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## Unobserved Components Models for Quarterly German GDP

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# Working Papers

## UNOBSERVED COMPONENTS MODELS FOR QUARTERLY GERMAN GDP

Gebhard Flaig

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## UNOBSERVED COMPONENTS MODELS FOR QUARTERLY GERMAN GDP

### Abstract

In this paper, an Unobserved Components Model is employed to decompose German real GDP into the trend, cycle and seasonal components and the working day effect. The most important findings are: 1) The growth rate of potential output declined from 4.2 per cent in the sixties to 1.4 per cent at the end of the nineties of the last century. 2) The business cycle is comprised of two subcycles with a period of about four and eight years, respectively. 3) The seasonal pattern is not constant over time and the number of working days contribute significantly to the short-term variability of output.

JEL Classification: C22, E20.

Keywords: trend, cycle, season, working day effect, output gap, unobserved components models.

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## 1 Introduction

The output gap, defined as the proportional deviation of realised from potential output, is one of the most important indicators of the cyclical state of the economy. In many models, the output gap is a key determinant of inflation and an argument in the Central Bank objective function (see, e.g., Clarida/ Galí/ Gertler 1999). From the viewpoint of time series analysis, the estimation of the output gap requires the decomposition of the observed output series into the non-stationary trend and the stationary cycle component. In many studies a variety of detrending techniques is used to carry out the trend-cycle decomposition (see e.g., Giorno et al. 1995, Canova 1998). In recent years, the use of ad hoc filters (e.g., the Hodrick-Prescott or the Baxter-King filter) has become more and more popular among macroeconomists. As is well known, ad hoc filters have a couple of disadvantages as they are typically independent of the time series under investigation (see Cogley/Nason 1995, Maravall 1995, Benati 2001).

In this study, an Unobserved Components (UC) Model is employed for decomposing German real GDP into the trend, cycle, seasonal and irregular components. As Harvey/Jaeger (1993) argue, this class of models provides a useful framework as they “are explicitly based on the stochastic properties of the data”. They are based on interpretable and well-defined models for the individual components, are very flexible in accomodating peculiar features of the time series and can be scrutinised by rigorous tests.

Unobserved Components Models have been used for an analysis of the GDP series of various countries (see, e.g., Watson 1986, Clark 1987, Harvey/Jaeger 1993, Flaig 2001 for the US, Kichian 1999 for Canada or Gerlach/Smets 1998 for the EMU area). For Germany, a first analysis has been presented in Flaig (2000). The novel feature in this study is a systematic exploration of how we should specify the different components (trend, cycle, season) in an optimal way and an investigation whether the total cycle can be broken up into several subcycles with different periodicity.

The organisation of the paper is as follows: In section 2, we present the econometric model and the alternative specification of the components. Section 3 provides the empirical results for German real GDP from the first quarter of 1960 to the second quarter of 2001. The final section contains a short summary and some concluding remarks.

## 2 An Unobserved Components Model for Quarterly Data

The basic assumption underlying Unobserved Components Models is that an observed time series  $y_t$  can be decomposed into several components which have an economic interpretation (for a general discussion see Harvey 1989; Maravall 1995). In the following, we decompose the logarithm of real GDP into the unobserved components trend  $T$ , season  $S$ , cycle  $C$ , the working day effect  $D$  and the irregular  $I$ :

$$(1) \quad y_t = T_t + S_t + C_t + D_t + I_t.$$

The trend component represents the long-run development of GDP and is specified as a random walk with a possibly time-varying drift rate  $\mu_t$ :

$$(2) \quad T_t = T_{t-1} + \mu_{t-1} + \gamma_1 DT_t + \varepsilon_t.$$

$DT_t$  (the level intervention) is a dummy variable which can take the values 0 or 1. If this variable is set to 1 in a specific period, the trend component jumps permanently by the amount  $\gamma_1$  from that period onwards. The level impulse  $\varepsilon_t$  is a white noise variable with mean zero and variance  $\sigma_\varepsilon^2$ .

The drift rate  $\mu_t$  is allowed to vary over time and is also defined as a random walk:

$$(3) \quad \mu_t = \mu_{t-1} + \gamma_2 DD_t + \xi_t.$$

$DD_t$  (the drift intervention) is a dummy variable which can take the values 0 or 1. If it is set to 1 in a specific period, the drift rate shows a jump and the level a kink. The drift impulse  $\xi_t$  is a white noise variable with variance  $\sigma_\xi^2$ .

Two different approaches are used for modelling the seasonal effects. The first formulation starts from the dummy variable method where it is assumed that the seasonal effects over four consecutive quarters sum to zero. We then simply add a white noise disturbance term to allow for changing seasonal patterns. The model is given in a recursive form by

$$(4) \quad S_t = -(S_{t-1} + S_{t-2} + S_{t-3}) + \omega_t,$$

where  $\omega_t$  is a white noise random variable with zero mean and variance  $\sigma_\omega^2$ .

The second formulation starts from the idea that the seasonal effect at time  $t$  can be specified as the sum of two cycles with the seasonal frequencies  $\pi/2$  (period of four quarters) and  $\pi$  (period of two quarters):

$$(5) \quad S_t = S_{t,1} + S_{t,2}.$$

Following Harvey (1989), we specify each seasonal cycle by a stochastic recursive formula:

$$(6) \quad \begin{pmatrix} S_{t,i} \\ S_{t,i}^* \end{pmatrix} = \begin{pmatrix} \cos \lambda_i & \sin \lambda_i \\ -\sin \lambda_i & \cos \lambda_i \end{pmatrix} \begin{pmatrix} S_{t-1,i} \\ S_{t-1,i}^* \end{pmatrix} + \begin{pmatrix} \omega_{t,i} \\ \omega_{t,i}^* \end{pmatrix}.$$

$S^*$  appears only by construction and has no intrinsic interpretation.

$\lambda_i \equiv \pi i/2$ ,  $i = 1, 2$ , denotes the seasonal frequencies and  $(\omega_{t,i}, \omega_{t,i}^*)$  are two uncorrelated white noise random variables with common variance  $\sigma_{\omega_i}^2$ . Note that for  $i = 2$ , equation (6) collapses to

$$(6a) \quad S_{t,2} = -S_{t-2,2} + \omega_{t,2}.$$

The cycle component is also specified by two different approaches. In the first approach, it is represented by a stationary autoregressive process of order  $p$ :

$$(7) \quad C_t = \phi_1 C_t + \phi_2 C_{t-1} + \dots + \phi_p C_{t-p} + \kappa_t.$$

The white noise variable  $\kappa_t$  with variance  $\sigma_\kappa^2$  is the cycle shock.

In the second approach, we specify the cycle  $C_t$  as the sum of  $M$  subcycles:

$$(8) \quad C_t = \sum_{i=1}^M C_{t,i}.$$

Each subcycle is specified as a vector AR (1) process:

$$(9) \quad \begin{pmatrix} C_{t,i} \\ C_{t,i}^* \end{pmatrix} = \rho_i \begin{pmatrix} \cos \lambda_i^C & \sin \lambda_i^C \\ -\sin \lambda_i^C & \cos \lambda_i^C \end{pmatrix} \begin{pmatrix} C_{t-1,i} \\ C_{t-1,i}^* \end{pmatrix} + \begin{pmatrix} \kappa_{t,i} \\ \kappa_{t,i}^* \end{pmatrix}.$$

The shocks  $\kappa_{t,i}$  and  $\kappa_{t,i}^*$  are assumed to be uncorrelated white noise variables with common variance  $\sigma_{\kappa_i}^2$ .

The period of subcycle  $i$  is  $2\pi/\lambda_i^C$ . The damping factor  $\rho_i$  with  $0 < \rho_i < 1$  ensures that  $C_{t,i}$  evolves as a stationary ARMA (2,1) process with complex roots in the AR-part (see Harvey 1989). This guarantees a quasi-cyclical behaviour of  $C_{t,i}$ . The shocks induce a stochastically varying phase and amplitude of the wave-like process. The cycle  $C_t$  is an ARMA (2 M, 2 M-1) process with restricted MA-parameters.

The working day effect  $D_t$  is specified by

$$(10) \quad D_t = \beta \ln(TD_t / \overline{TD}),$$

where  $TD_t$  is the number of working days in period  $t$  and  $\overline{TD}$  is the average number of working days over the estimation period. This formulation ensures that the working day effect has approximately a zero average over the estimation period.

The irregular component comprises a deterministic and a stochastic component:

$$(11) \quad I_t = \gamma_0 DI_t + u_t.$$

The deterministic component  $\gamma_0 DI_t$  (the impulse intervention) captures outliers which reflect identifiable events and  $u_t$  reflects temporary shocks which are modelled as a stochastic variable.  $u_t$  is assumed to be a white noise variable with variance  $\sigma_u^2$ .

It is assumed that all disturbances are normally distributed and are independent of each other. This is the usual assumption to assure the identification of the parameters (see, e.g., Watson 1986).

Estimation of the model parameters is carried out by maximum likelihood in the time domain. The initial values for the stationary cycle components are given by the unconditional distribution and for the nonstationary trend, drift and seasonal components by a diffuse prior. The filtered and smoothed values of the unobserved components are generated by the Kalman filter.

The unobserved components shown in figure 2 und figure 3 are the values from a fixed interval smoother (for details see Harvey 1989).

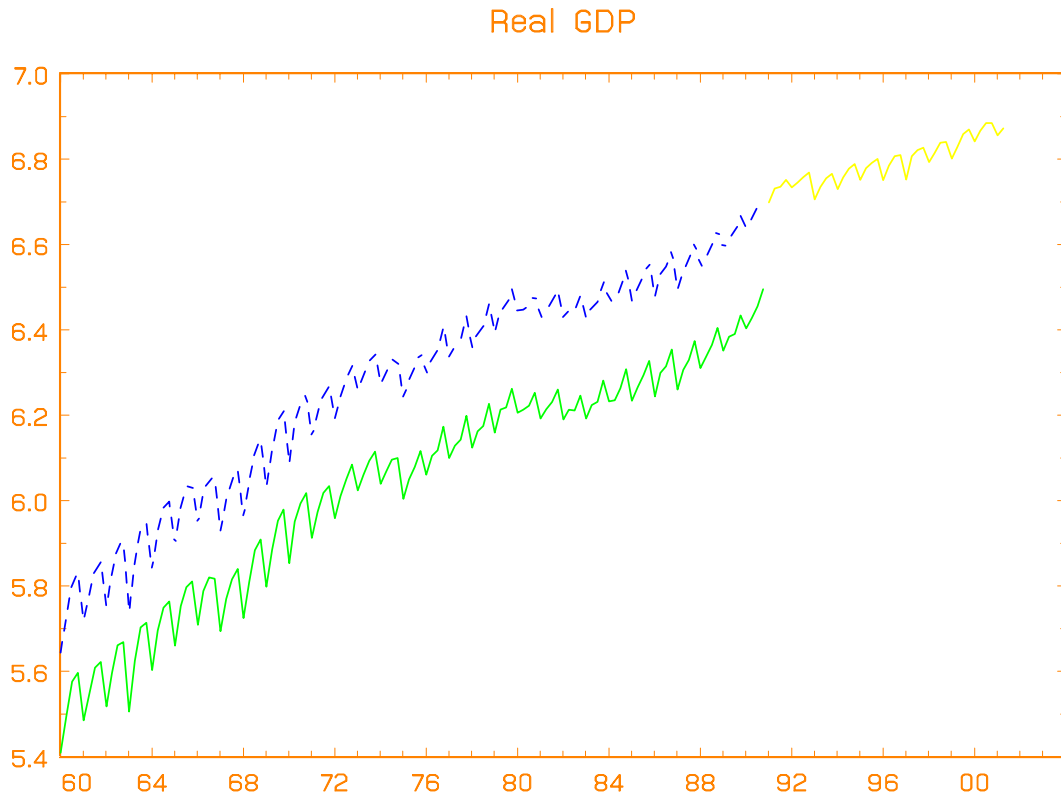
### 3 Empirical Analysis

#### 3.1 Data

In our empirical analysis we use quarterly data for German GDP from 1960:1 to 2001:2 (figure 1). The GDP series is represented in logs. The solid line shows the raw data used in the following study (Source: DIW, Vierteljährliche Volkswirtschaftliche Gesamtrechnung; Statistisches Bundesamt, Fachserie 18). The break in 1991:1 is attributable to three different reasons. Until 1990:4, GDP refers to West Germany, is measured in prices of 1991 and is defined according to the old System of National Accounts. From 1991:1 on, GDP covers unified Germany, is measured in prices of 1995 and defined according to the new European System of Accounts (see Strohm et al. 1999). The dashed line represents the fictitious development of the West-German GDP up to 1990 when it had the same value in 1991 as all-German GDP (actually, it was calculated by adding the value of 0.234, the logarithmic difference between the two GDP values in 1991). In order to allow for some flexibility, the basic models are estimated by including a level intervention  $DT_{91:1}$  in equation (2).

The number of working days are compiled by the Deutsches Institut für Wirtschaftsforschung (for a short discussion see Müller-Krumholz 1999).



**Figure 1: Real GDP (in logs)**

Note: The solid lines represent the original data for West Germany (left part) and all-Germany (right part), the dashed line the fictitious series obtained by proportional adjustment (for details see text).

A first look at figure 1 reveals some important characteristics of the GDP series which should be captured by a sensible model. First, the long-run growth rate is certainly not constant over time. The most important break seems to occur around 1973, but other changes in the drift rate may have happened in the late eighties and early nineties. Secondly, around the trend, there seems to exist a pronounced cycle with troughs in 1967, 1975, 1982 and 1993. It remains to be analysed whether some further cycles exist which cannot be detected by visual inspection of the time series. Thirdly, the seasonal variation is time varying. And fourthly, special events occur at irregular intervals which should be controlled for by specifying an impulse variable. One example is the unusual low value in the first quarter of 1963 which can be explained by an extremely cold winter.

### 3.2 Empirical Results

In a first step, we estimate a stochastic model without any drift intervention effects. The only deterministic elements are an impulse dummy in 1963:1 which captures an unusual cold weather effect and a level intervention in 1991:1 in order to control for an unobserved shift due to the already mentioned change in the data.

We carried out the estimation for each combination of two variants of seasonal specification (trigonometric versus dummy variables) and five versions of the cycle specification (trigonometric cycle with one, two and three subcycles and an autoregressive cycle with two and four lags, respectively). Summary statistics are presented in table 1.

Comparing the fit measures and test statistics between the top and bottom panel, one can see that the specification of the seasonal effect by a sum of sine-cosine waves with stochastically varying phase and amplitude clearly dominates the specification by a set of dummy variables plus a stochastic disturbance. With respect to the formulation of the cycle component, we note that an autoregressive cycle with four lags does not perform better than the model with only two lags. In case of a trigonometric specification of the cycle, the results are not so obvious. The information criteria select a model with just one cycle. A visual inspection of the estimated subcycles reveals that a model with two subcycles yields some interesting results whereas a third subcycle is negligible.

With respect to the cycle component, the subsequent analysis is carried out for a model with a trigonometric specification with two subcycles and an autoregressive model with two lags. In both cases, the seasonal component is specified as a trigonometric model. The aim is to modify the basic specification in order to improve the fit of the model in a parsimonious way.

In a first step, we examined the recursive residuals for large values which can be interpreted as outliers. The two most important events seem to have happened in 1979:2 and 1984:2. The latter date represents a strike in the metal industry. The interpretation of the first is not so obvious. A somewhat speculative explanation may be the then emerging fear of rising oil prices, and the announced increase of the VAT rate in the third quarter which may have led to higher purchases in the second quarter. Despite this interpretation problem, an

impulse dummy was included in the second quarter of 1979 and 1984, respectively.

**Table 1: Summary Statistics for the Model without a Drift Intervention**

	logLik	AIC	SIC	LB	JB	$\sigma_{RR}$
	Trigonometric Season					
Trigonometric Cycle						
1 Subcycle	480.4	-928.7	-879.4	3.7	0.5	0.0122
2 Subcycles	482.7	-927.3	-868.8	2.4	0.1	0.0120
3 Subcycles	483.9	-923.7	-855.9	2.3	0.1	0.0118
AR Cycle						
2 Lags	482.3	-932.4	-883.4	2.0	0.2	0.0120
4 Lags	482.4	-928.6	-873.2	2.0	0.2	0.0120
	Dummy Variable Season					
Trigonometric Cycle						
1 Subcycle	476.1	-922.1	-875.9	11.5	0.6	0.0125
2 Subcycles	478.2	-920.2	-864.8	10.3	0.6	0.0123
3 Subcycles	479.3	-916.7	-851.9	10.3	0.6	0.0121
AR Cycle						
2 Lags	478.0	-925.9	-979.7	9.3	0.4	0.0124
4 Lags	478.1	-922.1	-869.7	9.3	0.4	0.0123

Note: logLik denotes the maximised value of the likelihood function, AIC the Akaike information criterion, SIC the Schwartz information criterion, LB the Ljung-Box-statistic with 12 lags, JB the Jarque-Bera test statistic (critical value at 5% significance level: 6.0) and  $\sigma_{RR}$  the standard deviation of the recursive residuals.

In the following, we estimate three models which differ from each other with respect to the specification of the trend component. In all versions, the variance of the level shock  $\varepsilon_t$  turned out to be zero. The results in Tables A1 and A2 are for the restricted model where  $\sigma_\varepsilon^2$  is restricted to be zero.

*Model I* contains no deterministic drift intervention. The estimated parameters and summary statistics are presented in column 2 in Table A1 (in the appendix) for the trigonometric specification of the cycle component and in Table A2 for the purely autoregressive specification. The positive value of the standard deviation of the slope disturbance,  $\sigma_\xi$ , “allows” the trend component to smoothly adjust to the lower growth rates after 1973 and to their transitory increase in the late eighties (dashed lines in the left panel in Figure 2).

The smooth behaviour of the trend component is very problematic when in reality a sharp break occurred in a specific period. The most prominent candidate for such a break is the period of the first oil price shock.

To account for this event, *Model II* induces a drift intervention term in 1973:3. This dummy variable captures a large part of the variability of the trend growth in the model with a trigonometric specification of the cycle and even all in the model with the AR specification.

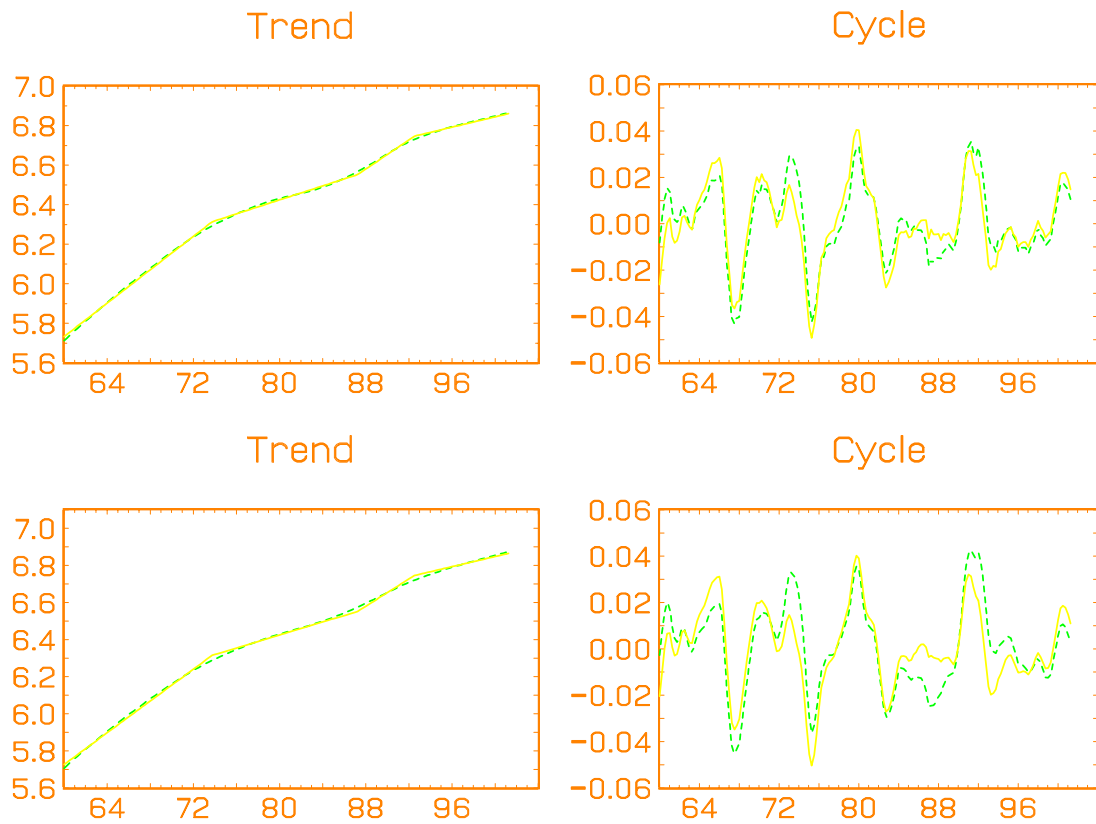
The fit of the model improves considerably (see the summary statistics in Tables A1 and A2). A somewhat disturbing effect is the implication for the cyclical component, especially for the model with the AR specification where  $\sigma_\xi$  has an estimated value of zero (see Table A2). The high growth rates in the late eighties / early nineties are (almost) totally attributed to the cycle which implies a very long and pronounced upswing during the eighties and an almost ten year cyclical downturn until 2000.

This somewhat implausible shape of the cycle can be avoided when we add in *Model III* a deterministically specified increase in the drift rate around the year 1989. Some experimentation yields as the optimal start point the second quarter of 1987 and as the end point the third quarter of 1992. The reason for this temporary increase in the trend growth rate may be the decline in oil prices in 1985/86 and an population increase since 1987 due to the immigration of native Germans from Eastern Europe. In addition, around 1990 we observe an unusual high investment boom induced by German unification.

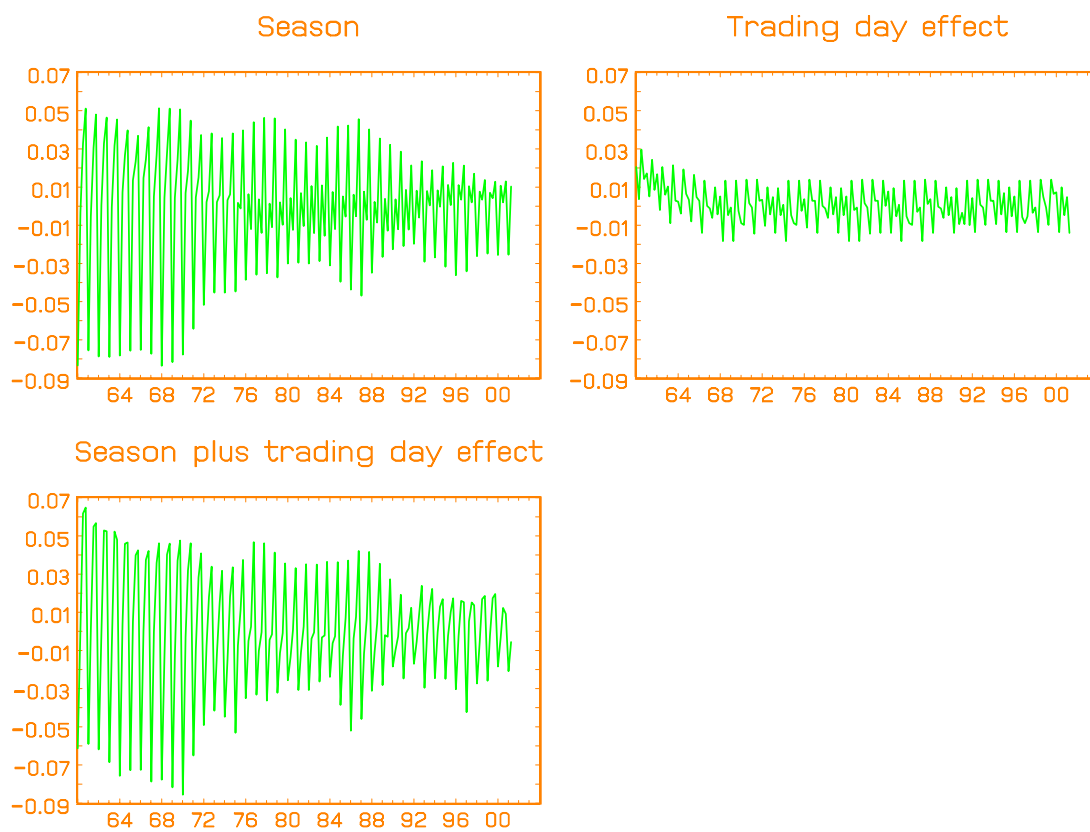
The inclusion of two drift interventions yields an impressive improvement in all fit criteria. Therefore, Model III is our preferred version. The trend and cycle components generated by this specification are shown in Figure 2 as the solid lines.

The trend component shows three kinks in 1973, 1987 and 1992. The cycle component generated by the two versions (trigonometric, AR) are almost indistinguishable. The trigonometric version identifies two cycles with a period of 4.1 and 8.1 years, respectively. This finding is compatible with “classical” ideas of the Kitchen-cycle (with a period of two to four years) and the Juglar-Cycle (with a period of eight to ten years).

The seasonal component and the working day effect are displayed in Figure 3. The most important change in seasonal pattern occurred about 1970 where the amplitude sharply decreased. The working day effect is smaller in magnitude, but is important for a good representation of the short-run fluctuations. Estimating models without incorporating a working day effect yields significantly worse fit criteria but leaves the trend and cycle components largely unaffected. The main effect is an increase in the variance of the irregular component.

**Figure 2: Trend and Cycle Components for Models I and III**

Note: The top panel represents the model with two subcycles in the trigonometric specification of the cycle component, the bottom panel the model with an (AR(2)-specification. The solid lines show the results for model III (drift with three deterministic interventions), the dashed lines for model I (purely stochastic model).

**Figure 3: Seasonal Component and Working Days Effect**

## 4 Conclusions

The aim of this paper is to characterise the most important salient features of German GDP before and after unification. By using an Unobserved Components Model, it is possible to decompose the observed GDP series into the trend, the cycle, the season and a working day effect.

The most important findings can be summarised as follows:

1. The trend component shows four different regimes. From the beginning of the sample in 1960 until 1973, the long run growth rate is about 4.2 %, between 1974 and 1987 about 1.8 %, between 1987 and 1992 about 3.6 %, and since then about 1.4 %. These figures show the dramatic decline in the growth rate of potential output over the last 40 years.
2. It seems that two cycles can be identified with a period of about four and eight years, respectively. The old idea about Kitchin- and Juglar-cycles, advocated by Schumpeter, Hansen, among others, has still merits for business cycle analysis.
3. The seasonal pattern is changing over time. The amplitude of the seasonal variations sharply declines in the early seventies and after re-unification. The last phenomenon may be partly attributable to some changes in the calculation of GDP. The empirical results show that a trigonometric seasonal model with two disturbance terms yields a significantly better fit than the stochastic dummy variable model with only one disturbance term. For a better short-run representation of the GDP series it is useful to account explicitly for the variation in the number of working days.

These findings have some important consequences for business cycle research and the modelling of macroeconomic data. First, great care should be exercised in specifying the short-run dynamics of GDP. The seasonal component shows a remarkable variability which cannot be reasonably captured by a stochastic dummy variable specification. In addition, the varying number of working days explains a great deal of the short-run variation of growth rates. The importance of this effect for macroeconomic models and VAR-systems is not clear and should be explored in future research. Secondly, all models with a constant drift rate are clearly misspecified. The real problem is not to choose between a deterministic linear trend and an autoregressive unit root with a constant drift rate. One needs no formal test to see that the mean growth rate is lower after 1973 than it was before. So, any reasonable model should allow for a variable drift



rate. The relevant question is whether the changes in the drift rate are caused by very infrequent events which would suggest an efficient specification as a deterministic intervention or by frequent and typically small shocks in which case a stochastic random walk specification of the drift rate is appropriate. The results in this paper suggest that the former approach yields a more satisfactory model (for similar results for US GDP see Flaig 2001). Thirdly, the analysis of cyclical regularities which are at the heart of classical business cycle theory promises new and important insights into the nature of capitalist market economies.

From a methodological standpoint, structural time series modelling provides a useful tool for time series analysis of macroeconomic data. This approach can be extended in several directions. The most promising direction may be a multivariate setting where several variables share common trends and/or common cycles (see Apel/Jansson 1999 or Flaig/Plötscher 2000).

## **Appendix: Estimation Results**

Tables A1 and A2 present the estimation results for three different specifications of the drift rate (the long-run growth rate):

Model I: No deterministic drift intervention.

Model II: Deterministic drift intervention in 1973 : 4.

Model III: Deterministic drift intervention in 1973 : 4; 1987 : 2, and 1992 : 3.

The rationale of the dates chosen for the interventions is explained in the text.

Table A 1: Estimation results for the trigonometric specification of the cycle

	Model I	Model II	Model III
<b>Trend</b>			
$\sigma_{\xi}$	0.0007 (4.8)	0.0003 (2.1)	—
$\gamma_2 (73 : 4)$	—	-0.0059 (2.8)	-0.0061 (21.2)
$\gamma_2 (87 : 2)$	—	—	0.0048 (7.9)
$\gamma_2 (92 : 3)$	—	—	-0.0060 (6.4)
<b>Season</b>			
$\sigma_{\omega_1}$	0.0023 (6.0)	0.0023 (6.2)	0.0024 (6.9)
$\sigma_{\omega_2}$	0.0011 (4.3)	0.0010 (4.5)	0.0011 (6.5)
<b>Cycle</b>			
$\rho_1$	0.9448 (41.1)	0.9595 (27.8)	0.9379 (40.2)
$\lambda_1$	0.3877 (10.9)	0.3968 (12.6)	0.3829 (11.1)
$\sigma_{\kappa_1}$	0.0033 (6.6)	0.0024 (1.6)	0.0037 (6.9)
$\rho_2$	0.9694 (52.1)	0.9641 (44.4)	0.9797 (75.2)
$\lambda_2$	0.1819 (6.7)	0.1493 (4.5)	0.1950 (11.4)
$\sigma_{\kappa_2}$	0.0037 (6.9)	0.0052 (2.7)	0.0027 (6.5)
<b>Irregular</b>			
$\sigma_u$	0.0000 (0.0)	0.0001 (0.1)	0.0001 (0.1)
$\gamma_3 (63 : 1)$	-0.0502 (5.2)	-0.0501 (4.9)	-0.0499 (5.6)
$\gamma_3 (79 : 2)$	0.0191 (3.1)	0.0193 (3.2)	0.0192 (3.4)
$\gamma_3 (84 : 2)$	-0.0203 (2.2)	-0.0202 (1.9)	-0.0204 (1.8)
<b>Working day effect</b>			
$\gamma$	0.3007 (6.4)	0.2929 (6.4)	0.2933 (6.6)
<b>LogLik</b>	491.4	492.8	504.8
<b>Akaike</b>	-944.7	-945.7	-967.6
<b>SIC</b>	-886.2	-884.0	-902.9
$\sigma_{RR}$	0.0114	0.0114	0.0106
<b>LB</b>	5.4	6.4	5.4
<b>JB</b>	0.5	1.1	2.8

Note: t-values in parentheses. The summary statistics are explained in Table 1.

Table A 2: Estimation results for the AR (2) specification of the cycle

	Model I	Model II	Model III
<b>Trend</b>			
$\sigma_{\xi}$	0.006 ( 3.9)	0.0000 ( 0.0)	—
$\gamma_2 (73 : 4)$	—	-0.0053 ( 6.4)	-0.0063 (10.7)
$\gamma_2 (87 : 2)$	—	—	0.0048 ( 4.4)
$\gamma_2 (92 : 3)$	—	—	-0.0058 ( 3.8)
<b>Season</b>			
$\sigma_{\omega_1}$	0.0025 ( 6.9)	0.0025 ( 6.8)	0.0026 ( 7.3)
$\sigma_{\omega_2}$	0.0012 ( 7.0)	0.0012 ( 8.6)	0.0012 ( 5.8)
<b>Cycle</b>			
$\phi_1$	1.6142 (30.0)	1.6039 (24.6)	1.6133 (42.2)
$\phi_2$	-0.6764 (15.6)	-0.6489 ( 9.6)	-0.7070 (19.1)
$\sigma_{\kappa}$	0.0046 (12.7)	0.0049 (22.4)	0.0042 (14.9)
<b>Irregular</b>			
$\sigma_u$	0.0000 ( 0.0)	0.0003 ( 0.1)	0.0003 ( 0.1)
$\gamma_3 (63 : 1)$	-0.0489 ( 4.3)	-0.0489 ( 4.2)	-0.0490 ( 4.9)
$\gamma_3 (79 : 2)$	0.0196 ( 3.7)	0.0196 ( 3.7)	0.0196 ( 4.0)
$\gamma_3 (84 : 2)$	-0.0207 ( 2.6)	-0.0207 ( 2.0)	-0.0208 ( 2.3)
<b>Working day effect</b>			
$\gamma$	0.3008 ( 7.2)	0.2997 ( 7.4)	0.2954 ( 7.2)
<b>LogLik</b>	492.3	496.1	502.1
<b>Akaike</b>	-952.5	-958.1	-968.1
<b>SIC</b>	-903.3	-905.7	-912.7
$\sigma_{RR}$	0.0113	0.0111	0.0107
<b>LB</b>	2.7	2.4	4.1
<b>JB</b>	0.3	0.7	1.7

Note: t-values in parentheses. The summary statistics are explained in Table 1.

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