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Investment opportunities in the source country and temporary migration

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Abstract

This paper examines how attractive investment opportunities available to temporary migrants in their country of origin affect their saving behavior and the optimal duration of stay abroad. The model predicts an inverse U-shaped relationship between migration duration and the expected rate of return on repatriated savings. A higher rate provides an incentive to go back earlier and consume less abroad, while it can also trigger emigration aimed at generating the savings required for investment after return. At a more general level, the paper illustrates how the behavior of temporary migrants reflects the interaction between their preferences and the opportunities available in the labor and capital markets of both countries.

JEL Classification: F22

Key Words: International migration, Remittances, Return migration

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1 Introduction

It is widely recognized that imperfections in the capital markets of developing countries impede the establishment and growth of small businesses. While the required initial investment in a wide range of informal-sector activities can be as low as one or two hundred dollars, even such small amounts are out of reach for most of the population in the developing countries. One way of reacting to the imperfections in the capital markets is by migrating temporarily: Going to work for a higher wage abroad, accumulating savings, and then returning to the source country to invest in a small business, purchase livestock, agricultural land or other productive assets.

A number of recent studies show that temporary migration is indeed an effective means of surmounting credit market imperfections. Mesnard’s (2004) analysis of a data set on Tunisian return migrants reveals that they faced difficulties in getting credit to invest in self-employment activities after returning to Tunisia. An overwhelming majority of them relied mainly on their own savings accumulated abroad. Dustmann and Kirchkamp (2002) present similar findings on the basis of a sample of Turkish immigrants returning from Germany. They find that 51.1% of them started microenterprises within four years after returning to Turkey. Only 1.2% reported bank credit as a major source of finance. For the case of Mexico, Woodruff and Zenteno (2007) find that remittances are an important source of financing for small-business activities. Durand, Kandel, Parrado and Massey (1996), Escobar-Patapi and Martinez-Castellanos (1991), and Taylor (1987) discuss the role of migration and remittances as a response to the lack of access to credit for Mexican families of modest means. Other contributions, including Ilahi (1999), Lucas (1987), Massey and Parrado (1998), McCormick and Wahba (2001) emphasize, as well, the role of savings accumulated abroad in enabling returnees to initiate small business activities or undertake investments that enhance the productivity of the family’s farm.¹

¹Some of these empirical studies also show that higher accumulated savings while abroad are associated with a higher probability of entering self-employment relative to wage labor [e.g., Mesnard (2004), Ilahi (1999), and Dustmann and Kirchkamp (2002)], while Mesnard and Ravaillon (2006) find that wealth increases the probability of starting up a business, but with diminishing returns over most of their data set on Tunisian migrants. See Docquier and Rapoport (2006) for a survey of this literature.
Recognizing this well-documented link between savings accumulated in the host country and self-employment activities of returning migrants in the source country, this paper provides a theoretical analysis of how investment opportunities at home affect the behavior of temporary immigrants while they live and work abroad. The model employed is similar to that developed by Djajić and Milbourne (1988) for the purpose of simultaneously determining the duration of foreign stay and saving behavior of temporary immigrant workers. The present study is an important extension of the Djajić-Milbourne analysis in that it takes into account not only the international wage differential as the driving force behind migration decisions, but also the international differential in the expected rates of return on capital. Migration and asset accumulation abroad can transform a migrant from being endowed only with labor (and hence attracted by the opportunities to sell labor services abroad) to becoming relatively well endowed with capital (and hence attracted by the opportunities to sell the services of capital back in the source country). Taking into consideration differences in the rates of return on capital at home and abroad is therefore essential to understanding a migrant’s decisions concerning saving behavior, duration of stay abroad and return migration.

Mesnard (2004) takes an important step in this direction. In the context of a very simple and elegant model, she looks at the duration of stay and consumption behavior of migrants, as well as their occupational choice after return, under the assumptions that there is no discounting (rates of interest and time preference are zero) and that investment projects available to returnees are lumpy (a certain minimum initial investment is required, à la Banerjee and Newman (1993), in order to initiate a small business in the source country). Migrant behavior in that setting is fundamentally driven by the objective to accumulate the minimum required savings. The present study assumes, instead, that migrants are not constrained to accumulate a specific stock of assets in order to start a business after return. They choose the scale of their investment as a function of their preferences, including time preference, and the opportunities available in the labor and capital markets of both countries.\footnote{The behavior of utility-maximizing temporary migrants with exogenously determined duration of stay abroad has been examined by Djajić (1989), with the analysis extended to the case of uncertainty concerning the prospect of return by Galor and Stark (1991).}

\footnote{As noted by Woodruff and Zenteno (2007), a wide range of investment opportunities...}
Another related study is that of Dustmann and Kirchkamp (2002), which uses numerical simulation to illustrate the behavior of migrants, including migration duration and the planned post-return activity. They simplify the analysis by assuming that if the migrant chooses self-employment upon return, the repatriated savings, $A$, are invested in an activity that generates a constant flow of income, $rA$, for the rest of the agent’s lifetime. There is no possibility of selling off parts of the investment over time so as to augment consumption. At the same time, it is assumed that the investment depreciates instantaneously at the end of the planning horizon.\footnote{4} I assume, instead, that the returnee invests in an enterprise that can be gradually sold off to support the optimal consumption program and I focus much more explicitly on the duration of stay and consumption behavior of migrants rather than on their occupational choice.

The remainder of the paper is organized as follows: The problem confronting a potential migrant is presented and analyzed in Section 2. He or she faces a relatively high wage abroad, where labor is scarce, and a relatively high return on capital at home, where labor is abundant. In the presence of constraints on credit, taking advantage of opportunities in both the labor and capital markets of both economies, requires of the migrant to work abroad and save, then return to the source country and invest the accumulated savings at a high rate of return. From the theoretical perspective, the key questions facing the migrant in that setting are the following: 1) How long should he or she work abroad, if at all, before returning to the source country with the accumulated savings, 2) what fraction of the foreign wage should be saved, and 3) at what rate should the accumulated assets be consumed after return.

Effects on the migrant’s behavior of changes in the source-country rate of return on investment and other exogenous variables, such as wages of both countries, are examined in Section 3. The analysis reveals an inverse U-shaped relationship between the optimal duration of stay abroad and the expected rate of return on investment in the source country. Moreover, are open to returning migrants, some requiring only a small amount of initial capital, while others call for larger accumulated savings.\footnote{4} These assumptions are perfectly suitable for an analysis of investment in human capital, but less so when it comes to physical capital.
an increase in the expected rate of return can trigger emigration aimed at generating the savings required for investment after return, while it also provides an incentive to go back earlier and save at a higher rate abroad. These effects are illustrated in Section 4 with the aid of numerical simulations using a specific utility function. The role of preferences and how they interact with market opportunities in influencing temporary migration is discussed in Section 5. Finally, Section 6 concludes the paper with a summary of the main findings.

2 The Model

Let us assume that a credit-constrained migrant leaves the source country at the age of 0 to work abroad until the age \( \tau \) at the wage \( w^* \). While abroad, s/he earns the foreign rate of return \( r^* \) on accumulated savings. After return to the source country, s/he works for the wage \( w < w^* \) and invests the repatriated savings in an activity that yields a rate of return \( r > r^* \). In

While there is very little empirical evidence on the rates of return on small business investments and self-employment activities of returning migrants, it is safe to assume that they can be considerably higher than the return on savings in the host country. In a study on remitting behavior of migrant labor in the South African gold mining industry, Penny (1986) finds that, in the absence of more attractive alternatives, most of the investment of migrant households is in the form of livestock. For livestock holders in the migrant-sending areas of Lesotho, Swallow and Brokken (1987) calculate the private rate of return on capital invested in cattle to be 9.4 percent, 8.7 percent for sheep, and 5.7 percent for goats. In the case of Egypt, Adams (1991) finds that in the 1980's, migrants’ remittances were mostly invested in land. He calculates the real rate of return on agricultural land in the study area to have averaged 9.5 percent per year over the period 1980-86.

A number of recent studies provide much higher estimates of rates of return on investment in small business activity in developing countries. De Mel, McKenzie and Woodruff (2007b) find the average return on capital of small and informal firms in Sri Lanka to be 68% per year. Udry and Anagal (2006) estimate returns on capital in traditional small-scale agricultural production in Ghana to be 50% per year. Using Mexican data, McKenzie and Woodruff (2006) observe rates of return on capital in small urban enterprises of around 15% per month. Measuring small enterprise profits in developing countries is a difficult task, however, as the vast majority of such businesses do not keep records and, if surveyed, are likely to understate revenues and overstate expenses [see de Mel, McKenzie and Woodruff (2007a)].
order to realize this return, we assume that the migrant must be physically present in the source country to monitor the investment and assure efficiency in the operations of the enterprise. This monitoring activity still allows the migrant to earn, in addition to the rate of return \( r \) on his or her investment, the market wage of the source country by working in one’s own enterprise or another establishment. We assume that foreign investors are not able to take advantage of small-business investment opportunities in the source country as they lack local information and networks which play a crucial role in enabling an investor to realize the rate of return \( r \).

The wage, investment income, and gradual liquidation of asset holdings after return serve to finance consumption over the remainder of the planning horizon from age \( \tau \) to \( T \). The problem facing the migrant is to maximize \( V \), the discounted utility from consumption abroad and at home, by choosing the consumption rate at each point in time and the optimal length of stay \( \tau \) in the foreign country.

\[
V = \int_0^\tau u(c_t^*) e^{-\delta t} dt + \int_\tau^T u(c_t) e^{-\delta t} dt,
\]

where \( c_t^* \) is the rate of consumption at time \( t \) while abroad, \( c_t \) is the rate of consumption after returning to the country of origin, \( \delta \) is the migrant’s constant rate of time preference, and the utility functions are concave and twice differentiable. Following Hill (1987) and Djajić and Milbourne (1988), I shall assume that the migrant prefers to consume in the source country rather than in the host country in the sense that \( u^*(x) < u^0(x) \), \( \forall x > 0 \) (i.e., the marginal utility of consumption at home is always higher than that associated with the same rate of consumption abroad).\(^6\)

Utility is maximized subject to the constraint that \( A \), the value of savings accumulated abroad is equal to the excess of discounted consumption over wage income after return. For simplicity, we shall assume that \( w^*, w, r^* \) and

\(^6\)As pointed out by Hill (1987) and Djajić and Milbourne (1988), there is evidence that migrants prefer to live and consume in their countries of origin because of the presence of family members as well as climate, culture, and life-style to which they are accustomed. Let \( R \) represent these other factors which, in addition to consumption, enter a migrant’s utility function. If \( c \) and \( R \) are Edgeworth complements, so that marginal utility of consumption is increasing in \( R \), and if \( R \) is higher in the source country than it is abroad, one would observe that migrants have a higher preference for consumption at home in the sense described in the text.
$r$ are all constant and the cost of migration is zero.\footnote{Djajić and Milbourne (1988) examine explicitly the role of migration costs in influencing consumption behavior and duration of stay. They find that an increase in such costs lengthens a migrant’s stay abroad. The present model generates the same outcome if migration costs are taken into account. An interesting extension, proposed by a referee, would be to assume that a credit-constrained migrant needs to accumulate savings before migration [rather than borrow at the interest rate $r$, as in Djajić and Milbourne (1988)], in order to finance migration. In such a model there would be an additional, initial phase of asset accumulation at home, with the opportunities available to the migrant in the subsequent two phases (i.e., employment abroad and entrepreneurial activity after return) affecting both the duration of and the saving rate in the first phase. 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While such an analysis would constitute an interesting extension of the present model, a satisfactory treatment would require considerably more space and would distract the reader from the main point of the paper.} Using the date of return $\tau$ as the point of reference, and noting that the rate of return on assets in the host country ($r^*$) differs from that of the source country ($r$), we may write the budget constraint as

$$A = \int_0^\tau (w^* - p_t^* c_t^*) e^{r^* t} dt = \int_\tau^T (w - p_t c_t) e^{-r(t-\tau)} dt,$$

where $p_t$ and $p_t^*$ are the prices at time $t$ of the single consumption good at home and abroad, measured in terms of the numeraire, say a unit of gold. The time 0 value of assets accumulated abroad is $A_0 = Ae^{r \tau}$. Defining the Lagrangian associated with the migrant’s maximization problem as

$$L = \int_0^\tau u^*(c_t^*) e^{-\delta t} dt + \int_\tau^T u(c_t) e^{-\delta t} dt + \lambda\left[\int_0^\tau (w^* - p_t^* c_t^*) e^{-r^* t} dt + e^{-r^* \tau} \int_\tau^T (w - p_t c_t) e^{-r(t-\tau)} dt\right].$$

The first order conditions are

$$\frac{\partial L}{\partial c_t} = u^'(c_t^*) e^{-\delta t} - \lambda p_t e^{-r(t+\tau-r^*)} = 0$$

$$\frac{\partial L}{\partial c_t^*} = u^*(c_t^*) e^{-\delta t} - \lambda p_t^* e^{-r^* t} = 0$$

$$\frac{\partial L}{\partial \tau} = u^*(c_t^*) e^{-\delta \tau} - u(c_t) e^{-\delta \tau} + \lambda [(w^* - p_t^* c_t^*) e^{-r^* \tau} - (w - p_t c_t) e^{-r(t-\tau)}] + \lambda (r - r^*) e^{-r^* \tau} \int_\tau^T (w - p_t c_t) e^{-r(t-\tau)} dt = 0,$$

and the budget constraint (2). These four equations enable us to solve for the values of the key endogenous variables: $c_t, c_t^*, \tau$, and the Lagrangian multiplier, $\lambda$.\footnote{Djajić and Milbourne (1988) examine explicitly the role of migration costs in influencing consumption behavior and duration of stay. They find that an increase in such costs lengthens a migrant’s stay abroad. The present model generates the same outcome if migration costs are taken into account. An interesting extension, proposed by a referee, would be to assume that a credit-constrained migrant needs to accumulate savings before migration [rather than borrow at the interest rate $r$, as in Djajić and Milbourne (1988)], in order to finance migration. In such a model there would be an additional, initial phase of asset accumulation at home, with the opportunities available to the migrant in the subsequent two phases (i.e., employment abroad and entrepreneurial activity after return) affecting both the duration of and the saving rate in the first phase. While such an analysis would constitute an interesting extension of the present model, a satisfactory treatment would require considerably more space and would distract the reader from the main point of the paper.}
3 Analysis of the Migrant’s Behavior

To simplify the analysis and the notation, let us assume that the migrant’s rate of time preference \( \delta = r^* \) and that \( p_t = p_t^* = 1 \). We can then write eqs. (3)-(5) as

\[
\begin{align*}
(6) & \quad u'(c_t) = \lambda e^{-(r-r^*)(t-\tau)}, \\
(7) & \quad u''(c_t^*) = \lambda, \\
(8) & \quad u(c_\tau) - u(c_\tau^*) + \lambda(r-r^*)A = \lambda[(w^* - c_\tau^*) - (w - c_\tau)].
\end{align*}
\]

With \( \delta = r^* \), we observe in eq. (7) that the marginal utility of consumption abroad is constant. The rate of consumption abroad, \( c_t^* \), is therefore also a constant, \( c^* \). In addition, eqs. (6) and (7) imply that upon return to the source country at time \( \tau \), consumption jumps to a higher rate (reflecting the migrant’s preference for consumption at home), while \( u'(c_{\tau+}) = u''(c_{\tau-}) \), so that the marginal utility of consumption is a continuous function of time when the migrant switches from the foreign to the home consumption stream.\(^9\) With the aid of (6) and (7), we may therefore write \( c_\tau = \phi(c^*) > c^* \), where \( \phi'(c^*) = u''(c_t^*)/u''(c_\tau) > 0 \). Moreover, eq. (6) shows that the marginal utility of post-return consumption is declining over time at the proportional rate \( r - r^* \). This implies that from time \( \tau \) to \( T \), consumption is increasing at the proportional rate equal to the product of \( r - r^* \) and the elasticity of intertemporal consumption substitution (ICS). We may therefore write

\[
(9) \quad c_t = \phi(c^*)e^{(r-r^*)(t-\tau)},
\]

where \( \theta = -u''(c_t)c_t/u'(c_t) > 0 \) is the elasticity of marginal utility with respect to consumption, assumed constant, and \( 1/\theta \) is the elasticity of ICS.

Eq. (8) sets the cost, in terms of forgone utility, of staying an extra unit of time abroad equal to the benefit. This makes the migrant indifferent between staying and returning to the source country, as must be the case when the

\(^8\)For the influence of international price differentials on the consumption-saving behavior of temporary migrants, see Djajić (1989).

\(^9\) \( c_{\tau-}^* \) and \( c_{\tau+} \) are the consumption rates just before and just after time \( \tau \).
date of return, \( \tau \), is chosen optimally. The cost of staying another instant (or the gain from returning a moment sooner) is the difference between \( u(c_{\tau}) \) and \( u^*(c^*) \), plus the utility value of \( (r - r^*) A \), which is the cost of waiting to invest the assets accumulated abroad at the relatively higher source-country rate of return. The benefit of staying longer, on the right-hand side of (8), is simply the utility value of the increase in wealth, due to accumulation rather than decumulation of assets associated with staying an extra unit of time abroad.\(^{10}\)

Eq. (8) may be rewritten with the aid of (7) and (9) to define

\[
G = \frac{u(\phi(c^*)) - u^*(c^*)}{u^*(c^*)} + (r - r^*) A - [w^* - c^* - w + \phi(c^*)] = 0,
\]

as the net gain (measured in terms of the numeraire, rather than utility) from returning to the source country a moment sooner. At the optimally chosen \( \tau, G = 0 \). Moreover, with the aid of (9), the budget constraint (2) may be written in an integrated form as the difference, \( B \), between assets accumulated abroad and assets decumulated after return as a result of over-spending.

\[
B = \left( \frac{w^* - c^*}{r^* - r} \right) (e^{r^* \tau} - 1) - \frac{\phi(c^*)}{g} (e^{g(T - \tau)} - 1) + \frac{w}{r} (1 - e^{-r(T - \tau)}) = 0,
\]

where \( g = \frac{r - r^*}{r} - r \) is the proportional rate of growth of the discounted (time-\( \tau \)) value of the post-return consumption rate, \( c_t \).

The system of eqs. (10) and (11) defines the internal temporary migration equilibrium. It yields the solutions for the two key variables of the model: the rate of consumption abroad, \( c^* \), and the duration of stay in the foreign country, \( \tau \), as functions of the exogenous variables, including \( w^*, w, \) and \( r \). Using \( G_x \) and \( B_x \) to represent \( \frac{\partial G}{\partial x} \) and \( \frac{\partial B}{\partial x} \), where \( x = c^*, \tau, w, w^* \) and \( r \) (these partial derivatives are presented and examined in the Appendix), we can write eqs. (10) and (11) in differentiated form as

\(^{10}\)An interesting implication of (8) is that a positive rate-of-return differential in favor of the source country increases the cost of postponing return migration. In that sense it contributes to a reduction in the duration of a migrant’s stay in the host country in relation to the benchmark case, examined by Djajić and Milbourne (1988), where \( r = r^* \). As we shall see in the next section, however, there is also an effect on \( \lambda \) that works in the opposite direction.
The determinant of the matrix of coefficients

\[ \begin{bmatrix} G_{c^*} & G_{r^*} \\ B_{c^*} & B_{r^*} \end{bmatrix} \cdot \begin{bmatrix} dc^* \\ d\tau \end{bmatrix} = \begin{bmatrix} -G_w dw - G_{w^*} dw^* - G_r dr \\ -B_w dw - B_{w^*} dw^* - B_r dr \end{bmatrix}. \]

The determinant of the matrix of coefficients \( \Delta = G_{c^*} B_{r^*} - B_{c^*} G_{r^*} \) is unambiguously positive if \( G_{c^*} > 0 \). As we shall see below, \( G_{c^*} \) is indeed positive in the neighborhood of an equilibrium so that \( \Delta > 0 \). The system of eqs. (12) yields the following comparative statics results:

\[
\begin{align*}
(13) & \quad \frac{d\tau}{dw} = \frac{B_{c^*} G_{w^*} B_{c^*} G_{r^*}}{\Delta} < 0, \\
(14) & \quad \frac{d\tau}{dw^*} = \frac{B_{c^*} G_{w^*} - B_{w^*} G_{r^*}}{\Delta} \geq 0, \\
(15) & \quad \frac{d\tau}{dr} = \frac{B_{r^*} G_{c^*} - B_{c^*} G_{r^*}}{\Delta} \leq 0, \\
(16) & \quad \frac{dc^*}{dw} = \frac{G_{r^*} B_{w^*} - G_{w^*} B_{r^*}}{\Delta} < 0, \\
(17) & \quad \frac{dc^*}{dw^*} = \frac{G_{r^*} B_{w^*} - G_{w^*} B_{r^*}}{\Delta} > 0, \\
(18) & \quad \frac{dc^*}{dr} = \frac{G_{r^*} B_{c^*} - G_{c^*} B_{r^*}}{\Delta} < 0,
\end{align*}
\]

with the elements in the numerators of these expressions presented and evaluated in the Appendix. Our findings can be summarized in the following propositions:

**Proposition 1:** a) An increase in \( w \) results in a downward shift of the migrant’s time profile of consumption and a decrease in the optimal length of stay abroad. b) An increase in \( w^* \) results in an upward shift of the migrant’s time profile of consumption, but it has an ambiguous effect on the optimal length of stay abroad. c) An increase in \( r \) results in a downward shift of the migrant’s time profile of consumption and it has an ambiguous effect on the optimal length of stay abroad.

As explained in the Appendix, the ambiguous sign of \( d\tau/dw^* \) in (14) stems from the interaction between the income and substitution effects arising from an increase in \( w^* \). Discussion of the other results, including the

\[11\] Moreover, on the dynamic assumptions that the migrant increases (decreases) \( c^* \) when \( B > 0 (B < 0) \) and lowers (raises) \( \tau \) if \( G > 0 (G < 0) \), stability of the equilibrium requires that \( \Delta > 0 \).
ambiguous effect of $r$ on $\tau$, is provided in the next section along with numerical simulations that help to illustrate the findings.

## 4 A Specific Utility Function

Let us suppose that the utility function is of the CRRA form

$$u^*(c_t^*) = \frac{c_t^{1-\theta}}{1-\theta},$$  
$$u(c_t) = \frac{\alpha c_t^{1-\theta}}{1-\theta},$$

where $\alpha > 1$ reflects the migrant’s preference for consumption at home.$^{12}$ We shall restrict our analysis to the case of $\theta < 1$. This guarantees that utility is positive and that the first-order conditions (6)-(8) are consistent with the assumption that the migrant prefers to consume in the source country.$^{13}$ Eq. (9) now becomes

$$c_t = c^* \alpha^\frac{1}{\theta} e^{(\frac{r-r^*}{\theta})(t-\tau)},$$

and the key eqs. (10) and (11) can be written as

$$G = c^* \frac{\theta}{1-\theta} (\alpha^\frac{1}{\theta} - 1) + (w - w^*) + \frac{c_t^* - c^*}{r^*}(w^* - c^*)(e^{r^*\tau} - 1) = 0$$

$^{12}$To my knowledge, there are no empirical studies that provide estimates of $\alpha$. A way of determining what might be realistic magnitudes of $\alpha$ is by asking what would $\alpha$ have to be if we are to observe permanent migration when $w^*$ is only 3 to 5 times higher than $w$, as is typically needed to trigger large-scale migration. When $w^*/w = 3$, it is simple to calculate on the basis of (19) that for $\theta$ in the range from .95 to .80, for example, the value of $\alpha$ that makes one indifferent between migrating permanently and staying at home is in the range between $\alpha = 1.05$ and $\alpha = 1.26$, while for $w^*/w = 5$, the corresponding range is from 1.08 to 1.38. These magnitudes can be interpreted to represent an upper bound on what might be a realistic value of $\alpha$. In the numerical simulations below, I use values of $\alpha$ in the range between 1.1 and 1.5. Dustmann and Kirchkamp (2002) use a value of $\alpha = 3$ in their simulation exercise, which is very much higher than what I consider to be plausible.

$^{13}$For the functional form given in (19), if $\theta > 1$, $u(c_t) - u^*(c_t^*) < 0$ at the rates of consumption abroad and at home that satisfy the first-order condition $u'(c_{t-}) = u^*(c_{t+})$. This implies a drop (rather than an increase) in utility of consumption at the moment of return to the source country. We therefore examine only the case of $\theta < 1$. The assumption that $\theta < 1$ is also consistent with the empirical estimates of the elasticity of intertemporal consumption substitution $(1/\theta)$ reported by Hansen and Singleton (1982), in the range between 1 and 1.5, and of Favero (2005), who reports an estimate of around 1.
Note that the partial derivatives of $G$ and $B$ with respect to $c^*$ and $\tau$ are now simply

\begin{align}
(20) \quad G_{c^*} &= \frac{\theta}{1-\theta}(e^{\frac{r^*}{\tau}} - 1) - \frac{\alpha \frac{1}{2}}{\theta} \frac{1}{\theta} (e^{\theta(T-\tau)} - 1) < 0,
(21) \quad G_{\tau} &= (r - r^*)(w^* - c^*) e^{r^* \tau} > 0,
(22) \quad B_{c^*} &= -\left[ \frac{1}{\tau^2} (e^{r^* \tau} - 1) + \alpha \frac{1}{2} \frac{1}{\theta} (e^{\theta(T-\tau)} - 1) \right] < 0,
(23) \quad B_{\tau} &= (w^* - c^*) e^{r^* \tau} + (c_{T\tau} - w_{T\tau}) > 0,
\end{align}

where $w_{T\tau} = we^{-r(T-\tau)}$ and $c_{T\tau} = \phi(c^*) e^{\theta(T-\tau)}$.

A diagrammatic solution to the migrant’s problem is provided in Fig. 1. The BB schedule represents the budget constraint \((11')\), with the slope \(dc^*/d\tau |_{BB} = -B_{\tau}/B_{c^*} > 0\). Anywhere above the BB schedule the migrant’s implied consumption path violates the budget constraint, while anywhere below BB, the migrant’s lifetime earnings are not completely spent. Equation \((10')\) is depicted by the $G = 0$ schedule with the slope $dc^*/d\tau |_{G=0} = -G_{\tau}/G_{c^*}$. For $\tau = 0$, $G_{c^*} > 0$. However, as $\tau$ increases along the $G = 0$ locus, $G_{c^*}$ diminishes in magnitude, while $G_{\tau}$ increases, resulting in a concave $G = 0$ schedule, which intersects BB before $G_{c^*}$ becomes negative. Accordingly, $G_{c^*}$ is positive in the neighborhood of the equilibrium. Anywhere to the right of the $G = 0$ locus, utility of the migrant is increased by returning earlier to the source country. Anywhere to the left, it pays to stay longer abroad. Accordingly, maximization of utility, consistent with the budget constraint, takes place at the point of intersection between the BB and $G = 0$ schedules.

Our simulations determine the equilibrium value of $\tau$ for various magnitudes of the exogenous variables, $w^*$ and $r$ , but also parameters of the utility function, $\theta$ and $\alpha$. For the purpose of this exercise, we shall initially assume that $w = 1$, $r^* = .03$, $\alpha = 1.2$, and consider what, on the basis of our earlier discussion, are empirically plausible values of $r(=.03, .04, .05, .07, .09, .12, .16, .21, \text{ and } .25)$ and $\theta(=.8, .85, .9, \text{ and } .95)$. On the basis of evidence on wage differentials between advanced and developing countries reported by Freeman and Oostendorp (2000) and Ashenfelter and Jurajda (2001), it is
realistic to consider the following values of \( w^* \) (=2, 3, 4, 5, 7, 10, 15, and 25). The planning horizon, \( T \), is assumed to be 50 years. The computations of the equilibrium values of \( \tau \) are presented in Table 1.

What the simulations illustrate very clearly, is that for the chosen range of parameters, an increase in \( w^* \) causes migrants to stay longer abroad. This is in contrast to the ambiguity suggested by expression (14). Here we get unambiguous results, because for the chosen values of \( \theta < 1 \), the absolute value of \( B_{w^*}/B_c \) is smaller than that of \( G_{w^*}/G_c \), guaranteeing that the upward shift of the \( G = 0 \) schedule in Fig. 1, due to an increase in \( w^* \), is larger than that of the \( BB \) schedule. This results in higher equilibrium values of both \( \tau \) and \( c^* \). The amount of repatriated savings can also be shown to increase with \( w^* \). In sum, in response to an increase in the foreign wage, migrants stay longer abroad, repatriate a larger stock of assets, and consume more at each point in time. Moreover, as shown in Table 1 (where an entry NM refers to "no migration"), in some cases a higher foreign wage may induce individuals to migrate when otherwise they would have chosen to stay in the source country. For example, with \( \theta = .95 \) at the bottom of the table, for \( r = .04 \) there is no migration if the foreign wage is at 2 or 3. However, if \( w^* = 4 \), it pays to migrate for .56 years, and increasingly more for higher values of \( r \).

For the effect of a change in \( w \), we recall that our comparative statics results indicate that \( dc^*/dw \) and \( d\tau/dw \) are unambiguously negative. This is confirmed by the simulation results (not shown) and consistent with the findings reported in Djajić and Milbourne (1988) for the case of \( r = r^* \). In terms of Fig. 1, an increase in \( w \) shifts both schedules to the left. The magnitude of the shift of the \( G = 0 \) schedule \( (-G_w/G_\tau) \) can be analytically shown in the general case to be larger than that of the \( BB \) schedule \( (-B_w/B_\tau) \), implying that an increase in \( w \) encourages migrants to return earlier and save a larger fraction of their foreign income.

We consider next the implications of a change in \( r \). Table 1 illustrates the interesting result that for any given \( \theta \), an increase in \( r \) can have a positive effect on \( \tau \) for relatively low values of \( w^* \) and \( r \), while it has an unambiguous negative effect on \( \tau \) for higher values of these same variables. For example,
with $\theta = .85$ and $w^* = 2$, we observe NM in the Table for $r = .03$. As $r$ increases across the table to .07, while other variables are held constant, $\tau$ rises to 3.93 years and then declines to 1.99 for $r = .25$. This can also be seen with the aid of Fig. 2, where an increase in $r$ shifts both schedules down. The magnitude of the shift of the $BB$ schedule is $-B_r/B_{c^*}$ and that of the $G = 0$ schedule is $-G_r/G_{c^*}$, where $G_r = A$, the stock of assets accumulated by the migrant while abroad. When $w^*$ is relatively low, so is $A$, resulting in a relatively small downward shift of the $G = 0$ schedule to $(G = 0)_1$. This makes it more likely that the intersection with $B'B'$ is at a higher value of $\tau$, such as $\tau_1$. Similarly, when the initial value of $r$ is close to $r^*$, $A$ is relatively small and $G_{c^*}$ relatively large, contributing once again to the possibility that an increase in $r$ results in a higher equilibrium value of $\tau$. Migrants then stay longer and save a larger fraction of their income abroad. For relatively greater values of $w^*$ and $r$, $A$ is larger at the point of return. The magnitude of the downward shift of the $G = 0$ schedule can then exceed that of the $BB$ locus, as illustrated by the intersection between $(G = 0)_2$ and $B'B'$ in Fig. 2. The optimal duration of stay abroad then declines from $\tau_0$ to $\tau_2$. Intuitively, for relatively higher values of $w^* - w$ and/or $r - r^*$, the migrant accumulates assets abroad at a higher rate and has a stock of assets at the point of return that is relatively larger. The larger the stock, the more powerful is the incentive to return sooner to the source country (relative to the incentive to stay abroad longer and save) in response to an increase in $r$. We therefore observe an inverted U-shaped relation between $\tau$ and $r$, with $\tau$ initially rising and then declining as $r$ increases relative to $r^* (= \delta)$.

As both schedules in Fig. 2 shift down in response to an increase in $r, dc^*/dr$ is unambiguously negative. The decline in $c^*$ in response to an increase in $r$ reflects the tendency for migrants to shift the time profile of their spending in favor of future (source-country) consumption and at the expense of consumption abroad. The effect on $c^*$ is obviously more pronounced the higher the elasticity of ICS.

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15The magnitude of the shift of BB is also smaller for relatively low values of $r$, but as our simulations indicate, not as much as that of the $G = 0$ schedule.
5 Role of Preferences and the Duration of Stay Abroad

The larger the elasticity of marginal utility of consumption ($\theta$), the lower is the elasticity of ICS ($1/\theta$), and the harder it is for individuals to substitute future for present consumption. That is, to give up utility early in life by going abroad to save for the purpose of investing the accumulated savings in the source country and enjoying more consumption in the future. We therefore observe in Table 1, for any given values of $w^*$ and $r$, a consistently negative relationship between $\theta$ and $\tau$. It is interesting to note that, not only does a lower $\theta$ entail a longer stay, but it can also induce temporary migration to occur under market conditions that result in no migration (NM) for somewhat higher values of $\theta$. For example, with $\theta = .95$ towards the bottom of the table, there is no migration for a range of relatively low values of $w^*$ and $r$. With $\theta = .9$ or .85, we find NM only for $w^* = 2$, while for $\theta = .8$, temporary migration of at least some duration occurs for all values of $w^*$ and $r$ displayed in the table.

Another key parameter of the utility function is $\alpha > 1$, reflecting the migrant’s preference for consumption in the source country. Preference for consumption at home deters workers from emigrating and draws them back home when migration does take place. The effect is very similar to that resulting from a commodity-price differential that makes consumption of any given bundle of goods cheaper in the source country. In the context of the present model, however, there is another motive for return: To benefit from a higher yield on repatriated assets. As we have seen above, the higher the value of $r$ relative to $r^*$, the greater the cost of remaining abroad with accumulated savings and not taking advantage of the high-yielding investment opportunities at home. What is interesting about this motive for return, in contrast to that associated with $\alpha > 1$, is that the positive gap between $r$ and $r^*(= \delta)$ not only serves to draw migrants back to the source country once they have a stock of savings abroad, but it also encourages them to migrate in the first place so as to initiate asset accumulation. It turns out to be a powerful motive for emigration, identified in some of the literature cited in the Introduction, as well as a motive for return.

The intuition behind this is straightforward, although somewhat subtle. As noted at the end of Section 4, a high rate of return on investment in
the source country (relative to \( \delta = r^* \)) encourages individuals to postpone consumption into the future. The advantage of consuming at home, where the utility of consuming any given bundle of goods is relatively higher, is then less significant for workers at the beginning of their planning horizon. At that point they are intensive savers (and more intensive, the lower the value of \( \theta \)), making the loss of utility associated with working and consuming abroad relatively small in comparison with the benefit of accumulating assets by working and saving in a high-wage economy. An increase in the value of \( r \) relative to \( r^* \) therefore lowers the attractiveness of staying permanently at home and encourages workers to migrate temporarily abroad. At the same time it encourages those working abroad to return sooner with accumulated savings so as to take advantage of the attractive investment opportunities in the source country.

In sum, migration decisions in the context of the present model, reflect the interaction between the preference for consuming at home (\( \alpha > 1 \)), and the opportunities available in both the labor and capital markets of the two countries. When \( r - r^* \) is large, it encourages workers to emigrate, save intensively while abroad, and return to the source country to take advantage of the high rate of return on repatriated savings. A high value of \( \alpha \), on the other hand, discourages out-migration and encourages return of those who do migrate, while a large \( w^* \) relative to \( w \), encourages emigration and a longer (sometimes permanent) stay abroad.

The interactions among these key variables for given values of \( w \) and \( r^* \) are illustrated in Table 2, where the equilibrium values of \( r \) are now displayed for various values of \( \alpha \), between 1.1 and 1.5, and a somewhat narrower range of values of \( \theta, w^* \) and \( r \) in relation to those displayed in Table 1. What we observe is that the greater the value of \( \alpha \) in the second column of Table 2, the more likely it is that for any given values of \( \theta, w^* \) and \( r \), migration does not take place. This is indicated by NM (no migration). Moreover, when migration does occur for a given set of values of \( \theta, w^* \) and \( r \), the entries in the table indicate that the length of stay abroad diminishes with an increase in \( \alpha \). We also observe that when \( \alpha \) is relatively low (e.g., \( \alpha=1.1 \)), and \( \theta \) relatively small (\( \theta = .80, .85 \)) migrants do not return to the source country (indicated by NR in Table 2) if \( w^* \) is relatively large. For any \( \alpha > 1 \), however, the pull back to the source country is stronger, the larger the value of \( \theta \). As \( \theta \) rises from .80 towards .95, we thus observe in Table 2 (as well as in Table 1) a
tendency for migrants to stay a shorter time abroad (or not to migrate at all when \(w^*\) is relatively low). In addition, it can be easily verified that the proportion of foreign income saved is larger with a higher value of \(\alpha\) as the expenditure pattern shifts in favor of consumption in the source country.

What Table 2 illustrates, in addition, is the point made earlier that an increase in \(r\) can induce temporary migration when it otherwise would not occur. For example, in any row where NM appears for a low value (or values) of \(r\), there is an equilibrium value of \(\tau > 0\) for higher values of \(r\). Higher values of \(r\) are also seen to encourage migrants to return with accumulated savings when otherwise they would prefer the "no return" option. For example, with \(w^* = 5, \alpha = 1.1,\) and \(\theta = .80\) or \(.85\), there is no return (NR) when \(r\) is relatively small (i.e., \(r = .03\) and \(.05\)). For higher values of \(r\), however, migrants return with accumulated savings to take advantage of the investment opportunities in the source country. In fact for \(r = .25\), rather than choosing NR, they stay abroad for a short period of time (i.e., \(\tau\) is equal to roughly 3.5 years). In sum, an increase in the rate of return on investment in the source country stimulates interest in temporary migration on the part of those who would otherwise prefer not to migrate, while also encouraging return migration of those who would have otherwise stayed permanently abroad.

It would be interesting to conduct an empirical investigation and test whether indeed temporary migration is more frequent from those developing countries where small businesses enjoy higher rates of return on investment, but face credit constraints due to lack of development of local financial markets. One might envisage a cross-country analysis with the proportion of the country’s labor force taking part in temporary migration as the dependent variable and the expected rate of return on entrepreneurial activity and a measure of access to credit as the regressors. Another key prediction of the model is the inverse U-shaped relationship between \(\tau\) and \(r\). Ideally, one

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16 Note that the utility from consuming the source country wage (\(w=1\)) is simply \(\alpha/(1-\theta)\), while the flow of utility enjoyed by a permanent migrant is \(u^* = w^*(1-\theta)/(1-\theta)\). For the two utilities to be equal, the foreign wage would have to take on the value \(w^* = \alpha^{1/(1-\theta)}\). For \(\alpha = 1.2\), this amounts to 2.48 when \(\theta = .80\), 3.37 when \(\theta = .85\), 6.19 when \(\theta = .90\), and 38.33 when \(\theta = .95\). Thus the greater the value of \(\theta\), the higher must be the value of \(w^*\) to attract permanent migrants from the low-wage source country. As Table 2 illustrates, the results are qualitatively similar in the case of temporary migrants.

17 This hypothesis can be tested in a quadratic regression model where the quadratic
would like to conduct a cross-country analysis, explaining the variations in the average duration of stay of temporary migrants by the differences between the expected rates of return on investments at home, as well as the gaps between wages in the host and source countries. Unfortunately, as noted in footnote 4, data on rates of return in the developing countries is sporadic at best, while availability of evidence on the duration of foreign stay of temporary migrants is even more limited. Any serious empirical analysis of the predictions of our model will have to wait until these data problems are resolved.

6 Concluding Remarks

The theoretical literature on consumption and duration-of-stay decisions of temporary migrants has focused primarily on the role of international wage differentials, price level differentials, human capital accumulation objectives, consumption preferences, and family commitments of migrants as the principal factors explaining behavior. Numerous other variables, however, may affect their key decisions. As suggested in the Introduction, return migration is often linked to the prospect of investing the savings accumulated abroad at an attractive rate of return in the source country. While other studies, both theoretical and empirical, have incorporated to some extent this motive into their analysis of international migration, none of them has looked explicitly at how the expected rate of return on investment at home affects the optimal saving and duration of stay decisions. Moreover, they have not examined how an increase in the expected rate of return triggers temporary migration or converts permanent into temporary migration. Thus the new element in this paper is an explicit analysis of how differences in investment opportunities across countries affect temporary migration and saving behavior of utility-maximizing agents.

The paper also illustrates the interaction between two important motives for return migration. Preference for consumption in the source country and a positive rate-of-return differential in favor of investment in the source country. It is interesting to note that the first motive pulls the migrants back to their country of origin or prevents them from leaving in the first place,

\[ \text{term} \times \text{dummy variable} = \begin{cases} 1 & \text{if wage and interest differentials are below a certain pre-specified threshold} \\ 0 & \text{if they are above} \end{cases} \]
while the second motive encourages workers to migrate, but discourages them from staying abroad for a long period of time. Our analysis shows that the duration of stay abroad should exhibit an inverted U-shaped relation to the rate of return ($r$) on investment in the source country. For low values of the international wage and interest rate differentials, it is found that an increase in $r$ has a positive effect on the duration of stay abroad, while having a negative effect for higher values of these same variables. Moreover, it is shown that an increase in $r$ may trigger temporary migration when it otherwise would not take place. It may also trigger return when migration would have otherwise been permanent.

There are possible policy implications that follow from the theoretical analysis of this paper. For example, host countries that rely on immigration to meet short-term shortages in the labor market, but wish to avoid permanent immigration, would find it in their interest to recruit temporary migrants from developing countries where (a) entrepreneurs are liquidity constrained in an environment characterized by high expected rates of return on small business investments and (b) where cultural, climatic, and cost-of-living differences between the host and source countries tend to attract migrants back home once they have accumulated the desired stock of assets. The analysis of this paper suggests that in the presence of such conditions migrants have a greater incentive to return, reducing the likelihood of permanent settlement in the host country. Another implication of the model is that a policy of addressing credit and liquidity constraints that face potential migrants in the developing countries will not necessarily result in an increase in the flow of migration. As pointed out in the literature, liquidity constraints, on the one hand, prevent many workers in the developing countries from realizing their migration plans. At the same time, our analysis suggests that liquidity constraints can also encourage others to develop such plans. Thus the impact on the flow of emigration associated with an improvement in the efficiency of credit markets in a source country is likely to be ambiguous.\footnote{Moreover, financial development can affect migration intentions in a number of subtle ways that go beyond the structure of the present model. See Docquier and Rapoport (2006) for a survey of the related literature and Rapoport (2002) for an interesting dynamic model of inequality, occupational choice and growth in an economy where individuals face liquidity constrains in financing migration as well as access to entrepreneurship. They present conditions under which migration and remittances give descendents of migrants access to entrepreneurship and allow the economy to move out of an initial underdevelopment trap to an efficient long-run equilibrium.}
Appendix

Partial derivatives of $G$

From eq. (10), the partial derivatives of $G$ with respect to the key variables of the model are

(A1) $G_{c^*} = (r - r^*) \frac{\partial A}{\partial c^*} + \frac{\partial}{\partial c^*}[w^* - c^* - w + \phi(c^*)] - \frac{\partial}{\partial c^*}(r - r^*)A \geq 0$

(A2) $G_r = (r - r^*)(w^* - c^*)e^{r\tau} > 0,$

(A3) $G_{w^*} = -\left[1 - \frac{r-r^*}{r^*} \left(e^{r\tau} - 1\right)\right] < 0,$

(A4) $G_w = 1,$

(A5) $G_r = A > 0,$

In eq. (A1), we note that $G_{c^*}$ is of ambiguous sign. Holding everything else constant, an increase in $c^*$ lowers $A$, thus lowering the opportunity cost of staying abroad and not investing accumulated assets in the home country at a relatively higher rate of return. This effect is captured by the first term which contributes to a negative $G_{c^*}$. An increase in $c^*$ also lowers the marginal utility of consumption and thus the utility value of the increase in wealth $[w^* - c^* - w + \phi(c^*)]$ associated with staying longer abroad. This corresponds to the second term and contributes to a positive value of $G_{c^*}$. Finally, the utility value of not investing accumulated savings in the source country also falls with an increase in $c^*$. This has a negative impact on $G$, as represented by the third term. Note that the combined effect of the last two terms is positive, as implied by condition (8), and needs to be compared with the first, negative, term to determine the sign of $G_{c^*}$.

$G_r$ in (A2), which is simply $(r - r^*) \frac{\partial A}{\partial r}$, has a positive value. That is, postponing return to a later date raises $G$ by extending the period of asset accumulation and increasing the stock of assets of which a proportion $(r - r^*)$ is sacrificed by staying longer abroad. An increase in $w^*$, however, seems to have an ambiguous effect on $G$. As shown in (A3), there is the negative, direct effect of a higher foreign wage ($-1$), but also a positive, indirect effect $\left[\frac{r-r^*}{r^*} (e^{r\tau} - 1)\right]$ as an increase in $w^*$ results in a higher $A$, of which a proportion $(r - r^*)$ is forfeited by staying abroad. Since it would never pay to
stay abroad to the point where the forgone earnings on accumulated assets exceed the rate of asset accumulation, \(w^* - c^*\), at the optimal \(\tau\) it must be the case that \(w^* - c^* > \frac{r\tau}{\tau^2} (e^{r\tau} - 1) (w^* - c^*)\) so that \(G_{w^*} < 0\). As for an increase in \(w\) in (A4), it obviously raises the gain from returning to the source country a moment sooner. The effect of a higher \(r\) is also positive, as shown in (A5), because an increase in the source-country rate of return on accumulated assets increases the urgency of return.

**Partial derivatives of \(B\)**

From eq. (11), the partial derivatives of \(B\) with respect to the key variables of the model are

\[
(A6) \quad B_{c^*} = -\left[ \frac{1}{\tau^2} (e^{r\tau} - 1) + \frac{\tau}{\tau^2} (e^{g(T-\tau)} - 1) \right] < 0
\]

\[
(A7) \quad B_r = (w^* - c^*)e^{r\tau} + (c_{T\tau} - w_{T\tau}) > 0
\]

\[
(A8) \quad B_{w^*} = \frac{1}{\tau^2} (e^{r\tau} - 1) > 0
\]

\[
(A9) \quad B_w = \frac{1}{\tau} (1 - e^{-r(T-\tau)}) > 0
\]

\[
(A10) \quad B_r = -w_{T\tau} \frac{(e^{r(T-\tau)} - 1)}{\tau^2} \right) + \left( \frac{\theta - 1}{\theta} \right) c_{T\tau} \left( \frac{T-\tau}{\tau} \right) - \frac{1-e^{-g(T-\tau)}}{\theta} > 0,
\]

where \(w_{T\tau} = w e^{-r(T-\tau)}\) and \(c_{T\tau} = \phi(c^*) e^{g(T-\tau)}\).

An increase in \(c^*\) unambiguously tightens the migrant’s budget constraint, as shown in (A6). It reduces the stock of assets accumulated abroad (the first term in the brackets) and increases spending after return (the second term) by shifting the entire time path of post-return consumption upward, as implied by eq. (9). As for an increase in \(\tau\), it relaxes the migrant’s budget constraint, as stated in (A7). It lengthens the period of saving (abroad) and shortens the period of overspending (at home). Similarly, the effect of an increase in \(w^*\) or \(w\) on the budget constraint is unambiguously positive, as indicated in (A8) and (A9). The effect of an increase in \(r\), shown in (A10), is ambiguous. While an increase in \(r\) lowers the present (time-\(\tau\)) value of wages received in the home country, it may either lower or raise the time-\(\tau\) value of the migrant’s post-return consumption stream. This can be seen by recalling that nominal consumption in the source country
grows at the proportional rate \( r \), while discounted consumption grows at the rate \( g = \frac{r - r^*}{\theta} - r \), which is positively related to \( r \) when \( 1/\theta > 1 \). Thus for \( 1/\theta > 1 \), \( B_r \) is unambiguously negative, as the present value of the migrant’s post-return labor income shrinks while that of consumption spending expands. For \( 1/\theta < 1 \), however, one cannot rule out the possibility that \( B_r > 0 \). For a sufficiently large value of \( \theta \), \( B_r > 0 \), as may be seen by evaluating \( B_r \) as \( \theta \to \infty \) and noting that \( \phi(c^*) > w \), and that \( g \to -r \) as \( \theta \to \infty \).\(^{19}\) Since empirical evidence suggests that \( \theta \) is not likely to be very large in relation to 1, we shall take \( B_r < 0 \) to be the relevant case. Note that \( B_r < 0 \) is a sufficient (but not necessary) condition for \( dc^*/dr < 0 \) in eq. (18).

Effects of a change in \( w^* \) on \( \tau \) and \( c^* \)

The ambiguous sign of \( d\tau/dw^* \) in (14) stems from the interaction between the income and substitution effects of an increase in \( w^* \). On the one hand, an increase in the foreign wage relaxes the budget constraint \((B_{w^*} > 0)\). This contributes to a decline in \( \tau \) as it enables the migrant to achieve a given consumption program with a shorter stay abroad. On the other hand, recalling that \( G_{w^*} < 0 \), an increase in \( w^* \) makes staying abroad more rewarding by increasing the migrant’s rate of asset accumulation at any given \( c^* \). Because this lowers \( G \), \( c^* \) must increase to reduce the marginal utility of consumption and hence the utility value of the higher savings rate so as to maintain \( G = 0 \). This increase in \( c^* \) has a negative, indirect impact on the migrant’s budget, amounting to \( B_{w^*}G_{w^*}/G_{c^*} \). If the degree of concavity of the utility function, \( \theta \), is relatively high, the increase in consumption needed to restore \( G = 0 \) after an increase in \( w^* \) is relatively small. In that case \( d\tau/dw^* < 0 \) as the direct effect on the budget, \( B_{w^*} \), dominates the indirect effect, \( B_{w^*}G_{w^*}/G_{c^*} \). For \( \theta < 1 \), the increase in consumption is larger and the indirect effect dominates. This is shown in our simulations in Section 4, where we consider values of \( \theta \) in the range between .80 and .95. The migrant then stays longer abroad in response to an increase in \( w^* \) in order to pay for the optimal consumption program.

\(^{19}\)Note that both expressions in the brackets of (21) are positive, the second one being so regardless of the sign of \( g \).
References


Figure 1. Solutions for the Optimal Duration of Stay and Consumption Abroad
Figure 2. Effect of an Increase in $r$ on $\tau$ and $c^*$
Table 1. Equilibrium values of $\tau$ for various values of $\theta$, $w^*$, and $r$ when $\alpha = 1.2$, $r^* = .03$ and $w = 1$

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Table 2. Equilibrium values of $\tau$ for various values of $\theta$, $\alpha$, $w^*$, and $r$, when $r^* =.03$ and $w = 1$.

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