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Incentives to (Irreversible) Investments Under Different Regulatory Regimes

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Working Papers

INCENTIVES TO (IRREVERSIBLE) INVESTMENTS UNDER DIFFERENT REGULATORY REGIMES

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Abstract

This paper addresses the issue of how regulatory constraints affect firm's investment choices when the firm has the option to delay investment. The *RPI-x* rule is compared to a profit sharing rule, which increases the *x* factor in case profits go beyond a given level. It is shown that these rules are identical in their impact on investment choices, in that the change in the option value exactly compensates the change in the "direct" profitability of investment. The result is then analysed in the light of option theory and explained on the basis of the "bad news principle".

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Incentives to (irreversible) investments under different regulatory regimes*

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Abstract

This paper addresses the issue of how regulatory constraints affect firm's investment choices when the firm has the option to delay investment. The $RPI - x$ rule is compared to a profit sharing rule, which increases the x factor in case profits go beyond a given level. It is shown that these rules are identical in their impact on investment choices, in that the change in the option value exactly compensates the change in the “direct” profitability of investment. The result is then analysed in the light of option theory and explained on the basis of the “bad news principle”.

1 Introduction

The literature on investment incentives in regulated industries indicates how a price cap such as the by now traditional $RPI - x$ provides the regulated firm appropriate incentives to invest. The idea is that the regulated price should increase at a rate equal to the difference between the expected inflation rate (the Retail Price Index, RPI) and an exogenously given component

*We would like to thank Giacomo Calzolari for comments on an earlier draft.

(x) which, roughly speaking, represents the expected increase of productivity the firm should attain. By making prices insensitive at the margin to firm's choices, the $RPI - x$ rule appears to eliminate the downward bias and the phenomenon known as "underinvestment". As Beesley and Littlechild (1989) put it when listing the main arguments in favor of $RPI - x$, "Because the company has the right to keep whatever profits it can earn during the specified period (and must also absorb any losses), this preserves the incentive to productive efficiency associated with unconstrained profit maximization".

However, we know from the UK experience that an $RPI - x$ scheme tends to leave the firm large profits, so that some authors¹ (among others, Sappington and Weisman, 1996; Burns, Turvey and Weyman Jones, 1998) have brought into consideration an alternative called "sliding scale" (or, less cryptically, "profit sharing"). According to this scheme, in case the firm's profits go beyond a pre-specified level, the x factor should be automatically adjusted upwards, making the price-cap more stringent; this re-distributes rents to the consumers, making the system more "fair" and more sustainable from a political viewpoint. This system has been criticized by some authors (see e.g., Mayer and Vickers, 1996) who - among other things - stress that if higher investment spurs a tighter price cap, then we have a dis-incentive to invest. The superiority of the $RPI - x$ system relative to profit sharing rules on the ground of technical efficiency was recognized also by advocates of profit sharing rules (e.g., Lyon, 1996), who only defend the PS system on the basis of overall allocative efficiency (profit sharing "typically" increases consumers surplus).

Weisman (1993) shows that when price cap rules incorporate an element of profit sharing, price caps may represent a worsening relative to a pure cost based regulation (a notoriously inefficient set-up). While most papers in the regulation literature implicitly consider fully reversible investments, we take a different approach which, following the modern theory of investment, stresses how these choices are typically irreversible². The consequence of irreversibility is that the decision to invest should consider the option value of investment. As an irreversible choice entails burning an opportunity, the value of waiting should be considered.

Our study shows that irreversibility of investment considerably changes the relative desirability of the aforementioned regulatory policies. With irreversible investments, we can show that the effect of $RPI - x$ and profit

¹This proposal, already debated in the UK, has also become popular among several policy makers, such as the Italian electricity regulator.

²As stated by Dixit and Pindyck (1994, p.3) "Most investment decisions share three important characteristics, investment irreversibility, uncertainty and the ability to choose the optimal timing of investment".

sharing on the incentives to invest may well be identical. The reason is that the introduction of the profit ceiling into a $RPI - x$ scheme decreases the net present value of the investment, but also decreases the value of waiting (i.e., the option value) by exactly the same amount. This is an application of the “bad news principle” (Bernanke, 1983), which indicates that, under investment irreversibility, uncertainty acts asymmetrically since only the unfavorable events affect the current propensity to invest. If, thus, profit sharing (i.e., the change in the x factor) occurs only in the good state, investment decisions are not affected.

This paper is linked to two streams of literature. The first one is the literature on investment irreversibility. Irreversibility may arise from ‘lemon effects’ (second-hand capital goods may be impossible to sell), and from capital specificity (see Dixit and Pindyck, 1994, and Trigeorgis, 1996). Even when brand-new capital can be employed in different productions, in fact, it may become specific once installed. Irreversibility may be caused by industry comovements as well: when a firm can resell its capital, but the potential buyers operating in the same industry are subject to the same market conditions, this comovement obliges the firm to resort to outsiders. Due to reconversion costs, however, the firm can sell the capital at a considerably low price than an insider would be willing to pay if it did not face the same bad conditions as the seller. The irreversibility of capital expenditures is even more obvious in markets subject to price regulation, which are typically natural monopolies; the scarcity (or total absence) of firms operating in the same sector and the public constraints coming from nature of the service may represent decisive factors in this respect³. Relative to this literature, we consider how investment is affected by different regulatory rules, showing how the option value of irreversible investments matters in determining the optimal regulatory policy.

The second stream of literature is the one on regulation and investment. Since Laffont and Tirole (1986), we know that optimal price schemes entail a distortion in firms’ choices. To tackle this problem, the rule labelled “ $RPI - x$ ” was introduced. In this case, investment does not affect price, so that this rule is supposed to have “minimum” distortionary effects on investment choices.

Beesley and Littlechild (1989) stress that the efficiency properties of this scheme may be undermined by two aspects. The first one is that the x factor

³ An idea of the empirical relevance of irreversible investments in regulated industries can be obtained looking at the so called “stranded costs”, i.e., at the value of assets that following liberalisation will hardly find a remuneration, but cannot be shifted to a different productive use. According to Lyon and Mayo (2000) these costs can be estimated for the US electricity sector “in the neighborhood of \$200 billion”.

is subject to periodic reviews (every 4-5 years), so that a cost decrease may be exploited by the regulator to decrease prices in the future. The second factor is the risk of expropriation, which means that, even before the review, the regulator may be tempted to intervene - following political pressure - in case firm's profits appear as "excessively" high.

In our paper we consider the option value of an investment of a given amount, to see how different regulatory schemes affect the timing of investment. Notice that here we do not introduce any of these elements indicated by Beesley and Littlechild (1989) as countervailing factors, so that one would expect $RPI - x$ to emerge as the more desirable regulatory rule. On the contrary, our main result is that, even in the extremely favorable case we examine, profit sharing does not underperform the purest version of $RPI - x$ ⁴.

The paper is organized as follows. The next section provides the simplest analytical set-up in which the problem of irreversible investments can be studied, recalling the "bad news principle". Section 3 provides the market model, stressing how the traditional view of investment indicates that profit sharing tends to disfavor investment. Section 4 introduces the option value of investment, showing how the result can change when we consider this aspect. Section 5 extends the previous analysis in continuous time, showing how previous results hold in this less specific set-up. Section 6 extends the model to the case of cost-reducing investments, showing how the same results follow. Section 7 analyses the ability of these rules to extract firms' rents for a given distortionary effect. Section 8 concludes the paper.

2 A simple discrete-time model

In this section we introduce a simple discrete-time infinite horizon model describing the behavior of a competitive risk-neutral firm. The model and the notation follow Dixit and Pindyck (1994, Ch. 2). In particular, we assume that risk is fully diversifiable and that the risk-free interest rate r is fixed.

A market is characterized by the inverse demand function $p_t = p(q_t)$, where p_t and q_t are price and quantity of the good at time t . Production takes place at a per-period cost given by $C = c_t q_t$. Furthermore, in order to

⁴This result can be usefully linked to some recent results of the empirical literature. For instance, Crew and Kleindorfer (1996) and the papers they review stress how the presumed superiority of optimal price rules does not emerge so clearly from experiences in different countries and sectors. The claim that $RPI - x$ rules lead to more efficient investment patterns than profit related regulatory schemes does not find a clear empirical support.

produce the firm needs to build an infrastructure. This could be the case, for instance, of an energy distributor that has to decide whether or not to invest in a new network (either a pipeline or wires) in order to serve a town, knowing that he will be subject to a certain regulatory regime. Thus we introduce the following:

- **Assumption 1** (investment). Production requires a one-off investment of a given amount I .

The important aspect of this assumption is that the amount of investments is given. Although firms often have the possibility to marginally adjust the value of their expenditures, it is also true that the size of most investment projects that utilities face is by and large determined by the size of the area they want to serve. Building a new electric line connecting two nodes of a transmission system to improve its reliability, or a pipeline to sell gas to a new city are choices that entail an expenditure that can only partially be controlled by the firm. This type of major investments is what we focus on.

Obviously, in these cases the firm is left with two major choices: whether or not to undertake the investment, and when to do so. Therefore, while the notion of “underinvestment” typically refers to the amount spent by the firm, in this context we will talk of underinvestment referring to the probability that a firm invests and to the date of the investment. The apparent difference between our notion and the usual one is simply due to the fact that we explicitly model uncertainty and time. A lower probability of investing decreases the *expected* value of investment. A delay in the decision decreases its *present* value.

To analyze the choice of when to invest we assume that the firm has an infinite time horizon and maximizes the (discounted) present value of future expected profit. If the firm operates (i.e., if it undertakes the investment), its per-period profits are denoted by

$$\Pi_t = [p(q_t) - c_t] q_t. \quad (1)$$

Both price and cost follow a deterministic trend. Price grows at a rate α (we will be more specific in this issue later on), so that we have $p_t = p_0(1+\alpha)^t$. Operating costs decrease at a rate γ , so that $c_t = c_0(1-\gamma)^t$. Per period profits can be rewritten as

$$\Pi_t = [p_0(1+\alpha)^t - c_0(1-\gamma)^t] q_t. \quad (2)$$

The values of the parameters α and γ must respect two conditions:

- $\sum_{t=0}^{\infty} \frac{[p_0(1+\alpha)^t - c_0(1-\gamma)^t]}{(1+r_t)^t} \geq 0$, so that production is worthwhile;
- $\partial [\Pi_t / ((1+r_t)^t q_t)] / \partial t < 0$ as $t \rightarrow \infty$, so that the series converges.
Namely, $\sum_{t=0}^{\infty} \frac{[p_0(1+\alpha)^t - c_0(1-\gamma)^t]}{(1+r_t)^t} < \infty$.

Initial price p_0 is set at a level, and it is convenient to define $m \equiv \frac{p_0 - c_0}{c_0} \geq 0$, i.e. $p_0 = (1+m)c_0$. Thus, since the moment of investment, we can write

$$\Pi_t = \begin{cases} \frac{m}{1+m} p_0 q_0 & \text{for } t = 0 \\ \left[(1+\alpha)^t - \frac{(1-\gamma)^t}{1+m} \right] p_0 q_t & \text{for } t \geq 1 \end{cases}$$

The second element we want to introduce is uncertainty on future demand. In this section, when time is discrete, we model uncertainty as follows:

- **Assumption 2** (demand). Demand follows a bivariate stochastic process. At time 0 demand is q_0 . At time 1, with probability ρ , demand will rise to $(1+u)q_0$, and with probability $(1-\rho)$ it will drop to $(1-d)q_0$, with $u, d > 0$ and $d < 1$. From then onwards demand follows a deterministic trend. More formally:

$$q_t = \begin{cases} (1+u)^t q_0 & \text{with probability } \rho, \\ (1-d)^t q_0 & \text{with probability } 1-\rho \end{cases} \quad (3)$$

Therefore⁵, at time 1 profit will be either $(1+u) \left[(1+\alpha) - \frac{1-\gamma}{1+m} \right] p_0 q_0$ or $(1-d) \left[(1+\alpha) - \frac{1-\gamma}{1+m} \right] p_0 q_0$. The value at $t = 1$ of the profit stream in case j is:

$$\sum_{t=1}^{\infty} \frac{\Pi_t}{(1+r)^t} = Y_j p_0 q_0$$

where

$$Y_j \equiv \left(\frac{1+r}{r-\alpha-j(1+\alpha)} - \frac{1+r}{r+\gamma-j(1-\gamma)} \cdot \frac{1}{1+m} \right) \quad (4)$$

with $j = -d, u$.

⁵Introducing a common demand trend with upward or downward movements from the trend would be possible, but it would introduce little additional substance.

2.1 The Bad News Principle

The typical investment problem considered in the regulation literature takes up the following form. A firm has the opportunity to make a (sequence of) investment(s), K , which reduce variable cost and whose level is determined by profit maximization: the marginal revenue from investment is equal to its marginal cost (r). If profit is $\pi = R(q) - rK - C(q, K)$, where $R(q)$ is revenue, q is output and C is variable cost, profit maximization entails $r = C'(q, K)$. Notice that this coincides with welfare maximization. In the presence of a regulatory constraint, price (and hence R) varies with marginal cost, so that an increase in K may decrease the firm's revenues. This introduces a distortion in the firm's choice known as "underinvestment".

The traditional analysis studies the investment decisions of a regulated firm concentrating on a one-period profit function, which summarizes the net present value of future cash flows. In particular, it implicitly assumes that investment is fully reversible⁶. This assumption is fairly unrealistic. Hence, in this section we will discuss irreversible choices, under a given investment time, and, in the next section, we will further extend the model allowing for delays in investment.

Moreover, here we want to consider a slightly different investment problem, namely one where the firm *needs* to invest in order to operate⁷. At time 0 the firm must decide whether or not to make an investment of given cost I . In this set-up, it is natural to look at "underinvestment" as a case in which investment of a given cost is either less likely to occur (underinvestment in expected value) or postponed (so that the present value of the investment is reduced).

⁶Full investment reversibility implies that the resale price of capital is equal to its purchase price, namely that the secondary market is efficient. Under reversibility, the firm installing capital acquires a put option and a call option of equal value; this is why the two options cancel out and there is no need to explicitly specify their presence.

Reselling the capital later is like exercising the put option. If, conversely, the firm expands installed capital the call option is exercised (see Abel *et al.*, 1996).

When investment is only partially reversible, i.s. the resale price of capital is less than its purchase price, the value of the put option is lower and the incentive to invest is reduced. When, finally, investment is irreversible, no resale is possible and the value of the put option is nil.

In studying investment decisions with partial reversibility, Abel and Eberly (1996) show that even when the difference between the purchase and resale prices of capital is small, the case of full irreversibility provides a better approximation to investment behaviour than the case with costless reversibility. This induces us to consider the irreversibility case as an approximation of the more plausible case of partial reversibility.

⁷We will later show that analogous conditions hold in the case of cost-reducing investments.

In order to make the problem “non banal” we assume that the following holds:

• **Assumption 3.**

$$Y_u p_0 q_0 > \frac{I}{1+r} > Y_{-d} p_0 q_0 \quad (5)$$

This means that the investment project will be profitable only in case of “good news”, i.e. if demand increases. Clearly, as the discounted cost of investment is larger than the present discounted value of future profits in the bad state, investment is not profitable if demand decreases. This implies that a downward jump in the firm’s profitability can be interpreted as “bad news”. If either side of the above double inequality did not hold, the problem would be trivial.

Let us now study the firm’s investment policy. The expected present value at time 0 of its future payoffs, net of the investment cost (NPV_0), is⁸

$$NPV_0 = \left[\frac{m}{1+m} + \rho Y_u + (1-\rho) Y_{-d} \right] p_0 q_0 - I \quad (6)$$

In the absence of any opportunity to delay investment, the firm’s decision is the one that maximizes the net present value of the whole investment opportunity, namely the firm chooses

$$\max \{NPV_0, 0\}. \quad (7)$$

In other words, the firm will invest if NPV_0 is positive, i.e. if

$$p_0 > \frac{1}{\frac{m}{1+m} + \rho Y_u + (1-\rho) Y_{-d}} \cdot \frac{I}{q_0} \quad (8)$$

Notice that when investment is reversible, given (5) if at $t = 1$ demand goes down ($j = -d$) the firm will divest.

⁸More formally the expected present value at time 0 of its future payoffs, net of the investment should be written as

$$NPV_0 = \max \left\{ \left[\frac{m}{1+m} + \rho Y_u + (1-\rho) Y_{-d} \right] p_0 q_0 - I, 0 \right\}.$$

However, given assumption (5), the above equation reduces to (6).

When, instead, investment is irreversible and the firm is *able to postpone investment*, rule (7) is incorrect, since a positive value of NPV_0 is not sufficient to invest. If the firm can postpone investment, in fact, it has the possibility of waiting for new information. This implies that the firm is endowed with an option⁹. To decide when to invest, therefore, the firm compares NPV_0 with the expected net present value of the investment opportunity at time 1, NPV_1

$$NPV_1 = \rho \left(Y_{up_0} q_0 - \frac{I}{1+r} \right) \quad (9)$$

which is positive given (5). Note that this implies that the firm invests at time 1 only if it receives good news (i.e. it faces an upward shift in profits)¹⁰.

While the firm's decision to invest at time 1 is "banal" because of complete information (the firm invests only in the good state), at time 0 the firm has to decide whether to invest immediately, or whether to wait until full information is obtained. If it invests now, it will enjoy the profit stream between time 0 and time 1. However, investing implies the exercise of the option to delay and entails paying an opportunity cost, equal to option value. The optimal decision is the one that maximizes the net present value of the whole investment opportunity, namely entails choosing:

$$\max \{NPV_0, NPV_1\}. \quad (10)$$

A comparison of problem (7) with problem (10) allows us to compute the value of the option to postpone. Subtracting (7) from (10) yields

$$\max \{NPV_0, NPV_1\} - \max \{NPV_0, 0\} = \max \{NPV_1, 0\} \quad (11)$$

which is the value of the option to postpone investment.

Of course, given assumption (5), the solution to (11) is (9). Therefore, the optimal stopping time is given by the comparison between NPV_0 and

⁹As shown by McDonald and Siegel (1985, 1986), the opportunity to invest is analogous to a perpetual call option.

¹⁰More formally the expected present value at time 1, net of the investment, should be written as

$$NPV_1 = \rho \max \left(Y_{up_0} q_0 - \frac{I}{1+r}, 0 \right) + (1-\rho) \max \left(Y_{-d p_0} q_0 - \frac{I}{1+r}, 0 \right).$$

However, given assumption (5), the above equation boils down to (9).

NPV_1 . If, therefore, $NPV_0 > NPV_1$, immediate investment is undertaken. If instead $NPV_1 > NPV_0$, then waiting one period is better.

The investment rule can be rewritten by comparing the alternative policies. Setting (6) equal to (9), and solving for p_0 we thus obtain

$$p_0^* \equiv \frac{1}{\frac{m}{1+m} + (1-\rho)Y_{-d}} \cdot \frac{1+r-\rho}{1+r} \cdot \frac{I}{q_0} \quad (12)$$

Equation (12) defines p_0^* as the trigger price in this reference case: namely when $p_0 > p_0^*$, immediate investment is preferred. As shown by (12), the investment decision depends on the extent of the downward demand shift, d , and its probability $(1-\rho)$, but is independent of the upward move's parameter.

This point is an immediate consequence of Bernanke's (1983) *Bad News Principle* (BNP): under investment irreversibility, uncertainty acts asymmetrically since only the unfavorable events affect the current propensity to invest¹¹. The BNP implies that the higher the investment cost I , the higher the return required to compensate for irreversibility and, consequently, the higher the trigger point.

3 Regulation and irreversibility

In the previous section we have restated a well known result in the theory of investment. Let us now turn to an explicit consideration of the price regulation, in order to establish how it affects a firm's decision.

3.1 Pure price cap

We will first assume that regulation follows the traditional price-cap rule known as $RPI - x$ (Beesley and Littlechild, 1989).

Definition 1 (*Price Cap*) *Under the Price Cap $RPI - x$ rule, if the firm starts producing at time $\tilde{t}$, the initial price $p_{\tilde{t}}$ is given, and its dynamics over an infinite period of time is defined by the difference between the inflation rate (changes in the retail price index, RPI) and an exogenous factor x :*

$$p_{t+1} = (1 + RPI - x)p_t \quad (PC)$$

for $t \geq \tilde{t}$.

¹¹ As stated by Bernanke (1983), "The investor who declines to invest in project i today (but retains the right to do so tomorrow) gives up short-run returns. In exchange for this sacrifice, he enters period $t+1$ with an "option" (...). In deciding whether to "buy" this option (...), the investor therefore considers only "bad news" states in $t+1$ (...)" [p. 92-3].

The factor x is linked to the productivity gain (cost reduction) that the regulator expects the firm to be capable of achieving every year, but is determined at the beginning and is thus exogenous to the firm. As already stressed, the logic of the $RPI - x$ rule is that, by making prices insensitive at the margin to firm's choices, it appears to eliminate underinvestment. Also notice that here we assume that price dynamics is given over an infinite horizon, so that current investments have no impact on prices either in the short- or in the long-run.

If the firm does not undertake the investment, it cannot produce and its profit is zero. Let us assume for the moment that investment can only take place at time 0, so that the firm has no option to delay it; we will relax this assumption in the next section.

If the firm invests, per-period profits are thus¹²

$$\Pi_t = [p_0(1 + RPI - x)^t - c_0(1 - \gamma)^t] q_t$$

When investment is irreversible but the firm *cannot* postpone it, the firm will invest if the expected net present value at time 0 of its future payoffs is positive:

$$NPV_0^C = -I + (p_0 - c_0) q_0 + \sum_{t=1}^{\infty} [\rho (\varphi_u^t p_0 - v_u^t c_0) + (1 - \rho) (\varphi_d^t p_0 - v_d^t c_0)] q_0 > 0 \quad (13)$$

where $\varphi_u \equiv \frac{(1+u)(1+RPI-x)}{1+r}$, $\varphi_d \equiv \frac{(1-d)(1+RPI-x)}{1+r}$, $v_u \equiv \frac{(1+u)(1-\gamma)}{1+r}$ and $v_d \equiv \frac{(1-d)(1-\gamma)}{1+r}$. If the initial price p_0 is set at a level above initial cost c_0 , so that $p_0 = (1 + m)c_0$, we solve (13) for p_0 obtaining

$$p_0^C \equiv \frac{1}{\frac{m}{1+m} + \rho \left(\frac{1}{1-\varphi_u} - \frac{1}{1-v_u} \frac{1}{1+m} \right) + (1 - \rho) \left(\frac{1}{1-\varphi_d} - \frac{1}{1-v_d} \frac{1}{1+m} \right)} \frac{I}{q_0} \quad (14)$$

This leads to the rule: invest if $p_0 > p_0^C$. Otherwise, investment is never undertaken. The term p_0^C thus denotes the trigger price in case of pure price cap regulation.

It is easy to check the following

Remark 1 *An increase in x increases p_0^C (i.e., $\frac{\partial p_0^C}{\partial x} > 0$). This implies that having a $RPI - x$ reduces the incentive to invest.*

¹²As we look at the consequences of different regulatory schemes on a firm's decisions, costs are considered known. The choice of the optimal price should instead consider asymmetric information, but this is beyond the scope of the current work.

3.2 Profit sharing

To tackle the fairness concern previously mentioned, an alternative to $RPI - x$ has been proposed, called *sliding scale* (or - less cryptically - profit sharing)¹³. This scheme is defined as follows:

Definition 2 (Profit sharing) *Under the Profit Sharing regulatory mechanism, the $RPI - x$ rule remains in place as long as profit remains below an exogenous level $\tilde{\Pi}$. If $\Pi_t > \tilde{\Pi}$, in period $t + 1$ the price decrease factor increases to $x' > x$:*

$$p_{t+1} = \begin{cases} (1 + RPI - x)p_t & \text{if } \Pi_t \leq \tilde{\Pi} \\ (1 + RPI - x')p_t & \text{if } \Pi_t > \tilde{\Pi}, \text{ with } x' > x \end{cases} \quad (\text{PS})$$

Thus, the price decrease factor remains constant as long as profits are considered “reasonable”. When they become “excessive”, this mechanism redistributes part of the surplus to the consumers. In this section we analyze this issue.

This switch point $\tilde{\Pi}$ will be reached depending on whether the state at time 1 will be good or bad. Define the switch time for the good state as

$$t_1 = t \mid \left\{ [p_0(1 + RPI - x)^{t-1} - c_0(1 - \gamma)^{t-1}] (1 + u)^{t-1} q_0 = \tilde{\Pi} \right\}$$

The definition of t_1 implicitly assumes that the starting price guarantees a profit level below $\tilde{\Pi}$. We also assume that if the bad state occurs, the switch point $\tilde{\Pi}$ will never be reached, namely

$$\nexists t_2^* = t \mid \left\{ [p_0(1 + RPI - x')^{t-1} - c_0(1 - \gamma)^{t-1}] (1 - d)^{t-1} q_0 = \tilde{\Pi} \right\} \quad (15)$$

Assumption (15) - which defines a “bad news” as one associated to a profit level bounded away from $\tilde{\Pi}$ - deserves a brief comment. The fact of reaching a switch point $\tilde{\Pi}$ is implicitly a good news.

Remark 2 *If we assumed that t_2^* exists, we would implicitly assume that the “bad news” simply consists of a delay in achieving high profits, namely a delay in good news. This is a contradiction in terms. Given the existence of t_2^* , in fact, the firm would face two alternative states, one better than the*

¹³Notice that one could also have an intervention rule based on the level of revenues instead of profits; see Sappington and Weisman (1996).

other. In both cases, however, the firm would gain from investing. It is quite obvious, given the established results on investment theory, that the behavior of the firm would not change because of the increase in the x factor. Our result would thus follow in quite a trivial way, since the firm would invest at time 0, irrespective of the toughness of the regulatory policy. Note that this assumption does not entail particular restrictions or a loss of generality. In any case, the following section presents the same result in a more general set-up.

Let us now write the NPV_0

$$NPV_0^S = -I + \left(\frac{m}{1+m}\right)p_0q_0 + (1-\rho)p_0q_0 \cdot \left[\sum_{t=1}^{\infty} \left(\varphi_d^t - \frac{v_d^t}{1+m} \right) \right] + \quad (16)$$

$$+ \rho p_0q_0 \cdot \left\{ \sum_{t=1}^{t_1} \left[\varphi_u^t - \frac{v_u^t}{1+m} \right] + \sum_{t=t_1+1}^{\infty} \left[\varphi_u^t - \frac{v_u^t}{1+m} \right] \right\}$$

with $\varphi_u \equiv \frac{(1+u)(1+RPI-x)}{1+r} > \varphi_u' \equiv \frac{(1+u)(1+RPI-x')}{1+r}$. Solving $NPV_0^S = 0$ for p_0 ,

one finds

$$p_0^S \equiv \frac{1}{\frac{m}{1+m} + \rho \left(\frac{1-\varphi_u^{t_1+1}}{1-\varphi_u} + \frac{\varphi_u'^{t_1+1}}{1-\varphi_u'} - \frac{1}{1-v_u} \frac{1}{1+m} \right) + (1-\rho) \left(\frac{1}{1-\varphi_d} - \frac{1}{1-v_d} \frac{1}{1+m} \right)} \frac{I}{q_0} \quad (17)$$

Considering (14), easy computations show that $p_0^S > p_0^C$. Therefore:

Proposition 3 (Underinvestment) *Consider a regulated monopoly which has to decide on an investment of given amount. When demand is uncertain as modelled in (3) and in the absence of any option to delay investment, correcting the $RPI - x$ rule with a Profit Sharing element makes investment less likely for any given value of p_0 .*

Therefore, in the absence of any option to delay, a profit-sharing formula might discourage irreversible investment. More precisely, if the initial price is between the two thresholds (i.e., if $p_0 \in [p_0^C; p_0^S]$) a project that would be carried out with a price cap regulation would not be initiated with a profit sharing system. This is because the extraction of a larger amount of monopoly rent reduces NPV_0 , thereby increasing the trigger point and, thus, discouraging investment.

Remark 3 We label the above result “underinvestment” in that, given initial price and a distribution of cost parameters, the expected value of investment is higher under Price Cap than under Profit Sharing. Notice that while the initial price p_0 is determined by the regulator (and thus is not a random element from the decision-maker’s point of view) cost parameters are not necessarily known to the regulator. Indeed, the $RPI - x$ rule finds its rationale in the existence of asymmetric information on the firm’s cost parameters, which from the viewpoint of the decision-maker are typically modelled as random variables (see, among others, Armstrong et al. (1995)).

This first result replicates the standard one, confirming that, under certain circumstances, Profit Sharing reduces the incentives to invest of a regulated monopoly. As will be shown in the next section, however, Proposition 3 fails to hold when the firm can choose when investing.

4 The value of the option to defer investment

When the firm has the opportunity to postpone investment, a positive value of NPV_0 is no longer a sufficient condition for investing. When the firm may postpone investment, in fact, delaying the decision to invest allows the firms to acquire further information. This implies that the option to delay is valuable. To decide when investing, therefore, NPV_0 must be larger than the sum of the effective cost (I) and the opportunity cost (i.e. the value of the option to wait). For simplicity, in this section we stick to the previous demand structure and we assume that the firm can choose whether investing at time 0 or at time 1 (or never), but is not allowed to further postpone investment (later on this assumption will be dropped, showing that the same result follows).

Under regulation, condition (5) must be modified as follows

$$p_0 q_0 \sum_{t=1}^{\infty} \left[\varphi_d^t - \frac{v_d^t}{1+m} \right] < \frac{I}{1+r} < p_0 q_0 \sum_{t=1}^{\infty} \left[\varphi_u^t - \frac{v_u^t}{1+m} \right] \quad (18)$$

Condition (18) implies that, in the good state, the present discounted value of future profits is larger than the present discounted value of the explicit investment cost, and vice versa. Obviously, if regulation is too heavy the second part of (18) does not hold and - trivially - no investment will be undertaken; we do not consider this case, where the market never opens.

4.1 Pure price cap

When the regulated firm is endowed with an option to delay, its problem is similar to (10), namely

$$\max \{NPV_0^{PC}, NPV_1^{PC}\} \quad (19)$$

As investment is irreversible, the NPV_0 is the same as in the problem without the option to delay (see (13)), i.e. $NPV_0^{PC} = NPV_0^C$. The option to defer investment matters instead in the calculation of the option value, which can be obtained considering that

$$\begin{aligned} NPV_1^{PC} &= \rho \left[-\frac{I}{1+r} + \sum_{t=1}^{\infty} \left(\varphi_u^t - \frac{v_u^t}{1+m} \right) p_0 q_0 \right] = \\ &= \rho \left[-\frac{I}{1+r} + \left(\frac{1}{1-\varphi_u} - \frac{1}{1-v_u} \cdot \frac{1}{1+m} \right) p_0 q_0 \right]. \end{aligned} \quad (20)$$

Notice that the possibility to defer investment creates an option value relative to the case where investment can be undertaken either at time 0 or never. The option expires at time 1: it can be exercised either at time 0 or at time 1.

Using equations (13) and (20), and solving for p_0 one thus obtains

$$p_0^{PC} \equiv \frac{1}{\frac{m}{1+m} + (1-\rho) \left(\frac{1}{1-\varphi_d} - \frac{1}{1-v_d} \frac{1}{1+m} \right)} \cdot \frac{1+r-\rho}{1+r} \cdot \frac{I}{q_0}, \quad (21)$$

which is the trigger point above which the present discounted value of current and future profits is greater than the summation of the explicit cost of investment I plus the opportunity cost (the call option). If $p_0 > p_0^{PC}$, therefore, investment is undertaken immediately. Otherwise, the firm finds it more profitable to postpone it.

It is easy to show that $p_0^{PC} > p_0^C$. This inequality is a straightforward implication of the introduction of the call option. This option is an opportunity cost which must be added to the explicit cost of investment I .

As can be seen, if the firm has the option to postpone investment, the investment decision depends on the size of the downward move, d , and its probability $(1-\rho)$, but is independent of the upward move's parameter u . The BNP implies that the higher the investment cost I , the higher is the risk premium of investment (and, consequently, the higher is the trigger point p_0^{PC}).

4.2 Profit sharing

Under profit sharing the firm's problem is to choose

$$\max \{NPV_0^{PS}, NPV_1^{PS}\},$$

where the NPV_0 is the same as in the previous section (see (16)), while NPV_1^{PS} is

$$NPV_1^{PS} = \rho \left[-\frac{I}{1+r} + \left(\frac{1 - \varphi_u^{t_1+1}}{1 - \varphi_u} + \frac{\varphi_u'^{t_1+1}}{1 - \varphi_u'} - \frac{1}{1 - v_u} \cdot \frac{1}{1 + m} \right) p_0 q_0 \right] \quad (22)$$

Setting (16) equal to (22) and solving for p_0 yields

$$p_0^{PS} \equiv \frac{1}{\frac{m}{1+m} + (1 - \rho) \left(\frac{1}{1 - \varphi_d} - \frac{1}{1 - v_d} \frac{1}{1+m} \right)} \cdot \frac{1 + r - \rho}{1 + r} \cdot \frac{I}{q_0} \quad (23)$$

The firm invests at time 0 whenever $p_0 > p_0^{PS}$. Comparing this expression with (21), one can easily see that

$$p_0^{PS} = p_0^{PC}.$$

We can thus summarize this result in the following Proposition.

Proposition 4 (Neutrality of profit sharing) *Consider a regulated monopoly which has to decide on an investment of a given amount. When demand is uncertain as modelled in (3) and the timing of investment is endogenous, correcting the $RPI - x$ rule with a profit sharing element does not make investments less likely relative to the pure price-cap device.*

Since the tightening of the price cap takes place only in case of “good news”, the BNP implies that, while profit sharing actually reduces the firm's rents, it does not interfere with its decision to invest relative to the pure price-cap rule. There are no investment projects that will be undertaken under one regime, but not under the other.

As will be shown in section 5, the result of Proposition 4 depends neither on the assumption that demand follows a bivariate stochastic process nor on the hypothesis that uncertainty vanishes after one period. Even when the firm's payoff is described by a continuous stochastic process, lasting to infinity, the same result holds.

The intuition for this result can be obtained considering how the option value of investment changes the firm's problem. Relative to the case of pure price cap, profit sharing reduces the *net* value of the project at time 0 (NPV_0) and its option value (NPV_1) *by the same amount*. Therefore the difference ($NPV_0 - NPV_1$) is unaffected by profit sharing, and so is the investment decision.

5 The model in continuous-time

So far we have used a very simple model, where investment could be undertaken only at time 0 or at time 1. Here we want to extend the previous framework to a continuous time set-up, which allows us to show how the previous result still holds in a more plausible model, where the firm can invest either at time 0 or at in any future date. Furthermore, using the findings of the ICAPM we can describe the behavior of a risk-neutral firm owned by risk-averse shareholders, thereby taking into account their risk premium [McDonald and Siegel (1985, 1986)].

Once investment is undertaken, current profits are

$$\Pi(t) = [p(t) - c(t)] q(t) \tag{24}$$

The firm has an infinite time horizon and maximizes the (discounted) present value of future expected profit. Price is subject to public regulation, and we will stick to the assumption that regulation follows the traditional price-cap rule known as $RPI - x$.

For simplicity we assume that $[p(t) - c(t)]$ has the following dynamics¹⁴

$$\frac{d[p(t) - c(t)]}{dt} = [p(t) - c(t)] (RPI - x + \gamma) \quad (25)$$

where the parameter γ captures possible cost reductions over time.

To model demand uncertainty, a natural extension of the bivariate structure (3) is the following

- **Assumption 2'** (demand). Demand follows a geometric Brownian motion

$$dq(t) = \alpha_q q(t) dt + \sigma_q q(t) dz_q \quad (26)$$

where α_q and σ_q are the growth rate and variance parameter, respectively.

Using equations (24), (25), and (26), and applying Itô's lemma we can obtain the profits' dynamics

¹⁴The assumed dynamics of $[p(t) - c(t)]$ is necessary for obtaining a closed-form solution. Equation (25) is a special case of a more general formulation obtained by assuming that output price and operating costs evolve according to the following equations:

$$p(t) = p_0 e^{(RPI - x)t}$$

and

$$c(t) = c_0 e^{-\gamma t}$$

To show this, let us use the above equations and differentiate $[p(t) - c(t)]$. We thus obtain

$$\begin{aligned} \frac{d[p(t) - c(t)]}{dt} &= \{(RPI + \gamma - x)[p(t) - c(t)] + \\ &\quad (RPI + \gamma - x)c(t) - \gamma[p(t) - c(t)]\} \end{aligned}$$

Setting term

$$\{(RPI + \gamma - x)c(t) - \gamma[p(t) - c(t)]\} = 0$$

we obtain (25). Note that the above equality implies that the mark-up is constant, i.e.

$$\frac{p(t) - c(t)}{c(t)} = \frac{RPI + \gamma - x}{\gamma}$$

$$d\Pi(t) = \alpha\Pi(t)dt + \sigma\Pi(t)dz \quad (27)$$

where $\alpha \equiv RPI - x + \gamma + \alpha_q$ is the expected growth rate of per-period profits and $\sigma = \sigma_q$ is the standard deviation¹⁵. Given the dividend rate δ (which must be positive in order for the net value of the firm to be bounded), if the shareholders are risk-neutral, we must have $r - \delta = \alpha$. If, conversely, shareholders are risk-averse, the difference $r - \delta$ takes account of a the risk premium¹⁶. Thus, with no loss of generality we assume $r - \delta = RPI - x + \gamma + \alpha_q$. Solving for the dividend rate we thus obtain

$$\delta(x) = r - RPI + x - \alpha_q - \gamma \quad (28)$$

Let us now use these results in the two cases we consider.

5.1 Pure price cap

The firm must solve a standard optimal stopping time problem, namely it must choose the timing of investment to maximize the expected present value of its payoff. The problem can be represented as follows

$$\max_t E \left[(V_{PC}(\Pi(t)) - I)e^{-rt} \right] \quad (29)$$

¹⁵The model could be easily extended by assuming that $[p(t) - c(t)]$ follows a stochastic process, i.e.

$$d[p(t) - c(t)] = [p(t) - c(t)] [(RPI + \gamma - x)dt + \sigma_{pc}dz_{pc}] .$$

This may be the case, for instance, if the RPI is a random variable and if technology shocks may take place. Thus, given the correlation coefficient $E[dz_q, dz_{pc}] = \kappa_{qpc}dt$ and using Itô's lemma, current profits are

$$d\Pi(t) = \Pi(t) \left\{ \left[\alpha + \frac{1}{2} \kappa_{qpc} \sigma_q \sigma_{pc} \right] dt + (\sigma_q dz_q + \sigma_{pc} dz_{pc}) \right\}$$

which is another standard geometric Brownian motion that can be studied in a similar way.

¹⁶According to the Intertemporal Capital Asset Pricing Model, in fact, the total expected rate of return $\mu = \delta + \alpha$ must satisfy the relationship $\mu = r + \lambda \sigma \rho_M$ where $\lambda \equiv (\mu_M - r) / \sigma_M$ is the market price of risk, with parameters μ_M, σ_M^2 and ρ_M representing the expected return, the variance of the market portfolio and the correlation coefficient between the rate of return on the asset and that on the portfolio, respectively. Under risk aversion, therefore, the equality $r - \delta = \alpha - \lambda \sigma \rho_M$ holds. As shown in Merton (1990, Ch. 15), the risk-adjusted drift $\alpha - \lambda \sigma \rho_M$ allows the valuation of the firm as if it were risk neutral.

where $E[\cdot]$ denotes the expectation operator, $V_{PC}(\Pi(t))$ is the project value under the price-cap regulation, i.e. the *NPV* of the project at time t . The solution of the problem, i.e. the optimal time of investments, will be defined as T^* . Problem (29) is therefore analogous to the discrete-time problem (19).

Using dynamic programming, the firm's value $V_{PC}(\Pi(t))$ can be written as

$$V_{PC}(\Pi(t)) = \Pi(t)dt + e^{-rdt} E[V_{PC}(\Pi(t) + d\Pi(t))]$$

Expanding the right-hand side and using Itô's lemma one obtains

$$rV_{PC}(\Pi(t)) = \Pi(t) + (r - \delta(x))\Pi V_{PC\Pi}(\Pi(t)) + \frac{\sigma^2}{2}\Pi^2 V_{PC\Pi\Pi}(\Pi(t)) \quad (30)$$

where $V_{PC\Pi} = \partial V_{PC}/\partial \Pi(t)$ and $V_{PC\Pi\Pi} = \partial^2 V_{PC}/\partial \Pi^2(t)$, respectively. For simplicity, hereafter, we will omit the time variable t .

To compute the value function, it is assumed that $V_{PC}(0, x) = 0$, namely when Π is very small, the project is almost worthless, and that no speculative bubbles exist¹⁷. Thus, equation (30) has the following solution

$$V_{PC}(\Pi, x) = \Pi/\delta(x) \quad (31)$$

As shown by Dixit and Pindyck (1994), the option function has the following form

$$O_{PC}(\Pi, x) = A\Pi^{\beta_1(x)} \quad (32)$$

where A is a parameter to be determined, and $\beta_1(x)$ is the positive root of the following characteristic equation¹⁸

$$\frac{\sigma^2}{2}\beta(\beta - 1) + (r - \delta(x))\beta - r = 0$$

It is easy to ascertain that $\frac{\partial \beta_1(x)}{\partial x} > 0$. The optimal investment timing can be computed using the Value Matching Condition (VMC) and the Smooth

¹⁷When Π is very small, in fact, the probability of it rising to the trigger point is close to zero. Therefore, the value function is almost worthless. For further details on the boundary conditions see Dixit and Pindyck (1994, Ch. 5 and 6).

¹⁸The positive root is

$$\beta_1(x) = \frac{1}{2} - \frac{r - \delta(x)}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{r - \delta(x)}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}}.$$

Pasting Condition (SPC). The former condition requires the net present value of the project to be equal to the option value to defer investment, $O_{PC}(\Pi, x)$, namely

$$V_{PC}(\Pi, x) - I = O_{PC}(\Pi, x) \quad (\text{VMC})$$

The second condition requires the slopes of the functions $[V_{PC}(\Pi, x) - I]$ and $O_{PC}(\Pi, x)$ to match

$$\frac{\partial [V_{PC}(\Pi, x) - I]}{\partial \Pi} = \frac{\partial O_{PC}(\Pi, x)}{\partial \Pi} \quad (\text{SPC})$$

Conditions *VMC* and *SPC* characterize optimal time T^* . Notice that, given (27), this value can be associated to a profit level Π^* : whenever current profit reaches Π^* , the firm invests.

To solve the optimal stopping time problem, let us substitute (31) and (32) into the *VMC* and the *SPC*. We thus obtain a two-equation system with two unknowns: the trigger point of Π , above which investment is profitable, and the coefficient A . It is easy to show that the trigger point is¹⁹

$$\Pi_{PC}^*(x) \equiv \frac{\beta_1(x)}{\beta_1(x) - 1} \delta(x) I \quad (33)$$

The option value multiple in equation (33), $\frac{\beta_1(x)}{\beta_1(x) - 1} > 1$, shows that the gross present discounted value $V_{PC}^*(\Pi_{PC}^*(x), x) = \frac{\Pi_{PC}^*(x)}{\delta(x)}$ must exceed the investment cost I to compensate for investment irreversibility. It is easy to ascertain that

$$\frac{\partial \left(\frac{\beta_1(x)}{\beta_1(x) - 1} \right)}{\partial x} = - \frac{1}{(\beta_1(x) - 1)^2} \frac{\partial \beta_1(x)}{\partial x} < 0 \quad (34)$$

namely a tightening of the price cap (increase in x) reduces the wedge $\frac{\beta_1(x)}{\beta_1(x) - 1}$ and, therefore, decreases the option value. However, this tightening also increases $\delta(x)$ and thus reduces the value of the project, $V_{PC}^*(\Pi, x)$. This means

¹⁹The computation of A is not necessary for our purposes. However, substituting $\Pi_{PC}^*(x)$ into the two-equation system one easily obtains

$$A = \frac{I}{\beta_1(x) - 1} (\Pi_{PC}^*(x))^{-\beta_1(x)}.$$

that an increase in x decreases both the expected profit and the opportunity cost of investing. If we define the elasticity of the wedge to x as

$$\eta(x) \equiv \frac{\partial \left(\frac{\beta_1(x)}{\beta_1(x)-1} \right)}{\partial x} \frac{x}{\frac{\beta_1(x)}{\beta_1(x)-1}} < 0$$

we can see that

$$\frac{\partial \Pi_{PC}^*(x)}{\partial x} \propto 1 + \eta(x) \frac{\delta(x)}{x}$$

As shown by Dixit and Pindyck (1994), who provide some numerical simulations, for reasonable values of the parameters, the sign of the derivative $\frac{\partial \Pi_{PC}^*(x)}{\partial x}$ is positive, namely an increase in the x factor increases the option function more than the value function, thereby tending to postpone investment. Again, we see that the $RPI - x$ rule is not neutral to investment decisions, as claimed by Beesley and Littlechild (1989).

5.2 Profit sharing

Let us now turn to the problem when regulation follows a Profit Sharing rule. If $\Pi < \tilde{\Pi}$ the Brownian motion is the same as in the previous section. If, instead, $\Pi > \tilde{\Pi}$, the Brownian motion describing the regulated payoff is²⁰

$$d\Pi = \alpha' \Pi dt + \sigma \Pi dz \quad (35)$$

with $\alpha' \equiv RPI - x' + \gamma + \alpha_q < \alpha$. If $\Pi > \tilde{\Pi}$, therefore, the dividend rate is given by equality $r - \delta(x') = \alpha' - \lambda \sigma \rho_M$, which implies the inequalities $\delta(x') > \delta(x) > \delta$. When a switch point $\tilde{\Pi}$ is introduced, both the option function and the value function must be solved separately for $\Pi < \tilde{\Pi}$ and $\Pi > \tilde{\Pi}$. Then, the values and derivatives of the functions are equated at the switch point $\Pi = \tilde{\Pi}$ (see Dixit and Pindyck, 1994, pp. 186-189).

In order to check whether an investment project is profitable, both explicit and opportunity costs must be taken into account. Thus, investment is profitable if (*and when*) the present discounted value of future profits, net of both costs, is positive. Similarly to the price-cap case, the firm's problem can be represented as follows

$$\max_t E \left[(V_{PS}(\Pi, x) - I) e^{-rt} \right] \quad (36)$$

²⁰Notice that it may well happen that profit first goes beyond $\tilde{\Pi}$, while at a later stage $\Pi < \tilde{\Pi}$. In this case - in line with the spirit of the mechanism at stake - this formulation guarantees that the price cap goes back to its original level.

Let us start with the analysis of the option value, which is a function of current profits, i.e. $O_{PS}(\Pi, x)$. In the $(0, \tilde{\Pi})$ region, condition $O_{PS}(0, x) = 0$ holds, and, therefore, the value function has the standard form $C_1 \Pi^{\beta_1(x)}$. In the $(\tilde{\Pi}, \infty)$ region, instead, the option function has the general form, namely it is given by the sum of $B_1 \Pi^{\beta_1(x')}$ and $B_2 \Pi^{\beta_2(x')}$ (with B_1 and B_2 to be determined), where $\beta_1(x')$ and $\beta_2(x')$ are the roots of the characteristic equation

$$\frac{\sigma^2}{2} \beta(\beta - 1) + (r - \delta(x'))\beta - r = 0$$

with $\beta_1(x') > 1$ and $\beta_2(x') < 0$ ²¹. To sum up, the option function is

$$O_{PS}(\Pi, x) = \begin{cases} C_1 \Pi^{\beta_1(x)} & \text{if } \Pi < \tilde{\Pi} \\ B_1 \Pi^{\beta_1(x')} + B_2 \Pi^{\beta_2(x')} & \text{if } \Pi > \tilde{\Pi} \end{cases} \quad (37)$$

By equating the values and the derivatives of the two components of the option function at point $\Pi = \tilde{\Pi}$, we can compute B_1 and B_2 as functions of C_1 . As shown in Appendix A, $B_1 \propto C_1$ and $B_2 \propto C_1$.

Let us now turn to the value function. The general solution of the value function is given by the sum of a perpetual rent, with discount rate $\delta(x)$, and a homogeneous (exponential) part. Again, it is assumed that $V_{PS}(0, x) = 0$ (namely when Π is very small, the project is almost worthless) and that no speculative bubbles exist. Thus the solution of the value function boils down to

$$V_{PS}(\Pi, x) = \begin{cases} \frac{\Pi}{\delta(x)} + V_1 \Pi^{\beta_1(x)} & \text{if } \Pi < \tilde{\Pi} \\ \frac{\Pi}{\delta(x')} + V_2 \Pi^{\beta_2(x')} & \text{if } \Pi > \tilde{\Pi} \end{cases} \quad (38)$$

As shown in Appendix A, stitching together the two components of the value function (38) at the switch point $\Pi = \tilde{\Pi}$, one obtains parameters V_1 and V_2 , where $V_1 < 0$ and $V_2 > 0$, respectively. Both V_1 and V_2 depend on the regulatory coefficients x and x' . In particular, $V_1 \Pi^{\beta_1(x)}$ is negative and

²¹The roots are

$$\beta_{1,2}(x') = \frac{1}{2} - \frac{r - \delta(x')}{\sigma^2} \pm \sqrt{\left(\frac{1}{2} - \frac{r - \delta(x')}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}},$$

and it is easy to ascertain that, given derivative $\frac{\partial \beta_1(x)}{\partial x} > 0$, inequality $\beta_1(x') > \beta_1(x)$ holds.

represents the present discounted value of future profit reduction due to the profit sharing (when Π goes beyond $\tilde{\Pi}$). $V_2\Pi^{\beta_2(x')}$ is positive and measures the present discounted value of the future increase in the profit participation when Π goes below $\tilde{\Pi}$ (in fact $\partial\Pi^{\beta_2(x')}/\partial\Pi < 0$).

As shown in the previous section, the firm's optimal investment timing is obtained by applying the *VMC* and *SPC*. Namely, substituting equations (37) and (38) into the *VMC* and *SPC* one obtains the trigger point and the unknown parameter of the option function. In the $(0, \tilde{\Pi})$ region, these conditions lead to the following system

$$\begin{aligned} \frac{\Pi}{\delta(x)} + V_1\Pi^{\beta_1(x)} - I &= C_1\Pi^{\beta_1(x)} \\ \frac{1}{\delta(x)} + V_1\beta_1(x)\Pi^{\beta_1(x)-1} &= C_1\beta_1(x)\Pi^{\beta_1(x)-1} \end{aligned} \quad (39)$$

which yields the same trigger point as the one obtained under the pure price-cap system in equation (33)²²

$$\Pi_{PS}^*(x) \equiv \frac{\beta_1(x)}{\beta_1(x) - 1} \delta(x) I, \quad (40)$$

and

$$\begin{aligned} C_1 &= V_1 + \frac{1}{\beta_1(x)} \frac{\Pi_{PS}^*(x)^{1-\beta_1(x)}}{\delta(x)} = \\ &= \frac{\beta_2(x') - 1}{\beta_1(x) - \beta_2(x')} \frac{\delta(x') - \delta(x)}{\delta(x')\delta(x)} \tilde{\Pi}^{1-\beta_1(x)} + \frac{1}{\beta_1(x)} \frac{\Pi_{PS}^*(x)^{1-\beta_1(x)}}{\delta(x)} > 0. \end{aligned}$$

The equality between $\Pi_{PS}^*(x)$ and $\Pi_{PC}^*(x)$ establishes the following:

²²The computation of C_1 is not necessary for our purposes. However, it is easy to show that it is equal to

$$\begin{aligned} C_1 &= V_1 + \frac{1}{\beta_1(x)} \frac{\Pi_{PS}^*(x)^{1-\beta_1(x)}}{\delta(x)} = \\ &= \frac{\beta_2(x') - 1}{\beta_1(x) - \beta_2(x')} \frac{\delta(x') - \delta(x)}{\delta(x')\delta(x)} \tilde{\Pi}^{1-\beta_1(x)} + \frac{1}{\beta_1(x)} \frac{\Pi_{PS}^*(x)^{1-\beta_1(x)}}{\delta(x)} > 0. \end{aligned}$$

Proposition 5 (Neutrality of profit sharing in continuous time) *Consider a regulated monopolist which has to decide on an investment of a given amount. When demand is uncertain as modelled in (26) and the timing of investment is endogenous, correcting the RPI – x rule with a profit sharing element does not affect the timing of investment.*

Remark 4 *Inequality $\Pi_{PS}^*(x) < \tilde{\Pi}$ is necessary for the above solution to exist. This is a totally natural assumption in our setting. To see why, remember that $\Pi_{PS}^*(x)$ is associated to an investment time T_{PS}^* . Having $\Pi_{PS}^*(x) > \tilde{\Pi}$ would mean that investment is undertaken (i.e. production begins) when the profit level is already above $\tilde{\Pi}$. This implies that profit sharing intervenes since the moment of investment itself. But then the RPI – x scheme would start from a value of x already equal to x' . This obviously contradicts the definition of profit sharing, i.e. the idea that regulation starts with a given value of x , which is made more stringent at a later stage, in case profit goes beyond a certain level²³.*

The neutrality (indifference) result is an extension of Proposition 4, and can be explained as follows. Profit sharing is equivalent to equity participation by the consumers. Recall, in fact, that when $\Pi > \tilde{\Pi}$, a given part of the surplus is redistributed to the consumers. When instead $\Pi < \tilde{\Pi}$, consumers do not share the bad result. Using the real option approach we can thus say that the profit sharing device is equivalent to a case where consumers are endowed with a put option with strike price $\tilde{\Pi}$, written on the firm's profits. If, therefore, the firm's return drops below $\tilde{\Pi}$ (bad result), consumers sell their equity participation at zero price. Then, they will re-buy (at zero price) their participation when the firm faces a good result, namely when $\Pi > \tilde{\Pi}$.

To clarify this point, let us concentrate on the $(0, \tilde{\Pi})$ region (with $\Pi_{PS}^*(x) < \tilde{\Pi}$), and recall equations (37), (38) and the solution of C_1 . The negative term $V_1 \Pi^{\beta_1(x)}$ measures the value of the consumers' put option, which must be added to both the project value and the option function. This addition is necessary because, irrespective of whether the firm is waiting or producing, a worthy put option is owned by the consumers. Since $V_1 \Pi^{\beta_1(x)}$ enters both functions, the difference $[V_{PS}(\Pi, x) - O_{PS}(\Pi, x)]$ is independent of the switch level $\tilde{\Pi}$, thereby making the profit-sharing device neutral.

²³Otherwise, the trigger point would have to be computed in the region $(\tilde{\Pi}, \infty)$. A non-linear system of equations would then be obtained. In this case, $\Pi_{PS}^*(x)$ and C_1 would be the unknowns, but the solution for C_1 would have a different from the one obtained in the $(0, \tilde{\Pi})$ region. Moreover, given the non-linearity of the above equations, the system should be solved numerically.

Easy computations show that, for a given payoff, the higher the switch point the greater the value of V_1 . This implies that $\frac{\partial C_1}{\partial \tilde{\Pi}} = \frac{\partial V_1}{\partial \tilde{\Pi}} > 0$ (with $V_1 < 0$). Therefore, an increase in $\tilde{\Pi}$ raises both the value and the option function by the same amount, and vice versa. Finally, note that the higher the switch point $\tilde{\Pi}$ the lower the put option value ($-V_1 \Pi^{\beta_1(x)}$). Of course, for $\tilde{\Pi} \rightarrow \infty$, profit sharing vanishes, and the put option turns to be nil.

6 Incremental investment and cost reduction

So far, we have considered investments which are necessary to produce. A second category of investment is at least as relevant (and analyzed in the literature), i.e. cost-reducing investments. In this section we want to extend the previous framework to accommodate for the latter category, showing how the same results follow. To this end, we modify Assumption 1 in the following way:

- **Assumption 1'** (investment). Production requires a one-off investment of a given amount I_1 , that is undertaken at time 0. At any time $t \geq 0$, the firm has the possibility to invest a further amount I_2 , which reduces its variable cost.

Notice that the firm is not obliged to undertake the total amount $I_1 + I_2$ of investment immediately. Rather, it can wait until the current payoff is sufficiently high. By investing I_1 , the firm receives a per-period payoff Π and acquires an (American call) option to invest again. When it undertakes investment I_2 , variable profits increase.

The cost reduction can be modelled as a downward jump in operating costs with a contemporaneous upward jump in profits. For simplicity we assume that the after-investment profit margin $[p(t) - c'(t)]$ follows the same dynamics as $[p(t) - c(t)]$, namely

$$\Pi(t) = [p(t) - c'(t)] q(t) = \Psi(t) [p(t) - c(t)] q(t) \quad (41)$$

where $\Psi(t)$ is

$$\Psi(t) = \begin{cases} 1 & \text{for } t < T \\ \Psi \geq 1 & \text{for } t > T \end{cases} \quad (42)$$

and T is the optimal time of investment I_2 , to be determined²⁴. Notice that (42) captures the idea that the second investment reduces costs, but a similar

²⁴The assumption on the profits' upward jump implies that, after investing I_2 , they evolve according the following

$$\Psi[d\Pi(t)] = \Psi\Pi(t)[\alpha dt + \sigma dz].$$

story could be told if investment contributes to expand revenues even further.

6.1 Price cap

Let us now compute the firm's value function. Recall equation (30). When investment I_1 is undertaken, given the boundary condition $V_{PC}^1(0, x) = 0$, the value function has a standard solution

$$V_{PC}^1(\Pi, x) = \frac{\Pi}{\delta(x)} + E_1 \Pi^{\beta_1(x)} \quad (43)$$

Namely, the firm's project is given by the perpetual rent $\Pi/\delta(x)$ plus the term $E_1 \Pi^{\beta_1(x)}$, which measures the value of the option to invest. Thus, the firm may find it profitable to invest if the net present value of investment I_1 is positive, i.e. $V_{PC}^1(\Pi, x) > I_1$. Therefore, the firm may decide to enter this market even if the present value of its current profits, are negative, i.e. $\frac{\Pi}{\delta(x)} < I_1$.

When the firm invests I_2 , per-period profits jump upward and the firm's project becomes $V_{PC}^2(\Pi, x)$. Since, by assumption, there are no bubbles and condition $V_{PC}^2(0, x) = 0$ holds, the firm's value is simply a perpetual rent

$$V_{PC}^2(\Pi, x) = \frac{\Psi \Pi}{\delta(x)} > \frac{\Pi}{\delta(x)} \quad (44)$$

Given equations (43) and (44), one can compute the trigger point above which incremental investment is profitable. It is worth noting that the project's function $V_{PC}^1(\Pi, x)$ includes the opportunity cost of undertaking investment I_2 . On the one hand, in fact, investment I_2 increases profits. On the other hand, it entails the loss of the perpetual rent earned with investment I_1 and the exercise of the call option. Thus, investment is profitable if the firm's project is at least equal to the sum of the explicit cost I_2 and the opportunity cost $V_{PC}^1(\Pi, x)$. In this case, therefore, the *VMC* and *SPC* define a two-equation system

$$V_{PC}^1(\Pi, x) = V_{PC}^2(\Pi, x) - I_2$$

$$\frac{\partial V_{PC}^1(\Pi, x)}{\partial \Pi} = \frac{\partial V_{PC}^2(\Pi, x)}{\partial \Pi}$$

where the trigger point Π_{PC}^* and parameter E_1 are the unknowns. Solving

the system in the same way already employed yields²⁵

$$\Pi_{PC}^* = \frac{\beta_1(x)}{\beta_1(x) - 1} \frac{\delta(x)}{\Psi - 1} I_2 \quad (45)$$

6.2 Profit sharing

Let us now turn to the Profit Sharing rule. If $\Pi < \tilde{\Pi}$ the Brownian motion is the same as in the previous section. In the $(0, \tilde{\Pi})$ region, the firm's project is given by the perpetual rent $\Pi/\delta(x)$ plus a term $F_1 \Pi^{\beta_1(x)}$, which measures the value of the option to reduce costs. In the $(\tilde{\Pi}, \infty)$ region, the value function has the general form, namely it is given by the sum of $G_1 \Pi^{\beta_1(x')}$ and $G_2 \Pi^{\beta_2(x')}$ (with G_1 and G_2 to be determined). To sum up, the value function is

$$V_{PS}^1(\Pi, x) = \begin{cases} \frac{\Pi}{\delta(x)} + F_1 \Pi^{\beta_1(x)} & \text{if } \Pi < \tilde{\Pi} \\ \frac{\Pi}{\delta(x)} + G_1 \Pi^{\beta_1(x')} + G_2 \Pi^{\beta_2(x')} & \text{if } \Pi > \tilde{\Pi} \end{cases} \quad (46)$$

By equating the values and the derivatives of the two components of the option function at point $\Pi = \tilde{\Pi}$, we can compute G_1 and G_2 as functions of F_1 . It is possible to show²⁶ that $G_1 \propto F_1$ and $G_2 \propto F_1$.

Let us now turn to the value function after investing I_2 . The general solution of the value function is given by the sum of a perpetual rent, with discount rate $\delta(x)$, and a homogeneous (exponential) part. Again, to compute the value function, it is assumed that $V_{PS}^2(0, x) = 0$ (namely when Π is very small, the project is almost worthless) and that no speculative bubbles

²⁵The unknown parameter is equal to

$$E_1 = \frac{\Psi - 1}{\beta_1(x)\delta(x)} \cdot \Pi_{PC}^{*1-\beta_1(x)} > 0.$$

Notice that $\frac{\partial E_1}{\partial \Psi} > 0$, namely the greater the increase in profitability, the more valuable the option to reduce costs is.

²⁶The two-equation system obtained is:

$$\begin{cases} \frac{\tilde{\Pi}}{\delta(x)} + F_1 \tilde{\Pi}^{\beta_1(x)} = \frac{\tilde{\Pi}}{\delta(x)} + G_1 \tilde{\Pi}^{\beta_1(x')} + G_2 \tilde{\Pi}^{\beta_2(x')}, \\ \frac{1}{\delta(x)} + F_1 \beta_1(x) \tilde{\Pi}^{\beta_1(x)-1} = \frac{1}{\delta(x)} + G_1 \beta_1(x') \tilde{\Pi}^{\beta_1(x')-1}. \end{cases}$$

Solving for G_1 and G_2 , one obtains $G_1 \propto F_1$ and $G_2 \propto F_1$.

exist. The function thus reduces to

$$V_{PS}^2(\Pi, x) = \begin{cases} \frac{\Psi\Pi}{\delta(x)} + L_1\Pi^{\beta_1(x)} & \text{if } \Pi < \tilde{\Pi} \\ \frac{\Psi\Pi}{\delta(x')} + L_2\Pi^{\beta_2(x')} & \text{if } \Pi > \tilde{\Pi} \end{cases} \quad (47)$$

As shown in Appendix B, parameters L_1 and L_2 are equal to $L_1 = \Psi V_1 < 0$ and $L_2 = \Psi V_2 > 0$, respectively. Recalling equation (38), we can thus show that $V_{PS}^2(\Pi, x) = \Psi V_{PS}(\Pi, x)$. Like the one-off case, both L_1 and L_2 depend on the regulatory coefficients x and x' . In particular, $L_1\Pi^{\beta_1(x)}$ is negative and represents the present discounted value of future profit reduction due to the profit sharing (when Π goes beyond $\tilde{\Pi}$). $L_2\Pi^{\beta_2(x')}$ is positive and measures the present discounted value of the future increase in the profit participation when Π goes below $\tilde{\Pi}$ (in fact $\partial\Pi^{\beta_2(x')}/\partial\Pi < 0$).

As shown in the previous section, the firm's optimal investment timing is obtained by applying the *VMC* and *SPC*. Substituting equations (46) and (47) into the *VMC* and *SPC* one obtains the trigger point and the unknown parameter of the option function. In the $(0, \tilde{\Pi})$ region, these conditions lead to the following system

$$\begin{aligned} \frac{\Psi\Pi}{\delta(x)} + L_1\Pi^{\beta_1(x)} - I_2 &= \frac{\Pi}{\delta(x)} + F_1\Pi^{\beta_1(x)} \\ \frac{\Psi}{\delta(x)} + L_1\beta_1(x)\Pi^{\beta_1(x)-1} &= \frac{1}{\delta(x)} + F_1\beta_1(x)\Pi^{\beta_1(x)-1} \end{aligned}$$

which yields (45)²⁷

$$\Pi_{PS}^{*'} = \frac{\beta_1(x)}{\beta_1(x) - 1} \cdot \frac{\delta(x)}{\Psi - 1} \cdot I_2 \quad (48)$$

Comparing conditions (45) and (48), it appears that

$$\Pi_{PC}^{*'} = \Pi_{PS}^{*'}$$

This can be summarized in the following way:

²⁷Easy computations show that

$$F_1 = L_1 + \frac{1}{\beta_1(x)} \frac{\Pi_{PS}^{*'}(x)^{1-\beta_1(x)}}{\delta(x)}.$$

Proposition 6 (Neutrality of profit sharing on cost-reducing investment) *Consider a regulated monopolist which has to decide on a cost-reducing investment of a given amount. When demand is uncertain as modelled in (26) and the timing of cost-reducing investment is endogenous, correcting the $RPI - x$ rule with a profit sharing element does not affect the timing of investment.*

This result simply reflects the parallel between loosening the price-cap and reducing costs. Both events represent “good news” and the endogeneity of the decision to reduce costs does not affect the neutrality result developed in the previous section.

7 Conclusions

Relative to the existing literature, which implicitly assumed reversible investment by regulated firms, our results appear significantly different. While current literature indicates that profit-sharing has a negative effect on investment decisions, our paper shows that this is not true. What makes a difference is the introduction of two fairly realistic assumptions: investment irreversibility and the firm’s ability to decide when undertaking it. This implies that the firm is endowed with a call option to delay, which expires when investment is undertaken. We have thus shown that a profit-sharing device reduces both the value of the project and the value of the option to wait by the same amount. According to the Bad News Principle, therefore, no additional distortion is introduced, with respect to price cap.

Moreover, it is worth noting that one of the regulators’ main targets is the rent extraction *per se*. After all, the very notion of profit-sharing comes from the idea that a scheme which yields an excessively imbalanced distribution of rents is undesirable, and the rate of return regulation scheme still prevailing in a large part of the US is based on the idea that restraining monopoly rents is a goal by itself.

What we show here, therefore, has an important policy implication. Since profit sharing has a greater ability to raise rents than price cap, but does not cause any additional distortion, it is possible to extract the *same* amount of rents with a *lower* value of x . Given the amount of rents extracted from the monopolist, under profit sharing the trigger point above which investment is profitable is thus lower than under a pure price cap; in other words, investment is undertaken earlier than under the pure price cap regime.

A question that still remains open, is the role that political uncertainty plays. One of the reasons why profit sharing has been proposed is that regulatory authorities are unable to commit not to intervene if the regulated

firm's profits turn out to be very high. By automatically curbing profits, a "sliding scale" device might decrease the regulator's incentive to intervene, thereby increasing the firm's incentive to invest. To what extent this argument really applies in the set-up we have developed, is a question mark that we leave for future investigation.

8 Appendices

8.1 Appendix A

In this Appendix we first compute parameters B_1 and B_2 as functions of C_1 . Then, parameters V_1 and V_2 are obtained.

Let us start with parameters B_1 and B_2 . Recall equation (38). Using the VMC and SPC at point $\Pi = \tilde{\Pi}$ (namely, equating the values and the derivatives of the two components of the option function at point $\Pi = \tilde{\Pi}$) we obtain

$$\begin{aligned} C_1 \tilde{\Pi}^{\beta_1(x)} &= B_1 \tilde{\Pi}^{\beta_1(x')} + B_2 \tilde{\Pi}^{\beta_2(x')} \\ C_1 \beta_1(x) \tilde{\Pi}^{\beta_1(x)-1} &= B_1 \beta_1(x') \tilde{\Pi}^{\beta_1(x')-1} + B_2 \beta_2(x') \tilde{\Pi}^{\beta_2(x')-1} \end{aligned}$$

Easy computations show that

$$B_1 = \frac{\beta_1(x) - \beta_2(x')}{\beta_1(x') - \beta_2(x')} \cdot C_1$$

$$B_2 = \frac{\beta_1(x') - \beta_1(x)}{\beta_1(x') - \beta_2(x')} \cdot \tilde{\Pi}^{[\beta_1(x) - \beta_2(x')]} \cdot C_1$$

Since $\beta_1(x') > \beta_1(x)$, we have: $B_1 \propto C_1$ and $B_2 \propto C_1$.

Let us now turn to the value function. Using the VMC and SPC at $\Pi = \tilde{\Pi}$ we obtain a system with two equations and two unknowns (V_1 and V_2):

$$\begin{aligned} \frac{\tilde{\Pi}}{\delta(x)} + V_1 \tilde{\Pi}^{\beta_1(x)} &= \frac{\tilde{\Pi}}{\delta(x')} + V_2 \tilde{\Pi}^{\beta_2(x')}, \\ \frac{1}{\delta(x)} + V_1 \beta_1(x) \tilde{\Pi}^{\beta_1(x)-1} &= \frac{1}{\delta(x')} + V_2 \beta_2(x') \tilde{\Pi}^{[\beta_2(x')-1]}. \end{aligned}$$

Solving for V_1 and V_2 yields

$$\begin{aligned} V_1 &= \frac{\beta_2(x')-1}{\beta_1(x)-\beta_2(x')} \cdot \frac{\delta(x')-\delta(x)}{\delta(x)\delta(x')} \cdot \tilde{\Pi}^{[1-\beta_1(x)]} < 0 \\ V_2 &= \frac{\beta_1(x)-1}{\beta_1(x)-\beta_2(x')} \cdot \frac{\delta(x')-\delta(x)}{\delta(x)\delta(x')} \cdot \tilde{\Pi}^{[1-\beta_2(x')]} > 0 \end{aligned}$$

Note that, in the region $(0, \tilde{\Pi})$, C_1 must be positive in order to have a positive option function. Otherwise, it would not have any economic meaning. Recall that

$$C_1 = V_1 + \frac{1}{\beta_1(x)} \cdot \frac{\Pi_{PS}^{*1-\beta_1(x)}}{\delta(x)}$$

Substituting the value of V_1 , one thus obtains

$$C_1 = \frac{1}{\beta_1(x)} \cdot \frac{\Pi_{PS}^*(x)^{1-\beta_1(x)}}{d} \cdot \left\{ 1 + \frac{\beta_1(x) [\beta_2(x') - 1] \delta(x') - \delta(x)}{\beta_1(x) - \beta_2(x')} \frac{\delta(x)}{\delta(x')} \left(\frac{\Pi_{PS}^*(x)}{\tilde{\Pi}} \right)^{[\beta_1(x)-1]} \right\}$$

8.2 Appendix B

In this Appendix we compute L_1 and L_2 . Stitching together the two components of the value function (47) at the switch point $\Pi = \tilde{\Pi}$, one obtains parameters

$$\begin{aligned} \frac{\Psi \tilde{\Pi}}{\delta(x)} + L_1 \tilde{\Pi}^{\beta_1(x)} &= \frac{\Psi \tilde{\Pi}}{\delta(x')} + L_2 \tilde{\Pi}^{\beta_2(x')} \\ \frac{\Psi}{\delta(x)} + L_1 \beta_1(x) \tilde{\Pi}^{[\beta_1(x)-1]} &= \frac{\Psi}{\delta(x')} + L_2 \beta_2(x') \tilde{\Pi}^{[\beta_2(x')-1]} \end{aligned}$$

Solving for L_1 and L_2 yields

$$\begin{aligned} L_1 &= \Psi V_1 = \Psi \frac{\beta_2(x')-1}{\beta_1(x)-\beta_2(x')} \cdot \frac{\delta(x')-\delta(x)}{\delta(x)\delta(x')} \cdot \tilde{\Pi}^{[1-\beta_1(x)]} < 0 \\ L_2 &= \Psi V_2 = \frac{\beta_1(x)-1}{\beta_1(x)-\beta_2(x')} \cdot \frac{\delta(x')-\delta(x)}{\delta(x)\delta(x')} \cdot \tilde{\Pi}^{[1-\beta_2(x')]} > 0 \end{aligned}$$

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