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# ON SUBSIDISING AUTO-COMMUTING\*

**Edward Calthrop** 

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## ON SUBSIDISING AUTO-COMMUTING

## **Abstract**

Evidence suggests that a considerable proportion of peak period trips are made for purposes other than commuting to or from work.given the different degrees of Hicksian complementarity with the labour market, optimal tax theory suggests that, in a second-best world, different trip purposes should be taxed at different rates. This paper explores this issue and argues for a uniform congestion toll (independent of trip-purpose) combined with a subsidy to auto-commuters. A numerical model suggests that while, in the absence of congestion tolls, commuting subsidies are welfare decreasing, an optimal pricing scheme entails auto-commuters receiving a subsidy of nearly 50 percent of the uniform road toll.

JEL Classification: H23, R41.

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### 1 Introduction

There is a common perception, reflected in the transportation economics literature, that most trips during the traditional 'rush hour' are made for the purpose of getting to - or returning from - the workplace. This perception seems to be at odds with recent data.

For instance, Figure 1 presents survey evidence from the United States<sup>1</sup>. Work trips comprise around a mere 37% of all trips during the two rush hour periods<sup>2</sup>. The most important non-work travel purposes seem to be shopping, family and personal motives, and recreational travel.

The evidence is not confined to the United States. A similar type of survey conducted in London suggests that around 60% of trips in the morning peak are work related and a mere 50% in the evening peak<sup>3</sup> (LRC, [26]).

It might seem that there is little reason for economists to be concerned with the motivation for travel. Theory teaches that, at least when government has a full set of instruments, urban road tolls should reflect marginal external costs. These costs, in turn, are independent of trip-purpose (see, for example, Walters [40]; Vickrey [39]; Kraus et al. [25]; Arnott, de Palma and Lindsey [1]; Small and Kazimi [37]).

But in reality government does not have a full set of instruments: revenues must be raised via distortionary taxes. Taxes on labour supply characterise most industrialised economies. In the presence of a labour supply tax, this paper

<sup>&</sup>lt;sup>1</sup> A copy of the 1995 National Personal Travel Survey (NPTS) [38] results can be downloaded from: http://www.bts.govt/ntda/npts. Figure 1 of this paper is a reproduction of NPTS Figure 12 (pp.14).

<sup>&</sup>lt;sup>2</sup>Note that the NPTS report [38] discusses a possible bias in the number of reported commuting trips due to trip-chaining: a shopping trip made during the return commute from work is categorised as a shopping trip (rather than a commute trip). Further comments are made on trip-chaining in the concluding section of this paper.

<sup>&</sup>lt;sup>3</sup>The survey suggests that in the morning peak (07.00-10.00), 49% of trips are work trips, 10% are employer's business trips, 11% are education trips, 6% are shopping trips, 8% are escort trips and general 'other trips' make up the remaining 16%. The respective figures for the evening peak (16.00-19.00) are: 36%,10%,5%,12%,6% and 30%.

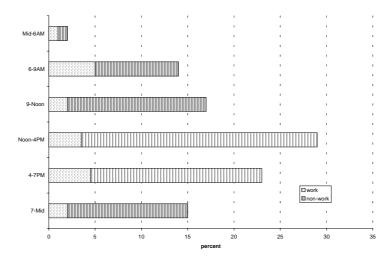


Figure 1: Time profile of commuting trips

shows that optimal road tolls are ideally differentiated between trip purposes. Commuting trips are charged at a lower rate than non-commuting, or leisure, trips. Given the difficulties of implementing a road toll based on trip purpose, this paper argues for a uniform congestion charge (identical for different trippurposes) plus a subsidy to commuting travel. This in turn casts a new light on an old debate concerning the merits of subsidising commuting: a policy common in several European countries including Germany and Scandinavia.

The logic for this structure of pricing is straightforward. Consider the welfare impact of raising the price of a commodity good above marginal cost, given distortionary taxes on labour. Clearly there is a distortion introduced on the commodity good market (a distortionary triangle). However, as stressed by Harberger [24], the welfare cost must also include the cost of exacerbating pre-existing distortions on other markets: in this case, the labour market. Increasing the price of the commodity results in a change in demand for leisure. The welfare loss on the leisure market is a rectangle: the pre-existing distortion (the marginal tax rate) multiplied by the change in demand.

Consider changing the price of either commuting transport or leisure trans-

port. The magnitude of the reaction on the leisure market will differ between the two transport markets. To the extent that the compensated cross-price elasticity of leisure differs between transport markets, so, *ceteris paribus*, the marginal welfare cost (from exacerbating pre-existing distortions on the labour market) varies between markets. The optimal tax rule accounts for this variation.

This point is well-known and forms the basis of the rule that efficient commodity taxes are set in function of the degree of complementarity between the good and the leisure market. (Corlett and Hague [15]; Diamond and Mirrlees [19] p.263; Atkinson and Stiglitz [4] Ch 12; Myles [29] p.124). In the presence of externalities, the same reasoning has been used to show that taking account of pre-existing tax distortions leads to modifications of simple Pigouvian analysis (see Sandmo [35]; Bovenberg and de Mooij [8]; Bovenberg and van der Ploeg [10]; Bovenberg and Goulder [9]; and Parry [30]).

This paper uses an approach developed in Bovenberg and van der Ploeg [10] to analyse optimal urban road tolls given differing trip purposes in a distorted economy. Two other papers have also dealt with taxation of transport in a world with distortionary taxes. Mayeres and Proost [28] derive optimal tax rules for personal transport markets and freight activities in the presence of road congestion. The authors assume a single trip purpose, and thus do not investigate the issue of the relative taxation of different trip purposes. Parry and Bento [32] consider a marginal tax reform exercise in a model with a single-trip purpose (commuting). The model results derived below are compared to both these contributions.

Although the desirability of tax deductions in general has been considered<sup>4</sup>, little attention has been paid to the case of commuting expenses. One exception

<sup>&</sup>lt;sup>4</sup>The discussion of tax-deduction has traditionally occurred as a part of a wider debate on the desirability of using indirect taxes in the presence of a non-linear direct tax: see Atkinson and Stiglitz [4] (Ch.14) for a discussion of the case of deducting medical and housing expenses from the income tax base. The standard result emerging from this analysis is that tax deductions are hard to justify if the commodity market is weakly separable from the leisure market (and an optimal non-linear income tax is in place). This paper can be seen as implicitly examining the case for tax deductions in the presence of a linear wage tax with identical individuals.

is a recent paper by Wrede [41]. He examines the effect of commuting subsidies when consumers choose both where to live and where to work. The optimal subsidy depends strongly on the assumptions made about mobility. However, the analysis assumes a fixed labour supply and a constant commute time to work (i.e. there is no road congestion).

In contrast to Wrede's analysis, this paper abstracts from issues of household and worker mobility and instead focuses on the impact of subsidies on labour supply. Furthermore, commuting is subject to congestion - a priori, a significant reason not to subsidise auto-commuting. An analytical model is presented in section 2, which derives the basic optimal structure of road tolls on commuting and non-commuting travel given a distortionary labour tax.

Section 3 introduces a numerical simulation model, which is used to investigate the impact of commuting subsidies. The model results (section 4) show that, for the benchmark scenario, auto-commuting receives an optimal subsidy of nearly 50 percent of the optimal uniform road toll. Some positive subsidy is shown to be optimal (in the presence of a uniform congestion toll) under a wide variety of assumptions on key parameters, though any level of subsidy is welfare decreasing in the absence of road tolls. Concluding remarks are drawn together in section 5.

## 2 The analytical model

I assume an economy with a fixed number of identical people, N. Each person chooses how much labour to supply (thus requiring a commute trip by car of a fixed distance,  $d_C$ ), and chooses between two private consumption goods: a non-commute trip, B (of fixed distance,  $d_B$ ) and a composite commodity good, A. The model captures a single mode of transport and two different trip purposes: a commuting trip and non-commuting trip.

Assuming a single mode of transport may be a good approximation to some urban economies - evidence for the US suggests that over 90% of commuting

trips are made by private car (Gordon and Richardson [22])<sup>5</sup>. This assumption is relaxed in the numerical model introduced below. In order to simplify the model, I focus only on congestion externalities - other externalities associated with auto use (e.g. air pollution, noise) can be included in a straightforward manner (see Calthrop and Proost [12] for a review of road transport externalities).

In order to supply a day's labour input, each individual must forego leisure time consisting of a fixed number of hours at work,  $t_w$ , plus the time required to commute to and from work. To ease notation, I set  $t_w = 1$ . Each individual chooses the number of days of labour to supply, L. Strict complementarity is assumed between the number of day's labour supply and commute trips, C, such that  $C = \alpha L$ . Henceforth  $d_L = \alpha d_C$ . To ease exposition, I assume  $\alpha = 1$ . The total volume of traffic per time period is given by  $N\left(d_BB + d_LL\right)$ . As is common in the congestion literature, the average (scaled) time required per unit distance travelled,  $\phi$ , is an increasing function of total traffic volume:  $\phi\left[N\left(d_BB + d_LL\right)\right]^6$  with  $\phi'\left[\cdot\right] > 0$ . A static representation of congestion is chosen to simplify matters - see Arnott et al. [1] for a dynamic model of congestion. To simplify notation, and without loss of generality, I assume henceforth that  $d_L = d_B = 1$ .

Pure leisure time, V, is given as the time remaining per period after working time, commuting time and non-commuting transport time have been subtracted from the total time endowment, denoted  $\overline{T}$ . Hence:

$$V = \overline{T} - L - \phi \left[ \cdot \right] (L + B)$$

Consumer preferences are represented by a utility function:

$$U = u\left[V, A, B, X\right] \tag{1}$$

 $<sup>^5</sup>$ See Table 14. The 1990 Nationwide Personal Travel Survey results show that 91.2% of trips made in the course of 'earning a living' are made by private transport. This compares to 87.1% in 1987.

<sup>&</sup>lt;sup>6</sup> Henceforth, square brackets contain the arguments of a function.

where X is the quantity of the public good consumed. The utility function is assumed to be quasi-concave and twice differentiable. The first-order derivative of the utility function with respect to argument j is given by  $u_j$ .

Assuming that producers face a linear constant-returns to scale technology in which output is proportional to employment, the material balance condition for the economy is given by:

$$\beta NL = \beta_a NA + \beta_b NB + \beta_c NC + \beta_x X \tag{2}$$

For good A and X, units are adjusted such that  $\beta_a = \beta_x = 1$ . The production price of good A acts as the numeraire. I assume  $\beta_b = \beta_c \equiv r$ , which ensures that the resource cost of a single unit of distance travelled in a car is independent of trip purpose.

#### 2.1 The controlled economy

Maximising (1) subject to (2) gives the following set of conditions:

$$\frac{u_B}{u_A} = r + w_s \left( \phi + \phi' N \left( L + B \right) \right) \tag{3}$$

$$\frac{u_B}{u_A} = r + w_s \left(\phi + \phi' N \left(L + B\right)\right)$$

$$\frac{u_V}{u_A} = w_s$$
(4)

$$\frac{Nu_X}{u_A} = 1 \tag{5}$$

where  $w_s$  is defined as the marginal social opportunity cost of leisure:

$$w_s = \frac{\beta - r}{1 + \phi + \phi' N \left( L + B \right)} \tag{6}$$

and is given by the return to labour per day (net of commuting costs) divided by the time input to society required for a day's work. Total time costs include the time supplied by the driver both at work and during the commute trip plus the time losses imposed on all other drivers from the marginal increase in average time per unit distance driven. At the optimum, the marginal rate of substitution

between leisure and good A (expression 4) is equal to the relevant marginal rate of transformation - the social opportunity cost of leisure consumption.

The marginal rate of transformation between goods B and A (expression 3) equals the resource cost of a trip (r) plus the social opportunity cost of the time required for making the trip. This time loss reflects both the loss to the driver directly from making an extra trip plus the time losses to all other drivers via the decrease in average speed.

Finally, the optimal condition for the provision of the public good (expression 5) is the usual Samuelsonian rule: the marginal utility from an extra unit of the private good A to an individual equals the marginal utility to all individuals from an extra unit of the public good X.

#### 2.2 The first-best market economy

Using the relationship between commuting and labour supply, the consumer budget constraint can be written as:

$$T + P_L L = P_B B + A \tag{7}$$

in which  $P_B = r(1 + t_B)$ ,  $P_L = \beta(1 - t_L) - r$ , T is a lump-sum transfer and the consumer price of good A is normalised to 1. An ad-valorem tax per unit distance driven is introduced,  $t_B$ . I also assume an ad valorem tax per hour of labour supply,  $t_L$ . The relationship between the labour supply tax and a commuting toll is elaborated upon below. Consumers are assumed to maximise (1) subject to a budget constraint (7) taking the speed of travel as parametric. At the optimum, consumers demand goods and supply labour such that:

$$\frac{u_B}{u_A} = P_B + w_p \phi \tag{8}$$

$$\frac{u_V}{u_A} = w_p \tag{9}$$

$$\frac{u_V}{u_A} = w_p \tag{9}$$

where  $w_p$  is defined as the marginal private opportunity cost of leisure:

$$w_p = \frac{P_L}{1+\phi} \tag{10}$$

The numerator gives the net private return from supplying an additional day's labour supply. The denominator gives the private time used in supplying an additional day's labour.

The relevant marginal transformation rate between leisure and the numeraire good (expression 9) equals the marginal private opportunity cost of leisure. The private cost of consuming a unit of good B comprises of the consumer price,  $P_B$ , plus time required to make a trip, which is valued at the marginal private opportunity cost of time multiplied by the private loss in time,  $\phi$ . In order to align private incentives with socially optimal incentives, the government uses the two available tax rates to equate the equations 8 and 9 with equations 3 and 4. This gives:

$$rt_B^* = \beta t_L^* = w_s \phi' N \left( L + B \right) \tag{11}$$

In a first best world, the optimal tax on both good B and labour supply reflect the marginal external cost of transport. By construction, the marginal external congestion cost of a commuting trip and a non-commuting, or leisure trip, is identical. The optimal tax paid on either type of trip equals the (social) value of time,  $w_s$ , multiplied by the marginal time losses imposed on other drivers,  $\phi'N(L+B)$ .

The model is closed by deriving the optimal level of public good from equation (5) and determining the level of T necessary to finance this expenditure net of externality revenues.

#### 2.3 The second-best economy

Assume that T = 0. The government must resort to distortionary taxes as lump-sum taxes are no longer available. For any required level of the public

good, the government's maximisation problem can be written as a function of the two tax instruments available:  $t_L$  and  $t_B$ . Annex 1 shows that the set of first order conditions to the government's maximisation problem is given by:

$$\lambda B - \mu \left\{ B + r(t_B - t_{BP}) \frac{\partial B}{\partial P_B} + \beta (t_L - t_{LP}) \frac{\partial L}{\partial P_B} \right\} = 0$$
 (12)

$$\lambda L - \mu \left\{ L - r(t_B - t_{BP}) \frac{\partial B}{\partial P_L} - \beta (t_L - t_{LP}) \frac{\partial L}{\partial P_L} \right\} = 0$$
 (13)

where  $\lambda$  is the marginal private utility of income and  $\mu$  is the marginal disutility from raising an additional unit of government revenue. Following Bovenberg and van der Ploeg [10], the term 'net social Pigouvian tax' is used to refer to:

$$t_{iP} = \frac{\sigma_i}{u_A} \frac{1}{\eta} \qquad i = B, L \tag{14}$$

in which  $\eta$  gives the marginal cost of public funds ( $\eta = \mu/\lambda$  and is derived further in equation (22) below) and:

$$r\sigma_{B} = \xi \phi' N \left\{ u_{V} \left( L + B \right) - \mu \left( rt_{B} \frac{\partial B}{\partial \phi} + \beta t_{L} \frac{\partial L}{\partial \phi} \right) \right\} = \beta \sigma_{L}$$
 (15)

where, the feedback effect,  $\xi$ , is given by:

$$\xi = \frac{1}{1 - \frac{\partial L}{\partial \phi} \phi' N - \frac{\partial B}{\partial \phi} \phi' N} \tag{16}$$

If there are no externalities present (i.e.  $t_{BP}=t_{LP}=0$ ), expressions 12 and 13 reduces to the standard first-order conditions to an optimal tax problem (c.f. Atkinson and Stiglitz, [4] Ch.12, or Auerbach [6]). In the presence of congestion externalities, the social contribution in terms of government revenues of additional demand from good B or supply of labour is measured as the net effect of a positive and a negative contribution. Increasing demand for non-work trips or labour supply boosts the tax base (and thus facilitates the provision

of public goods). However it also increases congestion levels. The net social contribution is measured as the difference between these two terms.

By construction, the net-social Pigouvian tax (per kilometre driven) is identical between different trip purposes. This is seen in equation 14 and 16. Note that the net-social Pigouvian tax falls, ceteris paribus, as a function of the marginal cost of public funds  $\eta$ . This point is well-known (see Bovenberg [7]; Sandmo [35]). The government uses taxes for two differing purposes: raising revenues and internalising environmental externalities. The higher the  $\eta$  in the economy, the more the optimal tax system focuses on generating revenues and less on internalising the externality. Indeed, the social cost of the externality, measured in public revenue terms, falls.

Annex 1 shows that, by exploiting the Slutsky matrix, expressions 12 and 13 can be rewritten to give each unknown tax rate as an implicit function of the compensated cross- and own-price elasticities of demand of the two goods:

$$\begin{pmatrix}
\frac{r(t_B - t_{BP})}{P_B} \\
\frac{\beta(t_L - t_{LP})}{P_L}
\end{pmatrix} = \begin{pmatrix}
\theta_B - \theta_{BP} \\
\theta_L - \theta_{LP}
\end{pmatrix} = \begin{pmatrix}
\frac{\mu - \lambda'}{\mu}
\end{pmatrix} \begin{pmatrix}
\varepsilon_{BB} & -\varepsilon_{BL} \\
-\varepsilon_{LB} & \varepsilon_{LL}
\end{pmatrix}^{-1} \begin{pmatrix}
-1 \\
1
\end{pmatrix}$$
(17)

where the marginal social utility of income,  $\lambda'$ , is given by:

$$\lambda' = \lambda + \mu \left( r \left( t_B - t_{BP} \right) \frac{\partial B}{\partial T} + \beta \left( t_L - t_{LP} \right) \frac{\partial L}{\partial T} \right) \tag{18}$$

In the absence of external costs, it can be shown that the marginal disutility of financing public spending  $(\mu)$  exceeds the marginal social utility of private income  $(\lambda')$  if public revenues are positive (c.f. Auerbach [6] pp.112). This is assumed to hold henceforth. Expression 17 can be solved for the two unknown tax rates. Denoting  $\mu/\lambda'$  by  $\eta'$ , the optimal tax rate on labour supply is given by:

$$\theta_L = \left(1 - \frac{1}{\eta'}\right) \left(\frac{\varepsilon_{LB} - \varepsilon_{BB}}{\varepsilon_{BL}\varepsilon_{LB} - \varepsilon_{BB}\varepsilon_{LL}}\right) + \frac{1}{\eta} \frac{\beta \sigma_L}{u_A P_L} \tag{19}$$

The labour tax comprises of a revenue-raising component (the 'Ramsey' component) plus a corrective component (the 'Pigouvian' component). This accords with the basic principle of 'additivity' - the net social Pigouvian damage from a commodity enters the tax formula for that commodity alone and in an additive manner (Sandmo [35])<sup>7</sup>. The negative semi-definite property of the Slutsky matrix implies that  $\varepsilon_{BL}\varepsilon_{LB} - \varepsilon_{BB}\varepsilon_{LL} > 0$ . Hence the Ramsey component is positive if  $\varepsilon_{LB} - \varepsilon_{BB} > 0$ . The optimal tax rate on non-commuting transport is:

$$\theta_B = (\theta_L - \theta_{LP}) \left( \frac{\varepsilon_{AL} - \varepsilon_{BL}}{\varepsilon_{AB} - \varepsilon_{BB}} \right) + \frac{1}{\eta} \frac{r\sigma_B}{u_A P_B}$$
 (20)

In accordance with additivity, this expression comprises of a Ramsey component plus a Pigouvian component. The sign of the Ramsey component depends on the relative Hicksian complementarity with the leisure market between the two consumption goods  $(\varepsilon_{AL} - \varepsilon_{BL})^8$ . If non-commuting transport is a relative complement to leisure  $(\varepsilon_{AL} > \varepsilon_{BL})$ , the Ramsey component is strictly positive. Only if  $\varepsilon_{AL} = \varepsilon_{BL}$  would the Ramsey component fall to zero on good  $B^9$ .

<sup>&</sup>lt;sup>7</sup> In a more general model, with non-identical individuals, Cremer *et al.* [14] show that the additivity property breaks down if the government is constrained to use linear commodity taxes (but can use a non-linear income tax).

<sup>&</sup>lt;sup>8</sup>I assume that  $\varepsilon_{AB} > 0$ .

<sup>&</sup>lt;sup>9</sup>The presence of congestion in the price of good B implies that we would not expect consumer preferences to be weakly separable from leisure. For example, allow utility to be additive in its components. Hence  $u = (1 - L - \phi[d_L L + d_B B]) + A + B$ . It is immediate that  $\frac{\partial \left(\frac{\partial u}{\partial A} / \frac{\partial u}{\partial A}\right)}{\partial L} \neq 0$ .

Note that, as discussed by Auerbach, [5], in a single consumer model (without a poll tax) weak separability of commodities from leisure alone does not imply that  $\varepsilon_{AL} = \varepsilon_{BL}$  nor is it necessary. It is however a sufficient condition given an additional restriction, namely that preferences are homothetic with respect to commodities. In a many person economy, Deaton [17] shows that weak-separability between goods and leisure plus parallel linear Engel curves for goods in terms of income is a sufficient condition for differentiated commodity tax to be unnecessary (given an optimal linear income tax). Finally, the most well-known result is that of Atkinson and Stiglitz [3]: given an optimal non-linear income tax schedule, weak separability alone is a sufficient condition for uniform commodity taxation.

If lump sum instruments are available, and hence the marginal cost of public funds equals 1, the optimal taxes on both markets collapse into the first-best expressions (11).

Expression 20 is essentially a special case of the optimal tax rule derived in Mayeres and Proost [28] (see their equation 22). Their model is in several aspects more general than that studied here: it allows for non-identical individuals, an additional tax instrument - a poll-tax, and a production sector that requires freight transport as an input. However, the authors only consider a single aggregate trip purpose. The optimal tax on road transport consists of a Ramsey component plus a net-social Pigouvian tax. The net-social Pigouvian tax contains all of the components of expression 14, although in addition the impact of congestion on productivity is accounted for. The Ramsey component of their optimal tax expression reflect the presence of the (optimal) poll-tax and the normalised price of labour. The Ramsey terms presented do not give any insight as to the importance of the relative relationship to the labour market. This point will be returned to in the next section, when optimal road tolls are investigated.

The government's maximisation problem can also be used to determine the optimal provision of the public good, X. The optimal condition can be written as  $^{10}$ :

$$\frac{Nu_X}{u_A} = \eta \left\{ 1 - Nr \left( t_B - t_{BP} \right) \frac{\partial B}{\partial X} - N\beta (t_L - t_{LP}) \frac{\partial L}{\partial X} \right\}$$
 (21)

As is well known, in a world of distortionary taxes, the marginal rate of transformation between private and public goods diverges from the Samuelsonian marginal rate of substitution (Atkinson and Stern [2]). Firstly, raising government provision of the public good may reduce the overall distortionary

 $<sup>^{-10}</sup>$ The derivation is obtained as follows. Differentiate the government's maximisation problem (given in Annex 1 equation [26]) with respect to a change in the public good X. Use the first-order condition of the consumer's problem to eliminate terms. Rewrite the total derivatives in terms of partial derivatives (i.e. incorporating a feedback term). Substitution of the partial derivative expressions plus some tedious manipulations give the expression shown.

tax burden by boosting the consumption of taxed goods. However, only the Ramsey component of taxes is socially relevant - the revenues from the Pigouvian tax compensate for increased congestion. Secondly, a higher marginal cost of public funds,  $ceteris\ paribus$ , acts to reduce the optimal provision of the public good (assuming that X is a normal good).

The first order condition for a change in the price of labour can be used to solve for the marginal cost of public funds in this model<sup>11</sup>, where the dash above the elasticity expressions denotes that the elasticities are in uncompensated terms i.e. changes in Marshallian demands are relevant.

$$\eta = \frac{1}{1 - (\theta_B - \theta_{BP}) \alpha_B \varepsilon_{BL}^{/} - (\theta_L - \theta_{LR}) \varepsilon_{LL}^{/}}$$
(22)

where  $\alpha_B$  denotes the share of private expenditures on good B. The marginal cost of public funds is greater than unity if the financing of additional public goods reduces the existing Ramsey tax base. As discussed by Bovenberg and van der Ploeg [10], this is the case if a positive labour tax coincides with an upward sloping Marshallian labour supply curve ( $\varepsilon'_{LL} > 0$ ) or if the net-social Pigouvian tax on non-commuting transport is positive ( $\theta_B > \theta_{BP}$ ) and private demand for non-commuting transport falls as the tax on labour supply increases ( $\varepsilon'_{BL} > 0$ ).

#### 2.4 Subsidising commuting

The analytical model derives two optimal tax rates over three goods. Although limited, the advantage of this approach is that it explicitly demonstrates the relationship between the optimal Ramsey tax rate on a commodity and the degree of Hicksian complementarity to the labour market. The optimal road toll per trip for non-commuting transport  $\tau_B$  can be written as:

<sup>&</sup>lt;sup>11</sup>Solve the first order condition for the tax on labour supply (equation 13) for  $\mu/\lambda$ . Dividing through by labour supply L and manipulating terms gives the expression shown.

$$\tau_B = rt_B = P_B \left(\theta_L - \theta_{LP}\right) \left(\frac{\varepsilon_{AL} - \varepsilon_{BL}}{\varepsilon_{AB} - \varepsilon_{BB}}\right) + \frac{1}{\eta} \frac{r\sigma_B}{u_A}$$
 (23)

The road toll contains two components: the first term on the right-hand side corrects for the distortionary costs of the labour supply tax (equivalent to a uniform commodity tax), while the second term corrects for the external cost of road use (via the net-social Pigouvian tax). This type of result in principle extends to a vector of different trip-purpose markets. Optimal road tolls would differ between different transport trip purposes according to their relative complementarity to the labour market, irrespective to external costs. As discussed above, the optimal road toll on non-commuting transport is similar to that derived in Mayeres and Proost [28], except that the Ramsey component of the urban road toll is not given in terms of the relative complementarity to the labour market.

The model also derives the optimal labour supply tax. The main disadvantage of this approach is that it does not uniquely determine the optimal commuting road toll: any combination of commuting toll plus a labour tax that summed to the optimal level (given by equation 19) would be sufficient to decentralise the social optimum. However, it is natural to interpret the net-social Pigouvian component of the optimal tax structure as that corresponding to the policy debate over the use of road-tolls to internalise the external cost of road use. Hence - arbitarily - I assume that the relevant tax rate on commuting transport is given by:

$$\tau_C = \beta t_L = \frac{1}{\eta} \frac{\beta \sigma_L}{u_A} = \frac{1}{\eta} \frac{r \sigma_B}{u_A} \tag{24}$$

where the equivalence between the last two terms follows from the assumption of identical marginal external costs captured in equation 15. It follows that  $\tau_C < \tau_B$  if non-commuting transport is the relative Hicksian complement to the leisure market.

The optimal toll on commuting transport (in the presence of a labour supply

tax) is derived in Parry and Bento [32]. They examine a marginal tax reform problem in a model with two commuting modes (an uncongested mode, rail, and a congested road). Marginal welfare gains from raising the road toll are positive up until the point that the toll equals marginal external congestion cost. This result contrasts with the net-social Pigouvian tax (equation 24) found in this paper.

Optimal pricing of urban road use in this model is reflected in equations 23 and 24. Clearly this can be implemented via differentiated road tolls. However, an alternative means of achieving the same pricing structure is a uniform road toll, equal to the optimal toll for non-commuting transport (equation 23) plus a subsidy to commuting transport equal to  $\tau_B - \tau_C$ . Given that administrative systems are in place in several countries to subsidise commuting expenses, plus the administrative complexities of a differentiated tolling system, the most practical means of implementing optimal road tolls would appear to be a uniform congestion toll (independent of trip purpose) plus a commuting subsidy. Note that in a model with two trip purposes, this scheme results in optimal prices. However, were the model to contain a vector of trip purposes, each with a varying relationship to the leisure market, such a scheme would be imperfect.

#### 3 The simulation model

The optimal tax rules are illustrated with a numerical model. The model is not designed to provide detailed city-specific results, but rather to demonstrate the welfare gains from a subsidy to auto-commuting transport in the presence of a uniform congestion toll. The numerical model assumes the presence of two modes of transport (congested auto and uncongested rail) and two trip purposes (commute and non-commute). The indeterminancy in the labour supply tax in the analytical model is resolved by simulating various congestion tolls and auto-commuting subsidy combinations for given levels of taxes on labour supply and rail use.

The model is constructed using MPSGE/GAMS and is similar to that de-

veloped by Tom Rutherford<sup>12</sup> and Parry and Bento [33]. A copy of the code is available from the author on request.

#### 3.1 Model structure

Each consumer maximises the following nested CES function:

$$U\left[V,A,B\right] = \left\{A^{\frac{\sigma_{U}-1}{\sigma_{U}}} + V^{\frac{\sigma_{U}-1}{\sigma_{U}}} + B^{\frac{\sigma_{U}-1}{\sigma_{U}}}\right\}^{\frac{\sigma_{U}}{\sigma_{U}-1}}$$

in which the aggregate number of non-commute trips, B, is given by a composite commodity of auto trips,  $B_A$  and rail trips,  $B_R$ .

$$B\left[B_{A},B_{R}\right] = \left\{B_{A}^{\frac{\sigma_{B}-1}{\sigma_{B}}} + B_{R}^{\frac{\sigma_{B}-1}{\sigma_{B}}}\right\}^{\frac{\sigma_{B}}{\sigma_{B}-1}}$$

The composite good, A, is produced as a Leontief function of labour supply, L, and a composite commuting good C:

$$C\left[C_{A}, C_{R}\right] = \left\{C_{A}^{\frac{\sigma_{L} - 1}{\sigma_{L}}} + C_{R}^{\frac{\sigma_{L} - 1}{\sigma_{L}}}\right\}^{\frac{\sigma_{L}}{\sigma_{L} - 1}}$$

which in turn is a CES function of commuting trips by auto,  $C_A$  and trips by rail,  $C_B$ .

The budget constraint of the consumer is given by:

$$A + r(1 + t_A)(C_A + B_A) + p_{TR}(C_R + B_R) = \beta(1 - t_L)L + srC_A + T$$

where  $p_{TR}$  gives the production price of a rail trip. Three tax rates exist: a uniform congestion toll at rate  $t_A$ , a tax on labour supply  $t_L$ , and a subsidy

 $<sup>^{-12}</sup>$  The model is constructed using MPSGE. See for the original version developed by Tom Rutherford at http://robles.colorado.edu/~tomruth/congest and www.rff.org/~parry/Links/transp3.htm for the version developed by Parry and Bento.

paid to auto-commuting at rate s. For simplicity, it is assumed that no taxes are set on rail trips. Finally, there exists a lump-sum transfer  $T^{13}$ .

The government budget constraint is given by:

$$N(\beta t_L L + r t_A (C_A + B_A) - r s C_A) = NT \tag{25}$$

#### 3.2 Calibration

Following Parry and Bento [33], the following functional form is adopted to model trip time (the inverse of trip speed) as a function of traffic volume:

$$\phi = \phi_0 \left\{ 1 + \gamma \left[ \frac{(C_A + B_A)}{CAP} \right]^k \right\}$$

where CAP is a measure of road capacity and  $\phi$  is the time required for a journey under freeflow (i.e. non congested) travel conditions. This specification is known as the 'Bureau of Public Roads formula' and has the appealing property that the marginal external time delay is k times the average delay. Following common practice, the values of  $\gamma=0.15$  and k=4 have been adopted - see Small [36] pp.69-72 for a discussion of the empirical evidence.

#### 3.2.1 Transportation Parameters

By assumption no taxes (or subsidies) are levied on transport markets in the benchmark ( $\overline{s} = \overline{t_A} = 0$ , where the upper bar indicates benchmark values of parameters).

Following Parry and Bento [33], I choose  $(C_A + B_A)/CAP$  such that peakperiod speed on the freeway is initially one-half of the free flow speed.

<sup>&</sup>lt;sup>13</sup> It is clear that in this formulation the lump-sum instrument can be used to raise revenues efficiently, leaving only a Pigouvian corrective role for congestion tolls. However, the model is being used to simulate the effect of the labour tax on congestion tolls: the lump-sum instrument is required to return tax revenues back to consumers - public good provision is not explicitly modelled.

Benchmark generalised transport expenditures are assumed to equal ten percent of GDP. Expenditures are assumed to be split equally between leisure and commuting trips, which accords with the empirical evidence for peak-period flows in London. This assumption is also the subject to sensitivity analysis. Further, it is assumed that expenditures on each trip purpose are equally split between the two modes.

Time expenditures comprise 50% of generalised expenditures for auto trips and 67% of rail trips: this accords with evidence for European cities used in the TRENEN models (see Calthrop *et al.* [13] and Proost and Van Dender [34]).

The model results are driven to a large extent by the choice of the various elasticities of substitution between modes ( $\sigma_B$  and  $\sigma_L$ ) in the demand structure. These parameter values are chosen such that the own-price elasticity of leisure auto demand equals -0.36. This appears to be in accordance with the general literature (see Calthrop *et al.* [13] Annex A; Goodwin [21] and Small [36]).

#### 3.2.2 Labour market parameters

The (average and marginal) benchmark labour tax rate is set at 0.38 (i.e. $\overline{t_L} = 0.38$ ). This is similar to other studies (see Lucas [27], Parry and Bento [32]). The labour supply elasticity is largely determined by the choice of  $\sigma_U$ . This parameter is chosen such that the (partial) uncompensated elasticity of labour supply equals 0.203. Sensitivity analysis is used to investigate the impact on results of alternative assumptions about this parameter.

#### 4 Simulation results

The model is used to compute welfare levels under a variety of assumptions concerning the uniform congestion toll  $(t_A)$  and the subsidy to auto-commuting trips (s). It is assumed that the real value of the benchmark transfer (T) to the consumer is held constant (recall equation 25). This implies that any increase in government revenue resulting from the road tolls (net of subsidy to auto-commuters)  $(t_A > \overline{t_A} = 0, s > \overline{s} = 0)$  is used to reduce existing labour taxes

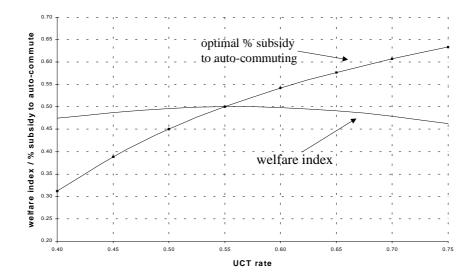


Figure 2: Optimal subsidy rates to auto-commuting

 $(t_L < \overline{t_L}).$ 

Figure 2 summarises the model results. The horizontal axis gives the rate of the uniform congestion toll (UCT). The vertical axis measures either a welfare index measure or the rate of subsidy to auto-commuting trips. The first curve of interest gives the optimal subsidy rate to auto-commuting for any given uniform congestion toll. This curve is rising: the higher the level of the uniform toll, the higher the optimal subsidy to auto-commuting. Thus, if the uniform congestion toll rate is 0.4, an optimal commuting subsidy would reimburse just over 30% of the toll fee.

Although not shown on Figure 2, the model results show that, in the absence of a uniform congestion toll, any positive commuting subsidy is welfare reducing. Moreover, this is the case for any uniform congestion toll below the rate of 0.24.

The possibility of subsidising auto-commuting affects the optimal uniform congestion toll. In the absence of commuting subsidies, model results show that the optimal uniform congestion toll is 0.42. However, the second curve shown on Figure 2 indicates that welfare is locally rising in the uniform congestion toll,

when combined with an (optimal) commuting subsidy. The optimal combination of instruments is a uniform congestion toll equal to 0.55 percent of the resource cost of a trip plus a subsidy to auto-commuting equal one-half of the congestion fee (0.275).

These model results accord with the intuition of the analytical model. Expression 23 shows that, in the optimum, road tolls are differentiated between different purposes in accordance with the compensated cross-price elasticities with the leisure market. The model results demonstrate that, for the benchmark scenario, optimal road tolls on commuting travel are nearly 50% lower than on non-commuting trips. This price structure is decentralised through a uniform congestion toll plus a toll-reimbursement to auto-commuters.

The analytical model is not well-suited to showing the optimal uniform congestion toll (in the absence of commuting-subsidies). Intuitively, the optimal toll is a weighted average of the optimal commuting- and non-commuting toll, where the weights depend on, amongst other factors, initial demand levels and demand reactions. The benchmark results indicate that the optimal uniform road toll is 0.42, compared to the optimal differentiated tolls of 0.55 (non-commuting) and 0.275 (commuting).

Table 1 shows the percentage change in transport demand levels in moving from the benchmark to the optimal set of taxes.

Table 1: percentage change in demand

$\max$ ket	% change
work_auto	-3
work_rail	+3.5
leisure_auto	-12.1
leisure_rail	+5.8

It is also clear from this table that labour supply increases (by around 0.25 percent). Peak period speed increase by 6.9 percent.

#### 4.1 Returning revenues lump-sum

If congestion toll revenues (net of any subsidy) are not used to reduce labour taxes, the net welfare cost of the toll increases. In terms of equation 25, any increase in congestion toll revenues (net of subsidies to commuting) are returned to the consumer via T rather than through reduced labour taxes: hence  $\tau = \overline{\tau}$ . The reduction in real wage from the increased toll produces a welfare loss on the (distorted) leisure market - the so-called 'tax-interaction' effect. Using the revenues to reduce labour taxes acts to reduce the net distortion on the leisure market - the tax-interaction effect is at least partially offset by a revenue recycling effect (see Goulder et al. [23] for more on this). Failing to take advantage of the revenue-recycling effect acts to increase the marginal cost of public funds in the model. Expression 14 suggests that, in this case, the net-social Pigouvian toll may fall.

Figure 3 confirms that this is the case in the model. Social welfare is maximised with a uniform congestion tax set at approximately 0.4, accompanied by a commuting subsidy of 68% of the uniform toll. The optimal commuting toll rate (equation 24) is approximately 0.32\*0.4=0.128. This is comparable to the case in which congestion tolls are used to reduce labour taxes: in the section above the optimal commuting toll was equal to 0.5\*0.55=0.275.

By using the revenues from congestion tolls less efficiently, it becomes, at the margin, less beneficial to shift drivers to more efficient modes of travel. To do so risks losing revenues at the margin, which are now more highly valued.

If a lower tier of government sets the road toll than that responsible for labour taxes, it seems plausible that congestion revenues will not be used efficiently. In this case, the benchmark model scenario suggest that uniform toll should be lower than they would otherwise be, and the optimal rate of subsidy on commuting travel is higher than otherwise.

Another striking model result concerns the optimal uniform congestion toll in the absence of a subsidy to auto-commuting: 0.28. This naturally differs from the 0.4 optimal rate in the presence of the subsidy.

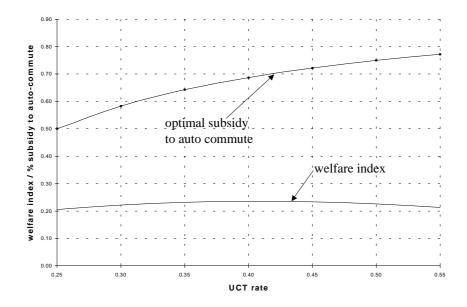


Figure 3: Subsidy to auto-commute (revenues returned lump-sum)

Table 2 contains the equivalent information to Table 2: changes in demands in moving from the benchmark to the new optimum (i.e. given lump-sum return of revenues).

Table 2: percentage change in demand

market	% change
work_auto	-0.7
work_rail	+0.8
leisure_auto	-9.4
leisure_rail	+4.5

Labour supply hardly rises. Speed on the congested road rises by 4.5 percent.

## 4.2 Sensitivity analysis

This section tests the extend to which the types of results derived for the benchmark model continue to hold under a wide variety of assumptions concerning

key parameters.

#### 4.2.1 Transport elasticities

A crucial parameter in determining the magnitude of optimal congestion fees is the elasticity of substitution between modes. The benchmark case was calibrated to an own-price elasticity of -0.36. An approximate span of elasticities reported in the literature might be taken to be -0.2 to -0.6. Table 3 shows the results under three different scenarios, each assuming that the congestion revenues are recycled via lower labour taxes. SC\_LOW is calibrated to an elasticity of -0.2; SC\_BNK repeats the benchmark results; and, SC\_HIGH is calibrated to an elasticity of -0.6.

Table 3: transport elasticities

	UCT	% subsidy to auto-commute
SC_LOW	0.69	0.71
SC_BNK	0.55	0.5
SC_HIGH	0.43	0.35

The level of the welfare maximising combination of a uniform congestion toll (UCT) and commuting subsidy vary significantly in function of the elasticity of mode-demand. The second column of the table shows that the optimal uniform toll falls as the absolute value of the elasticity is increased.

The third column shows that the optimal level of subsidy to commuting also falls in function of the elasticity. Simple calculations show that the effective tax rate on auto-commuting rises in function of the absolute value of the elasticity. For instance, under SC\_LOW, the effective tax rate on auto-commuting is (1-0.71)\*0.69=0.2. In contrast, for SC\_HIGH, the equivalent tax rate is (1-0.35)\*0.43=0.28.

#### 4.2.2 Elasticity of labour supply

The benchmark case was calibrated to a (partial) uncompensated labour supply elasticity of approximately 0.2. An approximate span of elasticities reported

in the literature might be taken to be 0.1 to 0.3. Table 4 shows the results under three different scenarios, each assuming that the congestion revenues are recycled via lower labour taxes. SC\_LOW is calibrated to an elasticity of 0.1; SC\_BNK repeats the benchmark results; and, SC\_HIGH is calibrated to an elasticity of 0.3.

Table 4: elasticity of labour supply

	UCT	% subsidy to auto-commute
SC_LOW	0.45	0.33
SC_BNK	0.55	0.50
SC_HIGH	0.65	0.57

The higher the labour supply elasticity, the larger the reaction on labour supply from a change in the real wage (via the commuting toll). It is therefore not surprising that the effective toll on auto-commuting is smaller for higher elasticities (e.g. SC\_HIGH = (1 - 0.57) \* 0.65 = 0.28; SC\_LOW = (1 - 0.33) \* 0.45 = 0.3). The same logic applies in reverse for the effect of an increase in the price of leisure travel.

## 5 Concluding remarks

A significant proportion of peak period road use is not related to commuting to- or from the workplace. It has been shown that, in the presence of distortionary taxes on labour supply, an optimal tolling system would set lower tolls on commuting trips than non-commuting trips. Trip purpose is not, however, observable to the government directly. This gives a rationale to a combination of a uniform congestion toll (paid by all auto-trips regardless of trip motive) plus a subsidy to auto-commuters.

The numerical model suggests that as much as 50 percent of the road toll should be reimbursed to commuters. Moreover, under a wide range of assumptions about key parameter values, some positive level of subsidy is welfare improving. Another very general result to emerge from the model is that in all

model versions, a subsidy to auto-commuting is welfare decreasing in the absence of a road toll. This casts doubt on the current policy of several European countries of reimbursing travel expenses in spite of the absence of road tolling.

Whilst demonstrating the general case for commuting subsidies, several caveats should be borne in mind before applying such results.

Standard economic demand theory does not deal explicitly with an important real-world phenomena: trip-chaining, whereby people combine several activities on one trip away from home. A more satisfactory formulation might be to allow utility to be a function of length of time spend in various activities (and goods consumed during that activity). The activities only occur at certain places and at certain times. This (non-convex) choice set is 'maximised' with respect to a budget constraint and a time constraint. The scheduling of activities becomes endogenous. This type of approach may help explain why a consumer may jump from working at A and shopping nearby at B, to working at Z and shopping at Y.

This is relevant to the findings of this paper. Should a driver that decides to go shopping on the way home from work be taxed differently from someone who proceeds directly home, changes his clothes, and then goes shopping? The simpler (and standard) formulation employed above does not give much insight into this problem.

Secondly, the link between commuting behaviour and labour supply is kept simple. If labour supply changes take the form of a fixed number of workers supplying extra hours per day, there are very different implications for congestion levels than if workers decide to work an extra day per week or non-workers are induced into the workforce. In addition, at least over the long term, changing the relative price of commuting may encourage some types of employees to work-at-home rather than commute.

Thirdly, the model has abstracted from issues of equity. However, this is clearly an important rationale for revenue raising (in addition to public good provision as modelled). Furthermore, the manner in which revenues are returned may be sensitive to degree of income inequality aversion in the social welfare function - see Mayeres and Proost [28] for a more detailed investigation of this issue.

Fourthly, the paper does not consider geographical space. As reviewed in the Introduction above, the case for commuting subsidies depends in part on the distortion to household location and work decisions. A more satisfactory model would integrate both quantity and location of labour with residential choice.

Finally, the numerical model is rather general. More detailed modelling is required to examine the impacts of actual (or proposed) policy measures.

#### 6 Annex 1: derivations

#### 6.1 Expressions 12 and 13

The Lagrangian to the government maximisation problem can be written as:

$$\Im [t_L, t_B] = u [V, A, B, X]$$

$$+ \lambda ((\beta (1 - t_L) - r) L - A - (1 + t_B) r B)$$

$$+ \mu (\beta L t_L + t_B r B - X/N)$$
(26)

Taking the first order conditions of this problem with respect to the two tax rates, and substituting from the first-order conditions of the consumer's maximisation problem (equations 8 and 9), gives the following two expressions:

$$\lambda B - \mu \left\{ B + rt_{B} \frac{dB}{dP_{B}} + \beta t_{L} \frac{dL}{dP_{B}} \right\} + u_{V} \phi^{'} \left[ \cdot \right] N \left( L + B \right) \left\{ \frac{dB}{dP_{B}} + \frac{dL}{dP_{B}} \right\} = 0$$

$$(27)$$

$$\lambda L - \mu \left\{ L - rt_{B} \frac{dB}{dP_{L}} - \beta t_{L} \frac{dL}{dP_{L}} \right\} - u_{V} \phi^{'} \left[ \cdot \right] N \left( L + B \right) \left\{ \frac{dB}{dP_{L}} + \frac{dL}{dP_{L}} \right\} = 0$$

In order to derive optimal tax expressions, the first order conditions need to be expressed in terms of partial rather than total derivatives. For example, the total effect on labour supply from an increase in the price of good B is:

$$\frac{dL}{dP_B} = \frac{\partial L}{\partial P_B} + \frac{\partial L}{\partial \phi} \frac{\partial \phi}{\partial L} \frac{dL}{dP_B} + \frac{\partial L}{\partial \phi} \frac{\partial \phi}{\partial B} \frac{dB}{dP_B}$$

Similar expressions can be derived for other effects. Solving two equations  $(dL/dP_B \text{ and } dB/dP_B)$  for the two unknowns, gives:

$$\begin{split} \frac{dB}{dP_B} &= \frac{\partial B}{\partial P_B} + \xi \phi^{'} \left[ \cdot \right] \left\{ \frac{\partial B}{\partial P_B} \frac{\partial B}{\partial \phi} + \frac{\partial L}{\partial P_B} \frac{\partial B}{\partial \phi} \right\} \\ \frac{dL}{dP_B} &= \frac{\partial L}{\partial P_B} + \xi \phi^{'} \left[ \cdot \right] \left\{ \frac{\partial L}{\partial P_B} \frac{\partial L}{\partial \phi} + \frac{\partial B}{\partial P_B} \frac{\partial L}{\partial \phi} \right\} \end{split}$$

in which the feedback term  $\xi$  is defined in equation 16. Symmetric expressions can be derived for the effect of an increase in the wage rate. These terms can then be substituted into the first-order conditions given in 27. Routine manipulations then gives the required expressions 12 and 13.

#### 6.2 Expression 17

Using the Slutsky decompositions  $(\partial I/\partial P_B) = S_{iB} - (\partial I/\partial T)B$  and  $(\partial I/\partial P_L) = S_{iL} + (\partial I/\partial T)L^{14}$ , where S represents the Slutsky matrix, expressions 12 and 13 can be re-written as:

$$\begin{pmatrix} (t_B - t_{BP}) \\ \beta(t_L - t_{LP}) \end{pmatrix} = -\left(\frac{\mu - \lambda'}{\mu}\right) \begin{pmatrix} S_{BB} & S_{LB} \\ S_{BL} & S_{LL} \end{pmatrix}^{-1} \begin{pmatrix} B \\ -L \end{pmatrix}$$
(28)

<sup>14</sup>This is a standard result from a model with a time constraint in which leisure is just the remainder after labour supply is subtracted from the time endowment. Marginally increasing the wage has two income effects: firstly, the cost function increases by an amount equal to the demand for leisure (the conventional income effect, negative if leisure is a normal good); secondly, it also increases the full-income available to the consumer by an amount equal to the endowment of time (the re-valuation of time effect, always positive). The net change in income is positive and equal to the time endowment minus leisure, i.e. labour supply. See discussion in Deaton and Muellbauer [18] pp.90 - and note the mistake in Varian [?] pp.146.

This model contains a more elaborate time constraint including travel times. However the derivate of the cost function (the change in income for the conventional income effect) is given by leisure plus travel time. Thus the net effect remains just labour supply.

in which the marginal social utility of income is defined in equation 18. Defining  $\varepsilon_{ik} \equiv P_k S_{ik}/I$  as the compensated elasticity of demand for commodity i with respect to the price of commodity k and using Slutsky symmetry ( $\varepsilon_{LB} = -\varepsilon_{BL} (P_B B/P_L L)$ ), expression 28 can be written as expression 17.

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