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The ignored performance measure

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The Ignored Performance Measure

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Abstract

This paper studies a setting in which a risk averse agent must be motivated to work on two tasks: (1) evaluates a new project and, if adopted, (2) manages it. While a performance measure which is informative of an agent’s action is typically valuable because it can be used to improve the risk sharing of the contract, this is not necessarily the case in this two-task setting. I provide a sufficient condition under which a performance measure that is informative of the second task is worthless for contracting despite the agent being risk averse. This shows that information content is a necessary but not a sufficient condition for a performance measure to be valuable.
1 Introduction

The controllability principle is one of the most discussed principles in the accounting literature. The conditional controllability principle is based on Holmström’s (1979) informativeness principle and states that managers should be evaluated on the basis of all information that provides insights into their actions (Antle and Demski 1988). The puzzle is, however, that firms often seem to disregard this principle. In his field study, Merchant (1987, 1989) explores performance measures in firms. He shows that firms typically use a strict subset of the available informative performance measures for evaluation purposes. While this is not consistent with the above literature which has focused on single-task settings, this paper develops a multi-task agency model that may rationalize this phenomenon.

I model a firm in which a risk averse agent must be motivated to pursue two tasks: (1) to evaluate a new project (evaluation task) and, if the project is adopted, (2) to manage it diligently in order to increase the expected outcome of the project (managerial task). When the agent works on the evaluation task, he obtains more accurate information regarding the quality of the project. The agent is supposed to adopt the project if the obtained information is favorable and to reject the project otherwise. The agent’s pay can be based on the outcome of the project and on an additional performance measure which is informative of the managerial task.

If the agent does not have to evaluate the project, the setting becomes a standard agency problem. In this case, the performance measure is valuable because it can be used to improve the risk sharing of the contract (Holmström 1979). However, if the agent must be motivated to work on both the evaluation and the managerial task, this result no longer holds. I provide a sufficient condition under which the performance measure is completely useless for contracting despite the agent being risk averse. The intuition for this result is as follows. In order to provide incentives for the evaluation task, the contract must impose risk on the agent (Lambert 1986). Given this risk, there are situations in which the contract does not have to reinforce the managerial
task. In other words, in this two-task setting, the incentive constraint of the managerial task might be slack. In this case, the performance measure cannot be used to improve the risk sharing of the contract since it is informative of the managerial task only: Using the performance measure to reduce the risk would eliminate incentives for the evaluation task. The performance measure is more likely to be useless if the first control problem (the evaluation task) is relatively severe compared to the second control problem (the managerial task). This result is intuitive since the problem of motivating the evaluation task is then more likely to dominate the problem of motivating the managerial task.

There is a large literature on the value of additional performance measures. Holmström (1979) shows that a performance measure is valuable for contracting purposes if it is informative, i.e., if it provides information not included in the existing measures (informativeness criterion). Feltham and Xie (1994) expand this result for multiple tasks. Starting with Holmström and Milgrom (1991), researchers discuss settings in which the informativeness criterion does not apply. Holmström and Milgrom show that using information to encourage a particular task may have an adverse impact on the performance of other tasks. Hence, it may be optimal to ignore this information in the incentive contract. Cremer (1995) and Yim (2001) allow for renegotiable contracts. In Cremer’s work, the principal may abstain from additional monitoring information because more information makes it more difficult to commit to threats and, therefore, makes it more costly to provide accurate incentives. Closest to my paper is Arya et al. (1998). They assume that the agent has to pursue two tasks and that two performance measures are available for contracting. One of the two measures is sensitive to both actions. Due to this spillover effect, it may be optimal to base the compensation only on this measure to induce both actions and to ignore the other. While Arya et al. assume risk neutrality, I consider managerial risk aversion. This allows me to characterize situations where additional information cannot be used to improve the risk sharing of the contract.
The remainder of this paper is organized as follows. Section 2 outlines the model and provides a formal statement of the problem under consideration. In Section 3, I analyze a benchmark situation in which the agent is only involved in the managerial task. In this case, the informative performance measure is always valuable. In Section 4, I present the main result that the performance measure may be worthless if the agent is involved in both the evaluation and the managerial task. Section 5 considers other modeling assumptions and shows that the main result remains to hold. Section 6 concludes.

2 Model

There are two parties: a risk neutral principal and a risk averse agent. The principal owns a project. Let $y \in \{0, 1\}$ be an indicator variable that denotes whether the project is undertaken or not. If $y = 0$, the project is rejected and the game ends. If $y = 1$, the project is undertaken and the attendant capital outlay is $I^o$. The project’s gross profits, $x \in (0, \infty]$, depend on the project quality, $\theta \in \{0, 1\}$, and the managerial effort level, $m \in (0, \infty]$, delivered by the agent. For simplicity, I assume that the production function has the form

$$x = \theta X(m), \quad (1)$$

where $X'(m) > 0$, $X''(m) \leq 0$, $X'(0) > 0$, $X(0) = 0$.$^1$ The agent’s choice of $m$ is unobservable to the principal and the disutility of exerting effort $m$ is simply $m$. The quality of the project is either bad, $\theta = 0$, or good, $\theta = 1$. The ex ante probability of the good and the bad quality is commonly known to be 0.5. Let $x_T \equiv X(m_T)$ be the target output requested by the principal if the investment is undertaken. That is, the principal requires the agent to deliver managerial input $m = m_T$. The target output

$^1$To ensure strictly positive managerial effort choices in equilibrium, I assume that $x'(0)$ is sufficiently large.


The agent is able to acquire better information about the quality of the project prior to investment. Let the agent’s evaluation effort, \( e \), be either 0 or 1. When the agent evaluates the project, \( e = 1 \), he receives a signal \( s \in \{ s_G, s_B \} \) that is informative of the quality of the project. The good (bad) signal indicates that the quality of the project is more likely to be good (bad). More precisely,

\[
\Pr[s = s_G|\theta = 1] = \Pr[s = s_B|\theta = 0] = 0.5 + i,
\]

where \( i \in (0, 0.5] \) is exogenous. If the posterior signal is \( s_G \) (\( s_B \)), then the project quality is good (bad) with probability \((0.5 + i)\) and bad (good) with probability \((0.5 - i)\). If \( i = 0.5 \), the signal \( s \) is a perfect proxy for the project quality. When \( i \) decreases the signal becomes more noisy but remains informative. Unless otherwise specified, I assume that the signal is not perfect, i.e., \( i < 0.5 \). The evaluation task is associated with effort cost \( k > 0 \). The principal knows the magnitude of \( k \) but cannot observe whether the agent has incurred the cost or not.

---

\(^2\)If the outcome lies in between these two values, the principal concludes that the agent has cheated and punishes him severely.
Moreover, I make the following assumptions: The agent observes the signal $s$ privately and has the authority to make the investment decision.³ I shall say that the agent implements the project if he chooses $y = 1$ and $m = m_T$. The principal wants the agent to implement the project, if $s = s_G$ and to reject the project, if $s = s_B$. The investment outlay $I^o$ is assumed to be sufficiently high so that the principal always finds it profitable to provide incentives for the evaluation task.

The risk neutral principal maximizes the expected value of $(x - I^o)y$ less the compensation payment $w$. The agent’s utility depends on his salary $w$ and the actions taken. His utility function is additively separable of the form $u(w) - ke - m$, where $u'(w) > 0$, $u''(w) < 0$, $u(0) = 0$. The outcome $x$ and the investment decision $y$ are verifiable, i.e., compensation payments can be based on $x$ and $y$. Given the structure of the problem, it is sufficient to consider three different wage payments. Let $w_H$ be the compensation payment if the agent provides the target output, $x \geq x_T$, and $w_L$ be the wage payment if he does not, $x < x_T$. If the agent rejects the project, he receives a so-called rejection wage, $w_0$. Clearly, given this wage contract, the agent either chooses $m = m_T$ or not to work at all, $m = 0$.

The objective of this paper is to discuss the value of a performance measure $M$ which is informative of $m$.⁴ For simplicity, I assume that the measure $M$ is a perfect proxy for $m$, i.e., $M = m$. Given the information content of $x$, the measure $M$ provides additional insights into the agent’s action $m$ since if the project quality is bad, $x$ tells us nothing about $m$. If the measure $M$ is available, the principal can base wage payments not only on $x$ and $y$ but on $M$. The measure $M$ allows the principal to enforce the desired managerial effort level simply by penalizing the agent if he chooses any effort level other than the desired one. Unless otherwise specified, I assume that

³Equivalently, the authority to make the investment decision may remain with headquarters. In this case, the manager is asked to report the signal $s$ and headquarters accepts or rejects the project according to a prespecified rule. See Melumad and Reichelstein (1987).

⁴Informative in the sense that $M$ adds information about $m$ that is not already conveyed by $x$. 

the performance measure $M$ is not available for contracting.

The interaction between the principal and the agent proceeds as follows. The principal offers the contract $(w_H, w_L, w_0, x_T)$. The agent chooses whether to participate in the relationship or not. He will do so if his expected utility exceeds his reservation utility which is normalized to zero. If the agent participates, he decides whether to evaluate the project and chooses to undertake $(y = 1)$ or to reject $(y = 0)$ it. If $y = 1$, the agent chooses his effort supply $m$. The investment decision, $y$, and the realized outcome, $x$, are observed publicly and the agent is reimbursed. The interaction between the principal and the agent is not repeated.

The following notation is helpful. Let $u_j$ denote the agent’s utility associated with a wage payment $w_j$, where $j \in \{H, L, 0\}$. Let $\Phi$ denote the inverse of the agent’s utility function $u$. Throughout the paper $\Phi'''$ is assumed to be nonnegative. This is equivalent to assuming that the risk tolerance $\rho \equiv -\frac{u'(w)}{\Phi''(w)}$ does not increase too fast with wealth, i.e., wealth effects are sufficiently small.\footnote{More precisely, one can show that $\Phi''' \geq 0$ is fulfilled if $\rho' \leq 2$.} This assumption is not required for the main results, but allows some comparative statics results to be presented in a simpler form. Let

$$
\Pi \equiv 0.5[0.5 + i)u_H + (0.5 - i)u_L - m_T] + 0.5u_0 - k
$$

denote the agent’s ex ante (expected) utility if he chooses to evaluate the project and to implement it $(y = 1$ and $m = m_T)$ if $s = s_G$ and to reject it $(y = 0$ and $m = 0)$ if $s = s_B$. Using this notation, the principal’s problem $(P)$ is the following
\[
\max_{m_T, u_H, u_L, u_0} \ 0.5[(0.5 + i)(X(m_T) - \Phi(u_H)) + (0.5 - i)(0 - \Phi(u_L)) - I^0] - 0.5\Phi(u_0) \quad (T)
\]

subject to

\[
\begin{align*}
\Pi & \geq 0.5u_H + 0.5u_L - m_T, \quad (IC_{e1}) \\
\Pi & \geq u_0, \quad (IC_{e2}) \\
u_0 & \geq u_L, \quad (IC_m) \\
\Pi & \geq 0. \quad (IR)
\end{align*}
\]

The objective function (T) reflects the principal’s desire to maximize firm value. The first two constraints ensure that the agent (weakly) prefers to evaluate the project rather than to implement it (IC_{e1}) or to reject it (IC_{e2}) without acquiring additional information. Constraint (IC_m) ensures that the agent (weakly) prefers to reject the project rather than to cause a project failure by choosing \( y = 1 \) and \( m = 0 \). Finally, the individual rationality constraint (IR) ensures that the agent receives his reservation utility of zero in expectation.

Note that there are some additional incentive constraints I have omitted. These constraints are redundant since they are implied by (IC_{e1}), (IC_{e2}), (IC_m) and (IR). Next, these constraints are briefly mentioned. Constraint \( \Pi_G \equiv (0.5 + i)u_H + (0.5 - i)u_L - m_T \geq u_0 \) ensures that the agent prefers to implement the project (\( y = 1 \) and \( m = m_T \)) after having obtained the good signal rather than to reject the project. This constraint is implied by (IC_{e2}). Constraint \( u_0 \geq \Pi_B \equiv (0.5 - i)u_H + (0.5 + i)u_L - m_T \) ensures that the agent prefers to reject the project after having obtained the bad signal rather than to implement it. This constraint is implied by (IC_{e1}). Constraint \( \Pi \geq u_L \) ensures that the agent will not cause a project failure by choosing \( y = 1 \) and \( m = 0 \). This is implied by (IC_{e2}) and (IC_m). Constraint \( \Pi_G \geq u_L \) guarantees that the agent prefers to implement the project after having obtained the good signal rather than to choose \( y = 1 \) and \( m = 0 \). This constraint is implied by (IC_{e2}) and (IC_m).
3 Motivating Managerial Effort

As a benchmark solution, consider a setting in which the agent is not supposed to evaluate the project. In this case, the sole motivational problem is with respect to the managerial task. Moreover, assume it is commonly known that \( s = s_G \), i.e., the project quality is good with probability \((0.5 + i)\) and bad with probability \((0.5 - i)\).

The principal’s problem \((PB)\) is the following

\[
\max_{m_T,u_H,u_L} \quad (0.5 + i)(X(m_T) - \Phi(u_H) + (0.5 - i)(0 - \Phi(u_L))) - I^o,
\]

subject to

\[
\Pi^{PB} \equiv (0.5 + i)u_H + (0.5 - i)u_L - m_T \geq u_L, \quad \text{(IC}^{PB}\text{)}
\]

\[
\Pi^{PB} \geq 0. \quad \text{(IR}^{PB}\text{)}
\]

The incentive constraint \((IC^{PB})\) ensures that the agent chooses \( m = m_T \) instead of \( m = 0 \). The individual rationality constraint \((IR^{PB})\) ensures that the agent receives his reservation utility of zero. The optimal solution to \((PB)\) satisfies

\[
\begin{align*}
  u_H &= \frac{m_T}{0.5 + i}, \quad u_L = 0 \quad \text{and} \\
  (0.5 + i)X'(m_T) &= \Phi'(u_H).
\end{align*}
\]

(The proof is in Appendix A.) The outcome is not informative of the agent’s action if the project quality is bad. Similar to standard agency models, the contract must impose some risk on the agent, \( u_H > u_L \), in order to motivate the managerial task. Since the agent is risk averse, he requires a risk premium that must be reimbursed by the principal. In order to limit this cost, incentives are muted and the second-best effort level is below the first-best level. Moreover, the optimal effort level is increasing in the precision of the signal \( s \). There are two reasons for this result. First, when the precision \( i \) increases, uncertainty decreases because it becomes less likely that the project fails although the agent has worked diligently. Hence, the required risk premium decreases and it becomes less costly to provide strong incentives. Second,
when $i$ increases, it gets less likely that the managerial effort is unproductive.\(^6\) Hence, the principal finds it advantageous to motivate a higher effort level. (See Appendix A for the proofs and for a more detailed discussion of these findings.)

The cost of motivating the managerial task can be avoided if the performance measure $M$ is available for contracting. In this case, the desired managerial input can be enforced without imposing risk on the agent and the first-best solution is achieved. Consequently, and consistent with standard agency problems, the informative performance measure $M$ is valuable to the firm because it improves the risk sharing of the contract.

**Proposition 1** *When the agent is involved in the managerial task only, the performance measure $M$ is valuable for contracting.*

### 4 Main Results

While the performance measure $M$ is always valuable if the agent is involved in the managerial task only, this is not the case if the agent is involved in both the managerial and the evaluation task. To determine the value of $M$, the easiest way to proceed is to compare the optimal contract under the assumption that $M$ is available (this relaxed problem is called $(P^r)$) with the optimal contract under the assumption that $M$ is not available (problem $(P)$). If the optimal solution to $(P)$ equals the optimal solution to $(P^r)$, the performance measure $M$ is not valuable to the firm.

If the measure $M$ is available for contracting, the sole motivational problem is with respect to the evaluation task. The optimization problem $(P^r)$ is the following: Maximize $(T)$ subject to $(IC_{e1})$, $(IC_{e2})$ and $(IR)$. The optimal solution to $(P^r)$ is given in the next lemma.

---

\(^6\)Recall that the managerial effort is unproductive when the project quality is bad.
Lemma 1 The optimal solution to $(P^r)$ satisfies

$$u_H = m_T + \frac{k}{i}, \ u_L = m_T - \frac{k}{i}, \ u_0 = 0$$ \hspace{1cm} (4)

$$(0.5 + i)X'(m_T) = (0.5 + i)\Phi'(u_H) + (0.5 - i)\Phi'(u_L).$$ \hspace{1cm} (5)

Proof: See Appendix B.

In order to provide incentives for the evaluation task, the principal must impose risk on the agent, i.e., $u_H > u_L$. From constraints $(IC_{e1})$ and $(IC_{e2})$ it follows that $u_H - m_T > u_0 > u_L - m_T$ must hold. If, for example, $u_H - m_T \geq u_L - m_T > u_0$, the agent always implements the project or, if $u_0 > u_H - m_T \geq u_L - m_T$, always rejects the project without making an effort to acquire better information. Note that $\frac{k}{i}$ can be used as a measure indicating the difficulty of providing incentives for the evaluation task. When the agent’s cost of gathering information, $k$, increases, the control problem gets more severe. When, on the other hand, $i$ increases, the incentive problem gets less severe since the signal the agents is supposed to acquire gets less noisy.

Consider now problem $(P)$ in which the measure $M$ is not available for contracting.

Lemma 2 The solution to $(P)$ is such that either

(i) all constraints are binding except for $(IC_{e1})$ or
(ii) all constraints are binding except for $(IC_m)$ or
(iii) all constraints are binding.

Proof: See Appendix B.

When the optimal solution to $(P)$ satisfies property (ii), the solution to $(P)$ equals the solution to the relaxed problem $(P^r)$. Hence, Lemma 2 states that there are situations in which the performance measure $M$ is not valuable for contracting. It remains to discuss when this is the case. Recall that constraint $(IC_m)$ is given by $u_0 \geq u_L$. This constraint becomes $m_T \leq \frac{k}{i}$ if wage payments are as in (4). Note that whenever this constraint is satisfied, cheating ($m = 0$) is not rewarding for the agent.
since $u_L \leq 0$. Constraint $(IC_m)$ is slack, if $m_T$ (determined by (4) and (5)) does not exceed the critical level $\hat{m} \equiv \frac{k}{i}$. In this case, the measure $M$ cannot be used to reduce the noise in the contract since this would eliminate incentives for the evaluation task. In other words, the performance measure $M$ does not improve the risk sharing of the contract and is therefore useless for contracting. The solution to $(P)$ satisfies property $(ii)$ if

$$Z(k, i) \leq 0,$$  \hspace{1cm} (6)

with $Z(k, i) \equiv (0.5 + i)X'(\hat{m}) - (0.5 + i)\Phi'(2\hat{m}) - (0.5 - i)\Phi'(0)$.

Condition (6) is therefore a sufficient condition under which $M$ is worthless for contracting.\footnote{Note that (6) implies that the effort level determined by (4) and (5) does not exceed the critical level $\hat{m}$.} To sum up: If (6) holds, the sole motivational problem is with respect to the evaluation task. Since $M$ is not informative of this task, it is useless for the control problem.

\textbf{Proposition 2} \textit{If (6) holds, the performance measure $M$ is worthless for contracting even though it is perfectly informative of the managerial task and the agent is risk averse.}

Proof: See Appendix B.

\textbf{Corollary 1}

$$\frac{\partial Z}{\partial k} < 0,$$

$$\frac{\partial Z}{\partial i} > 0,$$

$$\frac{\partial Z}{\partial v} > 0,$$

for the case of $X(m) = v\tilde{X}(m)$, where $\tilde{X}$ is increasing and concave.

Condition (6) is more likely to hold if the cost of evaluating the project, $k$, is high and the precision of the signal, $i$, and the productivity parameter, $v$, are low. When
$k$ becomes high and $i$ becomes low, it is difficult to motivate the evaluation task. A low productivity $v$ results in a low desired managerial effort supply. Hence, when $k$ is high and $i$ and $v$ are low, the problem of motivating the evaluation task is severe compared to the problem of motivating the managerial task so that the first incentive problem is likely to dominate the second incentive problem.

If, on the other hand, condition (6) is not satisfied, constraint $(IC_m)$ is binding in the optimal solution to $(P)$. The motivational problem is then with respect to both the evaluation and the managerial task. Since the measure $M$ is informative of the managerial task, it is useful for contracting. Similar to the single-task problem of Section 3, the performance measure is valuable then because it improves the risk sharing of the contract.

5 Robustness Considerations

In this section, I show that the main result that the performance measure $M$ may be useless for contracting is robust to other modeling assumptions.

**Multiple Evaluation Effort Choices:** So far, I have assumed that the evaluation effort choice is binary. However, the model can be extended by allowing for multiple evaluation effort choices. Consider a setting in which the agent receives the signal $s$ with probability $p(e) \in [0, 1]$, where $e$ is again the effort the agent devotes to the evaluation task. In other words, the agent chooses the probability of obtaining the signal $s$ by choosing the evaluation effort level $e$. For simplicity, let $p(e) = e$, with $e \in [0, 1]$. The agent’s private cost of exerting effort $e$ is $0.5ke^2$. Moreover, I assume that if the agent does not obtain signal $s$, the principal wants the agent to reject the project.

The finding that $M$ may be useless for contracting remains with this extension. However, the critical effort level changes and is now given by $\hat{m}_2 \equiv \frac{k}{i}e + 0.5ke^2$, where $e$ is the agent’s optimal response to the contract offered by the principal. In Appendix
C, I provide a formal statement of the problem under consideration and show that
\[(0.5 + i)X'(\hat{m}_2) \leq (0.5 + i)\Phi' \left(2\frac{k}{i}e\right) + (0.5 - i)\Phi'(0) \tag{7}\]
is a sufficient condition under which the measure \(M\) is worthless. The measure \(M\) is more likely to be worthless if, for example, the investment outlay \(I^o\) becomes small. In this case, the optimal evaluation effort level \(e\) is high because the project becomes more valuable to the principal. As a result, the problem of motivating the evaluation task tends to dominate the problem of motivating the managerial task.

**Lucky Agent:** In the basic model, the principal is able to infer \(m = m_T\) from \(x = x_T\). The goal of this section is to show that the qualitative results discussed so far are not an artifact of this simplifying assumption. To see this, assume that there is a positive probability, say \(q\), for which the outcome equals (or exceeds) the target outcome, i.e., \(x \geq x_T\), if the agent cheats on the managerial task and chooses \(m = 0\). Hence, in this case, if \(x \geq x_T\), the principal cannot tell for sure whether the agent has chosen \(m = m_T\) or whether he has cheated and simply was lucky.

Given this additional assumption, constraint \((IC_m)\) of problem \((P)\) becomes
\[u_0 \geq q(u_H - u_L) + u_L. \tag{IC\_new}\]
This constraint implies that the critical level is now \(\hat{m}_3 \equiv \frac{k}{t}(1 - 2q)\). The sufficient condition under which \(M\) is worthless for contracting becomes
\[(0.5 + i)X'(\hat{m}_3) \leq (0.5 + i)\Phi' \left(2\frac{k}{i}(1 - q)\right) + (0.5 - i)\Phi' \left(-2q\frac{k}{i}\right). \tag{8}\]
(See Appendix C for the proofs.) Comparing (8) with (6) reveals that the performance measure \(M\) is now less likely to be worthless. This result is intuitive since in the current setting the performance measure has "more" information content than in the setting discussed in Section 4.
**Wealth Constrained Agent:** In many recent agency models, the agent is assumed to have no private wealth (e.g., Lewis and Sappington 2000). An interesting question is therefore whether wealth constraints on part of the agent affect the main result of this paper. Interestingly, this is not the case and Proposition 2 remains to hold.

When the agent is wealth constrained, the problem under consideration differs from \((P)\) only in that it has additional nonnegativity constraints; \(w_j \geq 0\) for \(j \in \{H, L, 0\}\). I provide the optimal solution to this problem in Appendix C. The optimal solution is characterized by \(w_L, w_0 = 0\) and \(m_T \geq \hat{m}\). The reason why the target effort supply is no lower than \(\hat{m}\) is that it serves an additional purpose: It provides the agent with incentives to evaluate the project prior to implementation. These incentives arise from the agent’s wish to avoid wasting his managerial effort for a project of low quality.

### 6 Conclusion

This paper analyzes the optimal design of contracts in a setting where the principal wants to motivate a risk averse agent to evaluate a new project and, if adopted, to manage it. I find that, contrary to what conventional agency literature suggests, a performance measure that is informative of the managerial task may be completely useless for contracting despite the agent being risk averse. An informative performance measure is typically valuable to the firm because it can be used to improve the risk sharing of the contract. In this two-task model, however, this result no longer holds: Using the performance measure to reduce the risk imposed on the agent may eliminate incentives for project evaluation. Hence, this paper provides a setting where information content is a necessary but not a sufficient condition for a performance measure to be valuable.
Appendix A

First-best Solution

Consider problem \( (PB) \). If the performance measure \( M \) is available for contracting, then there is no incentive problem. In this case, a forcing contract is possible and the agent is asked to choose the desired action. The only constraint that must be considered is the participation constraint \( (IC^{PB}) \). The first-best managerial effort level is determined by

\[
(0.5 + i)X'(m_T^f) = \Phi'(m_T^f)
\]

and the compensation paid to the agent is \( \Phi(m_T^f) \).

Problem \( (PB) \)

Let \( \mu_1 \geq 0 \) and \( \mu_2 \geq 0 \) denote the Lagrangian multipliers associated with constraints \( (IC^{PB}) \) and \( (IR^{PB}) \). The Lagrangian of the problem \( (PB) \) is

\[
\max_{(PB)} L = (0.5 + i)(X(m_T) - \Phi(u_H) + (0.5 - i)(0 - \Phi(u_L)) - I^o
\]

\[
+ \mu_1((0.5 + i)u_H - (0.5 + i)u_L - m_T)
\]

\[
+ \mu_2((0.5 + i)u_H + (0.5 - i)u_L - m_T).
\]

The necessary conditions for a solution to \( (PB) \) include:

\[
\frac{\partial L}{\partial u_H} = -(0.5 + i)\Phi'(u_H) + \mu_1(0.5 + i) + \mu_2(0.5 + i) = 0, \quad (10)
\]

\[
\frac{\partial L}{\partial u_L} = -(0.5 - i)\Phi'(u_L) - \mu_1(0.5 + i) + \mu_2(0.5 - i) = 0, \quad (11)
\]

\[
\frac{\partial L}{\partial m_T} = (0.5 + i)X'(m_T) - \mu_1 - \mu_2 = 0. \quad (12)
\]

In the optimal solution to \( (PB) \) both \( (IR^{PB}) \) and \( (IC^{PB}) \) are binding, i.e., \( \mu_1 > 0 \) and \( \mu_2 > 0 \). The proof is by contradiction. Suppose that \( \mu_2 = 0 \). Then, (11) is violated since \( \Phi'(u_L) > 0 \) and \( \mu_1 \geq 0 \). Suppose that \( \mu_1 = 0 \). From (10) and (11) it follows that \( u_H = u_L \) which violates \( (IC^{PB}) \). Hence, in the optimal solution to \( (PB) \) the two constraints are binding. This implies

\[
u_H = \frac{m_T}{0.5 + i} \text{ and } u_L = 0.
\]
From (10) and (12) it follows that \((0.5 + i)X'(m_T) = \Phi'(u_H)\). Hence, the optimal solution is given by (2) and (3), i.e.,

\[
\begin{align*}
    u_H &= \frac{m_T}{0.5 + i}, \quad u_L = 0 \quad \text{and} \\
    (0.5 + i)X'(m_T) &= \Phi\left(\frac{m_T}{0.5 + i}\right).
\end{align*}
\]

The optimal solution can be nicely interpreted. First, note that the risk premium is defined as the difference between the expected wage payment and the security equivalent and is given by

\[
\pi = [(0.5 + i)\Phi(u_H) + (0.5 - i)\Phi(u_L)] - [\Phi((0.5 + i)u_H + (0.5 - i)u_L)].
\]

The security equivalent (the second term in square brackets) is the secure income for which the agent enjoys the same utility as if he would receive the risky wages \(w_H\) and \(w_L\). Inserting (2) in \(\pi\) and rearranging yields

\[
\pi = (0.5 + i)\Phi\left(\frac{m_T}{0.5 + i}\right) + (0.5 - i)\Phi(0) - \Phi(m_T).
\]

(13)

The risk premium is zero if the agent is risk neutral (i.e., if \(u''(w) = 0\)) and/or the signal \(s\) is perfect (\(i = 0.5\)). In such a situation, the first-best solution can be achieved. The risk premium is positive, if \(u''(w) < 0\) and \(i < 0.5\).

**Result 1:** Let \(m^*_T\) be the optimal (second-best) effort level. If the signal \(s\) is imperfect (\(i < 0.5\)), then \(m^*_T < m^*_T\).

The proof follows directly from (3) and (9). Note that the agent’s risk premium (13) is increasing in \(m_T\),

\[
\frac{\partial \pi}{\partial m_T} = \Phi'\left(\frac{m_T}{0.5 + i}\right) - \Phi'(m_T) > 0.
\]

(14)

The easiest way to see this is to consider the special case where \(u(w) = w\). In this case, \(w(u) = u\) and the risk premium simplifies to

\[
\pi = (0.5 + i)(\frac{m_T}{0.5 + i}) + (0.5 - i)(0) - (m_T),
\]

which is zero.
While the optimal wage payment $u_H$ is increasing in $m_T$, $u_L$ is independent of $m_T$ (see (2)). Hence, the difference between the two, $\Delta u \equiv u_H - u_L$, increases as $m_T$ increases. Intuitively, if $\Delta u$ increases, the agent faces a higher risk and therefore requires a higher risk premium.\(^9\) The principal trades off the cost of a high risk premium with the benefit of a high effort level. Since the agent requires a risk premium of zero in the first-best situation, the optimal effort level, $m^*_T$, is lower than the first-best effort level, $m^*_T$. In other words, the principal optimally mutes incentives in order to limit the agent’s risk premium.

**Result 2:** (a) $m^*_T$ is increasing in the precision of the signal $s$, $i$. (b) The difference between the first-best and second-best effort level is decreasing in $i$, i.e., $\frac{\partial (m^*_T - m^*_T)}{\partial i} < 0$.

The proof follows directly from (3) and (9). Note that the risk premium (13) is decreasing in the precision of the signal $s$, i.e.,

$$\frac{\partial \pi}{\partial i} = \Phi \left( \frac{m_T}{0.5 + i} \right) - \Phi' \left( \frac{m_T}{0.5 + i} \right) \frac{m_T}{0.5 + i} - \Phi(0) < 0.$$  

($\frac{\partial \pi}{\partial i}$ is negative since $\Phi'' > 0$.) As $i$ increases, it becomes less likely that the agent receives the low utility $u_L$ even though he chooses $m = m^*_T$. In other words, the risk declines.\(^10\) Hence, the risk premium required by the agent decreases with $i$. Since providing incentives becomes cheaper, the principal finds it profitable to induce a higher managerial effort level $m_T$. Note, however, that there is a second reason for Result 2: The probability that the effort has an impact on the outcome increases. Remember, with probability $(0.5 - i)$ the managerial effort has no influence on the outcome because the project quality is bad, i.e., $\theta = 0$. As it becomes more likely that the effort is productive, the principal wants to induce a higher effort level. The

\(^9\)Although the manager does not necessarily adapt himself to the variance, it is interesting to note that the variance, $\delta^2 = (\Delta u)^2 (0.5 - i)^2$, is increasing in $\Delta u$.

\(^10\)Note that the variance, $\delta^2 = (\Delta u)^2 (0.5 - i)^2$, decreases with $i$. 

first-best level, on the other hand, remains unchanged. Hence, Result 2b. If the signal is perfect \((i = 0.5)\), the first-best solution is achieved, \(m^*_T = m^f_T\).

Assume that there are two agents, called A and B. As already mentioned, the agent’s required risk premium is increasing in \(\Delta u\). If one agent, say agent B, is more risk averse than the other agent, A, it seems plausible to assume that the risk premium \(\pi_B\) of B increases more with an increasing \(\Delta u\) than the risk premium \(\pi_A\) of A does.

**Definition 1** If \(\frac{\partial \pi_B}{\partial \Delta u} > \frac{\partial \pi_A}{\partial \Delta u}\) for all \(\Delta u \geq 0\) and \(i < 0.5\), agent B is said to be more risk averse than agent A.

**Result 3:** Let \(x_{TA} \equiv X(m_{TA})\) and \(x_{TB} \equiv X(m_{TB})\) be the required output agent A and B is asked to provide, respectively. If agent B is more risk averse than agent A, then \((m^f_{TB} - m^*_T) > (m^f_{TA} - m^*_T)\) for all \(i \in (0, 0.5)\).

Result 3 asserts that the difference between the first-best effort level and the second-best effort level is higher for a more risk averse agent than for a less risk averse agent. Given Definition 1, the risk premium of B is higher than for A for all \(\Delta u > 0\). More importantly, from Definition 1 it follows that

\[
\frac{\partial \pi_2}{\partial m_T} > \frac{\partial \pi_1}{\partial m_T} \quad \text{for all } m_T,
\]

since \(\frac{d \Delta u}{d x_T} > 0\). Substituting the optimality condition (3) into (14) yields

\[
\frac{\partial \pi}{\partial m_T} = (0.5 + i)X'(m^*_T) - \Phi'(m^*_T) > 0.
\]

Remember that the first-best solution satisfies \((0.5 + i)X'(m^f_T) - \Phi'(m^f_T) = 0\). Hence, from (15) it follows that \((m^f_{TB} - m^*_T) > (m^f_{TA} - m^*_T)\). The greater the risk aversion of the agent the greater is the difference between the first-best and the second-best effort level. Intuitively, it is more costly to provide incentives that are close to first-best if the agent is more risk averse.
Appendix B

The optimal solution to Problem \((P)\) is as follows:

**Lemma 3** The optimal solution to \((P)\) is characterized by

(i)

\[
\begin{align*}
  u_H &= \frac{m_T + 2k}{0.5 + i}, \quad u_L = u_0 = 0, \\
  (0.5 + i)X'(m_T) &= \Phi'(u_H) \\
  \text{and } m_T &\geq \frac{k}{i}
\end{align*}
\]

(All constraints are binding except for \((IC_e1)\).)

or (ii)

\[
\begin{align*}
  u_H &= m_T + \frac{k}{i}, \quad u_L = m_T - \frac{k}{i}, \quad u_0 = 0, \\
  (0.5 + i)X'(m_T) &= (0.5 + i)\Phi'(u_H) + (0.5 - i)\Phi'(u_L) \\
  \text{and } m_T &\leq \frac{k}{i}
\end{align*}
\]

(All constraints are binding except for \((IC_m)\).)

or, if there does not exist a solution that satisfies (i) or (ii), (iii)

\[
\begin{align*}
  u_H &= 2\frac{k}{i}, \quad u_L = 0, \quad u_0 = 0, \\
  m_T &= \frac{k}{i}.
\end{align*}
\]

(All constraints are binding.)

Proof:

Let \(\lambda_1 \geq 0, \lambda_2 \geq 0, \lambda_3 \geq 0\) and \(\lambda_4 \geq 0\) denote the Lagrangian multipliers associated with constraints \((IC_{e1}), (IC_{e2}), (IC_m)\) and \((IR)\). The Lagrangian of the
Substituting (21) and (23) into (17) and rearranging yields
\[
\text{all Lagrangian multipliers are nonnegative. Suppose that binding and, consequently, then (22) implies that}
\]
\[
\text{In the optimal solution it must not be that } \lambda_1 = \lambda_3 = 0. \text{ Because if } \lambda_1 = \lambda_3 = 0, \text{ then (22) implies that } u_H = u_L \text{ which violates either } (IC_{e1}) \text{ or } (IC_{e2}).
\]
\[
\text{In the optimal solution } \lambda_2 > 0, \lambda_4 > 0, \text{ i.e., constraints } (IC_{e2}) \text{ and } (IR) \text{ are binding and, consequently, } u_0 = 0. \text{ The proof is by contradiction. Suppose that } \lambda_2 = \lambda_4 = 0. \text{ Then, (18) and (19) are violated since } \Phi'(u_H) > 0 \text{ and } \Phi'(u_L) > 0 \text{ and all Lagrangian multipliers are nonnegative. Suppose that } \lambda_2 > 0 \text{ and } \lambda_4 = 0. \text{ Substituting (21) and (23) into (17) and rearranging yields}
\]
\[
-0.5\Phi'(u_0) - (0.5 + i)0.5\Phi'(u_H) - (0.5 - i)0.5\Phi'(u_L) + 2\lambda_4 = 0.
\]
Hence, if $\lambda_4 = 0$, condition (17) is violated. Suppose that $\lambda_2 = 0$ and $\lambda_4 > 0$. Substituting (18) into (17) yields

$$0.5\Phi'(u_0) - 0.5\Phi'(u_H) = \lambda_1 \left(\frac{1}{0.5 + i}\right) - 2\lambda_2 + \lambda_3. \quad (24)$$

If $\lambda_2 = 0$, (24) implies $u_0 > u_H$. Note that $(IC_{e_1})$ and $(IC_{e_2})$ imply $u_H > u_L$. Hence, if $\lambda_2 = 0$, then $u_0 > u_H > u_L$ which violates constraint $(IC_{e_2})$. Therefore, it must be that $\lambda_2 > 0$ and $\lambda_4 > 0$ and $u_0 = 0$.

Suppose that $\lambda_1 = 0$. As already shown, in this case, it must be that $\lambda_3, \lambda_2, \lambda_4 > 0$. From $\lambda_3 > 0$ it follows that constraint $(IC_m)$ is binding, i.e., $u_0 = u_L = 0$. Hence, the participation constraint $(IR)$ implies

$$u_H = \frac{m_T + 2k}{0.5 + i} \text{ and } u_0 = u_L = 0. \quad (25)$$

From (19) and (21) and $\lambda_1 = 0$ it follows that

$$\lambda_3 = -(0.5 - i)0.5\Phi'(u_L) + (0.5 + i)(0.5 - i)0.5X'(m_T).$$

Inserting $\lambda_3$ in (23) yields

$$(0.5 + i)X'(m_T) = \Phi'(u_H). \quad (26)$$

If the solution given by (25) and (26) satisfies $m_T \geq \frac{k}{i}$, it is the optimal solution. In this case, constraint $(IC_{e_1})$ is indeed slack, i.e., $\lambda_1 = 0$.

Suppose that $\lambda_3 = 0$. Hence, $\lambda_1, \lambda_2, \lambda_4 > 0$. Since $(IC_{e_1})$, $(IC_{e_2})$ and $(IR)$ are binding, it follows that

$$u_H = m_T + \frac{k}{i}, \quad u_L = m_T - \frac{k}{i} \text{ and } u_0 = 0.$$

Since $\lambda_3 = 0$, (23) simplifies to

$$(0.5 + i)X'(m_T) = (0.5 + i)\Phi'(u_H) + (0.5 - i)\Phi'(u_L). \quad (28)$$

If the solution given by (27) and (28) satisfies $m_T \leq \frac{k}{i}$, it is the optimal solution. In this case, constraint $(IC_m)$ is indeed slack, i.e., $\lambda_3 = 0$. 22
If the solution given by (25) and (26) does not satisfy \( m_T \geq k \frac{t}{i} \) and the solution given by (27) and (28) does not satisfy \( m_T \leq k \frac{t}{i} \), it must be that \( \lambda_1, \lambda_2, \lambda_3, \lambda_4 > 0 \).

Hence, all constraints are binding and the optimal solution is

\[
\begin{align*}
  u_H &= m_T + \frac{k}{t}, \quad u_L = m_T - \frac{k}{t}, \quad u_0 = 0 \quad \text{and} \\
  m_T &= \frac{k}{t}.
\end{align*}
\]

**Proof of Lemma 1 and Lemma 2 and Proposition 2**

The proofs follow immediately from Lemma 3 and so are omitted.

**Appendix C**

**Multiple Evaluation Effort Choices**

Let

\[
\Pi_2 \equiv e0.5[(0.5 + i)u_H + (0.5 - i)u_L - m_T] + e0.5u_0 + (1 - e)u_0 - 0.5ke^2
\]

denote the agent’s ex ante utility. With this notation, the principal problem \((P2)\) is as follows

\[
\max_{m_T, u_H, u_L, u_0} e0.5[(0.5+i)(X(m_T)-\Phi(u_H))+(0.5-i)(-\Phi(u_L))-\Pi^o]-e0.5\Phi(u_0)-(1-e)\Phi(u_0)
\]

subject to

\[
(0.5 + i)u_H + (0.5 - i)u_L - m_T - u_0 - 2ke = 0, \quad (IC_P^{P2})
\]

\[
u_0 \geq 0.5u_H + 0.5u_L - m_T, \quad (IC_P^{P2})
\]

\[
u_0 \geq u_L, \quad (IC_P^{P2})
\]

\[
\Pi_2 \geq 0. \quad (IR_P^{P2})
\]

Constraint \((IC_P^{P2})\) is the agent’s first-order condition on evaluation effort \(e\). (I assume that the optimal effort \(e\) is in the interior of the action set.) Condition \((IC_P^{P2})\)
ensures that the agent will reject the project if he does not obtain the signal \( s \). Constraint \((IC^{P2}_m)\) ensures that the agent prefers to reject the project rather than to cause a project failure by choosing \( y = 1 \) and \( m = 0 \). Constraint \((IR^{P2})\) ensures that the agent receives his reservation utility of zero in expectation. Similar to problem \((P)\), there are several other constraints I have omitted since they are implied by \((IC_e^{P2}), (IC_r^{P2}), (IC_m^{P2})\) and \((IR^{P2})\).

Consider in the following the relaxed problem \((P2^r)\) in which constraint \((IC_m^{P2})\) is suppressed. If the solution to the relaxed problem \((P2^r)\) satisfies condition \((IC_m^{P2})\), the measure \( M \) is worthless for contracting.

Let \( \lambda_1 \geq 0, \lambda_2 \geq 0 \) and \( \lambda_3 \geq 0 \) denote the Lagrangian multipliers associated with constraints \((IC_e^{P2}), (IC_r^{P2})\) and \((IR^{P2})\). The Lagrangian of problem \((P2^r)\) is the following

\[
\max L = e^{0.5}[(0.5 + i)(X(m_T) - \Phi(u_H)) + (0.5 - i)(-\Phi(u_L)) - I^0] \\
-\Phi(u_0)(1 - 0.5e) \\
+\lambda_1(0.5(0.5 + i)u_H + (0.5 - i)u_L - m_T - u_0) - ke) \\
+\lambda_2(u_0 - 0.5u_H - 0.5u_L + m_T) \\
+\lambda_3(e^{0.5}(0.5 + i)u_H + (0.5 - i)u_L - m_T) + u_0(1 - 0.5e) - 0.5ke^2) .
\]
The necessary conditions for a solution to \((P2^r)\) include:

\[
\begin{align*}
\frac{\partial L}{\partial u_0} &= -\Phi'(u_0)(1 - 0.5e) - \lambda_10.5 + \lambda_2 + \lambda_3(1 - 0.5e) = 0, \\
\frac{\partial L}{\partial u_H} &= -e0.5(0.5 + i)\Phi'(u_H) + \lambda_10.5(0.5 + i) - \lambda_20.5 + \lambda_3e0.5(0.5 + i) = 0, \\
\frac{\partial L}{\partial u_L} &= -e0.5(0.5 - i)\Phi'(u_L) + \lambda_10.5(0.5 - i) - \lambda_20.5 + \lambda_3e0.5(0.5 - i) = 0, \\
\frac{\partial L}{\partial m_T} &= e0.5(0.5 + i)X'(m_T) - \lambda_10.5 + \lambda_2 - \lambda_3e0.5 = 0, \\
\frac{\partial L}{\partial e} &= 0.5((0.5 + i)(X(m_T) - \Phi(u_H)) + (0.5 - i)(-\Phi(u_L)) - I^c) \\
&\quad + 0.5\Phi(u_0) - \lambda_1k - \lambda_3((0.5 + i)u_H + (0.5 - i)u_L - m_T - u_0) - ke \\
&= 0.
\end{align*}
\]

In the optimal solution to \((P2^r)\), it must be that \(\lambda_2 > 0\). The proof is by contradiction. From (30) and (31) it follows that

\[
e(\Phi'(u_H) - \Phi'(u_L)) = \lambda_2 \frac{2i}{(0.5 - i)(0.5 + i)},
\]

If \(\lambda_2 = 0\), (34) implies that \(u_H = u_L\) for \(e > 0\). Substituting \(u_H = u_L\) into constraint \((IC_e^{P2})\) yields \(u_0 = u_H - m_T - 2ke\) which contradicts condition \((IC_e^{P2})\). Hence, \(\lambda_2 > 0\).

Since \((IC_e^{P2})\) binds and due to condition \((IC_e^{P2})\), it follows that

\[
u_H = m_T + u_0 + \frac{k}{i}e\quad\text{and}\quad u_L = m_T + u_0 - \frac{k}{i}e.
\]

Substituting (34) and (32) into (30) and rearranging yields

\[
(0.5 + i)X'(m_T) = (0.5 + i)\Phi'(u_H) + (0.5 - i)\Phi'(u_L).
\]

In the optimal solution it must be that \(\lambda_3 > 0\). The proof is by contradiction. Substituting (32) into (29) and rearranging yields

\[
\frac{\partial L}{\partial u_0} = -\Phi'(u_0)(1 - 0.5e) - 0.5e(0.5 + i)X'(m_T) + \lambda_3 = 0,
\]

which cannot be satisfies for \(\lambda_3 = 0\) since \(\Phi'(u_0) > 0\) and \(X'(m_T) > 0\). Hence, \(\lambda_3 > 0\).
From constraint \((IC_e^{P2})\) and from the participation constraint \((IR^{P2})\) (which is binding) it follows that \(u_0 = -0.5ke^2\).

If in the optimal solution to \((P2^r)\), \(u_L\) is nonpositive, then \((IC_m^{P2})\) is satisfied and the measure \(M\) is worthless for contracting. Hence, condition (7) is a sufficient condition under which \(M\) is useless.

**Lucky Manager**

The problem (called \((P3)\)) is now as follows: Maximize \((T)\) subject to \((IC_{e1}), (IC_{e2}), (IC_{new}^m)\) and \((IR)\). The optimal solution to \((P3)\) is given in the next lemma.

**Lemma 4** The optimal solution to \((P3)\) is characterized by (i)

\[
u_H = \frac{m_T + 2k}{(0.5 + i) - (0.5 - i)\frac{q}{1-q}}, \quad u_L = -\frac{q(m_T + 2k)}{0.5 + i - q}, \quad u_0 = 0,
\]

\(0.5 + i)\frac{0.5 + i - q}{1-q}X'(m_T) = (0.5 + i)\Phi'(u_H) - (0.5 - i)\frac{q}{1-q}\Phi'(u_L), \]

and \(m_T \geq \frac{k}{i}(1 - 2q),\)

or (ii)

\[
u_H = m_T + \frac{k}{i}, \quad u_L = m_T - \frac{k}{i}, \quad u_0 = 0,
\]

\(0.5 + i)X'(m_T) = (0.5 + i)\Phi'(u_H) + (0.5 - i)\Phi'(u_L), \]

and \(m_T \leq \frac{k}{i}(1 - 2q),\)

or, if there does not exist a solution that satisfies (i) or (ii), (iii)

\[
u_H = 2\frac{k}{i}(1 - q), \quad u_L = -2q\frac{k}{i}, \quad u_0 = 0,
\]

\[m_T = \frac{k}{i}(1 - 2q).\]

The proof is similar to the proof in Appendix B and so is omitted. Similar to problem \((P)\), if the solution to problem \((P3)\) satisfies property (ii), then constraint \((IC_{new}^m)\) is not binding and the measure \(M\) is worthless for contracting.
Wealth Constrained Agent

The problem in which the agent is wealth constrained (called \((P_4)\)) is similar to problem \((P)\) but has additional constraints: \(w_H, w_L, w_0 \geq 0\). Note that, since \(w_0 \geq 0\), the participation constraint \((IR)\) is implied by constraint \((IC_e2)\) and is therefore omitted in the following. Let \(\eta_1 \geq 0, \eta_2 \geq 0\) and \(\eta_3 \geq 0\) denote the Lagrangian multipliers associated with constraints \((IC_e1), (IC_e2)\) and \((IC_m)\). The Lagrangian of the problem \((P_4)\) is the following

\[
\max L = 0.5[(0.5 + i)(X(m_T) - \Phi(u_H)) + (0.5 - i)(0 - \Phi(u_L)) - I^0] - 0.5\Phi(u_0)
\]

\[
+ \eta_1(m_T + u_0 - (0.5 - i)u_H - (0.5 + i)u_L - 2k)
\]

\[
+ \eta_2((0.5 + i)u_H + (0.5 - i)u_L - u_0 - m_T - 2k)
\]

\[
+ \eta_3(u_0 - u_L).
\]

The necessary conditions for a solution to \((P_4)\) include:

\[
\frac{\partial L}{\partial u_0} \leq 0 \text{ and } u_0 \geq 0 \text{ and } \frac{\partial L}{\partial u_0}u_0 = 0, \tag{35}
\]

where

\[
\frac{\partial L}{\partial u_0} = -0.5\Phi'(u_0) + \eta_1 - \eta_2 + \eta_3,
\]

\[
\frac{\partial L}{\partial u_H} \leq 0 \text{ and } u_H \geq 0 \text{ and } \frac{\partial L}{\partial u_H}u_H = 0, \tag{36}
\]

where

\[
\frac{\partial L}{\partial u_H} = -0.5(0.5 + i)\Phi'(u_H) - \eta_1(0.5 - i) + \eta_2(0.5 + i),
\]

\[
\frac{\partial L}{\partial u_L} \leq 0 \text{ and } u_L \geq 0 \text{ and } \frac{\partial L}{\partial u_L}u_L = 0, \tag{37}
\]

where

\[
\frac{\partial L}{\partial u_L} = -0.5(0.5 - i)\Phi'(u_L) - \eta_1(0.5 + i) + \eta_2(0.5 - i) - \eta_3,
\]

\[
\frac{\partial L}{\partial m_T} \leq 0 \text{ and } m_T \geq 0 \text{ and } \frac{\partial L}{\partial m_T}m_T = 0, \tag{38}
\]

where

\[
\frac{\partial L}{\partial m_T} = 0.5(0.5 + i)X'(m_T) + \eta_1 - \eta_2.
\]

Consider the relaxed problem \((P_4^r)\) in which constraint \((IC_m)\) is suppressed. If the solution to the relaxed problem \((P_4)\) satisfies condition \((IC_m)\), the measure \(M\) is worthless for contracting. The optimal solution to \((P_4^r)\) is given in the next lemma.
Note that from this lemma it follows that (6) is a sufficient condition under which $M$ is worthless for contracting.

**Lemma 5** The optimal solution to $(P4^r)$ is either (i)

$$
\begin{align*}
    u_H &= m_T + \frac{k}{i}, \\
    u_L &= m_T - \frac{k}{i}, \\
    u_0 &= 0,
\end{align*}
$$

(39)

$$
(0.5 + i)X'(m_T) = (0.5 + i)\Phi'(u_H) + (0.5 - i)\Phi'(u_L)
$$

and $m_T \geq \frac{k}{i}$.

or, if there does not exist a solution that satisfies (i), (ii)

$$
\begin{align*}
    u_H &= 2\frac{k}{i}, \\
    u_L &= u_0 = 0 \text{ and } m_T = \frac{k}{i}.
\end{align*}
$$

(40)

Proof: In the relaxed problem $(P4^r)$ constraint $(IC_m)$ is dropped, i.e., $\eta_3 = 0$. Next, it is shown that in the optimal solution to $(P4^r)$ constraints $(IC_{e1})$ and $(IC_{e2})$ are binding, i.e., $\eta_1 > 0$ and $\eta_2 > 0$. The proof is by contradiction. Suppose that $\eta_1 = 0$. From (36) and (37) it follows that $u_H = u_L$. This violates either constraint $(IC_{e1})$ or constraint $(IC_{e2})$. Suppose that $\eta_2 = 0$. Then from (36) and (37) it follows that $u_H = u_L = 0$. This violates constraint $(IC_{e2})$. Hence, $\eta_1 > 0$ and $\eta_2 > 0$ must hold which implies that $(IC_{e1})$ and $(IC_{e2})$ are binding.

Note that since $X'(0) \gg 0$, it must be that $m_T > 0$. Hence, from (38) it follows that $\frac{\partial L}{\partial m_T} = 0.5(0.5 + i)X'(m_T) + \eta_1 - \eta_2 = 0$. Substituting this into (35) yields

$$
\frac{\partial L}{\partial u_0} = -0.5\Phi'(u_0) - 0.5(0.5 + i)X'(m_T) < 0.
$$

Since $\frac{\partial L}{\partial u_0} < 0$, from (35) it follows that $u_0 = 0$. Since $(IC_{e1})$ and $(IC_{e2})$ are binding and $u_0 = 0$, it follows that

$$
\begin{align*}
    u_H &= m_T + \frac{k}{i} \text{ and } u_L = m_T - \frac{k}{i} \text{ and } u_0 = 0.
\end{align*}
$$

(41)

Since $u_H > 0$, it follows from (36) that $\frac{\partial L}{\partial u_H} = 0$. Simplifying and rearranging yields

$$
\eta_2 = 0.5\Phi'(u_H) + \eta_1 \frac{0.5 - i}{0.5 + i}.
$$

(42)
Since $X'(0) \gg 0$, it must be that $m_T > 0$. Hence, $\frac{\partial L}{\partial m_T} = 0$ (see (38)), that is,

$$0.5(0.5 + i)X'(m_T) + \eta_1 - \eta_2 = 0. \quad (43)$$

Inserting (42) in (43) and rearranging yields

$$\eta_1 = (\Phi'(u_H) - (0.5 + i)X'(m_T)) \frac{0.5 + i}{4i}. \quad (44)$$

Inserting (42) and (44) in (37) and rearranging yields

$$(0.5 + i)X'(m_T) \leq (0.5 + i)\Phi'(u_H) + (0.5 - i)\Phi'(u_L). \quad (45)$$

The optimal solution is given by (41) and

$$(0.5 + i)X'(m_T) = (0.5 + i)\Phi' \left( m_T + \frac{k}{\imath} \right) + (0.5 - i)\Phi' \left( m_T - \frac{k}{\imath} \right)$$

if $m_T \geq \frac{k}{\imath}$. Otherwise, the optimal solution is given by (41) and $m_T = \frac{k}{\imath}$.

For the sake of completeness, the optimal solution to (P4) is given in the next lemma

**Lemma 6** The optimal solution to (P4) is characterized by either (i)

$$u_H = \frac{m_T + 2k}{0.5 + i}, \quad u_L = u_0 = 0,$$

$$(0.5 + i)X'(m_T) = \Phi'(u_H)$$

and $m_T \geq \hat{m}$,

or, if there does not exist a solution that satisfies (i), (ii)

$$u_H = 2\hat{m} \text{ and } u_L = u_0 = 0 \text{ and } m_T = \hat{m}.$$ 

The proof is similar to the proof of Lemma 5 and so is omitted.
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