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# THE KALAI-SMORODINSKY SOLUTION IN LABOR- MARKET NEGOTIATIONS

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# THE KALAI-SMORODINSKY SOLUTION IN LABOR-MARKET NEGOTIATIONS

## Abstract

Authors who consider efficient bargaining on the labor market predominantly focus on the Nash-bargaining solution. It seems, however, that actual labor market negotiations between an employers' federation and a labor union are often characterized by mutual concessions, which may be accounted for by an application of the Kalai-Smorodinsky solution to labor-market negotiations. Correspondingly, we investigate how a government can influence the equilibrium on the labor market by changing the reservation wage when the equilibrium is determined by the Kalai-Smorodinsky solution. We find that the induced employment effects may differ substantially when compared with the Nash bargaining solution. Hence, substituting the Kalai-Smorodinsky by the Nash bargaining solution is not innocuous, when actual negotiations are characterized by mutual incremental concessions.

JEL Code: C78, H39, H55, J40, J51.

Keywords: labor market negotiations, Kalai-Smorodinsky solution, Nash-bargaining solution, reservation wage, fiscal and social policies.

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# 1 Introduction

In view of substantial unemployment in most OECD countries, governments aim at increasing employment. Among others, various policies target at the difference between the net real wage of the low income groups and real compensation payments that accrue to the unemployed. These compensation payments encompass a variety of monetary and non-monetary benefits, depending on the specific institutions of the labor market, the tax system, the social security system *etc.* Taken together, these benefits, which may also include income earned in the shadow economy, determine a net real reservation wage below which laborers are not willing to give up leisure. Hence, for reasons of incentive compatibility, there must be a minimum wage differential between the low-income wage rate and the (all inclusive) unemployment benefits.

It seems plausible, that a reduction in the reservation wage will increase employment. Yet, any change of the reservation wage plausibly affects the market wage; and the sensitivity of the equilibrium wage rate with respect to the reservation wage hinges on the particular institutions of the labor market: How and when are equilibrium wages determined? If they are negotiated, who are the negotiating parties? What is their respective bargaining power during negotiations? And most importantly, how is the conflict ultimately settled? All of these institutional features arguably affect the speed and the extent of wage adjustments when public policies interfere with parameters determining the reservation wage.

In this paper we investigate the responsiveness of the equilibrium wage on changes in the reservation wage. In order to achieve this, we do not scrutinize the different channels through which a government may manipulate the reservation wage, but take the reservation wage as a policy variable of its own. Clearly, as long as the equilibrium of the labor market is established on the labor demand curve, any decrease in the reservation wage unambiguously leads to an increase in employment, provided that labor demand is decreasing. Thus, when labor market institutions are best described by, for example, the right-to-manage or the monopolistic-union model, a more restrictive social policy unambiguously results in lower wages and thus higher employment.

Yet, if we believe in the efficiency of our labor market institutions, *i. e.*, in the rationality of the bargaining parties, these labor market models are somewhat unsatisfactory as they lead to inefficient outcomes, which leave some scope for renegotiations. Also from a positive perspective, we may actually question whether labor market negotiations between an employers' federation and a labor union are best described by the right-to-manage model or the monopolistic-union model. Rather, it seems that these negotiations are better represented by a bargaining process in the course of which each party makes concessions with respect to its initial claim. Both parties gradually reduce their claims until an agreement is reached eventually. There is little reason to believe that both parties forego any potential gains, *i. e.*, that the outcome will be inefficient.

For these reasons, the descriptive (and hence explanatory) power of labor market models yielding efficient outcomes as the resolution of the labor-market conflict seems to be quite high for some OECD labor market institutions. Given this, efficient labor market models should be seriously considered in public finance analysis. McDonald and Solow (1981) were the first to study efficient bargaining in labor markets. Their focus was on the Nash bargaining solution (Nash, 1950) but they also considered the Kalai-Smorodinsky solution (Kalai and Smorodinsky, 1975) as an interesting alternative. However, to the best of our knowledge, the subsequent literature in both labor economics and public finance has exclusively studied the Nash bargaining solution, taking it as *the* representative for an efficient labor market outcome.<sup>1</sup> Part of the popularity of the Nash solution may be due to its analytic simplicity. However, while the Nash bargaining solution is characterized by the property of independence of irrelevant alternatives, other solution concepts are characterized by monotonicity properties which may be very appealing as well. In this context, the predominant concept is the Kalai-Smorodinsky solution, defined by equal proportional concessions of both parties, a feature which well reflects actual labor market negotiations in several countries.

Correspondingly, in this paper we investigate the responsiveness of the Kalai-

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<sup>1</sup>For recent contributions see Pissarides (1998), Fuest and Huber (2000), Garino and Martin (2000), Vannini and Bughin (2000), Petrakis and Vlassis (2000) and Strand (2002).

Smorodinsky solution with respect to changes in the reservation wage. We show that the comparative statics results for the Nash bargaining solution and the Kalai-Smorodinsky solution differ, and may even differ qualitatively, although both solutions yield Pareto efficient outcomes and under specific circumstances even induce identical outcomes. In particular, while a lower reservation wage unambiguously leads to higher employment (and to a lower wage rate) for the Nash bargaining solution, the same policy may have an ambiguous employment effect for the Kalai-Smorodinsky solution. Indeed there are specifications of the utility and production functions where a higher reservation wage leads to a *higher* employment level. Such a perverse effect may represent the exception rather than the rule if one restricts to standard utility and production functions frequently used in applications. However, in the universe of functions satisfying the standard differentiability and concavity assumptions a positive employment effect is not a pathological case. This shows that the Kalai-Smorodinsky solution cannot simply be substituted by the Nash bargaining solution on grounds of tractability, when actual labor market negotiations call for an application of the Kalai-Smorodinsky solution. Otherwise one may easily obtain distorted employment effects, and hence may derive biased policy recommendations. This finding is also backed by the analysis of two other bargaining solutions, the egalitarian bargaining solution proposed by Kalai (1977), where parties achieve equal gains over the disagreement point, and the equal-loss solution proposed by Chun (1988), which is defined by equal concessions. Again, the same ambiguity of the employment effect arises. As a consequence, the choice of a bargaining solution is far from being innocuous since the comparative statics effects may differ substantially.

The outline of the paper is as follows. In Section 2 we introduce the labor market model and define the Kalai-Smorodinsky solution for the resulting wage bargaining problem. Section 3 contains the comparative statics results for the Kalai-Smorodinsky solution which we compare with the corresponding effects for the Nash bargaining solution in Section 4. Finally, Section 5 concludes. All proofs are in Appendix A and a discussion of other common bargaining solutions can be found in Appendix B.

## 2 Equilibrium of the Labor Market

On the supply side of the labor market, there is a labor union representing all  $N$  laborer households, which we assume to have identical preferences. Each household is either employed at the full regular working time or has no job at all. An employed household receives a wage income equal to the net (real) wage  $w$  and obtains a corresponding utility level of  $v(w)$ , where  $v : \mathbb{R}_+ \rightarrow \mathbb{R}$  is the household's utility function. An unemployed household, however, attains some fixed utility from the consumption of leisure and unemployment benefits, say  $\bar{v}$ . We assume that  $v$  is twice continuously differentiable with  $v' > 0, v'' < 0$ , and that there exists  $\bar{w} \geq 0$  with  $v(\bar{w}) = \bar{v}$ . Hence,  $\bar{w} := v^{-1}(\bar{v})$  is the reservation wage below which no household is willing to work. That is, labor supply equals zero for all  $w < \bar{w}$ , equals  $N$  for all  $w > \bar{w}$ , and is indeterminate, *i. e.*,  $L \in [0, N]$  for  $w = \bar{w}$ .

Let a labor union act in accordance with its members' preferences. To be specific, assume that the labor union maximizes the sum of its members' utilities, or equivalently its representative members' expected utility:

$$U_1(w, L) := Lv(w) + (N - L)\bar{v},$$

where  $L$  denotes the number of workers who eventually find a job. Differentiation of the union's utility with respect to  $w$  and  $L$ , holding the level of  $U_1$  constant at, say,  $u_1$ , shows that the union's indifference curves are downward sloping whenever  $w$  is above the reservation wage  $\bar{w}$ :

$$\left. \frac{dw}{dL} \right|_{U_1=u_1} = -\frac{v(w) - \bar{v}}{Lv'(w)} < 0.$$

On the labor demand side, there is an employers' federation acting on behalf of all firms' interest, that is, it seeks to maximize firms' aggregate profits

$$U_2(w, L) := f(L) - wL,$$

where  $f : \mathbb{R}_+ \rightarrow \mathbb{R}_+$  is the firms' aggregate production function. We assume that  $f$  is twice continuously differentiable with  $f(0) = 0, f' > 0, f'(L) \rightarrow \infty$  for  $L \rightarrow 0, f'' < 0$ , and  $f(N)/N < \bar{w}$ . Differentiating the employers' profits with respect to  $w$  and  $L$ ,

holding the level of  $U_2$  constant at  $u_2$ , reveals that their indifference curves (firms' iso-profit curves) are increasing in  $L$  until  $f'(L) = w$ , and decreasing afterwards:

$$\left. \frac{dw}{dL} \right|_{U_2=u_2} = \frac{f'(L) - w}{L}.$$

Note that  $w = f'(L)$  characterizes the aggregate inverse labor-demand curve, the slope of which is  $f''(L) < 0$ .

## The Wage Bargaining Problem

In this paper we consider bargaining problems on the labor market when both the wage rate and the employment level are negotiated between the labor union and the employers' federation. An abstract two-party *bargaining problem* is characterized by a tuple  $(S, d)$ , where  $S \subset \mathbb{R}^2$  is a set of feasible utility allocations and  $d \in S$  is the *disagreement, dispute, or break off point*, which is the outcome of the bargaining game if the parties fail to agree on a utility allocation in the feasible set. More precisely,  $d$  represents the utility the parties receive during the dispute (in the context of a time-preference model) or the utility they receive in the event that the bargaining process breaks down (in the framework of an exogenous risk of breakdown).<sup>2</sup> In our case of wage-employment bargaining we naturally assume that the disagreement point is the utility allocation obtained when all workers are locked out, are at strike, or obtain a job in an other industry (depending on the particular context/interpretation). The wage bargaining game  $(S, d)$  is then given by  $d = (U_1(w, 0), U_2(w, 0)) = (N\bar{v}, 0)$  for arbitrary  $w \geq 0$  and

$$S = \{u \in \mathbb{R}^2 \mid u \leq U(w, L) \text{ for some } w, L, \text{ with } 0 \leq w, 0 \leq L \leq N, \text{ and } U(w, L) \geq d\},$$

where  $U(w, L) = (U_1(w, L), U_2(w, L))$ . That is, the feasible set consists of all utility allocations that are dominated by an individually rational utility allocation which can be achieved by an agreement on some wage rate and employment level.<sup>3</sup> Standard assumptions on a bargaining game  $(S, d)$  are that  $S$  is convex and closed, that the set

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<sup>2</sup>For more details confer Binmore, Rubinstein, and Wolinsky (1986).

<sup>3</sup>A utility allocation  $u \in S$  is *individually rational* if  $u \geq d$ .



of individually rational utility allocations  $\{u \in S \mid u \geq d\}$  is bounded and that both parties have a strong incentive to bargain, *i. e.*, that there exists  $u \in S$  with  $u_i > d_i$  for  $i = 1, 2$ . In Appendix A we prove that the wage bargaining game we defined indeed satisfies these assumptions.

A *solution* on a class of bargaining problems  $\mathcal{B}$  is a mapping  $\xi : \mathcal{B} \rightarrow \mathbb{R}^2$  such that  $\xi(S, d) \in S$  for all  $(S, d) \in \mathcal{B}$ . Many bargaining solutions have been studied in the literature (for an overview see Peters, 1992), the most prominent one being the (asymmetric) *Nash bargaining solution* (see Nash, 1950), defined as

$$\xi^N(S, d) = \operatorname{argmax} \{(u_1 - d_1)^\mu (u_2 - d_2)^{1-\mu} \mid u \in S, u \geq d\}, \quad (S, d) \in \mathcal{B},$$

where  $\mu \in (0, 1)$  denotes the bargaining strength of party one.<sup>4</sup> The Nash bargaining solution is often used in applications due to its analytical tractability. However, there are several other bargaining solutions with appealing properties selecting a different efficient outcome than the Nash solution. In particular, we believe that from a descriptive point of view the result of wage negotiations is better described by bargaining solutions that are based on some principle of proportional gains or concessions.

Hence, in this paper we assume that the outcome of labor market negotiations is determined by the solution concept introduced by Raiffa (1953) and Kalai and Smorodinsky (1975), *KS-solution* henceforth. The KS-solution is characterized by equal proportional concessions of both parties from their respective maximally attainable utility levels. More precisely, for a given bargaining problem  $(S, d)$  define the *utopia point*  $u^* = (u_1^*, u_2^*)$  by

$$u_i^* = \max\{u_i \mid u \in S, u_j \geq d_j \text{ for } j \neq i\} \quad i = 1, 2.$$

The KS-solution is then given by

$$\xi^{KS}(S, d) = d + \bar{\lambda}(u^* - d),$$

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<sup>4</sup>More precisely,  $\mu$  reflects either asymmetries in the bargaining procedure — for example, the time which elapses between  $i$ 's reaction to  $j$ 's proposal and the next instant in time at which  $i$  proposes to  $j$  — or asymmetries about the environment — for example, different estimates of the probability of an exogenous breakdown of the negotiations. For more details, confer again Binmore, Rubinstein, and Wolinsky (1986).

where  $\bar{\lambda} = \max\{\lambda \in \mathbb{R} \mid d + \lambda(u - d) \in S\}$ . If we define the *KS-curve* as the set of utility allocations  $u \geq d$  with

$$u_2 = d_2 + \frac{u_2^* - d_2}{u_1^* - d_1} (u_1 - d_1),$$

and the *Pareto-curve* as the set of Pareto efficient utility allocations, then the KS-solution is the intersection of the KS- with the Pareto-curve.<sup>5</sup> In the  $(w, L)$ -space the feasible part of the KS-curve is implicitly given by<sup>6</sup>

$$U_2(w, L) = d_2 + \frac{u_2^* - d_2}{u_1^* - d_1} (U_1(w, L) - d_1). \quad (1)$$

while the Pareto-curve is the set of  $(w, L)$  with  $U(w, L) \geq d$  for which both parties' indifference curves are tangent to each other, *i. e.*,

$$\frac{v(w) - \bar{v}}{v'(w)} = w - f'(L). \quad (2)$$

In order to determine the utopia point, we first calculate the maximal utility level the employers' federation may obtain when the utility of the labor union is at least  $d_1$ . That is, we solve

$$\max_{w \geq 0, 0 \leq L \leq N} U_2(w, L) \quad \text{s. t.} \quad U_1(w, L) \geq d_1 = N\bar{v}.$$

It is immediate to see that the constraint is binding at the solution,  $U_1(w, L) - N\bar{v} = 0$ , and hence, given our assumptions on  $f$ , the unique feasible utility allocation most favorable for the employers' federation is induced by the pair  $(\bar{w}, \bar{L})$  with  $0 < \bar{L} = f'^{-1}(\bar{w}) < N$  so that  $(d_1, u_2^*) = (N\bar{v}, U_2(\bar{w}, \bar{L}))$ . Similarly, the individually rational utility level most favorable to the labor union is achieved by the pair  $(\hat{w}, \hat{L})$  which solves

$$\max_{w \geq 0, 0 \leq L \leq N} U_1(w, L) \quad \text{s. t.} \quad U_2(w, L) \geq d_2 = 0.$$

It is straightforward to see that firms are left with zero profits, *i. e.*,  $f(\hat{L}) = \hat{w}\hat{L}$ . Moreover,  $(\hat{w}, \hat{L})$  must be Pareto efficient, *i. e.*, eqn (2) is satisfied.<sup>7</sup> The resulting

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<sup>5</sup> $u \in S$  is *Pareto efficient*, if there exists no  $u' \in S$  with  $u' \geq u$  and  $u' \neq u$ .

<sup>6</sup>Subsequently we restrict our attention to the feasible part of the KS-curve.

<sup>7</sup>To see this observe that the constraints  $0 \leq L \leq N$  are not binding at the solution  $\hat{L}$  since  $U_1(w, \bar{L}) > d_1$  for  $w = f(\bar{L})/\bar{L}$  and  $U_1(w, N) < d_1$  for  $w = f(N)/N$  because  $f(N)/N < \bar{w}$  by assumption.

utility allocation is given by  $(u_1^*, d_2) = (u_1^*, 0) = (U_1(\hat{w}, \hat{L}), U_2(\hat{w}, \hat{L}))$ . From the definitions of  $(\bar{w}, \bar{L})$  and  $(\hat{w}, \hat{L})$  it should be clear that  $d_1 = U_1(w, 0) = U_1(\bar{w}, L)$  for all  $L$  and  $w$  and that  $d_2 = U_2(w, 0) = U_2(\hat{w}, \hat{L})$  for all  $w$  must hold.

Now, substituting for  $u_1$ ,  $u_2$ ,  $u^*$ , and  $d$ , the KS- and the Pareto-curve in  $(w, L)$ -space are given by the set of points  $(w, L)$  that are individually rational ( $U(w, L) \geq d$ ) and satisfy

$$\phi_1(w, L) := (f(L) - wL)(v(\hat{w}) - \bar{v})\hat{L} - (f(\bar{L}) - \bar{w}\bar{L})(v(w) - \bar{v})L = 0, \quad (3)$$

$$\phi_2(w, L) := v(w) - \bar{v} - v'(w)(w - f'(L)) = 0, \quad (4)$$

respectively. Observe that individual rationality immediately implies that  $w \geq \bar{w}$ . Eqs (3) and (4) represent an equation system, the solution of which are the wage and the employment level negotiated by the labor union and the employers' federation. In order to reduce the notational effort we continue to use  $w$  and  $L$  for the *negotiated* wage and employment level.

If we treat the wage as a function of employment, then both curves are the graphs of well defined differentiable functions as we will show now. For the KS-curve this can be seen immediately. Implicitly differentiating the KS-curve with respect to  $w$  and  $L$  we obtain that the KS-curve is decreasing in  $L$  for  $(w, L)$  close to the Pareto-curve, in particular at the point of intersection of the KS- with the Pareto-curve:

$$\left. \frac{dw}{dL} \right|_{\phi_1=0} = \frac{[f'(L) - w][v(\hat{w}) - \bar{v}]\hat{L} - [f(\bar{L}) - \bar{w}\bar{L}][v(w) - \bar{v}]}{L \left( [v(\hat{w}) - \bar{v}]\hat{L} + [f(\bar{L}) - \bar{w}\bar{L}]v'(w) \right)} < 0. \quad (5)$$

Concerning the Pareto-curve it is straightforward to show that for all  $L \in [\bar{L}, \hat{L}]$  there exists a unique  $w = w(L) \geq \bar{w}$  such that  $(w, L)$  satisfies eqn (4) (see Appendix A).  $w(L)$  is increasing in  $L$  for  $L \in (\bar{L}, \hat{L}]$ , which follows from an application of the implicit function theorem:

$$w'(L) = \left. \frac{dw}{dL} \right|_{\phi_2=0} = \frac{f''(L)(v'(w))^2}{v''(w)(v(w) - \bar{v})} = \frac{f''(L)v'(w)}{v''(w)(w - f'(L))} > 0, \quad (6)$$

for all  $L \in (\bar{L}, \hat{L})$  and  $\lim_{L \rightarrow \hat{L}} w'(L) = \infty$ . Therefore, there exists no  $L > \hat{L}$  such that  $(L, w(L))$  is individually rational for the employers' federation. Hence, the Pareto

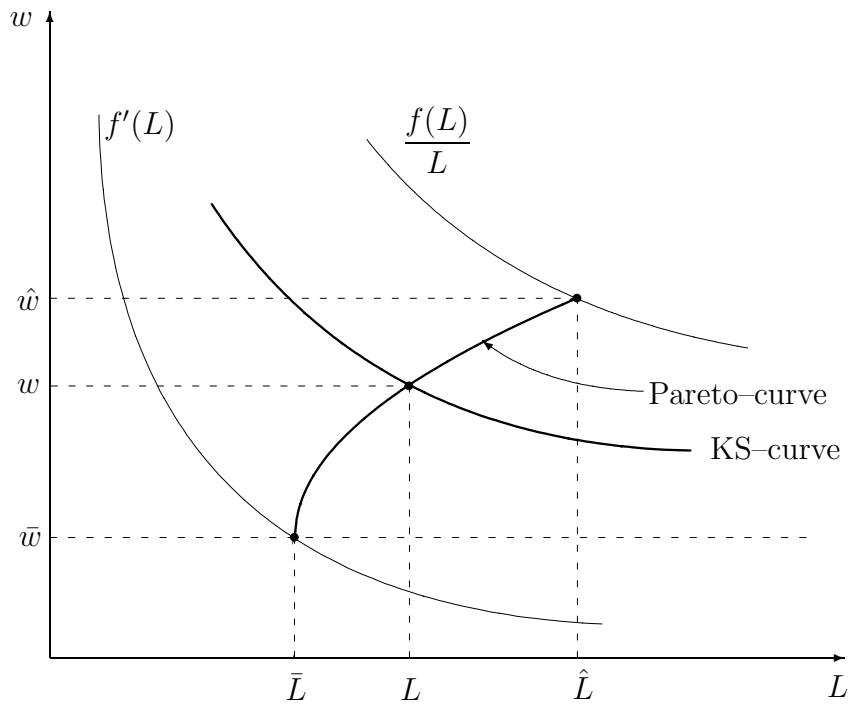


Figure 1: The KS-solution in  $(L, w)$  space.

curve is given by the set of points  $(L, w(L))$ , where  $L \in [\bar{L}, \hat{L}]$  and geometrically lies between the inverse labor demand curve, characterized by  $w = f'(L)$  and the firms' zero profit curve characterized by  $w = f(L)/L$ . Since the Pareto curve originates at  $(\bar{L}, \bar{w})$  — the equilibrium of a competitive labor market — wage-employment negotiations lead to both a higher wage and a higher employment level than for a competitive labor market — and so must the wage share, defined as  $\omega := wL/f(L)$ .<sup>8</sup>

The KS and the Pareto curve, as well as the resulting market equilibrium are illustrated in Figure 1.

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<sup>8</sup>This conclusion depends however on the implicit assumption that the economies we compare feature the same production technologies. This assumption may be questioned, though, for it seems that labor market institutions and production technologies are not independent of each other. Rather, empirical evidence suggests that there are specific combinations of labor-market institutions and technologies all of which lead to about the same wage share, as this number is roughly constant across most OECD countries.

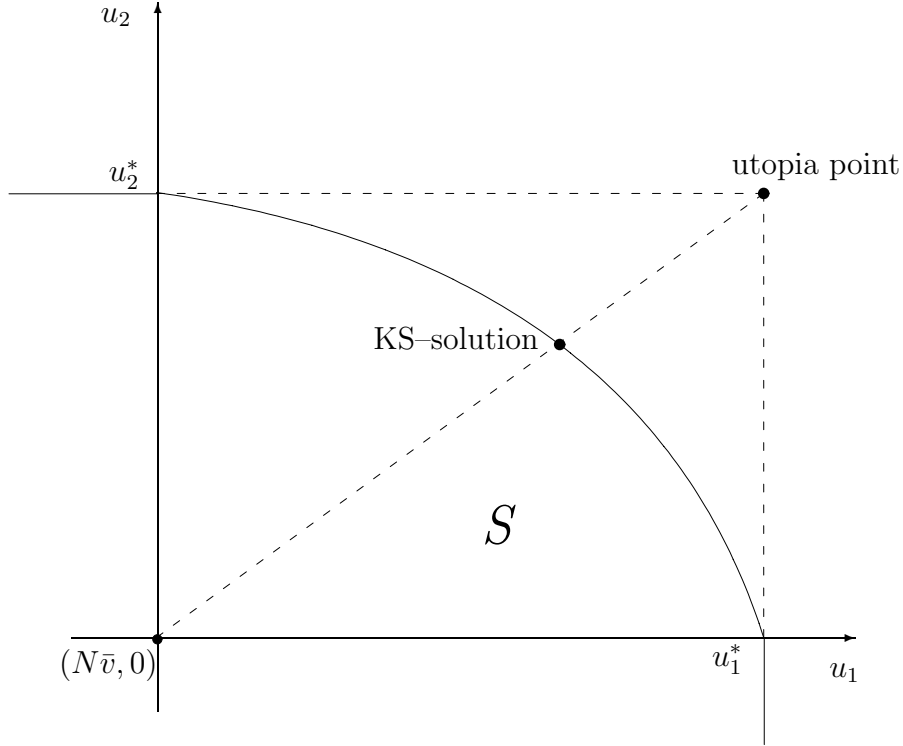


Figure 2: The KS-solution in utility space.

We will now scrutinize the shape of the Pareto-curve and the KS-curve in the utility space. Since we already know that the KS-curve is a straight line through  $d$  in the  $(u_1, u_2)$ -space, it remains to investigate the properties of the Pareto-curve. The Pareto efficient utility allocations  $(u_1, u_2)$  are given by

$$u_1 = U_1(w(L), L) = L(v(w(L)) - \bar{v}) + N\bar{v}, \quad (7)$$

$$u_2 = U_2(w(L), L) = f(L) - w(L)L, \quad (8)$$

where  $L \in [\bar{L}, \hat{L}]$ . Eqn (8) implicitly defines  $L$  as a function of  $u_2$ . Implicitly differentiating eqn (8) with respect to  $u_2$ ,  $0 < u_2 < u_2^*$ , gives

$$L'(u_2) = \frac{1}{f'(L) - w(L) - Lw'(L)} = \frac{v'(w)}{\bar{v} - v(w) - Lw'(L)v'(w)} < 0,$$

where we have written  $L = L(u_2)$ ,  $w = w(L)$  in order to simplify the notation. The Pareto-curve is the set of all points  $(u_1(u_2), u_2)$  with

$$u_1(u_2) := U_1(w(L(u_2)), L(u_2)) = L(u_2) [v(w(L(u_2))) - \bar{v}] + N\bar{v},$$

and we can determine the first and second order derivative of  $u_1(u_2)$  as

$$u_1'(u_2) = L'(u_2) [v(w) - \bar{v} + Lv'(w)w'(L)] = -v'(w) < 0, \quad (9)$$

$$u_1''(u_2) = -v''(w)w'(L)L'(u_2) < 0, \quad (10)$$

where again  $L = L(u_2)$  and  $w = w(L)$ . Hence, the Pareto curve is strictly decreasing and strictly concave in the utility space, as shown in Figure 2.

### 3 Comparative Statics

We first analyze how the negotiated employment and wage levels change with respect to the reservation wage  $\bar{w}$ , or equivalently, with respect to the reservation utility  $\bar{v}$ . This enables us to scrutinize the sensitivity of the equilibrium of the labor market with respect to changes in the reservation wage, or more generally with respect to public policy. To this end we rewrite eqn (3) as

$$(f(L) - wL)(u_1^* - N\bar{v}) = (v(w) - \bar{v})Lu_2^*. \quad (11)$$

Eqs (4) and (11) implicitly define  $w$  and  $L$  as functions of  $\bar{v}$ . In order to differentiate eqs (4) and (11) with respect to  $\bar{v}$ , we need to know the effect of a higher reservation utility level on the utopia point,  $u^*$ .

With a slight abuse of notation we write  $(u_1^*, u_2^*) = (U_1(\hat{w}, \hat{L}, \bar{v}), U_2(\bar{w}, \bar{L}, \bar{v}))$ . Recalling that  $(\hat{w}, \hat{L})$  and  $(\bar{w}, \bar{L})$  are the maximizers of  $U_1$  and  $U_2$  respectively, subject to individual rationality, we conclude that

$$\begin{aligned} \frac{du_1^*}{d\bar{v}} &= \frac{\partial U_1}{\partial \bar{v}}(\hat{w}, \hat{L}, \bar{v}) = N - \hat{L} \geq 0, \\ \frac{du_2^*}{d\bar{v}} &= \frac{\partial U_2}{\partial \bar{v}}(\bar{w}, \bar{L}, \bar{v}) = -\frac{\bar{L}}{v'(\bar{w})} < 0, \end{aligned}$$

by the envelope theorem. Now, using this result  $\frac{dd_1}{d\bar{v}} = N$  and  $\frac{dd_2}{d\bar{v}} = 0$ , differentiation of the KS- and the Pareto-curve with respect to  $\bar{v}$  yields

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \frac{dL}{d\bar{v}} \\ \frac{dw}{d\bar{v}} \end{bmatrix} = \begin{bmatrix} b \\ 1 \end{bmatrix}$$

where

$$\begin{aligned}
a_{11} &:= (u_1^* - d_1) \left( \frac{f(L)}{L} - f'(L) \right) > 0, & a_{12} &:= L (u_1^* - d_1 + u_2^* v'(w)) > 0, \\
a_{21} &:= v'(w) f''(L) < 0, & a_{22} &:= -v''(w) (w - f'(L)) > 0, \\
b &:= \frac{v(w) - \bar{v}}{v'(\bar{w})} L \bar{L} - (f(L) - wL) \hat{L} + Lu_2^* > 0.^9
\end{aligned}$$

Solving this for the desired derivatives we obtain

$$\frac{dw}{d\bar{v}} = \frac{1}{D_1} (a_{11} - a_{21}b) > 0, \tag{12}$$

$$\frac{dL}{d\bar{v}} = \frac{1}{D_1} (a_{22}b - a_{12}), \tag{13}$$

with  $D_1 := a_{11}a_{22} - a_{12}a_{21} > 0$ . While a higher reservation utility leads to a higher wage rate, the corresponding employment effect is ambiguous. Graphically, in an  $(L, w)$ -diagram an increase in the reservation utility (wage) shifts both the Pareto-curve and the KS-curve upwards. That is, for any given employment level  $L$  both the corresponding wage rate on the Pareto-curve as well as on the KS-curve increases:

$$\begin{aligned}
\left. \frac{dw}{d\bar{v}} \right|_{\substack{\phi_1=0 \\ L \text{ const.}}} &= - \frac{(f(L) - wL) \hat{L} - Lu_2^* - L \bar{L} \frac{v(w) - \bar{v}}{v'(\bar{w})}}{L(u_1^* - N\bar{v}) + Lu_2^* v'(w)} > 0, \\
\left. \frac{dw}{d\bar{v}} \right|_{\substack{\phi_2=0 \\ L \text{ const.}}} &= - \frac{1}{(w - f'(L)) v''(w)} > 0.
\end{aligned}$$

As both curves shift upward, a clear-cut increase in the bargained wage emerges. The employment effect, however, is unclear, just as our analysis shows. For many standard utility and production functions it will be positive. However, these functions are mainly chosen for mathematical convenience and it is impossible to justify them empirically. If instead one considers the whole class of utility and production functions that satisfy our assumptions, a positive employment effect is by no means a pathological exception. For any given utility and production function generating a negative employment effect we can modify the utility function on some closed interval such that it still satisfies our assumptions and such that the employment effect

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<sup>9</sup>That  $b$  is positive can be seen from eqn (11):  $\hat{L}(f - wL) = Lu_2^* \frac{v(w) - \bar{v}}{v'(\bar{w}) - \bar{v}} < Lu_2^*$ .

is reversed. In fact there exists a continuum of these modified utility functions. In Appendix A we provide such an example of a utility and production function where a higher reservation wage/utility level leads to higher employment.

This comparative statics effect is a notable result. It never occurs under the Nash bargaining solution as we will show in the following section.

## 4 A Comparison with the Nash-Bargaining Solution

In this section we contrast the comparative static effects of the labor market when the wage rate and the employment level are determined according to the KS-solution with those of the most popular solution concept in labor economics: the Nash-bargaining solution. We shall see that although both equilibrium concepts induce a Pareto efficient outcome and may in some cases (*e. g.*, linear utility) lead to the same equilibrium outcome, their comparative statics results differ in a significant way.

For the (asymmetric) Nash bargaining solution the negotiated contract is obtained by solving

$$\max_{w \geq 0, 0 \leq L \leq N} [U_1(w, L) - d_1]^\mu [U_2(w, L) - d_2]^{1-\mu} = [L(v(w) - \bar{v})]^\mu [f(L) - wL]^{1-\mu},$$

where the weight  $\mu \in (0, 1)$  reflects the distribution of bargaining power between the union and the firm. The two first-order conditions of this optimization problem yield the Pareto (or contract) curve, given by eqn (2), and the Nash curve, given by

$$w = \mu \frac{f(L)}{L} + (1 - \mu) f'(L). \quad (14)$$

The Nash bargaining solution thus induces an agreement on the contract curve, eqn (4), where the wage rate equals the weighted average of the marginal and average productivity of labor.

Implicit differentiation of eqs (4) and (14) with respect to  $\bar{v}$  yields

$$\begin{aligned} \frac{dw}{d\bar{v}} &= -\frac{1}{D_2} \left( \mu \frac{f'(L)L - f(L)}{L^2} + (1 - \mu) f''(L) \right) > 0, \\ \frac{dL}{d\bar{v}} &= -\frac{1}{D_2} < 0, \end{aligned}$$



where  $D_2 := (w - f'(L))v''(w) \left[ \mu \frac{f'(L)L - f(L)}{L^2} + (1 - \mu)f''(L) \right] - v'(w)f''(L) > 0$ . Hence, any increase in the reservation utility (wage rate) leads to a higher negotiated wage and reduces employment.

Contrasting this observation with our comparative statics results of the KS-solution, we arrive at the following result. Although both the KS- and the Nash-bargaining solution induce a labor market equilibrium on the Pareto curve, an increase in the reservation utility (wage rate) leads to a lower employment level under the Nash bargaining solution, but may possibly lead to a higher employment level for the KS-solution. Under both labor-market equilibrium concepts a higher reservation wage induces a higher wage rate.

It is important to note that the ambiguity concerning the employment effect is not an anomaly of the KS-solution. In Appendix B we show that the same ambiguity arises for two other well known bargaining solutions, namely the *egalitarian solution* introduced by Kalai (1977) and the *equal loss solution* introduced by Chun (1988). Since our analysis therewith has covered the most prominent bargaining solutions we can safely conclude that the Nash solution is exceptional in giving the clear prediction that the employment level decreases with the reservation wage. Observe that this is due to the fact that the Nash-curve, eqn (14), is independent of the reservation utility while the corresponding curves for the other bargaining solutions always shift with a change in the reservation utility.

Given that the predictions obtained for the Nash solution may be qualitatively different from those obtained for other standard bargaining solutions, our results should be taken as a serious warning against a routine application of the Nash bargaining solution in models of labor market negotiations on grounds of tractability. Contrary to what is sometimes suggested in the literature, an application of the Nash bargaining solution is not without loss of generality.

## Linear Utility

Finally we scrutinize an interesting limiting case, which we have ruled out so far: the case when an employed worker's utility is linear,  $v(w) = w$ . In this case, the slope of

the union's indifference curves equals  $-(w - \bar{w})/L$ , implying that the Pareto-curve reduces to

$$\bar{w} = f'(L).$$

Since this is independent of  $w$ , the Pareto-curve is vertical in the  $(L, w)$ -space at  $\bar{L}$ , from which we infer that  $L = \bar{L} = \hat{L}$  must hold. Straightforward calculations show that the KS-curve then reduces to

$$w = \frac{1}{2}(\bar{w} + \hat{w}) = \frac{1}{2}\left(f'(L) + \frac{f(L)}{L}\right),$$

where we have made use of the fact that  $f(L) = f(\hat{L}) = \hat{w}\hat{L} = \hat{w}L$ . The bargained wage equals the (unweighted) arithmetic average of the minimum and the maximum wage rate, or of the marginal and the average product of labor. Obviously, the employment level is independent of the reservation utility,  $\bar{v}$ , while the wage is increasing in  $\bar{v}$ .

If we are even more specific and apply a Cobb–Douglas production function,  $f(L) = L^\alpha$ , we obtain

$$\hat{w} = \frac{1}{\alpha} \bar{w} \quad \text{and} \quad w = \frac{1 + \alpha}{2\alpha} \bar{w}.$$

This also implies  $w = \frac{1+\alpha}{2} \frac{f(L)}{L}$ , or in terms of the wage share

$$\omega = \frac{1 + \alpha}{2}.$$

Since for a competitive labor market we have  $\omega = \alpha$ , wage-employment negotiations induce the same employment level but a higher wage and thus a higher wage share as compared with the competitive labor market.

Note that the linearity of the household's utility function implies that the wage and employment level (and hence the wage share) given by the KS-solution coincides with their corresponding levels given by the (symmetric) Nash bargaining solution. This is due to the fact under risk neutrality of households, *i. e.*,  $v$  linear, the Pareto-curve is affine-linear in utility space which we will show below. As is well known, all bargaining solutions that are symmetric, Pareto optimal, and invariant under positive

affine transformations of utility coincide on the class of bargaining problems with affine-linear Pareto-curve.<sup>10</sup>

Recall that the Pareto-curve in  $(w, L)$ -space is given by the set of points  $(w, L)$  that satisfy eqn (4). If  $v(w) = \alpha w + \beta$  ( $w \geq 0$ ) with  $\alpha > 0$  is linear, then  $(w, L)$  satisfies eqn (4) if and only if  $L = \bar{L}$ . Hence, in utility space, the Pareto-curve is given by all points  $(u_1, u_2)$ , satisfying

$$\begin{aligned} u_1 &= U_1(w, \bar{L}) = \bar{L}(v(w) - \bar{v}) + N\bar{v} = \bar{L}\alpha(w - \bar{w}) + N(\alpha\bar{w} + \beta), \\ u_2 &= U_2(w, \bar{L}) = f(\bar{L}) - w\bar{L}, \end{aligned}$$

from which it follows that

$$u_1(u_2) = K - \alpha u_2$$

for some constant  $K$ . That is, the Pareto-curve is affine-linear in utility space.

## 5 Conclusion

If we believe in the efficiency of our labor market institutions, we should expect the contracting parties — the labor union and the employers' federation — to negotiate for a Pareto efficient outcome when settling their conflict. In order to reach a unique Pareto efficient contract, the two parties may agree on several conceivable concepts. In bargaining theory, the two most popular and most widely used solution concepts are the Nash-bargaining and the Kalai-Smorodinsky solution. While the former has been frequently scrutinized by labor market economists, the latter has predominantly been ignored. This is amazing, as the Kalai-Smorodinsky solution, characterized by equal proportional concessions of both parties in a conflict, seems to be more intuitive than the Nash bargaining solution in modeling actual labor market institutions.

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<sup>10</sup>Let  $\xi$  be a bargaining solution on some class of bargaining problems  $\mathcal{B}$ . Then  $\xi$  is *symmetric* if for all  $(S, d) \in \mathcal{B}$ ,  $\xi(S', d') = (\xi_2(S, d), \xi_1(S, d))$ , whenever  $(S', d') \in \mathcal{B}$ , where  $S' = \{u' \in \mathbb{R}^2 \mid u'_1 = u_2, u'_2 = u_1 \text{ for some } u \in S\}$  and  $d' = (d_2, d_1)$ .  $\xi$  is *Pareto efficient* if  $\xi(S, d)$  is Pareto efficient for all  $(S, d) \in \mathcal{B}$ .  $\xi$  is *invariant under positive affine transformations of utility* if for all  $(S, d) \in \mathcal{B}$ ,  $\xi(S', d') = (v(\xi_1(S, d)), v(\xi_2(S, d)))$ , whenever  $v : \mathbb{R} \rightarrow \mathbb{R}$  is given by  $v(x) = ax + b$  ( $x \in \mathbb{R}$ ) for some  $a, b \in \mathbb{R}, a > 0$ , and  $(S', d') \in \mathcal{B}$ , where  $S' = \{u' \in \mathbb{R}^2 \mid u'_1 = v(u_1), u'_2 = v(u_2) \text{ for some } u \in S\}$  and  $d' = (v(d_1), v(d_2))$ .

Although both solution concepts yield a Pareto efficient outcome and may even induce identical outcomes their comparative statics results differ in a significant way. While a higher reservation utility (or wage) leads to a lower employment level and a higher wage for the Nash solution, it has an ambiguous employment effect but a positive wage effect for the Kalai-Smorodinsky solution. Further analysis of other (egalitarian-type) bargaining solutions shows that theoretical ambiguity of the employment effect is the rule rather than the exception. This is a remarkable result and it has important consequences for labor market policies that aim at reducing (involuntary) unemployment. Conventional wisdom may suggest that a government should seek to lower workers' reservation wage in order to reduce unemployment. However, this policy recommendation does not necessarily survive a formal analysis: Instead of reducing unemployment it may in fact increase it.

We do not claim that an increase in employment is the typical response to a higher reservation wage. Nevertheless, it may happen that employment increases in response to a higher reservation wage, if the labor market conflict is solved in accordance with another solution concept than the Nash bargaining solution. Hence, in these cases a very detailed knowledge about utility and production functions is required in order to determine the consequences of a change in the reservation wage.

The important conclusion to draw from our analysis is that the choice of a bargaining solution is far from being innocuous. The analytic results may not be robust to a change in the bargaining solution: quantitative effects typically differ, and qualitative statements may even be reversed. It seems that the applied literature is not aware of this fact since the choice of the bargaining solution is hardly ever discussed. Rather, the Nash solution is used as *the* representative of an efficient bargaining solution.

Whether real equilibrium bargaining patterns can adequately be described by the Nash or the Kalai-Smorodinsky solution or by any other bargaining solution is an open empirical question. As long as this question has not been answered, theorists are required to test the robustness of their results with respect to a change in the bargaining solution. At any rate, analytical tractability seems to be one of the weakest arguments in favor of a particular bargaining solution.

## Appendix A

**Lemma A.1** *Let a bargaining game  $(S, d)$  be given by  $d = (N\bar{v}, 0)$  and*

$$S = \{u \in \mathbb{R}^2 \mid u \leq U(w, L) \text{ for some } w, L, \text{ with } 0 \leq w, 0 \leq L \leq N \text{ and } U(w, L) \geq d\}.$$

*Then  $S$  is convex and closed. Moreover,  $\{u \in S \mid u \geq d\}$  is bounded and there exists  $u \in S$  with  $u_i > d_i$  for  $i = 1, 2$ .*

**Proof:** The convexity of  $S$  follows from the concavity of the Pareto curve in utility space (see eqn (10)) while the closedness of  $S$  is a consequence of the continuity of  $U_1(\cdot, \cdot)$  and  $U_2(\cdot, \cdot)$ . If  $u \in S$  is individually rational ( $u \geq d$ ), then  $u \leq u^*$ , where  $u^*$  is the utopia point (see p. 7). Hence  $\{u \in S \mid u \geq d\}$  is bounded. Finally, since  $U_2(\bar{w}, \bar{L}) = f(\bar{L}) - \bar{w}\bar{L} > 0 = d_2$  and  $U_1(\bar{w}, \bar{L}) = N\bar{v} = d_1$ , by continuity of  $U_1(\cdot, \cdot)$  and  $U_2(\cdot, \cdot)$  there exists  $w$  close to  $\bar{w}$  with  $U_i(w, \bar{L}) > d_i$  for  $i = 1, 2$ . Hence  $u$  with  $u_i = U_i(w, \bar{L})$  for  $i = 1, 2$ , satisfies  $u \in S$  and  $u_i > d_i$  for  $i = 1, 2$ . □

**Lemma A.2** *For all  $L \in [\bar{L}, \hat{L}]$  there exists a unique  $w = w(L) \geq \bar{w}$  such that  $(w, L)$  satisfies  $\phi_2(w, L) = 0$ .*

**Proof:** First observe that  $L > \bar{L}$  whenever  $(w, L)$  satisfies  $\phi_2(w, L) = 0$  and  $w > \bar{w}$ . In order to see this, assume  $L \leq \bar{L}$ . Then, by strict concavity of  $f$  and  $v$ ,

$$v(w) - \bar{v} = v'(w)(w - f'(L)) \leq v'(w)(w - f'(\bar{L})) = v'(w)(w - \bar{w}) < v(w) - \bar{v},$$

which is a contradiction. In particular, it is true that  $\hat{L} > \bar{L}$  and that  $w(\bar{L}) := \bar{w}$  is the unique  $w \geq \bar{w}$  satisfying  $\phi_2(w, \bar{L}) = 0$ . If there exists  $\tilde{L} > \bar{L}$  and  $\tilde{w} > \bar{w}$  with  $\phi_2(\tilde{w}, \tilde{L}) = 0$ , then for all  $L \in (\bar{L}, \tilde{L})$  there exists  $w, \bar{w} < w < \tilde{w}$ , such that  $\phi_2(w, L) = 0$ . This follows from the fact that  $\phi_2(\cdot, \cdot)$  is continuous and  $\lim_{w \searrow \bar{w}} \phi_2(w, L) = \phi_2(\bar{w}, L) > 0$ , while  $\lim_{w \nearrow \tilde{w}} \phi_2(w, L) = \phi_2(\tilde{w}, L) < 0$ . Moreover, the wage level  $w$  solving  $\phi_2(w, L) = 0$  for given  $L \geq \bar{L}$  is unique, since  $\partial\phi_2/\partial w < 0$  whenever  $w > \bar{w}$ . Hence, for all  $L \in [\bar{L}, \hat{L}]$  there exists a unique  $w = w(L) \geq \bar{w}$  such that  $(w, L)$  satisfies  $\phi_2(w, L) = 0$ . □

## An Example with a Positive Employment Effect

Let  $f(L) = L^{0.1}$  and

$$v(w) = \begin{cases} w^{0.25}, & w \leq \bar{w} \text{ or } w \geq \hat{w} \\ 0.3644530468 + 1.657044205w - 1.45w^2, & \bar{w} \leq w \leq w_{11} \\ 0.651597943 + 0.380844666w - 3.200051171 \cdot 10^{-2}w^2, & w_{11} \leq w \leq w_{12} \\ -0.714631886 + 3.164592642w - 1.45w^2, & w_{12} \leq w \leq w_{21} \\ 0.7114971634 + 0.3321615609w - 4.362905611 \cdot 10^{-2}w^2, & w_{21} \leq w \leq w_{22} \\ -1.013067158 + 3.446886045w - 1.45w^2, & w_{22} \leq w \leq \hat{w} \end{cases}$$

where

$$\begin{aligned} \bar{w} &= 0.4, & \hat{w} &= 1.108799908, \\ w_{11} &= 0.45, & w_{12} &= 0.9815758041, \\ w_{21} &= 1.007, & w_{22} &= 1.107362356. \end{aligned}$$

Then the wage and employment allocations corresponding to the utopia point of the firm and the union, respectively, are given by

$$\begin{aligned} (\bar{w}, \bar{L}) &= (0.4, 0.2143109957), \\ (\hat{w}, \hat{L}) &= (1.108799908, 0.891585777), \end{aligned}$$

and the KS-solution is given by

$$(w, L) = (1.005378035, 0.5276868336).^{11}$$

From

$$\begin{aligned} v(\bar{w}) &= 0.7952707288 & v'(\bar{w}) &= 0.4970442055 \\ v(w) &= 1.001341806 & v'(w) &= 0.2489963405 \\ v(\hat{w}) &= 1.026155779 & v'(\hat{w}) &= 0.2313663114 \end{aligned}$$

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<sup>11</sup>For comparison, the symmetric Nash Solution is given by  $(w, L) = (0.999075985, 0.515181416)$ .

one immediately verifies that the employment effect for the KS solution, given by eqn (13), is positive, *i. e.*,  $dL/d\bar{v} > 0$ .<sup>12</sup>

The reader will notice that the function  $v$  is not twice continuously differentiable at the endpoints of the intervals that appear in the piecewise definition. However, it is straightforward to see that one can smooth  $v$  and still have a positive employment effect.

## Appendix B

### The Egalitarian Solution

In the following we will perform comparative statics concerning the reservation wage for two other standard bargaining solutions. The *egalitarian solution*  $\xi^{EG}$  (Kalai, 1977) is characterized by equal gains over the disagreement point and is formally defined by

$$\xi^{EG}(S, d) = d + \hat{\lambda} \cdot (1, 1),$$

where  $\hat{\lambda} = \max\{\lambda \in \mathbb{R} \mid d + \lambda \cdot (1, 1) \in S\}$  for  $(S, d) \in \mathcal{B}$ .

Let the *EG-curve* be the set of utility allocations  $(u_1, u_2)$  with

$$u_2 - d_2 = u_1 - d_1.$$

In the  $(w, L)$  space the feasible part of the EG-curve is then given by the set of  $(w, L)$  satisfying  $U(w, L) \geq d$  and  $\phi_3(w, L) = 0$ , where

$$\phi_3(w, L) := f(L) - wL - L(v(w) - \bar{v}). \quad (\text{B.1})$$

Implicit differentiation yields

$$\left. \frac{dw}{dL} \right|_{\phi_3=0} = \frac{f'(L) - w - (v(w) - \bar{v})}{L(v'(w) + 1)} < 0, \quad (\text{B.2})$$

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<sup>12</sup>The function  $v$  was numerically approximated with a precision of  $10^{-9}$ . Analytically one starts from the utility function  $v(w) = w^{0.25}$  and changes the curvature of  $v$  in the KS-solution such as to generate a positive employment effect without changing the utopia points and the KS-solution.

*i. e.*, the EG–curve is decreasing. Differentiating the EG– and Pareto–curve (see eqn (4)) with respect to  $\bar{v}$  yields the equation system

$$\begin{bmatrix} \widehat{a}_{11} & \widehat{a}_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \frac{dL}{d\bar{v}} \\ \frac{dw}{d\bar{v}} \end{bmatrix} = \begin{bmatrix} \widehat{b} \\ 1 \end{bmatrix}$$

where  $a_{21} < 0$  and  $a_{22} > 0$  are defined on p. 12 and

$$\widehat{a}_{11} := w - f'(L) + v(w) - \bar{v} > 0,$$

$$\widehat{a}_{12} := L(v'(w) + 1) > 0$$

$$\widehat{b} := L > 0$$

Solving for the derivatives we obtain

$$\begin{aligned} \frac{dw}{d\bar{v}} &= \frac{1}{D_3} (\widehat{a}_{11} - a_{21}\widehat{b}) > 0, \\ \frac{dL}{d\bar{v}} &= \frac{1}{D_3} (a_{22}\widehat{b} - \widehat{a}_{12}), \end{aligned}$$

with  $D_3 := \widehat{a}_{11}a_{22} - \widehat{a}_{12}a_{21} > 0$ . Hence, again the wage effect is positive while the employment effect is ambiguous. As before, this is due to the fact that an increase in the reservation wage shifts both, the Pareto– and the EG–curve upward:

$$\left. \frac{dw}{d\bar{v}} \right|_{\substack{\phi_3=0 \\ L \text{ const.}}} = \frac{1}{1 + v'(w)} > 0. \quad (\text{B.3})$$

## The Equal Loss Solution

Contrary to the egalitarian solution, the *equal loss solution*  $\xi^{EL}$  (Chun, 1988) is characterized by equal losses from the utopia point. Formally,

$$\xi^{EL}(S, d) = u^* - \widehat{\lambda} \cdot (1, 1),$$

where  $\widehat{\lambda} = \max\{\lambda \in \mathbb{R} \mid u^* - \lambda \cdot (1, 1) \in S\}$  and  $u^*$  is the utopia point for  $(S, d) \in \mathcal{B}$ . Define the *EL–curve* as the set of utility allocations  $(u_1, u_2)$  with

$$u_2^* - u_2 = u_1^* - u_1.$$



In the  $(w, L)$  space the feasible part of the EL–curve is then given by the set of  $(w, L)$  satisfying  $U(w, L) \geq d$  and  $\phi_4(w, L) = 0$ , where

$$\phi_4(w, L) := u_2^* - f(L) + wL - u_1^* + L(v(w) - \bar{v}) + N\bar{v}. \quad (\text{B.4})$$

Implicit differentiation yields

$$\left. \frac{dw}{dL} \right|_{\phi_4=0} = \frac{f'(L) - w - (v(w) - \bar{v})}{L(v'(w) + 1)} < 0, \quad (\text{B.5})$$

*i. e.*, the EL–curve has the same slope as the EG–curve and is decreasing. Differentiating the EL– and Pareto–curve (see eqn (4)) with respect to  $\bar{v}$  yields the equation system

$$\begin{bmatrix} \widehat{a}_{11} & \widehat{a}_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \frac{dL}{d\bar{v}} \\ \frac{dw}{d\bar{v}} \end{bmatrix} = \begin{bmatrix} \widehat{b} \\ 1 \end{bmatrix}$$

where  $a_{21} < 0$  and  $a_{22} > 0$  (defined on p. 12),  $\widehat{a}_{12} > 0$  (defined on p. 21), and

$$\begin{aligned} \widehat{a}_{11} &:= v(w) - \bar{v} - f'(L) + w > 0, \\ \widehat{b} &:= L - \hat{L} + \frac{\bar{L}}{v'(\bar{w})}. \end{aligned}$$

Observe that the sign of  $\widehat{b}$  is ambiguous. Solving for the derivatives we obtain

$$\begin{aligned} \frac{dw}{d\bar{v}} &= \frac{1}{D_4} \left( \widehat{a}_{11} - a_{21}\widehat{b} \right), \\ \frac{dL}{d\bar{v}} &= \frac{1}{D_4} \left( a_{22}\widehat{b} - \widehat{a}_{12} \right), \end{aligned}$$

with  $D_4 := \widehat{a}_{11}a_{22} - \widehat{a}_{12}a_{21} > 0$ . Hence, for the equal loss solution both, the employment and the wage effect are ambiguous. This is due to the fact that an increase in the reservation wage may shift the EL–curve upward or downward, leading to an undetermined employment effect in the first and to an undetermined wage effect in the latter case:

$$\left. \frac{dw}{d\bar{v}} \right|_{\substack{\phi_4=0 \\ L \text{ const.}}} = \frac{L - \hat{L} + \frac{\bar{L}}{v'(\bar{w})}}{L(1 + v'(w))}. \quad (\text{B.6})$$

## References

- BINMORE, K., A. RUBINSTEIN, AND A. WOLINSKY (1986): “The Nash Bargaining Solution in Economic Modelling,” *Rand Journal of Economics*, 17, 176–188.
- CHUN, Y. (1988): “The Equal-Loss Principle for Bargaining Problems,” *Economics Letters*, 26, 103–106.
- FUEST, C., AND B. HUBER (2000): “Why Do Governments Subsidise Investment and Not Employment,” *Journal of Public Economics*, 78, 171–192.
- GARINO, G., AND C. MARTIN (2000): “Efficiency Wages and Union-Firm Bargaining,” *Economics Letters*, 69, 181–185.
- KALAI, E. (1977): “Proportional Solutions to Bargaining Situations: Interpersonal Utility Comparisons,” *Econometrica*, 45, 1623–1630.
- KALAI, E., AND M. SMORODINSKY (1975): “Other Solutions to Nash’s Bargaining Problem,” *Econometrica*, 43, 513–518.
- MCDONALD, I. M., AND R. M. SOLOW (1981): “Wage Bargaining and Employment,” *American Economic Review*, 71, 896–908.
- NASH, J. F. (1950): “The Bargaining Problem,” *Econometrica*, 18, 155–162.
- PETERS, H. J. M. (1992): *Axiomatic Bargaining Game Theory*. Theory and Decision Library, Dordrecht: Kluwer Academic Publishers.
- PETRAKIS, E., AND M. VLASSIS (2000): “Endogenous Scope of Bargaining in a Union-Oligopoly Model: When Will Firms and Unions Bargain over Employment?,” *Labour Economics*, 7, 261–281.
- PISSARIDES, C. A. (1998): “The Impact of Employment Tax Cuts on Unemployment and Wages; The Role of Unemployment Benefits and Tax Structure,” *European Economic Review*, 42, 155–183.

- RAIFFA, H. (1953): “Arbitration Schemes for Generalized Two-Person Games,” in *Contributions to the theory of games, II (Annals of Mathematics Studies, 28)*, ed. by H. Kuhn, and A. Tucker, pp. 361–387. Princeton University Press.
- STRAND, J. (2002): “Wage Bargaining and Turnover Costs with Heterogenous Labor and Perfect History Screening,” *European Economic Review*, 46, 1209–1227.
- VANNINI, S., AND J. BUGHIN (2000): “To Be (Unionized) or Not to Be? A Case for Cost-Raising Strategies under Cournot Oligopoly,” *European Economic Review*, 44, 1763–1781.

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