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# MINIMUM WAGE WITH OPTIMAL INCOME TAXATION

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# MINIMUM WAGE WITH OPTIMAL INCOME TAXATION

## Abstract

Minimum wage legislation is a standard policy tool in most countries. However, the overall merits of minimum wage are controversial due to its potential adverse effects on unemployment. In this paper we construct a simple model in which minimum wage plays an important re-distributive role, alongside income taxation, without generating adverse effects on unemployment.

JEL Classification: H2, D6.

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## 1. Introduction

Minimum wage legislation is a standard tool in the arsenal of policy measures in most developed countries<sup>1</sup>. It has attracted much attention by scholars and policy makers. The overall merits of minimum wage as a re-distributive tool, as it is commonly perceived by the public, are controversial due to its potential adverse effects on employment and high-school drop-out<sup>2</sup>. This is reflected manifestly in the ongoing debate revolving around the issue.

The literature on optimal re-distributive taxation following the seminal contribution of Mirrlees (1971) reckons with the possibility of low-skill people not working. Note however that in this literature, as there are no minimum wages, low-skill people are not forced out of the labor market, but rather choose not to work due to the high transfers granted at low incomes (thus there is no unemployment), and every worker receives the marginal value of product before taxes (in a competitive setting).<sup>3</sup> An exception to the literature is Allen (1989) who introduces a standard two-type economy and investigates the potential gains from using minimum wage legislation when an optimal income tax system is in place. Allen shows that with a linear tax system, minimum wage may be desirable when technology exhibits enough complementarity between the low skill and high skill workers. Allen shows further that with a non-linear system minimum wage is redundant. Notably, when technology exhibits perfect substitutability (as is the case with a standard Mirrlees' economy), minimum wage is not desirable.

In this paper we attempt to show that minimum wage has an important re-distributive role, while allowing for a linear technology (a la Mirrlees) and for an optimal general income tax system. We do it by introducing a signaling stage prior to the stage where labor market

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<sup>1</sup>The US Congress first instituted a minimum wage with the Fair Labor Standards Act of 1938 to ensure the workers a minimally adequate standard of living.

<sup>2</sup> The findings regarding the effect minimum wage legislation bears on employment are ambiguous. Many studies find that minimum wages reduce employment [see, e.g., Brown, Gilroy and Kohen (1982), Neumark and Wascher (1992, 1994) and Deere, Murphy and Welch (1995)], while other research has brought these findings into question [see, e.g., Card, Katz and Krueger (1994) and Card and Krueger (1994)], showing that minimum wages are associated with no change or even an increase in employment levels. For articles examining the effect of minimum wages on school enrollment status, see Neumark and Wascher (1992, 1996) and Evans and Turner (1995), amongst others.

<sup>3</sup> Saez (2002) examines how one can alleviate the lack of participation of low-skill workers in the labor force ("extensive margin problem") through earned income tax credits.

decisions are taken. We depart from the standard Mirrlees' setting, by allowing for asymmetric information between workers and employers with respect to workers' innate ability. In such a setup, workers may choose to signal their innate ability.<sup>4</sup> We show that in this case, a minimum wage may well be a desirable policy tool.

The organization of the paper will be as follows. Section 2 presents the model. Section 3 establishes the case for minimum wage. We conclude in section 4.

## 2. The Model

Consider a standard two-type Mirrlees' economy, where high type agents constitute a fraction  $0 < \gamma < 1$  of the population. For simplicity, we let  $\gamma = 1/2$ , and normalize the population of each type to unity. Earning abilities for the high type and low type individuals are denoted, respectively, by  $\bar{w} > \underline{w} > 0$ . Innate earning ability is assumed to be private information and, unlike the standard model, is observed neither by the firms that hire the workers nor by the government. Individuals, prior to the entry into the labor market, may choose to engage in some signaling activity (which could take the form of investment in education or taking some aptitude tests), thereby transferring the information to the would-be employers. It is assumed that the asymmetry in information of the standard model is however maintained, by assuming that signaling activities are unobserved (or difficult to interpret) by the government. It is assumed that the signaling activity is given by a binary choice, for simplicity, namely, the choice whether to purchase the signal or not. We simplify further by setting the acquisition cost of the signal at infinity for the low type while denoting by  $e > 0$  the finite signaling cost incurred by the high type agents, where cost is measured in forgone consumption terms.

The standard optimal tax model implicitly assumes a separating equilibrium in the signaling game. That is, firms observe agents' types. When signaling costs are sufficiently small, signals are acquired and the signaling constraint is non binding and thus does not affect the optimal tax system. It is however possible, as we indeed show below, that when signaling

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<sup>4</sup> A nice survey of signaling models is provided by Riley (1979) who also analyzes the Mirrlees' model in a signaling framework.

costs are not negligible, a pooling equilibrium may dominate the separating equilibrium cum optimal income tax system.

Differences in earning abilities are the only source of heterogeneity in the economy.

All agents share the same preferences given by some quasi-linear utility function:

$$(1) \quad U(c, l) = c - h(l),$$

Where  $c$  denotes consumption,  $l$  denotes labor and the function  $h$  is increasing, strictly convex and twice continuously differentiable.

As is common in the literature on optimal general income tax, one can describe the optimal tax system as a set of gross income - net income (consumption) bundles. Denoting the gross income by  $y$ , substituting into the utility function in equation (1), one can express the utility derived by an individual of type  $w$  choosing the income level  $y$  and the consumption level  $c$  as follows:

$$(2) \quad V^w(c, y) = c - h(y/w)$$

For later purposes we let  $\bar{y}, \underline{y}, \bar{c}$  and  $\underline{c}$  denote the income levels and consumption levels (before deducting signaling costs) derived by the high type and low type, respectively, in the optimal tax system. The social planner is seeking to maximize some welfare measure which strikes a balance between efficiency and equity considerations. For concreteness, we assume a CES welfare measure given by:

$$(3) \quad W = \left[ [V^w(\bar{c} - e, \bar{y})]^\rho + [V^w(\underline{c}, \underline{y})]^\rho \right]^{1/\rho};$$

with  $\rho$  measuring the degree of inequality aversion. Note that for  $\rho=1$  we obtain the *Utilitarian* welfare measure whereas for the limit case of  $\rho \rightarrow -\infty$ , we obtain the *Rawlsian* welfare measure. As the parameter  $\rho$  decreases the welfare measure exhibits a stronger preference for re-distribution. Assuming a separating equilibrium in the signaling stage, the social planner is seeking to solve the following program:

$$(4) \quad \max \left[ [V^w(\bar{c} - e, \bar{y})]^\rho + [V^w(\underline{c}, \underline{y})]^\rho \right]^{1/\rho}$$

subject to:

- (i)  $V^w(\bar{c} - e, \bar{y}) \geq V^w(\underline{c} - e, \underline{y}) \Leftrightarrow V^w(\bar{c}, \bar{y}) \geq V^w(\underline{c}, \underline{y})$
- (ii)  $V^w(\underline{c}, \underline{y}) \geq V^w(\bar{c}, \bar{y})$
- (iii)  $V^w(\bar{c} - e, \bar{y}) \geq V^w(\underline{c}, \underline{y}) \Leftrightarrow V^w(\bar{c}, \bar{y}) - V^w(\underline{c}, \underline{y}) \geq e$
- (iv)  $(\bar{y} - \bar{c}) + (\underline{y} - \underline{c}) \geq 0$

The inequalities (i)-(iv) describe the self selection constraints (for the high type and low type, correspondingly), the signaling constraint and the budget constraint. Suppose that the signaling constraint is not binding (although it may well be the case). Then, the standard results [see, e.g., Stiglitz (1982) and Balcer and Sadka (1982)] imply that the two binding constraints are the incentive constraint for the high type [given by (i)] and the budget constraint [given by (iv)]. Zero marginal tax rate (efficiency) at the top income and strictly positive marginal tax rate at the bottom income follow from the standard analysis of the first order conditions [see Balcer and Sadka (1982)].

### 3. Pooling equilibrium and minimum wage

#### 3.1 The case for pooling equilibrium

In the model presented above signaling is an endogenous choice. This may have two interesting implications on the optimal tax structure. First, it may be the case that the signaling constraint is binding, which limits the scope of re-distribution in the case of a separating equilibrium. Second, a pooling equilibrium, if it exists, may dominate the separating equilibrium cum optimal tax system. Clearly, in the case of a pooling equilibrium in the signaling game there's no scope for re-distribution via the tax system as we obtain full re-distribution by the compression of the wage system.<sup>5</sup> Full re-distribution, although it entails significant distortions, is something we can never get from an optimal tax system due to the informational constraints.

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<sup>5</sup> Note crucially that pooling of incomes (rather than wages) is undesirable in the two-type case, as it is always dominated by the separating equilibrium [See Stiglitz (1982) for a clear presentation of the result]. Note further that in the more general case, where there are more than two types, a case for bunching is possible [see Ebert (1992)].

Ignoring for the moment the question of whether the signaling constraint is binding, we turn to examine the case for pooling equilibrium. By pooling equilibrium in the signaling game we mean that the high type agents choose to eschew from signaling. Thus the firms are unable to distinguish between the low type workers and the high type ones and pay all workers the same wage rate which, by virtue of the zero profit and free entry conditions in equilibrium, would be equal to the average productivity. Formally, denoting the wage rate in the pooling equilibrium by  $\hat{w}$ , it follows:

$$(5) \quad \hat{w} = (\bar{w} + \underline{w}) / 2$$

We denote by  $\hat{y}$  the optimal income (gross and net, given the fact that a re-distributive tax system is redundant in the case of pooling) chosen by a typical individual faced with the wage rate  $\hat{w}$ . Denote the consumption level by  $\hat{c}(= \hat{y})$ . A pooling equilibrium does not have to exist. As the government only observes income and not the wage rate, a necessary condition for the existence of a pooling equilibrium is the following:

$$(6) \quad V^{\bar{w}}(\hat{c} - e, \hat{y}) < V^{\hat{w}}(\hat{c}, \hat{y}) \Leftrightarrow V^{\bar{w}}(\hat{c}, \hat{y}) - V^{\hat{w}}(\hat{c}, \hat{y}) < e$$

Namely, a typical high type agent would find it undesirable to invest in signaling and increase her wage rate while choosing the same income earned in the pooling equilibrium by all other agents. This is not a sufficient condition though, for the individual can set the income (hence consumption) at the optimal level rather than sticking to the economy-wide equilibrium level of income. This could reverse the sign of the inequality. It is clear however, that by pooling the incomes in equilibrium into a single level (levying high taxes on any other income different from  $\hat{y}$ ) the necessary condition becomes also a sufficient one.

The question we would like to address next is whether a pooling equilibrium could deliver a better social outcome than a separating equilibrium with an optimal tax system. To address the question we assume that to begin with (in a tax free environment) a separating equilibrium exists. We let the optimal income and consumption levels for the high type and



low type, in the tax free environment, be denoted respectively, by  $\bar{y}, \bar{y}, \bar{c}$  and  $\underline{c}$ . Formally, we require then,

$$(7) \quad V^{\bar{w}}(\bar{c}, \bar{y}) - V^{\underline{w}}(\underline{c}, \underline{y}) \geq e$$

We further require that a pooling equilibrium may be implemented. Thus condition (6) needs to be satisfied. It is easy to verify that the following holds:

$$(8) \quad V^{\bar{w}}(\bar{c}, \bar{y}) > V^{\bar{w}}(\hat{c}, \hat{y}) > V^{\hat{w}}(\hat{c}, \hat{y}) > V^{\underline{w}}(\underline{c}, \underline{y}).$$

Thus there exists a large set of signaling costs (lying within a well defined interval) for which both (6) and (7) are satisfied for any wage rates  $\bar{w} > \underline{w} > 0$ .

We turn next to compare between the two types of equilibrium. We prove the main theorem of the paper.

**Theorem:** When the welfare measure exhibits a sufficiently large degree of inequality aversion, then for moderate levels of inequality, the pooling equilibrium dominates the separating equilibrium with the optimal tax being implemented.

**Proof:** Denote the welfare measures for the separating (when the tax system is in the optimum) and pooling equilibrium, respectively, by  $W^{Sep}$  and  $W^{pool}$ . Using the notation above, it follows that:

$$(9) \quad W^{sep}(\bar{w}, \underline{w}) = \left[ [V^{\bar{w}}(\bar{c} - e, \bar{y})]^\rho + [V^{\underline{w}}(\underline{c}, \underline{y})]^\rho \right]^{1/\rho};$$

$$(10) \quad W^{pool}(\bar{w}, \underline{w}) = V^{\hat{w}}(\hat{c}, \hat{y})$$

We would show that when  $\rho$  lies sufficiently close to  $-\infty$ , and  $\underline{w}$  is sufficiently close to  $\bar{w}$ , both types of equilibrium exist for a large set of signaling costs, but the pooling equilibrium attains a higher level of welfare than the separating one with the tax system being set optimally.

In view of the maximization program in (4), there are two possibilities to consider in the case of a separating equilibrium. It is either the case where the signaling constraint is not binding or the one where it binds. We first assume that the constraint is not binding. We

consider the limiting *Rawlsian* case, where  $\rho \rightarrow -\infty$ . Note that for the *Rawlsian* case,  $W^{sep} = V^w(\underline{c}, \underline{y})$ . When  $\underline{w} = \bar{w}$ , it follows that  $\hat{w} = \bar{w}$  (hence  $\hat{y} = \underline{y} = \bar{y}$ ) and, obviously,  $W^{sep} = W^{pool}$ , as there is no redistributive taxation in place (no inequality to reduce) and (trivially) no signaling. Differentiating the welfare measures in both types of equilibrium with respect to  $\underline{w}$  and evaluating the derivatives at  $\underline{w} = \bar{w}$ , using the envelope theorem, it follows that:

$$(11) \quad \left. \frac{dW^{sep}}{d\underline{w}} \right|_{\underline{w}=\bar{w}} = U_c(\bar{c}, \bar{y}/\bar{w}) \cdot \frac{\bar{y}}{\bar{w}} > 1/2 \cdot U_c(\bar{c}, \bar{y}/\bar{w}) \cdot \frac{\bar{y}}{\bar{w}} = \left. \frac{dW^{pool}}{d\underline{w}} \right|_{\underline{w}=\bar{w}} > 0$$

It follows that for  $\underline{w}$  close enough to  $\bar{w}$  the pooling equilibrium attains a higher level of welfare than the separating one for any signaling cost which supports the two equilibria. In particular, we can define a neighborhood around  $\bar{w}$  for which the pooling equilibrium dominates the separating one, where this neighborhood does not depend on the signaling cost. Now fix some  $\underline{w}$  in this neighborhood. Then for any signaling cost which satisfies both (6) and (7), which we already showed to exist (there are infinitely many such costs) the claim of the theorem is established.

We turn next to the case where the signaling constraint is binding. Clearly, the welfare measure of the separating equilibrium is reduced relative to the calculated measure that ignored the fact that the constraint was binding, for the same set of parameters. This reinforces the advantage of the pooling equilibrium.

Last note that the result extends to welfare measures exhibiting a sufficient degree of inequality aversion by virtue of the continuity of the welfare measure with respect to  $\rho$ . This concludes the proof. QED

### 3.2 The case for minimum wage

In the game-theoretical literature on implementation there is a clear distinction between implementation and full implementation. The former refers to a framing of a game such that

one of its equilibria (potentially many) supports a certain outcome (say a social goal). The latter refers to the case where the outcome is implemented but there are no other equilibria of the game which support outcomes other than the desirable one. Implementing the pooling equilibrium is easy. We can set the tax system to pool all income levels at  $\hat{y}$ . Then a pooling equilibrium exists because high type agents have no motivation to distinguish themselves by virtue of (6). However this is not the end of the story, as there is still a possibility of a separating equilibrium (in the signaling game) but pooling at the income level, which, as we already mentioned, is strictly dominated by the separating equilibrium where the two types earn different incomes (the standard case). For instance, when the signaling constraint (iii) in the maximization formulation at (4) is not binding in the optimum, indeed such a pooling – separating equilibrium necessarily exists. To see that, note that when the constraint is not binding, then by virtue of the binding incentive compatibility constraint for the high type [(i) in (4)], the signaling constraint (iii) can be re-written as follows:  $V^{\bar{w}}(\underline{c}, \underline{y}) - V^w(\underline{c}, \underline{y}) > e$ . We need to show that the following holds:  $V^{\bar{w}}(\hat{c}, \hat{y}) - V^w(\hat{c}, \hat{y}) \geq e$ . It therefore suffices to show that:  $V^{\bar{w}}(\hat{c}, \hat{y}) - V^w(\hat{c}, \hat{y}) \geq V^{\bar{w}}(\underline{c}, \underline{y}) - V^w(\underline{c}, \underline{y})$ , which (by substitution) holds if and only if the following is satisfied:  $h(\hat{y}/\underline{w}) - h(\hat{y}/\bar{w}) \geq h(\underline{y}/\underline{w}) - h(\underline{y}/\bar{w})$ . To establish the last inequality, let  $g(y, \bar{w}, \underline{w}) = h(y/\underline{w}) - h(y/\bar{w})$ . It is easy to verify that by virtue of the convexity of  $h$ ,  $g'(y) > 0$ . The result follows from the fact that  $\hat{y} > \underline{y}$ . We provide a simple example where  $h$  is quadratic in the appendix, which illustrates the point. In the example, the pooling-separating equilibrium exists even when the signaling constraint is binding.

In order to ensure that the inferior separating equilibrium does not exist, an introduction of minimum wage is called for. Suppose we set the minimum wage just equal to  $\hat{w}$ . The minimum wage does not allow firms to offer low type agents a wage offer which equals their productivity in a separating equilibrium. This is not the end of the story yet, for this might lead to the possibility in which the low-type agents will be crowded out of the market into unemployment. This will form a separating equilibrium. To ensure that such

equilibrium does not exist we need to introduce a payroll tax levied on firms, which depends on the level of unemployment in the economy. Set the tax equal to zero when unemployment is zero, and sufficiently high otherwise. Setting the tax sufficiently high will preclude the possibility of unemployment in equilibrium.<sup>6</sup>

We conclude that in general, fully implementing the socially desirable pooling equilibrium requires the introduction of a minimum wage combined with employment dependent payroll tax system (experience rating). The appendix provides an illustrative example where minimum wage is desirable.

#### 4. Conclusions

We develop a model of wage setup with asymmetric information between workers and firms. In this model there could be two types of equilibrium: one is a separating equilibrium with each worker perfectly signaling her type and being remunerated accordingly. Another possibility is a pooling equilibrium where wages are pooled. We show that when inequality is moderate and the welfare measure exhibits enough inequality-aversion, the pooling equilibrium is preferable. This equilibrium obtains full redistribution (a goal that is never attained in a Mirrlees' model) on the one hand, while on the other hand entails inefficiency in the form of an excessive workload for the low skilled agents and a lower than optimum workload for the skilled individuals. A minimum wage is required in order to rule out the separating equilibrium and ensure that the pooling equilibrium prevails.

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<sup>6</sup> Formally, let  $l(w)$  denote the labor supply of an agent faced with a wage rate  $w$ . The profit that a firm would derive from a worker of high type is then bounded above by:  $\max_w [\bar{w} - w] \cdot l(w)$ . Setting the payroll tax higher than the solution to the maximization problem would establish the result.

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### Appendix: An Example

Consider the following simple example. Suppose that preferences are given by  $U(c, l) = c - l^2/2$ . Suppose a *Rawlsian* social planner. Further assume that  $\underline{w} = 100$ ,  $\bar{w} = 110$  and  $e = 550$ . Substituting for  $l$  into the utility function, one obtains that  $V^w(c, y) = c - \frac{y^2}{2w^2}$ .

The optimal tax program for the separating equilibrium (assuming that the signaling constraint is not binding, which we verify below) is then given by:

(A1)

$$\max_{\underline{c}, \underline{y}, \bar{c}, \bar{y}} \left\{ \underline{c} - \frac{\underline{y}^2}{2\underline{w}^2} \right\}$$

s.t.

$$(i) \quad \bar{y} - \bar{c} + \underline{y} - \underline{c} \geq 0$$

$$(ii) \quad \underline{c} - \frac{\underline{y}^2}{2\underline{w}^2} \geq \bar{c} - \frac{\bar{y}^2}{2\bar{w}^2}$$

$$(iii) \quad \bar{c} - \frac{\bar{y}^2}{2\bar{w}^2} \geq \underline{c} - \frac{\underline{y}^2}{2\underline{w}^2}$$

By substituting the binding constraints into the objective function, one can hence turn the constrained maximization in (A1) into an unconstrained maximization program given by:

$$(A2) \quad \max_{\underline{y}, \bar{y}} \left\{ \frac{\bar{y} + \underline{y}}{2} - \left[ \frac{\bar{y}^2 - \underline{y}^2}{4\bar{w}^2} \right] - \frac{\underline{y}^2}{2\underline{w}^2} \right\}$$

Formulating the first-order conditions yields:

$$(A3) \quad \bar{y} = \bar{w}^2$$

$$(A4) \quad \underline{y} = \frac{\underline{w}^2 \cdot \bar{w}^2}{2\bar{w}^2 - \underline{w}^2}$$

Substituting back into the objective function in (A2), re-arranging and simplifying, yields the following welfare measure for the case of a separating equilibrium:

$$(A5) \quad W^{sep} = \frac{\bar{w}^2}{4} + \frac{\underline{w}^2 \cdot \bar{w}^2}{4} \cdot \frac{1}{(2\bar{w}^2 - \underline{w}^2)} = 5,155$$

The welfare measure for the case of a pooling equilibrium is given by:

$$(A6) \quad W^{pool} = \frac{\hat{w}^2}{2} = \frac{\left[ \frac{\bar{w} + \underline{w}}{2} \right]^2}{2} = 5,512$$

Comparing (A5) and (A6) it follows that the pooling equilibrium dominates.

We turn next to verify that the signaling constraint is not binding. Formally, using the fact that the incentive constraint for the high type is binding in the optimum, we require that:

$$(A7) \quad \left[ \underline{c} - \frac{y^2}{2\bar{w}^2} \right] - \left[ \underline{c} - \frac{y^2}{2\underline{w}^2} \right] > e$$

In words – a high type agent will benefit from engaging in signaling, when the optimal tax system is in place. Substitution into (A7) yields that the left-hand-side is equal to 630. This establishes the result by recalling that  $e=550$ .

Next we turn to verify that condition (6) in the main body of the paper is satisfied. Namely, that the necessary condition for a pooling equilibrium exists. Formally, the condition is given by the following:

$$(A8) \quad \left[ \hat{c} - \frac{\hat{y}^2}{2\bar{w}^2} \right] - \left[ \hat{c} - \frac{\hat{y}^2}{2\hat{w}^2} \right] < e$$

In words – the condition requires that given that all income groups are pooled together, a high type agent would refrain from revealing his type via signaling in equilibrium. Substitution into (A8) yields that the left-hand-side is equal to 490. This establishes the result.

Last we show that minimum wage legislation is not redundant; namely, that otherwise a separating equilibrium in the signaling game (while a pooling of income groups) exists. Formally, such equilibrium exists if the following condition holds:

$$(A9) \quad \left[ \hat{c} - \frac{\hat{y}^2}{2\bar{w}^2} \right] - \left[ \hat{c} - \frac{\hat{y}^2}{2\underline{w}^2} \right] > e$$

Note that  $\hat{y} > \underline{y}$ . Thus the condition in (A7) implies the condition given in (A9). The result is thus established.



Last note that the range of signaling cost for which the results hold is given by the interval (490,630). Note further that in the example the high wage to low wage ratio is modest and given by  $110/100=1.1$ . This is done in order to construct a scenario where the signaling constraint is not binding – which requires limited wage dispersion. It is however easy to show that the dominance of the pooling equilibrium extends to significant levels of earnings inequality. Employing the same quasi-linear quadratic utility form and assuming again a *Rawlsian* planner, one can show numerically that by setting  $e=5,000$  and  $\bar{w}=190$ , the pooling equilibrium would still dominate the separating equilibrium. In this case the signaling constraint would be binding. Moreover, for the new parametric assumption, it would still hold that setting the minimum wage equal to the average productivity would be necessary to preclude the possibility of an inferior separating equilibrium, thereby (fully) implementing the pooling equilibrium.

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