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A Re-examination of the Link between Real Exchange Rates and Real Interest Rate Differentials

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A RE-EXAMINATION OF THE LINK BETWEEN REAL EXCHANGE RATES AND REAL INTEREST RATE DIFFERENTIALS

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Abstract

The real exchange rate - real interest rate (RERI) relationship is central to most open economy macroeconomic models. However, empirical support for the relationship, especially when cointegration-based methods are used, is rather weak. In this paper we reinvestigate the RERI relationship using bilateral real exchange rate data spanning the period 1978 to 1997. We first clarify the logic of applying cointegration methods to the RERI and propose an alternative way of testing the relationship. We demonstrate that the failure of earlier analyses to detect a stationary real interest rate is largely due to the low power of the tests employed.

JEL Code: E43, F31, F41.

Keywords: real exchange rates, real interest rates, cointegration.

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1 Introduction

Many well-known exchange rate models highlight the role of the real interest rate differential as a key determinant of real exchange rates. For example, sticky price models (see Dornbusch (1976) and Mussa (1984)) and optimising models (see, for example, Grilli and Roubini (1992) and Obstfeld and Rogoff (1996)) emphasize the effect of liquidity impulses on real interest rates and consequently the real exchange rate. This relationship is often summarised in the form of the real exchange rate - real interest rate (RERI) relationship.

However, despite its centrality to many open economy macro models, the empirical evidence on the RERI relationship, particularly when cointegration methods are used, has been rather mixed (this is discussed in more detail in section 2). In this paper we revisit the RERI relationship and suggest that the rather ambiguous extant evidence may reflect a failure to implement the relationship appropriately. This leads us to suggest a new way of testing the RERI model and our results indicate that the real interest rate differential is associated with the transitory part of the real exchange rate. Our empirical findings are also consistent with Baxter (1994) and Edison and Pauls (1993) who have emphasized that the link between real exchange rates and real interest differentials is in the business cycle domain, rather than in the low frequency domain. Our way of casting the RERI relationship into an empirical model also offers a perspective on cointegration-based studies of the relationship and helps shed light on a number of further issues, such as the relative volatility of the real exchange rate and the real interest rate differential, the persistence of real exchange rates, the low power of cointegration tests and the sometimes ambiguous sign of the correlation between real exchange rates and real interest rate differentials. In our analysis we use bilateral real exchange rates for the G7 countries. The sample period is 1978 quarter 2 to 1997, quarter 4.

The outline of the remainder of this paper is as follows. In the next section we consider the RERI relationship in some detail and, in particular, highlight some potential pitfalls in estimating the relationship using cointegration-based methods. We then go on to outline how the model may be estimated using the projections from a simple VAR model and also by using what we refer to as a trivially cointegrated framework. In section 3 we present a set of preliminary empirical results while in section 4 we examine the long-run relationship between real exchange rates and the real interest differential using a VAR-based approach. Section 5 is a concluding section.
2 The RERI relationship - some motivational issues and a proposed testing method.

In this section we consider some empirical puzzles that arise in trying to estimate the real interest rate parity condition. A number of studies have attempted to test the validity of the RERI relationship by cointegrating the real exchange rate with the real interest differential. The basic starting point of many of these studies is the following reduced form equation:

\[ q_t = \mu + \varphi (r_t - r_t^*) + w_t, \]

where \( q_t \) is the real exchange rate, \( r_t - r_t^* \) is the real interest differential and \( w_t \) is a disturbance term. The definition of the variables entering (1) and its derivation are considered in some detail below. Before discussing that derivation, however, we summarise the extant empirical evidence which exploits cointegration methods to test the RERI. Meese and Rogoff (1988), Edison and Pauls (1993), Throop (1994) and Coughlin and Koedijk (1990) use the Engle-Granger two-step method to test for cointegration between real exchange rates and real interest rates and are unable to reject the null of no cointegration. Somewhat more favourable evidence is reported when the maximum likelihood estimator of Johansen is employed (see, inter alia, Johansen and Juselius (1992), Edison and Melick (1995), MacDonald (1997)). Using panel cointegration methods MacDonald and Nagayasu (2000) find support for the RERI, while Chortareas and Driver (2001) find no evidence of a long-run relationship.

Additionally, some studies have added in an extra variable, deemed important for systematic movements of the real exchange rate, to the cointegrating set (see for example Meese and Rogoff (1988)) and this line of research also appears to offer mixed support for a cointegrating relationship amongst the variables. So in sum, the evidence in favour of a cointegrating relationship existing for the RERI relationship is somewhat ambiguous. Is there a root cause for this rather ambiguous evidence? As Baxter (1994) notes, studies which use a cointegration framework to test the RERI relationship are misplaced: 'the real exchange rate should not be cointegrated with the real rate differential'\(^4\). In motivating our own tests it is useful to demonstrate why this is the case (our discussion draws on Baxter (1994)).

The standard derivation of the RERI (see, for example, Meese and Rogoff (1988)) has as its starting point the familiar risk adjusted uncovered interest parity condition:

\(^1\text{Baxter (1994), page 29.}\)
\[ \mathbb{E}_t(s_{t+1} - s_t) = (i_t - i_t^*) + \sigma_t, \]  
\[ (2) \]

where \( s_t \) is the log of the spot exchange rate (home currency price of a unit of foreign exchange), \( i_t \) is the one period domestic interest rate, \( \mathbb{E}_t \) is the conditional expectations operator, \( \sigma_t \) is a stationary (time-varying) risk premium and an asterisk denotes a foreign magnitude. Assuming rational expectations, equation (2) may be rewritten as:

\[ s_{t+1} - s_t = (i_t - i_t^*) + \sigma_t + \epsilon_t. \]  
\[ (3) \]

where \( \epsilon_t \) is an iid random error.

The nominal exchange rate is usually thought of as an I(1) process and it therefore follows that the left hand side variable in (3), \( s_{t+1} - s_t \), must be I(0). Since \( \sigma_t + \epsilon_t \) is stationary, by assumption, it follows that the interest differential, \( i_t - i_t^* \), must also be stationary - the domestic interest rate must be cointegrated with the foreign interest rate. The balanced nature of this expression, in terms of the orders of integration, is a standard feature of arbitrage conditions and is the starting point of the cointegration testing methods first proposed by Campbell and Shiller (1987) for present value models. It turns out that translating (3) into the equivalent real interest parity condition produces a similar balance in terms of the integratedness of the right and left hand side variables. For example, by subtracting the expected inflation differential, \( \mathbb{E}_t(p_{t+1} - p_t) - \mathbb{E}_t(p_{t+1}^* - p_t^*) \), from both sides of (3), where \( p_t \) denotes the log of domestic price level, and assuming rational expectations the following expression may be obtained:

\[ q_{t+1} - q_t = (r_t - r_t^*) + \sigma_t + \epsilon_{t+1} + u_{t+1}, \]  
\[ (4) \]

where \( q_t = s_t + p_t^* - p_t \), \( r_t \) denotes the domestic real interest rate, defined as \( r_t = i_t - (\mathbb{E}_t(p_{t+1} - p_t)) \), and \( u_{t+1} \) is an iid inflation forecast error. Since the two disturbance terms - \( \epsilon_{t+1} \) and \( u_{t+1} \) - and the risk premium are stationary, it must follow, as in equation (3), that \( q_{t+1} - q_t \) and \( (r_t - r_t^*) \) are integrated of the same order. Since the real exchange rate is usually thought to be I(1), or close to I(1), \( q_{t+1} - q_t \) must be I(0) and therefore so too must \( (r_t - r_t^*) \). However, it follows from this that \( q_t \) and \( (r_t - r_t^*) \) cannot be cointegrated. Why then have a number of researchers, such as those noted above, nevertheless tried to cointegrate these variables? To gain insight into this we follow the derivation in Meese and Rogoff (1988) which is used in a number of the above-noted papers to test for a cointegrating relationship between real exchange rates and real interest rates. Meese and Rogoff consider the following adjustment equation for the real exchange rate:
\[ \mathbb{E}_t(q_{t+k} - \tilde{q}_{t+k}) = \theta^k (q_t - \tilde{q}_t), \quad 0 < \theta < 1, \quad (5) \]

where \( \tilde{q}_t \) is interpreted as the permanent component of the real exchange rate, or the long-run equilibrium. Meese and Rogoff then assume that \( \tilde{q}_t \) follows a random walk:

\[ \mathbb{E}_t \tilde{q}_{t+k} = \tilde{q}_t. \quad (6) \]

On substituting (6) in (5), the following expression may be obtained:

\[ q_t = \alpha_k (\mathbb{E}_t q_{t+k} - q_t) + \tilde{q}_t, \quad (7) \]

where \( \alpha_k \equiv 1/(\theta^k - 1) \). Noting that \( \alpha_k \) goes to \(-1\) as \( k \) tends to infinity, we get:

\[ \bar{q}_t = q_t + \lim_{k \to \infty} (\mathbb{E}_t q_{t+k} - q_t), \quad (8) \]

or, equivalently:

\[ \bar{q}_t = \lim_{k \to \infty} \mathbb{E}_t q_{t+k}. \]

On using the UIP condition at horizon \( k \) - \( \mathbb{E}_t(s_{t+k} - s_t) = (k i_t - k i^*_t) \) - where \( k i_t \) represents the nominal interest rates at time \( t \) on \( k \)-period bonds and on subtracting expected \( k \)-horizon relative inflation rates we obtain the \( k \)-period version of the real interest parity relationship, (4), as:

\[ (\mathbb{E}_t q_{t+k} - q_t) = (k r_t - k r^*_t), \quad (9) \]

where \( k r_t = k i_t - (\mathbb{E}_t p_{t+k} - p_t) \). Combining (7) with (9) we then get the reduced form equation which is the focus of the empirical studies discussed above, namely:

\[ q_t = \alpha_k (k r_t - k r^*_t) + \tilde{q}_t. \quad (10) \]

By assuming the long-run equilibrium real exchange rate in (10) is constant, we recover equation (1). Alternatively, the long-run equilibrium can be made time-varying by assuming it is a function of variables like net foreign assets and/ or a GDP differential (this is discussed further below). As in our discussions surrounding (3), it is immediately evident that (10) is an unlikely candidate for a cointegration-based study. This is because, irrespective of the order of integration of \( q_t \) and \( \tilde{q}_t \), equation (5) implies that \( q_t - \tilde{q}_t \) is stationary. If this is so, then equation (10), in turn, implies that \( k r_t - k r^*_t \) is also stationary. However, the empirical tests noted above, either implicitly or explicitly, always treat \( k r_t - k r^*_t \) as non-stationary. But if \( k r_t - k r^*_t \) is non-stationary then the basic theory used to derive (10) is clearly rejected. This
is a key observation first made by Baxter (1994). Hence trying to cointegrate the real exchange rate with the real interest differential, as Meese and Rogoff and others have done, would seem to be wrong because the relationship is unbalanced.

In this paper, we propose an alternative to the cointegration method which is more in the spirit of (10). To this end, let us rewrite (10) as:

$$\lim_{k \to \infty} E(q_{t+k} - q_t) = -\alpha_k(k r_t - k r^*_t),$$

(11)

This equation states that the current real interest rate differential contains sufficient information for forecasting the expected long-run change in the real exchange rate. Hence, while an econometrician may not have all the information that economic agents use to form expectations, equation (11) states that current real interest differentials embody all of that information. This is a familiar insight that was first proposed by Campbell and Shiller (1987) in the context of present value models but has not, to our knowledge, been used in the literature on the RERI relation. In particular, equation (11) indicates that past levels of the real interest rate differential should be included in the forecasting equation for real exchange rate changes. To obtain such a forecasting equation, we rewrite the expected long-run change in \( q \) as the sum of period-to-period changes:

$$\lim_{k \to \infty} E(q_{t+k} - q_t) = \sum_{k=1}^{\infty} E(\Delta q_{t+k}) = E(\Delta_\infty q).$$

(12)

A straightforward way to proxy the expectations in equation (12) is to use a forecast from a VAR that includes past levels of the real interest rate differential. We illustrate this in the context of a bi-variate system of the form:\(^2\)

$$A(L) \begin{bmatrix} r_t - r^*_t \\ \Delta q_t \end{bmatrix} = \varepsilon_t,$$

(13)

where \( A(L) \) is a matrix polynomial in the lag operator, and \( \varepsilon_t \) is an i.i.d. error vector with covariance matrix \( \Omega \).

For expositional purposes, let us consider a VAR(1) here. Then \( A(L) = A_1 \) and the VAR-approximation of \( E(\Delta q_{t+k}) \) is given by:

$$E(\Delta q_{t+k}) = \begin{bmatrix} 0 & 1 \end{bmatrix} A_1^k \begin{bmatrix} r_t - r^*_t \\ \Delta q_t \end{bmatrix}.$$

(14)

\(^2We now drop the index for the maturity horizon and use the shorthand notation \( r_t - r^*_t \) to denote long-term real interest rate differentials. We will henceforth adopt this simplified notation whenever the exact maturity horizon does not matter in our derivations.
To measure how closely \( \lim_{k \to \infty} E (q_{t+k} - q_t) \) is related to \((r_t - r_t^*)\) involves simply looking at correlations between the two series.

Our approach offers interesting perspectives on some of the earlier literature on the RERI. For example, Baxter (1994) was among the first to argue, in the context of the RERI derivation discussed above, that the real interest rate differential should be a stationary variable and therefore correlating it with a nonstationary variable does not make sense. Instead, she proposes correlating the real interest differential with the transitory, or stationary, component of the real interest differential extracted from the real exchange rate using a multivariate Beveridge-Nelson (1981) decomposition. This approach is shown to be successful in the sense that such correlations are significant and correctly signed. Although our approach also uses a permanent-transitory decomposition, it differs from Baxter’s in the important respect that our multivariate decomposition involves the real interest rate differential itself.\(^3\)

This feature of our decomposition also builds an interesting bridge to the literature, discussed above, that employs cointegration tests to analyse the RERI relationship. This is because cointegration methods may still be a useful way of testing the RERI if we are prepared to impose cointegration on the relevant vector to capture the theoretical necessity of \( r - r^* \) being \( I(0) \). That is, instead of estimating a VAR with \( q \) in differences and \( r - r^* \) in levels, we could instead consider an error correction model in

\[
X_t = \begin{bmatrix} r - r^* \\ q \end{bmatrix}'.
\]

We refer to such a system as ‘trivially cointegrated’ with unit vector. For example, if we find that \( \beta'X_t \) is \( I(0) \) (where \( \beta = [1 \ 0]' \)) we can call \( \beta \) a cointegrating vector for \( X_t \).\(^4\) This is a slightly unusual definition of cointegration. In fact, it is not encompassed by Engle and Granger’s original definition which requires all components of \( X_t \) to be \( I(1) \). However, Johansen (1995) has explicitly expanded the definition to allow for unit vectors as cointegration vectors and shows that all the standard representation and asymptotic theory can be used.

Below we provide evidence which shows that, while tests may not detect any cointegration between the real exchange rate and real differential at all (which may be due to low power), the preferred specification if cointegration

\(^3\)Baxter’s multivariate decomposition was derived from a bivariate VAR in monthly changes of the real exchange rate and inflation differential.

\(^4\)Although they did not use the term ‘trivial cointegration’, Edison and Mellick (1995) were, to our knowledge, the first to test the stationarity of the real interest rate in the RERI using cointegration methods.
is imposed would still be the trivial one, which is ultimately equivalent to
the mixed differences-levels specification (13).

In our empirical implementation, we consider two different information
sets. One is our 'baseline specification' (13) which only contains the real
exchange rate and the real interest rate differential. The second specification
considered recognizes that $\bar{q}_t$ may be time-varying and we therefore extend
the baseline information set to include a real per capita output differential,
$y - y^*$:

$$X_t = [r_t - r^*_t \quad q_t \quad y_t - y^*_t]'.$$

Following Bergstrand (1991), there are a number of arguments for includ-
ing the per capita output differential in a real exchange rate relationship.
First, according to the Balassa-Samuelson hypothesis, countries with a rela-
tively high per capita GDP have a relative productivity advantage in traded
goods compared to their trading partner(s) and this raises the relative price
of non-traded goods, thereby appreciating the real exchange rate de-
fined using CPIs. A second supply side influence on the internal price ratio involves
relative factor endowments. In the traditional Heckscher-Ohlin two factor,
two good, relative factor endowments model, nontraded (traded) goods are
assumed to be relatively labour-intensive (capital-intensive) in production.
High per capita income countries are assumed to have a comparative ad-
vantage in producing traded goods and so the relative price of non-traded
goods will be higher in countries with relatively high per capita income. In
addition to these supply side influences, there is also likely to be a demand
side effect on the internal price ratio if preferences for traded and non-traded
goods are non-homothetic (see, for example, Dornbusch (1988) and Neary
(1988)). In this paper we do not seek to separate the influence of these dif-
ferent sources on the real exchange rate. Rather we assume that they are
subsumed within our measure of per capita real income and focus on this as
the key determinant of $\bar{q}_t$.\(^5\)

3 A first pass at the RERI Relationship.

3.1 Data

Our data set consists of quarterly data for the G7 countries, the United
States, Japan, Germany, France, Italy, the United Kingdom and Canada,

\(^5\)There may also be other determinants of the systematic component of the real ex-
change rate, such as net foreign assets, but these are not considered in this paper.
over the period 1978:Q1 to 1997:Q4. All data are sourced from the IMF’s International Financial Statistics (IFS).

The nominal interest rates are long bond yields (line 61) and the price indices are consumer prices (line 64). We constructed bilateral CPI-based real exchange rates vis-a-vis the United States using average quarterly dollar exchange rates. The output data measure real GDP denominated in domestic currency (code 99B). These were converted into US dollars using the mean nominal exchange rate over the sample period. We then expressed GDP data in per capita terms using annual population data, also from the IFS, before constructing relative output levels, again vis-a-vis the U.S.

In order to obtain long-term real interest rates, we first constructed an estimate of average inflation expectations over the maturity horizon of the underlying government bonds (typically 10 years). This was achieved by running a univariate autoregression of CPI-inflation with 5 lags. We then generated forecasts of quarterly inflation 40 periods ahead. To generate the average expected annual inflation rate we finally divided the cumulative sum of inflation rates by the bond’s maturity horizon.

3.2 The RERI and Some Simple Correlations

We start our empirical analysis by examining the bivariate relationship between real exchange rates and the real interest rate differential. Our approach predicts a link between changes in the real exchange rate and the level of the real interest rate differential. In Figure 1 we plot the two variables for the six countries, vis-a-vis the United States, and this reveals that in some periods there is a striking similarity between the real exchange rate and the real interest differential, while in others the relationship is not at all clear. A formal correlation analysis also highlights the fact that the RERI relationship is not always in the data. For example, in Table 1, we provide correlations between observed changes in the real exchange rate at various time horizons (1 quarter, 1 year, 5 years, 10 years) and also between the levels of the two variables.

\textsuperscript{6}To check our results for robustness, we varied the lag length in the construction of expected inflation between 1 and 9 lags. All the results in the paper were found to be robust to this change in the construction of real interest rates.
Table 1: Comovement between real interest rates and real exchange rates

<table>
<thead>
<tr>
<th></th>
<th>$q_t$</th>
<th>$q_{t+1} - q_t$</th>
<th>$q_{t+4} - q_t$</th>
<th>$q_{t+20} - q_t$</th>
<th>$q_{t+40} - q_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canada</td>
<td>-0.19</td>
<td>-0.02</td>
<td>0.06</td>
<td>0.47</td>
<td>0.25</td>
</tr>
<tr>
<td>France</td>
<td>-0.19</td>
<td>-0.14</td>
<td>0.11</td>
<td>-0.21</td>
<td>-0.13</td>
</tr>
<tr>
<td>Germany</td>
<td>-0.67</td>
<td>-0.13</td>
<td>-0.00</td>
<td>0.34</td>
<td>0.74</td>
</tr>
<tr>
<td>Italy</td>
<td>-0.02</td>
<td>-0.09</td>
<td>0.31</td>
<td>0.10</td>
<td>-0.37</td>
</tr>
<tr>
<td>Japan</td>
<td>-0.20</td>
<td>-0.19</td>
<td>-0.10</td>
<td>0.53</td>
<td>0.53</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>-0.53</td>
<td>-0.22</td>
<td>-0.22</td>
<td>0.60</td>
<td>0.62</td>
</tr>
</tbody>
</table>

The numbers in the table reveal that the relationship between real exchange rates and the real interest rate would seem, at best, to be identified in the long-run. For example, for three out of the six countries the correlation at the 10-year differencing horizon is higher than 0.5. However, it is also noteworthy that all short-run correlations, i.e. at the 1-quarter horizon, as well as the level-correlations, are negative. The correlations between levels should, however, be interpreted with caution, since the real exchange rate is likely to be an integrated process.

### 3.3 Cointegration-based tests of the RERI

Despite the evident problems with applying cointegration methods to the RERI relationship, we noted in section two one way in which this may be justified. We therefore also apply multivariate cointegration methods to the RERI relationship. In specifying the appropriate lag length of the VAR, we relied on standard information criteria. Since all of those suggested the use of either 2 or 3 lags for all countries, we decided to estimate the VAR with 2 lags throughout and to include a set of seasonal dummies. Using a VAR specification with an unrestricted constant and without trend, we then proceeded to implement Johansen’s test for cointegration. The results are given in table 2.
Figure 1: U.S. bilateral CPI real exchange rates (solid line) and real interest differential (in \%*10^{-1}\)

\[\text{Figure 1: U.S. bilateral CPI real exchange rates (solid line) and real interest differential (in \%*10^{-1}\)\]
Table 2: Cointegration Tests

<table>
<thead>
<tr>
<th>No of CI-relations</th>
<th>Bi-variate model</th>
<th>Real Diff. Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Trace Test</td>
<td>Max. EV Test</td>
</tr>
<tr>
<td></td>
<td>h=1  h=2</td>
<td>h=1  h=2</td>
</tr>
<tr>
<td>Canada</td>
<td>8.06 0.51</td>
<td>7.55 0.51</td>
</tr>
<tr>
<td>France</td>
<td>13.38 3.53</td>
<td>9.84 3.53</td>
</tr>
<tr>
<td>Germany</td>
<td>15.83 4.03</td>
<td>11.80 4.03</td>
</tr>
<tr>
<td>Italy</td>
<td>10.72 4.14</td>
<td>6.58 4.14</td>
</tr>
<tr>
<td>Japan</td>
<td>7.77 2.29</td>
<td>5.48 2.29</td>
</tr>
<tr>
<td>United Kingdom</td>
<td><strong>23.45 5.38</strong></td>
<td><strong>18.07 5.38</strong></td>
</tr>
</tbody>
</table>

90% Crit. Values: 16.06 2.57 14.84 2.57 31.42 16.06 21.53 14.84
95% Crit. Values: 18.17 3.74 16.87 3.74 34.55 18.17 23.78 16.87

These results indicate that we accept the null of no cointegration for all countries with the exception of the UK. However, for this country we also reject the non-stationarity of the second linear relation, which would suggest that both variables are \( I(0) \). Similar results are obtained in the tri-variate 'real differential' system that also included relative per-capita output levels. These results confirm the evidence reported in other studies.

Table 3: Tests on cointegrating vectors

<table>
<thead>
<tr>
<th></th>
<th>bivariate system</th>
<th>real diff. model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \beta' = \begin{bmatrix} 1 &amp; 0 \end{bmatrix} )</td>
<td>( \beta' = \begin{bmatrix} 1 &amp; -1 \end{bmatrix} )</td>
</tr>
<tr>
<td>Canada</td>
<td>0.25 0.01</td>
<td>0.02</td>
</tr>
<tr>
<td>France</td>
<td>0.98 0.02</td>
<td>0.10</td>
</tr>
<tr>
<td>Germany</td>
<td>0.01 0.01</td>
<td>0.00</td>
</tr>
<tr>
<td>Italy</td>
<td>0.35 0.20</td>
<td>0.22</td>
</tr>
<tr>
<td>Japan</td>
<td>0.42 0.08</td>
<td>0.04</td>
</tr>
<tr>
<td>UK</td>
<td>0.00 0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

As discussed in section 2, in order to address the potentially low power of the cointegration tests, we also estimated our two models with one cointegrating relationship imposed. We then tested plausible hypotheses on the cointegrating vector. In particular, we test: i) the hypothesis of 'trivial' cointegration, i.e. the real interest rate differential is stationary (\( \beta' = \begin{bmatrix} 1 & 0 \end{bmatrix} \));
or, ii), that there is a genuine cointegrating relationship between the variables. In the bi-variate system we formalized this second hypothesis as \( \beta = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \), whereas in the real differential system, we explicitly allowed for a time-varying \( \beta \) that would not separately cointegrate with \( q \) by testing \( \beta = \begin{bmatrix} \beta_1 \\ 1 \\ -1 \end{bmatrix} \).

Interestingly, although our tests do not suggest the presence of cointegration, once we impose cointegration we find substantial evidence in favour of the first hypothesis, namely that the real interest differential is \( I(0) \). However, this evidence is largely confined to the bi-variate system. Table 3 gives the corresponding \( p \)-values for each country. In the bi-variate setup, the hypothesis is rejected for only two countries, namely Germany and the UK. For Italy and Japan, we can accept both hypotheses at the conventional 5 percent level. The fact that cointegration is not detected by conventional tests, but that plausible restrictions on the cointegration vector are accepted once cointegration is imposed, suggests that it is impossible to characterize the RERI relationship on purely statistical grounds. As the third column in table 3 demonstrates, adding a third variable in the form of the second real differential - relative outputs - does not help to impose more structure on the situation. Although it is interesting to note that even in the tri-variate system the ‘trivial cointegration’ hypothesis is accepted in two cases.

A more conventional way to test for the stationarity of the interest rate differential would of course have been to conduct univariate unit-root tests on that variable. These generally also reach the conclusion that \( r - r^* \) is \( I(1) \). However, we do not believe that unit-root tests are particularly informative about the RERI. The reason being that, according to the theory, \( r - r^* \) should be the transitory part of the real exchange rate. This requires us to examine the dynamic interaction of these variables: the two key variables should be considered jointly - the RERI cannot be examined by just considering the persistence of \( r - r^* \). Of course the Johansen method provides an appropriate method for testing the joint evolution of the two variables.

To address the question of whether standard cointegration or unit-root tests would pick up the stationarity of the real interest rate differential if it was truly stationary we constructed a Monte Carlo experiment, the results of which are reported in table 4. In particular, we used the parameter estimates from the VAR-specification in which the real interest rate differential features in levels, such as in (13), to generate 500 time series of the length of our sample \( (T = 74) \). We then run Johansen’s test on these artificial data. In the

\[ \text{Our results are consistent with Obstfeld and Taylor (2002) who provide ample evidence that the real interest rate differential is a stationary variable at longer horizons but demonstrate that in shorter samples the null of a unit root cannot be rejected.} \]
artificial data sets the real interest rate differential is \( I(0) \) by construction and the real exchange rate is \( I(1) \). In table 4 we report the rejection frequencies of the cointegration tests for both the ‘baseline’ and the ‘real differential’ specifications. It turns out that the null of no cointegration is generally rejected with a much lower frequency than the asymptotic nominal size of the test would suggest (i.e. in our case 95 percent). This suggests that the test may have particularly low power in the present application. We provide a rationale of why this may be the case in Section 4. We note also that the size distortion seems a bit less pronounced in the trivariate ‘real differential’ system.

Table 4: Monte Carlo results on tests for cointegration rank

<table>
<thead>
<tr>
<th></th>
<th>baseline</th>
<th></th>
<th>real differential</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Trace</td>
<td>MaxEV</td>
<td>Trace</td>
<td>MaxEV</td>
</tr>
<tr>
<td>Canada</td>
<td>0.02</td>
<td>0.01</td>
<td>0.18</td>
<td>0.20</td>
</tr>
<tr>
<td>France</td>
<td>0.69</td>
<td>0.49</td>
<td>0.68</td>
<td>0.57</td>
</tr>
<tr>
<td>Germany</td>
<td>0.43</td>
<td>0.26</td>
<td>0.77</td>
<td>0.73</td>
</tr>
<tr>
<td>Italy</td>
<td>0.45</td>
<td>0.26</td>
<td>0.93</td>
<td>0.81</td>
</tr>
<tr>
<td>Japan</td>
<td>0.68</td>
<td>0.53</td>
<td>0.80</td>
<td>0.61</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>0.68</td>
<td>0.56</td>
<td>0.96</td>
<td>0.88</td>
</tr>
</tbody>
</table>

The results in table 4 suggest that cointegration tests may provide very little guidance in analysing the RERI because they will not reject the null of non-stationarity when it is likely to be false. The Monte Carlo results confirm that it may be necessary to override cointegration test results to uncover the RERI from the data. Doing so provides further evidence in favour of a stationary real interest rate differential.

In table 5, we have imposed one cointegrating relationship in the estimation of both the baseline and the real differential specifications. The cointegrating vector is left unrestricted. We then calculate the correlation of
the cointegrating error, $\beta'X_t$, with the real interest rate differential. As is evident, this correlation is very high in almost all cases. This provides further strong evidence that, if cointegrating methods are used, the data indicate that what should be stationary is indeed the interest rate differential.

Table 5:

<table>
<thead>
<tr>
<th></th>
<th>Correlations of $r - r^*$ with $\beta'X_t$ in the cointegrated setup</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>baseline</td>
<td>real differential</td>
</tr>
<tr>
<td>Canada</td>
<td>0.90 (0.05)</td>
<td>0.42 (0.10)</td>
</tr>
<tr>
<td>France</td>
<td>1.00 (0.01)</td>
<td>0.86 (0.06)</td>
</tr>
<tr>
<td>Germany</td>
<td>0.78 (0.08)</td>
<td>0.70 (0.08)</td>
</tr>
<tr>
<td>Italy</td>
<td>0.80 (0.07)</td>
<td>0.66 (0.09)</td>
</tr>
<tr>
<td>Japan</td>
<td>0.90 (0.05)</td>
<td>0.35 (0.10)</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>0.61 (0.09)</td>
<td>0.02 (0.11)</td>
</tr>
</tbody>
</table>

Standard errors in parentheses

In view of our results, we suggest economic theory should provide guidance on how to proceed. As we have argued in the previous section, assuming that both real interest rates and the real exchange rate are integrated processes is inconsistent with the simple theoretical model we discussed.

In the remainder of this paper, we therefore maintain the real interest rate differential as a stationary variable. As we demonstrate, this gives rise to very plausible, economically interpretable results. Treating the real interest rate as a stationary variable also allows us to generate very high correlations between VAR-generated expectations of exchange rate changes and the real interest rate differential itself. Because the real interest rate differential will typically be the only stationary variable in the systems we consider, these systems are viewed as trivially cointegrated with a unit-vector. In such a setup, the real interest rate differential can be viewed as the transitory component of the real exchange rate that indicates to what extent a currency is over- or undervalued.
In contrast to most of the earlier literature, the method we have proposed in the previous section does not require us to directly examine the link between observed real exchange rates and the real interest rate differential. Rather, we ask: to what extent do real interest rate differentials reflect market expectations of long-run exchange rate changes? At the same time, we also allow for persistent deviations of the actual real exchange rate from this expectational anchor. We apply and further develop our methods in the next section.

4 A Second Pass at the RERI: Alternative Long-run Relationships.

As the last section has demonstrated, we do not, in general, find evidence of cointegration for the RERI, although there is evidence of ‘trivial cointegration’. This is true for both the base-line bi-variate relationship, as well as for the system that contains relative outputs. In this section we propose an alternative way of measuring the long-run link between real exchange rates and the long-term real interest rate differential.

It is important to emphasize that theory itself does not predict a direct link between the observed real exchange rate and the real interest rate differential. Rather, the real interest rate differential should reflect the expected rate of change of the real exchange rate. In testing this relation, most of the extant literature assumes that the market’s expectation of the real exchange rate and its actual realization differ only by an i.i.d. error term and then proceeds to tests a link between the levels of these variables. Hence, the conventional tests that examine the relation between the real exchange rate and the real interest rate differential are joint tests of market efficiency and the long-run link we are interested in. If we are willing to accept that misalignments, defined as possibly very persistent non-i.i.d. deviations of the real exchange rate from its long-run level, play a role in actual data, the traditional way of conducting the analysis is likely to be flawed. As we noted in section 2, we use a simple VAR framework to proxy the long-run expected rate of change of the real exchange rate.

8Such persistence could arise from the error in the UIP condition. For example, Frankel and Froot (1987) demonstrate that survey-based exchange rate expectations are persistently biased. One interpretation for this bias could be the existence of noise traders in foreign exchange markets, as in the model of De Long et al (1990). Jeanne and Rose (1999) and Devereux and Engel (2001) present models in which the (non-systematic) error in a UIP equation stems from the behaviour of noise trading.
In Table 6 we provide the correlations of $E(\Delta q)$ with $r - r^*$ from both the baseline specification and the 'real differential' model.

### Table 6: Correlations of $E(\Delta q_{t+\infty,t})$ with $r - r^*$

<table>
<thead>
<tr>
<th></th>
<th>Baseline</th>
<th>'Real Differential'</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canada</td>
<td>0.30 (0.11)</td>
<td>0.19 (0.12)</td>
</tr>
<tr>
<td>France</td>
<td>0.60 (0.09)</td>
<td>0.63 (0.09)</td>
</tr>
<tr>
<td>Germany</td>
<td>0.86 (0.06)</td>
<td>0.88 (0.06)</td>
</tr>
<tr>
<td>Italy</td>
<td>0.96 (0.03)</td>
<td>0.87 (0.06)</td>
</tr>
<tr>
<td>Japan</td>
<td>-0.24 (0.11)</td>
<td>-0.02 (0.12)</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>-0.20 (0.12)</td>
<td>0.54 (0.10)</td>
</tr>
</tbody>
</table>

Standard errors in parentheses

Except for Japan and the United Kingdom, the correlations of the forecast from the baseline model are reasonably high, in spite of the parsimony of our model specification. The 'real differential' model considerably improves the approximation for the United Kingdom but does little for the other countries.

One important issue relating to our forecast equation for $\Delta q$ is that it also contains changes in the real exchange rate that economic agents may consider to be permanent. In our current specification, these changes might erroneously affect our long-run forecast of $\Delta q$. Following Campbell and Shiller (1988) and Froot and Ramadorai (2001), we could, for example, allow for an error term in equation (9). In equation (4), we referred to such an error as a risk premium, although a more neutral interpretation would be the 'excess return' in holding a particular country’s currency. By solving the one-period real interest rate parity condition forward we get:

$$q_t = E_t \left\{ \sum_{s=0}^{\infty} - \left( r_{t+s} - r^*_{t+s} \right) \right\}.$$  

In the terminology of Campbell and Shiller (1988), the first term in parentheses, i.e. the expected changes in future real interest differentials, is the 'cash-flow' news, whereas the second term is called 'expected returns' news. In stock market data, Campbell and Shiller (1988) demonstrate that expected return news are the dominant source of the long-run variation in stock returns and Froot and Ramadorai (2001) report similar results for the real exchange rate of currencies.

Suppose, the sum of expected returns is essentially a random walk (as the evidence by Froot and Ramadorai would suggest), but also contains a transitory component that is serially correlated, but not too persistent. Then
observed changes in the real exchange rate would contain both changes in
the random walk component and changes in the long-term real interest rate
differential. Using observed real exchange rate changes in the forecasting
equation for $\Delta q$ could then lead to measures of expected appreciation or
depreciation that are poorly correlated with the real interest rate differential.
As long as we do not have a good proxy of movements in expected returns,
the link between real exchange rates and real interest rates may therefore
remain blurred.

Given this, we reformulate the RERI relation as a conditional one: will
common shocks to the real exchange rate and the real interest rate differential
give us a long-run forecast of $\Delta q$ that is correlated with the real interest rate
differential? If this is the case, we should be able to improve our proxy of
market expectations if we explicitly condition our forecasts only on changes
in the real exchange rate that are linked to changes in the real interest rate
differential. Hence, in forming our forecast of $\Delta q$, we propose using current
changes in $\Delta q$ that are common to both the real interest rate and the real
exchange rate.

In the framework of the VAR models we use in our analysis, we can iden-
tify the common component in $\Delta q$ and $r - r^*$ via a Choleski-decomposition
of the reduced-form covariance matrix, $\Omega$. To see this, let the elements of $\Omega$
be denoted by $\{\omega_{ij}\}_{i,j=1,2}$ and note that the expectation of the error-term in
the real exchange rate regression, conditional on the error in the real interest
rate equation, is given by:
$$
\mathbb{E}[\varepsilon_2|\varepsilon_1] = \frac{\omega_{21}}{\omega_{11}}\varepsilon_1.
$$

Note further that the Choleski factorisation of $\Omega$, $S = \{s_{ij}\}_{i,j=1,2}$, has the
form
$$
S = \begin{bmatrix}
\sqrt{\omega_{11}} & 0 \\
\omega_{21}\sqrt{\omega_{11}} & \sqrt{\omega_{22} - \omega_{21}^2/\omega_{11}}
\end{bmatrix}.
$$

Therefore,
$$
\varepsilon_2 = \frac{\omega_{21}}{\sqrt{\omega_{11}}}\varepsilon_1 + e_2\sqrt{\omega_{22} - \omega_{21}^2/\omega_{11}},
= \frac{\omega_{21}}{\omega_{11}}\varepsilon_1 + e_2\sqrt{\omega_{22} - \omega_{21}^2/\omega_{11}},
= \varepsilon_2|\varepsilon_1 + \text{residual}.
$$

Hence, the impulse response of the real exchange rate to the first of the
two orthogonal shocks (i.e. $e_1$) reflects the common component in both
the real interest rate differential and the real exchange rate. The second
shock can then be interpreted as non-fundamental in the sense that it reflects changes in expected ‘excess returns’ on holding the currency.

Table 7: Correlations using conditional forecasts.

<table>
<thead>
<tr>
<th>Country</th>
<th>2 variables</th>
<th>'Real Differential'</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canada</td>
<td>0.48 (0.10)</td>
<td>0.30 (0.11)</td>
</tr>
<tr>
<td>France</td>
<td>0.74 (0.08)</td>
<td>0.65 (0.09)</td>
</tr>
<tr>
<td>Germany</td>
<td>0.88 (0.06)</td>
<td>0.87 (0.06)</td>
</tr>
<tr>
<td>Italy</td>
<td>0.98 (0.02)</td>
<td>0.89 (0.05)</td>
</tr>
<tr>
<td>Japan</td>
<td>-0.25 (0.11)</td>
<td>0.00 (0.12)</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>-0.27 (0.11)</td>
<td>0.55 (0.10)</td>
</tr>
</tbody>
</table>

Standard errors in parentheses

In table 7 we provide the correlations between \( r - r^* \) and \( E(\Delta_{\infty} q) \) using this ‘conditional’ procedure. Under this procedure \( E(\Delta_{\infty} q) \) is now formed with knowledge only of \( \epsilon_1 \) and \( \epsilon_2 | \epsilon_1 \) (and past values of \( r - r^* \) and \( \Delta q \), of course) rather than \( \epsilon_1 \) and \( \epsilon_2 \) as in table 4. Again, the second column gives the results we obtain from the tri-variate ‘real differential’ model using the analogous procedure in which we condition on shocks to both output and the real interest rate.

For France, Germany and Italy the correlations are again high and there is little difference between the relative performance of the bivariate and ‘real differential’ models. For Canada, the base-line model performs slightly better in approximating the ups and downs of the real interest differential, whereas again for the UK the real differential model is clearly better. For Japan, again, neither estimate of \( E(\Delta_{\infty} q) \) reflects the movements in \( r - r^* \).

Table 8: Correlations from best model.

<table>
<thead>
<tr>
<th>Country</th>
<th>Correlation</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canada</td>
<td>0.48</td>
<td>2 vars, cond.</td>
</tr>
<tr>
<td>France</td>
<td>0.74</td>
<td>2 vars, cond.</td>
</tr>
<tr>
<td>Germany</td>
<td>0.88</td>
<td>2 vars cond or real diff uncond.</td>
</tr>
<tr>
<td>Italy</td>
<td>0.98</td>
<td>2 vars, uncond.</td>
</tr>
<tr>
<td>Japan</td>
<td>0.00</td>
<td>real diff, cond.</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>0.55</td>
<td>real diff, cond.</td>
</tr>
<tr>
<td>Mean</td>
<td>0.60</td>
<td></td>
</tr>
</tbody>
</table>
We summarise the results from this section as demonstrating that for most countries in our sample, very parsimonious models do reasonably well in providing measures of long-run expected changes in the real exchange rate that are highly correlated with real interest rate differentials. In table 8 we summarise for each country the highest correlation between \( r - r^* \) and \( E(\Delta q) \) generated by any of the four models. In all cases, except Japan, the best model generates correlations between \( r - r^* \) and \( E(\Delta q) \) that are considerably above 0.5. For Germany and Italy we even reach correlations of around 0.9. The average correlation attained is 0.6. Taking account of the relative parsimony of our models, we believe that these results should be viewed as very encouraging as they would seem to confirm that the RERI is in the data.

As we have argued earlier, the approach we have suggested in this section offers the advantage that it does not require proxies of \( E(\Delta q) \) and actual realisations of \( \Delta q \) to move closely together, at least not in the short- to medium-run. Hence, our procedure implicitly allows for the presence of persistent risk premia or other deviations from uncovered interest parity.

### 4.1 Cointegration, relative volatilities and exchange rate persistence

As we demonstrated in Section 3, the interaction of real interest rate differentials and the real exchange rate is reasonably well characterized by a trivially cointegrated system. Representing the RERI relation in the form of a trivially cointegrated system is also quite useful in understanding some closely related issues, such as the evident persistence of real exchange rates (the so-called 'PPP puzzle' of Rogoff (1995)) and why the transitory component of the real exchange rate is more volatile than the real interest differential. To see this we use the insight from Gonzalo and Granger (1995), Proietti (1997) and Johansen (1997) that the permanent and transitory parts of a multivariate time series can be expressed as a linear combination of the data themselves. Because the original decomposition by Gonzalo and Granger (1995) is also the most tractable analytically, we use it here \(^9\). The fundamental idea of Gonzalo and Granger (1995) is to decompose the non-stationary vector \( X_t \) according to

\[
X_t = (I - \Phi) X_t + \Phi X_t,
\]

(18)

\(^9\)For comparison we also calculated the transitory components according to the Proietti and Johansen procedures, with virtually identical results.
where \([I - \Phi]X_t\) is \(I(1)\), \(\Phi X_t\) is \(I(0)\). It must follow that \(\Phi = \psi\beta'\). By choosing \(\psi = \alpha(\beta'\alpha)^{-1}\) it must further follow that \([I - \Phi] = \beta_\perp(\alpha'_\perp\beta_\perp)^{-1}\alpha'_\perp\) and:

\[
X_t = \beta_\perp(\alpha'_\perp\beta_\perp)^{-1}\alpha'_\perp X_t + \alpha(\beta'\alpha)^{-1}\beta'X_t. \quad (19)
\]

In our trivially cointegrated system, it is now easy to verify that with \(X = [r_t - r_t^*\quad q_t]\) and \(\alpha = [\alpha_1\quad \alpha_2]'\), the transitory part of the real exchange rate, \(q_T^t\), can be written as

\[
-q_T^t = \lim_{k \to \infty} E(q_{t+k} - q_t) = -\frac{\alpha_2}{\alpha_1}(r_t - r_t^*) = \psi_1(r_t - r_t^*) \quad (20)
\]

Hence, the size of the transitory components of the real exchange rate is determined by the ratio of the loading coefficients that are associated with the error correction term. A closer look at equation (20) allows us to provide an integrated explanation of: i) the low power of cointegration tests in the context of the RERI; ii) why the transitory part of the real exchange rate is much more volatile than the real interest rate; and iii) why the real exchange rate may appear excessively persistent in univariate representations.

4.1.1 Power issues

The power of cointegration tests depends on the speed of error-correction. That is, the 'length' of the vector \(\alpha\) determines whether a cointegration test will reject the null or not. As is well known, there is a lot of unpredictable short-term volatility in real exchange rates, and therefore we should not be surprised to find that little error correction is detected in quarterly data.

4.1.2 Relative volatilities

While the overall speed of error-correction will determine the power of cointegration tests, the relative volatility of the transitory part of the real exchange rate and the real interest rate differential depends on the ratio of the adjustment coefficients, \(\alpha_2/\alpha_1\). Hence, a low power in the detection of the 'trivial' cointegrating relationship that represents the real interest rate differential is entirely compatible with a very sizable and highly volatile transitory component.
In table 9, we provide the relative volatilities of the transitory component of the real exchange rate and the real interest rate differential, calculated in two ways: first, using our forecast-based method, which we refer to as the 'Beveridge-Nelson decomposition', and one based on the decomposition (19), where we have imposed one cointegrating vector in the estimation of the VAR, but not restricted it to the unit vector. The most interesting result in table 9, is that our measures of expected changes in $q$ are in most cases much more volatile than the real interest rate differential itself (the bi-variate system for Canada and France being an exception). It is well known that real exchange rates are more volatile than any plausible fundamental (see, for example, Mussa (1986)). In fact, a transitory component of $q$ that is many times more volatile than the real interest rate differential itself is perfectly consistent with the RERI relationship. To see this note that under the maintained hypothesis of trivial cointegration the relative volatilities provide an estimate of $\alpha_k$ in equation (7). Recall that:

$$\alpha_k = \frac{1}{\theta^k - 1},$$

where $\theta$ is the persistence from the basic Meese-Rogoff (1988) adjustment equation (5). This implies that the more persistent deviations from the long-
run equilibrium exchange rate become, the more volatile should they appear relative to the real interest rate differential.

While in line with the theory, our results corroborate and extend those of Baxter (1994). Using univariate decompositions of the real exchange rate, Baxter finds that the transitory component reacts less than one-to-one with movements in real interest rate differentials, and she generally finds that $q^T$ is more volatile than $r - r^*$ in the multivariate decomposition.

### 4.1.3 Exchange rate persistence

Rogoff (1996) has argued that deviations from purchasing power parity have a half-life of between 3-5 years. Assuming a half-life of 15 quarters is in line with this observation and would amount to a value of $\theta$ of around 0.95. From equation (21) this could imply that the long-term real interest rate differential (based on 10 year bonds, i.e. $k = 40$) should be roughly as volatile as the misalignment itself, i.e. $a_{40} \approx -1$. But already with $\theta = 0.99$ the transitory part of $q$ should be more than 3 times as volatile as the real interest rate differential. Our measures of relative variability are mostly in excess of 3, implying values for $\theta$ that are extremely close to unity and that imply half-lives that are much too long to appear plausible. This result may seem to cast doubt on the empirical validity of the RERI relationship. However, we note that in the RERI relationship both variables are endogenous. By using a VAR and cointegration methods, we acknowledge that the eventual speed of adjustment depends on the dynamic interaction of the two variables. The estimates of $\theta$ that would be implied by the relative volatilities in table 9 are therefore best seen as indicative and are likely to be highly misleading as indicators of exchange rate persistence. We also note that the estimates of $\theta$ reported in the literature are surrounded by a huge degree of uncertainty. This is why, throughout the paper, we have focused on correlation measures in assessing the RERI, rather than on regression coefficients.

### 5 Summary of Conclusions

In this paper we have re-examined the real exchange rate - real interest rate (RERI) relationship using data for six US dollar bilateral exchange rates, over the period 1978 to 1997. Many previous tests of this relationship have involved attempting to cointegrate measures of a real exchange rate with a measure of a country’s real interest differential. However, following Baxter (1994) the derivation of the RERI relationship suggests that such a method is likely to be flawed since if the real exchange rate is integrated of order one,
the real interest differential must be stationary. One way of justifying the use of cointegration methods in the context of the RERI is in terms of what we have called a trivially cointegrated system, and in this paper we documented substantial evidence for this approach. For example, estimating a bi-variate system of the real exchange rate and the real interest rate differential with one unrestricted cointegrating vector imposed regularly generates a transitory component of the real exchange rate which is highly correlated with the real interest rate differential. The same result is obtained when the expected real exchange rate, derived from an estimated VAR model, is correlated with the real interest differential. Such correlations follow naturally from the model derivation. We have also shown that a failure of earlier analyses to detect a stationary real interest differential may be due to the extremely low power of cointegration tests in this particular environment. These findings indicate that the real interest rate differential can indeed be characterized as a stationary process that forms the transitory component of the real exchange rate.

As an alternative to cointegration-based tests of the RERI, we proposed a VAR-based approach. This involves taking the projection for the change in the real exchange rate from a bivariate VAR, consisting of the change in the real exchange rate and the real interest differential, and correlating this with the real interest differential. We argued that this kind of test is much closer to the spirit of the RERI relationship than many extant tests. A more refined variant of this test involved correlating the component of the change in the real exchange which is common to both the real interest rate and the real exchange rate. We demonstrated how the common component could be derived from a bivariate VAR of the real interest differential and the change in the exchange rate using a Choleski decomposition. In sum, our correlations-based approach produced measures of long-run expected changes in the exchange rate which are highly correlated with real interest rate differentials. Finally, we also demonstrated that the expected changes in the exchange rate are, in the majority of cases, more volatile than the real interest differential itself and we show that this is consistent with the RERI model. Finally, using a measure of persistence derived from an adjustment equation, we have shown the value of using a correlation-based approach, rather than one based on a regression analysis, to assessing the RERI.
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