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Working Paper

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CESifo Working Paper, No. 634

Provided in Cooperation with:
Ifo Institute – Leibniz Institute for Economic Research at the University of Munich

Suggested Citation: Epstein, Gil S.; Nitzan, Shmuel (2002) : Effort and Performance in Public-Policy Contests, CESifo Working Paper, No. 634

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EFFORT AND PERFORMANCE IN PUBLIC-POLICY CONTESTS

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CESifo Working Paper No. 634 (2)

January 2002

Category 2: Public Choice

An electronic version of the paper may be downloaded
• from the SSRN website: www.SSRN.com
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EFFORT AND PERFORMANCE IN PUBLIC-POLICY CONTESTS

Abstract

Government intervention often gives rise to contests in which the possible ‘prizes’ are determined by the existing status-quo and some new public-policy proposal. In this paper we study the general class of such two-player public-policy contests and examine the effect of a change in the proposed policy, a change that may affect the payoffs of the two contestants, on their effort and performance. We extend the existing comparative statics studies that focus on the effect of changes either in the value of the prize in symmetric contests or in one of the contestants’ valuation of the prize in asymmetric contests. Our results hinge on the relationship between the strategic own-stake (“income”) effect and the strategic rival’s-stake (“substitution”) effect. This relationship is determined by three types of ability and stakes asymmetry between the contestants. In particular, we specify the asymmetry condition under which a more restrained government intervention that reduces the contestants’ prizes has the perverse effect of increasing their aggregate lobbying efforts.

Keywords: public-policy contests, policy reforms, lobbying efforts, strategic own-stake effect, strategic rival’s-stake (“substitution”) effect.

JEL Classification: D72, D6.

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I. Introduction

Tournaments, conflict, patent races and rent seeking have been modeled as contests in which participants exert efforts to increase their probability of winning a prize. A significant element in such contests is the function that provides each player’s probability of winning for any given combination of the efforts made by the contestants, the so called contest success function (CSF). In the literature specific CSF’s have been employed without any particular reason other than analytical convenience. Skaperdas (1996) provided a rationale for employing a particular CSF by axiomatizing the general class of additive CSF’s. A large class of contests has thus been rationalized. In most studies of contests, however, the source of the prize or, more generally, the source of the contestants’ prize valuations has been ignored. Despite the fact that in some studies the source of the prize system has been based on the existence of monopoly profits (rents) or various forms of protective trade-policies, Mueller (2002), the general role of public policy as a determinant of the contest prize system was not adequately studied. The main objective of this paper is to fill this gap.

Our argument is that quite often the prize system is determined by government intervention that takes the form of a policy proposal that constitutes an alternative to an existing status-quo policy. The possible ‘prizes’ or stakes of the affected interest groups are equal to the differences in the payoffs corresponding to the status-quo and the proposed policy reform. Whether the proposed policy is implemented or not depends on the outcome of a political contest in which these groups exert efforts to increase the probability that their preferred policy, the existing policy or the proposed policy, is the outcome of the contest. The existing status-quo policy is the realized contest outcome if the proposed policy is rejected. The alternative proposed policy is the implemented contest outcome if the proposed policy is approved. Typically, therefore, in a two-player contest one interest group supports the rejection of the proposed policy because it prefers the status-quo policy while the other interest group supports the approval of the alternative proposed policy. For example, a tax reform may be supported by one industry and opposed by another. Existing pollution standards may be defended by the industry and challenged by an environmentalist interest group. A monopoly can face the opposition of a customers coalition fighting
for appropriate regulation.\textsuperscript{1} Capital owners and a workers union can be engaged in a contest that determines the minimum wage, and so on.\textsuperscript{2} The outcome of such contests depends on the contestants’ exerted efforts (fighting, lobbying or rent-seeking efforts), that depend, in turn, on the parameters of the contest and, in particular, on the contestants’ payoffs in the event that the public-policy proposal is approved or rejected. Some of the above examples are elaborated in the sequel to illustrate the effect of a policy reform on the payoff structure of the contestants.

A major concern in the contest literature has been the issue of how do changes in the parameters of the contest (number, valuations and abilities of the contestants and the nature of the information they have) affect their equilibrium efforts and the extent of relative prize dissipation, Hillman and Riley (1989), Hurley and Shogren (1998), Nitzan (1994). In addition, attention has been paid to the effect of changes in these parameters on the contestants’ expected payoffs, Baik (1994), Gradstein(1995) and Nti (1997, 1999) and on their aggregate expected payoff, Epstein and Nitzan (2001b). The main comparative statics concern of this study is the clarification of the effect of changes in public policy that determine the prize system on the contestants’ efforts and performance: probability of winning the contest. Earlier studies examined the sensitivity of total efforts to changes either in the value of the prize or in the prize valuation of one of the contestants. Our extended comparative statics analysis focuses on the effects of a change in the proposed public policy that generates simultaneous modifications in the prize valuations of all the contestants. Nevertheless, even in those cases that such a change only modifies the prize valuation of a single contestant, we generalize the existing results that dealt with special forms of our general contest success function.

\textsuperscript{1} A special case of this setting is studied by Baik (1999) who analyzes the welfare effect of consumer opposition to the existence of monopoly rents.

\textsuperscript{2} Two recent examples from U.S politics that illustrate the public-policy contest that we study are the congressional reviews of late-term Clinton administration actions on ergonomics and environmental regulations on land use in national forests. Both regulations were reviewed and criticized by the new Congress, and both could have been squelched. But the ergonomics regulations (a decade in the making) were overturned under the Congressional Review Act of 1996, while the environmental regulations were allowed to stand. The explanations for these outcomes can be traced to the strength of the interest groups supporting the regulations (organized labor and the environmental lobby,
We first present the general framework of binary public-policy contests with two possible states of nature (approval and rejection of the proposed policy) allowing the general contest success functions axiomatized by Skaperdas (1996). This framework has numerous possible applications such as contests on the approval or rejection of a proposed minimum wage, monopoly regulation, tax reform, protection by tariff or some new environmental policy. The rest of the paper is then devoted to the comparative-statics properties of the public policy contest and, in particular, to the clarification of the role of three types of asymmetry between the contestants on the sensitivity of effort and performance to the proposed public policy.

Changes in the asymmetry between the contestants can give rise to perverse incentives for a politician who designs a contest. Baye et. al. (1993) have shown that a politician wishing to maximize political rents may find it in his interest to exclude certain lobbyists from participation in the lobbying process - particularly lobbyists valuing most the prize - because this increases the lobbying efforts of the remaining contestants. More recently, Che and Gale (1998) have proved that asymmetric limits on exerted effort can also have the effect of increasing the total efforts of the contestants. We show that in a public-policy contest, under certain conditions of asymmetry between the contestants, a more restrained government intervention that reduces the prizes of the two contestants has the perverse effect of increasing their total exerted efforts.

In section II we introduce the public-policy contest. In section III we present the function that generates the prize system (the stakes) of the contest and illustrate its applicability. Section IV contains the comparative statics analysis that focuses on the effect of changes in the proposed public policy on the equilibrium asymmetries between the contestants and, in turn, on the equilibrium effort of the interest groups and on their probability of winning the contest. Section V contains brief concluding remarks.

II. The Public Policy Contest

In our contest the players are two interest groups that are differently affected by the approval and rejection of a proposed policy. In general, one group derives a higher benefit than the other from the realization of its preferred policy. We therefore refer respectively).
to one player as the Low-Benefit (LB) player and to the other player as the High-Benefit (HB) player. The interest groups engage in a contest that determines the probabilities of approval and rejection of the proposed policy.

Player $i$’s preferred policy is approved in probability $Pr_i$. The present discounted value of this policy to this player is equal to $u_i$ and its value to his opponent player $j$ is equal to $v_j$. By assumption then, for each player, approval of his preferred policy is associated with a positive payoff, that is, $u_i > v_i$. Note that, in general, the four values $u_L, v_L, u_H$ and $v_H$, viz., the players’ payoffs corresponding to the approval and rejection of the policy $I$ proposed by the government (a ruling politician or a bureaucrat) depend on $I$.

Let $x_i$ denote the effort of the risk-neutral player $i$. The expected net payoff of $i$ is given by:

$$E(w_i) = Pr_i u_i(I) + Pr_j v_j(I) - x_i, i \neq j$$

Given the contestants’ efforts, the probabilities of approval and rejection of the proposed policy, $Pr_L$ and $Pr_H$, are obtained by the contest success function. As in Skaperdas (1992), it is assumed that $\frac{\partial Pr_i(x_i, x_j)}{\partial x_i} > 0$, $\frac{\partial Pr_i(x_i, x_j)}{\partial x_j} < 0$ and $\frac{\partial^2 Pr_i(x_i, x_j)}{\partial x_i^2} < 0$ (the latter inequality ensures that the second order conditions are satisfied). Since $Pr_i(x_i, x_j) + Pr_j(x_j, x_i) = 1$, $i \neq j$, it holds that

$$\frac{\partial^2 Pr_i(x_i, x_j)}{\partial x_i \partial x_j} = -\frac{\partial^2 Pr_j(x_j, x_i)}{\partial x_i \partial x_j}.$$
The ability of a contestant \( j \) to convert effort into probability of winning the contest can be represented by the marginal effect of a change in his effort on his winning probability. By assumption, this marginal effect is declining with his own effort. A change in his effort also affects, however, the marginal winning probability of his opponent \( i \). The opponent \( i \) has an advantage in terms of ability if a change in \( j \)’s effort positively affects his marginal winning probability. In other words, a positive (negative) sign of the cross second-order partial derivative of \( \frac{\partial^2 Pr_i}{\partial x_j \partial x_i} \), implies that \( i \) has an advantage (disadvantage) when \( j \)’s effort changes. At some given combination of efforts \((x_i, x_j)\), the ratio between the effect of a change in \( j \)’s effort on the marginal winning probability of \( i \) and the effect of a change in \( j \)’s effort on his own ability, \( \frac{\partial^2 Pr_i}{\partial x_i \partial x_j} \), is therefore a local measure of the asymmetry between the abilities of \( i \) and \( j \). This asymmetry together with two types of stakes-asymmetry that are presented below play a crucial role in determining the comparative statics effects on which this study focuses.

Denote by \( n_i = (u_i - v_i) \) the stake of player \( i \) (his real benefit from winning the contest), (see Baik, 1999, Epstein and Nitzan, 2001b and Nti, 1999). A player’s stake is secured when he wins the contest, that is, when his preferred policy is the outcome of the contest. Recall that for one player the desirable outcome is associated with the approval of the proposed policy while for the other player the desirable outcome is realized when the proposed policy is rejected. The expected net payoff (surplus) of interest group \( i \) can be rewritten as follows:

\[
E(w_i) = v_i(I) + Pr_i n_i(I) - x_i
\]  

(3)

In general, the stakes of the contestants are different, that is, one of them has an advantage over the other in terms of his benefit from winning the contest. With no loss of generality, we assume that \( n_L \leq n_H \). The ratio \( n_L / n_H \) is a measure of the asymmetry between the stakes of the contestants.
By our assumptions, both players participate in the contest ($x_L$ and $x_H$ are positive). We therefore focus on interior Nash equilibria of the contest. Solving the first order conditions \[
\frac{\partial E(w_L)}{\partial x_L} = 0 \quad \text{and} \quad \frac{\partial E(w_H)}{\partial x_H} = 0
\] we obtain:

\[
(4) \quad \Delta_i = \frac{\partial \Pr_i(x_i, x_j)}{\partial x_i} n_i(I) - 1 = 0 \quad \forall \ i \neq j \quad \text{and} \quad i, j = L, H
\]

Thus, the first order conditions require\(^6\) that:

\[
(5) \quad \frac{\partial \Pr_i}{\partial x_i} = \frac{1}{n_i(I)} \quad \forall \ i = L, H
\]

By the expressions in (5) that determine the equilibrium efforts of the players and their probabilities of winning the contest and by the assumed properties of the CSF, we directly obtain that under a symmetric contest success function\(^7\) ($\forall x_i, x_j, \Pr_i(x_i, x_j) = \Pr_j(x_j, x_i)$), the player with the higher stake makes a larger effort and has a higher probability of winning the contest. The probability of the socially more efficient outcome of the contest is thus higher than the probability of the less efficient outcome. For a similar result see Baik (1994) and Nti (1999). This type of efficiency criterion has been used by Ellingsen (1991), Fabella (1995) and, more recently, by Hurley (1998).

III. Public Policy and the Prize System (The Contestants’ Stakes)

A change in the policy instrument $I$ has an effect on the stakes of the players and thus on their efforts and on their probability of winning the contest\(^8\). In this section we examine how a change in the proposed policy affects the prize system, that is, the

---

\(^6\) It can be easily verified that the second order conditions hold.

\(^7\) Such symmetry implies that the two players share an equal ability to convert effort into probability of winning the contest.

\(^8\) Note that the domain of the policy instrument $I$, the closed interval $I \in [L, T]$, may reflect economic feasibility or political feasibility.
contestants’ stakes, assuming that the functions \( n_i(I) \) (\( i = L, H \)) are continuous and twice differentiable in \( I \). A policy reform may affect the stake of one of the contestants or the stakes of both of them. Denoting the effect of a change in \( I \) on \( n_i \) by \( n'_i, \ n'_i = \frac{\partial n_i}{\partial I} \), our subsequent analysis relates to all of the following five possible types of public-policy effects on the stakes of the interest groups:

**Table 1: The Possible Types of a Policy Reform**

<table>
<thead>
<tr>
<th>Type</th>
<th>( n'_i )</th>
<th>( n'_j )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i)</td>
<td>&gt;0</td>
<td>&lt;0</td>
</tr>
<tr>
<td>(ii)</td>
<td>&gt;0</td>
<td>=0</td>
</tr>
<tr>
<td>(iii)</td>
<td>=0</td>
<td>&lt;0</td>
</tr>
<tr>
<td>(iv)</td>
<td>&gt;0</td>
<td>&gt;0</td>
</tr>
<tr>
<td>(v)</td>
<td>&lt;0</td>
<td>&lt;0</td>
</tr>
</tbody>
</table>

In reforms of type (ii) and (iii), a change in \( I \) only affects the stake of one interest group. The incidence of the proposed policy reform in these cases is therefore partial. A change in \( I \) can be interpreted as a more (less) restrained government intervention if it reduces (increases) the affected stake. Clearly, such a change also affects the stakes-asymmetry between the contestants. For example, in type (ii) reform where \( i=H \), an increase in \( I \) represents a less restrained intervention that increases the asymmetry between the stakes of the contestants.

In the remaining types the incidence of the proposed reform is complete because a change in \( I \) affects the stakes of the two contestants. In reforms of type (i) a change in the policy instrument \( I \) has opposite effects on the stakes of the two players. If \( i=H \), such a change positively affects the asymmetry between the stakes of the contestants. If \( i=L \), the asymmetry between the stakes is inversely related to a change in \( I \). In both cases a change in the proposed policy can be considered as a more restrained government intervention if it reduces the sum of the stakes \( n_L + n_H \).

In reforms of type (iv) and (v) a change in \( I \) has a similar positive or negative effect on the stakes of the players. In both of these cases, therefore, such a change can
be unambiguously interpreted as a more restrained or a less restrained government intervention. In these cases the effect of a change in $I$ on the stakes-asymmetry depends on the relationship between the elasticities of the stakes with respect to $I$.

Specifically, the effect of a change in $I$ on $n_L/n_H$ depends on whether $\frac{\eta_L}{\eta_H}$ is greater or smaller than 1, where $\eta_j = \frac{\partial n_j}{\partial I} n_j, j = L, H$. In a type (iv) reform the asymmetry in the stakes is positively related to a change in $I$ if $\frac{\eta_L}{\eta_H} < 1$. In a type (v) reform the asymmetry in the stakes is positively related to a change in $I$ if $\frac{\eta_L}{\eta_H} > 1$.

Usually, a change in public policy affects the stakes of the two contestants. The applicability of the corresponding reforms of type (i), (iv) and (v) is illustrated by the following examples.

**Monopoly price regulation:** Several authors have pointed out that consumers oppose government protection of a monopoly in attempting to defend their surplus, Baik (1999), Ellingsen (1991), Fabella (1995). The government is assumed to make a binary decision: regulating the monopoly or not, that is, force the firm to charge the competitive price or let it charge the profit-maximizing monopoly price. In this example therefore the firm is the $LB$ player and the consumers’ representative is the $HB$ player. A change in the proposed price positively affects the stakes of both players, provided that the monopoly price ranges between the competitive price and the profit-maximizing monopoly price. Under this example the reform is of type (iv). If the monopoly price is restricted to the range of prices exceeding the profit-maximizing monopoly price, then a change in price inversely affects the firm’s stake and positively affects the consumers’ stake. In such a case the example of monopoly price regulation is a type (i) reform where $i=H$.

**Public-Good Provision:** The government considers building a park on the border of a residential neighborhood. In order to finance the project, a general tax is levied on all residents of the country who may benefit from the provision of the proposed park. In addition to the tax, the local residents, the individuals who reside close to the park, are subjected to another “tax”: the negative externalities (increased congestion, noise, etc.) associated with living close to a public park that attracts a large number of
visitors all year round. Suppose that the policy instrument of the government is the tax paid by the residents of the country. We consider three possible cases:

(1) The collected taxes are allocated to the building of the park. An increase in the lump-sum tax or in the tax rate implies an increase in the size of the park. We assume that such an increase raises the benefit of the non-local residents, however, it reduces the benefit of the local residents. In contrast to example 1, in the present example it is not clear who is the LB and the HB player. It is clear nevertheless that as the size of the park increases, the benefit of the non-local residents from approval of the proposed tax and from the corresponding park and the benefit of the local residents from the rejection of this proposal increase with the tax and, in turn, with the size of the park. As in the previous example the proposed reform is of type (iv).

(2) Although an increase in taxes increases the park size, we now assume that beyond a certain point, an increase in the size of the park does not increase the benefit of the general public. In such a case, an increase in taxes and, in turn, in the size of the park may reduce the benefit associated with the approval of the proposed change in the tax for the non-local residents, while increasing the net benefit associated with the rejection of the proposed tax change for the local residents. In such a case, the proposed reform is of type (i) or type (ii).

(3) Suppose that the collected taxes are used to finance the park as well as to compensate the local residents for the negative externalities. In such a case it is possible that an increase in the proposed tax results in a decrease in the benefit associated with the rejection of the proposed tax change for the local residents while reducing the benefit associated with the approval of the proposed tax change for the non-local residents. The reform can be of type (v).

Protection by Tariff: The study of trade policy determination has often applied rent seeking or contest models, Hillman (1989), Mueller (2002). Consider, for example, a local producer who wishes to be protected by a tariff on the import of the product he produces. Such a protection creates rents for the producer, however, it reduces the welfare of the consumers. This simple example is similar to the first example of monopoly price regulation. The producer is the LB player and the representative of the consumers is the HB player. An increase in the proposed tariff increases the benefit of the consumers from disapproval of the proposed tariff change. It also
increases the benefit of the producer from the approval of the proposed change in the degree of protection he enjoys. The proposed reform is of type (iv) and, subject to the required modifications in the interpretation of the $LB$ and $HB$ players, the conclusions obtained in this example are similar to those of the first example.\footnote{If the policy instrument is a protective general tariff - which is imposed on all imported products including the imported inputs, the representatives of the local producers and the consumers are, respectively, the $LB$ and $HB$ players. In such a case, rejection of a proposed increase in the tariff increases the net benefit of the local producers. Approval of such a proposed increase in the protective tariff may increase or decrease the benefit (the rent) of the local producers. The latter possibility occurs when production costs become sufficiently high due to the increased tariff on the imported inputs. Under this latter possibility, the proposed reform is of type (i). Otherwise we are back to a type (iv) reform.}

Clearly, numerous other applications come to mind. In fact, any setting where public policy affects the payoffs of two agents (interest groups), such that one agent is interested in the approval of the proposed policy and the other agent supports the rejection of that proposal can serve as an illustration to our model. One can easily construct other examples assuming, for example, that the policy instrument is the quality of a public good provided by the government, the degree of privatization of a particular publicly-owned company or any control variable that results in income transfer from one agent (interest group) to another.

IV. Public Policy, Efforts and Winning Probabilities

The effort exerted in the public-policy contest deserves attention because it can be interpreted as social costs and, therefore, serve as a measure of inefficiency. Understanding how public policy affects this effort, which is often referred to as rent dissipation, is the main goal of the proposed theory of public-policy contests.

By differentiation of the first order conditions (see (4)), we get that the Nash equilibrium efforts satisfy the following conditions:

$$\frac{\partial x_i^*}{\partial I} = \frac{\frac{\partial \Delta_j}{\partial x_j} \frac{\partial \Delta_i}{\partial x_j} - \frac{\partial \Delta_j}{\partial x_i} \frac{\partial \Delta_i}{\partial x_j}}{\frac{\partial \Delta_j}{\partial x_i} - \frac{\partial \Delta_i}{\partial x_i}} \quad \forall i \neq j, \ i, j = L, H$$

We thus obtain that:
Rewriting (7) together with (5), we obtain the fundamental equation that generates all the comparative statics results:

\[
\frac{\partial x^*_i}{\partial I} = \frac{n_i}{n_j} \left( \frac{\partial^2 P_{r,i}}{\partial x_i^2} \frac{\partial n_j}{\partial x_j} - n_j \frac{\partial^2 P_{r,j}}{\partial x_j^2} \frac{\partial n_i}{\partial x_i} \right) - n_j \frac{\partial^2 P_{r,j}}{\partial x_j^2} \frac{\partial n_i}{\partial x_i} \frac{\partial n_j}{\partial x_j} \forall i \neq j, \ i, j = L, H
\]

where \(B = n_i n_j \left( \frac{\partial^2 P_{r,j}}{\partial x_j^2} \frac{\partial^2 n_i}{\partial x_i^2} - \frac{\partial^2 P_{r,i}}{\partial x_i^2} \frac{\partial^2 n_j}{\partial x_j^2} \right)\) and all second-order partial derivatives are computed at the Nash equilibrium \((x^*_H, x^*_L)\). The first term in (8) represents the strategic rival’s-stake (“substitution”) effect. The sign of this term is equal to the sign of \(\frac{\partial^2 P_{r,j}}{\partial x_j^2} \frac{\partial n_i}{\partial x_i} \). The second term represents the own-stake (“income”) effect. The sign of this term is equal to the sign of \(\eta_i\). By assumption, \(\frac{\partial^2 P_{r,i}}{\partial x_i^2}(x_i, x_j) < 0\) and, by (2), \(\frac{\partial^2 P_{r,i}}{\partial x_i \partial x_j} \frac{\partial^2 P_{r,j}}{\partial x_j \partial x_i} < 0\). Hence, \(B > 0\).

A. Public Policy, Efforts and Winning Probabilities

When a change in \(I\) only affects the stake of one of the contestants, as in reforms type (ii) and (iii), \(\eta_i\) or \(\eta_j\) is equal to zero and (8) reduces to

\[
(8') \quad \frac{\partial x^*_i}{\partial I} = \frac{1}{B} \left( \frac{\partial^2 P_{r,i}}{\partial x_i^2} \eta_j n_i \right) \quad \text{or} \quad \frac{\partial x^*_j}{\partial I} = \frac{1}{B} \left( - \frac{\partial^2 P_{r,j}}{\partial x_j^2} \eta_i n_j \right) \forall i \neq j, \ i, j = L, H
\]

In these cases the change in player \(i\)’s effort corresponding to the change in \(n_j\) is equal to the strategic rival’s-stake (“substitution”) effect, when \(i \neq j\), or to the strategic...
own-stake ("income") effect, when \( i=j \). The former effect is ambiguous, depending on the sign of the cross-partial derivative of the contest success function. The latter effect is clear-cut, due to our assumption that the marginal winning probability of a contestant is declining in his own effort.

In case (ii) with \( i=H \) and case (iii) with \( i=L \),

\[
\frac{\partial x^*_H}{\partial I} = \frac{1}{B} \left( \frac{\partial^2 \Pr_L}{\partial x_H^2} \eta_H n_L \right) \quad \text{and} \quad \frac{\partial x^*_L}{\partial I} = \frac{1}{B} \left( \frac{\partial^2 \Pr_L}{\partial x_H^2} \eta_H n_L \right).
\]

In cases (iii) with \( i=H \) and case (ii) with \( i=L \),

\[
\frac{\partial x^*_H}{\partial I} = \frac{1}{B} \left( \frac{\partial^2 \Pr_H}{\partial x_L^2} \eta_L n_H \right) \quad \text{and} \quad \frac{\partial x^*_L}{\partial I} = \frac{1}{B} \left( \frac{\partial^2 \Pr_H}{\partial x_L^2} \eta_L n_H \right).
\]

We therefore obtain

**Proposition 1:**

In case (ii) with \( i=H \) and case (iii) with \( i=L \),

\[\text{Sign} \left( \frac{\partial^* x_H}{\partial I} \right) = \text{Sign} \left( \eta_H \right) \quad \text{and} \quad \text{Sign} \left( \frac{\partial^* x_L}{\partial I} \right) = \text{Sign} \left( \frac{\partial^2 \Pr_L}{\partial x_H \partial x_L} \eta_H \right).\]

In cases (iii) with \( i=H \) and case (ii) with \( i=L \),

\[\text{Sign} \left( \frac{\partial^* x_L}{\partial I} \right) = \text{Sign} \left( \eta_L \right) \quad \text{and} \quad \text{Sign} \left( \frac{\partial^* x_H}{\partial I} \right) = \text{Sign} \left( \frac{\partial^2 \Pr_H}{\partial x_L \partial x_H} \eta_L \right).\]

Proposition 1 directly yields the following general comparative statics result that focuses on the sensitivity of a contestant’s effort to a change in his or his rival’s stake:

**Corollary 1.1:**

\[
\frac{\partial x^*_H}{\partial n_H} > 0, \quad \frac{\partial x^*_L}{\partial n_L} > 0, \quad \text{Sign} \left( \frac{\partial x^*_H}{\partial n_H} \right) = \text{Sign} \left( \frac{\partial^2 \Pr_H}{\partial x_H \partial x_L} \right) \quad \text{and} \quad \text{Sign} \left( \frac{\partial x^*_L}{\partial n_L} \right) = \text{Sign} \left( \frac{\partial^2 \Pr_H}{\partial x_L \partial x_H} \right).
\]

By this first corollary, under our general contest success function, the effort exerted by a contestant is positively related to his stake. That is, the strategic own-stake ("income") effect is always positive (effort of every player is a "normal good"). In contrast, the effort exerted by a player can be positively or negatively related to the stake of his rival. It can also be independent of the rival’s stake. When the marginal
winning probability of a contestant in equilibrium is positively (negatively) related to his rival’s effort, his strategic substitution effect is positive (negative). Following Bulow, Geanakoplos and Klemperer (1985), in such a case we say that a contestant’s effort is a strategic complement (substitute) to his rival’s effort. When the cross-partial derivative of the contest success function is equal to zero the contestants’ efforts are independent. Note that, by (2), in our setting the strategic substitution effects are asymmetric; if a player’s effort is a strategic complement to his opponent’s effort, then his opponent’s effort is a strategic substitute to his effort.

In the symmetric case where, $\forall x_H$ and $x_L$, $Pr_H(x_L, x_H) = 1 - Pr_H(x_H, x_L)$, there exists a pure strategy Nash equilibrium, such that $x_H^* > x_L^*$ and, in equilibrium,

$$\text{Sign} \left( \frac{\partial^2 Pr_H}{\partial x_H \partial x_L} \right) = \text{Sign} \left( x_H^* - x_L^* \right) > 0,$$

which implies that $\frac{\partial^2 Pr_L}{\partial x_L \partial x_H} < 0$. Hence, by Corollary 1.1,

**Corollary 1.2:** If, $\forall x_H$ and $x_L$, $Pr_H(x_L, x_H) = 1 - Pr_H(x_H, x_L)$, then

$$\frac{\partial x_H^*}{\partial n_H} > 0, \quad \frac{\partial x_L^*}{\partial n_L} > 0, \quad \frac{\partial x_H^*}{\partial n_L} > 0 \quad \text{and} \quad \frac{\partial x_L^*}{\partial n_H} < 0.$$

This second corollary generalizes the result obtained by Nti (1999) where $Pr_i$ is assumed to take the particular symmetric logit form, as in Tullock (1980), $Pr_i(x_i, x_j) = \frac{x_i'}{x_i' + x_j'}$, $I = L, H$. In this special case of symmetric lobbying abilities of the contestants, the $HB$ player can be referred to as the favored player and the $LB$ player can be referred to as the underdog, see Dixit (1987). Corollary 1.2 establishes that effort of the favored player increases with both his own stake (valuation of the contested prize) and with the stake (prize valuation) of the underdog. Effort of the underdog increases with his stake (prize valuation), but decreases with the stake (prize valuation) of the favored player.

Another more general asymmetric form of the logit contest success function is:

$$Pr_H = \frac{\sigma h(x_H)}{\sigma h(x_H) + h(x_L)}, \quad \text{where} \quad \sigma > 0, \quad h(0) \geq 0 \quad \text{and} \quad h(x_i) \text{ is increasing in } x_i,$$

10 In this special case we keep the assumption that a contestant’s marginal winning probability is
Baik (1994). Here the parameter $\sigma$ represents the asymmetry between the lobbying abilities of the two players. Note that when $\sigma < 1$ the $HB$ player has an ability disadvantage relative to the $LB$ player. It can be shown that under this particular contest success function, $\text{Sign} \left( \frac{\partial^2 \Pr_L}{\partial x_L \partial x_H} \right) = \text{Sign} \left( \Pr_L - \Pr_H \right)$ and, therefore, for some $\sigma^* < 1$, $\Pr_L = \Pr_H = 1/2$ and $\frac{\partial^2 \Pr_L}{\partial x_L \partial x_H} = \frac{\partial^2 \Pr_H}{\partial x_H \partial x_L} = 0$. By Corollary 1.1 we get

**Corollary 1.3:** If $\Pr_H = \frac{\sigma h(x_H)}{\sigma h(x_H) + h(x_L)}$, where $\sigma > 0$, $h(0) \geq 0$ and $h(x_i)$ is increasing in $x_i$, then

$$\frac{\partial x_H}{\partial n_H} > 0, \quad \frac{\partial x_L}{\partial n_L} > 0 \quad \text{and} \quad \frac{\partial x^*_L}{\partial n_L} \geq 0 \iff \frac{\partial x^*_H}{\partial n_H} \leq 0 \iff \Pr_H \leq 1/2.$$ 

This third corollary generalizes Proposition 1 in Baik (1994).

**B. Complete incidence: Policy reforms affecting both stakes**

When a change in $I$ affects the stakes of the two contestants, as in reforms type (i), (iv) and (v), $\eta_L$ and $\eta_H$ are positive or negative. By the fundamental equation (8), when the contestants’ efforts are independent, the sensitivity of every contestant’s effort with respect to a proposed policy reform is always unequivocal. When the contestants’ efforts are not independent, the sensitivity of one of the contestants’ effort with respect to a proposed policy reform is always unequivocal because the sign of his strategic rival’s-stake (“substitution”) effect is equal to the sign of his strategic own-stake (“income”) effect. The sensitivity of his opponent’s effort with respect to the proposed policy reform is ambiguous, depending on whether his strategic own-stake (“income”) effect is larger than, equal to or smaller than his strategic rival’s-stake (“substitution”) effect. Using (8) we thus get
**Proposition 2**: In cases (i), (iv) and (v),

(a) If \( \frac{\partial^2 \text{Pr}_i}{\partial x_j \partial x_i} = 0 \), then \( \text{Sign} \left( \frac{\partial x^*_i}{\partial I} \right) = \text{Sign} \left( \eta_i \right) \).

(b) If \( \frac{\partial^2 \text{Pr}_i}{\partial x_j \partial x_i} \neq 0 \), then (1) \( \text{Sign} \left( \frac{\partial x_i}{\partial I} \right) = \text{Sign} \left( \eta_i \right) \) \( \iff \) \( \text{Sign} \left( \frac{\partial^2 \text{Pr}_i}{\partial x_j \partial x_i} \eta_j \right) = \text{Sign} \left( \eta_i \right) \)

and

\[
(2) \quad \frac{\partial x_i}{\partial I} > 0 \iff -\frac{\partial^2 \text{Pr}_i}{\partial x_j^2} \eta_j n_j > -\frac{\partial^2 \text{Pr}_i}{\partial x_j \partial x_i} \eta_i n_j
\]

By Proposition 2 (a), if the contestants are symmetric in equilibrium in terms of their abilities, then the strategic rival’-stake (“substitution”) effects vanish (efforts are independent) and the positive strategic own-stake (“income”) effect solely determines the direct effect of a change in I on a contestant’s effort. In the perfectly symmetric case where, \( \forall x_H \) and \( x_L \), \( \text{Pr}_H(x_L, x_H) = 1 - \text{Pr}_H(x_H, x_L) \) and \( n_H = n_L = n \), there exists a symmetric pure strategy Nash equilibrium, \( x^*_H = x^*_L \), and \( \frac{\partial^2 \text{Pr}_H}{\partial x_H \partial x_L} = 0 \), see Dixit (1987). Hence, by Proposition 2 (a),

**Corollary 2.1**: If, \( \forall x_H \) and \( x_L \), \( \text{Pr}_H(x_L, x_H) = 1 - \text{Pr}_H(x_H, x_L) \) and \( n_H = n_L = n \), then

\[
\frac{\partial x^*_H}{\partial n} = \frac{\partial x^*_L}{\partial n} > 0.
\]

This corollary generalizes a similar result established by Nti (1999), assuming a particular contest success function of the logit form.

Proposition 2(b) can be used to determine the sensitivity of the contestants’ efforts in all possible situations corresponding to the three types of policy reforms affecting both stakes and \( \frac{\partial^2 \text{Pr}_i}{\partial x_j \partial x_i} \neq 0 \). Consider for example a type (i) policy reform and suppose that \( \frac{\partial^2 \text{Pr}_H}{\partial x_H \partial x_L} < 0 \), that is, the effort of the HB player is a strategic substitute of the effort of the LB player. By Proposition 2 (b), in such a case,
\[ \frac{\partial x^*_H}{\partial I} > 0 \text{ and } \frac{\partial x_L}{\partial I} \geq 0 \iff \frac{\partial^2 \Pr_{H}}{\partial x_H^2} \eta_L n_H > -\frac{\partial^2 \Pr_{L}}{\partial x_L \partial x_H} \eta_H n_L. \]

Notice that by Proposition 2(b), the conditions resolving the ambiguity regarding the sensitivity of \( j \)'s effort to a proposed policy reform involve the three elements of asymmetry between the contestants introduced in sections II and III:

\[ A^1_j = \frac{\partial^2 \Pr_j}{\partial x_j \partial x_i} \left( \frac{\partial^2 \Pr_i}{\partial x_i^2} \right), \quad A^2_j = \frac{n_i}{n_j} \quad \text{and} \quad A^3_j = \frac{\eta_j}{\eta_i}. \]

In fact, the comparison between the strategic rival’s-stake (“substitution”) effect and the strategic own-stake (“income”) effect depends on the relationship between the ability-asymmetry represented by \( A^1_j \) and the normalized stakes-asymmetry represented by \( A^3_j / A^2_j = \frac{\eta_j}{\eta_i}/ \frac{n_j}{n_i} \).

Specifically, by Proposition 2, it can be easily verified that

\textbf{Corollary 2.2:} \[ \frac{\partial x_L}{\partial I} < 0 \implies \frac{\partial x_L}{\partial I} > 0 \iff A^1_j > A^3_j \quad \text{and} \quad A^2_j > A^3_j. \]

\[ \frac{\partial x_L}{\partial I} > 0 \implies \frac{\partial x_L}{\partial I} < 0 \iff A^1_j < A^3_j \quad \text{and} \quad A^2_j < A^3_j. \]

To illustrate the economic interpretation of this corollary, suppose, for example, that the \( HB \) player has a disadvantage in terms of his equilibrium ability (marginal winning probability), that is, \( \frac{\partial^2 \Pr_H}{\partial x_H \partial x_L} < 0 \). By Proposition 2, when the proposed reform is of type (iv), an increase in \( I \) induces the \( LB \) player to increase his effort. In this case the \( HB \) player’s effort is a strategic substitute to the \( LB \) player’s effort, so the strategic substitution effect induces the \( HB \) player to reduce his effort. However, his effort is a “normal” good, so the increase in his stake induces him to increase his effort. The latter effect is dominant and the \( HB \) player also increases his effort, if his advantage in terms of stakes, which is represented by the stake-asymmetry measure

\[ \frac{A^3_H}{A^3_L} = \frac{\eta_H}{\eta_L} \frac{n_H}{n_L} \]
ability-asymmetry measure \( A^1_H = \frac{\partial^2 \Pr_H}{\partial x_H \partial x_L} \left( \frac{\partial^2 \Pr_L}{\partial x_L^2} \right) \). Similar economic interpretations can be given to the conditions in Corollary 2.2 in all other possible situations corresponding to the three types of reforms affecting the two players, given that the \( HB \) player is advantageous or disadvantageous in terms of his equilibrium ability.

In our setting, the response of one contestant to a change in the proposed policy is ambiguous. A change in \( I \) may differently affect therefore the aggregate efforts of the contestants. This implies that when \( \frac{\partial^2 \Pr}{\partial x_i \partial x_j} \neq 0 \), under any type of a proposed reform the effect of a change in \( I \) on the aggregate effort \( X^* = x^*_H + x^*_L \) is ambiguous. Since \( \frac{\partial X^*}{\partial I} = \frac{\partial x^*_H}{\partial I} + \frac{\partial x^*_L}{\partial I} \), by (8’) and Corollary 1.1 we get:

**Proposition 3:**

\[
\frac{\partial^2 \Pr_i}{\partial x_i \partial x_j} > 0 \Rightarrow \frac{\partial X}{\partial n_j} > 0 \quad \text{and} \quad \frac{\partial X}{\partial n_i} > 0 \Leftrightarrow -\frac{\partial^2 \Pr_j}{\partial x_j^2} \eta_j n_j > \frac{\partial^2 \Pr_j}{\partial x_i \partial x_j} \eta_i n_j.
\]

That is, if \( i \)'s effort is a strategic complement to \( j \)'s effort, then aggregate effort increases with an increase in \( j \)'s stake. Aggregate effort also increases with \( i \)'s stake, if the positive strategic own-stake ("income") effect of player \( i \) is larger than the negative strategic rival's-stake ("substitution") effect of player \( j \).

By (2) and (8) we obtain that:

\[
\frac{\partial X^*}{\partial I} = \frac{\partial x^*_H}{\partial I} + \frac{\partial x^*_L}{\partial I} =
\]

\[
(9) \quad 1 \left( \frac{\partial^2 \Pr_H}{\partial x_H \partial x_L} (\eta_L n_H - \eta_H n_L) \right) \left( \frac{\partial^2 \Pr_H}{\partial x_H^2} \eta_H n_H + \frac{\partial^2 \Pr_L}{\partial x_L^2} \eta_L n_L \right)
\]

Hence,
This proposition states the condition that resolves the ambiguity regarding the sensitivity of aggregate effort with respect to a change in $I$. The condition clarifies the role of stakes-asymmetry and ability-asymmetry between the contestants. It implies, in particular, that even under the most restrained type-(v) policy reform that reduces the stakes of the two contestants, it is possible that the two contestants are induced to increase their aggregate effort. This occurs when the negative rival’s-stake (“substitution”) effect of the contestant who is induced to increase his effort more than counterbalances the sum of the two positive own-stake (“income”) effects and his opponent’s positive substitution effect. Alternatively, if the $LB$ player’s effort is a substitute to the $HB$ player’s effort, a sufficiently high reduction in the normalized stakes-asymmetry, a sufficiently high value of $\frac{A^3_H}{A^2_H}$, would induce the $LB$ player to increase his effort such that aggregate effort is increased. When the $HB$ player’s effort is a substitute to the $LB$ player’s effort, a sufficiently small reduction in the normalized stakes-asymmetry, a sufficiently small value of $\frac{A^3_H}{A^2_H}$, would induce the $HB$ player to increase his effort such that aggregate effort is increased. The fact that a reduction in stakes-asymmetry can give rise to perverse effort incentives have been noticed by Baye et. al. (1993) and by Che and Gale (1998). The former scholars have noticed that a politician who designs a contest may find it in his interest to exclude certain lobbyists from participation in the lobbying process - particularly lobbyists valuing most the prize - because this increases the lobbying efforts of the remaining contestants. The latter scholars have noticed that asymmetric limits on exerted effort can also have the effect of increasing the aggregate efforts of the contestants. We show that in a public-policy contest, under a sufficiently high or a sufficiently low reduction in stakes-asymmetry, the most restrained government intervention that takes the form of a type-(v) policy reform that reduces the prizes of the two contestants may have the perverse effect of increasing their aggregate efforts.

Let us finally consider how a change in the proposed policy affects the performance of the contestants, their probability of winning the contest:
\[ \frac{d \Pr^*_L}{d I} = \frac{\partial \Pr_L}{\partial x^*_L} \frac{\partial x^*_L}{d I} + \frac{\partial \Pr_L}{\partial x^*_H} \frac{\partial x^*_H}{d I} \]

Note that \( \frac{\partial \Pr_L}{\partial x^*_H} = - \frac{\partial \Pr_H}{\partial x^*_L} \), \( \frac{\partial^2 \Pr_L}{\partial x^*_L \partial x^*_L} = - \frac{\partial^2 \Pr_H}{\partial x^*_L \partial x^*_L} \) and \( \frac{\partial \Pr_L}{\partial x^*_H} = \frac{1}{n_i(I)} \). Thus we may rewrite (10) as:

\[ (11) \quad \frac{d \Pr^*_L}{d I} = \frac{1}{B n_L n_H} \left( n_L n_H \frac{\partial^2 \Pr_H}{\partial x^*_L \partial x^*_L} (\eta_H + \eta_L) - \left( \frac{\partial^2 \Pr_H}{\partial x^*_L \partial x^*_L} \eta_H n^2_H - \frac{\partial^2 \Pr_L}{\partial x^*_L \partial x^*_L} \eta_L n^2_L \right) \right) \]

This gives

**Proposition 5:**

\[ \frac{d \Pr^*_L}{d I} > 0 \text{ if } \frac{\partial^2 \Pr_L}{\partial x^*_L \partial x^*_L} (\eta_H + \eta_L) > \left( \frac{\partial^2 \Pr_H}{\partial x^*_L \partial x^*_L} \eta_L n^2_L - \frac{\partial^2 \Pr_H}{\partial x^*_L \partial x^*_L} \eta_H n^2_H \right) \]

By this proposition we get:

**Corollary 5.1:**

(a) Under a type-(i) reform with \( i=H \), \( \frac{d \Pr^*_L}{d I} < 0 \text{ if } \frac{\partial^2 \Pr_L}{\partial x^*_L \partial x^*_L} \leq 0 \);

(b) Under a type-(iv) reform with \( i=L \), \( \frac{d \Pr^*_L}{d I} > 0 \text{ if } \frac{\partial^2 \Pr_L}{\partial x^*_L \partial x^*_L} \geq 0 \);

(c) Under a type-(iv) reform,

\[ \frac{d \Pr^*_L}{d I} < 0 \text{ if } \frac{\partial^2 \Pr_L}{\partial x^*_L \partial x^*_L} \left( \frac{\partial^2 \Pr_H}{\partial x^*_L \partial x^*_L} \right) > \eta_L n^2_L \text{ and } \frac{\partial^2 \Pr_H}{\partial x^*_L \partial x^*_L} \leq 0 \]

(d) Under a type-(v) reform,

\[ \frac{d \Pr^*_L}{d I} < 0 \text{ if } \frac{\partial^2 \Pr_H}{\partial x^*_L \partial x^*_L} \left( \frac{\partial^2 \Pr_L}{\partial x^*_L \partial x^*_L} \right) > \eta_L n^2_L \text{ and } \frac{\partial^2 \Pr_L}{\partial x^*_L \partial x^*_L} \leq 0. \]
Under a type (i) reform with $i=H$, an increase in the proposed policy increases the stakes-asymmetry between the contestants. In other words, such a reform tends to increase the disadvantage of the LB player in terms of stakes. If he is also disadvantageous in terms of ability (marginal contest winning probability), then, by Corollary 5.1 (a), the proposed increase in $I$ reduces both his effort (see Proposition 2) and his probability of winning the contest. Notice that this is the case, despite the possible decline in the effort exerted by the HB player. Under a type-(i) reform with $i=L$, an increase in the proposed policy reduces the stakes-asymmetry between the contestants. That is, the LB player becomes less disadvantageous in terms of the contest stakes. If he also has a disadvantage in terms of ability (marginal contest winning probability), then, by Corollary 5.1 (b), the proposed increase in $I$ increases his probability of winning the contest, despite the fact that his effort need not rise (see Proposition 2).

V. Conclusion

Government intervention often gives rise to contests in which the possible prizes are determined by the existing status-quo and some new public-policy proposal. Since a proposed policy reform has different implications for different interest groups, these groups make efforts to affect in their favor the probability of approval of the proposed public policy. A change in the proposed policy modifies the stakes of the interest groups who take part in the contest on the approval or rejection of the proposed policy. Such a change has two effects on the nature of the public-policy contest. On the one hand, it affects the degree of competition by increasing or decreasing the sum of the potential prizes (stakes). On the other hand, it also affects the contest degree of competition by increasing or decreasing the asymmetry between the contestants’ stakes (prize valuations). What determines the contestants’ effort response to the proposed policy reform and, in turn, the change in their probability of winning the contest, are three asymmetry factors: The existing stakes-asymmetry; the asymmetry in the effect of a proposed reform on the existing stakes; and the ability-asymmetry: the asymmetry in the effect of a change in a contestant’s effort on his own and on his opponent’s marginal probability of winning the contest.

We studied a general class of two-player public-policy contests and examined the effect of a change in the proposed policy, a change that may affect the payoff of one contestant or the payoffs of the two contestants, on their effort and performance.
Proposition 1 and its corollaries generalize the comparative statics results of Baik (1994) and Nti (1999) that focus on the effect of changes either in the value of a contest prize in symmetric contests or in one of the contestants’ valuation of the prize in asymmetric contests, assuming special forms of our general contest success function. Propositions 2 – 5 present the comparative statics results in the extended setting where the proposed public policy determines the prize system: the stakes of the two interest groups. All the results hinge on a fundamental equation that stresses the significance of the relationship between the strategic own-stake (“income”) effects and the strategic rival’s-stake (“substitution”) effects corresponding to any change in the proposed public policy. We clarified how this relationship is determined by the three types of asymmetry between the contestants. In particular, we specified the asymmetry conditions under which a more restrained government intervention that reduces the contestants’ prizes has the perverse effect of increasing their aggregate lobbying efforts. This result complements the related findings of Baye et. al (1993) and Che and Gale (1998) that were established in the context of all-pay auctions. While these scholars focused, respectively, on constraints on the set of contestants and on caps on lobbying expenditures as possible means of reducing the asymmetry between the contestants, we emphasize the role of public policy reforms in generating direct changes in stakes-asymmetry and indirect changes in ability-asymmetry between the contestants.

The public-policy contest that we studied has numerous applications. It seems to us that it can constitute a useful basic building block in a more general theory of endogenous public-policy determination. In such a theory the contestants themselves could determine the two competing policies. Alternatively, another player, naturally the government, could design the contest and control the prize system by determining the proposed policy reform. Such a designer might be solely concerned with the aggregate efforts of the contestants (the lobbying expenditures or the campaign contributions), as in Baye et. al. (1993). But, of course, he may have broad objectives that take into account the aggregate efforts of the contestants as well as their welfare (expected payoffs), as in Epstein and Nitzan (2001a), Grossman and Helpman (1994) and Hillman (1982).
References


