

Breuer, Michael

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Author's addresses      Michael Breuer  
E-mail: [mbreuer@soi.unizh.ch](mailto:mbreuer@soi.unizh.ch)

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E-mail: [soilib@soi.unizh.ch](mailto:soilib@soi.unizh.ch)

# Deductible or Co-Insurance: Which is the Better Insurance Contract under Adverse Selection?

Michael Breuer\*  
University of Zurich

## ABSTRACT

The standard solution to adverse selection is the separating equilibrium introduced by Rothschild and Stiglitz. Usually, the Rothschild-Stiglitz argument is developed in a model that allows for two states of the world only. In this paper adverse selection is discussed for continuous loss distributions. This gives rise to the new problem of finding the proper form of an insurance contract to impose partial insurance of the low risks.

This paper contributes to the discussion on optimal insurance. It analyzes two basic forms of insurance contracts: A contract with a deductible and a contract imposing a positive co-insurance rate. Since high risks can always self-reveal themselves as high risks and buy the optimal insurance contract at high risks' premiums the Pareto-superior insurance contract is the one that leaves the low risks with higher expected utility while deterring high risks from joining the contract that is designed for low risks. The deductible contract turns out to be superior if premiums contain a sufficiently high loading.

*Keywords:* Insurance, Adverse Selection, Deductible, Co-Insurance

*JEL-Classification:* D81, D82, D62

\* Address for correspondence: Socioeconomic Institute, University of Zurich, Hottingerstr. 10, 8032 Zurich, Switzerland. E-mail: [mbreuer@soi.unizh.ch](mailto:mbreuer@soi.unizh.ch). Phone: +41 1 634 45 95.

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## **1. Introduction**

Insurance economists have paid a lot of attention to adverse selection. Starting with the seminal contribution by Rothschild and Stiglitz (1976), adverse selection is now seen as one of the most severe problems of insurance markets under asymmetric information, causing welfare losses or even a complete breakdown of insurance markets. The notion of a separating equilibrium in competitive insurance markets under asymmetric information, providing full insurance to the high risks while restricting the low risks to partial insurance, is well understood. Rothschild and Stiglitz employ the Nash equilibrium concept to get their results. Other equilibrium concepts such as Wilson (1977) or Riley (1979) have come to different results but have also contributed to our understanding of insurers' behavior and how insurers are restricted in designing policies in private insurance markets under adverse selection. However, if there is perfect competition on the insurance market, the analysis by Rothschild and Stiglitz is still relevant.

The Rothschild-Stiglitz argument is developed in a model that allows for two states of the world only (namely one state of loss occurrence and one state with no loss).<sup>1</sup> Premiums in the Rothschild-Stiglitz do not contain a loading but are actuarially fair. However, in the light of the vast literature about optimal insurance tariffs for continuous loss distributions it is somewhat surprising that the literature remains silent about the adequate form of insurance contract to enforce this separating equilibrium if loss distributions are continuous. Therefore, this paper analyzes adverse selection if loss distributions are continuous and premiums contain a linear loading. While the generalizations of the model's participation constraints and incentive-compatibility constraints to continuous distributions of losses are straightforward, the agents are now confronted with a new decision parameter, namely the form of contract to impose partial insurance. This is not a trivial problem. In fact, insurers have to look for the form of contract that leaves the low risks with a level of expected utility as high as possible while discouraging high risks from opting for the contract tailored for the low risks.

The following analysis concentrates on two basic types of contracts: Full coverage of losses above a nonzero deductible on the one hand and co-insurance contracts, on the other hand. This allows to contrast the design of an insurance contract which is known to be optimal when premiums contain a linear loading (the deductible contract) with another standard design of insurance contracts. Since high risks can always opt for full coverage in the Rothschild-Stiglitz model the insurance contract that leaves the low risks with higher expected utility while deterring high risks from joining the contract that is designed for low risks is in fact Pareto-superior.

## 2. The model

Let the insured population consist of two types of risks, high risks ( $h$ ) and low risks ( $l$ ). Both risk types may suffer a loss of  $x \cdot L$ . The maximum loss is  $L$ .<sup>2</sup> The distribution of  $x$  is described by the density functions  $f_h(x)$  and  $f_l(x)$ . Expected losses are therefore  $\int_0^1 xL f_i(x) dx$  for  $i = h, l$ . Low risks are assumed to be better risks in the sense of first order stochastic dominance (FOSD), that is

$$(1) \quad F_l(\tilde{x}) = \int_0^{\tilde{x}} f_l(x) dx \geq \int_0^{\tilde{x}} f_h(x) dx = F_h(\tilde{x}) \quad \forall x, > \text{ for at least one } x,$$

with  $F_i(x)$ ,  $i = h, l$  representing the cumulative density functions. As usual in models of the Rothschild-Stiglitz type, utility functions  $U$  ( $U' > 0$ ,  $U'' < 0$ ) and initial wealth  $w$  are the same for both risks. Without any insurance, high and low risk individuals obtain expected utility  $EU_{h,1}$  and  $EU_{l,1}$ , respectively, defined as

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<sup>1</sup> Berger and Cummins (1992) are an exception in this respect. In the second part of their paper they allow for continuous loss distributions and characterize high and low risks by mean preserving spreads. However, in such a model adverse selection problems only arise if insurers are not risk neutral.

<sup>2</sup> The maximum amount of the loss may well be as high as the individual's wealth. Assuming a maximum loss therefore is not a very restrictive assumption. However, it will turn out to be extremely helpful for comparisons of the two types of contracts.

$$(2) \quad EU_{i,1} = \int_0^1 U(w - xL) f_i(x) dx \text{ for } i = h, l.^3$$

Individuals are fully aware of their own risk type.

Insurers, on the other hand, cannot identify an individual's risk type. Acting in a competitive environment (and profits restricted to be non-positive) they rather have to rely on self-selection of insured to establish a sustainable separating equilibrium, if there is one.<sup>4</sup> An insurance contract is defined by a premium  $P$  and an indemnity schedule  $I$ . The premiums are assumed to meet the non-profit-constraint. While Rothschild and Stiglitz (1976) assume the premiums to be actuarially fair, that is they meet the expected value of the indemnity, premiums in this paper may contain a constant loading. Rothschild and Stiglitz (1976) have shown that under perfect competition the only possible equilibrium is a separating one, providing at most partial coverage for the low risks. Let the two contracts  $(P_h^*, I_h^*)$  and  $(P_l^*, I_l^*)$  for high and low risks be a separating equilibrium. Written formally in the most general form, insurers have to find a pair of insurance contracts that maximize the low risks' expected utility under the following restrictions:

$$(3) \quad EU_i = \int_0^1 U(w - P_i^*(f_i(x)) - xL + I_i^*(xL)) f_i(x) dx \geq EU_{i,1} \text{ for } i = h, l$$

$$(4) \quad \int_0^1 U(w - P_h^*(f_h(x)) - xL + I_h^*(xL)) f_h(x) dx \geq \int_0^1 U(w - P_l^*(f_l(x)) - xL + I_l^*(xL)) f_h(x) dx$$

Condition (3) is the participation constraint stating that for both types of risk the expected utility obtained from the insurance policy tailored for each of them must not be lower than the expected utility from remaining uninsured. The (only binding) incentive compatibility constraint on the other hand is given by (4): High risks must have an incentive to reveal themselves as high risks and opt for the

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<sup>3</sup> The situation of no insurance coverage is labelled by the index 1 for reasons of compatibility to the later section: It is equivalent to a co-insurance rate or a deductible of 1.

<sup>4</sup> As in the original model by Rothschild and Stiglitz, there might be no stable separating equilibrium if the fraction of high risks in the entire insurance population is low.

contract tailored for them instead of enjoying the lower premiums of the low risks' contract.

However, until now nothing has been said about the indemnity schedule that is appropriate to reduce low risks' insurance coverage. In what follows we concentrate on two basic forms of insurance tariffs, namely contracts showing a deductible and contracts with a constant co-insurance rate. High risks can always rely on getting their reservation level of expected utility  $EU_{h,\min}$  from optimal insurance at high risks' premiums by choosing their optimal co-insurance rate or deductible. Low risks' expected utility in equilibrium, on the other hand, is higher if one can find the form of insurance contract that minimizes the low risks' loss in expected utility caused by the reduction of coverage that is necessary to push the high risks expected utility from buying the low risks' contract down to  $EU_{h,\min}$ . In fact, the better contract in this sense would be Pareto-superior. In what follows it will turn out that the optimal form of an insurance contract highly depends on the amount of the loading factor. However, it is useful to begin the analysis for the case of actuarially fair premiums that do not contain a loading.

### 3. Actuarially fair premiums

#### 3.1 Insurance contracts with a constant co-insurance rate

Let the insurance contract demand a co-insurance rate  $c$  of every loss, so that the coverage reduces to  $(1-c)$  of every loss.<sup>5</sup> Indemnity then is simply

$$(5) \quad I_c = (1-c)xL$$

and the actuarially fair premium is given by

$$(6) \quad P_{i,c} = \int_{x=0}^1 (1-c)xL f_i(x) dx \quad \text{for } i = h, l.$$

Differentiating (6) w.r.t  $c$  yields:

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<sup>5</sup> In the literature it is more common to label the replacement rate  $c$  and the co-insurance rate  $(1-c)$ . The terminology used here will make it easier to compare the results of this section with findings on contracts of the deductible type, analyzed in section 3.2.



$$(7) \quad \frac{\partial P_{i,c}}{\partial c} = - \int_{x=0}^1 xL f_i(x) dx \quad \text{for } i = h, l.$$

According to (7), the reduction of the premium due to a higher co-insurance rate is linear. If both risk types buy the contract tailored for them they obtain expected utility of

$$(8) \quad EU_{i,c} = \int_{x=0}^1 U(w - P_{i,c} - cxL) f_i(x) dx \quad \text{for } i = h, l.$$

Differentiating (8) w.r.t.  $c$  and using (7) yields:

$$(9) \quad \frac{\partial EU_{i,c}}{\partial c} = \int_{x=0}^1 U'(w - P_{i,c} - cxL) \left( - \int_{x=0}^1 -xL f_i(x) dx - xL \right) f_i(x) dx \leq 0$$

for  $i = h, l$ .

Differentiating (9) again w.r.t.  $c$  yields

$$(10) \quad \frac{\partial^2 EU_{i,c}}{\partial c^2} = \int_{x=0}^1 U''(w - P_{i,c} - cxL) \left( - \int_{x=0}^1 -xL f_i(x) dx - xL \right)^2 f_i(x) dx < 0,$$

which proves that  $EU_{i,c}$  is concave in  $c$ . At  $c = 0$ , (9) can be rewritten as

$$(11) \quad \left. \frac{\partial EU_{i,c}}{\partial c} \right|_{c=0} = \int_{x=0}^1 \left[ -U'(w - P_{i,c}) xL f_i(x) + U'(w - P_{i,c}) \int_{x=0}^1 xL f_i(x) dx \right] dx = 0;$$

consequently,  $EU_{i,c}$  has a maximum  $c = 0$  (see Mossin 1968). Furthermore, the function will decrease more rapidly as self-insurance rates get higher.

However, if high risks opt for the low risks' contract (paying premium  $P_{l,c}$ ) they obtain expected utility of:

$$(12) \quad EUL_{h,c} = \int_0^1 U(w - P_{l,c} - cxL) f_h(x) dx.$$

Observe that at  $c = 0$  high risks can enjoy the same expected utility as low risks by buying the same insurance contract. Since insurers are not able to observe an individual's risk they cannot prevent any high risk from doing so. Consequently a full insurance contract would result in a pooling of the risks and high risks would obtain expected utility that is higher than the expected utility they would enjoy

with full insurance for actuarially fair premiums ( $EU_{h,c}$ ). Since insurers do not receive premiums from the high risks that cover their expected losses, this form of pooling risks is, of course, not sustainable. However, it is the high risks' threat to join the low risks' contract that urges the insurer to offer only partial coverage for low risks.

The obvious solution to this kind of asymmetric information problem is to offer contracts that combine low risks' premiums with a positive co-insurance rate to discourage high risks from opting for the low risks' contract and to let them buy full coverage in exchange to high risks' premiums. In a separating equilibrium  $EUL_{h,c} = EU_{h,c}$ . Consequently, for actuarially fair premiums the co-insurance rate has to be high enough to push  $EUL_{h,c}$  down to  $EU_{h,c}$  at  $c = 0$ . Unfortunately, a higher co-insurance rate reduces the low risks expected utility too. The marginal effect of a higher co-insurance rate on low risks' expected utility  $EU_{l,c}$  can be obtained by applying (9) for  $i = l$ . Differentiating  $EUL_{h,c}$  w.r.t.  $c$  on the other hand yields:

$$(13) \quad \frac{\partial EUL_{h,c}}{\partial c} = \int_0^1 U'(w - P_{l,c} - c xL) \left( - \int_0^1 -xL f_l(x) dx - xL \right) f_h(x) dx < 0.$$

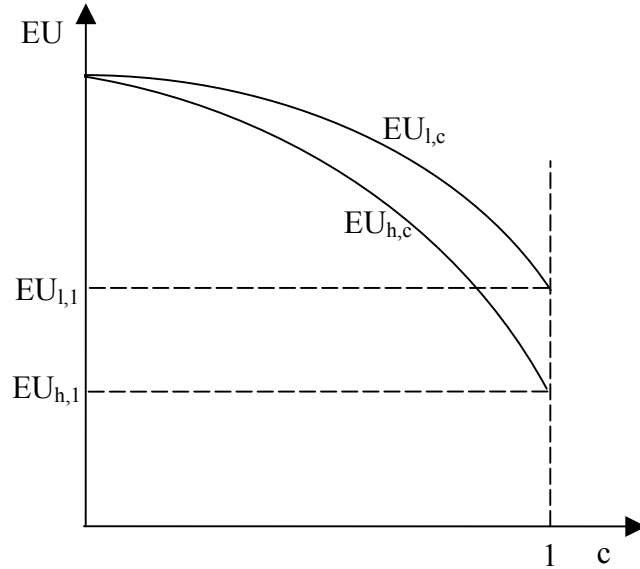
At  $c = 0$  (13) reads as

$$(14) \quad \left. \frac{\partial EUL_{h,c}}{\partial c} \right|_{c=0} = \int_0^1 U'(w - P_{l,c}) \left( - \int_0^1 -xL f_l(x) dx - xL \right) f_h(x) dx < 0,$$

so that the slope of the  $EUL_{h,c}$ -curve at  $c = 0$  is negative. Comparing (13) with (9) for  $i = l$  furthermore reveals that  $EUL_{h,c}$  has more weight on high losses and correspondingly on higher marginal utilities than  $EU_{l,c}$ . Therefore, the marginal negative effect of a decrease of  $c$  on expected utility is unambiguously stronger for high risks holding the low risks' policy than for low risks – irrespective of the level of  $c$ . Since both risk types enjoy the same expected utility at  $c = 0$ , high risks' expected utility must always be lower than low risks' expected utility if  $c > 0$  and the difference between both expected utilities must increase monotonically with  $c$ . Expected utility as a function of the co-insurance rate  $c$  for both risks will therefore look as illustrated in figure 1.

The difference in the effect on both risk types is stronger the more risk averse individuals are and the more weight high risks' density function has on higher losses compared to low risks' density function.

Figure 1: Expected utility as a function of co-insurance rate



### 3.2 Insurance contracts with a deductible

Now, let the insurance contract prescribe a deductible  $D$  with  $0 \leq D \leq 1$ . The variable  $D$  represents the deductible as a fraction of the maximum loss  $L$ . This allows convenient comparisons between deductible and co-insurance contracts. By multiplication with the maximum loss  $L$ ,  $D$  can easily be translated into a number giving the absolute deductible fixed in an insurance policy. Consequently, indemnity payments are

$$(15) \quad I_D(x) = \begin{cases} 0 & \text{for } x \leq D \\ xL - DL & \text{for } x > D \end{cases}$$

The corresponding actuarially fair premiums for high and low risks are given by

$$(16) \quad P_{i,D} = \int_{x=D}^1 L(x-D)f_i(x) dx \quad \text{for } i = h, l.$$

Differentiating (16) w.r.t.  $D$  yields:

$$(17) \quad \frac{\partial P_{i,D}}{\partial D} = \int_{x=D}^1 -L f_i(x) dx = -L(1 - F_i(D)) \quad \text{for } i = h, l.$$

The premium reduction due to reducing the level of insurance therefore is not linear any more (as it was in the case of a co-insurance contract) but is highest at low deductibles and diminishing with higher  $D$ . If both risk types opt for the contract tailored for them their expected utilities read as:

$$(18) \quad EU_{i,D} = \int_{x=0}^D U(w - P_{i,D} - xL) f_i(x) dx + U(w - P_{i,D} - D \cdot L) \cdot \int_{x=D}^1 f_i(x) dx$$

for  $i = h, l$ .

In analogy to the previous section, differentiating (18) w.r.t.  $D$  and using (17) proves that under actuarially fair premiums both risk types would prefer a deductible of zero that is for full insurance coverage:

$$(19) \quad \frac{\partial EU_{i,D}}{\partial D} = \int_{x=0}^D U'(w - P_{i,D} - xL) \left( - \int_{x=D}^1 -L f_i(x) dx \right) f_i(x) dx +$$

for  $i = h, l$ .

$$U'(w - P_{i,d} - DL) \cdot \left( - \int_{x=D}^1 -L f_i(x) dx - L \right) \int_{x=D}^1 f_i(x) dx$$

Condition (19) is zero at  $D = 0$  and  $D = 1$  only. To see if we can expect a maximum at one of these two extreme points it is necessary to differentiate (19) once more w.r.t.  $D$ . This yields at  $D = 0$

$$(20) \quad \left. \frac{\partial^2 EU_{i,D}}{\partial D^2} \right|_{D=0} = \int_{x=0}^D U'(w - P_{i,d}) L f_i(0) dx - U'(w - P_{i,d}) L f_i(0) \int_{x=0}^1 f_i(x) dx = 0$$

and at  $D = 1$ :

$$(21) \quad \left. \frac{\partial^2 EU_{i,D}}{\partial D^2} \right|_{D=1} = \left( \int_{x=0}^1 -U'(w - xL) f_i(x) dx + U'(w - L) \right) \cdot L f_i(1) > 0.$$

Consequently, there is a minimum at  $D = 1$ . Since there are no other extreme points, we have a corner solution for the maximum of  $EU_{i,D}$  at  $D = 0$ .

If, however, high risks opt for the low risks' contract they obtain expected utility of

$$(22) \quad EU_{h,D} = \int_{x=0}^D U(w - P_{l,D} - xL) f_h(x) dx + \int_{x=D}^1 U(w - P_{l,D} - DL) f_h(x) dx.$$

Again, at  $D=0$  the high risks can enjoy the same expected utility as the low risks simply by buying the same insurance contract.

The marginal effect of an increasing deductible on  $EU_{l,D}$  can be obtained by applying (15) and (19) for  $i=l$ . Rewriting yields:

$$(23) \quad \begin{aligned} \frac{\partial EU_{l,D}}{\partial D} &= \int_0^1 U'(w - P_{l,D} - xL + I_D(x)) \left( - \int_D^1 -L f_l(x) dx \right) f_l(x) dx + \\ &\quad \int_D^1 U'(w - P_{l,D} - DL) \cdot (-L) f_l(x) dx \\ &= L(1 - F_l(D)) \int_0^1 U'(w - P_{l,D} - xL + I_D(x)) f_l(x) dx - \\ &\quad L(1 - F_l(D)) U'(w - P_{l,D} - DL) \end{aligned}$$

The effect of a higher deductible on high risks opting for the low risks' contract on the other hand can be obtained by differentiating (22) w.r.t.  $D$  and applying the same transformations:

$$(24) \quad \begin{aligned} \frac{\partial EU_{h,D}}{\partial D} &= \int_0^1 U'(w - P_{l,D} - xL + I_D(x)) \left( - \int_D^1 -L f_l(x) dx \right) f_h(x) dx + \\ &\quad \int_D^1 U'(w - P_{l,D} - DL) \cdot (-L) f_h(x) dx \\ &= L(1 - F_l(D)) \int_0^1 U'(w - P_{l,D} - xL + I_D(x)) f_h(x) dx - \\ &\quad L(1 - F_h(D)) U'(w - P_{l,D} - DL) \end{aligned}$$

Note that both, (23) and (24) have extreme points at  $D=0$  and  $D=1$  only. Furthermore, differentiating (24) again w.r.t  $D$  gives

$$(25) \quad \left. \frac{\partial^2 EU_{h,D}}{\partial D^2} \right|_{D=1} = \int_{x=0}^1 -U'(w - xL) f_h(x) dx \cdot L f_l(1) + U'(w - L) L f_h(1) > 0,$$

because if  $f_l(1) > f_h(1)$ , the low risks' loss distribution would not dominate the high risks' loss distribution in the sense of FOSD. Therefore  $EU_{h,D}$  has a minimum at  $D=1$ , so that we have a corner maximum at  $D=0$ .

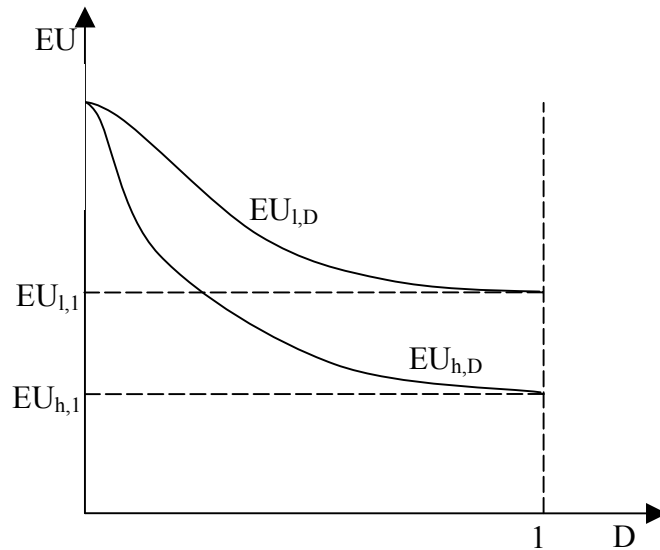
Since both risk types' expected utilities have a maximum at  $D=0$  they must decrease at all  $0 < D < 1$ . Therefore, for  $D > 0$ ,  $\partial EU_{l,D}/\partial D < 0$  and  $\partial EU_{h,D}/\partial D < 0$ , with a minimum somewhere between zero and one. Expected utility as a function of the deductible  $D$  for both risks therefore is first concave and then convex as  $D$  increases.

Concentrating on the last transformations of equations (23) and (24) it is clear that their second terms must dominate the first terms, which restrain the negative marginal effect of an increasing  $D$  on expected utility. By assumption,  $f_l(x)$  in (23) has less weight on higher losses and therefore on higher marginal utilities than  $f_h(x)$  in (24), so that the integral in (24) exceeds its counterpart in (23) for risk averse individuals. The effect on the integral is the stronger the more weight the high risks' density function has on high losses compared to the low risks' density function. Furthermore, differences between both integrals are greater the more risk adverse individuals are.

Having a closer look at the second terms one finds that the marginal negative effect of a higher deductible for a certain  $D$ ,  $0 < D < 1$ , is the stronger for high risks relative to low risks the more the values of the distribution functions  $F_l(D)$  and  $F_h(D)$ ,  $F_l(D) \geq F_h(D)$ , differ at this  $D$ . Low and high risks' expected utilities as functions of the deductible  $D$  therefore highly depend on the loss distribution functions. They differ more distinctive for small losses and small deductibles if low risks' loss distribution function has not too much weight on low losses.

To illustrate, figure 2 sketches high and low risks' expected utility as function of  $D$ .

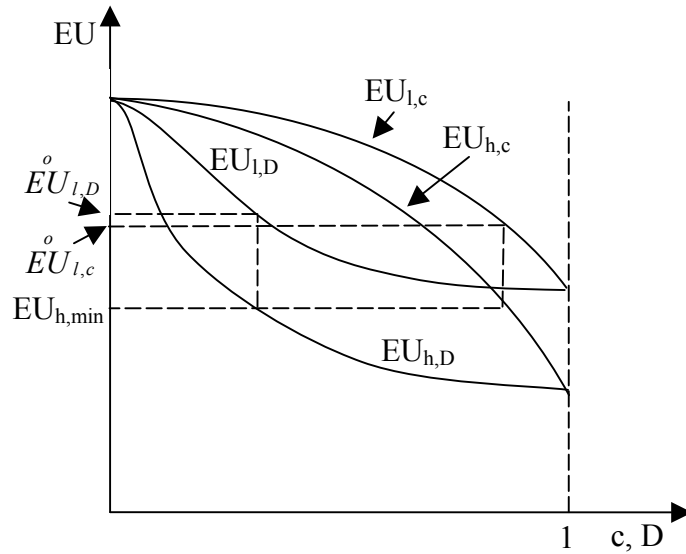
Figure 2: Expected utility as a function of deductible



### 3.3 Which type of insurance policy is the better one under adverse selection if premiums are actuarially fair?

The analysis in sections 2.1 and 2.2 has shown that expected utility as a function of the co-insurance rate  $c$  is strictly concave for both risks. Expected utility as a function of a deductible  $D$  for both risks, on the other hand, is first concave and then convex as  $D$  increases. From this follows that the appropriate co-insurance rate to push high risks' expected utility from buying the low risks' contract down to  $EU_{h,\min}$  is always higher than the deductible (defined as proportion of the maximal loss  $L$ ) that leaves the high risks at the same level of expected utility, as illustrated in figure 3.

Figure 3: Comparing deductible and co-insurance



However, to find the Pareto-superior insurance contract one has to compare the levels of expected utility low risks can reach in both insurance schedules when pushing high risks' expected utility down to  $EU_{h,\min}$ , the level of expected utility high risks get from buying full insurance at high risks' premiums. Note that  $EU_{h,\min}$  itself depends on the differences between high and low risks' loss distributions: The more alike both distributions are the less high and low risks' premiums differ and the higher is  $EU_{h,\min}$ .

Unfortunately, it turns out that neither form of schedule is the best in all situations. Whether low risks prefer a deductible to a co-insurance rate highly depends on the distribution functions ( $F_l(x)$  and  $F_h(x)$ ) and the level of expected utility high risks can obtain from full insurance at high risks' premiums,  $EU_{h,\min}$ . E.g., if differences between both risks' loss distribution functions are not very large (and therefore  $EU_{h,\min}$  is relatively high) and  $F_l(x)$  has not too much weight on small losses, a deductible may be the low risks' first choice of insurance contract. If, on the other hand, low risks' loss distribution function has almost all weight on small losses, a co-insurance rate is more likely to be optimal.



## 4. Premiums containing a loading

The somewhat negative result obtained in section 3 changes if we allow premiums to contain a loading. More specifically, assume premiums to be actuarially fair times a constant loading of  $1 + \kappa$ , with  $\kappa$  representing the loading factor. Under symmetric information the optimal insurance contract would now be a contract with a deductible as has been shown by Arrow (1971) and others.<sup>6</sup> However, as will be shown below, this result holds unambiguously under adverse selection only if the loading factor exceeds a minimum amount up to a certain point.

### 4.1 Insurance contracts with a constant co-insurance rate

As in section 3.1 premiums depend on the co-insurance rate  $c$ , but now additionally contain the loading  $(1 + \kappa)$ :

$$(26) \quad P_{i,c} = (1 + \kappa) \int_{x=0}^1 (1 - c)xL f_i(x) dx \quad \text{for } i = h, l.$$

From differentiating (26) w.r.t.  $c$  it is clear, that premiums still depend on  $c$  in a linear way:

$$(27) \quad \frac{\partial P_{i,c}}{\partial c} = -(1 + \kappa) \int_{x=0}^1 xL f_i(x) dx \quad \text{for } i = h, l.$$

Since individuals' utility function does not change, it is easy to obtain the insured's expected utility by inserting (27) into (8). Differentiating this new function w.r.t.  $c$  yields

$$(28) \quad \frac{\partial EU_{i,c}}{\partial c} = \int_{x=0}^1 U'(w - P_{i,c} - c xL) \left( (1 + \kappa) \int_{x=0}^1 xL f_i(x) dx - xL \right) f_i(x) dx$$

for  $i = h, l$ ,

which is greater than zero at  $c = 0$ . Consequently, under symmetric information every insured gets a higher expected utility by deciding for a positive co-insurance rate rather than a full insurance contract. It is interesting to observe that (28) can even be positive at  $c = 1$  if  $\kappa$  is sufficiently high, indicating a corner so-

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<sup>6</sup> See e.g. Raviv 1979 or Gollier and Schlesinger (1996).

lution at  $c=1$ , since  $EU_{i,c}$  is concave in  $c$  as can be shown by differentiating (28) again w.r.t.  $c$ :

$$(29) \quad \frac{\partial^2 EU_{i,c}}{\partial c^2} = \int_{x=0}^1 U''(w - P_{i,c} - c xL) \left( (1 + \kappa) \int_{x=0}^1 xL f_i(x) dx - xL \right)^2 f_i(x) dx$$

for  $i = h, l$ .

For the case of asymmetric information this also implies that the high risks' reservation level of expected utility by revealing themselves as high risks and accepting the higher premiums is not the one they obtain at  $c=0$ , but the level they obtain at their optimal (positive) level of  $c$ .

Unfortunately, without further assumptions on the utility function and the loss distributions no further statements about the optimal level of  $c$  for high and low risks can be made. Under symmetric information the optimal amount of  $c$  for high risks can well be higher or lower than the optimal amount of  $c$  for low risks. The optimal level of co-insurance does not even have to increase monotonically with a higher loading factor  $\kappa$ .<sup>7</sup> However, as noted before, there is a level of  $\kappa$  that induces individuals to buy no insurance coverage. Furthermore, since an increase of  $c$  reduces the transfers high risks receive from the low risks in form of subsidized premiums, low risks can always push the high risks down to their reservation level of expected utility by choosing an adequate level of  $c$ , which might be  $c=1$ .

Let  $\tilde{\kappa}_c$  be the minimum value of  $\kappa$  that leads the low risks to choose the corner solution at  $c=1$ , which means no insurance protection at all. High risks who buy the insurance contract tailored for the low risks, on the other hand, could still opt for partial insurance ( $0 < c < 1$ ) at a loading of  $1 + \tilde{\kappa}_c$ , in particular as long as their premiums are subsidized by the low risks. Consequently,  $\tilde{\kappa}_c$  is the critical value for a separating equilibrium in a co-insurance contract that allows the low risks to obtain at least partial insurance protection. If  $\kappa \geq \tilde{\kappa}_c$  low risks opt out and do not

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<sup>7</sup> On the one hand, an increase of the loading factor makes insurance coverage more expensive. On the other hand it reduces the insured's wealth and thereby increases their risk aversion if utility functions show decreasing absolute rate of risk aversion (DARA). See Eeckhoudt and Gollier (1995, chapter 10) for this argument.

buy any insurance, irrespective of the existence of high risks and informational asymmetries. Insurers, on the other hand, then can be sure that their insured are high risks and adjust their premium and the co-insurance rate appropriately.

#### 4.2 Insurance contracts with a deductible

By introducing a constant loading the premium function (16) changes to

$$(30) \quad P_{i,D} = (1 + \kappa) \int_{x=D}^1 L(x-D) f_i(x) dx \quad \text{for } i = h, l.$$

Inserting the new premium function into the expected utility function (17) and differentiating w.r.t.  $D$  gives

$$(31) \quad \frac{\partial EU_{i,D}}{\partial D} = \left\{ \int_{x=0}^D U'(w - P_{i,D} - xL)(1 + \kappa) \cdot f_i(x) dx + \right. \\ \left. U'(w - P_{i,D} - DL) \cdot \left( (1 + \kappa) \int_{x=D}^1 f_i(x) dx - 1 \right) \right\} \cdot L \int_{x=D}^1 f_i(x) dx \\ \text{for } i = h, l.$$

Note that the loading changes one of the extreme points of (19): While (19) and (31) both have extreme points at  $D=1$ , it is true that  $\frac{\partial EU_{i,D}}{\partial D} > 0$  at  $D=0$  for any positive  $\kappa$ . As expected, full insurance coverage is not optimal any more when premiums are not actuarially fair. However, from the FOC (31) it is not clear if there are any other extreme point between  $D=0$  and  $D=1$  and if these extreme points are maxima. To tackle these questions, (31) is differentiated once more w.r.t.  $D$ :

$$(32) \quad \frac{\partial^2 EU_{i,D}}{\partial D^2} = \left\{ \int_{x=0}^D U''(w - P_{i,D} - xL) \cdot (1 + \kappa) f_i(x) dx - \right. \\ \left. U''(w - P_{i,D} - DL) \cdot \left( -(1 + \kappa) \int_{x=D}^1 f_i(x) dx + 1 \right) \right\} \cdot (-L f_i(D)) \\ + \int_{x=0}^D U''(w - P_{i,D} - xL) \cdot (1 + \kappa)^2 \cdot \left( -L \int_{x=D}^1 f_i(x) dx \right)^2 f_i(x) dx \\ + U''(w - P_{i,D} - DL) \cdot \left( -(1 + \kappa) \int_{x=D}^1 -L f_i(x) dx - L \right)^2 \cdot \int_{x=D}^1 f_i(x) dx$$

for  $i = l, h$ .

As has already been noted by Mossin (1968), (32) is negative, if the first derivative (31) is positive or zero.<sup>8</sup> Consequently, depending on the amount of the loading factor  $\kappa$ ,  $EU_{i,D}$  has a maximum at some  $D$  between  $D=0$  and  $D=1$  or increases monotonically within this range, in which case it is maximized at the corner solution  $D=1$ . Call the loading factor at which the low risks do not buy a deductible contract any more  $\tilde{\kappa}_D$ .

Again, high risks' reservation level of expected utility they can get from accepting the higher premiums is not the one they obtain at  $D=0$ , but the one they obtain at their optimal (positive) level of  $D$ . Moreover, the optimal level of  $D$  does not necessarily increase with  $\kappa$ .<sup>9</sup> However, it remains true that low risks can always push the high risks down to their reservation level of expected utility by increasing  $D$  and thereby reducing their transfers to the high risks. High risks could still wish to buy the low risks' deductible contract at a loading factor of  $\kappa \geq \tilde{\kappa}_D$ , in which case insurers can be sure about their insured's type and adjust the premiums and the deductible appropriately.

### 4.3 Which type of insurance policy is the better one under adverse selection if premiums include a loading?

The results derived in section 4 so far do not look encouraging: For any loading factor  $\kappa$  below  $\tilde{\kappa}_c$  or  $\tilde{\kappa}_D$ , the amounts of loading that induce low risks to buy no insurance if the insurance contract is of the co-insurance of deductible type, respectively, no general conclusions about the superiority or inferiority of either type of contract can be derived. Loading factors above  $\tilde{\kappa}_c$  or  $\tilde{\kappa}_D$ , on the other hand, lead the low risks to abstain from buying a positive amount of insurance in at least one of the two contracts.

However, one standard result from the literature on optimal insurance policies is that an insurance contract prescribing a deductible is superior to other designs of

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<sup>8</sup> The first derivative is zero or positive iff the terms in parentheses of (30) or (31) are zero or positive. In (31) these terms are multiplied by a negative term while the other terms in (31) are always negative.

<sup>9</sup> Footnote 7 applies.

insurance contracts when the loading is linear. This includes that  $\tilde{\kappa}_D \geq \tilde{\kappa}_c$ , because otherwise there would be an amount of loading leading to partial insurance in a co-insurance contract and to no insurance in a deductible contract. Due to risk averseness of the individuals this would contradict the claim that the deductible contract is the optimal design of an insurance contract.

Consequently, we have to distinguish between three levels of the loading: If the loading factor is  $0 \leq \kappa \leq \tilde{\kappa}_c$ , the results of section 3 apply and it is not possible to derive any general conclusions about the superiority or inferiority of either type of contract without additional information about the loss distributions. If  $\tilde{\kappa}_c \leq \kappa \leq \tilde{\kappa}_D$  the deductible contract (weakly) dominates the co-insurance contract, since the former might allow the low risks to get at least a partial insurance in a separating equilibrium while the latter leaves the low risks with no insurance protection at all. If, finally,  $\tilde{\kappa}_D \leq \kappa$  low risks will not buy any positive amount of insurance. In this situation, any individual that buys insurance protection can be assumed to be of the high-risk type; the problem of asymmetric information ceases and the market equilibrium is efficient. Of course, the loading can be so high that even high risks abstain from buying insurance protection, in which case the insurance market (efficiently) breaks down completely.

## **5. Conclusion**

Generalizing the Rothschild-Stiglitz model by allowing for continuous distributions of losses does not change its principal result: The only possible sustainable equilibrium is a separating contract. High risks get full insurance while low risks are restricted to partial coverage to discourage high risks from buying the low risks' contract. However, the new question arising is, which kind of insurance contract is desirable to impose partial coverage: a contract showing a deductible or a contract with a constant co-insurance rate. The analysis has shown that both forms of contract may be favourable under certain circumstances, highly depending on the premiums' loading factor. If the loading factor is zero or sufficiently small, no general statement about the Pareto-superiority of either of the two forms of insurance contracts can be made without further information about both risks' loss distributions. A co-insurance contract, which is inferior under symmetric in-

formation and a linear loading might dominate a deductible contract. For higher but not too high loading factors, however, the deductible contract is unambiguously Pareto-superior since low risks would give up insurance protection completely if provided by a co-insurance contract. Finally, for extremely high co-insurance rates, both risk types do not buy insurance protection any more and the insurance market breaks down completely.

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