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Barter, Liquidity and Market Segmentation

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BARTER, LIQUIDITY AND MARKET SEGMENTATION

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BARTER, LIQUIDITY AND MARKET SEGMENTATION

Abstract

This paper explores the private and social benefits from barter exchange in a monetized economy. We first prove a no-trade theorem regarding the ability of firms with double-coincidences-of-wants to negotiate improvements in trade among themselves relative to the market outcomes. We then demonstrate that in the presence of liquidity shocks, introducing a non-monetary exchange avoids this limitation and enhances trade by (1) generating liquidity and (2) by segmenting the market place into low-demand and high-demand customers in a manner which is impossible with pure monetary exchange. We provide comparative statics illustrating the importance of each effect and relevant extensions.

Keywords: barter, exchange.

JEL Classification: C78, D82, L11.

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1 Introduction

It is with good reason that the typical economic trade involves the exchange of goods for money; a commonly accepted money serves to overcome Jevon's (1875) constraint of a "double coincidence of wants" that plagues non-monetary exchange. Despite money's apparent advantage, it remains the case that even in monetized economies, a large number of trades do not involve monetary transfers for goods and services. The purpose of this paper is to understand the continued existence of barter in circumstances where a commonly accepted form of money already exists.

Barter between firms is significant even in monetized economies. First, it has been estimated that between ten and twenty percent of world trade is characterized by some form of countertrade, where imports into a country are tied to exports of similar value.[1] Second, a considerable amount of trade between firms occurs on organized commercial barter markets. At last count, more than 500 such exchanges were operating in the United States with approximately 300,000 firms participating as active traders.[2] These barter networks trade a great diversity of goods and services. Some examples include rental cars, hotel rooms, office equipment, business services, printing, and vehicle maintenance.[3] The International Reciprocal Trade Association estimates that the annual value of barter trade by North American companies is approximately $10 billion.[4]

[1]While these arrangements may involve some cash transfers, they typically have a large component that is either a simultaneous exchange of goods or a sequential transaction such as a buy-back agreement (where in exchange for a purchase of plant and equipment, the selling firm agrees to buy back some portion of output for some specified period in the future) or a counter-purchase agreement (where transfers of technologically unrelated goods are promised in the future). See Hennart (1989), and Hennart and Anderson (1993), Marin and Schnitzer (1995), Ellingsen and Stole (1996), Prendergast and Stole (1996), and sources cited therein for various theoretical and empirical analyses of countertrade. The 10-20 percent estimates of countertrade are based on data prior to 1990 when a large portion of world trade involved Soviet-block economies interacting with free-market economies; today's estimates are likely to be substantially lower.

[2]IRTA (2000) and *Kiplinger's Personal Finance*, February 1996 (Roha and Schulhof (1996, p. 103)).

[3]The exchanges are largely self regulated, although the IRTA provides industry standards for IRTA-member barter networks. The typical barter exchange charges 10-15 percent per transaction as well as an annual membership fee of $100-$600 and possibly a monthly maintenance fee of $6-$30. Transaction prices within the exchange are (in theory at least) identical to outside retail prices, except that the transactions take place in barter scrip.

[4]These barter exchanges typically operate by creating their own currencies, referred to as barter dollars or scrip. The standard economic hazard of accepting such scrip is then that it is not generally used as currency so that agents are restricted to purchasing from the available supply on the barter market. It is the inherent thinness of the barter exchanges that causes the absence of a double coincidence of wants. In this sense, the barter markets also share a similarity to local currencies, where communities (or more commonly, businesses within communities) issue coupons or scrip which become traded as a form of private money. For instance, there is approximately $60,000 of community money circulating in Ithaca, New York (known as Ithaca Hours), which is acceptable in some but not all stores; Greco (1994) and Glover (1995) provide an extensive (but largely non-economic) discussion of such local currencies today. Locally-accepted private currencies were also quite widespread during the Depression era in the United State (Mitchell and Shafer (1984)); indeed, Irving Fisher (1933) was a notable proponent of private community money as a means of ending the Depression. We should emphasize that our principal interest in this paper is how such seemingly inefficient barter-like arrangements can survive in a fully monetized economy (i.e., one with a *universally* accepted money), and *not* in why the supplier of the private money is willing to do so. It is clear that the ability to issue private money offers the issuer seniorage revenues (see for example Kashyap's (1995) case study of Canadian Tire for a interesting example of private coupons creating substantial seniorage for the issuing company). We focus in this paper on understanding the specific questions of why private monies which are not universally accepted will

There is by now a large literature on how barter fares as an alternative to fiat money.[5] However, this literature typically treats barter as an inefficient form of exchange that is ultimately dominated by a commonly accepted form of fiat currency. The purpose of this paper is to consider a continued role for barter arrangements in an economy which already has paper money. To do so, we rely on the twin themes of liquidity shocks and market power. In particular, we illustrate that an economy with liquidity shocks is characterized by two separate reasons for non-monetary exchange. First, and rather straightforward, barter becomes a means of creating liquidity for cash-constrained firms. Second, barter becomes a means of market segmentation which firms can use to increase their profits and welfare. This latter effect is our primary interest. We will illustrate simple conditions under which each effect dominates.[6]

Consider first the effect of market power on the ability of agents to barter in the absence of liquidity problems. We begin by illustrating how market power in isolation severely restricts barter (or more generally the bundling of trades). If firms price at marginal cost, it is obvious that barter trades can be consummated if, for each agent, the consumption value of the available good desired by an agent exceeds the production cost of the good that they produce. This is the standard double coincidence of wants condition. However, when price exceeds marginal cost, equilibrium barter becomes harder to sustain in the presence of a cash market; this is because the opportunity cost of barter is now the production cost and the potential lost cash sale. A firm may not barter with another even if it is common knowledge that consumption value exceeds production costs, for the reason that the firm rationally anticipates that it may sell the good to the other firm at a greater profit for cash if it refuses.

This problem becomes particularly acute when a willingness to barter reveals information about whether the customer would have purchased on the cash market in the absence of barter. We illustrate this effect in a general framework in Section 2 by showing that *if a firm's valuation for its desired good is independent of its costs of production*, then there is no mechanism which can improve upon the standard market allocation without bundling. In other words, there is no possibility to tie trades in a way that increases trading efficiency while simultaneously satisfying the incentive

nonetheless circulate in equilibrium and how their circulation will ultimately affect resource allocation.

[5] See Kiyotaki and Wright (1989, 1991, 1993), Trejos and Wright (1991), Matsuyama, et al. (1993), Williams and Wright (1994), and Banerjee and Maskin (1996).

[6] We should be clear at the outset that our interest in non-monetary exchange focuses on situations where the means of exchange affects ultimate resource allocation. There are some instances where agents could choose not to use money to trade, but where there is no effect on welfare. For instance, consider the case where two agents both buy and sell to each other, but where they both are commonly known to have valuations above the price charged by the other, implying that in absence of barter both would buy on the cash market. In such a setting, there is no loss (or gain) from simply swapping the goods: barter does not facilitate trade. In our view, this is a rather uninteresting form of barter as the ultimate resource allocation does not depend on the means of exchange. Consequently, we restrict attention to situations where barter affects the ultimate resource allocation in a way that at least one party finds desirable.

and participation constraints of each firm. To see the power of this conclusion, consider a simple setting in which two producing agents have demands for each other's output and suppose that it is known with probability one that each agent's valuations exceed the production costs, which are the same for both. In a world without cash markets, it is obvious that barter increases welfare so that the first best can be attained by swapping goods. Yet this can never happen in the presence of cash markets for the simple reason that any firm that proposes such a contract reveals that it would purchase on the "cash market" otherwise. Given this, the other firm has no reason to agree to the barter arrangement.[7]

The consequence of this no-trade-creation theorem is that monopoly power in cash markets constrains the ability of agents to barter. This, of course, does not get us very far in explaining how such arrangements can survive in a monetized economy; instead it reinforces the likelihood of money trades. However, Section 3 illustrates that with the addition of possible liquidity constraints, our no-trade result no longer applies. We introduce liquidity constraints in Section 3, where firms must sell their goods to generate cash. They receive information on the random demands from their cash customers, but where they place goods on a barter market before knowing the resolution of demand for sure. In this setting, we illustrate two roles for barter. First, firms may choose to barter goods for the simple reason that they see it as unlikely that they will have cash to buy. Thus barter provides liquidity to firms which otherwise are constrained from trading. Second, barter can be used as a means of market segmentation in a way that increases firm surplus.

How can barter provide a means of firms segmenting markets on the basis of willingness to pay, given the result of Section 2? The key is that liquidity constraints give rise to a natural correlation between valuations (denominated in cash) and opportunity costs which firms can exploit when marketing their goods. Consider a firm which believes that its cash paying customers are unlikely to buy. This firm consequently believes its ability to pay for the good is low. Yet the fact that they cannot sell for cash also means that they have excess inventory: Thus, its opportunity cost of selling is low and there emerges a natural positive correlation between demands and costs which can be exploited by a selling firm.

How does such a correlation generate a role for barter in equilibrium? In the absence of any correlation, any agent who proposes barter would have purchased on the cash market with at least as great a probability as suggested by the firm's prior. But this is no longer necessarily true when there are liquidity constraints; instead, a firm which is likely to have a cash-paying customer will

[7]This no-trade-improvement result was originally anticipated in the barter game of Ellingsen and Stole (1996). They demonstrated that if each firm plays a simple simultaneous swapping game (where swaps occur iff both firms agree to them), additional trade creation cannot take place if the original cash prices set by the firms are the monopoly profit maximizing prices. The analysis in section 2 of the present paper extends their result to general trading mechanisms and market structures, confirming their conjecture.

be unwilling to commit its goods to the barter market for the reason that the good may be taken before the cash-paying customer arrives. On the other hand, those firms which believe themselves to be cash constrained will be happier to make such a commitment as the opportunity cost of the goods is lower, thereby revealing themselves to be customers on whom the selling firm should accept less. In other words, a willingness to supply a good can reveal a low valuation in a way that breaks the nexus of the no-trade theorem in Section 2.[8]

The paper therefore illustrates two salient functions of barter exchange: first, as a means of creating liquidity for cash-constrained firms, and second, as a means of market segmentation. In Section 3, we identify the importance of each effect on two dimensions. First, the inherent efficiency of barter exchange is important, by which we mean the likelihood that an agent receives his desired good on the barter market. Increases in the efficiency of barter exchange increases the liquidity role of barter markets for an obvious reason, as the costs of forgoing the cash market falls. However, increasing the efficiency of barter markets has an ambiguous effect on the ability to segment the market. The reason is that the equilibrium market segmentation requires that barter be desirable to the cash-constrained firms but not to the unconstrained customers. Yet an efficient barter market makes avoiding monopoly cash pricing more attractive. Thus for sufficiently efficient exchange, equilibrium market segmentation disappears. The second dimension on which we consider two effects is the dispersion of liquidity shocks across firms. At one extreme, all firms are equally likely to have cash customers (low dispersion) and at the other we consider the case where some firms are almost certainly constrained and others are almost certain to have cash-paying customers (high dispersion). Here we find that as the dispersion of liquidity shocks increase, (i) the likelihood of trade for liquidity reasons falls, and (ii) the likelihood of trade for segmentation reasons rises. Thus in environments where idiosyncratic shocks to liquidity are common, the primary catalyst for barter becomes market segmentation.

Section 4 considers the robustness of our results. Until this point we have assumed the inefficiency of the barter market. In this section, we endogenize the efficiency of barter, by allowing multiple customers on the barter market. No pair of customers is assumed to have a double coincidence of wants, implying that demands can be fully satisfied only if enough agents arrive on the barter market. We show that (i) the two functions of barter remain, and (ii) their extent depends on the value ascribed to consuming the wrong set of goods. In this section, we also extend our

[8]Note that the literature on barter exchanges emphasizes the characteristics of trading firms in these terms. For instance, Greco (1994) describes the dual desires of "disposing of excess inventory" and "conserving their cash" (p.86), precisely the ingredients we rely upon for our results. The dual roles of barter exchanges in providing both liquidity and the marketing of excess inventory has also been noted by the International Reciprocal Trade Association (2000). The IRTA (2000) notes, for example, that "sellers [in barter exchanges] legally extend credit to the collective membership [of the exchange]" and "barter is a marketing tool, not a tax tool."

results to other demands structures, by using a Hotelling model of differentiation. Here the private information on demand relates to elasticities of substitution (as in Hotelling), where high valuation customers have strong preferences for particular goods, but low valuation customers are closer to indifferent over which good they obtain. Beyond showing that our effects remain in this section, we identify an alternative role for barter via market segmentation, as low valuation customers are happier with the random allocation of goods that barter entails. Thus this subsection illustrates how barter can separate customers on elasticities of substitution.

Another issue of robustness is whether our results are proof from competitive forces. Section 5 considers the effect of multiple barter markets with different levels of efficiency. We argued above that market segmentation requires some level of inefficiency in barter exchange. But will more efficient markets arise which ultimately drive out this reason for barter? We show here that dual barter markets can co-exist in the sense that a perfectly efficient barter market does not drive out a less efficient counterpart. We see both these extensions as illustrating the robustness of our results. As noteworthy, this section also illustrates that market segmentation plays an important role as liquidity generation always moves agents into the most efficient barter market.

We begin in Section 2 by considering the effect of monopoly on the ability of agents to bundle purchases, of which barter is obviously an example, arriving at our basic no-trade-creation theorem in the absence of liquidity constraints. Section 3 illustrates how liquidity constraints give rise to the dual roles of barter through market segmentation and liquidity generation, and the effect of parameters on each effect. Section 4 illustrates the robustness of our results to endogenizing barter efficiency while Section 5 allows efficient barter exchange to co-exist with a less efficient counterpart.

2 The Limitations of Nonmonetary Exchange: A No-Trade-Improvement Result

Throughout the paper, we argue that the interaction between market power and liquidity shocks gives rise to interesting reasons for the prevalence of barter. With this in mind, we begin by considering the difficulty of generating barter-like arrangements in a monopolistic setting without liquidity constraints. Specifically, we demonstrate that if (i) payoffs to market participants are quasi-linear in money and (ii) consumption-related private information is independently distributed relative to production-related private information, then each firm's optimal pricing policy with respect to its population of consumers is immune to renegotiation by any two firms with potential double-coincidences of wants who meet each other and attempt to improve bilateral trade. This result is not dependent upon the market structure (whether the firms are oligopolists or monopolists

in their respective markets) or the dimensionality of production or uncertainty. It only requires that the initial pricing strategies are optimal given the distribution of demand parameters of consumers (and "consuming" firms) and that preferences are decomposable and can be aggregated (which is implied by (i) and (ii) above).

Initially, our analysis is limited to two firms and uncertainty over demand parameters, but the results are then generalized. Consider two firms, A and B, each a monopolist on his own separated product market; there is no competitive externality as they do not share customers. Each potential buyer in product market i has private information regarding his willingness to pay which is given by a demand parameter, v_i, independently drawn from set $\mathcal{V}^i$ according to some commonly known distribution. The consumer surplus for a buyer of type v_i on market i is given by the quasi-linear payoff function

$$U(v_i) = u(q, v_i) - t,$$

where q is quantity consumed and t is a total transfer or payment made to the firm. Firms are risk neutral and produce output at a constant marginal cost, $C(q) = cq$. As is well known, an optimal nonlinear pricing contract can be implemented with a direct revelation mechanism of the form $\mu_i = \{q_i(v), t_i(v)\}_{v \in \mathcal{V}^i}$. Let $U(\mu_i(v')|v) \equiv u(q_i(v'), v) - t_i(v')$ be a consumer's surplus from participating in the mechanism μ_i with type v while reporting type v'.

It is useful to begin by considering the case where there is no possibility that firms could have demands for each other's product. Let μ_i^* denote the optimal mechanism that a such profit-maximizing monopolist would choose, which we assume to be unique and deterministic.[9] Specifically,

$$\mu_i^* = \arg\max_{\mu_i \in \mathrm{IC} \cap \mathrm{IR}} \ \mathrm{E}_{v_i}[t_i(v_i) - cq(v_i)],$$

where $\mu_i \in \mathrm{IC} \cap \mathrm{IR}$ implies that μ_i is (interim) incentive compatible and individually rational. As is well known, μ_i^* will typically induce consumption distortions across buyers in order to reduce the information rents to the inframarginal customers. In the simple case in which $u(q, v) = vq$ and $q \in [0, 1]$, we have the familiar monopoly distortion in which a single price, $p > c$, is set for $q = 1$. More generally, the optimal monopoly offering will contain multiple price-quantity pairs and exhibit nonlinearities, as in Mussa and Rosen (1978), for example.

[9] We could easily extend the analysis to incorporate random mechanisms without much difficulty. The assumption of uniqueness, however, allows us to reach a rather powerful conclusion regarding the renegotiation of these optimal pricing schemes – the schemes are strongly renegotiation proof (i.e., there does not exist any equilibrium to the renegotiation game under which the original pricing schemes are renegotiated). Without uniqueness, strong renegotiation proofness is no longer clear as a firm may choose to renegotiate to a new trading allocation which has identical utility outcomes for itself but possibly different outcomes for other firms in the economy. Note, however, that in the case of monopolists, non-linear pricing schemes are typically unique under simple concavity assumptions (e.g., Mussa-Rosen (1978)).

We now extend the model and assume that each firm is also a potential customer on the other firm's market. To be precise, firm A (B) draws a demand valuation v for firm B's (A's) good from the same distribution as other customers of firm B (A). In addition, the firms retain their own other customers, the set a for firm A, and the set b for firm B. As a consequence, each firm faces two potential types of buyers – buyers who do not also offer a product which the firm wishes to consume (we refer to this potential buyer as simply a firm's own "customer") and a potential buyer who is a monopolist on the other product market and offers a good for sale in which the firm is possibly interested (we refer to this buyer as a consuming "firm"). Graphically, firms A and B offer goods to each other and to their captive customers a for firm A and b for firm B; arrows indicate potential consumers of a firm's output.

Figure 1. Potential selling relationships

In this setting, could the two firms design an alternative trading mechanism between themselves which has less economic distortion than the monopoly mechanism, $\mu^* \equiv (\mu_A^*, \mu_B^*)$? One reasonable conjecture is that the monopolists could "trade" their monopoly power with each other and simultaneously reduce their (possibly nonlinear) prices between themselves. We show that this conjecture is incorrect, however, by demonstrating that the original mechanism μ^* is robust to renegotiation by the firms: in particular, there does not exist any Bayesian equilibrium to a class of renegotiation games in which the original monopoly mechanism is renegotiated.

To understand the robustness of the monopoly mechanism μ^*, we investigate whether, after learning their valuations, there is an alternative mechanism which either firm can suggest, which is acceptable to the other, and which possibly reduces the consumption distortions inherent in μ^*.

To do so, we begin by first considering whether the monopoly mechanism, μ^*, is interim incentive efficient, as defined in Holmström and Myerson (1983). An incentive compatible mechanism μ is interim incentive efficient if and only if there does not exist another incentive compatible mechanism μ' which interim dominates μ: i.e., $U_i(\mu'|v_j) \geq U_i(\mu|v_j) \;\; \forall\, i, j \neq i, \; \forall\, v_j \in \mathcal{V}^j$ with at least one strict inequality, where U_i is the expected utility of agent i with demand type v_j for firm j's

product assuming the incentive compatible mechanism μ is played truthfully by both players.[10] If a mechanism is interim incentive efficient when positive welfare weights are restricted to those of a single firm, we say that the mechanism is *undominated* for that firm. We find that μ^* is indeed interim incentive efficient and undominated for each firm, and in addition, it is incentive "safe" in the sense of Myerson (1983) (i.e., μ is *safe* iff, fixing i, for every type $v_j \in \mathcal{V}^j$ μ is incentive compatible given $\{v_j\}$ is known by all players). The proof of all theorem is supplied in the appendix.

Theorem 1 *The bilateral monopoly mechanism, μ^*, is interim incentive efficient, undominated for each firm, and safe.*

The logic of theorem 1 is simple: the worst type firms (i.e., firms that receive the least amount of rents) strictly prefer μ^* to any other mechanism because it uniquely maximizes their aggregate payoffs subject to incentive compatibility and participation. Safety follows immediately from the decomposition of the information constraints into consumption and production components. Accordingly, in the appendix we consider an even richer set of market environments with an arbitrary number of firms, each exclusively controlling a vector of goods and having demands which allow for interactions in consumption. Providing that demand and production parameters are separate and independently distributed and marginal costs of production are constant, we can extend Theorem 1 appropriately.

Theorem 2 *The general mechanism, μ^*, which is associated with the Nash equilibrium in pricing strategies among a finite number of firms with independent demand and production preferences is interim incentive efficient, undominated for each firm, and safe.*

We now consider non-cooperative games in which one of the firms offers a new mechanism as a bilateral renegotiation offer. Following Myerson's (1983) *inscrutability principle*, we can view the mechanism-selection game as a communication game induced from the original Bayesian game and apply the revelation principle. In short, we can focus on incentive compatible mechanism proposals by an informed party which do not reveal any information at the proposal stage. Given that μ^* is safe and interim undominated (what Myerson refers to as a "strong solution"), a powerful result applies.

Theorem 3 ***(Myerson, 1983)*** *The initial monopolistic pricing mechanism, μ^*, is the unique strong solution (with respect to interim expected utilities). Moreover, any other mechanism $\hat{\mu} \neq \mu^*$ which is proposed by firm i such that the set $S = \{v \in \mathcal{V}^j | U_i(\hat{\mu}|v) > U_i(\mu^*|v)\}$ is non-empty is not incentive compatible for types in S, and hence not inscrutable.*

[10] In the present setting, note that the i subscript denotes firm i and hence the valuation parameter is drawn from the distribution for firm j.

Myerson's result is quite intuitive. To understand the proof, suppose that S, the preferred set of mechanisms, were nonempty and $\hat{\mu}$ incentive compatible over S. In such a case, a new incentive compatible mechanism could be constructed (μ^* for all types not in S and $\hat{\mu}$ for all types in S) which interim dominates μ^*. Because μ^* is interim incentive efficient, this is not possible, and so $\hat{\mu}$ must not be incentive compatible. To understand uniqueness, suppose to the contrary that $\tilde{\mu}$ was also undominated and safe and yielded different interim expected utilities. Then a better mechanism could be constructed by combining the two mechanisms, using $\tilde{\mu}$ whenever it yielded more expected interim utility than μ^* and vice versa, generating a dominating mechanism.[11] As such, μ^* is unique in interim utilities, as well as being immune to replacement.

The import of the three preceding theorems is that, if either firm was in a position to propose a new mechanism as an alternative μ^* (i.e., different interim utilities), the proposal should be rejected by the other firm. To make this statement precise, consider a specific extensive form renegotiation game. Suppose one firm is selected with a randomizing device to make a take-it-or-leave-it renegotiation offer to the another. If the offer is accepted, the new mechanism is played between the two firms; if it is rejected, the original mechanism is played. Myerson (1983) has demonstrated that there exists beliefs for the non-offering firm such that a strong solution is a Bayesian equilibrium outcome. In the present setting, because μ^* is unique, we can apply the inscrutability principle and deduce that μ^* is in fact the only outcome which can emerge in a perfect Bayesian equilibrium.

Theorem 4 *The initial monopolistic pricing mechanism, μ^*, is strongly renegotiation proof.*

A few remarks are in order. First, in any equilibrium of the renegotiation game in which the allocation is renegotiated in some subgame, we can replace the equilibrium allocation with an inscrutable mechanism μ' which incorporates the various outcomes of the game and allows the offering principal to make an announcement of private information to select a particular sub-allocation. It is without loss of generality to restrict attention to such direct and inscrutable mechanisms and equilibria in which truth-telling is an equilibrium strategy. Theorem 3, however, says that any mechanism which provides positive utility gains for some offerer over the status quo mechanism cannot be incentive compatible, and hence it is not inscrutable. Such a mechanism cannot be offered and accepted in any perfect Bayesian equilibrium. What remains is that a mechanism which has equivalent utility consequences for the offering firm (but distinct consequences

[11] Other results concerning μ^* can also be shown. It is durable (Holmström and Myerson (1983)), it is an expectational equilibrium (Myerson (1983)), and a neutral optimum (Myerson (1983)). All of these notions underscore the likelihood that bilateral monopolists will be unable to renegotiate their initial customer contracts in order to give each other a price reduction.

for the other firm(s)) will not be offered. Because we have assumed that μ_i^* is unique in maximizing firm i's expected profit, we can eliminate this possibility.[12]

Second, given the intuition behind Theorem 3, it seems likely that other non-cooperative renegotiation games will have similar difficulties reaching a new trading allocation. Along these lines, for example, one can demonstrate that μ^* is "durable" in the terminology of Holmström and Myerson (1983), and therefore μ^* is weakly renegotiation proof in a game in which all players simultaneously decide whether or not to replace μ^* with a given alternative and equilibria are required to be trembling hand perfect.[13] In sum, barter and related bundling arrangements are severely restricted by the ability of agents to extract rents through their individual monopoly pricing. It is for this reason that we now turn to the role of liquidity constraints to soften this no-trade-creation outcome.

3 Barter in a Liquidity-Constrained Trading Framework

Conceptually the only change that we make from the previous section is to address the possibility that cash customers will not purchase for exogenous reasons, implying that the firm does not have the required cash to purchase goods. We furhter restrict attention to a simpler model in which $q \in \{0, 1\}$, and $C(q) = cq$ if the firm has production capacity and $C(q) = \infty$ otherwise. We assume each firm has a production capacity of one. We maintain two product markets with a single monopolist on each market. As a further simplification, we assume that in each market the customer's marginal valuation for consumption is distributed independently and identically with prior probability $\hat{\alpha}$ of value $\overline{v}$ and corresponding probability $(1 - \hat{\alpha})$ that value is $\underline{v} < \overline{v}$.

Given this description of the product market, the optimal product market monopoly mechanism in absence of any bilateral trading opportunities is to set a price at either $p = \underline{v}$ or $p = \overline{v}$, depending upon the posterior probability of $v = \overline{v}$; it is never optimal to reduce the value of the good (e.g., sell probabilities of consumption), so price discrimination is never an optimal pricing strategy. Later,

[12] This theorem is also closely related to Maskin and Tirole's (1992) analysis of a class of renegotiation games led by an informed principal. Because μ_i^* is the unique maximizing mechanism for the lowest type proposer, it is part of a Rothschild-Stiglitz-Wilson allocation in Maskin and Tirole's (1992) terminology. In their more structured economic environment, μ^* is would therefore strongly renegotiation proof.

[13] A simple example taken from Ellingsen and Stole (1996) makes the logic for this result clear. Consider the case of two firms where each has a unit demand for the other's good with valuation v uniformly distributed on $[0, 1]$; costs are taken to be zero. In this setting, it is common knowledge that productive efficiency requires trade to take place. Optimal monopoly prices for each good are $p = \frac{1}{2}$. Now consider a simple trading game in which each firm simultaneously announces "barter" or "no barter". In the case of both firms announcing "barter," the goods are immediately swapped; otherwise, the firms are free to purchase from one another on the traditional marketplace. When each firm is evaluating its announcement, it takes the other firms announcement as "barter" (since otherwise the barter game is irrelevant). But conditional on saying "barter," in any equilibrium the other firm's valuation must be higher for having chosen "barter" than not choosing barter. As such, the largest set of barter trades which can be supported as an equilibrium is for each firm to announce barter iff its valuation exceeds $\frac{1}{2}$; but whenever trade would occur via barter, it would have otherwise occurred on the traditional marketplace. Trade cannot be improved.

we will introduce assumptions on the underlying parameters of the model to assure that $p = \overline{v}$ in all pricing subgames so as to ensure a monopoly distortion in the cash market. In such a case, $\mu_i^* = \{(0,0),(1,\overline{v})\}$.

Our results rely on the assumption that firms which suffer low demand are likely to be both cash constrained and hold excess inventory. We formalize this association by assuming that (i) a firm has only a single unit of capacity, and (ii) a firm can only get money to buy a good by selling its good for money. As a result, it will only buy a good on the product market if both its valuation is not less than the posted price *and* it has cash from a previous sale. Absent a sale, a firm is effectively liquidity constrained and cannot purchase on the cash market.

A barter offer occurs when either firm offers a unit of output in a one-for-one exchange with another firm. Nonetheless, there may be some loss in exchange, represented by a final valuation of consumption for the bartered good of λv, where $\lambda \leq 1$; we will leave the determination of λ unmodelled at this point but return to the issue in section 4. As such, the net physical value to a firm from a successful bartered exchange is $\lambda v - c$. Standard double coincidence of wants would then be satisfied if $\lambda v \geq c$. However, as we emphasized above, under monopoly there will be an additional cost of trading on the barter market, namely, the opportunity cost of a (possibly) lost sale on the product market. This can occur either through the cash customer otherwise willing to buy or through the "firm customer" buying on the cash market after it makes a sale to its own cash customer.

The timing of the market game is as follows. First, nature draws for each firm a demand parameter for the other firm's good, $v \in \{\underline{v},\overline{v}\}$, according to the prior distribution $\hat{\alpha}$. Additionally, each firm receives a market forecast for its own customer's demand parameter. The forecast is either high demand, $\overline{\alpha}$, or low demand, $\underline{\alpha} < \overline{\alpha}$, where $\overline{\alpha}$ and $\underline{\alpha}$ represent posterior estimates of the probability that $v = \overline{v}$. Thus, we will say firms with $\alpha = \underline{\alpha}$ are illiquid (or liquidity-constrained) firms and those with $\alpha = \overline{\alpha}$ are liquid (or not liquidity-constrained). Note that these descriptions are made before the ultimate realization of demand. The forecast is low with probability ϕ and high with probability $(1-\phi)$. The prior estimate is therefore $\hat{\alpha} = \phi\underline{\alpha} + (1-\phi)\overline{\alpha}$. We assume that this forecast applies only to the firm's own customer; firm demands are assumed to be drawn according to the prior, $\hat{\alpha}$, regardless of the forecast for the captive customer markets.

Following the realization of the demand posterior, each firm makes a decision as to whether or not to offer its unit on the barter exchange. If both firms A and B post offers, exchange take place; otherwise, no barter occurs.

Finally, if the firms retain their unit of production capacity for any reason, they enter their cash markets and set prices. Product market trades take place over two sub-periods. In the first

sub-period, the each firm's own customer decides whether or not to purchase a good. In the second sub-period, any firm having made a cash sale in the first sub-period and having a valuation at least as great as the market price buys a good from the other firm. At this point, the trading game ends. We assume throughout the analysis that arbitrage of goods is not possible between the cash customers and the consuming firms. Figure 2 summarizes the timing.

Figure 2. Timing

Before considering the role of barter, we initially address how firms set prices. First, in the absence of barter, a firm will always find it optimal to price at $p = \overline{v}$ providing

$$(\underline{\alpha} + (1 - \underline{\alpha})\hat{\alpha}^2)(\overline{v} - c) > \underline{v} - c. \tag{1}$$

Here the probability of a sale is the probability that a firm's own customer purchases the good (which is at least $\underline{\alpha}$) plus the probability that if the firm's customer does not purchase the good (with complementary probability $(1 - \underline{\alpha})$) and that the other firm has $v = \overline{v}$ (with probability $\hat{\alpha}$) and its customer will purchase from it (with probability $\hat{\alpha}$) thereby inducing the firm to purchase itself. We maintain this assumption throughout the paper so that a cash market without a barter exchange exhibits a monopoly distortion.

This in itself is not sufficient to rule out price changes after the firm has been to the barter market. For instance, if a firm realizes that by failing to exchange goods on the barter market it is dealing with a low valuation firm, then it may wish to change its price to $p = \underline{v}$. In this sense, the barter market can serve a role as a learning device to provide firms information on the valuations of firm-customers on the other side of the market which can be used when setting prices. We do not believe that this is an integral role of barter markets, so we desire to exclude post-barter price revisions. To this end, note that price revisions arise because the firms' beliefs about the valuations of customers change when a firm fails to show at the barter market by enough to make a revision desirable. We mute these effects by assuming that with some probability, efficient barter trades do not occur as the two parties simply do not meet. If this probability is sufficiently large, then

there is little revision in the beliefs of firms from the fact that no trade occurred on the barter market, thus retaining $\overline{v}$ as the chosen monopoly price, given the assumption above. We make this assumption solely to exclude what we feel is an unreasonable role for barter which is a contrivance of our stylized timing.[14]

3.1 Equilibria

At the time of deciding whether or not to post a good on the barter exchange, each firm privately knows its demand forecast v and its own demand parameter α: thus, its type is $(v, \alpha) \in \{(\overline{v}, \overline{\alpha}), (\overline{v}, \underline{\alpha}), (\underline{v}, \overline{\alpha}), (\underline{v}, \underline{\alpha})\}$. When making a decision whether or not to enter the barter exchange, a firm should take as given that the other firm has decided to barter, for otherwise the decision to barter is irrelevant. In order to determine the opportunity cost of barter, firm A must make an assessment as to the probability that the firm B would have purchased on the cash market had firm A not posted a good on the barter market. We denote this equilibrium probability as $\Phi \equiv \text{Prob}[v = \overline{v} \text{ \& firm has cash}|\text{barter}]$. Note that Φ will generally exceed the unconditional probability of $v = \overline{v}$ and the firm having cash. This is a form of a winner's curse, and is precisely the economics behind the no-trade results of section 2.

Having defined Φ, the decision to barter is made by a firm iff

$$\Delta(v, \alpha) \equiv \lambda v - c - [\alpha + (1 - \alpha)\Phi](\overline{v} - c) > 0. \tag{2}$$

The first two terms represent the physical net benefit from barter; the third term represents the opportunity cost on the product market. Specifically, with probability α the firm receives demand from its captive customer, and with probability $(1 - \alpha)\Phi$ the firm does not sell to its captive customer but does sell to the other firm; in both events, the profit earned is $(\overline{v} - c)$. Given that $\Phi \in [0, 1]$ is determined as part of the equilibrium in the barter-cash trading game, $\Delta(v, \alpha)$ is an increasing function in v and a nonincreasing function of α. As such, whenever $(\underline{v}, \alpha)$ prefers

[14] Allowing for such price revisions does not alter the qualitative nature of our results. We have also solved a model where illiquid firms would like to change their prices to $\underline{v}$ if the absence of trade on the barter market reveals that the "firm-customer" has valuation $\underline{v}$. In this extension we additionally assume that (i) $\overline{\alpha}(\overline{v} - c) \geq \underline{v} - c$, so liquid firms prefer to charge $\overline{v}$, (ii) those firms who sell at $\underline{v}$ do not have sufficient cash to buy the other good at $\overline{v}$, and (iii) cash customers get first choice on the cash market. In this setting, we show that the same set of equilibria continue to arise as described below, though the exact parameterizations change in some instances.

There are other equally valid approaches to muting the post-barter price-revision effect. For instance, if we assume each firm has a large number, n, of customers in the captive market who value the good at $v = \overline{v}$ with probability one and only one customer with truly uncertain demand, then $p = \overline{v}$ is optimal for any posterior on the nth marginal buyer; in such a model we would have to make an additional assumption that a firm with value $v = \overline{v}$ will only buy at price $p = \overline{v}$ if it has made $n + 1$ sales and that capacity is $n + 1$. More generally, we simply need an assumption that the effect of the barter market on beliefs is small, which is reasonable when the barter market is small relative to the cash market.

to barter, so does $(\overline{v}, \alpha)$; and whenever $(v, \overline{\alpha})$ prefers to barter, so does $(v, \underline{\alpha})$. Thus, excluding the case in which no firm barters (which is always an equilibrium for the usual search-externality reason), there are five potential classes of pure-strategy equilibria in the barter game: (1) $(\overline{v}, \underline{\alpha})$ type barters, (2) both $(\cdot, \underline{\alpha})$ types barter, (3) both $(\overline{v}, \cdot)$ types barter, (4) $(\overline{v}, \overline{\alpha})$, $(\overline{v}, \underline{\alpha})$ and $(\underline{v}, \underline{\alpha})$ barter, (5) all four firm types barter. The following lemma states that only the first three classes of equilibria are possible.

Lemma 1 *Pure-strategy equilibria of classes (4) and (5) do not exist.*

The impact of the lemma is that there are three equilibria configurations of interest, ignoring the equilibrium in which no one goes the barter exchange.

Figure 3. Possible equilibria configurations

First consider the case in which all high valuation $\overline{v}$ firms barter. Then both liquid and illiquid firms enter the barter market for the reason that the barter exchange is serving a pure liquidity role, allowing high demand customers to exchange goods in the absence of money. We label this the "liquidity equilibrium". By contrast, in the case in which all low liquidity, $\alpha = \underline{\alpha}$, firms barter, the barter exchange is serving a role in segmenting the market place and increasing consumption. In particular, firms which are liquidity constrained will enter the barter market, selling on the cash market only if they find no one there. The danger that the illiquid firms face in going to the barter market is that the exchange may also attract high valuation, liquid firms. But the correlation between willingness to pay and opportunity cost can serve to resolve this concern because those firms which do not face liquidity constraints do not believe themselves likely to have the excess inventory that could be used on the barter market. With such correlation, firms may be able to market their goods on the barter market without fearing that their liquid customers will be tempted

to enter. In this way, non-monetary exchange can segment the market; we refer to this equilibrium as the "segmentation equilibrium".

Note most importantly that this segmentation cannot occur without the barter market. This is for the simple reason that in order to illustrate a low willingness to pay cash, the firm must be willing to sacrifice a unit of its output on the barter market. Thus, forgoing the prospect of cash sales serves to illustrate that willingness to pay is low and so potential sellers should be willing to accept less from them. Note also that barter here increases the propensity to trade as low valuation firms are now consuming the good when before they were excluded. The equilibrium in which only the high-valuation, low-demand firms trade amongst themselves is a hybrid of both these variations. We consider each of these cases below in our characterization theorem.

Theorem 5 *The following is a complete characterization of pure-strategy barter equilibria.*

1. *An equilibrium in which firm type $(\overline{v}, \underline{\alpha})$ exclusively posts goods on the barter exchange exists iff the following parameter condition is satisfied:*

$$\frac{1}{\overline{v}}[c+(\overline{v}-c)(2-\underline{\alpha})\underline{\alpha}] \leq \lambda \leq \min\left\{\frac{1}{\underline{v}}[c+(\overline{v}-c)(2-\underline{\alpha})\underline{\alpha}], \frac{1}{\overline{v}}[c+(\overline{v}-c)(\underline{\alpha}+(1-\underline{\alpha})\overline{\alpha})]\right\} < 1.$$

2. *A low-demand equilibrium in which firms with $\alpha = \underline{\alpha}$ exclusively post goods on the barter exchange exists iff the following parameter conditions are satisfied:*

$$\frac{1}{\underline{v}}[c+(\overline{v}-c)[\underline{\alpha}+(1-\underline{\alpha})\underline{\alpha}\hat{\alpha}] \leq \lambda \leq \frac{1}{\overline{v}}[c+(\overline{v}-c)(\overline{\alpha}+(1-\overline{\alpha})\hat{\alpha}\underline{\alpha}] < 1.$$

3. *A high-valuation equilibrium in which firms with $v = \overline{v}$ exclusively post goods on the barter exchange exists iff the following parameter conditions are satisfied:*

$$\frac{1}{\overline{v}}[c+(\overline{v}-c)[\overline{\alpha}+(1-\overline{\alpha})\hat{\alpha}] \leq \lambda \leq 1.$$

Additionally, the parameter values which support equilibria of class (3) are inconsistent with those of classes (1) and (2).

Theorem 5 characterizes the three types of equilibria. First, there is the equilibrium where all high valuations customers barter $(\overline{v}, .)$, the liquidity equilibrium. The only case where market segmentation does not occur is in this equilibrium. Second, there is a range of parameters where all constrained firms barter $(\cdot, \underline{\alpha})$. Finally, there is an equilibrium where only high valuation, cash constrained firms barter. The parameter values which satisfy the requirements for the liquidity equilibrium are disjoint from those of the other two. Note finally that there is no equilibrium

where all firms barter, so that the desire to create liquidity is not large enough to warrant all firms bartering. In other words, monopoly pricing continues to act as a constraint on trade, as in Section 2.

3.2 Comparative Statics: Barter Efficiency and the Dispersion of Shocks

Our interest is ultimately in determining when each effect is important. Tos do so, we consider the effect of two relevant variables. First, we address the effect of changing the intrinsic efficiency of the barter market, λ. Second, we consider the effect of heterogeneity of the population by adopting the following lower-dimension parameterization of the model, measuring the dispersion of demand shocks to firms. Specifically, we let $\delta \equiv \overline{\alpha} - \underline{\alpha}$ represent a parameter of mean-preserving dispersion so that $\delta = 0$ corresponds to $\underline{\alpha} = \overline{\alpha} = \hat{\alpha}$ and $\delta = 1$ corresponds to $\underline{\alpha} = 0$ and $\overline{\alpha} = 1$ for a fixed $\hat{\alpha}$.[15] Thus $\delta = 0$ implies that all firms are identical while $\delta = 1$ implies that some firms are certain to have cash customers and others have no possibility of such customers.

We do not provide formal theorems on the comparative statics of the model, but instead provide parameterizations to illustrate what we feel are robust observations. In particular, we set $\hat{\alpha} = \sqrt{\frac{\underline{v}-c}{\overline{v}-c}}$ to guarantee that absent the barter market, each firm would choose to set $p = \overline{v}$: i.e., $(\underline{\alpha} + (1 - \underline{\alpha})\hat{\alpha}^2)(\overline{v} - c) > \underline{v} - c$ for all $\underline{\alpha} \in (0, 1)$. For a given set of parameters $\overline{v}$, $\underline{v}$ and c, we then graphically illustrate the entire regions of δ and λ for which each equilibria exist. This is done below in Figure 4 for the case in which $\overline{v} = 4$, $\underline{v} = 1$ and $c = 0$.

[15] Such a parameterization requires that $\underline{\alpha} = (1 - \delta)\hat{\alpha}$, $\overline{\alpha} = (1 - \delta)\hat{\alpha} + \delta$, and $\phi = 1 - \hat{\alpha}$.

Figure 4. Equilibria under various Parameters

Figure 4 identifies the parameter values under which the three equilibria can be supported. Most importantly, the effects of the two variables differ depending on whether we consider the liquidity equilibrium $(\overline{v}, \cdot)$ or either of the equilibria which involve market segmentation, $(\cdot, \underline{\alpha})$ or $(\overline{v}, \underline{\alpha})$. First consider the effect of changing λ. This unambiguously increases the the likelihood of firms trading for liquidity reasons (i.e., for any level of dispersion, increasing λ makes the $(\overline{v}, \cdot)$ equilibrium more likely). This should not be surprising as more efficient barter markets will result in liquid firms being more willing to forego the cash market for the barter market. But remember that the segmentation results rely on barter being desirable for (at least some) illiquid firms but undesirable for the liquid firms. As a result, initially increasing λ from zero makes market segmentation more likely (as the illiquid firms are more likely to enter) but ultimately, if λ increases too much market segmentation becomes impossible (as the liquid firms now find it attractive). Therefore, moderate efficiency levels for barter aid market segmentation at the cost of liquidity generation, while highly efficient barter markets will eliminate market segmentation.

Second, note that the effect of dispersion differs between the two types of equilibria. Increases in δ unambiguously reduce the likelihood of the liquidity equilibrium. This arises for the simple reason that as dispersion rises, the probability of a cash trade increases for the liquid firms, which are thus less willing to place goods on the barter market. By contrast, this aids the ability of firms to segment their market. More specifically, as dispersion rises in the $(\cdot, \underline{\alpha})$ equilibrium configuration, the illiquid firms find the barter market more attractive (as they are less likely to have

cash customers) while the liquid customers increasingly prefer the cash market. Both effects lead to easier market segmentation so holding λ constant, increases in δ increase the likelihood of the market segmentation equilibria with $(\cdot, \underline{\alpha})$ firms bartering. The existence of the other segmentation equilibrium – only type-$(\overline{v}, \underline{\alpha})$ firms bartering – does not depend monotonically on δ. For low levels of dispersion, increases in dispersion increase the likelihood of existence; but for moderate levels of dispersion, further increases in δ reduce the likelihood of existence. This latter effect arises because higher dispersion induce type-$(\underline{v}, \underline{\alpha})$ firms to also enter, destroying the single-firm segmentation equilibrium.

4 Multilateral Barter and the Source of Barter Frictions

So far, we have left the inefficiency of barter $(1 - \lambda)$ as an exogenous variable. Ultimately, this reflects the absence of a double coincidence of wants. But the likelihood of a double coincidence of wants depends on the supply of firms in the barter market, which itself depends on the equilibrium strategies of the parties. As a result, we consider the robustness of our results to endogenizing λ. To do so, we add a third firm, C, and make the trading relationships amongst the firms cyclic. Introducing the third firm and the cyclic nature of demand generates a danger that a firm which places its good on the barter market, trades its output to another firm with an undesirable good. Figure 5 gives the trading relations between the agents.

Figure 5. Potential selling relationships

Thus, firm A has good B as its preferred good, but only firm C has good A as its preferred

option. Thus there is no bilateral double coincidence of wants. Each has its own independent cash customer, i, for firm I. The timing is unaltered except for an additional third sub-period in the cash game to allow for a firm's own-customer cash sale to inject liquidity into the cash market and circulate fully. For example, suppose all firm's are $\overline{v}$ types, but only firm A sells for cash in the first sub-period; in the second sub-period, A buys from C; in the third sub-period, C buys from B.

The barter market is assumed to operate as follows. First, if only one agent shows up, no trade is consummated and that firm returns to the product market in the hope of selling its good. Second, if all three firms show up, efficient trade occurs where each party gets its preferred good. Finally, if only two firms show up, we assume that trade is consummated between the two firms. Note that this implies that one of the firms must must obtain its less preferred good. Thus, if firms A and C arrive on the barter market, firm C obtains its preferred good while firm A is left with its less preferred alternative.[16] The equilibrium probability that the desired good is indeed obtained by a firm is λ, though remember that here we more generally allow an additional value to barter trade through as possible value from consuming (or storing) the "wrong" good. Specifically, we ascribe a value to the utility generated by the less preferred good, which we denote by $s(v)$, with $0 \leq s(v) \leq \underline{v}$. We will consider different parameterizations of $s(v)$ below.

Introducing a third firm has two effects on the model. First, we endogenize λ through the possibility that the firm is left holding its less preferred good. Consider the incentives of firm A, which desires good B. It receives $s(v)$ if it receives good C, which occurs if A and C enter but B does not. Let ρ measure the equilibrium probability that a firm goes to the barter market. Then conditional on A entering, there is a probability $2\rho - \rho^2$ that at least one other firm will also enter and so some trade will be consummated. Therefore, given some trade being consummated, there is a conditional probability $\frac{\rho(1-\rho)}{2\rho-\rho^2} = \frac{1-\rho}{2-\rho}$ that trade will involve only A and C, and so firm A ends up with its less preferred good. Let $\gamma = \frac{1-\rho}{2-\rho}$. Then the probability of A getting its preferred good is $1 - \gamma$.

The second effect of introducing multilateral barter is that the character of Φ is more complex as in some cases the opportunity cost of the barter market is a lost sale on a firm who is not on the barter market, whereas in the previous case it was only necessary to evaluate this cost conditional on the potential firm customer being on the barter market. How this affects the updating depends on the particular equilibrium being played.

[16] The workings of the barter market are essentially a metaphor for scrip-in-advance requirements. That is, typically commercial barter exchanges require firms to sell goods for barter scrip before purchasing goods. The risk of such a transaction is that the demanded good is never posted on the exchange but the posting firm loses its unit of output nonetheless.

Given these effects, the net value of barter is now represented by

$$\Delta(v, \alpha) \equiv (1-\gamma)v + \gamma s(v) - c - [\alpha + (1-\alpha)\Phi](\overline{v} - c) > 0,$$

with Φ and γ suitably determined by the equilibrium configuration of bartering firms. We now consider various parametrizations of $s(v)$, where we do not provide formal proofs of our results; instead, we simply provide simulations of the model to illustrate what we feel are the important effects.

4.1 No Value to the Less Preferred Good

We begin by considering the case where there is no value to the less preferred good, $s(v) = 0$. When the non-preferred good has low salvage value, market segmentation is likely to become less common. The reason for this is that low valuation consumers find the cash market increasingly preferable. Consistent with this prediction, simulations of our model illustrate that the the segmentation equilibrium becomes less likely as the salvage value falls to zero, suggesting that the value of the alternative goods on the barter market directly affects the ability of barter to segment the market for customers.

4.2 Positive Salvage Value

Now consider the more interesting case where the less preferred good has some value which is independent of the valuation the firm places on its preferred good. For example, it may be possible to sell it at a discount to another possible buyer, or it may be that the firm can hold onto barter scrip which can be used at a future date. We assume that the salvage value is give by $s(v) = s$. The key difference between this and the previous case it that the barter market becomes more attractive and so opportunities for market segmentation open up.

Example. As in the case of bilateral exchange, we can lower the dimension of our problem by fixing $\hat{\alpha}$ and parameterizing the degree of dispersion, $\delta \equiv \overline{\alpha} - \underline{\alpha}$. This is done below in Figure 6 for the case in which $\overline{v} = 4$, $\underline{v} = 1$, $c = 0$, and the salvage value of a non-ideal barter good is $\frac{3}{4}\underline{v}$.

Figure 6. Equilibria under Various Parameters

The three components of this figure give parameter configurations where each type of equilibrium arises. The bottom graph identifies the case where the liquidity equilibrium arises. Note however that in this case it is not possible to exogenously vary the efficiency of the barter market, since this is tied down by the probability of entering the barter market and the nature of the equilibrium;

the efficiency of the barter market, however, is independent of δ. In this numerical example, the endogenous inefficiency of the barter market for the $(\overline{v}, \underline{\alpha})$-equilibrium is $\lambda = .57$ (in other words, with probability 0.43, each agent receives the wrong good on the barter market); the inefficiency of the barter market for the $(\cdot, \underline{\alpha})$-equilibrium is $\lambda = .69$; and the inefficiency of the barter market for the $(\overline{v}, \cdot)$-equilibrium is $\lambda = .64$. We can address the feasibility of the equilibria by varying δ at each value of λ. The liquidity equilibrium (i.e., $(\overline{v}, \cdot)$-types barter) can be supported so long as δ does not exceed δ_3^*, at which point the high valuation liquid firms find the cash market too attractive. The middle graph gives parameter values under which the segmentation equilibrium $(\cdot, \underline{\alpha})$ exists; here for the equilibrium λ, segmentation arises if δ exceeds δ_2^*. The top graph provides the wide range of parameters for which the equilibrium where only $(\underline{v}, \underline{\alpha})$ trades. Note also that the same comparative statics on δ continue to hold here, where more dispersion makes market segmentation more likely and the liquidity equilibrium less likely. Therefore, allowing salvage value for goods implies results similar to those in Section 3.

4.3 Product Differentiation

In this section, we extend our results to another commonly used demand structure, where issues of product differentiation become important. Specifically, we have characterized v as the absolute valuation for the good which naturally has an interpretation in terms of the elasticity of demand. However, an alternative approach to modeling demand (as in Hotelling) is to consider the goods as substitutes where v proxies for the elasticity of *substitution* between them. In particular, assume that v now measures the specificity of tastes for one good over another, in the spirit of Hotelling. More specifically, assume that firm i has demands for either good i or good j. Each market here is modeled as a unit line segment with each supplying firm situated at one of the endpoints. Each consumer is predisposed to like one good more than the other in the following sense. Consider firm A. In the previous subsection it valued good B at v and good C at $s(v)$. Here we assume that v measures the relative merits for two goods, where a consumer with $v = 1$ has good B as his ideal good, while those with type $v = 0$ have good C as the ideal good. Following Hotelling, we assume that a consumer with "location" v receives consumer surplus of $k + v$ from his preferred good and $k - v$ from his less preferred good. This is a standard Hotelling set-up where we can consider the situation of firm A as ideal for type $v = 1$ while consumers who have $v = \frac{1}{2}$ are indifferent between the two goods. In our previous description of the multilateral barter market, we lacked a double coincidence of wants by assuming that no bilateral exchange involved both parties getting their preferred good. We retain this analogy here by assuming that for each agent $v > \frac{1}{2}$, so that all agents have different preferred goods; A prefers B over C, B prefers C over A, and C prefers A over

B. However in this case as v varies, agents are becoming closer to indifferent over their ultimate allocation. As a consequence, the net value of the barter market is given by

$$\Delta(v, \alpha) \equiv k + (1-\gamma)v - \gamma v - c - [\alpha + (1-\alpha)\Phi](\overline{v} + k - c) > 0,$$

where now the cash market price is assumed to be the monopoly-distorted price of $p = \overline{v} + k$ rather than the $p = \overline{v}$ given previously.

Once again, we can show that barter can be used both to provide liquidity and to segment the market. However, an important aspect of this demand structure is that market segmentation becomes more likely than in the case with simple salvage values which are independent of type. The reason for this is that with differentiated products, barter can serve to segment the market not only on willingness to buy but additionally on the elasticity of substitution between goods. In effect, those who value one good less value others more, something that makes segmentation easier, an effect which was absent in the previous section. Essentially the barter market is akin to a randomization over the goods that the firms ultimately attain. Firms which have valuation $\underline{v}$ are closer to indifferent between which good they receive, so it is easier to induce them to accept a lottery over goods than for a firm with more specific tastes (i.e., $v = \overline{v}$). Therefore, we have yet another reason why firms can use barter to segment the market; here they segment based on the fact that some agents have lower elasticities of substitution than others, and barter markets have the feature that they involve more random allocations that cash markets, thus allowing separation.

Example: Consider the following numerical example in which $\underline{v} = 1$, $\overline{v} = 10$, $c = 0$, and $k = \overline{v}$.

Figure 7. Equilibria with Product Differentiation

5 Does Competition Eliminate Segmentation?:Dual Barter Market Equilibria

A recurring theme in economics is that better institutions tend to ultimately dominate their less efficient counterparts. In this section, we consider whether the introduction of a more efficient barter market (λ close to 1) drives out its less efficient counterpart. As we have argued above, barter markets must be sufficiently inefficient for market segmentation to operate. If this is the

case, does the advent of a more efficient barter exchange (where supplies are posted, markets are thick, and swaps can occur at little deadweight loss) necessarily lead to the disappearance of less efficient (and hence market segmenting) barter markets?

We study this problem in the following way: suppose that in addition to the market in Section 3, as also add a barter market which is characterized by complete efficiency in swaps, so that agents always get their desired good if they trade; i.e., $\lambda = 1$. Since this is the most efficient market possible, it will attract high valuation customers with the greatest likelihood as they have little to lose in consumption value. In this setting, if the inefficient ($\lambda < 1$) barter market survives, in the sense that some firms use it rather than the efficient market, we can be confident that our results are robust even to the introduction of an extremely efficient barter exchange. This is indeed the case.

Rather than consider the extensive set of possible equilibrium configurations (there are 50 pure-strategy configurations), we content ourselves at this early stage of research with examples of three distinct configurations which are pure-strategy equilibrium outcomes for a wide range of parameter values: (i) type $(\overline{v}, \overline{\alpha})$ firm barters in the efficient exchange while type $(\overline{v}, \underline{\alpha})$ barters on the inefficient exchange; (ii) type $(\overline{v}, \overline{\alpha})$ firm barters in the efficient exchange while types $(\overline{v}, \underline{\alpha})$ and $(\underline{v}, \underline{\alpha})$ barter on the inefficient exchange; (iii) type $(\overline{v}, \overline{\alpha})$ firm barters in the efficient exchange while the remaining three types barter on the inefficient exchange.

A simple example of these three equilibrium configurations as a function of parameters of dispersion, δ, and barter inefficiency, λ, is provided in Figure 7 below for the case in which $\underline{v} = 1$, $\overline{v} = 4$, and $c = 0$.

Figure 7. Dual Equilibria

Even with this limited set of example configurations, a few important results present themselves. First, there are a number of different instances in which the inefficient barter market continues to operate in conjunction with its efficient counterpart. In these instances, the efficient market does not drive out its less efficient counterpart. This necessarily implies a market segmentation role for barter trade, since if trade occurred for liquidity reasons, all firms would go to the efficient market. Second, in the three configurations, the inefficient barter market can only survive if there is a sufficiently large dispersion of shocks, so that once again we feel that market segmentation is most likely to arise in circumstances where firms face idiosyncratic liquidity shocks.

6 Conclusions

Barter typically plays a rather secondary role as a straw man in economic theory, generally being used as an inefficient benchmark against which the merits of money are usually compared. Yet through barter exchanges, local currencies and countertrade, it continues to play an active role in economic exchange. Our objective in this paper has been to understand why such a supposedly poor means of exchange continues to play a role in monetized economies. Our interest here is as much conceptual as empirical, since from an early stage of our careers, economists are taught to shun the swapping of good for other goods. In this paper we argue that barter plays two roles which may in fact make it a desirable means of exchange. First, and rather obvious, for agents who have no money, they can attempt to use their own goods as the means of exchange as they have

little else. This liquidity generating role for money is of little conceptual interest if that was the only purpose that barter serves. We illustrate that a second reason for barter is that it can serve to segment markets on the basis of willingness to pay. Put simply, liquidity constrained firms can only plausibly prove themselves to be such by being willing to place excess inventory on the barter market.

Although we know of no other research which has identified the choice over different means of exchange as a mechanism for facilitating trade via segmentation, there are nonetheless a few related literatures which are worth noting. First, following Kiyotaki and Wright (1989), a sizeable literature has addressed the role for fiat money in various matching and search-theoretic environments and its value over barter in eliminating the double-coincidence-of-wants problem.[17] In this research, the emphasis is on how money can overcome search or matching frictions. Williams and Wright (1994), and Banerjee and Maskin (1996) provide related models of fiat money which also provide theoretical foundations for Gresham's law. Both papers demonstrate that in absence of fiat money, socially-inefficient low-quality goods are produced which serve as a form of commodity money in a barter economy. More relevant, Ellingsen (1998) notes that barter can be used as a liquidity device in tandem with money, allowing firms without money to purchase goods using costly barter transactions. In a second line of research, the work of Ellingsen and Stole (1996) is similar to our own in that they begin with a limited no-trade-creation result and then consider the strategic choice by national governments to mandate barter (i.e., countertrade) relationships between their domestic firms and their foreign counterparts; the present work shares a similar starting point, but is concerned only with voluntary barter relationships where the means of exchange is endogenous between the parties to the transaction. Third, a literature has emerged which examines intermediated trade in matching-theoretic models; Rubinstein and Wolinsky (1987), Bhattacharya and Hagerty (1989), Bose and Pingle (1995), and Bose (1996). This literature shares similarities with this paper in that we allow intermediation in the form of a barter market as a means of facilitating trade.[18] Here, however, we emphasize the important information role of barter in contrast to simple intermediation as the key source of market segmentation. A fourth literature has explored the prevalence of trade credit among small businesses. (See, for example, Petersen and Rajan (1997), Biais and Grollier (1997), and Mian and Smith (1994) for evidence, theoretical models, and related citations.) It is worth mentioning here that according to Mian and Smith (1994), that "for 3550

[17]For example, Kiyotaki and Wright (1991,1993), Trejos and Wright (1991), and Aiyagari and Wallace (1992). Matsuyama, et al. (1993), build on this work to understand which of multiple national monies will emerge in equilibrium exchange between nations.

[18]Bose and Pingle (1995) also generate market segmentation in a dual exchange equilibrium. In their complete information setting, patient firms trade in bilateral negotiated-price meetings and the remainder of the economy visits "stores" to obtain goods at intermediated prices. We consider a different set of trading institutions which differ over the means of exchange rather than the form of intermediation.

non-financial NASDAQ firms covered by COMPUTSTAT, accounts payable were 26% of corporate liabilities at the end of 1992." Hence, the importance of trade credit is not to be discounted, and the insights of the present work may have a more general value.

It is worth concluding here by making a few remarks on our modeling strategy and some extensions of the paper. First note that we have required firms to place goods on the barter market before the ultimate resolution of demand. If we were to reverse the order, so that firms know for sure whether they have excess inventory, then there would be an obvious liquidity role for barter, but by assumption. We feel that this would be unreasonable as we would be simply saying that barter arises solely for liquidity reasons. There would no longer be a role for barter as a means of market segmentation, since the relevant single crossing property is not satisfied. Consequently, we chose the timing both to reduce the likelihood of barter and also to reflect the primary concern of firms entering the barter market; namely, the danger that the item that was placed on the barter exchange might have been purchased on the cash market.

Second, note that we have allowed firms to have no access to credit markets. However, a natural extension of our model would be to allow bilateral credit arrangements, where selling firms could delay payment on goods until the time that the buyer has cash. This strategy, which is addressed in a different context by Kiyotaki and Moore (1997), could provide an alternative way for firms to overcome their liquidity constraints, and so not rely on the barter market. But remember here that in some cases welfare is increased by barter arrangements as it allows them to overcome the no-trade theorem result of Section 2. Consequently, it is unclear under what terms firms would indeed offer such credits to one another and whether such credits will eliminate barter markets. Another tack would be to consider one-way trades and the role of trade credit as a mechanism to price discriminate. A simple description of this sort of price discrimination strategy in which a monopolistic seller's offer of trade credit reduces the effective price to low-quality borrowers who by assumption are more price sensitive is provided in Petersen and Rajan's (1997) study of trade credit relationships among small business. They find empirical evidence consistent with this strategy: firms with high margins are more likely to extend trade credit. A more general theory along the lines of the the present work would shed light on the effects of networks of trade credit and perhaps the effect of product differentiation (local monopoly power) on final product and trade-credit allocations.

Third, we have not allowed firms to exercise monopoly power on the barter markets. In other words, firms must swap one-for-one on this market. Since relative cash prices are unity, the barter exchange rate is consistent with the market rate. But firms could conceivably try to demand better terms of trade on these markets in the same way as they do in the cash market. It is not clear how

this market will find equilibrium where all firms are stating prices in terms of the goods of others, who in turn are stating monopoly terms for their goods. It remains the case that the intermediated barter exchanges that we have alluded to in the paper make attempts to constrain price demands on the barter market, by requiring that prices cannot be higher than on cash markets (in our model, this implies a one-for-one swap). However, at a basic conceptual level, it would be useful to consider the effects of such monopoly power, to identify whether monopoly distortions vary with the medium of exchange.

Finally, note that throughout the paper we have assumed that prices were chosen always to equal $\overline{v}$. This plays two roles in the model. First, it implies that there are monopoly distortions, which is a central theme that generates welfare effects of barter. However, it also plays a secondary role in that firms which are credit constrained do not change their prices. Therefore, at a basic level they have some control over their cash constraints as they could reduce prices if they chose. This is important here as we require that agents must go to the barter market in some cases to reveal that they are truly liquidity constrained. But in an environment where firms change their prices if they are liquidity constrained, this need not be the case as firms may be able to identify whether they are liquidity constrained from their prices. Our assumptions ensure that this is not the case, so that barter is the only means by which to reveal type. More generally, however, this paper addresses reason why credit constraints cannot simply be corrected by changes in the price level; in monopoly circumstances, such price reductions may simply not be incentive compatible.

Appendix: Proofs of Results

Proof of Theorem 1: Let $U_i(v_A, v_B)$ represent the indirect utility (consumer and producer surplus) of firm i given it has demand parameter v_i and the other firm has parameter v_j; let $U_i(v_j)$ represent the unconditional expectation of this utility over the other firm's private information. By definition,

$$U_A(v_A, v_B) + U_B(v_A, v_B) \equiv S_A(q_A(v_A, v_B), v_A) + S_B(q_B(v_A, v_B), v_B).$$

Define consumer surplus as

$$CS_i(v_A, v_B) \equiv U_i(v_A, v_B) - \min_{v_j} U_i(v_A, v_B),$$

where $\min_{v_j} U_i(v_A, v_B)$ is the indirect utility of the worst type. For simplicity, define $\underline{v}_i \equiv \arg\min_{v_i} U_j(v_A, v_B)$; note that $\underline{v}_i$ is independent of v_j given our separability assumptions. Then, substitution and taking expectations yields

$$U_A(\underline{v}_B) + U_B(\underline{v}_A) \equiv \mathrm{E}_{v_A,v_B}[S_A(q_A(v_A, v_B), v_A) - CS_B(v_A)] + \mathrm{E}_{v_A,v_B}[S_B(q_B(v_A, v_B), v_B) - CS_A(v_B)].$$

By definition, μ_i^* uniquely maximizes $\mathrm{E}_{v_A,v_B}[S_i(q_i(v_A, v_B), v_i) - CS_j(v_i)]$ subject to incentive compatibility and participation constraints, and so μ^* uniquely maximizes the right-hand side above subject to incentive compatibility and participation constraints. As such, μ^* is interim incentive efficient. Moreover, it is undominated for firm i because it maximizes firm i's expected profits subject to participation of firm j (a necessary condition being that the worst type of firm j receives utility equal to that provided by μ^*).

The second result of the theorem that the mechanism is "safe" follows immediately from the fact that the firm's private information only enters directly in the determination of the firm's consumer surplus and not its profits. As such, given any monopoly nonlinear pricing allocation, μ^*, if it is known that firm j has a demand parameter of v_i, the mechanism is still incentive compatible – i.e., firm j will nonetheless select the allocation $\{q_j(v_i), t_j(v_i)\}$ and firm i's selection from firm j's nonlinear pricing schedule is unchanged by firm j's demand parameter; the latter would not be the case if the either firm's payoffs were not linear in money and therefore the marginal rate of substitution between q and money depended upon the level of money. ‖

Proof of Theorem 2: (sketch) We first need to introduce some additional notation to make our multi-firm strategic pricing game precise. Let θ_i be firm i's private information vector (note we use a subscript to denote firm association with the information rather than the product, in contrast to elsewhere in the paper). Each firm is assumed to exclusively control some subset of products (firm i's product line). This implies, among other things, that only firm i can contract upon $q_{i,j}$, the amount of i's good consumed by firm j. Let I_b be the set of potential buyers of firm i's product line; let I_s be the set of potential sellers to firm i. Because we have restricted attention to deterministic mechanisms and constant marginal costs of production, each firm i's pricing mechanism offered to firm j can be represented by a nonlinear pricing schedule, $P_{i,j}(q_{i,j})$; note that this formulation allows for second- and third-degree price discrimination. We consider the problem of renegotiation from the point of view of two firms, i and j, which are a subset of N firms, $k = 1, \ldots, N$. From firm i's point of view, the residual utility function of firm k ($k \neq i$) is simply

$$\hat{U}_k(q_{i,k}, \theta_k) \equiv \max_{q_{-i,k}} u_k(q_{-i,k}, q_{i,k}, \theta_k) - \sum_{l \neq i} P_{l,k}(q_{l,k}).$$

The associated consumer surplus is

$$\hat{CS}_k^i(q_{i,k}, \theta_k) \equiv \hat{U}_k(q_{i,k}, \theta_k) - P_{i,k}(q_{i,k}),$$

and the associated surplus for firm i's goods is

$$\hat{S}_k(q_i, \theta) \equiv \sum_{k \in I_b} \hat{U}_k(q_{i,k}, \theta_k) - c_i(\theta_i)' q_i - F_i,$$

where we use have used our assumption of constant marginal costs. Thus, firm i's profit on its product line is

$$\Pi_i(q_i, \theta) \equiv S_i(q_i, \theta) - \sum_{k \in I_b} \hat{CS}_k^i(q_{i,k}, \theta_k),$$

and expected profit is simply $\mathrm{E}_{\theta_{-i}}[\Pi_i(q_i, \theta)]$. By assumption, μ_i^* maximizes expected profit subject to incentive compatibility and participation constraints of its own mechanism, taking μ_{-i}^* as given. (Note that off the equilibrium path, firm i may cause μ_{-i}^* to induce false reporting of types or non-participation.)

Now consider firm i and j meeting among themselves in a renegotiation game. Note that the sum of their joint surpluses is simply

$$\hat{U}_i(q_{i,j}, \theta_i) + \hat{U}_j(q_{j,i}, \theta_j) - c_i(\theta_i)' q_{i,j} - c_j(\theta_j)' q_{j,i} = U_i(\underline{\theta}_i^c, \theta_i^p) + \hat{CS}_i^j(\theta_i^c) + U_j(\underline{\theta}_j^c, \theta_j^p) + \hat{CS}_j^i(\theta_j^c),$$

where $\underline{\theta}_i^c \equiv \arg\min_{\theta_i^c} U_i(\theta_i)$, and the c and p superscripts on the information parameters indicate whether the information is related to the marginal value of consumption or production, respectively. Note that because of our assumptions on separability, $\hat{CS}_i^j$ and $\underline{\theta}_i^c$ are both independent of θ_i^p. Rearranging the terms of the above expression, we have

$$U_i(\underline{\theta}_i^c, \theta_i^p) + U_j(\underline{\theta}_j^c, \theta_j^p) = \hat{U}_i(q_{i,j}, \theta_i) + \hat{U}_j(q_{j,i}, \theta_j) - c_i(\theta_i)' q_{i,j} - c_j(\theta_j)' q_{j,i} - \hat{CS}_i^j(\theta_i^c) - \hat{CS}_j^i(\theta_j^c).$$

Following similar arguments as in the proof of Theorem 1, we have again that μ^* is interim incentive efficient and undominated for each firm. Safety follows from identical arguments as in Theorem 1. ‖

Proof of Theorem 4: We sketch a proof here; the analysis is similar to the second part of Myerson's proof of his Theorem 2, (p. 1790). Choose any equilibrium in which there is renegotiation to another allocation other than μ^* for some type profile of types in the economy. Using the inscrutability principle, we can replace the equilibrium outcome with an inscrutable mechanism which does not reveal the offering firm's type at the time of offer, but only later in the simultaneous direct revelation game. Such a mechanism must provide at least as much utility for all types of the offering firm as the status quo mechanism, μ^*, which can always be implemented (since it is a strong solution). But since μ^* is undominated, any renegotiated allocation must provide exactly the same interim utility for all types of the offering firm. Lastly, because μ^* is the component-wise uniquely optimal profit-maximizing pricing strategy for the firms, no other mechanism exists which provides identical interim utility for any individual firm. Hence, it cannot be renegotiated in any equilibrium. ‖

Proof of Lemma 1: Consider the 3-way barter equilibrium. If types $(\overline{v}, \overline{\alpha})$, $(\overline{v}, \underline{\alpha})$ and $(\underline{v}, \underline{\alpha})$

barter in equilibrium, the value of Φ is given by $\Phi = \frac{\hat{\alpha}^2}{\hat{\alpha}+(1-\hat{\alpha})\phi} > \hat{\alpha}^2$. Thus,

$$\Delta(\underline{v}, \underline{\alpha}) = \lambda\underline{v} - c - (\underline{\alpha} + (1-\underline{\alpha})\Phi) < \lambda\underline{v} - c - (\underline{\alpha} + (1-\underline{\alpha})\hat{\alpha}^2) \leq \underline{v} - c - (\underline{\alpha} + (1-\underline{\alpha})\hat{\alpha}^2) < 0,$$

where the last inequality comes from our assumption that absent barter, the monopoly price is optimally set at $p = \overline{v}$. Thus, the $(\underline{v}, \underline{\alpha})$ type does not wish to barter.

Consider the 4-way barter equilibrium. In this case, $\Phi = \hat{\alpha}^2$, and so a similar argument as above implies that the $(\underline{v}, \underline{\alpha})$ type does not wish to barter.‖

Proof of Theorem 5: Lemma 1 states that only 3 classes of equilibria are potential candidates for barter. Consider each in turn.

Case 1: $\{(\overline{v}, \underline{\alpha})\}$ barters. If only the $(\overline{v}, \underline{\alpha})$-firm barters, the conditional probability of a firm selling a good to a rejected barterer on the cash market is $\Phi^1 = \underline{\alpha}$, the probability that the rejected firm obtains a cash sale from his own customer. The following three incentive conditions are necessary and sufficient to insure the existence of this type of equilibrium:

$$\begin{aligned}
\Delta(\overline{v}, \underline{\alpha}) &= \lambda\overline{v} - c - [\underline{\alpha} + (1-\underline{\alpha})\underline{\alpha}](\overline{v} - c) \geq 0, \\
\Delta(\overline{v}, \overline{\alpha}) &= \lambda\overline{v} - c - [\overline{\alpha} + (1-\overline{\alpha})\underline{\alpha}](\overline{v} - c) \leq 0, \\
\Delta(\underline{v}, \underline{\alpha}) &= \lambda\underline{v} - c - [\underline{\alpha} + (1-\underline{\alpha})\underline{\alpha}](\overline{v} - c) \leq 0.
\end{aligned}$$

Because $\Delta(v, \alpha)$ is decreasing in α, the third condition implies that $\Delta(\underline{v}, \underline{\alpha}) < 0$, so we may ignore this latter condition. Simplifying these conditions in terms of their implications for λ, we have

$$\frac{1}{\overline{v}}[c + (\overline{v} - c)(2 - \underline{\alpha})\underline{\alpha}] \leq \lambda \leq \min\left\{\frac{1}{\underline{v}}[c + (\overline{v} - c)(2 - \underline{\alpha})\underline{\alpha}], \frac{1}{\overline{v}}[c + (\overline{v} - c)(\underline{\alpha} + (1-\underline{\alpha})\overline{\alpha})]\right\} < 1.$$

Case 2: $\{(\underline{v}, \underline{\alpha}), (\overline{v}, \underline{\alpha})\}$ barter. If only the $\underline{\alpha}$-firms barter, the conditional probability of a firm selling a good to a rejected barterer on the cash market is $\Phi^1 = \hat{\alpha}\underline{\alpha}$, the probability that the rejected firm has a high valuation $v = \overline{v}$ and obtains a cash sale from his own customer. The following two incentive conditions are necessary and sufficient to insure the existence of this type of equilibrium:

$$\begin{aligned}
\Delta(\underline{v}, \underline{\alpha}) &= \lambda\underline{v} - c - [\underline{\alpha} + (1-\underline{\alpha})\underline{\alpha}](\overline{v} - c) \geq 0, \\
\Delta(\overline{v}, \overline{\alpha}) &= \lambda\overline{v} - c - [\overline{\alpha} + (1-\overline{\alpha})\underline{\alpha}](\overline{v} - c) \leq 0.
\end{aligned}$$

Because $\Delta(v, \alpha)$ is increasing in v, the first condition implies that $\Delta(\overline{v}, \underline{\alpha}) > 0$ and the second condition implies that $\Delta(\underline{v}, \overline{\alpha}) < 0$, so we may ignore these two additional requirements. Simplifying the above two inequalities in terms of their implications for λ, we have

$$\frac{1}{\underline{v}}[c + (\overline{v} - c)(\underline{\alpha} + (1-\underline{\alpha})\underline{\alpha}\hat{\alpha})] \leq \lambda \leq \frac{1}{\overline{v}}[c + (\overline{v} - c)(\overline{\alpha} + (1-\overline{\alpha})\hat{\alpha}\underline{\alpha}] < 1.$$

Case 3: $\{(\overline{v}, \underline{\alpha}), (\overline{v}, \overline{\alpha})\}$ barter. If only the $\overline{v}$-firms barter, the conditional probability of a firm selling a good to a rejected barterer on the cash market is $\Phi^1 = \hat{\alpha}$, the probability that the rejected firm obtains a cash sale from his own customer. The following two incentive conditions are necessary and sufficient to insure the existence of this type of equilibrium:

$$\begin{aligned}
\Delta(\overline{v}, \overline{\alpha}) &= \lambda\overline{v} - c - [\overline{\alpha} + (1-\overline{\alpha})\underline{\alpha}](\overline{v} - c) \geq 0, \\
\Delta(\underline{v}, \underline{\alpha}) &= \lambda\underline{v} - c - [\underline{\alpha} + (1-\underline{\alpha})\underline{\alpha}](\overline{v} - c) \leq 0.
\end{aligned}$$

Because $\Delta(v, \alpha)$ is decreasing in α, the first condition implies that $\Delta(\overline{v}, \underline{\alpha}) > 0$ and the second condition implies that $\Delta(\underline{v}, \overline{\alpha}) < 0$, so we may ignore these two additional requirements. Simplifying the above two expressions in terms of their implications for λ, we have

$$\frac{1}{\overline{v}}[c + (\overline{v} - c)[\overline{\alpha} + (1 - \overline{\alpha})\hat{\alpha}] \leq \lambda \leq \frac{1}{\underline{v}}[c + (\overline{v} - c)[\underline{\alpha} + (1 - \underline{\alpha})\hat{\alpha}].$$

Given our maintained assumption that absent a working barter exchange, a firm would always set $p = \overline{v}$ (i.e., $(\underline{\alpha} + (1 - \underline{\alpha})\hat{\alpha})(\overline{v} - c) > \underline{v} - c$), the righthand side of the above inequality is greater than 1, giving us the expression of the proposition. Examining the lefthand side of this expression, it is greater than the righthand sides of the expressions of cases 1 and 2. Hence, the above conditions are inconsistent with other barter equilibria. ‖

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