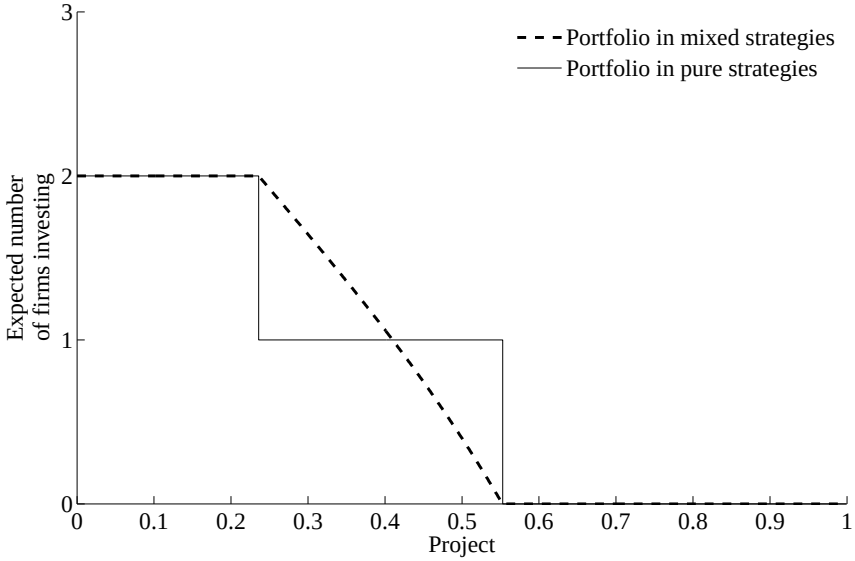

Figure 8: Symmetric mixed strategy equilibrium.

7.5 Limited budget and costly financing

This section considers the case where firms face an exogenous constraint on their research budgets. This constraint can take the form of a budget constraint, or it can (equivalently) take the form of costly financing for research. The main result is that a binding budget constraint or a costly source of financing imposes a positive opportunity cost on investments in research projects, but that the main mechanics of the model remain unchanged.

First, suppose that there are two firms in a market and that each firm has a budget B and suppose that the budget is binding, in the sense that firms would want to invest more in research if they had more resources.[16] Then the following result is obtained:

Proposition 8 (Equilibrium in a game with limited budget)**.**
Suppose that Assumptions 1 and 2 hold, and that there are two firms with a budget B. Then, an equilibrium always exists, the induced equilibrium market portfolio is unique and any investment plan which induces a portfolio identical to the market equilibrium portfolio is itself an equilibrium. Furthermore, there exists a unique $\beta > 0$ such that:

1. *the maximum number of firms investing in any project m^b is given by*

$$m^b = \max_{\{1,2\}} n \qquad s.t. \qquad R(n) - r(n-1, N) - C(0) > \beta.$$

[16]Formally, if $m = 2$ then $2B < 2\int_0^{\alpha_2} C(j)dj + \int_{\alpha_2}^{\alpha_1} C(j)dj$ and if $m = 1$ then $2B < \int_0^{\alpha_1} C(j)dj$.

2. Firm frontiers are determined by

$$\begin{aligned} R(1) - r(0,N) - C(\alpha_1^b) = \\ R(m^b) - r(m^b - 1, N) - C(\alpha_m^b) = \ \beta. \end{aligned}$$

3. Let $\alpha_{m+1}^b = 0$ *and* $\alpha_0^b = 1$. *The total expenditure is*

$$m \int_0^{\alpha_m^b} C(j)dj + (m-1) \int_{\alpha_m^b}^{\alpha_{m-1}^b} C(j)dj = 2B.$$

Then the equilibrium portfolio $n^b(j)$ *is given by*

$$n^b(j) = k \quad \text{if} \quad j \in [\alpha_{k+1}^b, \alpha_k^b).$$

As can be seen from conditions 1. and 2., the basic form of the market equilibrium portfolio will remain unchanged. The only difference is that the budget constraint will impose positive opportunity cost β on the choice of research projects, as opposed to the unconstrained equilibrium where the opportunity costs was 0. In the scenario where firms can borrow unlimited funds at some positive price, the equilibrium characterized above still holds, but now β is exogenously given and as a function of the cost of financing.

8 Conclusion

Economists have long studied how the level of investment in innovation changes as the intensity of competition in the market changes. This paper starts with the observation that studying solely the level of investment in innovation is incomplete. As an alternative, a model where firms explicitly choose their investment projects from a set of heterogeneous projects is developed and the equilibrium is characterized.

It is shown that, while the effect of an increase in competition on the total level of investment in innovation is ambiguous, the increase in competition increases the variety of approaches to innovation and increases the probability that an innovation is discovered. The policy recommendation drawn from this conclusion is that competition authorities should take into account this negative effect on the investment in innovation when reviewing merger cases.

Comparing the equilibrium market portfolio with the optimal equilibrium portfolio, it is shown that the market will tend to underinvest in drastic innovation. This underinvestment will be more severe the higher the potential benefit from innovation and the lower the overall intensity of competition in the industry. This suggests that R&D subsidies

should be targeted at high cost and high potential benefit projects (so-called blue sky projects) — especially in the industries with few firms and low intensity of competition.

Appendix A Proofs

A.1 Proof of Proposition 1

I prove each of the three statements contained in Proposition 1 in turn.

Lemma 1 (Existence)**.** *An equilibrium in pure actions always exists.*

I provide a constructive proof of Lemma 1 in three steps. Step 1 constructs the candidate equilibrium. Step 2 proves that no firm can increase its expected profits by making additional investments. Step 3 proves that no firm can increase its expected profits by reducing investments. Finally, notice that any deviation from the investment plan I^* can be written as a collection of investments and divestments and by Steps 2 and 3, each such investment and divestment decreases expected profits and hence any such collection must decrease expected profits. Thus, no firm can profitably deviate from the investment plan I^* and then, by definition, I^* is an equilibrium.

Step 1. *Constructing the candidate equilibrium.*

Given a game, define m such that

$$m = \max_{\{1,\dots,N\}} n$$

$$\text{s.t. } R(n,N) - r(n-1,N) - C(0) > 0$$

Since by assumption $R(1,N) - r(0,N) - C(0) > 0$, a solution to this maximization problem always exists.

Next, calculate each $\alpha_1, \alpha_2, \dots, \alpha_m$ such that the following condition holds:

$$\begin{aligned} R(1,N) - r(0,N) - C(\alpha_1) &= \\ R(2,N) - r(1,N) - C(\alpha_2) &= \\ R(3,N) - r(2,N) - C(\alpha_3) &= \\ &\vdots \\ R(m,N) - r(m-1,N) - C(\alpha_m) &= 0. \end{aligned}$$

Since rewards are finite and costs of innovation approach infinity as $j \to 1$, values $\alpha_1, \alpha_2, \dots, \alpha_m$ always exist. Furthermore, since $C(j)$ is increasing and by Assumption 1 the reward of innovation are non-increasing it is easy to see that $\alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_m$.

Observe that $N \geq m$. For each $i \in \{1, \ldots, m\}$, let $I_i^* = [0, \alpha_i)$. For each $i \in \{m+1, \ldots, N\}$ let $I_i^* = \emptyset$. I will demonstrate that I^* is an equilibrium.

Step 2. *Suppose that I^* is constructed as above. Then no firm can increase its expected profits by making additional investments.*

Proof. First observe that since $\forall j \in (\alpha_1, 1)$, by construction $R(1, N) - r(0, N) - C(j) < 0$, no firm has an incentive to invest beyond the technology frontier. I will consider separately the firms which in I^* have some investment and those firms which do not.

First, fix a firm $i' \in \{1, \ldots, m\}$ and take any feasible investment interval $L \subseteq [\alpha_{i'}, \alpha_1)$. It must be that $\min(L) \in [\alpha_k, \alpha_{k-1})$[17] for some $k \leq i'$ and $k \geq 2$ and $\sup(L) \in (\alpha_{k'}, \alpha_{k'-1}]$ for some $k' \leq i'$ and $k' \geq 2$, with $k' \leq k$.

First consider the case where $k' = k$. Then $L \subseteq [\alpha_k, \alpha_{k-1})$. Observe that $R(k, N) - r(k-1, N) - C(\alpha_k) = 0$ and $n(j, I^*) = k - 1$ for all $j \in [\alpha_k, \alpha_{k-1})$ by construction. Since $C(\cdot)$ is assumed to be strictly increasing, then $R(k, N) - r(k-1, N) - C(j) < 0$ for all $j \in (\min(L), \sup(L))$. Hence,

$$-\int_L C(j)dj + \int_L R(k, N)dj - \int_L r(k-1, N)dj < 0$$

and the firm i' has no incentive to invest in the interval L.

Next consider the case where $k' < k$. Then we can write $L = [\min(L), \alpha_{k-1}) \cup [\alpha_{k-1}, \alpha_{k-2}) \cup \cdots \cup [\alpha_{k'}, \sup(L))$. Denote these subintervals as L^{k-1}, $L^{k-2}, \ldots, L^{k'-1}$. Observe that by construction, the following statements hold:

$$\begin{aligned}
R(k, N) - r(k-1, N) - C(\alpha_k) = 0 \text{ and } n(j, I^*) = k - 1 \qquad & \text{for all } j \in L^{k-1} \\
R(k-1, N) - r(k-2, N) - C(\alpha_{k-1}) = 0 \text{ and } n(j, I^*) = k - 2 \qquad & \text{for all } j \in L^{k-2} \\
\vdots & \\
R(k', N) - r(k'-1, N) - C(\alpha_{k'}) = 0 \text{ and } n(j, I^*) = k' - 1 \qquad & \text{for all } j \in L^{k'-1}
\end{aligned}$$

Since $C(\cdot)$ is assumed to be strictly increasing, the following statements hold:

$$\begin{aligned}
R(k, N) - r(k-1, N) - C(j) < 0 \qquad & \text{for all } j \in L^{k-1} \\
R(k-1, N) - r(k-2, N) - C(j) < 0 \qquad & \text{for all } j \in L^{k-2} \\
\vdots & \\
R(k', N) - r(k'-1, N) - C(j) < 0 \qquad & \text{for all } j \in L^{k'-1}
\end{aligned}$$

[17] If $\alpha_k = \alpha_{k-1}$, let $L = \{\alpha_k\}$ and $\min(L) = \alpha_k$.

But then it holds

$$\int_L R(n(j,I^*)+1,N) - r(n(j,I^*),N) - C(j)dj =$$

$$= \left(\int_{L^{k-1}} R(k,N) - r(k-1,N) - C(j)dj\right) + \left(\int_{L^{k-2}} R(k-1,N) - r(k-2,N) - C(j)dj\right) + \cdots$$

$$\cdots + \left(\int_{L^{k'-1}} R(k',N) - r(k'-1,N) - C(j)dj\right) < 0$$

and the firm i' has no incentive to invest in the interval L.

Next, fix a firm $i' \in \{m+1,\ldots,N\}$ and take any feasible investment interval $L \subseteq [0,\alpha_1)$. Observe that we can write L as a union of two sets, $L = L' \cup L''$ where $L' \subseteq [0,\alpha_m)$ and $L'' \subseteq [\alpha_m,\alpha_1)$. By the same argument as above, it holds that any investment in the set L'' cannot be profitable. Consider now an investment in the set L'. By construction, m is the maximum number of firms that can profitably invest in the project $j=0$. Since $C(j)$ is strictly increasing it is also the maximum number of firms that can invest in any project. By construction, there are m firms investing in all projects in $[0,\alpha_m)$ and as a result the firm i' cannot profitably invest in the set L'. Thus, the investment in the set L cannot be profitable.

□

Step 3. *Suppose that I^* is constructed as above. Then no firm can increase its expected profits by decreasing investments.*

Proof. First observe that all firms $i > m$ have zero investments by construction and hence cannot decrease their investments. Fix a firm $i' \in \{1,\ldots,m\}$ and take any feasible investment interval $L \subseteq [0,\alpha_{i'})$. We can write L as a union of two sets, $L = L' \cup L''$ where $L' \subseteq [0,\alpha_m)$ and $L'' \subseteq [\alpha_m,\alpha_{i'})$. I will consider the profitability of disinvestments from sets L' and L'' in turn.

Consider first a disinvestment from the set L'. By construction $R(m,N) - r(m-1,N) - C(\alpha_m) = 0$. Since $C(j)$ is strictly increasing $R(m,N) - r(m-1,N) - C(j) > 0$ for all $j \in [0,\alpha_m)$. It follows that

$$\int_{L'} R(m,N) - r(m-1,N) - C(j)dj > 0.$$

and the firm has no incentive to divest from the set L'.

Next, consider a disinvestment from the set L''. It must be that $\min(L'') \in [\alpha_k,\alpha_{k-1})$ for some $k \geq i'$ with $k \leq m+1$ and $\alpha_{m+1} = 0$ and $\sup(L'') \in (\alpha_{k'},\alpha_{k'-1}]$ for some $k' \geq i'$ and $k' \leq k$.

First consider the case where $k' = k$. Then $L'' \subseteq [\alpha_k, \alpha_{k-1})$. Observe that $R(k-1,N) - r(k-1,N) - C(j) > 0$ and $n(j, I^*) = k-1$ for all $j \in (\alpha_k, \alpha_{k-1})$ by construction. Hence,

$$\int_{L''} R(k-1,N) - r(k-2,N) - C(j)dj > 0.$$

and the firm i' has no incentive to divest from the interval L''.

Next consider the case where $k' < k$. Then we can write $L'' = [\min(L''), \alpha_{k-1}) \cup [\alpha_{k-1}, \alpha_{k-2}) \cup \cdots \cup [\alpha_{k'}, \sup(L''))$. Denote these subintervals as L^{k-1}, $L^{k-2}, \ldots, L^{k'-1}$. Observe that by construction, the following statements hold:

$$\begin{aligned}
R(k-1,N) - r(k-2,N) - C(j) > 0 \text{ and } n(j,I^*) = k-1 \quad & \text{for all } j \in L^{k-1} \\
R(k-2,N) - r(k-3,N) - C(j) > 0 \text{ and } n(j,I^*) = k-2 \quad & \text{for all } j \in L^{k-2} \\
\vdots & \\
R(k'-1,N) - r(k'-2,N) - C(j) > 0 \text{ and } n(j,I^*) = k'-1 \quad & \text{for all } j \in L^{k'-1}
\end{aligned}$$

But then it holds

$$\int_{L''} R(n(j,I^*),N) - r(n(j,I^*)-1,N) - C(j)dj =$$

$$\begin{aligned}
= \int_{L^{k-1}} R(k-1,N) - r(k-2,N) - C(j)dj + \int_{L^{k-2}} R(k-2,N) - r(k-3,N) - C(j)dj + \\
+ \cdots + \int_{L^{k'-1}} R(k'-1,N) - r(k'-2,N) - C(j)dj > 0
\end{aligned}$$

and the firm i' has no incentive to divest from the interval L''.

As firms do not have an incentive to divest from intervals L' and L'' they also have no incentive to invest from the interval L. □

Lemma 2. *If I^* is an equilibrium and $0 < n(j, I^*) < N$ for some $j \in [0,1)$, then infinitely many equilibria exist.*

Proof. Let I^* be an equilibrium and fix some $j \in [0,1)$ such that $0 < n(j, I^*) < N$. Then there exist firms i and i' such that $j \in I_i$ and $j \notin I_{i'}$. Then there must exist some $\epsilon > 0$ such that $[j, j+\epsilon) \cap I_i = [j, j+\epsilon)$ and $[j, j+\epsilon) \cap I_{i'} = \emptyset$.

Consider an investment plan $\hat{I}$ such that $\hat{I}_{i''} = I^*_{i''}$, for all $i'' \neq i, i'$. For i and i' let $\hat{I}_i = I^*_i \setminus [j, j+\epsilon)$ and $\hat{I}_{i'} = I^*_{i'} \cup [j, j+\epsilon)$. In words, only transfer the ownership of investment in projects $[j, j+\epsilon)$ from firm i to firm i' and leave everything else unchanged. I will demonstrate that $\hat{I}$ is also an equilibrium and hence, since there is an infinite number of ways to choose ϵ, there exists an infinity of equilibria.

Suppose that $\hat{I}$ is not an equilibrium. Then, there exists a firm that can profitably change its investment plan. This means that there exists an interval $L \subset [0,1)$ and a firm i^l such that firm can increase its expected profits by either investing in the interval L or divesting from the interval L. Consider first those firms $i'' \neq i, i'$. By construction $n(j, I^*) = n(j, \hat{I})$ for all $j \in [0,1)$. From Equation 1, it is clear that strategic effects only influence the expected profit through $n(j, I)$. Thus, if a firm can profitably deviate from $\hat{I}$ it can also profitably deviate from I^*.

Next, consider firms i and i'. Since their investment plans are unchanged in the set $[0, j) \cup [j+\epsilon, 1)$ by an argument identical to the one above, if they could profitably deviate in this set from $\hat{I}$, they could also profitably deviate from I^*. Now consider the set $[j, j+\epsilon]$. Firm i' can deviate in this set only by not investing. Suppose that there exists an interval $L' \subseteq [j, j+\epsilon]$, such that not investing in this set increases the expected profits of firm i'. Then it must be the case that

$$\int_{L'} R(n(j, \hat{I}), N) - r(n(j, \hat{I}) - 1, N) - C(j)dj < 0.$$

But in this case, firm i could profitably deviate from I^* by not investing in the interval L'. Next, firm i can deviate in the set $[j, j+\epsilon]$ only by investing. Suppose that there exists an interval $L' \subseteq [j, j+\epsilon]$, such that investing in this set increases the expected profits of firm i. Then it must be the case that

$$\int_{L'} R(n(j, \hat{I}) + 1, N) - r(n(j, \hat{I}), N) - C(j)dj > 0.$$

But in this case, firm i' could profitably deviate from I^* by investing in the interval L'.

Thus, in each case, a profitable deviation from $\hat{I}$ implies a profitable deviation from I^* which contradicts the initial assumption that I^* is an equilibrium. □

Lemma 3. *If there are multiple equilibria they all result in the same market portfolio of investment in innovation. That is, if I_1^* and I_2^* are equilibrium investment plans, then $n(j, I_1^*) = n(j, I_2^*)$ for all $j \in [0,1)$. If I_1^* is an equilibrium then any investment plan I_3^* such that $n(j, I_1^*) = n(j, I_3^*)$ for all $j \in [0,1)$ is also an equilibrium.*

I prove this Lemma in two steps, each proving one part of the Lemma.

Step 1. *If there are multiple equilibria they all result in the same market portfolio of investment in innovation. That is, if I_1^* and I_2^* are equilibrium investment plans, then $n(j, I_1^*) = n(j, I_2^*)$ for all $j \in [0,1)$.*

Proof. Suppose not. Then, there exists a point $j \in [0,1)$ such that $n(j, I_1^*) \neq n(j, I_2^*)$. Suppose, without loss of generality, that $n(j, I_1^*) > n(j, I_2^*)$. Fix a firm i and a point

$\epsilon > 0$ such that it holds $[j, j+\epsilon) \cap I^*_{1,i} = [j, j+\epsilon)$ and $[j, j+\epsilon) \cap I^*_{2,i} = \emptyset$ and $n(l, I^*_1) = \text{const}$, $n(l, I^*_2) = \text{const}, \forall l \in [j, j+\epsilon)$. Such a firm and a point always exist.

(1) Suppose $R(n(j, I^*_1), N) - r(n(j, I^*_1) - 1, N) \geq C(j+\epsilon)$. Then there exists an $\epsilon' > 0$ such that $R(n(j, I^*_1), N) - r(n(j, I^*_1) - 1, N) > C(l)$ for all $l \in [j+\epsilon-\epsilon', j+\epsilon)$. By Assumption 1 it holds $R(n(j, I^*_2)+1, N) - r(n(j, I^*_2), N) > C(l)$ for all $l \in [j+\epsilon-\epsilon', j+\epsilon)$. Then it holds

$$\int_{j+\epsilon-\epsilon'}^{j+\epsilon} R(n(l, I^*_2) + 1, N) - r(n(l, I^*_2), N) - C(l)dl > 0.$$

Then I^*_2 cannot be an equilibrium as firm i could increase its expected profits by investing in the interval $[j+\epsilon-\epsilon', j+\epsilon)$.

(2) Suppose $R(n(j, I^*_1), N) - r(n(j, I^*_1) - 1, N) < C(j+\epsilon)$. Then there exists an $\epsilon' > 0$ such that $R(n(j, I^*_1), N) - r(n(j, I^*_1) - 1, N) < C(l)$ for all $l \in [j+\epsilon-\epsilon', j+\epsilon)$. Then it holds

$$\int_{j+\epsilon-\epsilon'}^{j+\epsilon} R(n(l, I^*_2), N) - r(n(l, I^*_2) - 1, N) - C(l)dl < 0.$$

Then I^*_1 cannot be an equilibrium as firm i could increase its expected profits by not investing in the interval $[j+\epsilon-\epsilon', j+\epsilon)$. □

Step 2. *If I^*_1 is an equilibrium then any investment plan I^*_3 such that $n(j, I^*_1) = n(j, I^*_3)$ for all $j \in [0, 1)$ is also an equilibrium.*

Proof. Suppose not. Then in the investment plan I^*_3 exists a firm i and an interval L such that firm i would be better off by either investing in the interval L or by divesting from interval L.

(1) Suppose that the firm i can profitably invest in the interval L. Then there exists $L' \subseteq L$ such that $R(n(j, I^*_3)+1) - r(n(j, I^*_3), N) > C(l)$ for all $l \in L'$. But then there exists a firm i'' and a set $L'' \subseteq L'$ such that $L'' \cap I^*_{1,i''} = \emptyset$. Then I^*_1 cannot be an equilibrium as the firm i'' could profitably deviate by investing in the interval L''.

(2) Suppose that the firm i can profitably divest from the interval L. Then there exists $L' \subseteq L$ such that $R(n(j, I^*_3), N) - r(n(j, I^*_3 - 1), N) < C(l)$ for all $l \in L'$. But then there exists a firm i'' and a set $L'' \subseteq L'$ such that $L'' \cap I^*_{1,i''} = L''$. Then I^*_1 cannot be an equilibrium as the firm i'' could profitably deviate by divesting from the interval L''. □

A.2 Proof of Proposition 2

Proof. Observe that by Assumption 2, for all $n \geq 1$ we have $r(n, N) = 0$. Let I^* be the equilibrium constructed in the proof of Lemma 1. If $n^*(j)$ as constructed in Proposition 2 is equal to $n(j, I^*)$ for all $j \in [0, 1)$, then by statement 3 in Proposition 1 it characterizes the equilibrium market portfolio of research projects.

I here show that $n^*(j) = n(j, I^*)$. First, as noted in the proof of Lemma 1, observe that $\alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_m$. First suppose that $j \in [\alpha_1, 1)$ Then it must be that $j \geq \alpha_k$ for all $k \in \{1, \ldots, m\}$. Hence $n^*(j) = 0$. By construction, $n(l, I^*) = 0$ for all $l \in [\alpha_1, 1)$. Next, suppose that $j \in [0, \alpha_1)$. Then

$$n^*(j) = \max_{1,\ldots,m} k$$

$$\text{s.t. } j < \alpha_k$$

Let $\hat{k} = n^*(j)$. It holds that $j < \alpha_{\hat{k}} \leq \alpha_{\hat{k}-1} \leq \cdots \leq \alpha_1$. By construction, each firm i such that $i \in \{1, \ldots, \hat{k}\}$ invests in j. Hence, $n(j, I^*) = \hat{k}$. □

A.3 Proof of Proposition 3

Proof. Two statements need to be proven. First that the maximum number of firms investing weakly increases. Second that the variety of projects developed weakly increases, hence the probability of developing an innovation also weakly increases. Let the number of firms in the pre-innovation market increase from N to N'. Denote the maximum number of firms investing in two cases as m and m' and the one-firm frontiers as α_1 and α_1'.

By Proposition 2 we have $m = \max_{\{1,\ldots,N\}} n$ such that $R(n) - r(n-1, N) - C(0) > 0$. Observe that $m \in \{1, \ldots, N\} \subseteq \{1, \ldots, N'\}$. If $n = 1$, by assumption 3 we have $R(n) - r(n-1, N') - C(0) \geq R(n) - r(n-1, N) - C(0)$. If $n > 1$, by assumption 2 we have $R(n) - r(n-1, N') - C(0) = R(n) - r(n-1, N) - C(0)$. Hence m is chosen from a subset from which m' is chosen and it satisfies a stricter condition. Thus m' cannot be lower than m.

By Proposition 2 the variety of projects developed in the two equilibria is equal to the sets $[0, \alpha_1)$ and $[0, \alpha_1')$ and the probability of successfully developing an innovation is equal to α_1 and α_1'). Thus we need to show that $\alpha_1 \leq \alpha_1'$. By Proposition 2 we have $R(1) - r(0, N) = C(\alpha_1)$ and $R(1) - r(0, N') = C(\alpha_1')$. By assumption 3 we have $r(n-1, N') \leq r(n-1, N)$ hence $C(\alpha_1') \geq C(\alpha_1)$. Since $C(j)$ is assumed to be increasing this implies $\alpha_1' \geq \alpha_1$.

□

A.4 Proof of Proposition 4

Proof. suppose the intensity of competition increases from (R, r) to (R', r'). Denote the respective equilibrium investment plans as I and I'. Then the following holds by direct

application of Proposition 2:

$$
\begin{aligned}
m' &\begin{cases} = m & \text{if } R'(m,N) - C(0) > 0 \\ < m & \text{otherwise} \end{cases} \\
\alpha'_k &< \alpha_k \quad \forall k \in \{2, \ldots, m'\} \\
\alpha'_1 &> \alpha_1
\end{aligned}
$$

and

$$
\begin{aligned}
n(j, I') &\leq n(j, I) && \text{for all } j \in [0, \alpha_1) \\
n(j, I') &> n(j, I) && \text{for all } j \in [\alpha_1, \alpha'_1) \\
n(j, I') &= n(j, I) = 0 && \text{for all } j \in [\alpha'_1, 1).
\end{aligned}
$$

Since $\alpha'_1 > \alpha_1$ the variety of research projects undertaken and the probability of discovering an innovation increase. Since $\alpha'_k < \alpha_k \quad \forall k \in \{2, \ldots, m'\}$ there are some projects which are developed by fewer firms than with less intense competition. Hence the amount of duplication of research decreases.

□

A.5 Proof of Proposition 5

Proof. The portfolio given in Proposition 5 can always be constructed. I show that it is optimal. Suppose not. Then, there exists a project $j \in [0, 1)$ such that investing either more or less than $n^o(j)$ marginally increases the expected welfare. There are two cases: (1) there exists a possibility to profitably increase investment in some project and (2) there exists a possibility to profitably decrease investment in some project.

(1) Suppose that there exists a possibility to profitably increase investment in some project j. Then there exists some n such that $n^0(j) < n \leq N$ and

$$W(n) - nC(j) > W(n^0(j)) - n^0(j)C(j).$$

Then we can write

$$\left[\sum_{k=n^o(j)+1}^{n} W(k) - W(k-1) - C(j) \right] + W(n^0(j)) - n^0(j)C(j) > W(n^0(j)) - n^0(j)C(j)$$

$$\sum_{k=n^o(j)+1}^{n} W(k) - W(k-1) - C(j) > 0.$$

Suppose $n^o(j) = m^o$. Then, $\forall k > n^o(j)$ it holds:

$$\Big(W(k) - W(k-1) - C(0)\Big) + \Big(C(0) - C(j)\Big) \leq 0$$

the first bracketed expression is by construction not positive while the second is not positive since the function $C(\cdot)$ is increasing. A sum of non-positive elements cannot be positive. A contradiction.

Suppose now that $n^o(j) < m^o$. By construction it holds

$$W(n^0(j)+1) - W(n^0(j)) - C(\alpha^o_{n^0(j)+1}) = 0$$

and for every $k > n^0(j)$ by assumption holds

$$W(k) - W(k-1) - C(\alpha^o_{n^0(j)+1}) \leq 0$$

By construction $j > \alpha^o_{n^0(j)+1}$, so that $C(j) > C(\alpha^o_{n^0(j)+1})$. Plugging it into the expression above, it follows

$$W(k) - W(k-1) - C(j) \leq 0 \quad \forall k n^0(j).$$

Again, a sum of non-positive elements cannot be positive. A contradiction.

(2) Suppose that there exists a possibility to profitably decrease investment in some project j. Then there exists some n such that $0 \leq n < n^0(j)$ and

$$W(n) - nC(j) > W(n^0(j)) - n^0(j)C(j).$$

Then we can write

$$W(n) - nC(j) > W(n) - nC(j) + \left[\sum_{k=n+1}^{n^o(j)} W(k) - W(k-1) - C(j)\right]$$

$$0 > \sum_{k=n+1}^{n^o(j)} W(k) - W(k-1) - C(j).$$

By construction $W(n^o(j)) - W(n^o(j)-1) - C(j) > 0$ and by assumption it holds for any $k < n^o(j)$ that

$$W(n^o(j)) - W(n^o(j)-1) - C(j) > 0$$

A sum of positive elements has to be positive. A contradiction.

□

A.6 Proof of Corollary 1

Using the notation of Propositions 2 and 4 the variety of R&D projects in the market portfolio is $[0, \alpha_1]$ and the variety of R&D projects in the optimal portfolio is $[0, \alpha_1^o]$. Thus the market will underinvest in the variety of R&D projects if and only if $\alpha_1 < \alpha_1^o$. Since $C(\cdot)$ is increasing this is equivalent to $C(\alpha_1) < C(\alpha_1^o)$. By Propositions 2 and 4 it then holds $R(1) - r(0, N) < W(1) - W(0)$. Decomposing $W(1)$ into $CS(1) + R(1)$ and $W(0)$ into $CS(0) + Nr(0, N)$ yields the desired result. Overinvestment and optimal investment cases are proven analogously.

A.7 Proof of Proposition 6

Since assumptions 1, 2 and 4 hold, by Corollary 1 this market will underinvest in the variety of R&D projects if and only if $CS(1) - W(0) + r(0, N) > 0$. Denote with q_1 the quantity supplied by a monopolist with the innovation and with q_0 the quantity supplied by a single firm if no innovation is developed. Since the innovation is drastic $P(q_1) < P(Nq_0)$ or equivalently $q_1 > Nq_0$. Since we can write the consumer surplus as the difference between total utility and the total expense paid by consumers it holds $CS(1) = \int_0^{q_1} P(s)ds - P(q_1)q_1$. Welfare is total utility less the total cost of production, so it holds $W(0) = \int_0^{Nq_0} P(s)ds - N\bar{c}(q_0)$. Then this market will underinvest if and only if:

$$\int_0^{q_1} P(s)ds - P(q_1)q_1 - \left[\int_0^{Nq_0} P(s)ds - N\bar{c}(q_0)\right] + r(0, N) > 0.$$

Subtracting the integrals and rearranging terms gives:

$$\int_{Nq_0}^{q_1} P(s)ds - P(q_1)q_1 + N\bar{c}(q_0) + r(0, N) > 0.$$

By assumption $P'(\cdot) < 0$ so that $\int_{Nq_0}^{q_1} P(s)ds \geq (q_1 - Nq_0)P(q_1)$. The inequality above will hold whenever the following inequality holds:

$$(q_1 - Nq_0)P(q_1) - P(q_1)q_1 + N\bar{c}(q_0) + r(0, N) > 0.$$

Rearranging gives:

$$N\bar{c}(q_0) - Nq_0P(q_1) + r(0, N) > 0.$$

By assumption $\bar{c}'(\cdot) \geq 0$ so that $\bar{c}(q_0) = \int_0^{q_0} \bar{c}'(s)ds \geq (q_0 - 0)\bar{c}'(0)$. The inequality above will hold whenever the following inequality holds:

$$Nq_0(\bar{c}'(0) - P(q_1)) + r(0, N) > 0.$$

Since $r(0,N) \geq 0$ by rationality of firms and $\bar{c}'(0) > P(q_1)$ by definition of a drastic process innovation, the above inequality always holds.

A.8 Proof of Corollary 2

Consider first the case where $\alpha_k^o < \alpha_k$. Since $C(\cdot)$ is increasing then it holds $C(\alpha_k^o) < C(\alpha_k)$. Since assumptions 1, 2 and 4 hold, then Propositions 2 and 5 hold. Applying them yields $W(k) - W(k-1) < R(k)$ and decomposing the expression for $C(\cdot)$ yields $kR(k) + CS(k) - (k-1)R(k-1) - CS(k-1) < R(k)$. Rearranging gives:

$$\delta(k) = \Big[(k-1)\big(R(k) - R(k-1)\big)\Big] + \Big[CS(k) - CS(k-1)\Big] < 0.$$

Hence $\alpha_k^o < \alpha_k$ if and only if $\delta(k) < 0$. The other cases follow by substituting the inequality sign.

A.9 Proof of Proposition 2a

Observe that the proof of Proposition 1 does not require Assumption 2. Proof of Proposition 2a exactly mirrors the proof of Proposition 2, except without setting $r(n,N) = 0$ for all $n \geq 1$. In essence, Proposition 2 is a special case of Proposition 2a.

A.10 Proof of Proposition 7

Observe that the function $F(j)$, by the assumptions made on the function $C(j)$, is both increasing and differentiable. denote with $f(j)$ its probability density function.

Suppose, without loss of generality, that the firm 2 invests according to the function F and the firm 1 chooses some pure action x_1. Then the profit of the firm 1 can be expressed as:

$$\begin{aligned}\pi_1(x_1, F_2) = &- \int_0^{x_1} C(j)dj + \int_0^{x_1} \left[\int_0^{x_2} R(2)dj + \int_{x_2}^{x_1} R(1)dj + \int_{x_1}^{1} r(0,2)dj\right] f(x_2)dx_2 + \\ &+ \int_{x_1}^{1} \left[\int_0^{x_1} R(2)dj + \int_{x_1}^{x_2} r(1,2)dj + \int_{x_2}^{1} r(0,2)dj\right] f(x_2)dx_2.\end{aligned}$$

Deriving:

$$\begin{aligned}\frac{d\pi_1(x_1,F_2)}{dx_1} =& -C(x_1)+\left[\int_0^{x_1} R(2)dj+\int_{x_1}^{x_1} R(1)dj+\int_{x_1}^{1} r(0,2)dj\right] f(x_1)- \\ &-\left[\int_0^{x_1} R(2)dj+\int_{x_1}^{x_1} R(1)dj+\int_{x_1}^{1} r(0,2)dj\right] f(x_1)+ \\ &+\int_0^{x_1}\left[R(1)-r(0,2)\right] f(x_2)dx_2+\int_{x_1}^{1} R(2)f(x_2)dx_2\end{aligned}$$

and simplifying:

$$\frac{d\pi_1(x_1,F_2)}{dx_1} = -C(x_1)+\left[R(1)-r(0,2)\right]F(x_1)+R(2)\left(1-F(x_1)\right).$$

Observe that, for x_1 such that $\frac{C(j)-R(2)}{R(1)-r(0,2)-R(2)} < 0$ it follows that $\frac{d\pi_1(x_1,F_2)}{dx_1} > 0$, for x_1 such that $\frac{C(j)-R(2)}{R(1)-r(0,2)-R(2)} \in [0,1]$ it follows that $\frac{d\pi_1(x_1,F_2)}{dx_1} = 0$, and for x_1 such that $\frac{C(j)-R(2)}{R(1)-r(0,2)-R(2)} > 1$ it follows that $\frac{d\pi_1(x_1,F_2)}{dx_1} < 0$. Hence, firm 1 is maximizing its expected profit by choosing any action x_1 in the support of the function $f(x)$. Thus (f,f) is a profile of equilibrium mixed strategies.

A.11 Proof of Proposition 8

Lemma 4. *An equilibrium inducing portfolio equivalent to the one characterized in the Proposition can always be constructed.*

Proof. Either $m = 2$ or $m = 1$. If $m = 1$, then it holds $\int_0^{\alpha_1^b} C(j)dj = 2B$. Then there exists a point x such that $0 < x < \alpha_1^b$ and $\int_0^x C(j)dj = B$ and $\int_x^{\alpha_1^b} C(j)dj = B$. Let one firm invest in the interval $[0,x)$ and the other firm in the interval $[x,\alpha_1^b)$. This investment plan generates a portfolio equivalent to the one characterized.

If $m = 2$, then it holds $2\int_0^{\alpha_2^b} C(j)dj + \int_{\alpha_2^b}^{\alpha_1^b} C(j)dj = 2B$. Then there exists a point x such that $\alpha_2^b \le x \le \alpha_1^b$ and $\int_0^x C(j)dj = B$ and $\int_0^{\alpha_2^b} C(j)dj + \int_x^{\alpha_1^b} C(j)dj = B$. Let one firm invest in the interval $[0,x)$ and the other firm in the set $[0,\alpha_2^b)\cup[x,\alpha_1^b)$. This investment plan generates a portfolio equivalent to the one characterized. □

Lemma 5. *β always exists and is unique.*

Proof. Define functions $\psi^1(\beta):[0,\beta^1]\to\mathbb{R}^+$, $\psi^2(\beta):[0,\beta^2]\to\mathbb{R}^+$ such that

$$\psi^1(\beta) = \int_0^{C^{-1}\left(R(1)-r(0,2)-\beta\right)} C(j)dj$$

$$\psi^2(\beta) = \int_0^{C^{-1}\left(R(2)-\beta\right)} C(j)dj + \int_0^{C^{-1}\left(R(1)-r(0,2)-\beta\right)} C(j)dj$$

with $\beta^1 = R(1) - r(0,2) - C(0)$ and $\beta^2 = R(2) - C(0)$. Since $C(\cdot)$ is continuous, strictly increasing and defined on an interval, its inverse is continuous and strictly increasing as well. Hence both $\psi^1(\beta)$ and $\psi^1(\beta)$ are continuous and strictly decreasing. Furthermore, by Assumption 1 it holds $\beta^1 \geq \beta^2$.

Either (i) $\psi^1(\beta^2) \geq 2B$ or (ii) $\psi^1(\beta^2) < 2B$. If (i) is true, $\psi^1(\beta^2) \geq 2B$ and $\psi^1(\beta^1) = 0 < 2B$. By the Intermediate Value Theorem there exists some $\beta^* \in [\beta^2, \beta^1)$ such that $\psi^1(\beta^*) = 2B$ and furthermore β^* is unique since $\psi^1(\beta)$ is strictly decreasing. Observe that $\beta^* \in [R(2) - C(0), \beta^1)$, hence $R(1) - r(0,2) - C(0) > \beta^*$ and $R(2) - C(0) \leq \beta^*$. Thus, by the condition 1. of Proposition 8 we have $m^b = 1$. By the condition 2. the firm frontier is $\alpha_1^b = C^{-1}\big(R(1) - r(0,2) - \beta^*\big)$. Finally, the condition 3. holds since $\int_0^{\alpha_1^b} C(j)dj = 2B$ by construction. Hence, β^* uniquely satisfies all three conditions of the Proposition 8.

If (ii) is true, then $\psi^2(\beta^2) < 2B$ and $\psi^2(0) > 2B$, by the assumption of the binding budget constraint. By the Intermediate Value Theorem there exists some $\beta^* \in (0, \beta^2)$ such that $\psi^2(\beta^*) = 2B$ and furthermore β^* is unique since $\psi^2(\beta)$ is strictly decreasing. Observe that $\beta^* \in (0, \beta^2)$, hence $R(2) - C(0) > \beta^*$. Thus, by the condition 1. of Proposition 8 we have $m^b = 2$. By the condition 2. the firm frontiers are $\alpha_1^b = C^{-1}\big(R(1) - r(0,2) - \beta^*\big)$ and $\alpha_2^b = C^{-1}\big(R(2) - \beta^*\big)$. Finally, the condition 3. holds since $\int_0^{\alpha_2^b} C(j)dj + \int_0^{\alpha_1^b} C(j)dj = 2B$ by construction. Hence, β^* uniquely satisfies all three conditions of the Proposition 8. □

Lemma 6. *The investment plan constructed in Lemma 4 is an equilibrium and any investment plan inducing the same portfolio is an equilibrium as well.*

Proof. The proof is analogous to the proof of Proposition 1, with the opportunity cost equal to β as opposed to 0. □

Lemma 7. *The portfolio constructed in Proposition 8 and the portfolio induced in Lemma 4 are equivalent.*

Proof. The proof is analogous to the proof of Proposition 2. □

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