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The provision point mechanism with reward money

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Abstract

We introduce reward money into the provision point mechanism with refunds. Reward money is distributed among the contributors in proportion to their contributions only when the provision point is not reached. In environments without aggregate uncertainty, the provision point is always reached in equilibrium as competition for reward money and preference for the public good induce sufficient contributions. Importantly, the mechanism not only ensures allocative efficiency but also distributional. At a specific level of reward money, we obtain a unique equilibrium, where all consumers contribute the same proportion of their private valuations. The advantages of the mechanism are also demonstrated for collective action problems.

Keywords: Public goods; private provision; provision point mechanism; distributional efficiency; collective action problem *JEL codes:* D82, H41.

1 Introduction

This paper proposes a new mechanism for the provision of threshold public goods. The mechanism extends the provision point mechanism with refunds by an additional clause. The clause specifies a sum of reward money to be distributed among the contributors in

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proportion to their contributions if the sum of contributions is below the provision point. Hence, in the event of insufficient contributions each contributor gets his contribution refunded and, additionally, a share of the promised reward money. In environments without aggregate uncertainty, the distribution of reward money never occurs in equilibrium. Competition for reward money and preference for the public good induce contributions up to the level where the provision point is reached. Importantly, this mechanism not only resolves the free-riding problem but can also implement the public good in the unique Nash equilibrium.

In equilibria, obtained under the proposed mechanism, every consumer obtains a pay-off from the public good at least as high as that from the share of the reward money assigned to him if he deviates. Therefore, the effect of the introduction of reward money is a reduction of the set of individually rational strategies that can be supported in equilibrium. A higher level of reward money implies a smaller set of equilibrium strategies. With reward money set at the net value of the project, the mechanism uniquely implements the public good project.

The unique equilibrium has a special feature. Every consumer contributes the same proportion of his valuation, which is equal to the ratio of the cost of the public good and its total value. Therefore, the mechanism ensures not only allocative but also distributional efficiency. Taken from a different perspective, the mechanism effectively levies a Lindahl tax and can be expressed as a demand to pay a proportional tax on the private valuation for the public good. The reward money ensures that consumers have the right incentives to reveal their privately known valuations truthfully. The main condition for uniqueness is the absence of aggregate uncertainty about the value of the public good. We also discuss the performance of the mechanism in environments with aggregate uncertainty.

As it is well known, free riding is likely to emerge when externalities are not internalized. Reward money can be viewed as a device to compensate consumers for externalities they create. For the same reason, we argue that the suggested mechanism can prove useful in other situations where the problem that externalities are not internalized arises. Specifically, we demonstrate this on a collective action problem, where participation in

a project is individually rational only when a critical mass of participants is reached. Reward money effectively eliminates undesirable equilibria leaving only the efficient one, which, by design, does not lead to the distribution of reward money. Moreover, the mechanism implements the efficient outcome in weakly dominant strategies. In the case of negative externalities, e.g., the problem of the commons, the mechanism fails to achieve the efficient outcome without distributing reward money. The reason is that the efficient outcome is not individually rational with negative externalities unlike in the case with positive externalities.

Generally, the idea behind our mechanism relates to the augmented revelation principle of Mookherjee and Reichelstein (1990) (also see Ma et al. (1988)). They show that the revelation principle augmented with specially designed transfer payments eliminates the undesirable equilibria produced by the direct mechanism. At the same time, as in our mechanism, transfer payments are never paid in equilibrium. Taken from this more general perspective, our mechanism when equivalently reformulated as a direct mechanism can be seen as a practically applicable example of the augmented revelation principle.

The literature on the private provision of public goods and, specifically, on the provision point mechanism is immense to be discussed in any greater detail here. The provision point mechanism with refunds has a practical appeal as it is simple for implementation. Benjamin Franklin applied it successfully already in the 18th century. This mechanism is formally introduced and analyzed in Bagnoli and Lipman (1989) (see Palfrey and Rosenthal (1984) for a discrete version). They show that under complete information it uniquely implements the efficient outcome in undominated perfect equilibrium, but it certainly gives rise to a multiplicity of Nash equilibria including inefficient ones. Experimental studies reveal that this mechanism implements the public good in about 50 percent of cases (Isaac et al. (1989), Cadsby and Maynes (1999), Marks and Croson (1999)) but the problem of free riding is sizable (see Ledyard (1995) and Chen (2008) for reviews). In the field, the implementation rate is much lower (Rose et al. (2002)). With the introduction of seed money, i.e., significant first-move donations, the efficiency of the provision point mechanism improves (List and Lucking-Reiley (2002)), but a multiplicity

ity of equilibria, including free riding, is still a problem (Andreoni (1998)). Attempts are made to improve the performance of the provision point mechanism by introducing different rebate rules of contributions exceeding the provision point such as proportional rebate, winner-takes-all, etc. For experimental evidence, see Marks and Croson (1998), Rondeau et al. (1999), Spencer et al. (2009), who show improvements in allocative efficiency, but there are concerns regarding distributional efficiency. All this calls for further effort on improving the provision point mechanism, in which the current paper attempts to make a contribution.

On the theoretical side, this paper is closest to public good games with rewards to contributors. Falkinger (1996) proposes a mechanism that rewards contributors with above-average contributions. Morgan (2000) studies the mechanism that induces contributions with the help of lotteries. Goeree et al. (2005) demonstrate the advantages of the all-pay auction design in soliciting contributions. For experimental evidence on the performance of these mechanisms, see Falkinger et al. (2000), Morgan and Sefton (2000), Lange et al. (2007), and Corazzini (2010), who all report improved allocative efficiency. However, distributional efficiency may be failed. In the case of the lottery mechanism, it can happen (and it is empirically supported, see Kearney (2005)) that it is poorer people who end up financing the public good, i.e., lotteries are regressive. Morgan (2000) points out that adverse distributional effects may override allocative gains, leaving this problem open. The same applies to the mechanisms of Falkinger (1996) and Goeree et al. (2005). Lastly, the mechanisms that reward contributors, discussed above, lead to the distribution of promised rewards in equilibrium, which is not the case with the mechanism proposed in the current paper.

The remainder of the paper is organized as follows. After introducing a set-up in Section 2, we study the performance of the mechanism in two different environments: (i) without aggregate uncertainty (Section 3) and (ii) with aggregate uncertainty (Section 4). In the environment with aggregate uncertainty, the focus is on comparative analysis of the proposed mechanism and the standard provision point mechanism with refunds. Section 5 deals with an application of the proposed mechanism to a collective action

problem. The last section concludes the study.

2 Set-up

There is an economy that consists of a set $N = \{1, \dots, n\}$ of consumers with quasi-linear utility functions

$$U_i = w_i + v_i h(G). \quad (1)$$

In (1), w_i denotes the wealth of consumer i in the numeraire good, and $v_i h(G)$ denotes his utility from the public good G provided. The public good cannot be provided in amounts below a threshold of $G_{\min} > 0$. The function $h(G)$ is strictly increasing and concave, $h'(\cdot) > 0$ and $h''(\cdot) < 0$. Valuation $v_i > 0$ is privately known, which is drawn from a common prior distribution $F(\cdot)$ for every $i \in N$. The public good is provided by transforming the numeraire good into G on a one-for-one basis. We assume that the marginal utility of the public good, $v_i h'(G)$, is smaller than 1 for every i so that no consumer finds it individually beneficial to increase the amount of the public good already provided. Throughout the paper, wealth constraints are assumed to be non-binding for all consumers.

In the economy, there is a public authority that seeks to implement the public good project of a given size G . The authority has a capacity to raise a budget of at most B , which is, however, insufficient to provide the public good in any amount, i.e., $B < G_{\min}$. To raise the required funds G , the authority turns to the public with the following mechanism. Consumers are asked to make voluntary contributions toward the public good; let $\mathbf{g} = (g_1, \dots, g_n) \in \mathbb{R}_+^n$ denote a profile of their contributions. If $\sum_j g_j \geq G$, the public good is financed out of the contributions collected, with the excess amount $\sum_j g_j - G$ wasted (assumed for the ease of exposition). If $\sum_j g_j < G$, the public good is not provided, the contributions are refunded, and the authority distributes an ex-ante promised level of reward money R among the consumers in proportion to their contributions.

For brevity, we label the mechanism by its amount of reward money, R . Until further

notice, we shall ignore the constraint that $R \leq B$ and assume that the authority can credibly promise any level of reward money R . For ease of exposition, we normalize $h(G) = 1$ so that v_i denotes consumer i 's willingness to pay for the public good G . Let G_{-i} denote the sum of all contributions of consumers other than i . The payoff to consumer i under a mechanism R is given by

$$\pi_i = \begin{cases} \mathcal{I}(g_i + G_{-i} \geq G) [v_i - g_i] + \mathcal{I}(g_i + G_{-i} < G) \left[\frac{g_i R}{g_i + G_{-i}} \right] & \text{if } g_i + G_{-i} > 0 \\ 0 & \text{if } g_i + G_{-i} = 0, \end{cases} \quad (2)$$

where $\mathcal{I}(\cdot)$ is an index function.

In the next two sections, we analyze the performance of the suggested mechanism in two different environments, where the total value of the project is (i) certain and (ii) uncertain, respectively.

3 No Aggregate Uncertainty

Here, we study the situation when the total value of the public good project can be estimated without knowing the exact valuations of consumers. This assumes the knowledge of the first moment of the empirical distribution for private valuations. When the number of consumers is large then, by the law of large numbers, this first moment can be approximated by the first moment of the common prior $F(\cdot)$. But even without reversion to the law of large numbers, one can think of situations where the total value of the project is known. For instance, the total value of, e.g., a park, can be inferred from expected price changes of the nearby property.¹

We let V denote the total value of the public project G and assume

Assumption 1 *V is known with certainty.*

By Assumption 1, we have $V = \sum_j v_j$ known, while the rest of the incomplete-information

¹The assumption of the known sum of private characteristics is not uncommon in economic modeling. E.g., Bergstrom and Varian (1985) present a general result; Marks and Croson (1999) discusses it in relationship to the performance of the provision point mechanism.

framework is retained. We assume that consumers choose contributions (without randomizing) that constitute a Nash equilibrium of the game induced by a mechanism R .

Definition 1 *A profile of contributions \mathbf{g} is a Nash equilibrium if for each i g_i maximizes π_i given G_{-i} .*

We also make the Nash assumption that each consumer believes that the contributions of others are independent of his own.

Next, given Assumption 1, we characterize the set of pure-strategy Nash equilibria under a mechanism $R > 0$. Denote this set by $\Gamma(R)$.

Proposition 1 *If $0 < R \leq V - G$, then $\Gamma(R) = \left\{ \mathbf{g} : \forall i, g_i \leq \frac{G}{R+G}v_i, \sum_j g_j = G \right\}$. If $R > V - G$, then $\Gamma(R) = \{\emptyset\}$. If $R' > R$, then $\Gamma(R') \subseteq \Gamma(R)$.*

Proof. When $R > 0$, there is no equilibrium \mathbf{g} such that $\sum_j g_j < G$. For every $i \in N$, let \tilde{G}_{-i} denote consumer i 's beliefs about the sum of other consumers' contributions. Consumer i contributes $g_i = \max\{0, G - \tilde{G}_{-i}\}$ to have the public good provided if

$$\tilde{G}_{-i} \geq G - \frac{G}{R+G}v_i. \quad (3)$$

The condition in (3) follows from the individual rationality condition $v_i - g_i \geq \frac{g_i}{G}R$, where the right-hand side is the upper-bound utility of the consumer when he contributes marginally less than needed to have the public good provided. The largest individually rational contribution leading to the provision of the public good is given for every i by

$$g_i \leq \frac{G}{R+G}v_i. \quad (4)$$

Summing up (4) we see that the public good can be provided in equilibrium only if $G + R \leq V$. Next, we check the consistency of beliefs in (3). From (4) we see that it is rational to expect $\tilde{G}_{-i} \leq \frac{G}{R+G}\sum_{j \neq i} v_j = \frac{G}{R+G}(V - v_i)$. It immediately follows that beliefs in (3) are consistent also if $G + R \leq V$. Thus, if $R > V - G$ then $\Gamma(R) = \{\emptyset\}$. If $R \leq V - G$, then $\Gamma(R) = \left\{ \mathbf{g} : \forall i, g_i \leq \frac{G}{R+G}v_i, \sum_j g_j = G \right\}$. The last part of the proposition follows

from the observation that if (4) holds for R' it also holds for $R < R'$. But the reverse is not true. Hence, $\Gamma(R') \subseteq \Gamma(R)$. ■

With a promise to reward the contributors in the event the provision point is not reached, the mechanism actually induces a sufficient amount of contributions for the public good to be provided. The reason is that when $R \leq V - G$ there is always a consumer willing to increase his contribution to have either a larger share of the reward money or the public good provided. In equilibrium, each consumer needs to obtain a utility level from the public good at least as high as that obtained from the share of the reward money the consumer is entitled to if he deviates. A higher level of reward money implies a more profitable deviation and, thus, a higher level of utility for each consumer in equilibrium, which reduces the set of equilibria. However, if the promised amount is too generous, it makes consumers seek utility from the reward money rather than from the public good, which results in no pure strategy equilibrium. (With a discrete contribution space, $R > V - G$ would result in the equilibrium outcome $\sum_j g_j = G - \varepsilon$, where ε is the smallest currency unit.)

When reward money is set at the net value, $R = V - G$, the mechanism implements the public good in the unique equilibrium. This equilibrium has a special feature that all consumers contribute the same proportion of their private valuations, $g_i = \frac{G}{V} v_i$. Hence, the ratio G/V can be interpreted as a voluntary Lindahl tax, levied on consumers' private valuations for the public good. The reward money $R = V - G$ induces the consumers to reveal their private valuations truthfully if they believe that others do the same. Formally, the uniqueness result can be explained by the fact that at the point of provision the payoff π_i is continuous and concave in own contribution. For the analogous reason, we have a multiplicity of equilibria when $R < V - G$ because of the discontinuity of the payoff π_i at the point of provision.

Even though reward money is never distributed in equilibrium, the capacity of raising it needs to be credible. There are several possible sources of reward money, the simplest of which is seed money generated from individual donors. If the source of reward money is the budget of the authority, then it has to be that $R \leq B$, restricting the set of feasible

mechanisms. And if unique implementation is a desirable property, then this constraint on reward money can be binding for public good projects of large size. Under this circumstance, the public authority would have to either reduce the amount of the public good sought in order to preserve uniqueness or proceed without unique implementation.

If the authority raises its budget through taxes imposed on consumers, then there is also a question when the promised reward money needs to be raised: before the announcement of the fund-raising campaign or after it. It is important because the timing of taxation can have an effect on consumer payoffs in (2). To illustrate our argument, suppose that the authority can levy a lump-sum tax $\tau = R/N$ from every consumer. Because of quasi-linear preferences, ex-ante taxation has no effect on consumer preferences for the public good. Therefore, Proposition 1 continues to hold in its entirety.²

Ex-post taxation, however, implies a change in consumer payoffs in (2) as the second term becomes $\mathcal{I}(g_i + G_{-i} < G) \left[\frac{g_i R}{g_i + G_{-i}} - \tau \right]$. Analogously to (4), the individually rational contribution has to satisfy

$$g_i \leq \frac{G}{R + G}(v_i + \tau). \quad (5)$$

Thus, the upper bound on individually rational contributions increases with ex-post taxation. The reason is that, when the provision point is reached, the consumer also avoids paying the tax, making his gain from the public good being $v_i + \tau$ rather than v_i . But for the same reason, however, with ex-post taxation we can obtain equilibria with contributions larger than valuations. Consumers with $v_i < \frac{G}{N}$ may pledge contributions above their valuations to increase the likelihood of reaching the provision point done to avoid the tax. Finally, with ex-post taxation the existence of equilibria is independent of the condition $R \leq V - G$ as potential gains from reward money are diminished by taxes. The outcome of the unique equilibrium is, accordingly, not preserved. Despite these disadvantages, ex-post taxation has an advantage that it is reverted to only in the non-equilibrium event of the distribution of reward money, whereas with ex-ante taxation the cost of raising taxes needs to be incurred immediately.

²If the authority sets the provision point at $G - R$ rather than at G , supplying the remaining funds from tax revenues when the provision point is reached, then we have the public good provided whenever $V \geq G$ (i.e., independently of R) and, correspondingly, a multiplicity of equilibria.

It is also worthwhile to discuss the negative side of the mechanism, which is the “bad” non-equilibrium outcome, when the promised reward money needs to be distributed. It hardly has any impact on the social welfare (none, in fact, with quasi-linear preferences). On the individual level, the “bad” outcome is not, however, without an element of justice. Unlike in the typical scenario of the private provision of public goods, the mechanism with reward money leaves free-riders worse off than contributors, who then can be thought of receiving a “compensation” for the public good being not provided in proportion to their revealed preference for it.

4 Aggregate Uncertainty

In the previous section, the mechanism with reward money is analyzed under the assumption of no aggregate uncertainty. It is interesting to see what implications on contributing behavior the introduction of reward money can have in environments with aggregate uncertainty. Without the structure imposed by Assumption 1 the simple characterization of equilibria given in Proposition 1 may no longer hold. The consistency of beliefs about the sum of contributions collected by others can be infringed if the condition $V \geq R + G$ is not met. Certainly, if for every V in the support of the distribution for total valuations we still have that $V \geq R + G$, then Proposition 1 continues to hold. And so it will when $V \geq R + G$ holds in expectation as consumers are risk neutral.

In what follows, we study the effects of the introduction of reward money on the symmetric equilibrium strategies obtained under the standard provision point mechanism with refunds that lead to the provision of the public good with a positive probability. (Clearly, with aggregate uncertainty the zero-contribution equilibrium is also eliminated by reward money.) We apply the concept of symmetric Bayesian Nash equilibrium to predict the outcomes of the mechanisms studied. Consumer i ’s strategy, $g_i(v_i)$, is a one-to-one mapping from the set of valuations to the set of contributions. We impose symmetry so that all consumers use the same strategy, denoted by $g(v)$. Let $\Phi(\cdot)$ be the distribution of the sum of $n - 1$ random variables $g(v)$ with its probability density

function $\phi(.) > 0$ and the infimum of its support \underline{G} .

Definition 2 *A strategy $g(v)$ is a symmetric Bayesian Nash equilibrium of the game induced by a mechanism R if*

$$g(v) = \arg \max_g (1 - \Phi(G - g))(v - g) + \int_{\underline{G}}^{G-g} \phi(\gamma) \frac{g}{\gamma + g} R d\gamma. \quad (6)$$

The strategy $g(v)$ maximizes a consumer's expected utility when others play the same strategy. In (6), $1 - \Phi(G - g)$ stands for the probability that the provision point is reached when the consumer contributes g , and the second term stands for the expected share of the reward money when the provision point is not reached. In order to apply the first-order approach, we impose the following regularity condition.

Assumption 2 *The inverse hazard rate of $\Phi^*(.)$ is monotonically decreasing.*

Under the mechanism $R = 0$, the equilibrium strategy $g^0(v)$ is implicitly determined by the first-order condition

$$g^0(v) = v - \frac{1 - \Phi^0(G - g^0(v))}{\phi^0(G - g^0(v))}. \quad (7)$$

By Assumption 2, the second-order condition is met and $g^0(v)$ is uniquely determined. Next, we introduce reward money $R > 0$ into the mechanism to see what effect it has on the equilibrium strategy $g^0(v)$. Intuitively, it will give rise to two countervailing effects. First, an increase in contribution raises the probability of the provision point being reached and, accordingly, lowers the probability of obtaining a share of the reward money. Second, an increase in contribution raises the expected share of the reward money. Formally, the sum of the two effects is given by

$$-\phi^R(G - g^R(v)) \frac{g^R(v)}{G} R + \int_{\underline{G}}^{G-g^R(v)} \phi^R(\gamma) \frac{\gamma}{(\gamma + g^R(v))^2} R d\gamma \equiv A(v), \quad (8)$$

where $g^R(v)$ is the equilibrium strategy under the mechanism $R > 0$. The expression in (8) is obtained by taking the derivative of the second term of (6).

In comparative analysis that follows, we shall assume that R is not too big so that $\Phi^R(.) \approx \Phi^0(.)$, i.e., distributional changes are only of second order compared to the direct effects of reward money. Having said this, we can immediately establish

Proposition 2 $g^0(v) \leq g^R(v)$ if $A(v) \geq 0$.

A positive $A(v)$ means that the second effect of the introduction of reward money dominates the first one, as defined above, leading to a higher contribution toward the public good. Analogously, we obtain a lower contribution if $A(v)$ is negative. To see under what circumstances $A(v) > 0$ is more likely to arise, we simplify its expression by using that the term $\frac{\gamma}{(\gamma + g^R(v))^2}$ in the integrand of (8) is decreasing in γ (we assume that $\underline{G} > g^R(v)$). Then, after rearrangement, we get

$$A(v) > \frac{R}{G} \left(-g^R(v) + \frac{\Phi^R(G - g^R(v))}{\phi^R(G - g^R(v))} \left(1 - \frac{g^R(v)}{G} \right) \right). \quad (9)$$

From (9) we can see that $A(v)$ is more likely to be positive for (i) smaller values of $g^R(v)$ or, assuming monotonicity, smaller private valuations v and (ii) a smaller likelihood of an individual contribution being pivotal. In words, if individual actions bear little impact on the success probability of reaching the provision point, then we are more likely to observe increases in contributions from the introduction of reward money. This, in turn, is more probable with a larger degree of uncertainty about the sum of contributions of others. Furthermore, consumers with low contributions are more prone to revise upward their contributions than those with high contributions. Finally, related to the previous discussion about the effects of timing of taxation, ex-post taxation prompts consumers to contribute more as the tax enters positively the right-hand side of (9).

Welfare implications of the introduction of reward money into the provision point mechanism with refunds are not straightforward to assess in environments with uncertainty about the net value of the public good. As before, the main advantage is the elimination of the zero-contribution equilibrium.³ Conditional on the symmetric efficient

³In an experimental study, Marks and Croson (1999) show that with uncertainty the efficiency rate of the provision point mechanism remains similar to that obtained for the case without uncertainty, which means the rate of about 50 percent.

equilibrium strategies played, reward money can increase or decrease the probability of the implementation of the public good project depending on the degree of uncertainty. Thus, when reward money has a positive effect on contribution levels, ex-post efficient and inefficient projects are both implemented with a higher probability. The joint effect depends on the relative likelihood of efficient and inefficient projects.

5 Collective Action Problem

In the private provision of public goods, the purpose of reward money can also be viewed as a way to compensate consumers for externalities they create. The problem that externalities are not internalized arises in many different situations hindering the achievement of socially optimal outcomes. Next, we apply our mechanism to one such situation, specifically, a collective action problem, where participation in a project is individually rational only when a critical mass of participants is reached (see Myatt and Wallace (2009) for a recent discussion on collective action problems).

Imagine a government that plans to populate a new area with a capacity of at most M settlers. The individual cost of settling in this area is fixed at c , whereas the benefit, denoted by $v(m)$, depends on the total number of settlers, m . Let the benefit be increasing in m implying positive externalities from settlement. Assume that it is individually rational to settle in the area only if there are at least $\underline{m} - 1$ other settlers, where $\underline{m} = \min\{m : v(m) \geq c\}$ and let $1 < \underline{m} < M$. There is a large population of people who simultaneously decide whether to file an application for a settlement. Applications are contractually binding and in case more than M applications are filed, M of them are randomly selected. As $v(1) < c$, we can have two equilibrium outcomes (i) “bad” – nobody settles and (ii) “good” – there are M settlers.

A much applied way to eliminate the “bad” equilibrium in similar problems is via subsidies. In our example, the government can offer $\underline{m} - 1$ subsidies of size s , which are randomly distributed if there are more than $\underline{m} - 1$ applications. However, for the subsidy scheme to eliminate completely the “bad” equilibrium it has to be that $s = c - v(1)$ with

the total budget of $(\underline{m} - 1)(c - v(1))$ required. Now suppose that the government applies a mechanism with reward money. The government announces that if the number of settlers is smaller than \underline{m} then the settlers equally share a pre-specified endowment of R ; if $m \geq \underline{m}$ then no money from the government is distributed. Obviously, for R sufficiently large the only equilibrium outcome is when there are M settlers. The threshold \underline{R} such that with $R > \underline{R}$ there is only “good” equilibrium is determined by

$$\underline{R} = \max_{m \leq \underline{m}-1} m(c - v(m)). \quad (10)$$

To see this, if $R \geq c - v(1)$ then there will be at least one settler, if $R \geq \max\{c - v(1), 2(c - v(2))\}$ at least two, if $R \geq \max\{c - v(1), 2(c - v(2)), 3(c - v(3))\}$ at least three, and so on until we establish (10).

Unlike in the case of subsidies, no money is distributed in equilibrium under the mechanism with reward money. Furthermore, as $\underline{R} < (\underline{m} - 1)(c - v(1))$ the budget at stake is lower than that with subsidies (provided, though, sufficiently many settlers claim the subsidy). The mechanism with reward money compensates settlers for the externalities they create as long as the critical mass is not attained (after which externalities play no important role). Moreover, it is straightforward to see that the promise of reward money $R > \underline{R}$ has an implication that filing an application is a weakly dominant strategy.

An interesting question is whether similar effects can be achieved when the mechanism is applied to problems with negative externalities, e.g., the problem of the commons. The answer, however, is no. The mechanism with reward money is designed in such a way that the events of distribution of reward money and of achievement of the social optimum are exclusive. In the case with positive externalities, the social optimum is also individually optimal, i.e., is a Nash equilibrium, but it is not in the case with negative externalities. Therefore, it is impossible to achieve the social optimum without the distribution of reward money.

6 Conclusion

In this paper, we propose a modification of the provision point mechanism with refunds that leads to significant improvements in performance. The modification is reward money introduced to benefit contributors in case the provision point is not reached. In environments without aggregate uncertainty the proposed mechanism leads to the unique outcome, where the public good is provided (and no reward money needs to be distributed). Moreover, the mechanism with a specific level of reward money implements the efficient outcome uniquely with every consumer contributing the same proportion of his private valuation. Thus, the proposed mechanism not only achieves allocative efficiency but also distributional. We also apply the mechanism to a collective action problem and show that it can implement the efficient outcome in weakly dominant strategies. Lastly, this mechanism remains fairly simple and can be experimentally tested.

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