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A Confidence Interval for Combined Univariate Economic Forecasts

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Abstract

Research in combining of economic forecasts made by several research institutes on the same economic variable has focused on estimation, hoping that the combined forecast will be improved by taking into account the expert opinions of the institutes. We provide a confidence interval on the combined forecast. A scoring of the individual institutes is proposed using their historical performance in forecasting.

Key words: Heterogeneous variances; Expert opinion; Confidence interval.

1 Introduction

Consider the situation where one has several forecasts on the same quantity, e. g. several economic research institutes forecast an important economic variable. Interest is in combining the individual forecasts to improve the accuracy of the forecast. Since by way of combining several forecasts one takes into account several expert opinions, one expects a better performance of the combined forecast. However, it is rarely observed that the simply average of the individual forecasts is beaten by more sophisticated methods. Klapper (1998, 1999, 2000) discusses rank-based techniques for combining forecasts. There is little known about confidence intervals on the combined forecast in this area.

In this paper, we give several confidence intervals on the combined forecast which are of approximate nature. The confidence intervals are derived using analysis of variance as a main tool. Some simulation results on the properties of the confidence intervals are provided. Finally, the confidence intervals are applied on German economic data.

2 A statistical model for combining forecasts

We consider the following model

$$y_i \sim N(\mu, \alpha_i), \tag{2.1}$$

where y_i denotes the forecast of the i -th institute, and μ represents the true unknown quantity, $i = 1, \dots, K$, $K > 2$. Note that we assume that the variances α_i are heteroscedastic. Confidence intervals and tests are discussed in a related heteroscedastic ANOVA model by Hartung and Argaç (2002a, 2002b) and Hartung and Knapp (2000).

We estimate the mean μ by a weighted average of the individual forecasts y_i ,

$$y = \sum_{i=1}^K b_i^2 \cdot y_i, \quad (2.2)$$

where b_i^2 denotes the weight which is given to the i -th institute by some scoring process. We assume that the weights are normed

$$\sum_{i=1}^K b_i^2 = 1, \quad (2.3)$$

and we also assume that $b_i^2 < 1/2$, that is we exclude the possibility that one particular institute dominates the rest. Consider now the following quadratic form, see also Hartung, Böckenhoff and Knapp (2002),

$$u_{ib}^2 = b_i^2 \cdot \left(y_i - \sum_{j=1}^K b_j^2 \cdot y_j \right)^2, \quad (2.4)$$

which can be interpreted as a weighted quadratic deviation of each individual forecast from the weighted average of the individual forecasts of each institute. Let $\beta_i = b_i^2 \cdot \alpha_i$, and one can derive explicitly the important moments of the quadratic form u_{ib}^2 ,

$$E(u_{ib}^2) = (1 - 2 \cdot b_i^2) \cdot \beta_i + b_i^2 \cdot \sum_{j=1}^K b_j^2 \cdot \beta_j =: e_i(b, \beta), \quad (2.5)$$

$$\text{var}(u_{ib}^2) = 2 \cdot e_i(b, \beta)^2, \quad (2.6)$$

$$\text{Cov}(u_{ib}^2, u_{jb}^2) = 2 \cdot b_i^2 \cdot b_j^2 \cdot \left(\sum_{k=1}^K b_k^2 \cdot \beta_k - \beta_i - \beta_j \right) =: 2 \cdot e_{ij}(b, \beta)^2. \quad (2.7)$$

We need to estimate the variance of the combined forecast and of each individual forecast, and for this purpose we use quadratic functions of the individual forecasts as estimators. The variance of the combined forecast is estimated with an unbiased positive (PSD–MINQUE) variance estimator which is given by, see also Hartung, Böckenhoff and Knapp (2002),

$$\widehat{var}(y)_{psd} = \frac{1}{1 + \sum_{k=1}^K \frac{b_k^4}{1 - 2b_k^2}} \cdot \sum_{i=1}^K \frac{b_i^2}{1 - 2b_i^2} \cdot u_{ib}^2 \quad (2.8)$$

$$= \sum_{i=1}^K d_i \cdot u_{ib}^2. \quad (2.9)$$

The estimator of the variance of each forecast is given by a positive minimum biased variance estimator (PSD–MINQMBE) given by

$$\widehat{\alpha}_i = \frac{\sum_{j=1}^K u_{jb}^2 + \sum_{j=1}^K d_j \cdot u_{jb}^2}{\sum_{j=1}^K b_j^2 \cdot \tilde{\alpha}_j} \cdot \tilde{\alpha}_i, \quad (2.10)$$

where

$$\tilde{\alpha}_i = \frac{1}{b_i^2} \cdot \frac{(1 - b_i^2)^2}{(1 - b_i^2)^4 + b_i^4 \sum_{j \neq i} b_j^4} \cdot u_{ib}^2. \quad (2.11)$$

Note here also that both estimators are given explicitly, see also Hartung, Böckenhoff and Knapp (2002) for a detailed derivation. Under suitable conditions these estimators are consistent, exist always and are unique. Surprisingly, the variance of the combined forecast can be unbiasedly estimated without any information about the variance of the individual forecasts.

We will construct the confidence interval using a pivotal quantity, and for this purpose we have to determine the distribution of the variance estimators. We

will approximate the distribution of the variance estimators by a suitable χ^2 -distributions using moment matching, see Satterthwaite (1946) and Patnaik (1949). This leads to

$$\nu \cdot \sum_{i=1}^K d_i \cdot u_{ib}^2 / E\left(\sum_{i=1}^K d_i \cdot u_{ib}^2\right) \sim \chi_\nu^2, \quad (2.12)$$

where

$$\nu = 2 \frac{[E(\sum_{i=1}^K d_i \cdot u_{ib}^2)]^2}{\text{var}(\sum_{i=1}^K d_i \cdot u_{ib}^2)}. \quad (2.13)$$

Now, this gives us the following pivot (nominator and denominator are independent)

$$\frac{y - \mu}{\sqrt{\sum_{i=1}^K d_i \cdot u_{ib}^2}} \sim t_\nu, \quad (2.14)$$

where

$$\nu = \frac{\left\{ \sum_{i=1}^K d_i \cdot e_i(b, \beta) \right\}^2}{\sum_{i=1}^K d_i^2 \cdot e_i(b, \beta)^2 + \sum_{i=1}^K \sum_{j=1, j \neq i}^K d_i \cdot d_j \cdot e_{ij}(b, \beta)^2}. \quad (2.15)$$

In practice, the unknown quantities have to be replaced by estimators, and here we use $\hat{\beta}_i = b_i^2 \cdot \hat{\alpha}_i$.

Now, we are in the position to derive the confidence intervals which are of course of approximate nature only.

$$K_1 : \quad y \mp t_{\nu, 1-\alpha/2} \cdot \sqrt{\sum_{i=1}^K d_i \cdot u_{ib}^2} \quad (2.16)$$

$$K_2 : \quad y \mp u_{1-\alpha/2} \cdot \sqrt{\sum_{i=1}^K d_i \cdot u_{ib}^2} \quad (2.17)$$

$$K_3 : \quad y \mp t_{K-1;1-\alpha/2} \cdot \sqrt{\sum_{i=1}^K d_i \cdot u_{ib}^2} \quad (2.18)$$

The first interval is the approximate interval derived using the t -distributed pivot, the second version is obtained if one replaces the quantile of the t -distribution with the quantile of the standard normal distribution for large degrees of freedom. The last version is obtained if one ignores the heteroscedastic variances. Now, we have to specify the weights b_i^2 . We suggest the following choice of the weights:

$$b_i^2 = \frac{\sum_{j=1}^L (y_{ij} - y_j^w)^{-2}}{\sum_{i=1}^K \sum_{j=1}^L (y_{ij} - y_j^w)^{-2}}, \quad (2.19)$$

where y_{ij} denotes the forecast of the i -th institute for some economic variable in the j -th time period or year and y_j^w denotes the true realized value in the j -th time period or year. Hence, each institute is scored taking into account its historical performance in forecasting in the last L time periods or years. We consider quadratic deviations of each forecast from the true value.

3 Monte Carlo results

Since the intervals we constructed are of approximate nature, we conducted a simulation experiment to check their validity with respect to the actual confidence coefficients and lengths of the proposed confidence intervals. We considered $K = 7$ and $K = 14$ institutes. The weights b_i^2 are chosen to be equal, in a second scenario the weight of one institute is large compared to the other weights which are chosen to be equal, and finally, the weights are chosen to be almost equal. The variances of the individual forecasts are homoscedastic and heteroscedastic. We paired small variances with small weights and small variances with large weights. We draw samples from the normal and from a centered χ^2 -distribution with a few degrees of freedom to cover also the case of non-normal observations. The number of repetitions in the simulations is 1000. In the simulations, we give summary statistics on the degrees of freedom of the approximate t-distribution. We report the bias, the standard deviation and the root mean square error (RMSE) of the variance estimators given in (2.10). We provide the empirical confidence level of the intervals at the nominal level of 90% and 95%, the lower and the upper confidence limits and the corresponding widths of the confidence intervals.

The main result is that, in general, the intervals K_1 and K_3 are conservative, the confidence interval K_2 also attains acceptable levels, but might become liberal under certain circumstances.

4 Data analysis

We applied the confidence intervals to German economic data. We used the data on GDP from seven major economic research institutes from 1984–1996. The data are described in Klapper (1998). For the first forecast, that is for 1987, we used the data from 1984–1986. The true values are given in the next table:

Table 1: True values of GDP.

Year	Value
1987	1.9
1988	3.7
1989	3.3
1990	4.7
1991	3.7
1992	1.6
1993	-1.7
1994	2.4
1995	1.9
1996	1.4

We give the weights b_i^2 , the variance estimators given in (2.10) and (2.11), the variance of the combined forecast given in (2.8), the degrees of freedom of the approximate t–distribution, the combined forecast given in (2.2), and finally, the confidence intervals at nominal level of 90% and 95%.

Results

forecast for the year: 1987

bi²:

0.036 0.069 0.332 0.036 0.343 0.033 0.151

a_tilde:

1.119 0.085 0.405 0.240 0.507 0.112 0.293

a_hat:

0.995 0.076 0.360 0.213 0.450 0.100 0.260

variance:

0.107

df_hat:

1.447214

forecast(95%):

forecast	K1.l	K1.u	K2.l	K2.u	K3.l	K3.u
	2.525	0.452	4.599	0.452	4.599	1.724 3.326

forecast(90%):

forecast	K1.l	K1.u	K2.l	K2.u	K3.l	K3.u
	2.525	1.267	3.783	1.987	3.064	1.889 3.161

forecast for the year: 1988

bi²:

0.040 0.067 0.241 0.028 0.466 0.048 0.111

a_tilde:

0.533 0.559 0.933 0.091 0.102 0.012 0.051

a_hat:

0.441 0.462 0.772 0.075 0.085 0.010 0.042

variance:

0.049

df_hat:

1.119492

forecast(95%):

forecast	K1.l	K1.u	K2.l	K2.u	K3.l	K3.u
	1.706	-0.478	3.89	1.274	2.138	1.167 2.245

forecast(90%):

forecast	K1.l	K1.u	K2.l	K2.u	K3.l	K3.u
	1.706	0.541	2.871	1.343	2.069	1.278 2.134

forecast for the year: 1989

bi²:

0.040 0.067 0.241 0.028 0.464 0.048 0.111

a_tilde:

0.005 0.036 0.007 0.194 0.012 0.579 0.006

a_hat:

0.005 0.034 0.007 0.183 0.011 0.546 0.006

variance:

0.004

df_hat:

1.270905

forecast(95%):

forecast	K1.l	K1.u	K2.l	K2.u	K3.l	K3.u
2.43	1.931	2.929	2.305	2.556	2.274	2.587

forecast(90%):

forecast	K1.l	K1.u	K2.l	K2.u	K3.l	K3.u
2.43	2.145	2.715	2.325	2.536	2.306	2.555

forecast for the year: 1990

bi^2:

0.042 0.068 0.240 0.029 0.460 0.048 0.112

a_tilde:

0.234 0.001 0.002 0.001 0.003 0.059 0.033

```

a_hat:
0.217 0.001 0.002 0.001 0.003 0.055 0.031

variance:
0.001

df_hat:
1.454133

forecast(95%):
forecast  K1.l  K1.u  K2.l  K2.u  K3.l  K3.u
          3.034  2.825  3.243  2.969  3.099  2.953  3.115

forecast(90%):
forecast  K1.l  K1.u  K2.l  K2.u  K3.l  K3.u
          3.034  2.907  3.161  2.979  3.089  2.969  3.099

forecast for the year: 1991
bi^2:
0.044 0.069 0.240 0.030 0.458 0.049 0.112

a_tilde:
0.017 0.017 0.024 0.146 0.039 0.492 0.168

a_hat:
0.016 0.016 0.022 0.136 0.036 0.457 0.156

```

variance:

0.013

df_hat:

1.258859

forecast(95%):

forecast	K1.l	K1.u	K2.l	K2.u	K3.l	K3.u	
	3.373	2.484	4.262	3.153	3.593	3.099	3.647

forecast(90%):

forecast	K1.l	K1.u	K2.l	K2.u	K3.l	K3.u	
	3.373	2.868	3.878	3.189	3.557	3.155	3.591

forecast for the year: 1992

bi²:

0.082 0.067 0.248 0.029 0.432 0.043 0.099

a_tilde:

1.074 0.243 0.330 0.001 0.654 0.059 0.032

a_hat:

1.002 0.227 0.308 0.001 0.611 0.055 0.030

variance:

0.17

df_hat:
1.138552

forecast(95%):
forecast K1.l K1.u K2.l K2.u K3.l K3.u
1.966 -1.983 5.914 1.157 2.774 0.957 2.975

forecast(90%):
forecast K1.l K1.u K2.l K2.u K3.l K3.u
1.966 -0.161 4.092 1.287 2.644 1.164 2.767

forecast for the year: 1993

bi^2:
0.061 0.173 0.300 0.028 0.304 0.034 0.101

a_tilde:
1.000 0.272 0.005 0.321 0.005 0.964 0.366

a_hat:
0.917 0.250 0.004 0.295 0.004 0.884 0.335

variance:
0.013

df_hat:
3.085219

forecast(95%):

forecast	K1.l	K1.u	K2.l	K2.u	K3.l	K3.u
-0.053	-0.407	0.301	-0.274	0.169	-0.329	0.224

forecast(90%):

forecast	K1.l	K1.u	K2.l	K2.u	K3.l	K3.u
-0.053	-0.316	0.21	-0.239	0.133	-0.272	0.167

forecast for the year: 1994

bi²:

0.063 0.173 0.299 0.028 0.303 0.034 0.100

a_tilde:

1.176 0.307 0.392 0.237 0.484 0.001 0.019

a_hat:

1.029 0.269 0.343 0.207 0.423 0.001 0.016

variance:

0.085

df_hat:

1.819994

forecast(95%):

forecast	K1.l	K1.u	K2.l	K2.u	K3.l	K3.u
0.525	-0.859	1.909	-0.047	1.098	-0.19	1.24


```
forecast(90%):
forecast   K1.l  K1.u    K2.l    K2.u  K3.l  K3.u
          0.525 -0.388 1.439    0.045    1.006 -0.042 1.093
```

forecast for the year: 1995

```
bi^2:
0.063 0.173 0.298 0.029 0.302 0.034 0.100
```

```
a_tilde:
1.340 0.012 0.286 0.009 0.016 0.590 0.104
```

```
a_hat:
1.209 0.011 0.258 0.008 0.014 0.532 0.094
```

```
variance:
0.03
```

```
df_hat:
1.614924
```

```
forecast(95%):
forecast   K1.l  K1.u    K2.l    K2.u  K3.l  K3.u
          3.095  2.145 4.044    2.753    3.436 2.668 3.521
```

```
forecast(90%):
forecast   K1.l  K1.u    K2.l    K2.u  K3.l  K3.u
          3.095 2.493 3.696    2.808    3.381 2.756 3.433
```

forecast for the year: 1996

bi²:

0.167 0.154 0.264 0.026 0.268 0.030 0.090

a_tilde:

0.826 0.001 0.011 0.544 0.081 0.548 0.453

a_hat:

0.734 0.001 0.009 0.483 0.072 0.487 0.402

variance:

0.028

df_hat:

1.904998

forecast(95%):

forecast	K1.l	K1.u	K2.l	K2.u	K3.l	K3.u
1.78	1.029	2.531	1.454	2.106	1.373	2.187

forecast(90%):

forecast	K1.l	K1.u	K2.l	K2.u	K3.l	K3.u
1.78	1.277	2.283	1.506	2.054	1.457	2.103

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Simulation results for K=7 (normal distribution)

empirical level of CI (normal distr.)

bi² : 0.143 0.143 0.143 0.143 0.143 0.143 0.143

variances : 2 2 2 2 2 2 2

repetitions: 1000

df:

df_mean:

3.275

df_std:

0.919

df_quantiles:

0%	25%	50%	75%	100%
1.063	2.559	3.292	3.933	6.035

bias(variance estimators):

0.0849 -0.0310 -0.0120 0.0015 -0.0831 -0.1153 0.2338

std(variance estimators):

3.2120 2.7277 2.8178 3.1083 2.8052 2.6790 3.2227

RMSE:

3.2131 2.7279 2.8178 3.1083 2.8064 2.6814 3.2312

RMSE.total:

20.5861

conf.level(95%):

K1	K2	K3
99.8	92.5	97.7

width:

K1	K2	K3
0.452	0.27	0.337

conf.level(90%):

K1	K2	K3
96.5	84.7	92.2

width:

K1	K2	K3
0.329	0.226	0.267

empirical level of CI (normal distr.)

bi² : 0.143 0.143 0.143 0.143 0.143 0.143 0.143

variances : 2 4 6 8 10 12 14

repetitions: 1000

df:

df_mean:

3.033

df_std:

0.947

df_quantiles:

0%	25%	50%	75%	100%
1.083	2.336	2.973	3.669	5.992

bias(variance estimators):

-0.0242 0.2960 0.2216 0.2688 -0.1576 -0.4110 1.0639

std(variance estimators):

2.7411 6.0115 9.0319 11.0558 13.9781 16.0869 21.2612

RMSE:

2.7412 6.0187 9.0347 11.0591 13.9790 16.0922 21.2878

RMSE.total:

80.2126

conf.level(95%):

K1	K2	K3
99.8	92.7	98.6

width:

K1	K2	K3
0.986	0.541	0.675

```
conf.level(90%):
  K1    K2    K3
97.3  84.5 92.4
```

```
width:
  K1    K2    K3
0.699 0.454 0.536
```

empirical level of CI (normal distr.)

```
bi^2 : 0.49 0.085 0.085 0.085 0.085 0.085 0.085
variances : 2 2 2 2 2 2 2
repetitions: 1000
```

```
df:
df_mean:
1.209
```

```
df_std:
0.556
```

```
df_quantiles:
  0%  25%  50%  75% 100%
  1 1.007 1.019 1.085 5.709
```

```
bias(variance estimators):
-0.0051 0.2343 0.1295 0.1702 -0.0069 0.1486 0.1629
```


std(variance estimators):

2.8543 3.1827 2.9185 2.9561 2.7382 3.1266 3.2277

RMSE:

2.8543 3.1913 2.9214 2.9610 2.7382 3.1301 3.2318

RMSE.total:

21.0282

conf.level(95%):

K1	K2	K3
99.9	97.3	98.8

width:

K1	K2	K3
5.686	0.921	1.15

conf.level(90%):

K1	K2	K3
98.8	94.5	97.1

width:

K1	K2	K3
2.861	0.773	0.913

empirical level of CI (normal distr.)

bi² : 0.49 0.085 0.085 0.085 0.085 0.085 0.085

variances : 2 4 6 8 10 12 14

repetitions: 1000

df:

df_mean:

1.416

df_std:

0.688

df_quantiles:

0% 25% 50% 75% 100%

1.001 1.029 1.091 1.43 5.284

bias(variance estimators)(variance estimators):

-0.0938 0.2995 -0.0674 0.5797 -0.0439 -0.6084 0.0709

std(variance estimators):

2.8515 6.0550 8.7045 11.9132 13.6275 16.0967 20.2054

RMSE:

2.8530 6.0624 8.7047 11.9272 13.6275 16.1082 20.2056

RMSE.total:

79.4887

```
conf.level(95%):  
  K1    K2   K3  
99.8    95 98.6
```

```
width:  
  K1    K2   K3  
5.093  0.914 1.142
```

```
conf.level(90%):  
  K1    K2   K3  
99.1   89.3 94.8
```

```
width:  
  K1    K2   K3  
2.628  0.767 0.907
```

empirical level of CI (normal distr.)

```
bi^2 :  0.49 0.085 0.085 0.085 0.085 0.085 0.085  
variances :  14 12 10 8 6 4 2  
repetitions:  1000
```

```
df:  
df_mean:  
1.136
```

```
df_std:  
0.411
```

df_quantiles:

	0%	25%	50%	75%	100%
1	1.004	1.01	1.044	4.424	

bias(variance estimators):

-1.2423	0.4273	0.4654	0.2545	0.1702	0.6977	0.6605
---------	--------	--------	--------	--------	--------	--------

std(variance estimators):

18.3193	17.8309	15.3039	11.0667	8.5818	6.7522	3.5595
---------	---------	---------	---------	--------	--------	--------

RMSE:

18.3613	17.8361	15.3110	11.0696	8.5835	6.7881	3.6202
---------	---------	---------	---------	--------	--------	--------

RMSE.total:

81.5698

conf.level(95%):

	K1	K2	K3
99.9	97.9	99.5	

width:

	K1	K2	K3
14.673	2.332	2.911	

conf.level(90%):

	K1	K2	K3
99.6	96.4	97.9	

width:

	K1	K2	K3
	7.347	1.957	2.312

empirical level of CI (normal distr.)

bi² : 0.163 0.163 0.163 0.128 0.128 0.128 0.128

variances : 2 2 2 2 2 2 2

repetitions: 1000

df:

df_mean:

3.055

df_std:

0.934

df_quantiles:

	0%	25%	50%	75%	100%
	1.153	2.372	3.019	3.647	5.935

bias(variance estimators):

-0.1225 0.1601 0.1224 0.1058 0.0082 -0.1449 0.1971

std(variance estimators):

2.7621 2.9168 2.8044 2.8081 2.8088 2.5920 3.1021

RMSE:
2.7648 2.9212 2.8071 2.8101 2.8088 2.5960 3.1084

RMSE.total:
19.8164

conf.level(95%):
K1 K2 K3
99.6 91.7 97.8

width:
K1 K2 K3
0.513 0.284 0.354

conf.level(90%):
K1 K2 K3
97.1 83.4 91.3

width:
K1 K2 K3
0.365 0.238 0.281

empirical level of CI (normal distr.)

bi² : 0.163 0.163 0.163 0.128 0.128 0.128 0.128
variances : 2 4 6 8 10 12 14
repetitions: 1000

df:

df_mean:

3.164

df_std:

0.94

df_quantiles:

	0%	25%	50%	75%	100%
	1.119	2.485	3.105	3.783	5.749

bias(variance estimators):

0.0408 0.0216 0.6190 -0.0224 -0.3049 0.3677 -0.2523

std(variance estimators):

2.9522 5.8892 9.6222 11.1032 13.5997 16.8349 20.1514

RMSE:

2.9525 5.8892 9.6421 11.1032 13.6031 16.8389 20.1530

RMSE.total:

80.1819

conf.level(95%):

	K1	K2	K3
	99.7	92.1	97.9

```
width:
  K1    K2    K3
0.879  0.504  0.63
```

```
conf.level(90%):
  K1    K2    K3
96.4   86.7  91.9
```

```
width:
  K1    K2    K3
0.633  0.423  0.5
```

empirical level of CI (normal distr.)

```
bi^2 :  0.163  0.163  0.163  0.128  0.128  0.128  0.128
variances :  14 12 10 8 6 4 2
repetitions: 1000
```

```
df:
df_mean:
2.653
```

```
df_std:
0.889
```

```
df_quantiles:
  0%   25%   50%   75%  100%
1.108 1.993 2.579 3.255 5.986
```


bias(variance estimators):

0.4523 0.7456 -0.2133 0.0324 0.2825 0.0681 0.0174

std(variance estimators):

23.0188 17.4906 12.9867 11.2607 8.7093 5.8048 2.7633

RMSE:

23.0233 17.5065 12.9885 11.2607 8.7139 5.8052 2.7634

RMSE.total:

82.0614

conf.level(95%):

K1	K2	K3
99.6	93.3	98.7

width:

K1	K2	K3
1.303	0.613	0.765

conf.level(90%):

K1	K2	K3
97.5	86.5	93.1

width:

K1	K2	K3
0.881	0.514	0.608

Simulation results for K=14 (normal distribution)

empirical level of CI (normal distr.)

bi² : 0.071 0.071 0.071 0.071 0.071 0.071 0.071 0.071 0.071
0.071 0.071 0.071 0.071 0.071

variances : 2 2 2 2 2 2 2 2 2 2 2 2 2 2

repetitions: 1000

df:

df_mean:

5.748

df_std:

1.424

df_quantiles:

	0%	25%	50%	75%	100%
	1.681	4.792	5.743	6.799	10.331

bias(variance estimators):

0.0618	-0.0999	0.0779	-0.0179	0.0227	0.0256	-0.1004	0.1329
-0.0441	-0.0642	0.0477	0.0600	0.1040	0.0923		

std(variance estimators):

2.8247	2.8607	2.9056	2.8659	2.7845	2.8692	2.6589	2.9051	2.9288
2.7215	2.8942	2.8175	2.9424	2.9869				

RMSE:

2.8253 2.8624 2.9067 2.8659 2.7845 2.8694 2.6608 2.9081 2.9291
2.7223 2.8946 2.8181 2.9442 2.9883

RMSE.total:

39.9798

conf.level(95%):

K1	K2	K3
98	94.7	96

width:

K1	K2	K3
0.131	0.101	0.111

conf.level(90%):

K1	K2	K3
94.7	87	91.2

width:

K1	K2	K3
0.103	0.085	0.091

empirical level of CI (normal distr.)

bi² : 0.071 0.071 0.071 0.071 0.071 0.071 0.071 0.071 0.071
0.071 0.071 0.071 0.071 0.071

variances : 2 4 6 8 10 12 14 2 4 6 8 10 12 14

repetitions: 1000

df:

df_mean:

5.097

df_std:

1.48

df_quantiles:

	0%	25%	50%	75%	100%
	1.502	4.067	5.026	6.071	10.179

bias(variance estimators):

-0.0497	-0.3323	-0.2174	-0.2721	0.1960	0.4987	-0.0362	0.0259
0.0075	0.1093	0.6663	0.4455	0.1394	0.2580		

std(variance estimators):

2.7687	4.8413	8.4074	10.8751	14.8004	16.9099	19.2347	2.6014
5.7870	8.5633	11.8345	14.6627	17.8034	19.9322		

RMSE:

2.7692	4.8527	8.4103	10.8785	14.8017	16.9172	19.2347	2.6015
5.7870	8.5640	11.8532	14.6695	17.8040	19.9339		

RMSE.total:

159.0772

conf.level(95%):

K1	K2	K3
99.3	95.5	97.2

width:

K1	K2	K3
0.275	0.201	0.222

conf.level(90%):

K1	K2	K3
96.3	89.3	92.3

width:

K1	K2	K3
0.213	0.169	0.182

empirical level of CI (normal distr.)

bi² : 0.49 0.039 0.039 0.039 0.039 0.039 0.039 0.039 0.039 0.039
0.039 0.039 0.039 0.039 0.039

variances : 2 2 2 2 2 2 2 2 2 2 2 2 2 2

repetitions: 1000

df:

df_mean:

1.074

df_std:

0.403

df_quantiles:

	0%	25%	50%	75%	100%
--	----	-----	-----	-----	------

1	1.001	1.002	1.008	6.745	
---	-------	-------	-------	-------	--

bias(variance estimators):

0.0039	0.2393	0.0639	0.2580	0.2457	0.1169	0.0545	0.0699
--------	--------	--------	--------	--------	--------	--------	--------

0.0032	0.0379	0.1534	0.1163	0.2099	-0.0010		
--------	--------	--------	--------	--------	---------	--	--

std(variance estimators):

2.7053	3.1131	2.9087	3.2083	3.2643	2.9553	2.8958	3.1724	2.9733
--------	--------	--------	--------	--------	--------	--------	--------	--------

2.9048	3.2233	2.9696	3.1931	2.8789				
--------	--------	--------	--------	--------	--	--	--	--

RMSE:

2.7053	3.1223	2.9094	3.2187	3.2735	2.9576	2.8964	3.1731	2.9733
--------	--------	--------	--------	--------	--------	--------	--------	--------

2.9051	3.2270	2.9719	3.2000	2.8789				
--------	--------	--------	--------	--------	--	--	--	--

RMSE.total:

42.4123

conf.level(95%):

	K1	K2	K3
--	----	----	----

99.9	99.2	99.5	
------	------	------	--

```
width:
  K1      K2      K3
5.988    0.93    1.026
```

```
conf.level(90%):
  K1      K2      K3
99.7     98.8    99.1
```

```
width:
  K1      K2      K3
2.981    0.781    0.841
```

empirical level of CI (normal distr.)

```
bi^2 : 0.49 0.039 0.039 0.039 0.039 0.039 0.039 0.039 0.039 0.039
        0.039 0.039 0.039 0.039 0.039
variances : 2 4 6 8 10 12 14 2 4 6 8 10 12 14
repetitions: 1000
```

```
df:
df_mean:
1.215
```

```
df_std:
0.76
```

df_quantiles:

	0%	25%	50%	75%	100%
1	1.003	1.008	1.042	7.84	

bias(variance estimators):

-0.0080	0.5225	0.1500	-0.3747	-0.2556	-0.1500	0.1654	0.0679
0.2000	-0.1521	-0.2405	0.5913	0.1039	0.1053		

std(variance estimators):

2.8348	6.3810	7.9408	10.9674	13.8247	16.6863	19.8173	3.0228
5.7593	8.1164	10.7338	14.9750	17.5106	20.4718		

RMSE:

2.8349	6.4023	7.9422	10.9738	13.8271	16.6870	19.8180	3.0235
5.7628	8.1179	10.7365	14.9867	17.5109	20.4720		

RMSE.total:

159.0957

conf.level(95%):

	K1	K2	K3
99.6	98.4	98.9	

width:

	K1	K2	K3
5.818	0.92	1.014	

conf.level(90%):

K1	K2	K3
99.3	96.6	97.5

width:

K1	K2	K3
2.909	0.772	0.831

empirical level of CI (normal distr.)

bi² : 0.49 0.039 0.039 0.039 0.039 0.039 0.039 0.039 0.039 0.039
0.039 0.039 0.039 0.039 0.039
variances : 14 12 10 8 6 4 2 14 12 10 8 6 4 2
repetitions: 1000

df:

df_mean:
1.066

df_std:

0.401

df_quantiles:

0%	25%	50%	75%	100%
1	1	1.001	1.005	7.254

```

bias(variance estimators):
-0.5070 -0.0808  2.0567  0.3658  0.5928  0.7026  0.7820  0.3977
 1.1400  0.8301  0.8617  0.8067  0.4196  0.8316

std(variance estimators):
18.8285 15.9962 16.9769 11.4816  9.6429  6.8297  3.8885 20.0774
18.9652 14.8103 12.4895  9.3268  5.7532  3.9510

RMSE:
18.8353 15.9964 17.1010 11.4874  9.6611  6.8657  3.9663 20.0813
18.9994 14.8336 12.5192  9.3616  5.7685  4.0375

RMSE.total:
169.5144

conf.level(95%):
  K1      K2    K3
100    99.3 99.3

width:
  K1      K2    K3
15.49    2.4 2.646

conf.level(90%):
  K1      K2    K3
99.7    98.9 99

```

width:

	K1	K2	K3
	7.706	2.014	2.169

empirical level of CI (normal distr.)

bi² : 0.082 0.082 0.082 0.082 0.082 0.082 0.064 0.064 0.064
0.064 0.064 0.064 0.064 0.064
variances : 2 2 2 2 2 2 2 2 2 2 2 2 2 2
repetitions: 1000

df:

df_mean:
5.189

df_std:

1.534

df_quantiles:

	0%	25%	50%	75%	100%
	1.32	4.089	5.087	6.169	9.834

bias(variance estimators):

0.0183	0.2387	-0.0449	-0.0247	0.0637	0.0782	-0.0954	0.0967
0.0945	0.0123	0.1122	-0.0044	0.1358	-0.0229		

std(variance estimators):

2.7978 3.2477 2.7921 2.8415 2.8609 2.8032 2.7845 2.9860 2.8381
2.7730 2.9123 2.7994 2.9603 2.9343

RMSE:

2.7979 3.2565 2.7925 2.8416 2.8616 2.8043 2.7861 2.9876 2.8397
2.7731 2.9145 2.7994 2.9634 2.9344

RMSE.total:

40.3525

conf.level(95%):

K1	K2	K3
99	93.5	96.8

width:

K1	K2	K3
0.144	0.106	0.117

conf.level(90%):

K1	K2	K3
95	85.4	89.2

width:

K1	K2	K3
0.112	0.089	0.096

empirical level of CI (normal distr.)

bi² : 0.082 0.082 0.082 0.082 0.082 0.082 0.064 0.064 0.064
0.064 0.064 0.064 0.064 0.064

variances : 2 4 6 8 10 12 14 2 4 6 8 10 12 14

repetitions: 1000

df:

df_mean:

4.913

df_std:

1.568

df_quantiles:

	0%	25%	50%	75%	100%
	1.424	3.773	4.87	5.938	10.294

bias(variance estimators):

0.0021	0.1168	0.1399	-0.0487	0.0748	0.4408	-1.8022	-0.0412
-0.1266	-0.2446	0.0432	-0.5136	-0.0758	-0.0226		

std(variance estimators):

2.6662	5.8799	8.8537	10.5506	13.9117	18.5455	16.5929	2.8618
5.8081	8.3145	11.2268	12.7527	17.8100	21.0710		

RMSE:

2.6662	5.8811	8.8548	10.5507	13.9119	18.5507	16.6904	2.8621
5.8094	8.3181	11.2268	12.7630	17.8102	21.0710		

RMSE.total:

156.9666

conf.level(95%):

K1	K2	K3
99.4	95.5	97.6

width:

K1	K2	K3
0.287	0.203	0.223

conf.level(90%):

K1	K2	K3
97.1	89.9	92.7

width:

K1	K2	K3
0.219	0.17	0.183

empirical level of CI (normal distr.)

bi² : 0.082 0.082 0.082 0.082 0.082 0.082 0.064 0.064 0.064

0.064 0.064 0.064 0.064 0.064

variances : 14 12 10 8 6 4 2 14 12 10 8 6 4 2

repetitions: 1000

df:

df_mean:

4.594

df_std:

1.457

std_quantiles:

	0%	25%	50%	75%	100%
	1.317	3.526	4.569	5.552	9.163

bias(variance estimators):

1.9064	-0.5692	0.0767	0.0588	-0.2077	0.2297	0.0106	-0.5260
-0.5235	0.9935	0.4286	-0.1073	-0.0044	0.1365		

std(variance estimators):

22.1807	16.6132	13.3367	11.3082	8.4413	5.7210	2.6936	19.5792
16.6148	15.5968	11.2331	8.4723	5.6918	2.9436		

RMSE:

22.2624	16.6230	13.3369	11.3083	8.4439	5.7256	2.6937	19.5863
16.6230	15.6284	11.2413	8.4730	5.6918	2.9467		

RMSE.total:

160.5844

conf.level(95%):

	K1	K2	K3
	98.9	94.1	96.9

```
width:
  K1    K2    K3
0.313  0.216  0.238
```

```
conf.level(90%):
  K1    K2    K3
95.8   86.8  90.7
```

```
width:
  K1    K2    K3
0.238  0.181  0.195
```


Simulation results for K=7 (centered χ^2 -distribution)

empirical level of CI (Chisquare)

bi² : 0.143 0.143 0.143 0.143 0.143 0.143 0.143

df : 1 1 1 1 1 1 1

repetitions: 1000

df:

df_mean:

3.359

df_std:

1.507

df_quantiles:

0%	25%	50%	75%	100%
1.038	2.005	3.29	4.601	6.818

bias (variance estimators):

-0.4009 0.3490 -0.0869 -0.2389 0.1519 -0.1593 -0.0323

std (variance estimators):

4.2679 7.0795 6.3895 6.3215 9.8191 6.0000 6.6633

RMSE:

4.2867 7.0881 6.3900 6.3260 9.8202 6.0021 6.6633

RMSE.total:

46.5766

conf.level(95%):

K1	K2	K3
92.2	83	89.3

width:

K1	K2	K3
0.563	0.241	0.301

conf.level(90%):

K1	K2	K3
84.1	76.5	82.6

width:

K1	K2	K3
0.367	0.202	0.239

empirical level of CI (Chisquare)

bi² : 0.143 0.143 0.143 0.143 0.143 0.143 0.143

df : 1 2 3 4 5 6 7

repetitions: 1000

df:

df_mean:

2.939

df_std:

0.944

df_quantiles:

0%	25%	50%	75%	100%
1.033	2.257	2.906	3.612	5.754

bias (variance estimators):

-0.0471 -0.3609 -0.0926 -1.1107 -0.1310 1.0688 -0.7785

std (variance estimators):

8.7001 9.3991 13.6465 12.5846 17.9840 26.8254 21.7252

RMSE:

8.7003 9.4061 13.6468 12.6335 17.9845 26.8467 21.7391

RMSE.total:

110.957

conf.level(95%):

K1	K2	K3
99.3	89.7	96.6

width:

K1	K2	K3
1.028	0.515	0.643

```
conf.level(90%):  
  K1    K2    K3  
94.8  83.2 89.3
```

```
width:  
  K1    K2    K3  
0.707 0.433 0.511
```

empirical level of CI (Chisquare)

```
bi^2 : 0.49 0.085 0.085 0.085 0.085 0.085 0.085  
df : 1 1 1 1 1 1 1  
repetitions: 1000
```

```
df:  
df_mean:  
1.147
```

```
df_std:  
0.447
```

```
df_quantiles:  
  0%  25%  50%  75% 100%  
1.001 1.006 1.018 1.061 5.776
```

```
bias (variance estimators):  
0.1116 -0.1553 0.3741 -0.0544 0.0611 0.1048 -0.0784
```

std (variance estimators):

8.5120 5.6313 8.1074 7.3199 6.4857 8.6732 5.3924

RMSE:

8.5127 5.6334 8.1161 7.3201 6.4860 8.6738 5.3930

RMSE.total:

50.135

conf.level(95%):

K1	K2	K3
99.7	97.5	98.7

width:

K1	K2	K3
4.95	0.804	1.004

conf.level(90%):

K1	K2	K3
99.1	95.2	97.3

width:

K1	K2	K3
2.493	0.675	0.797

empirical level of CI (Chisquare)

bi² : 0.49 0.085 0.085 0.085 0.085 0.085 0.085

df : 1 2 3 4 5 6 7

repetitions: 1000

df:

df_mean:

1.393

df_std:

0.649

df_quantiles:

	0%	25%	50%	75%	100%
	1.001	1.051	1.129	1.376	4.829

bias (variance estimators):

-0.3588 -0.3678 0.1325 1.1266 0.0367 0.6134 0.1670

std (variance estimators):

4.9605 9.5840 13.1359 19.1095 17.9362 27.2881 27.0886

RMSE:

4.9734 9.5911 13.1366 19.1426 17.9363 27.2950 27.0891

RMSE.total:

119.1641

```
conf.level(95%):
  K1      K2   K3
99.4   92.9 97.1
```

```
width:
  K1      K2   K3
4.144  0.779 0.973
```

```
conf.level(90%):
  K1      K2   K3
98.3   85.4 92.9
```

```
width:
  K1      K2   K3
2.165  0.654 0.772
```

empirical level of CI (Chisquare)

```
bi^2 :  0.49 0.085 0.085 0.085 0.085 0.085 0.085
df :    7 6 5 4 3 2 1
repetitions: 1000
```

```
df:
df_mean:
1.127
```

df_std:

0.374

df_quantiles:

	0%	25%	50%	75%	100%
1	1.003	1.011	1.049	4.384	

bias (variance estimators):

0.0238 1.6984 1.7597 0.4762 0.9604 1.2631 1.0971

std (variance estimators):

25.0118 26.8882 22.1192 14.8699 14.1669 13.1838 11.3971

RMSE:

25.0119 26.9418 22.1891 14.8775 14.1994 13.2442 11.4498

RMSE.total:

127.9136

conf.level(95%):

	K1	K2	K3
100	98.4	99.4	

width:

	K1	K2	K3
14.735	2.345	2.927	


```
conf.level(90%):
  K1    K2    K3
99.7  95.9 98.3
```

```
width:
  K1    K2    K3
7.381 1.968 2.325
```

empirical level of CI (Chisquare)

```
bi^2 : 0.163 0.163 0.163 0.128 0.128 0.128 0.128
df : 1 1 1 1 1 1 1
repetitions: 1000
```

```
df:
df_mean:
3.064
```

```
df_std:
1.246
```

```
df_quantiles:
  0%  25%  50%  75% 100%
1.034 1.922 3.008 4.083 6.005
```

```
bias (variance estimators):
0.4249 0.3321 -0.2552 -0.3396 0.2818 -0.1030 -0.0455
```

std (variance estimators):

9.1910 9.8628 5.2837 4.4549 7.3616 7.5874 8.6069

RMSE:

9.2009 9.8684 5.2899 4.4678 7.3670 7.5881 8.6070

RMSE.total:

52.389

conf.level(95%):

K1	K2	K3
95.2	82.2	90.4

width:

K1	K2	K3
0.635	0.256	0.319

conf.level(90%):

K1	K2	K3
85.4	75.8	81.5

width:

K1	K2	K3
0.406	0.214	0.253

empirical level of CI (Chisquare)

bi² : 0.163 0.163 0.163 0.128 0.128 0.128 0.128

df : 1 2 3 4 5 6 7

repetitions: 1000

df:

df_mean:

3.088

df_std:

1.014

df_quantiles:

	0%	25%	50%	75%	100%
	1.081	2.297	3.086	3.823	5.903

bias (variance estimators):

0.0201 -0.4296 0.8007 -1.4969 0.6285 0.2590 -1.0156

std (variance estimators):

6.2827 7.6358 16.2171 11.8507 21.4044 29.1257 20.8245

RMSE:

6.2828 7.6479 16.2368 11.9448 21.4137 29.1268 20.8493

RMSE.total:

113.5021

```
conf.level(95%):  
  K1    K2   K3  
99.3  89.9 95.8
```

```
width:  
  K1    K2   K3  
0.941 0.481 0.601
```

```
conf.level(90%):  
  K1    K2   K3  
93.5  82.6 89.9
```

```
width:  
  K1    K2   K3  
0.651 0.404 0.477
```

empirical level of CI (Chisquare)

```
bi^2 : 0.163 0.163 0.163 0.128 0.128 0.128 0.128  
df : 7 6 5 4 3 2 1  
repetitions: 1000
```

```
df:  
df_mean:  
2.536
```

df_std:

0.856

df_quantiles:

	0%	25%	50%	75%	100%
	1.035	1.875	2.463	3.059	5.407

bias (variance estimators):

0.6707	0.3385	0.3536	0.2057	0.4185	-0.2960	0.3251
--------	--------	--------	--------	--------	---------	--------

std (variance estimators):

28.0134	21.8782	22.0783	17.5604	14.2897	11.3791	9.6280
---------	---------	---------	---------	---------	---------	--------

RMSE:

28.0214	21.8808	22.0812	17.5616	14.2959	11.3829	9.6335
---------	---------	---------	---------	---------	---------	--------

RMSE.total:

124.8573

conf.level(95%):

	K1	K2	K3
	99.8	91.7	98

width:

	K1	K2	K3
	1.381	0.603	0.753

conf.level(90%):

K1	K2	K3
97.8	83.2	91

width:

K1	K2	K3
0.911	0.506	0.598

Simulation results for K=14 (centered χ^2 -distribution)

empirical level of CI (Chisquare)

bi² : 0.071 0.071 0.071 0.071 0.071 0.071 0.071 0.071 0.071
0.071 0.071 0.071 0.071 0.071

df : 1 1 1 1 1 1 1 1 1 1 1 1 1 1

repetitions: 1000

df:

df_mean:

4.697

df_std:

2.708

std_quantiles:

0%	25%	50%	75%	100%
1.082	2.549	4.02	6.323	12.344

bias (variance estimators):

0.1161	-0.1868	0.1990	0.2943	0.0806	-0.2641	-0.0478	0.0153
-0.0703	-0.2341	0.1063	0.0230	0.0951	-0.3336		

std (variance estimators):

8.0426	5.0594	11.5179	11.1385	8.3347	5.6707	6.7102	6.4682
8.5697	5.4499	6.6220	7.8788	7.6240	6.9652		

RMSE:

8.0435	5.0629	11.5196	11.1424	8.3351	5.6769	6.7104	6.4682
8.5700	5.4549	6.6228	7.8789	7.6245	6.9732		

RMSE.total:

106.0831

conf.level(95%):

K1	K2	K3
91.1	86.9	89.5

width:

K1	K2	K3
0.177	0.093	0.103

conf.level(90%):

K1	K2	K3
86.1	81.8	83.9

width:

K1	K2	K3
0.123	0.078	0.084

empirical level of CI (Chisquare)

bi² : 0.071 0.071 0.071 0.071 0.071 0.071 0.071 0.071 0.071
0.071 0.071 0.071 0.071 0.071

df : 1 2 3 4 5 6 7 1 2 3 4 5 6 7

repetitions: 1000

df:

df_mean:

4.632

df_std:

1.675

df_quantiles:

	0%	25%	50%	75%	100%
	1.256	3.363	4.602	5.915	11.366

bias (variance estimators):

0.1096	-0.2397	-0.3474	0.7535	0.3557	0.2619	-0.8744	0.2431
0.0070	0.2901	-0.4068	0.5526	0.4600	-0.7067		

std (variance estimators):

8.3947	9.9169	11.7673	19.9040	19.2651	23.9713	24.3522	9.5834
10.5655	16.9995	18.5879	26.9952	26.6536	26.2663		

RMSE:

8.3954	9.9198	11.7724	19.9182	19.2684	23.9727	24.3679	9.5865
10.5655	17.0020	18.5924	27.0008	26.6575	26.2758		

RMSE.total:

253.2953

conf.level(95%):

K1	K2	K3
97.5	93	94.7

width:

K1	K2	K3
0.302	0.196	0.217

conf.level(90%):

K1	K2	K3
93.6	86.8	90.1

width:

K1	K2	K3
0.225	0.165	0.178

empirical level of CI (Chisquare)

bi² : 0.49 0.039 0.039 0.039 0.039 0.039 0.039 0.039 0.039 0.039
0.039 0.039 0.039 0.039 0.039

df : 1 1 1 1 1 1 1 1 1 1 1 1 1 1

repetitions: 1000

df:

df_mean:

1.067

df_std:

0.401

df_quantiles:

	0%	25%	50%	75%	100%
1	1.001	1.003	1.007	7.839	

bias (variance estimators):

-0.1052	0.5221	0.0120	-0.2031	0.0908	0.0649	-0.0410	-0.0910
-0.1046	0.0694	0.2752	0.4972	0.1763	0.7091		

std (variance estimators):

6.0611	12.8983	5.6311	5.5860	6.7873	6.9731	7.4593	6.7069
6.2153	5.9869	7.1581	8.0291	7.9955	13.3629		

RMSE:

6.0620	12.9088	5.6311	5.5897	6.7879	6.9734	7.4594	6.7075
6.2162	5.9873	7.1634	8.0444	7.9975	13.3817		

RMSE.total:

106.9104

conf.level(95%):

	K1	K2	K3
99.9	99.9	99.1	99.2

```
width:
  K1      K2      K3
5.048  0.785  0.866
```

```
conf.level(90%):
  K1      K2      K3
99.7    98.4  98.8
```

```
width:
  K1      K2      K3
2.514  0.659  0.71
```

empirical level of CI (Chisquare)

```
bi^2 :  0.49 0.039 0.039 0.039 0.039 0.039 0.039 0.039 0.039 0.039
        0.039 0.039 0.039 0.039 0.039
df :    1 2 3 4 5 6 7 1 2 3 4 5 6 7
repetitions: 1000
```

```
df:
df_mean:
1.143
```

```
df_std:
0.564
```

df_quantiles:

	0%	25%	50%	75%	100%
1	1.006	1.011	1.029	7.869	

bias (variance estimators):

-0.3721	0.0963	0.5852	-0.0775	1.1802	-0.5679	-1.1394	-0.0250
0.6170	0.1463	-0.3930	-0.2446	1.0341	0.2649		

std (variance estimators):

5.0853	10.3437	13.7329	15.2421	22.5533	21.3652	22.7735	6.1834
11.9011	14.0667	16.4910	16.8193	28.4762	23.4579		

RMSE:

5.0989	10.3442	13.7453	15.2423	22.5841	21.3728	22.8019	6.1835
11.9171	14.0675	16.4957	16.8211	28.4949	23.4594		

RMSE.total:

228.6286

conf.level(95%):

	K1	K2	K3
99.8	98.9	99.3	

width:

	K1	K2	K3
4.823	0.767	0.845	

conf.level(90%):

	K1	K2	K3
99.4		98	98.7

width:

	K1	K2	K3
2.416	0.644	0.693	

empirical level of CI (Chisquare)

bi^2 :	0.49	0.039	0.039	0.039	0.039	0.039	0.039	0.039	0.039	0.039				
		0.039	0.039	0.039	0.039	0.039								
df :	7	6	5	4	3	2	1	7	6	5	4	3	2	1
repetitions:	1000													

df:

df_mean:

1.047

df_std:

0.312

df_quantiles:

	0%	25%	50%	75%	100%
1	1	1	1.001	1.004	6.369

bias (variance estimators):

1.5573	2.4006	0.9161	0.9757	0.3818	0.8075	0.9463	-0.2010
0.4716	0.8197	0.1363	0.9830	1.1622	0.5475		

std (variance estimators):

29.1755	29.7733	21.3582	18.7988	12.6091	10.0351	6.4408	25.6372
21.5856	20.5570	15.4451	13.7668	11.4395	5.9937		

RMSE:

29.2170	29.8699	21.3779	18.8241	12.6149	10.0675	6.5100	25.6380
21.5908	20.5733	15.4457	13.8018	11.4984	6.0187		

RMSE.total:

243.0479

conf.level(95%):

K1	K2	K3
99.9	99.5	99.7

width:

K1	K2	K3
16.319	2.527	2.786

conf.level(90%):

K1	K2	K3
99.8	99.2	99.2

width:

	K1	K2	K3
	8.118	2.121	2.284

empirical level of CI (Chisquare)

bi ² :	0.082	0.082	0.082	0.082	0.082	0.082	0.082	0.064	0.064	0.064
	0.064	0.064	0.064	0.064	0.064					
df :	1	1	1	1	1	1	1	1	1	1
repetitions:	1000									

df:

df_mean:

4.587

df_std:

2.422

df_quantiles:

	0%	25%	50%	75%	100%
	1.088	2.539	4.076	6.543	11.72

bias (variance estimators):

-0.2142	-0.0946	-0.0837	0.1369	-0.2922	-0.0499	-0.2427	0.0269
-0.3982	0.1428	-0.0146	-0.3882	-0.1507	0.0678		

std (variance estimators):

5.1084	5.9582	6.3212	6.6632	5.2090	5.7063	7.2672	10.2402
4.2995	7.3587	6.6523	8.7553	6.6601	6.5954		

RMSE:

5.1129	5.9590	6.3218	6.6646	5.2172	5.7066	7.2713	10.2402
4.3179	7.3601	6.6523	8.7639	6.6618	6.5957		

RMSE.total:

92.8454

conf.level(95%):

K1	K2	K3
91.1	86.7	88.6

width:

K1	K2	K3
0.182	0.096	0.106

conf.level(90%):

K1	K2	K3
85.6	81.6	84.1

width:

K1	K2	K3
0.126	0.08	0.087

empirical level of CI (Chisquare)

bi² : 0.082 0.082 0.082 0.082 0.082 0.082 0.064 0.064 0.064
0.064 0.064 0.064 0.064 0.064

df : 1 2 3 4 5 6 7 1 2 3 4 5 6 7

repetitions: 1000

df:

df_mean:

4.515

df_std:

1.666

df_quantiles:

	0%	25%	50%	75%	100%
	1.308	3.25	4.513	5.785	8.958

bias (variance estimators):

0.5394	-0.1500	-0.1449	-0.5557	-0.2473	0.8610	-0.6024	0.3752
-0.4230	0.6961	0.5339	0.3338	-0.6406	1.3720		

std (variance estimators):

10.3663	9.8152	13.2064	15.0877	18.9813	28.3542	22.8702	9.1346
7.7884	16.2544	21.3924	20.6429	21.1151	35.6943		

RMSE:

10.3803	9.8164	13.2072	15.0980	18.9829	28.3672	22.8782	9.1423
7.7999	16.2693	21.3991	20.6456	21.1248	35.7206		

RMSE.total:

250.8317

conf.level(95%):

K1	K2	K3
97.9	92.9	95.8

width:

K1	K2	K3
0.315	0.2	0.221

conf.level(90%):

K1	K2	K3
94.4	86.3	88.9

width:

K1	K2	K3
0.233	0.168	0.181

empirical level of CI (Chisquare)

bi² : 0.082 0.082 0.082 0.082 0.082 0.082 0.064 0.064 0.064

0.064 0.064 0.064 0.064 0.064

df : 7 6 5 4 3 2 1 7 6 5 4 3 2 1

repetitions: 1000

df:

df_mean:

4.336

df_std:

1.627

df_quantiles:

	0%	25%	50%	75%	100%
	1.217	3.079	4.327	5.503	9.646

bias (variance estimators):

0.8380	0.4453	0.6705	-0.1161	0.7208	-0.0810	-0.2193	-0.5960
0.5554	-1.0513	1.3427	0.5341	-0.2364	0.1986		

std (variance estimators):

28.5691	25.2573	23.1433	17.6375	18.3094	12.3501	5.1858	23.4702
23.9417	15.7911	26.2190	18.4062	10.8732	10.2991		

RMSE:

28.5813	25.2612	23.1531	17.6379	18.3236	12.3504	5.1905	23.4777
23.9481	15.8261	26.2534	18.4140	10.8758	10.3011		

RMSE.total:

259.5941

conf.level(95%):

	K1	K2	K3
	97.8	91.9	94.3

```
width:
  K1      K2      K3
0.346  0.213  0.234
```

```
conf.level(90%):
  K1      K2      K3
92.6   85.9  88.9
```

```
width:
  K1      K2      K3
0.253  0.179  0.192
```