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OPTIMAL FACTOR INCOME TAXATION IN THE PRESENCE OF UNEMPLOYMENT

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Abstract

According to conventional wisdom internationally mobile capital should not be taxed or should be taxed at a lower rate than labour. An important underlying assumption behind this view is that there are no market imperfections, in particular that labour markets clear competitively. At least for Europe, which has been suffering from high unemployment for a long time, this assumption does not seem appropriate. This paper studies the optimal factor taxation in the presence of unemployment which results from the union-firm wage bargaining both with optimal and restricted profit taxation when capital is internationally mobile and labour immobile. In setting tax rates the government is assumed to behave as a Stackelberg leader towards the private sector playing a Nash game. The main conclusion is that in the presence of unemployment, the conventional wisdom turns on its head; capital should generally be taxed at a higher rate than labour.

Keywords: Optimal factor taxes, union wage bargaining, Stackelberg and Nash games

JEL Classification: H21, J51, C70
1. Introduction

In the literature on the optimal taxation the conditions for production efficiency were identified in a seminal paper by Diamond and Mirrlees (1971). According to their result, it is not desirable to distort production decisions as a part of the optimal tax package if (i) pure profits or pure rents are fully taxed away, and (ii) a full set of commodity taxes exists so that there will be one tax instrument for each margin of substitution in the consumer's utility function. Production efficiency is achieved if taxes on factors of production are not differentiated.

For various reasons, however, such as imperfect observability, legal constraints, etc. it may not be possible to fully tax pure rents, in which case the government is forced to rely on distortionary taxes. In the theory of optimal taxation, the Ramsey rule and its special case, the 'inverse elasticity rule', tell how distortionary taxes should then be designed so as to minimize the excess burden of the tax system. According to the inverse elasticity rule the government should levy the highest tax on the most inelastic activity. This argument lies behind the conventional wisdom concerning the taxation of capital income: internationally mobile capital should not be taxed or should be taxed at a lower rate than labour because capital is more sensitive than labour to changes in its own tax rates.¹

The application of the Ramsey rule or the inverse elasticity rule usually assumes that there are no market imperfections, in particular that the labour market clears competitively. At least in the case of Europe, which has been suffering from high unemployment for a long time, this assumption does not seem appropriate. Hence the question arises: should capital still be exempted from taxation or be taxed at a lower rate than labour in the presence of unemployment?

This paper shows that there are several reasons to argue that this conventional wisdom might not hold in the presence of unemployment. Firstly, under involuntary unemployment the supply of labour is locally infinitely elastic. According to the inverse elasticity rule this would suggest that labour should not be taxed at a higher rate than capital. Secondly, involuntary unemployment due to the wage rate being higher than the competitive wage rate means that the private marginal cost of labour exceeds the social marginal cost of labour. A way to increase employment and hence welfare is to

¹ Cf. e.g. Eggert and Haufler (1999) for a recent discussion of this argument.
subsidize labour input relative to capital input, for which social marginal cost equals the world interest rate.

We use the framework which has been developed by Koskela and Schöb (1998) to analyse the employment and welfare effects of a revenue-neutral factor tax reform, which increases the source-based capital tax and reduces the labour tax, to analyze the conditions for an optimal factor tax system. The model considers a small open economy, where the domestic production is represented by a single firm facing monopolistic competition from abroad. Capital is assumed to be perfectly mobile across countries, while labour is internationally immobile. Wage and thereby unemployment determination is modelled by the 'right-to-manage' approach, according to which the wage rate is negotiated in a bargaining process between the representative trade union and the firm and the firm then unilaterally determines employment. The government levies taxes subject to various constraints so as to maximize total surplus, which is linear in workers' net-of-tax wage income, the money-metric utility which the unemployed derive from leisure and unemployment benefit payments, and the net-of-tax profits.

We study the rules for optimal factor taxes in the presence of unemployment by starting from the benchmark case where the government is not restricted in taxing pure profits. Then we move on to the more realistic case of restricted profit taxation and explore its implications, both for individual factor taxes and for the structure of factor taxation. In the analysis of the optimal taxation we consider first a model with a Stackelberg game structure where the government chooses tax rates first, and the labour market organizations then determine the wage rate in a wage negotiation, taking the tax rates as given. The results derived from the Stackelberg game will then be compared with the results derived from a Nash game between the government and the labour organizations.

The paper is organized as follows: Section 2 presents the basic model and some qualitative results, while Section 3 sets up the social welfare maximization problem under the appropriate constraints. Welfare maximizing factor taxes with optimal profit taxation are derived in Section 4, while the properties of optimal factor taxes with restricted profit taxation are presented in Section 5. In Section 6 we relate our results to the existing literature. Finally, Section 7 concludes.
2. The model

We apply a framework which has been used by Koskela and Schöb (1998) to analyze the employment and welfare effects of a revenue-neutral tax reform which increases the source-based capital income tax and reduces labour taxes. We consider a small open economy, where domestic production is represented by a single monopolistic firm which produces good $Y$ with capital $K$ and labour $L$ as inputs. Capital is assumed to be perfectly mobile between countries while labour is internationally immobile. Technology is assumed to be linear-homogeneous and is represented by a constant elasticity of substitution production function

$$ Y = f(L, K). $$

(1)

The monopolistic firm faces output demand $D(p)$, which is decreasing in the output price $p$ and is assumed to be isoelastic, i.e.

$$ Y = D(p) = p^{-\varepsilon} $$

(2)

with $\varepsilon \equiv -\left(D(p)/\partial p\right) : p/Y$ denoting the output demand elasticity. The closer substitutes for good $Y$ on the world market are, the more elastic output demand becomes. The firm maximizes profits, which are given by

$$ \pi = p(Y)Y - \tilde{r}K - \tilde{w}L. $$

(3)

The firm considers the gross interest rate $\tilde{r}$ as given. It is given by the net-of-tax interest rate plus a source-based capital tax, i.e. $\tilde{r} = (1+t_r)r$ with $t_r$ denoting the capital tax rate. The gross wage $\tilde{w}$ consists of the net-of-tax wage $w$, which is negotiated between a trade union and the firm, plus the labour tax, i.e. labour taxes and social security contributions $t_w$, so that $\tilde{w} = (1+t_w)w$.

To guarantee a profit maximum, the output demand elasticity must exceed unity, i.e. $\varepsilon > 1$, in which case profit maximization implies that the firm will set a price which exceeds the constant marginal cost $c(\tilde{w}, \tilde{r})$ by a constant mark-up factor $\varepsilon/(\varepsilon - 1) > 1$.

All $N$ workers of the economy are represented by a trade union which maximizes its $N$ members' net-of-tax income. Each member supplies one unit of labour if employed, or zero labour if unemployed. The net-of-tax income of a working member hence equals the net-of-tax wage rate $w$. 
Being unemployed, a trade union member has an outside option $b$ which depends on the utility derived from leisure and the unemployment benefit transfers from the government. The objective function of the trade union can thus be written as

$$V^* = wL + b(N - L).$$

The wage rate is determined in a bargaining process between the trade union and the firm and the firm then unilaterally determines employment. This is modelled by using a 'right-to-manage' model which represents the outcome of the bargaining by an asymmetric Nash bargaining. The fall-back position of the trade union is given by $V^0 = bN$, i.e. if the negotiations break down, all members receive their reservation wage equal to the outside option. The fall-back position of the firm is given by zero profits, i.e. $\pi^0 = 0$. Using $V = V^* - V^0$, the Nash bargaining maximand can be written as

$$\Omega = V^\beta \pi^{1-\beta},$$

with $\beta$ representing the bargaining power of the trade union. The first-order condition with respect to the net-of-tax wage rate is

$$\Omega_w = 0 \iff \beta \frac{V}{V} + (1 - \beta) \frac{\pi}{\pi} = 0.$$

Using a CES production technology we will apply the explicit formulation of the wage elasticity of labour demand,

$$\eta_{Lw} \equiv L_w \tilde{w} / \tilde{w} = -\sigma + s(\sigma - \epsilon),$$

with $\sigma$ being the elasticity of substitution between labour and capital and $s = \tilde{w}L / cY$ being the cost share of labour (cf. Koskela and Schöb 1998) to further develop condition (6),

$$\Omega_w = 0 \iff (w - b)(\beta \eta_{Lw} + (1 - \beta)s(1 - \epsilon)) + w\beta = 0.$$

Equation (7) implicitly determines the negotiated net-of-tax wage from Nash bargaining as a function of the tax policy parameters $t_w$ and $t_r$ so that we have $w = w(t_w, t_r)$.
To derive the optimal tax formulas we have first to know how wage negotiations are affected by the tax system. We therefore provide some comparative statics results we will use later on. The effect of a change in the labour tax rate on the net-of-tax wage rate is

\[ W_{t_w} = -\frac{\Omega_{w_t}}{\Omega_{ww}} = -\frac{(w-b)zw}{y + (w-b)z(1 + t_w)}, \]

with \( y = \beta(1 + \eta_{L,z}) + (1 - \beta)(1 - \varepsilon)s \) and \( z = [\beta(\sigma - \varepsilon) + (1 - \beta)(1 - \varepsilon)]s_{z} \). As the second-order condition is assumed to hold throughout, i.e. \( \Omega_{ww} = y + (w-b)z(1 + t_w) < 0 \), we can infer that \( \text{sign}(w_{t_w}) = \text{sign}(z) = \text{sign}(-s_{w}) \) if labour and capital are price complements \( \sigma < \varepsilon \), as we will assume in what follows. (Note that \( \varepsilon > 1 \)). For a CES production technology, the partial derivative of the cost share of labour with respect to the gross wage rate is given by

\[ s_{w} = \frac{s}{w} (1 - s)(1 - \sigma) \begin{cases} > & \sigma < 1 \\ < & \sigma = 1 \\ \leq & \sigma > 1 \end{cases}, \]

so that we have

\[ W_{t_w} \begin{cases} < 0 & \text{as } \sigma < 1 \\ = 0 & \text{as } \sigma = 1 \\ > 0 & \text{as } \sigma > 1 \end{cases}. \]

If the elasticity of substitution is less than one, an increase in the labour tax rate will lead to an increase in the cost share of labour \( s \). A larger share \( s \) implies that a one percent change in the wage rate induces a larger increase in total and marginal cost and results in a lower output. This will lead firms to lay off more workers. Hence, the trade union benefits less from demanding higher wages. By contrast, if the cost share of labour \( s \) increased due to higher labour taxes, profits would fall at a higher rate if the trade union succeeded in increasing the net-of-tax wage. Therefore the firm will oppose wage increases more strongly and demand lower wages. An increase in the labour tax rate weakens the trade union’s bargaining position and strengthens the firm’s bargaining position at the same time. Both effects work in the same direction and the net-of-tax wage will fall. If, on the contrary, substitutability is high, e.g. \( \sigma > 1 \), the net-of-tax wage rate will rise.
An exogenous increase in the capital tax rate has an effect on the cost share of labour opposite to that of the increase in the labour tax rate.\textsuperscript{4} Hence, depending on the elasticity of substitution, the total effect of an increase in $t_r$ is:

$$w_r = \begin{cases} > 0 & \text{as } \sigma < 1 \\ = 0 & \text{as } \sigma = 1, \\ < 0 & \text{as } \sigma > 1 \end{cases}$$  

Finally, we consider the government budget. The government requires a fixed amount of tax revenues to finance the public good $G$ and, in addition, it has to pay unemployment benefits $b^0$ to all unemployed workers. Denoting the total number of workers by $N$, the number of unemployed workers is given by $N - L$. The government levies the labour tax $t_w$ on wage income and a source-based tax on domestic capital input $t_r$. In addition there is a profit tax $t_\pi$ on domestic profits so that the government budget constraint is given by

$$t_w wL + t_r rK + t_\pi \pi = G + b^0 (N - L).$$  

To focus on efficiency aspects of the optimal tax structure only, we consider the total surplus as an appropriate social planner’s objective function (cf. Summers, Gruber and Vergara 1993). (Distributional considerations enter only in that we allow for an unemployment benefit payment to sustain the existence minimum of unemployed workers.) Total surplus consists of the wage income equal to $wL$, which accrues to workers, $b(N - L)$, the money metric-utility unemployed derive from leisure and unemployment benefit payments, and the net-of-tax profit income $(1 - t_\pi)\pi$. As we hold $G$ constant we suppress the term $G$ in the welfare function. Furthermore, the income from the domestic capital stock is also assumed to be constant and therefore is not explicitly considered in the welfare function either. All domestic profits go to domestic capitalists.\textsuperscript{5} Hence, the welfare function is given by

$$S = wL + b(N - L) + (1 - t_\pi)\pi.$$  

\textsuperscript{4}This can be seen from deriving the cost share of capital $(1 - s)$ with respect to the capital tax rate (cf. Koskela and Schöb 1998).

\textsuperscript{5}For an analysis when foreigners receive a fraction of domestic profits, see Huizinga and Nielsen (1997).
3. Welfare maximization: basic setup

We start by considering a model with a Stackelberg game structure, where the government chooses tax rates first and the labour organizations then determine the wage rate in a wage negotiation, taking the tax rates as given. Later on, we examine the implications of a Nash game. Hence, the government maximizes welfare (12) subject to the budget constraint of the government (11), the outcome of the wage negotiation, which is implicitly given by the first-order condition of the Nash bargaining (7), and the constraint on the profit tax rate (14):

\[
\max_{t_w, t_r, t_z} S = wL + b(N - L) + (1 - t_z)\pi ,
\]

s.t.

\[t_w wL + t_r rK + t_z \pi = G + b^0(N - L) .
\]

\[\Omega_w = 0 \iff (w - b)\left(\beta \eta_{L,w} + (1 - \beta)s(1 - \epsilon)\right) + w\beta = 0 .
\]

\[t_z \leq \tilde{t}_z
\]

The Lagrangian for the welfare maximization is

\[
\mathcal{L} = wL + b(N - L) + (1 - t_z)\pi - \lambda (G + b^0(N - L) - t_w wL - t_r rK - t_z \pi) - \mu \Omega_w + \varphi (\tilde{t}_z - t_z) ,
\]

where \(\lambda\), \(\mu\) and \(\varphi\) describe the shadow prices of the constraints (11), (7) and (14), respectively.

The first-order conditions with respect to the profit tax rate, the two factor tax rates and the net-of-tax wage rate can be expressed (after some manipulations) as follows:

\[\lambda t_z = 0 \iff \pi(\lambda - 1) = \varphi , \tag{16a}\]

\[\lambda t_w = \left[w - (b - \lambda b^0) + \lambda t_w w\right]L\eta_{L,w} + \lambda t_r rK\eta_{K,w} + (\lambda - 1)(1 - t_z)\tilde{w}L - \mu \Omega_{w,t_z}(1 + t_w) = 0 , \tag{16b}\]

\[\lambda t_r = \left(w - (b - \lambda b^0) + \lambda t_w w\right)L\eta_{L,r} + \lambda t_r rK\eta_{K,r} + (\lambda - 1)(1 - t_z)\tilde{r}K - \mu \Omega_{w,t_r}(1 + t_r) = 0 , \tag{16c}\]

\[\lambda w = \left(w - (b - \lambda b^0) + \lambda t_w w\right)L\eta_{L,w} + \lambda t_r rK\eta_{K,w} - (\lambda - 1)(t_z - t_w/(1 + t_r))\tilde{w}L - \mu \Omega_{w,t_r}w = 0 . \tag{16d}\]
using the following expressions of the factor demand elasticities: \( \eta_{K,\bar{w}} = K - \bar{w}/K = s(\sigma - \varepsilon) \), 
\( \eta_{L,\bar{r}} = L - \bar{r}/L = (1 - s)(\sigma - \varepsilon) \) and 
\( \eta_{K,\bar{r}} = -\sigma + (1 - s)(\sigma - \varepsilon) \). By inspecting the complementary slackness condition

\[ \bar{r}_n - t_n \geq 0, \quad \varphi \geq 0, \quad \varphi(\bar{r}_n - t_n) = 0, \]

we can distinguish two cases. If \( \varphi = 0 \), the profit tax constraint is not binding and the government can choose the profit tax rate optimally. This case will be analysed in Section (4). The other case, where the constraint is binding, i.e. \( \varphi > 0 \), will be discussed Section (5).

4. Welfare maximization with optimal profit taxation

If \( \varphi = 0 \), the first-order condition with respect to the profit tax rate (16a) reduces to

\[ \lambda = 1. \] (17)

The shadow price \( \lambda \) represents the marginal cost of public funds. A marginal cost of public funds equal to one indicates that the government can raise taxes to meet its revenue requirement without imposing any cost on society. Hence, the economy faces no tax distortion but is left with labour market distortions. To analyse how these labour market distortions affect welfare, we subtract (16d) from (16b), using \( \lambda = 1 \). This yields

\[ -\mu \Omega_{w,\bar{w}} \left[ -\frac{\Omega_{w,\bar{w}}(1 + t_{\bar{w}})}{\Omega_{w,\bar{w}}} + 1 \right] = 0. \] (18)

As we know from the second-order condition, \( \Omega_{w,\bar{w}} < 0 \), the shadow price \( \mu \) must be equal to zero if the terms in brackets are non-zero. The first term in brackets represents the net-of-tax wage elasticity with respect to the labour tax. As can be seen from equations (8) and (10), this elasticity has a non-negative value for \( \sigma \geq 1 \). However, it takes a negative value for \( \sigma < 1 \), but as long as the labour tax rate is not fully shifted onto workers, i.e. as long as an increase in the labour tax rate increases the gross wage rate, the absolute value of the elasticity is below one. It is shown formally in Appendix 1 that the net-of-tax wage elasticity with respect to the labour tax is indeed always larger than \(-1\), which is also in conformity with empirical studies (cf. e.g. Lockwood and Manning 1993
and Holm, Honkapohja and Koskela (1994). Therefore, the terms in brackets must always be negative and condition (13) holds only if the shadow price $\mu = 0$.

This result suggests that if the government can use profit taxation without any restriction, i.e. apply non-distortive taxation, the Nash bargaining constraint is not binding. Intuitively, whatever net-of-tax wage rate is fixed in the wage negotiation between the trade union and the firm, the government can choose an appropriate wage tax or subsidy to obtain the gross wage which optimizes social welfare.

Solving the equation system (16b) and (16c) with respect to the factor tax rates and making use of $\varphi = 0$, $\lambda = 1$ and $\mu = 0$, we obtain:

$$t_c = 0,$$

and

$$t_w = \left( \frac{w - (b - b^0)}{w} \right).$$

These two tax rates ensure that both gross factor prices equal their social opportunity cost. From equation (1) it follows that the marginal productivity of capital, which equals the gross interest rate, also equals the world interest rate, $\bar{r} = r$, which is the true opportunity cost of domestic capital consumption. Any tax on capital would be borne by labour and profit income. As the capital owner always receives $r$, less capital input in domestic production due to a capital tax rate would thus reduce labour income and profits and hence welfare. It should be noted (with respect to the later analysis) that the production technology has no impact on this result.

**PROPOSITION 1:** If the government can set the profit tax optimally, it should not levy any capital tax.

This proposition has also been derived in the special case of the monopoly union model by Boeters and Schneider (1999) and by Richter and Schneider (2000). It is also a well-known property of optimal taxation for economies with competitive labour markets (see e.g. Bucovetsky and Wilson 1991).
Substituting into equation (20) the definition of the gross wage rate yields \( \tilde{w} = b - b^v \), i.e. the gross wage equals the disutility of labour, in other words the social cost of labour. Whatever the outcome in the wage negotiation is, the government can introduce a wage subsidy which guarantees that the marginal productivity of labour equals the marginal social cost of labour and thus maximizes social welfare. The wage subsidy is equal to the mark-up between the net-of-tax wage rate and the marginal revenue product of labour the wage negotiation yields, given this subsidy. This establishes full employment in the sense that there is no involuntary unemployment anymore. Hence, we have

**PROPOSITION 2:** If the government can set the profit tax optimally, it should levy a wage subsidy which completely offsets the mark-up between gross and net-of-tax wage rate as determined in the wage negotiations.

Proposition 2 confirms for a unionized labour market the result by Guesnerie and Laffont (1978) that in a first-best world, the output of a price maker should be subsidized such that the market price equals the marginal cost. Here, the trade union exercises monopoly power in the labour market by increasing the net-of-tax wage rate above the disutility of labour. Whatever mark-up is determined in the wage negotiations, the government can always subsidize labour so that the marginal product value of labour equals the marginal opportunity cost of labour. Hence, with unrestricted profit taxation, the government can always achieve production efficiency and thus restore the first-best allocation.

### 5. Welfare maximization with restricted profit taxation

So far we have focused on unrestricted profits. In practice, however, this is for several reasons the exceptional rather than the normal case. Firstly, tax authorities may have difficulties in distinguishing between pure profits and return to capital investments. Secondly, optimal profit taxation may be impossible if there are institutional or legal constraints. Hence, we now turn to the more relevant case where \( \varphi > 0 \), i.e. the profit tax constraint is binding and the profit tax rate is set at the upper bound for the profit tax rate \( \bar{\tau} \).
In this case, as profits are always positive, it can be seen directly from equation (16a) that \( \lambda > 1 \), i.e. the marginal cost of public funds exceeds unity. This means that the government has to apply distorting taxes to raise revenues for the finance of public goods. But this is not the only distortion the economy faces. As can be shown, the labour market constraint also becomes binding, i.e. the government cannot offset the inefficiency caused by setting the net-of-tax wage rate \( w \) above the social cost of working, \( b - b^0 \). Intuitively, the government has to apply distorting taxes to finance the wage subsidy. While a marginal mark-up due to the wage negotiation has only a second-order effect on welfare, the lower tax revenue requirement generates a first-order welfare gain. This is a standard second-best result according to which, in the presence of more than one distortion, it is not optimal to establish the first-best solution in only one sector.

Formally, the shadow price \( \mu \), which represents the social cost of labour market imperfection, can be signed by subtracting (16d) from (16b):

\[
\mu \Omega_{ww} w \left[ -\frac{\Omega_{w} (1 + t_r)}{\Omega_{ww}} + 1 \right] = -(\lambda - 1) wL < 0. \tag{21}
\]

As has been discussed in Section 3, the terms in brackets on the left-hand side are positive. Hence, condition (21) can hold only if \( \mu > 0 \), i.e. reducing the labour market distortion due to wage negotiations is always welfare improving (cf. Appendix 2). The lower the net-of-tax wage rate as a result of the wage negotiation, the lower the welfare loss of distortive taxes will be. This will be true irrespective of the question of whether the net-of-tax wage rate changes as a consequence of a tax rate change.

### 5.1 Optimal factor taxes when the net-of-tax wage rate remains unchanged

If the elasticity of substitution is unity, which is the case for a Cobb-Douglas production function, the net-of-tax wage rate is independent of the tax rates (cf. Section 2), so we have \( \Omega_{w} = 0 \). Using the calculations given in Appendix 3, the optimal factor tax formulas for this case are

\[
\left( \frac{t_r}{1 + t_r} \right)_{\sigma = 1} = \frac{1}{\varepsilon} \left( 1 - \frac{1}{\lambda} \right) (1 - t_r), \tag{22}
\]
and
\[
\left( \frac{t_w}{1+t_w} \right)_{\sigma=1} = -\frac{1}{\lambda} \left( \frac{w-(b-\lambda b^z)}{\tilde{w}} \right) + \frac{1}{\epsilon} \left( 1 - \frac{1}{\lambda} \right) \left( 1 - \tilde{r}_x \right).
\]  
\text{(23)}

Equation (22) shows that the capital tax becomes strictly positive. The positive capital tax results from the restrictive profit taxation, which forces the government to rely on distortionary taxation. The capital tax rate is higher, the lower the feasible profit tax rate \( \tilde{r}_x \) and the higher the marginal cost of public funds \( \lambda \).

A comparison of equation (23) with the optimal labour tax formula for unrestricted profit taxation, equation (20), shows that the labour tax rate is also higher. The first term on the right-hand side, which represents the subsidy component of the tax rate, is increasing in the marginal cost of public funds \( \lambda \). The reason for this effect is that the internalization has to be financed by distorting taxes and becomes more costly, the higher the marginal cost of public funds is. There is a second positive term, which one might refer to as the Ramsey component of the labour tax rate. This tax component represents the optimal tax one should levy on labour to minimize the excess burden of taxation. The wage subsidy part is at least partially offset by the Ramsey component. Hence, in the case of Nash wage bargaining with restricted profit taxation, a positive labour tax is possible as a part of the optimal tax treatment of factors of production. These results can be summarized in the following two propositions.

**PROPOSITION 3:** If the government cannot set the profit tax optimally and factor taxes will have no effect on wage negotiation, the government should levy a positive capital tax.

**PROPOSITION 4:** If the government cannot set the profit tax optimally and factor taxes will have no effect on wage negotiation, the optimal tax treatment of labour will consist of a subsidy component and a Ramsey tax component.

As a comparison with the capital tax formula shows that the Ramsey component of the labour tax equals the capital tax rate. Hence, as long as the net-of-tax wage is not affected by the tax system
the only difference in factor taxes stems from the wage subsidy paid to reduce the labour market distortion:

\[
\left( \frac{t_r}{1 + t_r} \right)_{\alpha=1} - \left( \frac{t_w}{1 + t_w} \right)_{\alpha=1} = \frac{1}{\lambda} \left( \frac{w - (b - \lambda b^o)}{\tilde{w}} \right) > 0, \quad (24)
\]

This is worth emphasizing in an additional proposition.

**Proposition 5:** If the government cannot set the profit tax optimally and the factor taxes will have no effect on wage negotiation, the capital tax rate should be higher than the labour tax rate.

Proposition 5 implies that in the absence of any labour market distortions, factor tax rates should be equal. The reason for equiproportional Ramsey components can be seen from applying the so-called 'inverse elasticity rule', according to which the government should levy the highest tax rate on the most inelastic activity. In the standard literature on taxing mobile capital (see e.g. Bucovetsky and Wilson 1991, Eggert and Haufler 1999), this argument has been put forward to justify a zero tax on capital, which is infinitely elastic in supply, and a positive tax on labour, whose supply elasticity is finite. However, in the presence of unemployment the result no longer holds. Firstly, under involuntary unemployment the supply of labour is locally infinitely elastic, which suggests according to the inverse elasticity rule that labour should not be taxed at a higher rate than capital. Secondly, there is a distortion in the labour market and the net-of-tax wage rate exceeds the marginal disutility of labour. This is an argument for the government to subsidize labour relative to capital.

### 5.2 Optimal factor taxes when the net-of-tax wage rate changes

Now we turn to the more general case where the elasticity of substitution between factors of production differs from one. In this case the outcome of the wage negotiation is affected by changes in factor taxation as we showed in Section 3. Solving the system of equations (16b)-(16c) for the CES production function case with respect to the tax rates and making use of \( \lambda > 1 \) and \( \mu > 0 \), we obtain the general optimal factor tax formulas.
\[
\left( \frac{t_c}{1 + t_c} \right) = \frac{1}{\varepsilon} \left( 1 - \frac{1}{\lambda} \right) \left( 1 - \bar{t}_x \right) + \frac{\mu}{\lambda} \left( \frac{\Omega_{stc} (1 + t_c)}{(1 - s)cY\sigma} \right) \tag{25a}
\]

and

\[
\left( \frac{t_w}{1 + t_w} \right) = -\frac{1}{\lambda} \left( w - (b - \lambda b) \frac{s}{\bar{w}} \right) + \frac{1}{\varepsilon} \left( 1 - \frac{1}{\lambda} \right) \left( 1 - \bar{t}_x \right) - \frac{\mu}{\lambda} \left( \frac{\Omega_{stc} (1 + t_w)}{s c Y \sigma} \right) \tag{25b}
\]

where

\[
\Omega_{stc} = (w - b) \left( \frac{s}{1 + t_w} \right) (1 - s)(1 - \sigma) \left[ \beta \sigma - \varepsilon + 1 - \beta \right] \begin{cases} < 0 \iff \sigma > 1 \end{cases}.
\tag{26}
\]

Compared with the formulas presented in Section 5.1, there is a new term – the second and third terms on the right-hand side in (25a) and (25b) respectively – which captures the effect that changes in the wage rate will have on the optimal factor taxes. Since we have already discussed the other terms, we will focus on these new terms only. From equation (25a) we can deduce

**Proposition 6:** If the government cannot set the profit tax optimally and factor taxes will affect the wage negotiation, the optimal capital tax should fall short of (exceed) the Ramsey component if the elasticity of substitution between capital and labour is smaller (greater) than one.

This result has a natural interpretation. If the elasticity of substitution between capital and labour is less than one, a fall in the capital tax rate decreases the net-of-tax wage rate as the outcome of Nash bargaining. The labour market distortion due to the difference between the net-of-tax wage \( w \) and the social marginal cost of labour becomes smaller. On the contrary, if the elasticity of substitution exceeds one, then a rise in the capital tax rate will decrease the net-of-tax wage rate and thereby reduce the labour market distortion.

With respect to the labour tax rate, we obtain

**Proposition 7:** If the government cannot set the profit tax optimally and factor taxes affect the wage negotiation, the optimal labour tax should exceed (fall short of) the Ramsey component plus
the wage subsidy if the elasticity of substitution between capital and labour is smaller (greater) than one.

Proposition 7 has an interpretation analogous to Proposition 6. With the elasticity of substitution being less than one, a rise in the labour tax rate decreases the net-of-tax wage rate so that the labour market distortion becomes smaller and vice versa. Then the labour market distortion can be decreased by raising the labour tax rate.

While the capital tax unambiguously exceeds the labour tax rate both in the presence of unrestricted profit taxation and in the case of a Cobb-Douglas production function, the general case needs further elaboration. Subtracting equation (25b) from equation (25a), we obtain

\[
\left( \frac{t_c}{1 + t_c} \right) - \left( \frac{t_w}{1 + t_w} \right) = \frac{1}{\lambda} \left( \frac{w - (b - \lambda b^*)}{\tilde{w}} \right) + \frac{\mu}{\lambda} \left( \frac{\Omega w_s (1 + t_w)}{c Y \sigma s (1 - s)} \right)
\]

(27)

where the second term on the right-hand side depends on the elasticity of substitution. Proposition 5 shows that when the factor taxes have no effect on the wage negotiation, the capital tax rate should be higher than the labour tax rate in the presence of unemployment. If the wage negotiation is affected by the factor taxes, then one should increase the capital tax and decrease the labour tax even further if the elasticity of substitution exceeds one. If \( \sigma \) is less than one, it is optimal to increase the labour tax rate and decrease the capital tax rate to alleviate the labour market distortion. For the latter case, it cannot be ruled out that the labour tax rate exceeds the capital tax rate. These findings are summarized in

PROPOSITION 8: If the government cannot set the profit tax optimally, the capital tax rate should be higher than the labour tax rate if the elasticity of substitution is greater than or equal to one. If the elasticity of substitution is less than one, then the relative size of optimal factor taxes remains ambiguous a priori.
5.3 A Nash game

So far we have assumed that the government acts as a Stackelberg leader. In this section we will briefly illuminate the alternative Nash assumption, where the government taxes the net-of-tax wage rate determined in wage negotiations as given and the labour organizations in turn take the tax rates as given (cf. Hersoug 1984).

The Nash equilibrium conditions for the determination of the labour tax rate by the government and of the wage rate as the outcome the wage bargaining can be easily derived. For the Nash bargaining solution we have already assumed that the labour organizations take the tax rates as given. Different to the maximization problem in Section 3, however, we now have to maximize welfare with respect to conditions (11) and (7) only because the government takes the net-of-tax wage rate as given. As this is equivalent to the maximization problem where the labour market distortion constraint is not binding, the optimal tax formulas are:

\[
\left( \frac{t_r}{1 + t_r} \right)_{\text{Nash}} = \frac{1}{\epsilon} \left( 1 - \frac{1}{\lambda} \right) (1 - \bar{t}_r), \tag{28}
\]

and

\[
\left( \frac{t_w}{1 + t_w} \right)_{\text{Nash}} = -\frac{1}{\lambda} \left( \frac{w - (b - \lambda b^*)}{\bar{w}} \right) + \frac{1}{\epsilon} \left( 1 - \frac{1}{\lambda} \right) (1 - \bar{t}_r). \tag{29}
\]

This can be summarized in a proposition.

**PROPOSITION 9:** If the government cannot set the profit tax optimally and the government and the labour organizations play Nash, capital taxes should always be non-negative and exceed the labour tax rate.

Note that if the profit tax rate can be set optimally, the capital tax rate should be zero and labour should be subsidized.\(^6\) In this case, the marginal cost of public funds is unity and the tax formulas are the same as in the Stackelberg game [equations (19) and (20)].

\(^6\) It is shown in Appendix 4 that under the assumptions made the Nash equilibrium is unique and stable.
6. Related literature

There is recent literature which deals with the optimal factor taxation in the presence of unemployment. One of the first papers dealing with optimal taxation in the presence of involuntary unemployment is Bovenberg and van der Ploeg (1996). Although they do not consider capital taxation but study optimal taxation, optimal provision of public goods and environmental policy, their analysis with respect to the optimal labour tax rate can easily be applied to models with internationally mobile capital as considered here. They show that (in the absence of any externalities) the optimal labour tax rate strikes a balance between two objectives. Firstly, the labour tax serves the purpose of raising tax revenues. Therefore, the labour tax rate should be higher (the labour subsidy lower), the higher the marginal cost of public funds and the lower the profit tax rate, ceteris paribus. Secondly, the subsidy component is used to offset the labour market rationing. As their results are derived for the case of rigid net-of-tax wage rates, our Propositions 1 and 4 generalize their result to the case of endogenous wage determination where the tax system might affect the net-of-tax wage rate.

Using a monopoly union model, Richter and Schneider (2000) show that if profit taxation is restricted, the capital tax may be used as an indirect tool to reduce the labour market distortion, i.e. the union’s ability to raise the net-of-tax wage above the marginal cost of labour, if it affects the labour demand elasticity and hence the monopoly power of the trade union. This result (see their Proposition 7(ii)) is in line with our Proposition 6 and shows that non-zero capital tax rates are in general desirable (i) to minimize the excess burden of taxation if profit taxation is restricted and (ii) to reduce the labour market distortion due to monopoly union power if the net-of-tax wage rate is affected by the capital tax rate.

Apparently, however, a result derived by Boeters and Schneider (1999) and Richter and Schneider (2000) for the monopoly union case and Fuest and Huber (1999) for the Nash bargaining for the optimal capital tax rate in the case of unrestricted profit taxation seems to be in sharp contrast to our Proposition 1. The reason for that is that they assume that profits are fully taxed away, i.e. that the profit tax rate is fixed at 100%. Hence, they actually consider a special case of restricted profit taxation rather than optimal profit taxation. This restriction applies if the tax revenue requirement
exceeds the profit income in the economy (which is only possible if factor taxes affect the net-of-tax wage rate). In this case the marginal cost of public funds is above unity and the labour market distortion constraint becomes binding. By contrast, in deriving Propositions 1 and 2, we allow the government to adjust the profit tax rate optimally, i.e. we assume that profit incomes exceed the government’s tax revenue requirement. In this case the Nash bargaining constraint is not binding and the first-best solution can be restored. If one would allow the government to set the profit tax optimally, our Proposition 2 could be confirmed in their models as well.\(^7\) As long as profit taxation is not restricted, labour market distortions do not justify a positive tax on mobile capital. Therefore, the well-known results of optimal taxation in economies with competitive labour markets (see e.g. Bucovetsky and Wilson 1991, Razin and Sadka 1991 and for a recent discussion Eggert and Haufler 1999) can be generalized if profits can be fixed optimally.

Boeters and Schneider (1999) also compare the model where the government is a Stackelberg leader with the model where there is a Nash game between the government which sets the tax rates, and the monopoly union which sets the net-of-tax wage rate.\(^8\) They show that under the Nash assumption the capital income should not be taxed and labour should be subsidized. This is a special case of our Proposition 9 as they assume that \(\tilde{t}_s = 1\). The more general result of Proposition 9 shows, however, that whenever profit cannot fully be taxed away, the capital tax rate should be positive. This confirms the results derived by Bruce (1992), Mintz and Tulkens (1996) and Huizinga and Nielsen (1997) for the case of competitive labour markets, namely, that if profit income cannot be fully taxed, a source-based capital tax serves as a tool to tax profit indirectly.

\(^7\) A proof is available upon request.

\(^8\) Fuest and Huber (1999) also analyze the Nash game between the government and the labour organizations. However, they assume that the government takes the gross wage as given. Although it does not matter whether one assumes that the net-of-tax wage or the gross wage is determined in wage negotiations for the Stackelberg game, the Nash outcome crucially depends on what the government considers to be unaffected by its own actions.
7. Conclusions

It is well known that if it is not possible to tax pure profits fully, the government is forced to rely on distortionary taxes. In the theory of optimal taxation, the Ramsey rule and its special case, the 'inverse elasticity' rule, tell how the distortionary taxes should be then designed so as to minimize the excess burden of the tax system. The inverse elasticity rule requires that the government levies the highest tax rate on the most inelastic activity. This argument lies behind the conventional wisdom that internationally mobile capital should not be taxed or should be taxed as a lower rate than labour.

Applications of the Ramsey rule or of the inverse elasticity rule usually assume that there are no other market imperfections, in particular that labour markets clear competitively. At least for Europe, which has been suffering from high unemployment for a long time, this assumption does not seem appropriate. Hence, it is important to ask whether the conventional wisdom, according to which capital should be taxed at a lower rate than labour, still holds in the presence of unemployment.

In this paper we have studied the optimal factor taxation in the presence of unemployment which results from the union-firm wage bargaining both with optimal profit taxation and with restricted profit taxation when capital is internationally mobile and labour immobile. Our main conclusion is that in the presence of unemployment the conventional wisdom turns on its head; capital should generally be taxed at a higher rate than labour. The optimal levels of factor taxes depend on specific features of the situation, like the game structure between the government and the private sector, the properties of production technology and the question of whether the unrestricted profit taxation is feasible or not. Countries with rigid labour markets should therefore be very careful in adopting tax policies which are appropriate for countries where labour markets are sufficiently flexible.
Appendix 1: Net-of-tax wage elasticity

Using the explicit formulations from the CES production function for the second derivatives, \( \Omega_{ww} = y + (w-b)z(1+t_w) < 0 \) and \( \Omega_{wtw} = y + (w-b)zw \) with \( y = \beta(1+\eta_{Lw'}) + (1-\beta)(1-\epsilon)s \) and \( z = [\beta(\sigma-\epsilon) + (1-\beta)(1-\epsilon)]k'_{e_w} \) so that the change in the net-of-tax wage rate due to a change in the labour tax rate is given by:

\[
W_{tw} = -\frac{\Omega_{wtw}}{\Omega_{ww}} - \frac{(w-b)zw}{y + (w-b)z(1+t_w)}. \tag{A1}
\]

Substituting this into the definition of the net-of-tax wage elasticity yields

\[
\omega_{tw} \equiv \frac{W_{tw}}{w} (1-t_w) = \frac{xz}{y(1+t_w)^{-1} + xz}.
\]

The condition \( \omega_{tw} > -1 \) holds if \( y < 0 \). Calculating the net-of-tax-wage rate from condition (7) yields

\[
w = y(\beta)^{-1}b \tag{A2}
\]

As \( w > b \) it follows immediately from inspection of (A2) that \( \beta > 0 \) implies \( y < 0 \). Hence, \( \omega_{tw} > -1 \). Q.E.D.

Appendix 2: \( \mu > 0 \) with restricted profit taxation

Subtracting (16d) from (16b) implies that the following condition must hold:

\[
\mu[\Omega_{ww}w - \Omega_{wtw}(1+t_w)] = -\mu(1)wL < 0 \tag{A3}
\]

so that the left-hand side of (A3) must be negative as well. Its bracket term can be rewritten as follows:

\[
\Omega_{ww}w - \Omega_{wtw}(1+t_w) = \Omega_{ww}w \left[ -\frac{\Omega_{wtw}(1+t_w)}{\Omega_{ww}w} + 1 \right] \tag{A4}
\]

where the first term in brackets on the right-hand side is the net-of-tax wage elasticity with respect to the labour tax rate. As has been proved in Appendix 1, it is larger than \( -1 \). Hence (A4) is negative and the condition (A3) holds only if \( \mu > 0 \). Q.E.D.

Appendix 3: Derivation of the optimal factor tax formulas

For the case \( \varphi = 0 \) and hence \( t_e = \bar{t}_e \), rearranging the equations (16b) and (16c) yields
\[
\begin{align*}
\begin{pmatrix}
  wL_n_{L,\tilde{\nu}} & rK_n_{K,\tilde{\nu}} & \lambda_{sL} \\
  wL_n_{L,\tilde{\nu}} & rK_n_{K,\tilde{\nu}} & \lambda_{sL}
\end{pmatrix}
= & \begin{pmatrix}
  (1-\lambda)(1-\tilde{t}_x)\tilde{w}L - \frac{(w-(b-\lambda b^0))}{w}wL_n_{L,\tilde{\nu}} + \mu\Omega_{w\tilde{\nu}}(1+t_w) \\
  (1-\lambda)(1-\tilde{t}_x)\tilde{r}K - \frac{(w-(b-\lambda b^0))}{w}wL_n_{L,\tilde{\nu}} - \mu\Omega_{w\tilde{\nu}}(1+t_w)
\end{pmatrix}, \\
\end{align*}
\]

(A5)

with \( \Omega_{w\tilde{\nu}} = -\Omega_{w\tilde{\nu}}(1+t_w)/(1+t_r) \) (cf. Koskela and Schöb 1998). Applying Cramer’s rule and using the fact that the determinant of the left-hand side matrix is equal to \( \Delta = wLrK\sigma\varepsilon \) yields

\[
\begin{align*}
\lambda_{sL} = & -\left(\frac{w-(b-\lambda b^0)}{w}\right) + \frac{(1-\lambda)(1-\tilde{t}_x)}{wL\sigma\varepsilon}\left[\tilde{w}L_n_{K,\tilde{\nu}} - \tilde{r}K_n_{K,\tilde{\nu}}\right] - \frac{\mu}{\lambda}\frac{\Omega_{w\tilde{\nu}}(1+t_w)}{\tilde{w}L_n_{K,\tilde{\nu}} - \tilde{r}K_n_{K,\tilde{\nu}}}, \\
\lambda_{sL} = & +\frac{(1-\lambda)(1-\tilde{t}_x)}{rK\sigma\varepsilon}\left[\tilde{r}K_n_{L,\tilde{\nu}} - \tilde{w}L_n_{L,\tilde{\nu}}\right] + \frac{\mu}{\lambda}\frac{\Omega_{w\tilde{\nu}}(1+t_w)}{\tilde{r}K_n_{L,\tilde{\nu}} - \tilde{w}L_n_{L,\tilde{\nu}}}.
\end{align*}
\]

(A6)

(A7)

Using the explicit elasticity formulas, we have

\[
\begin{align*}
\tilde{w}L_n_{K,\tilde{\nu}} - \tilde{r}K_n_{K,\tilde{\nu}} = cY\left(s\eta_{K,\tilde{\nu}} - (1-s)\eta_{K,\tilde{\nu}}\right) = cY\theta, \\
\tilde{r}K_n_{L,\tilde{\nu}} - \tilde{w}L_n_{L,\tilde{\nu}} = cY\left((1-s)\eta_{L,\tilde{\nu}} - s\eta_{L,\tilde{\nu}}\right) = cY\sigma(1-s).
\end{align*}
\]

(A8a)

(A8b)

Hence, we end up with conditions (25a) and (25b).

Appendix 4: Stability and uniqueness of the Nash game equilibrium

The optimal factor tax rates with restricted profit taxation under the Nash game between the government and the partners of the wage negotiation are given in equations (28) and (29). This appendix shows that under the assumptions made, the equilibrium of this game is unique and stable.

We first substitute the expression for the optimal capital tax into the equation for the optimal tax treatment of labour to derive – after some manipulation – the reaction function of the government given the wage rate and the optimal capital tax

\[
\begin{align*}
\tilde{w}L_n_{K,\tilde{\nu}} - \tilde{r}K_n_{K,\tilde{\nu}} = cY\left(s\eta_{K,\tilde{\nu}} - (1-s)\eta_{K,\tilde{\nu}}\right) = cY\theta, \\
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\end{align*}
\]

Hence, we end up with conditions (25a) and (25b).
\[
\left. \frac{dw}{dt} \right|_{t_w = 0} = -\frac{w}{1 + t_w} < 0 \tag{A13}
\]

and

\[
\left. \frac{dw}{dt} \right|_{\Omega_w = 0} = \frac{\Omega_{tw}}{\Omega_{ww}} \begin{cases} > 0 & \text{as } \sigma \begin{cases} < \sigma \end{cases} \right] = 1. \tag{A14}
\]

The stability of the equilibrium of this Nash game requires that the reaction curve for the wage bargaining must be flatter than that for the government. More precisely, the stability condition can be written as

\[
\Delta = \bar{w}_{t_w} \Omega_{ww} - \bar{w}_{w_w} \Omega_{ww} = -w \Omega_{ww} + (1 + t_w) \Omega_{ww} = -w \Omega_{ww} \left( 1 - \frac{\Omega_{ww} (1 + t_w)}{\Omega_{ww}} \right) > 0 \tag{A15}
\]

and it holds in our model irrespective of the size of the elasticity of substitution and the relative bargaining power of the trade union and the firm. It is easy to see that the stability condition is also independent of whether we have optimal or restricted profit taxation.

References


