

Brunner, Johann K.; Pech, Susanne

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Johann K. Brunner
Susanne Pech

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CESifo

Center for Economic Studies & Ifo Institute for Economic Research
Poschingerstr. 5, 81679 Munich, Germany

Tel.: +49 (89) 9224-1410

Fax: +49 (89) 9224-1409

e-mail: office@CESifo.de



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Abstract

This paper investigates the effect of adverse selection and price competition on the private annuity market in a model with two retirement periods. In this framework annuity companies can offer contracts with different payoffs over the periods of retirement. Varying the time structure of the payoffs affects annuity demand and welfare of individuals with low and high life expectancy in different ways. By this, annuity purchasers can be separated according to their survival probabilities. Our main finding is that a Nash-Cournot equilibrium may not exist; if one exists, it will be a separating equilibrium. On the other hand, even if a separating equilibrium does not exist, a Wilson pooling equilibrium exists

Keywords: Annuity markets, adverse selection, uncertain lifetimes, pooling equilibrium, separating equilibrium

JEL Classification: D82, D91, G22

Johann K. Brunner
University of Linz
Department of Economics
Altenberger Strasse 69
4040 Linz
Austria
e-mail: Jk.brunner@jk.uni-linz.ac.at

Susanne Pech
University of Linz
Department of Economics
Altenberger Strasse 69
4040 Linz
Austria
e-mail: susanne.pech@jk.uni-linz.ac.at

1. Introduction

Private life-annuity markets are frequently recognized as being weak. That is, less life-annuities are demanded than one could expect, given the need to insure against uncertainty about the duration of life, in order to smooth consumption appropriately over one's lifetime.

To the extent that the low demand is explained by a bequest motive or by the existence of a public pension system, the weakness is not attributed to an intrinsic problem of this market. However, there is a further reason put forward in the literature, namely asymmetric information which leads to adverse selection: The fact that individuals have more information about their survival probability than annuity companies induces higher annuity demand of persons with long life expectancy, which in turn drives down the rate of return on annuities below the rate corresponding to the average probability of survival.¹ As a consequence of this phenomenon, a loss of welfare arises for persons who cannot buy an appropriate annuity contract. This shortcoming of the annuity market is supposed to become increasingly important, because in many countries the existing public pension system, organized according to the pay-as-you-go method, is expected to allow only a reduced replacement-ratio in the future, hence increased private insurance will be required.

In the present paper we point at a further consequence of the asymmetric information problem, in addition to the adverse-selection problem described so far: The time structure of the payoffs matters. Individuals with low life expectancy will put less weight on the payment they may not receive in the last period of life than individuals with high life expectancy do. This fact can be used by firms to offer annuity contracts which are favourable for low-risk individuals but not for high-risk individuals.² We show in a theoretical model that this fact has important consequences on the functioning of the annuity market.

In the model usually employed for the analysis of annuity markets (see Pauly (1974), Abel (1986) and Walliser (1998)), there is one period of retirement, and there are two groups of individuals with differing life expectancy. Competition takes place via prices (i.e. via the rate of return, that is the pension payment per unit of annuity), which are fixed by the firms.

¹ Empirical evidence suggests that none of these three reasons alone (but only the interaction of adverse selection, public pension system and bequest motives) can explain the weakness of the market. See, e.g., Friedman and Warshawsky (1988, 1990), Walliser (1998), Mitchell et al. (1999).

² In a recent empirical study of the annuity market in the U.K., Finkelstein and Poterba (1999) present some evidence that long living individuals indeed prefer contracts with payoffs increasing over time in nominal terms compared to contracts with constant nominal payoffs.

Individuals can buy as many annuities as they want. As it is well-known, in this framework only a pooling equilibrium is possible, where all individuals receive the same rate of return.

We extend this model by introducing two periods of retirement and by assuming that the payoffs need not be the same in both periods. This implies that contracts are characterized by two prices, set by the firms. As already noted above, the important aspect in this extended model is that annuity demand as well as welfare of the individuals are sensitive with respect to the time structure of the payoffs. This makes it possible for firms to separate individuals according to their survival probabilities. It turns out that in such a market no Nash-Cournot equilibrium may exist. If one exists, it will be a separating equilibrium. These findings can be interpreted as a further explanation of weakness of annuity markets mentioned above.³

The Nash-Cournot equilibrium in insurance markets is studied by Rothschild and Stiglitz (1976). In their framework firms offer a number of different contracts which specify both a price and a quantity. Individuals who prefer a higher quantity, are willing to pay a higher price for it. A prerequisite for the existence of price and quantity competition is that individuals can buy at most one contract, which may be a reasonable assumption for some insurance markets, e.g. insurance against accidents, but seems difficult to apply to the annuity market.⁴ Consequently, in our model individuals are free to buy as many annuities as they want. Separation becomes possible because firms can fix two prices instead of a price and a quantity.

As a potential answer to the question what happens in an insurance market, if no Nash-Cournot-equilibrium exists, Wilson (1977) introduced a different equilibrium concept which is based on specific beliefs of firms concerning the reaction of other firms to new contract offers. We show that a Wilson equilibrium always exists in our model.

Our paper proceeds as follows: In Section 2 we introduce the basic model of annuity demand under asymmetric information with two periods of retirement. We analyze the effect of a variation in the time structure of the payoffs on annuity demand and on welfare of an

³ A related explanation for the weakness of the annuity market is offered by Townley and Boadway (1987). They also quit the assumption of a single period of retirement but model the duration of retirement in continuous time. They allow annuity companies to offer contracts characterized by two parameters, namely the term, i.e. the duration of the contract, and the price (i.e. the constant rate of return). Within this framework they show that no equilibrium may exist, if it exists, it is either a pooling equilibrium or a separating equilibrium.

⁴ Eckstein, Eichenbaum and Peled (1985) make indeed this assumption for the annuity market and derive the same results as Rothschild and Stiglitz (1976).

individual under uncertain lifetime. In Section 3 we turn to the investigation of equilibria. Section 4 contains concluding remarks.

2. Annuity demand in a model with two periods of retirement

2.1 The basic model with asymmetric information

Consider an economy with N individuals who live for a maximum of three periods $t = 0, 1, 2$. In the working period $t = 0$ individual i earns a fixed labour income w , spends an amount A^i on annuities and consumes an amount c_0^i . This gives the budget equation for period 0:

$$c_0^i = w - A^i. \quad (2.1)$$

The individuals retire at the end of period 0. Through the purchase of annuities they make provision for future consumption in the periods of retirement $t = 1, 2$. An annuity contract is characterized by the payoffs (q_1, q_2) : An annuity $A^i = 1$ pays q_t units of money to the individual in the retirement periods $t = 1, 2$, if she survives. In order to concentrate on the design of the annuity contracts, we assume that neither an interest-bearing saving instrument nor a public pension system exist. This does not affect the qualitative results and simplifies the analysis. Hence, for individual i the budget equations for the two retirement periods are

$$c_1^i = q_1 A^i, \quad (2.2)$$

$$c_2^i = q_2 A^i. \quad (2.3)$$

Survival to period $t = 1$ is uncertain and occurs with probability π_1^i , $0 < \pi_1^i < 1$. In the same way, given that an individual is alive in period 1, survival to period 2 occurs with probability π_2^i , $0 < \pi_2^i < 1$. Each individual decides on her consumption plan over the uncertain duration of her retirement by maximizing expected utility from a time-separable utility function U^i ,

$$U^i = (1 - \pi_1^i)u(c_0^i) + \pi_1^i(1 - \pi_2^i)(u(c_0^i) + u(c_1^i)) + \pi_1^i\pi_2^i(u(c_0^i) + u(c_1^i) + u(c_2^i)), \quad (2.4)$$

subject to conditions (2.1), (2.2) and (2.3). In (2.4) the function $u(c_t^i)$ describes utility of consumption per period, where we assume $u'(c_t^i) > 0$, $u''(c_t^i) < 0$ and $\lim_{c \rightarrow 0} u'(c) = \infty$. Notice

that the specification in (2.4) embodies that the individuals have no bequest motive and do not discount future consumption for any reason other than risk aversion. (2.4) can be reduced to

$$U^i = u(c_0^i) + \pi_1^i u(c_1^i) + \pi_1^i \pi_2^i u(c_2^i) . \quad (2.4')$$

Inserting (2.1), (2.2) and (2.3) into (2.4') and differentiating with respect to A^i yields the first order condition of this maximization problem as

$$-u'(c_0^i) + \pi_1^i q_1 u'(c_1^i) + \pi_1^i \pi_2^i q_2 u'(c_2^i) = 0 . \quad (2.5)$$

From (2.2) and (2.3) we know that c_1^i and c_2^i depending on q_1 and q_2 . Let $A^i(q_1, q_2)$ be the annuity demand determined by (2.5), for given (q_1, q_2) .

From now on we assume that the otherwise identical individuals are divided into two groups $i = L, H$, characterized by different risks of a long life, i.e. by different probabilities of survival $\pi_t^H > \pi_t^L$ for $t = 1, 2$. Let γ and $(1 - \gamma)$ denote the share of the high-risk and low-risk individuals, resp., with $0 < \gamma < 1$. The probabilities π_t^i and γ are public information, known by the annuity companies. But it is the private information for each individual to know her type, i.e. her probability of survival. As a consequence, there is an adverse-selection problem in the annuity market. This is illustrated by the following lemma, which shows that high-risk individuals buy more annuities than low-risk individuals, given any contract (q_1, q_2) .

Lemma 1: For any contract (q_1, q_2) an individual with high survival probabilities will demand a larger quantity of annuities than an individual with low survival probabilities, i.e. $A^H(q_1, q_2) > A^L(q_1, q_2)$.

Proof: Annuity demand $A^L(q_1, q_2)$ of a type-L individual, given that a contract (q_1, q_2) is offered, is determined by the first order condition (2.5) for $i = L$. We consider the derivative of type H's expected utility at $A^L(q_1, q_2)$, i.e.

$$-u'(c_0^L) + \pi_1^H q_1 u'(c_1^L) + \pi_1^H \pi_2^H q_2 u'(c_2^L) . \quad (2.6)$$

Using (2.5) for $i = L$ and $\pi_1^H > \pi_1^L$, $\pi_2^H > \pi_2^L$, it follows that (2.6) is positive, which means that utility of a type-H individual will rise, if she increases her annuity demand above $A^L(q_1, q_2)$. By

doing so, she reduces consumption in period 0 (marginal utility of consumption will rise due to concavity of the instantaneous utility function u), but increases consumption in periods 1 and 2 (marginal utility of consumption will fall). Altogether, optimal annuity demand $A^H(q_1, q_2)$ of a type-H individual must be above $A^L(q_1, q_2)$. Q.E.D.

2.2 Separating and pooling contracts

An annuity contract is said to be *individually fair*, if expected payoffs equal its price, i.e. if (q_1, q_2) fulfill

$$1 - \pi_1^i q_1^i - \pi_1^i \pi_2^i q_2^i = 0, \quad i = L, H. \quad (2.7)$$

Obviously, this implies that the annuity companies make zero expected profits, given that identical individuals buy these contracts. We show that among all individually fair annuity contracts the one with equal payoffs is preferred.

Lemma 2: Among all individually fair contracts, the one with $q_1 = q_2$ is preferred.

Proof: We maximize lifetime utility (2.4') with respect to q_1 and q_2 , subject to (2.7). Using (2.2) and (2.3), the first-order conditions of this problem are

$$\pi_1^i A^i u'(c_1^i) + \lambda \pi_1^i = 0, \quad (2.8a)$$

$$\pi_1^i \pi_2^i A^i u'(c_2^i) + \lambda \pi_1^i \pi_2^i = 0, \quad (2.8b)$$

where λ is the Lagrange multiplier associated with the constraint (2.7). From (2.8a) and (2.8b), we find that maximization requires $u'(c_1^i) = u'(c_2^i)$, which implies $q_1 = q_2$ for any arbitrarily given A^i . Q.E.D.

In the following, we indicate the most preferred individually fair contract by $(\hat{q}_1^i, \hat{q}_2^i)$, where $\hat{q}_1^i = \hat{q}_2^i = 1 / (\pi_1^i + \pi_1^i \pi_2^i)$ (see (2.7)). Note that with $(\hat{q}_1^i, \hat{q}_2^i)$ an individual i chooses not only the same level of consumption in the two retirement periods 1 and 2, but also in her working period 0. This can be seen from the fact that, as the consumption levels are the same in periods 1 and 2, (2.5) reduces to $u'(c_0^i) = \hat{q}_1^i (\pi_1^i + \pi_1^i \pi_2^i) u'(c_1^i)$, which implies $c_0^i = c_1^i = c_2^i$.

From our assumption $\pi_t^L < \pi_t^H$, $t = 1, 2$, it follows that individual fairness for each group can be fulfilled only with two *separate contracts*. In contrast, a contract (q_1, q_2) which is bought by both groups, is called a *pooling contract*. For such a contract, the zero-profit condition reads (for shortness we use A^i instead of $A^i(q_1, q_2)$)

$$(1 - \gamma)A^L(1 - q_1\pi_1^L - q_2\pi_1^L\pi_2^L) + \gamma A^H(1 - q_1\pi_1^H - q_2\pi_1^H\pi_2^H) = 0, \quad (2.9)$$

which can also be written as

$$1 + \rho - q_1(\pi_1^L + \rho\pi_1^H) - q_2(\pi_1^L\pi_2^L + \rho\pi_1^H\pi_2^H) = 0, \quad (2.9')$$

where ρ is defined by $\rho(q_1, q_2) \equiv (\gamma A^H(q_1, q_2)) / ((1 - \gamma)A^L(q_1, q_2))$, that is the ratio of annuity demand of both groups. Note that ρ depends on (q_1, q_2) , but for shortness, we usually do not indicate this dependency. Our assumption on the survival probabilities implies

$$1 - q_1\pi_1^L - q_2\pi_1^L\pi_2^L > 1 - q_1\pi_1^H - q_2\pi_1^H\pi_2^H. \quad (2.10)$$

It follows that the LHS in (2.10) is positive and the RHS is negative, otherwise the LHS in (2.9) would be non-zero. From this we conclude that for the low-risk individuals expected returns from a pooling contract are lower than required for individual fairness, while for the high-risk individuals they are higher.

In the Lemmas 3 and 4 below, we consider a pooling contract and investigate the effect of a marginal change in the payoffs on expected utility and on annuity demand of an individual of type $i = L, H$. Clearly, if q_1 (or q_2) is increased alone, then both groups profit and buy more annuities. However, such an increase would produce a loss for the annuity companies. Hence, the interesting case is when q_1 is increased at the expense of q_2 (or vice versa), such that the zero-profit condition (2.9) remains fulfilled.

Starting from a contract with $q_1 = q_2$, we characterize the first-round effect on expected utility and on annuity demand of a marginal increase of q_1 , when the associated change of q_2 , such that (2.9') remains fulfilled, is calculated under the assumption of a constant ratio ρ of annuity demand of the two groups.

Lemma 3: Consider a contract with $q_1 = q_2$ which together with $\bar{A}(q_1, q_2)$, $i = L, H$ fulfills the zero-profit condition (2.9'). A marginal increase of q_1 (and thus a marginal decrease of q_2), where (2.9') for fixed p remains fulfilled, makes an individual with high survival probabilities worse off and an individual of low survival probabilities better off.

Proof: Substituting (2.2) and (2.3) into (2.4') we get (apply the Envelope Theorem)

$$\frac{\partial U^i}{\partial q_1} = \pi_1^i A^i u'(c_1^i) + \pi_1^i \pi_2^i A^i \frac{\partial q_2}{\partial q_1} u'(c_2^i).$$

Making use of the fact that $c_1^i = c_2^i$, given that $q_1 = q_2$, it follows

$$\left. \frac{\partial U^i}{\partial q_1} \right|_{q_1 = q_2} = \pi_1^i A^i u'(c_1^i) \left(1 + \pi_2^i \frac{\partial q_2}{\partial q_1} \right). \quad (2.11)$$

Implicit differentiation of the zero-profit condition (2.9') gives

$$\pi_2^i \frac{\partial q_2}{\partial q_1} = -\pi_2^i \frac{\pi_1^L + p\pi_1^H}{\pi_1^L \pi_2^L + p\pi_1^H \pi_2^H}. \quad (2.12)$$

It is straightforward to see that the RHS of (2.12) is smaller than -1 for $i = H$, and greater than -1 for $i = L$, i.e. $\pi_2^H \partial q_2 / \partial q_1 < -1$ and $\pi_2^L \partial q_2 / \partial q_1 > -1$. As a consequence, the RHS in (2.11) is negative for $i = H$ and positive for $i = L$, which proves the Lemma. Q.E.D.

The foregoing Lemma describes the first-round effect, which is responsible for the negative result concerning the existence of a pooling contract in equilibrium (see Section 3.1). As one expects, an individual with low survival probabilities prefers a pooling contract (q_1, q_2) with $q_1 > q_2$, compared to a contract that offers her equal payoffs, while the opposite holds for an individual with high survival probabilities. Thus, the annuity companies have an incentive to design separate contracts for the two groups.

The intuitive reason why a low-risk individual finds a shift of consumption from period 2 to period 1 attractive (starting from $q_1 = q_2$) can be explained as follows: If q_1 is increased by one, q_2 is decreased by $|\partial q_2 / \partial q_1|$, and this decrease is weighted by the individual probability of survival π_2^L . Since with a pooling contract the associated decrease of q_2 goes more to the expense of the high-risk individuals, it turns out from (2.9') that $|\partial q_2 / \partial q_1| < 1/\pi_2^L$ (for

constant ρ). As a result, the expected loss in period 2, $\pi_2^L |\partial q_2 / \partial q_1|$ is lower than one and type-L individuals profit from a shift towards increasing q_1 . (Note that due to $q_1 = q_2$, marginal utility is equal in both periods.) By the same reasoning type-H individuals, who expect to live longer, are better off by a shift towards reducing q_1 .

Remark: Inspection of the proof of the forgoing Lemma shows that an increase of q_1 at the expense of q_2 improves welfare of low-risk individuals, if $c_1^L \leq c_2^L$. It follows that their most preferred pooling contract exhibits $c_1^L > c_2^L$, i.e. $q_1 > q_2$. By similar reasoning one finds that the most preferred pooling contract for the high-risk individuals exhibits $c_1^H < c_2^H$, i.e. $q_1 < q_2$.

The next Lemma characterizes the effect of a marginal change of q (and q_2) on annuity demand. Let R be the Arrow-Pratt coefficient of relative risk aversion, $R = -c_t^i u''(c_t^i) / u'(c_t^i)$.

Lemma 4: Consider a contract with $q_1 = q_2$ which together with $A(q_1, q_2)$, $i = L, H$, fulfills the zero-profit condition (2.9'). The effect of a marginal increase in q_1 on the annuity demand of each individual $i = L, H$, where (2.9') for fixed ρ remains fulfilled, depends on the relative risk aversion in the following way:

- (i) If $R < 1$, then $\left. \frac{dA^H}{dq_1} \right|_{q_1 = q_2} < 0$ and $\left. \frac{dA^L}{dq_1} \right|_{q_1 = q_2} > 0$.
- (ii) If $R = 1$, then $\left. \frac{dA^H}{dq_1} \right|_{q_1 = q_2} = 0$ and $\left. \frac{dA^L}{dq_1} \right|_{q_1 = q_2} = 0$.
- (iii) If $R > 1$, then $\left. \frac{dA^H}{dq_1} \right|_{q_1 = q_2} > 0$ and $\left. \frac{dA^L}{dq_1} \right|_{q_1 = q_2} < 0$.

Proof: dA^i / dq_1 is determined by implicit differentiation of the first-order condition for annuity demand, $\partial U^i / \partial A^i = 0$, with respect to q_1 as

$$\frac{dA^i}{dq_1} = - \frac{\partial^2 U^i / \partial A^i \partial q_1}{\partial^2 U^i / \partial^2 A^i}. \quad (2.13)$$

Since the denominator of the RHS of (2.13) is negative due to the second-order condition of the maximization problem, dA^i / dq_1 has the same sign as the numerator of the RHS of (2.13).

Substituting (2.1), (2.2) and (2.3) into (2.4') we obtain

$$\frac{\partial^2 U^i}{\partial A^i \partial q_1} = \pi_1^i (u'(c_1^i) + q_1 A^i u''(c_1^i)) + \pi_1^i \pi_2^i \frac{\partial q_2}{\partial q_1} (u'(c_2^i) + q_2 A^i u''(c_2^i)). \quad (2.14)$$

Using the fact that $c_1^i = c_2^i$ for $q_1 = q_2$ and substituting R into (2.14), gives

$$\left. \frac{\partial^2 U^i}{\partial A^i \partial q_1} \right|_{q_1 = q_2} = \pi_1^i (1 + \pi_2^i \frac{\partial q_2}{\partial q_1}) u'(c_1^i) (1 - R). \quad (2.15)$$

If $R = 1$, then (2.15) and thus (2.13) are zero for individuals of both types $i = L, H$. Otherwise we determine, as in the proof of Lemma 3, $\partial q_2 / \partial q_1$ from the zero-profit condition (2.9') and find that $\pi_2^H \partial q_2 / \partial q_1 < -1$ and $\pi_2^L \partial q_2 / \partial q_1 > -1$. Thus, given that $R < 1$, (2.15) is negative for $i = H$ and positive for $i = L$. The opposite is true for $R > 1$. Q.E.D.

This result follows from the fact that, per definition, the effect of an increase of q on $q_1 u'(q_1 A^i)$, i.e. on the marginal utility of A^i in period 1, can be written as $(1 - R)$, and the same applies to period 2 (note that $q_1 = q_2$, initially). Hence, whether an increase of q_1 increases or decreases expected marginal utility of A^i (in both retirement periods together) depends on $(1 + \pi_2^i \partial q_2 / \partial q_1) (1 - R)$. As was argued above, the first term is positive for $i = L$ and negative for $i = H$. Finally, in order to see the effect on annuity demand, one concludes easily that an increase (decrease) of expected marginal utility of A^i in both periods of retirement means that demand for annuities is raised (reduced, resp.).

3. Equilibria

Introducing two instead of one retirement period in the model allows annuity companies to offer contracts which differ in the division of the payoffs over time. In this section it is shown that this implies the possibility of a separating equilibrium, which means that annuity companies separate individuals according to their survival probabilities. To obtain this result we make use of the well-known concept of a Nash-Cournot equilibrium, which was studied by Rothschild and Stiglitz (1976) in the context of insurance markets. Our result is in contrast to studies considering one period of retirement only, which find that under price competition there will be a pooling equilibrium. In Subsection 3.3 we extend the analysis by introducing

the concept of the Wilson (1977) equilibrium, where it is assumed that firms anticipate reactions of the other firms to new contract offers, viz. that they will withdraw unprofitable existing contracts. Since these expectations make a new contract offer less attractive, we find that in this setting a pooling equilibrium exists, even if a separating equilibrium does not exist.

3.1 The non-existence of a pooling equilibrium

We call a contract (q_1, q_2) a *pooling equilibrium*, if together with $A^i(q_1, q_2)$, $i = L, H$, the zero-profit condition (2.9) is fulfilled and if no other contract exists, which is preferred to (q_1, q_2) by at least one group $i \in \{L, H\}$ and which allows a nonnegative profit. Our main result is that in general no pooling equilibrium exists. As a preparation we show that a pooling contract (q_1, q_2) , which fulfills the zero-profit condition (2.9), produces positive profits, if it is bought only by low-risk individuals. By continuity this is true for contracts with payoffs close to (q_1, q_2) as well.

Lemma 5: Let (q_1, q_2) be a pooling contract which together with $A^i(q_1, q_2)$, $i = L, H$, fulfills the zero-profit condition (2.9). Any contract $(q_1 + \delta q_1, q_2 + \delta q_2)$, which is close enough to (q_1, q_2) and which is chosen only by group L (i.e., $A^H = 0$) allows a nonnegative profit.

Proof: We have already argued that with a pooling contract (q_1, q_2) the low-risk individuals receive less expected returns than required for individual fairness (see the considerations following (2.10)). This in turn means that the profit for an insurance company is positive, given that only this group chooses the contract (q_1, q_2) .

By continuity, this holds for any contract $(q_1 + \delta q_1, q_2 + \delta q_2)$ in the neighbourhood of (q_1, q_2) .

Q.E.D.

We now introduce a further assumption on U^i , in addition to strict concavity of the instantaneous utility function u . Let indirect utility $U^i(q_1, q_2)$ for any contract (q_1, q_2) be defined in the usual way as utility attained with annuity demand $A^i(q_1, q_2)$. We assume that indifference curves in the (q_1, q_2) -space satisfy the "single-crossing" condition

$$-\frac{\partial U^L / \partial q_1}{\partial U^L / \partial q_2} < -\frac{\partial U^H / \partial q_1}{\partial U^H / \partial q_2} \quad \text{for all } (q_1, q_2). \quad (3.1)$$

This condition, which is familiar from other models with asymmetric information, requires that the slope of an indifference of a low-risk individual is always steeper than that of a high-risk individual. Hence, indifference curves of the two groups can cross only once. Using the Envelope Theorem, (3.1) reduces to $u'(q_1 A^L)/(\pi_2^L u'(q_2 A^L)) > u'(q_1 A^H)/(\pi_2^H u'(q_2 A^H))$, and one observes that the condition is certainly fulfilled for the logarithmic or for any isoelastic utility function u , as $\pi_2^L < \pi_2^H$. Single-crossing is needed for a concise formulation of Proposition 1 only; in the remark after the proof the significance of the condition will be discussed thoroughly, and it will be argued that in general the Proposition holds without this assumption.

Proposition 1: No pooling equilibrium exists, given the single-crossing condition (3.1).

Proof: Let some contract (q_1, q_2) with associated $\dot{A}(q_1, q_2)$, $i = L, H$, be given, such that the zero-profit condition (2.9) is fulfilled. We find the effect δU^i of a marginal change $(\delta q_1, \delta q_2)$ of the contract on group i 's utility as

$$\delta U^i = \frac{\partial U^i}{\partial q_1} \delta q_1 + \frac{\partial U^i}{\partial q_2} \delta q_2, \quad i = L, H, \quad (3.2)$$

The single-crossing condition implies that the RHS's of the two equations (3.2) are linearly independent (i.e. there is no k such that $\partial U^L / \partial q_1 = k \partial U^H / \partial q_1$ and $\partial U^L / \partial q_2 = k \partial U^H / \partial q_2$), hence the two equations (3.2) have a unique solution. Choosing some $\delta U^L > 0$, $\delta U^H < 0$ and solving (3.2) for $\delta q_1, \delta q_2$, one finds a new contract $(q_1 + \delta q_1, q_2 + \delta q_2)$, which is preferred by group L , but not by group H . By the foregoing Lemma, it also allows a non-negative profit. (As δU^L and δU^H can be chosen arbitrarily close to zero, δq_1 and δq_2 can be taken as arbitrarily close to zero as well.) Hence (q_1, q_2) is not a pooling equilibrium.

Q.E.D.

This result can be illustrated in a diagram where the payoffs q_1 and q_2 are drawn on the axis (see Figure 1). The dashed line ZP denotes the zero-profit condition (2.9) for a pooling contract, with slope $-(\pi_1^L + \rho \pi_1^H)/(\pi_1^L \pi_2^L + \rho \pi_1^H \pi_2^H)$, where ρ depends on (q_1, q_2) . Consider any contract (q_1, q_2) fulfilling (2.9), i.e. any point on ZP . Due to the single-crossing condition the slope of the indifference curve U^L corresponding to the low-risk group is steeper than that of U^H , the indifference curve of the high-risk group. Therefore one can find a contract $(q_1 + \delta q_1, q_2 + \delta q_2)$, close to (q_1, q_2) , which is preferred by the low-risk individuals only - and is,

therefore, profitable for the annuity companies, as Lemma 5 tells us. Hence (q_1, q_2) does not represent a pooling equilibrium.

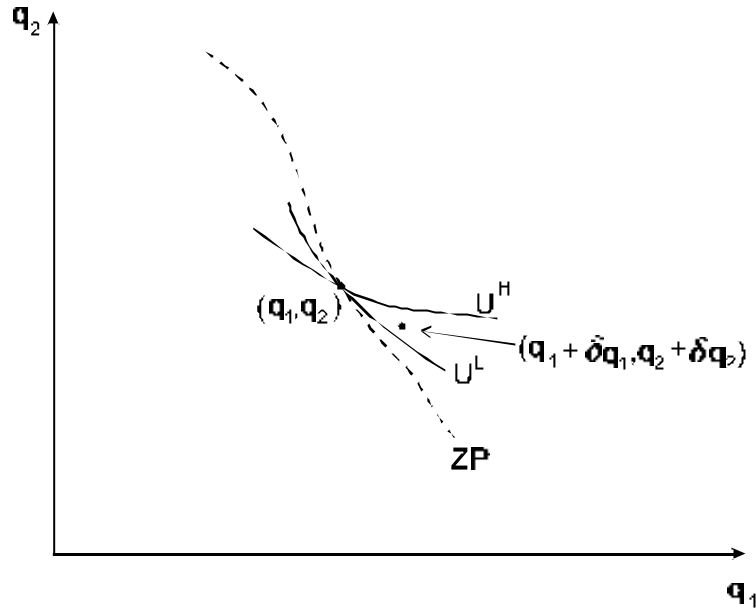


Figure 1

By means of Figure 1 the significance of the single-crossing condition can be discussed. One observes immediately that the result of Proposition 1 certainly holds as long as the slopes of U^L and U^H differ in (q_1, q_2) , independently of which one is steeper. Even if U^L and U^H have the same slope, the result holds, given that the slope of ZP is different. In this case one can find another pooling contract $(q_1 + \delta q_1, q_2 + \delta q_2)$ close to (q_1, q_2) which is preferred by both groups and produces non-negative profits. Only if there exists a point on ZP in which the slopes of ZP , U^L and U^H are identical, this represents a pooling equilibrium. Clearly, this case can occur for very specific parameter constellations only, a small perturbation of γ or of the π_t^i would destroy the equilibrium. From these considerations we can conclude that in general Proposition 1 holds without assuming the single-crossing condition.

3.2 The possibility of a separating equilibrium

We call a set of two contracts (q_1^L, q_2^L) , (q_1^H, q_2^H) a *separating equilibrium*, if each fulfills the respective zero-profit condition (2.7), if group L does not prefer (q_1^H, q_2^H) to (q_1^L, q_2^L) and vice versa, i.e. if

$$U^H(q_1^H, q_2^H) \geq U^H(q_1^L, q_2^L), \quad (3.3)$$

$$U^L(q_1^L, q_2^L) \geq U^L(q_1^H, q_2^H), \quad (3.4)$$

and if no other contract exists, which is preferred to (q_1^i, q_2^i) by at least one group $i \in \{L, H\}$ and which allows a nonnegative profit.

We show that a separating equilibrium may, but need not exist, by referring to the logarithmic utility function. With that, lifetime utility (2.4') for an individual $i = L, H$ reads

$$U^i = \ln(c_o^i) + \pi_1^i \ln(c_1^i) + \pi_1^i \pi_2^i \ln(c_2^i). \quad (3.5)$$

(3.5) has two convenient properties: (i) As mentioned above, the single-crossing condition (3.1) is fulfilled, since at any (q_1, q_2) the slope of the indifference curve, which is $-q_2 / (\pi_2^i q_1)$, is flatter for a type-H individual than for a type-L individual. (ii) Annuity demand of any individual $i = L, H$ does not depend on the payoffs, since the coefficient of relative risk aversion R is equal to one (see Lemma 4 and (A1) in the Appendix). These properties help to keep the analytical and graphical analysis simple.

Proposition 2: For appropriate γ and π_t^i , $t = 1, 2$, $i = L, H$, a separating equilibrium $(\hat{q}_1^H, \hat{q}_2^H)$, $(\bar{q}_1^L, \bar{q}_2^L)$ with the properties

- (i) the zero-profit conditions (2.7) for $i = L, H$ are fulfilled for each contract,
 - (ii) $\hat{q}_1^H = \hat{q}_2^H$, $\bar{q}_1^L > \bar{q}_2^L$,
 - (iii) type-H individuals are indifferent between $(\hat{q}_1^H, \hat{q}_2^H)$ and $(\bar{q}_1^L, \bar{q}_2^L)$,
- exists.

Proof: A numerical example for the existence of such an equilibrium is provided in the Appendix. Q.E.D.

An intuition for Proposition 2 is derived from geometric arguments (see Figure 2). Each contract of a separating equilibrium must fulfill the zero-profit conditions (2.7) for the specific group, drawn as ZP^i with slope $-1/\pi_2^i$, $i = L, H$. Observe that $(\hat{q}_1^H, \hat{q}_2^H)$, the contract which, among all individually fair contracts (i.e. those on ZP^H), is most preferred by type-H individuals (see Lemma 2), must be part of the equilibrium: Any other contract on ZP^H is dominated by $(\hat{q}_1^H, \hat{q}_2^H)$, and firms need not care whether type-L individuals might choose that contract, because this would only increase the profit.

However, firms supplying a separate contract to the type-L individuals must care that this contract is not chosen by the high-risk individuals, because then they would make a loss.

The properties of the separating equilibrium correspond to familiar findings for other models with asymmetric information: Individuals in the "best" group (in our case: the long-living individuals) can buy their "first-best" contract, while individuals in the other group can only buy a "distorted" contract, in order to keep the former away from buying the contract designed for the latter, i.e., to avoid pooling.

Proposition 3: For appropriate γ and π^i_t , $t = 1, 2$, $i = L, H$, no separating equilibrium exists.

Proof: A numerical example for the non-existence of such an equilibrium is provided in the Appendix. Q.E.D.

We show that the contract set $(\hat{q}_1^H, \hat{q}_2^H)$ and $(\bar{q}_1^L, \bar{q}_2^L)$ may not be an equilibrium, because there may exist a pooling contract that allows a non-negative profit and is preferred by both groups $i = L, H$. The argument is demonstrated graphically in Figure 2. Consider some pooling contract that lies above the indifference curves \hat{U}^H and \bar{U}^L , but on or below the dashed line ZP, again indicating the zero-profit condition (2.9) for pooling-contracts (Note that in case of logarithmic utility, ZP is indeed a straight line, since annuity demand \bar{A} and thus ρ do not depend on (q_1, q_2)). Obviously, any such pooling contract, e.g. $(\tilde{q}_1, \tilde{q}_2)$, dominates the potential separating equilibrium $(\hat{q}_1^H, \hat{q}_2^H)$, $(\bar{q}_1^L, \bar{q}_2^L)$ and produces a non-negative profit. The existence of such a contract is less likely, the greater the difference between the survival probabilities of both groups $i = L, H$ and the higher γ . For example, the zero-profit line ZP in Figure 2 simply shifts to the left for a higher share of type-H individuals. If it does not cross \bar{U}^L , no dominating pooling contract exists.

3.3 The Wilson equilibrium

In Subsections 3.1 and 3.2 we have analyzed the existence of Nash-Cournot equilibria. These are defined on the basic assumption that firms, when offering a new contract, take the other firms' contract offers as given. Obviously, various different beliefs of firms concerning the reaction of other firms can be formulated. Wilson (1977) introduced the following approach: Let a set of existing contracts be offered. A firm, considering a new contract offer, believes that existing contracts are withdrawn, if they become unprofitable due to the new contract offer. As a consequence, former buyers of the existing contracts will turn to the new offer, which influences profitability of the latter. Accordingly, a *Wilson pooling equilibrium* (q_1, q_2) has to fulfill the property that no other contract exists which is preferred by at least one group $i = L, H$ and allows a nonnegative profit, given that (q_1, q_2) is withdrawn if it becomes unprofitable. The analogous qualification has to be added to the definition of the separating equilibrium in order to describe a *Wilson separating equilibrium*.

One observes immediately that this qualification makes the definition less restrictive (new contract offers are less attractive). As a consequence, any Nash-Cournot equilibrium is also a Wilson equilibrium. Moreover, we have:

Proposition 4: A Wilson equilibrium exists, even if the separating equilibrium does not exist. It is a pooling equilibrium, denoted by $(\tilde{q}_1, \tilde{q}_2)$, with the following properties:

- (i) The zero-profit condition (2.9) is fulfilled.
- (ii) $(\tilde{q}_1, \tilde{q}_2)$ is the most preferred pooling contract for the type-L individuals.

Proof: The proof is derived from geometric arguments (see Figure 2). Consider the pooling contract $(\tilde{q}_1, \tilde{q}_2)$. We show that no firm has an incentive to deviate from $(\tilde{q}_1, \tilde{q}_2)$: In case that a contract $(\tilde{q}_1 + \delta q_1, \tilde{q}_2 + \delta q_2)$ is offered which is preferred by the low-risk, but not by the high-risk individuals (compare Figure 1), the original contract $(\tilde{q}_1, \tilde{q}_2)$, being then purchased by the high-risk individuals only, makes negative profits and will be withdrawn from the market. Consequently, the type-H individuals will also accept the contract $(\tilde{q}_1 + \delta q_1, \tilde{q}_2 + \delta q_2)$, which therefore will turn out to be unprofitable and will not be offered. As a result, $(\tilde{q}_1, \tilde{q}_2)$ is a Wilson pooling equilibrium. Q.E.D

A numerical example for a Wilson pooling equilibrium is provided in the Appendix.

4. Concluding remarks

Considering a life-cycle model with more than one period of retirement allows the formulation of an additional important aspect of the annuity market: It is an attractive strategy for companies to offer annuity contracts, for which the pension payoffs are not constant over the periods of retirement, since individuals with different life expectancies will put different weights on the payment they may or may not receive in the last period of life. In the present study we have analyzed the consequence of this possibility on the existence of equilibria in the private annuity market under price competition and asymmetric information. Our main finding was that in this framework a Nash-Cournot equilibrium may not exist; if one exists, it will be a separating equilibrium. On the other hand, even if a separating equilibrium does not exist, a Wilson pooling equilibrium exists.

By assuming only one period of retirement, previous studies have neglected the fact that the time structure of the payoffs matters, which lead then to the conclusion that under price competition and adverse selection a pooling equilibrium always exists. So, in a general perspective, annuity markets are actually more complicated than it has been supposed so far

and the existence of a stable outcome is less likely. This complexity may be seen as a further explanation why annuity markets are weak.

A further consequence of our extended model is that it should change the view guiding empirical studies. Usually, they start from the premise that the annuity market should ideally offer a pooling contract for all risks, and study the adverse-selection phenomenon by comparing life-expectancy of annuity purchasers with the average life-expectancy of the population. By looking at a specific annuity contract, the magnitude of adverse selection is measured by the difference between the expected rate of return for the general population and the expected rate of return for the subpopulation of annuitants.⁵ Instead, our result suggests that a primary object of investigation should be the question of whether separating indeed occurs and to which extent.

In a recent empirical paper, Finkelstein and Poterba (1999) study the selection effects across three different types of annuity contracts: fixed nominal payoffs, five percent annually escalating nominal payoffs and inflation-indexed payoffs. Since the authors do not have contract-specific mortality probabilities, they compute the expected present value of the payoffs, based on the average population mortality. They show that the expected present value of inflation-linked annuities is about eight percent lower than that of fixed nominal annuities and that of the escalating nominal annuities is about five percent lower than that of fixed nominal annuities. The authors take this result as an indirect evidence that index-linked and escalating annuities are selected by individuals with high life-expectancies: Only these individuals have an incentive to buy such contracts, because for them the expected present value of the payoffs, based on their low mortality rates, is higher and may exceed that of annuities with fixed nominal payoffs. This first evidence from the U.K., which is consistent with our theoretical results, points into the direction that selection across different types of annuity contracts is of some relevance for the annuity market.

Private annuity insurance is becoming more important, because of the expected decline of the replacement ratio offered by the public pension system in many countries. Our contribution adds to the set of studies expressing doubts on the adequate functioning of the annuity market. Clarifying this issue further, appears to be a prominent task for future theoretical and empirical research.

⁵ See, e.g., Friedman and Warshawsky (1988, 1990), Walliser (1998), Mitchell et al. (1999).

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Appendix: Numerical illustration of the (non-)existence of separating equilibria

For logarithmic utility (see(3.5)), annuity demand is computed from (2.5) as

$$A^i = \frac{\pi_1^i(1+\pi_2^i)}{1+\pi_1^i(1+\pi_2^i)} w, \quad (A1)$$

which is independent from the rates of return (q_1, q_2) , as mentioned in the text.

The separating equilibrium contracts $(\hat{q}_1^H, \hat{q}_2^H)$ and $(\bar{q}_1^L, \bar{q}_2^L)$ are computed as follows: Solving the zero- profit condition (2.7) for $i = H$ and setting $q_1^H = q_2^H$ yields

$$\hat{q}_1^H = \frac{1}{\pi_1^H(1+\pi_2^H)}. \quad (A2)$$

The contract $(\bar{q}_1^L, \bar{q}_2^L)$ for type-L individual, is determined by the self-selection constraint (3.3), and the zero-profit-condition (2.7) for $i = L$. Assuming equality, one derives from (3.3) (making use of (A1), (A2), (2.2), (2.3) and (3.5))

$$q_1^{L(1+\pi_2^H)/\pi_2^H} - \frac{1}{\pi_1^L} q_1^{L/\pi_2^H} + \pi_2^L \hat{q}_1^{H(1+\pi_2^H)/\pi_2^H} = 0. \quad (A3)$$

(A3) can be solved to compute \bar{q}_1^L , then \bar{q}_2^L follows from (2.7).

In order to proof that the contracts $(\hat{q}_1^H, \hat{q}_2^H)$ and $(\bar{q}_1^L, \bar{q}_2^L)$ indeed constitute an equilibrium, we have to show that there is no pooling contract which fulfills the zero-profit condition (2.9') and is preferred by individuals of both types $i = L, H$. To do so, we concentrate on the pooling contract $(\tilde{q}_1, \tilde{q}_2)$ which together with (A1) fulfills the zero-profit condition (2.9') and is preferred most by a type-L individual. This is the accurate procedure, since an individual of type H is certainly better off with the pooling contract $(\tilde{q}_1, \tilde{q}_2)$ than with her own contract $(\hat{q}_1^H, \hat{q}_2^H)$, given a type-L individual prefers $(\tilde{q}_1, \tilde{q}_2)$ to $(\bar{q}_1^L, \bar{q}_2^L)$. Maximization of (3.5) for $i = L$ subject to (2.9') gives

$$\tilde{q}_1 = \frac{1+\rho}{(1+\pi_2^L)(\pi_1^L + \rho\pi_1^H)}, \quad \tilde{q}_2 = \frac{\pi_2^L(1+\rho)}{(1+\pi_2^L)(\pi_1^L\pi_2^L + \rho\pi_1^H\pi_2^H)} \quad (A4)$$

where $\rho = (\gamma A^H)/((1-\gamma)A^L)$. Thus, whenever the low-risk individuals are worse off at $(\tilde{q}_1, \tilde{q}_2)$, the contracts $(\hat{q}_1^H, \hat{q}_2^H)$ and $(\bar{q}_1^L, \bar{q}_2^L)$ constitute an equilibrium. Otherwise they do not and the contract $(\tilde{q}_1, \tilde{q}_2)$ is the pooling equilibrium according to the definition of Wilson.

In Table 1 we provide numerical examples, for which annuity demand A^i , the contracts $(\hat{q}_1^H, \hat{q}_2^H)$, $(\bar{q}_1^L, \bar{q}_2^L)$ and $(\tilde{q}_1, \tilde{q}_2)$, as well as expected utility of individuals of both types $i = H, L$, U^H and U^L , at these contracts are calculated explicitly. We choose three different scenarios, which differ in the share γ of the high risk individuals (scenario 1 and 2) and in the survival probability π_2^L of the type-L individuals in period 2 (scenario 1 and 3). In scenario 1 the contracts $(\hat{q}_1^H, \hat{q}_2^H)$ and $(\bar{q}_1^L, \bar{q}_2^L)$ constitute an equilibrium. Taking this as a reference point, we show that a lower share γ of type-H individuals (scenario 2) and a higher survival probability π_2^L of the type-L individuals in period 2 (scenario 3) entail that there is no separating equilibrium in a competitive annuity market. In these both scenarios $(\tilde{q}_1, \tilde{q}_2)$ constitute the Wilson pooling equilibrium.

Table 1: Numerical illustration of the (non-)existence of separating equilibria

Scenario 1: Existence of separating equilibrium			Scenario 2: Non-existence of a separating equilibrium			Scenario 3: Non-existence of a separating equilibrium		
$w = 1000, \gamma = 0.5$ $\pi_1^H = 0.8, \pi_2^H = 0.6, A^H = 561.4$ $\pi_1^L = 0.6, \pi_2^L = 0.2, A^L = 418.6$			$w = 1000, \mathbf{g} = 0.2$ $\pi_1^H = 0.8, \pi_2^H = 0.6, A^H = 561.4$ $\pi_1^L = 0.6, \pi_2^L = 0.2, A^L = 418.6$			$w = 1000, \gamma = 0.5$ $\pi_1^H = 0.8, \pi_2^H = 0.6, A^H = 561.4$ $\pi_1^L = 0.6, \pi_2^L = \mathbf{0.5}, A^L = 473.7$		
contracts	U^H	U^L	contracts	U^H	U^L	contracts	U^H	U^L
$\hat{q}_1^H=0.781, \hat{q}_2^H=0.781$	13.871	10.534	$\hat{q}_1^H=0.781, \hat{q}_2^H=0.781$	13.871	10.534	$\hat{q}_1^H=0.781, \hat{q}_2^H=0.781$	13.871	11.588
$\bar{q}_1^L=1.620, \bar{q}_2^L=0.232$	13.871	10.826	$\bar{q}_1^L=1.620, \bar{q}_2^L=0.232$	13.871	10.826	$\bar{q}_1^L=1.541, \bar{q}_2^L=0.252$	13.871	11.656
$\tilde{q}_1=1.166, \tilde{q}_2=0.511$	13.987	10.724	$\tilde{q}_1=1.282, \tilde{q}_2=0.792$	14.273	10.833	$\tilde{q}_1=0.941, \tilde{q}_2=0.838$	14.053	11.721