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PRIVATIZATION UNDER  
ASYMMETRIC INFORMATION

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## PRIVATIZATION UNDER ASYMMETRIC INFORMATION

### Abstract

This paper models privatization as a cooperative game between the government, a trade union and the private shareholders. These players know that privatization increases the efficiency of a firm, but only the management of the firm knows the exact value of the relevant productivity-increasing parameter. This incomplete information changes many of the results which were attained in Bös (1991) in a full-information setting.

Keywords: Privatization, asymmetric information

JEL Classification: D82, L33

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# 1 Introduction

It has become well-known that privatization is high on the agenda of governments, but low on the agenda of trade unions. Therefore, any government which privatizes has to cope with the trade unions' political postulates. This paper is devoted to the question how the government compromises with a representative trade union when it comes to dealing with privatization and with its consequences for the workers in the formerly public firm. We assume that a representative trade union acts as antagonist to the government. We exclude the possibility that several trade unions representing various groups of the firm's employees each follow different objectives and have to coordinate their different interests in a complex system of hierarchical order. The representative trade union is interested in the firm preserving jobs, even if this leads to more inefficiency in production and less profit. It is, however, willing to agree to at least some firing of employees if the remaining employees receive part of the increased profits and if there is a 'social safety net' for the dismissed. Therefore, when deciding on privatization, the government enters into negotiations with the trade union about a plan of employee shares and financial compensation for the dismissed.

The government in this paper is perceived as an institution which has an ideological interest in privatization, but which also wants to draw money from selling its property and so willingly cooperates with private stockholders who share the government's interests. For an explanation we can assume that the government is interested in reducing the tax pressure, increasing particular public expenditures, reducing the public debt.

In the course of their negotiations government and trade union have to anticipate how the board of the firm will adjust to their compromise. This adjustment will be described as a cooperative game between the (private and public) stockholders and the representatives of the trade union in the firm. Moreover, government, trade union and the board of the firm will have to anticipate how the management of the firm adjusts to the decisions related to privatization and general firm policy.

The present paper builds on Bös (1991, pp. 149-176). This chapter of my book on the theory of privatization assumes perfect information of all actors. Several interesting results are derived, in particular:

- partial privatization never happens; either the firm remains in public ownership or it is fully privatized;
- private stockholders never get shares free of charge;
- the firm always employs more workers than is efficient (even if it is fully privatized).

The present paper abandons the assumption of perfect information and shows that the results of my previous approach change drastically.

Assume that a firm is to be privatized. The government, the trade union, the board of the firm and the private stockholders correctly anticipate that the productivity of the firm will

increase.<sup>1</sup> They also anticipate that the productivity increase will be more pronounced if more shares are sold to the public, that is, a fully privatized firm can be expected to realize higher productivity increases than a partially privatized firm. However, the government, the trade union, the board of the firm and the private stockholders do not precisely know how much this productivity will increase for a given degree of privatization. This is private knowledge of the management of the firm. We assume that the board of the firm, which is responsible for hiring the management, applies a direct mechanism to extract this private knowledge from the management. For this purpose it pays an incentive income to the management which includes an information rent. The inclusion of the management's incentive compatibility and participation constraints shapes the new results of the present paper.

The paper is organized as follows. In section 2 we deal with the firm which is a candidate for privatization. We describe its technology, the profit-sharing arrangements, and the decision making of the board and of the management. Section 3 presents the basic features of the three-stage game. In a first stage it is determined whether the firm should remain public or should be partially or fully privatized, at which price the shares are to be sold to private stockholders, and how many shares should be given to employees free of charge. In a second stage the board chooses the general course of production, that is, it decides on labor and capital inputs and on the price at which the output is to be sold. It also stipulates the management's incentive income. Finally, in a third stage the management chooses its effort, production takes place and the product is sold. The following sections 4 and 5 present the detailed analysis of the three stages of the game. A brief summary concludes.

## 2 The Firm

### *The Technology*

We assume a one-product monopolistic firm that produces output  $z$  according to the following production function:

$$z = g(\alpha\theta, e, K, N); \quad g_{\alpha\theta}, g_e, g_K, g_N > 0, \quad (1)$$

where subscripts denote partial derivatives. The firm uses as inputs managerial effort  $e$ , capital input  $K$  and labor input  $N$ . The technology also depends on the degree of privatization  $\alpha \in [0, 1]$ . If  $\alpha = 0$ , we have a fully public firm, if  $\alpha = 1$  the firm is fully privatized. Partial privatization is not excluded from our model. Privatization leads to changes in the organization of the firm and hence to changes in the technology.<sup>2</sup> However, to which

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<sup>1</sup>For empirical evidence see, for instance, Kikeri et al. (1992), Martin and Parker (1997), and Meggison et al. (1994).

<sup>2</sup>The technology  $g(\alpha\theta, \cdot)$  is a reduced-form representation of these organizational changes. In Bös and Peters (1988) we explicitly model how a technology  $g(\alpha, K, N, z)$  results endogenously from changes in the composition of the 'technological management' in a firm. Such an endogenization of  $g(\cdot)$  could also be done in the present model, however, this would imply a four-stage game instead of the present three-stage game, and considering the Bös and Peters (1988) specifications of the technological management would not change the qualitative results of the present paper.

extent the privatization changes the technology, depends on some productivity parameter  $\theta \in [\underline{\theta}, \bar{\theta}]$ , where  $\underline{\theta}$  is the worst type. The actual value of the productivity parameter is private knowledge of the management of the firm; it is not known by the government, the trade union, and the stockholders, who only know the density  $f(\theta)$ .<sup>3</sup>

The production function can be inverted to obtain an effort-requirement function

$$E = E(\alpha\theta, K, N, z); \quad E_{\alpha\theta}, E_K, E_N < 0, E_z > 0. \quad (2)$$

This function determines the minimal effort which is necessary in a firm of type  $\theta$  to combine inputs and outputs. We need the effort-requirement function for the definition of the management's incentive problem. Moreover, we shall need a third representation of the technology, namely the labor-requirement function<sup>4</sup>

$$N \equiv N(\alpha\theta, E(\cdot), K, z). \quad (3)$$

The firm covers all demand for its output,

$$z(p) = g(\alpha\theta, e, K, N), \quad (4)$$

where  $z(p)$  is the market demand which is decreasing in the price.<sup>5</sup> Producing output and selling it to the market leads to a profit

$$\Pi := pz - p_K K - p_N N - t, \quad (5)$$

where the interest rate  $p_K$  and the wage rate  $p_N$  are exogenously given, whereas the management's incentive income  $t$  will be endogenized.

### *Sharing the Profit*

The profit is shared between employees, private stockholders, and the government. The sharing depends on the extent of non-employee shares, denoted by  $\phi \in [0, 1]$  and on the degree of privatization, denoted by  $\alpha$ . The profit is shared as described in what follows.

The *employees* get  $(1 - \phi)$  percent<sup>6</sup> of the profit on the basis of their employee shares.<sup>7</sup> The employees do not have to pay for the shares; on the other hand, they have no voting rights in the stockholders' meeting. Hence the employee shares are the basis of some

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<sup>3</sup>Private information about costs is the best-known paradigm of regulation under asymmetric information. The paper which introduced this paradigm is Baron and Myerson (1982); well-known further papers are Freixas and Laffont (1985), Laffont and Tirole (1986, 1990). Laffont (1994) gives a good overview of the problem.

<sup>4</sup>This is a simple identity, for details compare Bös (1994), p. 307, footnote 5.

<sup>5</sup>We assume a price elasticity of demand  $z_p p/z < -1$  for  $p > P^o$  where  $P^o$  is an arbitrarily chosen high price level. For prices  $p > P^o$  revenue decreases with increasing price. This assumption is necessary to guarantee the existence of the Nash equilibrium in this paper.

<sup>6</sup>It is convenient to speak of  $\phi$  percent and of  $\alpha$  percent although  $\phi$  and  $\alpha$  are figures between zero and one. Purists may prefer always to multiply  $\phi$  and  $\alpha$  by 100.

<sup>7</sup>For special problems of employee shares in the case of privatization see, for instance, Grout (1984), and Bös and Nett (1991).

percentage of profits transferred to employees. They do not influence the firm's decision process in any direct way. (Indirectly, of course, it matters for the firm's decisions how much of the profit remains with the government and with the private stockholders.) The employees' share in the profit is evenly spread over the working force which was active when the privatization started,  $N^o$ . All employees who worked in the firm prior to privatization are given shares. This implies a sort of 'social safety plan' for those who are dismissed in the course of privatization,  $N^o - N$ . For the other employees it is a simple transfer.

The *private stockholders* get  $\alpha\phi$  percent of the profit: this are the dividend payments they receive. Let the shares be indexed by  $h$  and assume that a share represents an ownership of  $\alpha_h$  percent of the firm,  $0 \leq \alpha_h \leq 1$ ,  $\sum \alpha_h = \alpha$ . The ownership right  $\alpha_h$  entitles stockholder  $h$  to claim dividend payments of  $\alpha_h\phi\Pi$ . Consequently, a potential purchaser of an ownership right in  $\alpha_h$  percent of the firm is willing to pay up to  $\alpha_h\phi\Pi$ . We denote the purchaser's payment by  $\alpha_h s\Pi$ , where

$$0 \leq s \leq \phi. \quad (6)$$

In the following we will speak of  $s$  as the issue price of the shares.

Finally, the *government* gets the remaining  $(1 - \alpha)\phi$  percent of the profit. It has to trade off these revenues from its remaining shares and the revenue from selling shares,  $s\alpha\Pi$ . Therefore, the government enters the privatization game only if its revenues do not fall below some threshold  $R^o$ :<sup>8</sup>

$$(1 - \alpha)\phi\Pi + s\alpha\Pi \geq R^o; \quad R^o > 0. \quad (7)$$

The government revenue constraint has some interesting implications. First, the positive revenue requirement implies that only profitable firms will be privatized. (Given our assumptions on  $s, \alpha$ , and  $\phi$ , positive  $R^o$  requires positive  $\Pi$ .) Second, the government revenue requirement excludes  $\phi = 0$ . In this case the government would neither receive money from its remaining shares ( $\phi = 0$ ) nor receive money from selling shares. ( $s = 0$  is implied by  $\phi = 0$ , otherwise private stockholders would not be willing to accept the shares.) Hence  $R^o$  could not be positive and the government would not play the game. This result sounds reasonable. We would not expect a privatization campaign to lead to such an intensive participation of employees that they are awarded all the profits. Note, by the way, that  $s = 0$  is not a priori excluded by the government revenue requirement.

### *The Board of the Firm*

Decisions on prices, labor and capital inputs, and on the management's compensation are made on the basis of a compromise between representatives of the (public and private) stockholders and representatives of the trade union. This follows Aoki (1980, 1982) who models the firm as a *stockholder-employee cooperative game*. The cooperative game can

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<sup>8</sup>In the simple partial model of this paper we do not explicitly explain how  $R^o$  came about, but take it as exogenously given.

be implemented by various institutional arrangements. First, the stockholder representatives can be identified as the board of the firm, which compromises with the trade-union representatives. Second, if the German model of ‘codetermination’ is applied, both stockholder and trade-union representatives are part of the board, so that the compromise takes place within the board. It will be convenient not to distinguish between these different institutional arrangements, but always to speak of a decision made by the board with the understanding that this decision is the result of a cooperative game between stockholder and trade-union representatives.

In our model there are public and private stockholders to whom we ascribe the same objective function: there is a permanent coalition between government and the private stockholders. The government is not interested in a welfare objective, which would most probably be challenged by the private stockholders: rather, the government is interested in profit, and so are the private stockholders. Taking together their shares in the profit of the firm, the government and the private stockholders calculate the non-employee share in profits as  $\phi\Pi$ . The government’s interest in high profits of the firm goes back to some basic insight which can be attributed to a conservative government. Higher profits are thought of as leading to a long-run improvement of the private economy, in contrast to short-run Keynesian policies. Moreover, the (conservative) government has an ideological interest in privatization and this interest is fully shared by the (conservative) private stockholders. Hence all of the stockholders in the firm can be characterized by an objective function

$$V = V(\phi\Pi, \alpha); \quad V_1, V_2 > 0. \quad (8)$$

The employees in the firm are represented by a trade union which is interested in realizing high dividends from employee shares, but is also interested in keeping jobs at the firm. Hence we impute to the trade union an objective function

$$W = W\left(\frac{(1-\phi)\Pi}{N^o}, N\right); \quad W_1, W_2 > 0. \quad (9)$$

This function captures the conflict of interests inherent in the trade union. On the one hand, the union is interested in high profits, because the dividends from employee shares are higher, the more profitable the firm. On the other hand, the union is interested in jobs, even if this reduces profits.

The compromise between stockholders and trade union is modelled as a cooperative Nash equilibrium. Both players have concave von Neumann-Morgenstern utility functions. If cooperation fails, the worst that can happen are utility levels  $V^o$  and  $W^o$  which we treat as given in the model. We assume the utilities which result from cooperation exceed these reservation levels:  $V^* > V^o$ ,  $W^* > W^o$ . As in Aoki (1980), this implies that both players have appropriate threat potential so that the utilities of the cooperative solution exceed the utilities of open conflict.

It will be convenient to use the following abbreviations for the bargaining powers of the



two players:<sup>9</sup>

$$b_{V_i} := \frac{V_i}{V - V_o} f(\theta), \quad b_{W_i} := \frac{W_i}{W - W_o} f(\theta); \quad i = 1, 2, \quad (10)$$

where  $b_{V_i}, b_{W_i} > 0$  result from our assumptions.

### *The Management*

We impute to the management a utility function of the following type<sup>10</sup>

$$U = t - \psi(e). \quad (11)$$

The disutility from labor is measured by the money metric  $\psi(e)$ . The management dislikes high effort,  $\psi'(e) > 0$ , with an ever-increasing intensity,  $\psi''(e) > 0$ .<sup>11</sup>

The optimization of the management's utility must be taken into account by the board of the firm. However, there is asymmetric information: the management is better informed about the technology than the government, the private stockholders and the trade union. The other parties cannot observe the management's effort (hidden action) and the productivity parameter (hidden information). On the other hand, we shall apply a sort of Laffont-Tirole approach<sup>12</sup> by assuming that the other parties are able to observe capital and labor inputs, and the output price. To overcome the problem of asymmetric information about  $e$  and  $\theta$ , the board of the firm applies a direct mechanism, that is, it stipulates a contract with the following properties:

- if the management announces the correct productivity parameter, it gets an income which depends on the announced value of the parameter. Whether the announcement is correct, is checked by the planner: the board's observations must be equal to the input quantities  $K, N$  and the price  $p$  which are derived from the board's optimization, given the announced value of the productivity parameter;
- no income is paid if the management is caught lying;
- the income is defined in such a way that truth-telling is the management's dominant strategy (*incentive compatibility*, IC):<sup>13</sup>

$$\dot{U}(\theta) = -\psi' \alpha E_{\alpha\theta} \begin{cases} > 0 & \text{if } \alpha > 0, \\ = 0 & \text{if } \alpha = 0. \end{cases} \quad (12)$$

It is plausible that the income has to be chosen in such a way that the management's utility is increasing in the productivity parameter. The management is inclined to understate the parameter. However, since this reduces its utility, it will not do so. Note that

<sup>9</sup>See Aoki (1980: 605).

<sup>10</sup>This type of utility function is very usual in the literature, see for instance Laffont and Tirole (1993).

<sup>11</sup>Additionally  $\psi''' \geq 0$ . This technical assumption prevents the optimality of stochastic incentive schemes. See Laffont and Tirole (1990: 5).

<sup>12</sup>As introduced in Laffont and Tirole (1986).

<sup>13</sup>The detailed derivation of the IC condition is presented in appendix 1. This appendix also contains the assumptions needed to satisfy the second-order IC condition, that is, to avoid bunching.

there is only one case where the managerial utility is not increasing in  $\theta$ , that is, where the management does not get an information rent for the revelation of the productivity parameter: this occurs if there is no privatization and, therefore, any revelation of the productivity parameter is worthless.

Finally, the board has to consider the management's participation constraint

$$U(\theta) \geq U^o, \tag{13}$$

where  $U^o$  is the management's reservation utility.

### 3 The Three-Stage Game

#### *Timing of Events*

We consider the following three-stage setting:

- The first stage is a cooperative game where the government and a representative trade union determine the degree of privatization, the issue price at which shares are sold to private stockholders, and the extent of employee shares.
- At the second stage the board of the firm decides on the general course of production, that is, on the amount of capital and labor inputs and on the price at which the output is to be sold. The board also chooses the management's incentive income. As already mentioned, this decision of the board is always based on a compromise between stockholder representatives and trade-union representatives, which is modelled as a cooperative game.
- In the third stage the management of the firm announces the value of  $\theta$  and chooses its effort level. The output is produced and sold to the consumers.

In the subgame-perfect solution at every stage the subsequent stages of the game must perfectly be anticipated. In this paper this is done in the following way: Stage 3, the management's decision, is anticipated at stage 2 by explicitly considering the management's incentive-compatibility and participation constraints. Stage 2, the firm's decision, and stage 3, the management's decision, are anticipated at stage 1 *because the stage-one and the stage-two game are described by the same optimization approach*. In the following we will explain why this is the case.

#### *The Optimization Approach of Stages 1 and 2*

It is immediately plausible to impute to the trade-union representatives at stage 1 the same objective function  $W(\cdot)$  which the representatives of the same trade union employ at stage 2. Now recall our assumption of a permanent coalition between government and private stockholders. Therefore, it is equally plausible to impute to the government representatives at stage 1 the same objective function  $V(\cdot)$  which the public and private stockholders employ at stage 2. Let us further assume that the security levels  $V^o$  and  $W^o$  are identical at stages 1 and 2. This amounts to assuming that the cooperation in the stage-one game is cancelled if the trade union and the stockholders fail to reach

cooperation at the firm level. Therefore, in the stage-one and in the stage-two subgame the same objective function is maximized:

$$\int_{\underline{\theta}}^{\bar{\theta}} [\log(V - V^o) + \log(W - W^o)] f(\theta) d\theta. \quad (14)$$

This is the usual formula for a cooperative Nash solution. However, since at stages 1 and 2 the players are not informed about the true value of the productivity parameter  $\theta$ , they maximize an expected value. This accounts for the anticipation of stage 3 of the game, that is, the management's announcement of the productivity parameter.

Next we have to show that the constraints of the optimization approaches at stage 1 and at stage 2 can be chosen identically. Let us present all relevant constraints, adding in brackets the associated Lagrangean parameters.<sup>14</sup> We have:

$$z(p) = g(\alpha\theta, e, K, N), \quad (\lambda) \quad (15)$$

$$s \leq \phi, \quad (\nu) \quad (16)$$

$$R^o \leq (1 - \alpha)\phi\Pi + s\alpha\Pi, \quad (\pi) \quad (17)$$

$$\alpha \leq 1, \quad (\xi) \quad (18)$$

$$\phi \leq 1, \quad (\tau) \quad (19)$$

$$dU/d\theta = -\psi' \alpha E_{\alpha\theta}, \quad (\mu) \quad (20)$$

$$U(\underline{\theta}) = U^o, \quad (21)$$

$$U(\theta) \geq U^o \quad \text{for all } \theta > \underline{\theta}. \quad (22)$$

To simplify the notation, in the above constraints we have suppressed the explicit dependence of all instrument variables on the productivity parameter  $\theta$ , that is,  $\alpha(\theta)$ ,  $p(\theta)$ , etc. We have also suppressed the explicit dependence of the Lagrangean multipliers on the productivity parameter  $\theta$ , that is,  $\lambda(\theta)$ ,  $\nu(\theta)$ , etc. (The multipliers depend on  $\theta$  because the constraints are defined for every single  $\theta$ , for example, there is not only one government revenue requirement, but there are as many constraints as there are  $\theta$ 's.) Moreover, the non-negativity of  $s$  and  $\alpha$  is not introduced as explicit structural constraints.<sup>15</sup>

Are all of these constraints relevant at both stage 1 and stage 2? Let us begin with stage 2, the decision at the board level. It is evident that the constraint on the market equilibrium and the input policy is relevant. The particular constraints on  $s$ ,  $\alpha$ , and  $\phi$  can be added to the board's optimization problem without doing harm, since  $s$ ,  $\alpha$ , and  $\phi$  are exogenously given in the stage-two game and no board instruments enter these three constraints. It is also evident that the management's incentive-compatibility and participation constraints are relevant for the board's decision. However, what about the government revenue requirement? We assume that the government has the threat potential to ensure that its

<sup>14</sup>We do not mention associated Lagrangean parameters of the management's participation constraints (21) and (22) because (21) is considered as initial condition of the control-theoretic approach and (22) follows from (21) and the incentive-compatibility condition (20) since  $-\psi' \alpha E_{\alpha\theta} \geq 0$ .

<sup>15</sup>These constraints will be taken care of in the necessary optimum conditions. See Panik (1976: 297) for this sort of procedure.

revenue constraint is accepted when the decision is made by the board. The private stockholders and the trade union's representatives know that privatization and employee shares are worthwhile for the government only if its revenue requirements are met. If this constraint is not accepted, the government will reverse its policy which would be detrimental to the private stockholders and the trade union. – Let us next turn to stage 1. It is immediately clear that the government revenue requirement and the constraints on  $s, \alpha$ , and  $\phi$  are relevant. However, since the players at stage 1 anticipate the board's and the management's adjustment, the stage-one players must also take account of the market equilibrium and the input policy of the board and of the management's decision.

Given the formal identity of the stage-one and the stage-two optimization approaches the *strategic connection of the two stages* is as follows:

(i) In the stage-one subgame, the adjustment of the board and of the management are anticipated. Accordingly, the results of stages 2 and 3 of the game are taken into account. Hence at stage 1 the optimization problem is solved with respect to *all* unknown variables. Calculating the optimal values of  $s, \alpha$ , and  $\phi$  is the basis of setting these instruments on the basis of the compromise between government and trade union. Calculating the optimal values of  $N, K$ , and  $p$  means the anticipation of the board's adjustment to the stage-one game. These variables are only *calculated* in the stage-one game, but are *not set* on the basis of this game. The players of the stage-one game take the role of a Stackelberg leader, fully exploiting their knowledge of the adjustment of the board. Similarly, calculating the optimal value of effort  $e$  means the anticipation of the management's adjustment to stages 1 and 2 of the game.

(ii) The board always plays second and therefore can only take the role of a Stackelberg follower. The two players of the stage-two subgame are not established prior to the setting of  $s, \alpha$ , and  $\phi$ . Hence they cannot strategically anticipate the adjustment of the privatization policy to the board's behavior. The best they can do is to find the optimal policy given some extent of privatization, and of employee shares. Therefore, in the stage-two game the optimal values of  $N, K$ , and  $p$  are calculated as the basis of setting these instruments by the board. However, calculating the management's effort level  $e$ , once again only means anticipating the management's adjustment.

In the following sections we shall present the results of our model and give an economic interpretation. The details of the optimization and all proofs can be found in appendix 2. It should be possible to understand the results of the model without necessarily reading the proofs.

## 4 Privatization and Employee Shares

Let us begin with the optimization approach of stage 1, the decision on privatization and employee shares. There are two main determinants of the economic results of the stage of the game, namely, the lack of information about the privatization productivity and the bargaining powers of the two players.

### Lack of Information

The results of the model depend decisively on the players' lack of information about the productivity parameter  $\theta$ . This influence is captured by the variable  $\Omega$ :

$$\Omega := b_{V2} - \lambda g_{\alpha\theta} - \mu \psi' \left[ E_{\alpha\theta} + \alpha\theta \frac{\partial E_{\alpha\theta}}{\partial(\alpha\theta)} \right]. \quad (23)$$

Several of the following propositions will decisively depend on the sign of  $\Omega$ , and it can clearly be seen that this sign, in turn, depends on various incentive-correction terms, that is, terms which result from the differentiation of the management's incentive-compatibility constraint (12). Note that it is the IC-constraint which is decisive. This clearly shows that it is really the asymmetric-information problem which shapes the results. In appendix 2 it is shown that the following two conditions are sufficient to ensure  $\Omega > 0$ :

**Condition 1:**  $\partial E_{\alpha\theta} / \partial(\alpha\theta) \geq 0$ .

**Condition 2:**  $\partial E_{\alpha\theta} / \partial K \leq 0$ .

(Condition 2 is needed to ensure that the Lagrangean multiplier of the market-clearing condition  $\lambda < 0$ .)

What is the economic meaning of the conditions 1 and 2? Condition 1 implies that an increase in  $\alpha\theta$  decreases the absolute value of the marginal rate of transformation between effort  $e$  and privatization productivity  $\alpha\theta$  in the labor-requirement function: an increase in the privatization productivity makes it more difficult for the management to exploit the improvement in  $\alpha\theta$  by reducing effort  $e$ . This can be proved as follows.<sup>16</sup> Consider the labor-requirement function,

$$N \equiv N(\alpha\theta, E(\alpha\theta, K, N, z(p)), K, z(p)). \quad (24)$$

Differentiation with respect to  $\alpha\theta$  yields<sup>17</sup>

$$0 = \frac{\partial N}{\partial(\alpha\theta)} + \frac{\partial N}{\partial e} E_{\alpha\theta}, \quad (25)$$

or, equivalently,

$$E_{\alpha\theta} = - \frac{\partial N / \partial(\alpha\theta)}{\partial N / \partial e} =: MRT(e, \alpha\theta). \quad (26)$$

This marginal rate of transformation between effort and privatization productivity  $\alpha\theta$  is negative which fits in with the assumption  $E_{\alpha\theta} < 0$ . It consists of two effects.  $\partial N / \partial(\alpha\theta) < 0$  because the labor requirement is lower the better the type of the firm. In the same way  $\partial N / \partial e < 0$  because the labor requirement is lower the higher the management's effort. Condition 2 can be interpreted in a similar way.

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<sup>16</sup>Cfr. Laffont and Tirole (1990: 15-6).

<sup>17</sup>This is a comparative-static analysis at the optimum: how does an infinitesimal change of  $\alpha\theta$  influence the labor-requirement function *ceteris paribus*, that is, holding constant  $K, N$  and  $z(p)$  at their optimal level.

## Bargaining Powers

The relation between the government's and the trade union's bargaining power can be used to distinguish three different types of firms:

- the conservative firm with a positive value of firm power  $F$ ,

$$F := \left[ b_{V1} - \frac{b_{W1}}{N^o} - \pi(1 - \alpha) \right], \quad (27)$$

- the trade-union-dominated firm with  $F < 0$ ,
- the neutral firm with  $F = 0$ .

If the firm is fully privatized, the sign of  $F$  depends only on the comparison between the government bargaining power  $b_{V1}$  and the trade union bargaining power  $b_{W1}/N^o$ . If parts or all of the firm remain in public ownership, the trade union has to trade off its interest in profit and its interest in jobs, where the latter is measured by  $\pi$ , the Lagrangean parameter associated with the government budget constraint.  $\pi$  is associated with the trade union because it is the only player in the game whose interest in jobs may imply an interest in lower profits and hence in a binding government revenue requirement. The government itself is interested in a high value of the firm. Its revenue requirement is a minimum constraint and it is quite happy if the constraint is not binding because the firm is very profitable. Therefore, the government has no particular interest in the revenue constraint being binding.

We are now in a position to present and interpret the various propositions which result from maximizing our model with respect to the degree of privatization, the issue price at which the shares are sold to private stockholders, and the extent of employee shares. The details of the optimization and all proofs can be found in appendix 2.

### Proposition 1 (Degree of Privatization)

- 1.1 If  $\Omega > 0$ , partial privatization never happens; either the firm is fully privatized or it remains in full public ownership. If  $\Omega = 0$ , the firm may be fully privatized, partially privatized, or remain in full public ownership. If  $\Omega < 0$ , the firm always remains in full public ownership.
- 1.2 A trade-union-dominated firm never remains in full public ownership; it is either partially or totally privatized. The private stockholders get nothing. The government gets its minimum revenue requirement  $R^o$ .
- 1.3 For conservative or neutral firms, the following cases may occur if  $s > 0$ :
  - $\pi < 0, \nu = 0$ : the firm remains in full public ownership. The government gets its minimum revenue requirement  $R^o$ ;
  - $\pi < 0, \nu < 0$ : privatization where the private stockholders get nothing. The profit is only high enough to meet the government revenue requirement;

–  $\pi = 0, \nu = 0$ : if  $\Omega > 0$ , the firm is fully privatized. The zero Lagrangean multipliers imply that any combination of binding and not-binding revenue constraint and issue price constraint is possible.

### *Economic Interpretation 1*

This proposition is decisively different from its counterpart in Bös (1991), pp. 158-162, which showed that in the perfect-information setting partial privatization could never happen. We now show that in the asymmetric-information setting partial privatization can be optimal if  $\Omega = 0$ . To understand this result, let us consider the marginal condition  $\mathcal{H}_\alpha$ , which results from differentiating the Hamiltonian function of the optimization approach  $\mathcal{H}$  with respect to the degree of privatization  $\alpha$ . We obtain

$$\mathcal{H}_\alpha = \underbrace{b_{V2} - \lambda g_{\alpha\theta}\theta - \mu\psi'}_{\Omega} \left[ E_{\alpha\theta} + \alpha\theta \frac{\partial E_{\alpha\theta}}{\partial(\alpha\theta)} \right] + \pi(\phi - s)\Pi + \xi \leq 0 \quad (28)$$

where the Lagrangean parameter  $\pi$  refers to the government budget constraint and  $\xi$  refers to the ‘ $\alpha$  never exceeds 1’ constraint. Hence, if  $\pi < 0$ , the government budget constraint is binding, and if  $\xi < 0$ , total privatization is optimal.

The factors which are traded-off are the following:

- the ideological interest in privatization, measured by  $b_{V2}$ ,
- the asymmetric information, as expressed by  $\partial E_{\alpha\theta}/\partial(\alpha\theta)$  and by  $\partial E_{\alpha\theta}/\partial K$ , which determines the sign of  $\lambda$  (see conditions 1 and 2 above),
- the influence which privatization and the productivity parameter exert on the technology of the firm, as expressed by  $g_{\alpha\theta}\theta$  (and weighted by  $\lambda$ ),
- the union’s interest in jobs, transferred into the interest in a binding government revenue constraint and measured by  $\pi$ ,
- the interest of private stockholders, measured by  $(\phi - s)$ ,
- the interest in profit  $\Pi$ , which is shared by all.

### *Lack of Information*

The first decisive trade-off is that between the ideological interest in privatization and the problem of asymmetric information. The asymmetric information makes privatization more costly because an information rent has to be paid to the management.<sup>18</sup> This rent is higher the higher the degree of privatization because the value of the information about the productivity parameter  $\theta$  increases in this degree. Accordingly, the asymmetric information has a negative effect on the desirable degree of privatization: in the asymmetric-information model there will be more cases where the government retains the firm in full public ownership. Recall the definition of  $\Omega$ . If  $\Omega > 0$ , the ideological interest

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<sup>18</sup>If the firm remains in full public ownership, no information rent is to be paid because the value of an information about the productivity parameter  $\theta$  is zero.

in privatization  $b_{V2}$  exceeds the costs of the information rent. The higher  $\Omega > 0$ , the more likely  $\xi < 0$ , which implies full privatization. If, on the other hand  $\Omega < 0$ , the information rent is particularly costly compared with the ideological interest and, therefore, the firm always remains in full public ownership. In between, there is the case of  $\Omega = 0$ , that is, balance of ideological interest and asymmetric-information costs. It is only this special balance which may lead to partial privatization.

Note that our model does not lead to the simple result that  $\Omega > 0$  implies full privatization,  $\Omega = 0$  all kinds of privatization, including partial privatization, whereas  $\Omega < 0$  implies full public ownership. However, the trade-off between ideological interest in privatization and costs of the information asymmetry is only one part of the various components which influence the result on privatization. Therefore, it is fully plausible that there may be a situation where we have full privatization for all  $\Omega > \hat{\Omega} > 0$ , whereas the firm remains in public ownership for all  $\Omega < \hat{\Omega}$ .

### *Bargaining Powers*

The second decisive trade-off refers to the players' bargaining powers. If the shares are sold at a positive issue price, *trade-union dominance* leads to equating issue price and dividend incomes because the trade union wants to minimize the private stockholders' net returns from their investment in shares,  $(\phi - s)\Pi$ . Moreover, the strong trade union makes sure that the government only gets its minimum revenue requirement  $R^o$ . If the firm remained in full public ownership, no employee shares would be issued and the first argument of the trade union's objective function would be zero. Hence, the trade union is always interested in achieving at least partial privatization.

In the *conservative or neutral firm* the issue price may fall below the dividends. Then we have a basic trade-off which is responsible for the results. This trade-off is the comparison

$$\Omega + \pi(\phi - s)\Pi \stackrel{\geq}{<} 0. \quad (29)$$

*Total privatization* is achieved if  $\Omega$  exceeds the product of the other interests. This happens if either the trade union or the private stockholders fail to promote their interests. The trade union's promotion of jobs is achieved if  $\pi < 0$  because then the government revenue requirement is binding. Otherwise there may be excess profit which could have been used to create jobs. Of course the trade union's interest in the employees' share in the profit might fully be satisfied, and if the trade union is not very much interested in jobs, it might be content. However, the profit interest has overcompensated the job interest; therefore it is fair to say that not achieving  $\pi < 0$  means failing to promote the interest in jobs. The private stockholders fail to promote their interest if  $\phi = s$ , because in that case they do not earn anything. We should not be too surprised if the private stockholders' interest is not promoted in the stage-one game. After all, they are not even players in that game.

Note that it is impossible to simultaneously promote the job interest of the trade union and the earning interest of the private stockholders. As is shown in proof 1.3 in appendix



2,  $\pi < 0$  and  $\phi > s$  (which implies  $\nu = 0$ ) cannot hold simultaneously if at least one share is sold. On the other hand,  $\pi < 0$  and  $\phi > s$  ( $\nu = 0$ ) can be fulfilled simultaneously if *no privatization* takes place. Economically, in that case, private stockholders have failed to promote their own interests.  $s < \phi$  does not mean anything to them if  $\alpha = 0$ . They do not earn anything from such a formal fixing of  $s$ . Thus it is possible that the firm remains in full public ownership. Employee shares may have been issued in that case.

It is a little paradoxical that trade-union dominance always implies privatization whereas the conservative firm might fully remain in public ownership. Note, however, that we defined a conservative firm with respect to its interest in the value of the firm ( $b_{V1}$ ), not with respect to its ideological interest in privatization ( $b_{V2}$ ). Therefore, the government may reject privatization if its ideological interest is not too pronounced and its interest in profit cannot be satisfied in negotiations with the trade union. In the trade-union-dominated firm there is no comparable trade-off between profit interest and ideological interest. Hence, the trade union reduces as far as possible both stockholders' and government's revenues. Of course, this result crucially depends on the assumption that the trade union retains its influence in the privatized firm. Hence, the union is willing to accept a form of privatization which excludes financial gains of non-employee stockholders, depresses the government to its reservation level of revenues from privatization, but implies guaranteed jobs and high wages plus transfers to the employees by means of employee shares.

**Proposition 2** (Issue Price of Shares)

- 2.1 If  $\Omega > 0$ , private stockholders never get shares free of charge. If  $\Omega = 0$ , private stockholders may get shares free of charge.
- 2.2 Consider  $\Omega > 0$  and let the issue price be equated to the dividend,  $s = \phi$ . Then the firm never remains in public ownership but is always fully privatized.

*Economic Interpretation 2*

This proposition is shaped by the influence of the players' lack of information. The results are the same as in Bös (1991), pp. 161-162, but now they are only valid for  $\Omega > 0$ . This is the case if the ideological interest in privatization exceeds the costs of the information rent paid to the management. As already mentioned in the economic interpretation 1, this is the case where full privatization is very likely. A completely new result refers to  $\Omega = 0$ : if the ideological interest in privatization and the costs of the management's information rent are fully balanced, a give-away of shares may turn out to be the optimal policy. (The case of  $\Omega \leq 0$  can be forgotten since in this case there is no privatization and, therefore, the question of an issue price is irrelevant.)

A give-away of shares must not be chosen if  $\Omega > 0$ . In our model the government wants to earn at least a minimal revenue  $R^o > 0$ . Revenues from retaining shares in the firm are traded off against revenues from selling shares. Now recall that partial privatization is excluded if  $\Omega > 0$ . The government, therefore, has two options to meet the revenue

requirement: first, it can leave the firm in full public ownership and enjoy the dividend income; second, it can fully privatize the firm and forgo any dividend income. In the latter case the revenue requirement  $R^o > 0$  can be met only if the private shareholders pay a (positive) price for the shares. Giving away the shares free of charge would leave the government with no revenues at all.

Let us next deal with the second part of proposition 2. If the issue price is equated to the dividend, the private stockholders are excluded from sharing in earnings.<sup>19</sup> This makes the privatization particularly attractive for both the government and the trade union. Hence the firm always is fully privatized and never left in public ownership. Note that the issue price  $s$  is the only variable set in the stage-one game which does not enter the objective function that characterizes the Nash solution. Hence  $s$  can easily be increased if necessary because there is no countervailing trade-off in the objective function. As we shall see in the following, it depends on the relative bargaining power of the trade union and the government who gains most from excluding the private stockholders from earning anything. If the trade union is more powerful, its interest in high labor inputs ( $b_{W2}$ ) and its willingness to accept the ensuing profit reductions will lead to a binding government revenue requirement, withdrawing the money from the government. If the government is more powerful, the revenue requirement may be not binding, withdrawing the money from the creation of further jobs.

**Proposition 3** (Employee Shares)

- 3.1 A conservative firm never issues employee shares.
- 3.2 In a trade-union-dominated firm the private stockholders earn nothing; the government gets its minimum revenue requirement  $R^o$ . Any percentage of employee shares which is compatible with the government revenue requirement can be obtained.
- 3.3 In the neutral firm any combination of private stockholders' earnings and employee shares can occur.

*Economic Interpretation 3*

If conservative interests dominate, the value of the firm  $\phi\Pi$  is maximized by maximizing  $\phi$ . This excludes employee shares. On the other hand, if trade-union interests dominate, the issue price of the shares is increased because then the government revenue requirement can be met by higher revenues from selling shares and lower dividends from the remaining shares. Hence the trade-union-dominated policy consists of equating  $s$  to  $\phi$  and reducing  $\phi$  until the government revenue requirement is binding. The trade union makes use of the substitutive relation of  $\phi$  and  $s$  along a binding government revenue constraint.

## 5 Efficiency, Pricing and Effort

Let us now solve the optimization approach of stage 2. At this stage, the ownership situation has been decided upon and so has the sharing of the profit. The board of the

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<sup>19</sup>If we choose  $s \leq \phi - \epsilon$ , there is always some minimal gain for the private stockholders.

firm chooses optimal inputs and an optimal price. The results on efficiency and pricing are summarized in the following two propositions.

**Proposition 4** (Efficiency)

- 4.1 If the firm remains public, it employs more workers than is efficient.
- 4.2 If the firm is privatized, it may even employ less workers than is efficient.

*Economic Interpretation 4*

The trade union's interest in high employment leads to a distortion in the labor-capital inputs. For any two inputs, cost minimization requires an equalization of the ratio of marginal productivities and the ratio of factor prices, in our case

$$g_N/g_K = p_N/p_K. \quad (30)$$

However, if the firm remains in full public ownership, the ratio of marginal productivities falls below the factor prices,

$$g_N/g_K < p_N/p_K. \quad (31)$$

If we interpret  $g_N$  and  $g_K$  as firm-internal shadow prices, either the shadow price of labor  $g_N$  is too low compared with the market price of labor  $p_N$ , or the shadow price of capital  $g_K$  is too high compared with the market price of capital  $p_K$ , or both. Consequently, the firm employs more workers than is efficient.

If the firm is partially or totally privatized, then the acquisition of information changes the above result. Whether we have overmanning or overcapitalization in this case depends on the incentive-correction terms

$$\mathcal{I}_k = \frac{\mu\psi'\alpha}{p_k} \cdot \frac{\partial E_{\alpha\theta}}{\partial k}; \quad k = K, N. \quad (32)$$

We obtain:

$$g_N/g_K < p_N/p_K \quad \text{if } (\mathcal{I}_N - \mathcal{I}_K) < b_{W2}/p_N; \quad (\text{overmanning}) \quad (33)$$

$$g_N/g_K > p_N/p_K \quad \text{if } (\mathcal{I}_N - \mathcal{I}_K) > b_{W2}/p_N; \quad (\text{overcapitalization}). \quad (34)$$

Recall that in condition 1 we assumed  $\partial E_{\alpha\theta}/\partial K \leq 0$  and, therefore,  $\mathcal{I}_K \geq 0$ . Hence, overcapitalization occurs if  $\mathcal{I}_N > \mathcal{I}_K \geq 0$ , that is, the marginal rate of transformation between effort and privatization productivity<sup>20</sup> must be more sensitive to changes in the workforce than to changes in capital investment. This result is plausible: since capital investments are less 'dangerous' with respect to the management's effort reduction, overcapitalization is the natural choice of the board of the firm.

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<sup>20</sup>See equation (26) above.

**Proposition 5** (Pricing)

5.1 The general price formula reflects the incentive-correction terms  $\mathcal{I}_K$  and  $\mathcal{I}_p$ :

$$\left[ p - \frac{\lambda p_K}{\lambda g_K + \mathcal{I}_K p_K} \right] z_p = -z - \frac{\mathcal{I}_p p_K}{\lambda g_K + \mathcal{I}_K p_K}, \quad (35)$$

where we have defined

$$\mathcal{I}_p = \mu \psi' \alpha \frac{\partial E_{\alpha\theta}}{\partial z} \frac{\partial z}{\partial p}. \quad (36)$$

5.2 If there is no privatization, the firm sets the price like a profit-maximizing monopolist:

$$\left( p - \frac{p_K}{g_K} \right) z_p = -z. \quad (37)$$

*Economic Interpretation 5*

The price formula of the public enterprise, equation (37), can directly be compared with the usual monopoly price formula

$$(p - C_z) z_p = -z. \quad (38)$$

This comparison shows that  $p_K/g_K$  can be treated as the marginal costs of producing  $z$ . The firm behaves like a profit-maximizing monopolist who specifies the marginal costs as  $p_K/g_K$ . Note that  $p_K/g_K$  depends on the bargaining power of both players, including the trade union's accentuation of jobs,  $b_{W2}$ .<sup>21</sup> Hence the 'marginal costs'  $p_K/g_K$  take account of the particular interests of the stockholders and the trade union's representatives, and therefore the price  $p$  typically will be different from the price set by a pure profit maximizer.<sup>22</sup>

Let us now turn to the general price formula (35). The left-hand side shows that instead of the 'marginal costs'  $p_K/g_K$  a modified cost term is used, namely  $\lambda p_K/(\lambda g_K + \mathcal{I}_K p_K)$ . Let us now apply condition 2 and assume that  $\mathcal{I}_K \geq 0$ , which implies  $\lambda < 0$ . Moreover, we assume  $|\lambda g_K| > \mathcal{I}_K p_K$ , otherwise we would have negative 'marginal costs.' Then, in the general price formula, the firm behaves like a profit-maximizing monopolist who overestimates its production costs which are reflected by  $p_K/g_K$ . This is plausible: the acquisition of information is costly and this is reflected in the modified cost term. – Unfortunately, the new term on the right-hand side escapes such an easy and plausible interpretation.

We finally turn to stage 3 of the game where the management chooses its effort level.

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<sup>21</sup>This can directly be seen from the conditions  $\mathcal{H}_N = 0$  and  $\mathcal{H}_K = 0$  in appendix 2.

<sup>22</sup>If this price is considered too high by the government, it could impose an *RPI - X* constraint. For details see Bös (1991), pp. 164-165.

## Proposition 6 (Effort)

6.1 To achieve the optimal effort, the following condition must hold for a marginal effort increase:

$$\begin{aligned} \text{marginal increase in productivity} = & \text{marginal reduction in managerial utility} \\ & \text{minus} \\ & \text{marginal increase in managerial utility} \\ & \text{caused by a compensating income increase.} \end{aligned}$$

In this equation all three terms are weighted so as to be measured in terms of the cooperative Nash-equilibrium objective function of stockholders and trade union.

6.2 The effort is not chosen in a cost-minimizing way.

### *Economic Interpretation 6*

The positive side of an effort increase is the associated increase in productivity. However, the effort increase also directly reduces the managerial utility and, in order to guarantee the IC-condition in the presence of increased effort, the management's incentive income has to be increased. This explains the three terms in the equation presented in proposition 6. The details are rather complicated and therefore relegated to appendix 2.

The effort is not chosen in a cost-minimizing way, regardless of whether the players are fully informed about the privatization productivity parameter or not. The reason are the distortions caused by the objective function which results from the cooperative Nash equilibrium.

## 6 Summary

This paper shows how decisive the explicit consideration of incomplete information shapes the results of economic modelling. Many of the results which were attained in Bös (1991) break down if incomplete information about the productivity consequences of privatization is introduced in the otherwise unchanged model. Recall the three main results mentioned in the introduction of this paper. They are changed as follows.

First, in Bös (1991) partial privatization never happens; either the firm remains in public ownership or it is fully privatized. This result does not hold any longer in the incomplete-information setting because of a trade-off between the government's ideological interest in privatization and the costs of the information rent which has to be paid to the management for the revelation of the true value of the privatization-productivity parameter. Therefore, the asymmetric information has a negative effect on the desirable degree of privatization. There will be more cases where the government retains the firm in full public ownership. And it is also possible that a particular balance of ideological interest and asymmetric-information costs implies that partial privatization becomes optimal.

Second, in Bös (1991) private stockholders never get shares free of charge. In the present paper this is only the case if the ideological interest in privatization exceeds the costs of the information rent paid to the management. A give-away of shares is possible if there is a particular balance of ideological interest and information costs.

Third, in Bös (1991) the firm always employs more workers than is efficient (even if it is fully privatized). This result breaks down because of the incomplete information of the players. Whether we have overmanning or overcapitalization in a privatized firm depends on the incentive-correction terms which measure in how far changes in capital and labor inputs make it easier for the management to reduce its effort if the productivity parameter increases. Overcapitalization occurs if the marginal rate of transformation between effort and privatization productivity reacts more sensitively to changes in the workforce than to changes in capital investment: since capital investments are less ‘dangerous’ with respect to the management’s effort reduction, overcapitalization is the natural choice of the board of the firm.

## Appendices

### Appendix 1: The Management’s Optimization

The board of the firm wants to induce the management to announce the correct value of the productivity characteristic  $\theta$ . This is achieved by choosing a management reward scheme

$$\text{incentive pay} = \begin{cases} t(\hat{\theta}) & \text{if } K = K(\hat{\theta}), N = N(\hat{\theta}) \text{ and } p = p(\hat{\theta}); \\ 0 & \text{otherwise,} \end{cases} \quad (39)$$

where  $\hat{\theta}$  is that value of the characteristic which has been announced by the management.

Given this offer, the management decides to announce a particular value  $\hat{\theta}$ . The management would not have any degree of freedom if the board knew the actual value of  $\hat{\theta}$  and could monitor the management’s behavior according to this knowledge. But since this is not the case, the management could announce a false  $\theta$  if that value implies higher utility. However, when telling lies, the management must be cautious. The board is able to observe the capital and labor inputs and the output price. Hence, the effects of effort, and the actual and announced values of  $\theta$  must always be consistent with the input quantities and the output price which the board will be able to observe. This characterization of the incentive-pay schedule presupposes that inputs and price are set by the management. If the board chooses these variables as control variables, he does so to anticipate the management’s setting of these variables.<sup>23</sup> The actual instrument of the regulator, however, is the managerial incentive income.

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<sup>23</sup>In a similar way effort is chosen as control variable to anticipate the management’s behavior.

### *Incentive Compatibility*

The management's preferred announcement  $\hat{\theta}$  results from the minimization of the distance function

$$\mathcal{D} := \left[ t(\theta) - \psi(E(\alpha\theta, K(\theta), N(\theta), z(p(\theta)))) \right] - \left[ t(\hat{\theta}) - \psi(E(\alpha\hat{\theta}, K(\hat{\theta}), N(\hat{\theta}), z(p(\hat{\theta}))) \right]. \quad (40)$$

Capital and labor inputs and the output price are observable by the regulator and are used as control variables, therefore, they depend on the productivity parameter. The observability, however, restricts the management when announcing some  $\hat{\theta}$ . Consider a management that announces a false  $\hat{\theta}$ , that is, it pretends to work in a worse type of firm than is actually the case. This allows a reduction of effort. However, to be trustworthy, the management has to choose the effort in such a way that in spite of its working in a better firm and in spite of the reduced effort, the firm uses just those labor inputs  $N$  which are observed and which correspond to the falsely announced  $\hat{\theta}$ . More formally, the management chooses effort  $e(\theta, \hat{\theta})$  in such a way that  $N(\hat{\theta})$  results. Therefore, a consistent lie of the management implies a labor-requirement function

$$N(\hat{\theta}) = N(\alpha\theta, e(\theta, \hat{\theta}), K(\hat{\theta}), z(p(\hat{\theta}))), \quad (41)$$

where  $e(\theta, \hat{\theta})$  has been chosen in such a way that it exactly compensates for the influence of the actual  $\theta$  as given by the first argument of the labor-requirement function.

Minimizing the distance function  $\mathcal{D}$  with respect to the announced  $\hat{\theta}$  yields a first-order condition

$$\frac{dt}{d\hat{\theta}} - \psi' \left[ \frac{\partial E}{\partial K} \frac{\partial K}{\partial \hat{\theta}} + \frac{\partial E}{\partial N} \frac{\partial N}{\partial \hat{\theta}} + \frac{\partial E}{\partial z} \frac{\partial z}{\partial p} \frac{\partial p}{\partial \hat{\theta}} \right] = 0. \quad (42)$$

Moreover, we consider changes in utility which result from changes in the actual  $\theta$ :

$$\dot{U}(\theta) := \frac{dU}{d\theta} = \frac{dt}{d\theta} - \psi' E_{\alpha\theta} \alpha - \psi' \left[ \frac{\partial E}{\partial K} \frac{\partial K}{\partial \theta} + \frac{\partial E}{\partial N} \frac{\partial N}{\partial \theta} + \frac{\partial E}{\partial z} \frac{\partial z}{\partial p} \frac{\partial p}{\partial \theta} \right]. \quad (43)$$

At the optimum we have  $\theta = \hat{\theta}$ , and observability implies

$$\frac{dK}{d\theta} = \frac{dK}{d\hat{\theta}}; \quad \frac{dN}{d\theta} = \frac{dN}{d\hat{\theta}}; \quad \frac{dz}{dp} \frac{dp}{d\theta} = \frac{dz}{dp} \frac{dp}{d\hat{\theta}}. \quad (44)$$

Since inputs and outputs are observable, the management must always lie in such a way that (44) holds. Moreover, we have

$$\frac{dt}{d\theta} = \frac{dt}{d\hat{\theta}}, \quad (45)$$

as a property of the income schedule which is set by the board and therefore trivially is observable. Hence we can substitute equation (42) into equation (43) to obtain

$$\dot{U}(\theta) = -\psi' \alpha E_{\alpha\theta} \begin{cases} > 0 & \text{if } \alpha > 0, \\ = 0 & \text{if } \alpha = 0, \end{cases} \quad (46)$$

which is the management's incentive-compatibility constraint.

The second-order condition  $U_{\hat{\theta}\hat{\theta}}^2 \geq 0$  takes the form<sup>24</sup> (at  $\theta = \hat{\theta}$ )

$$\begin{aligned} & - \psi'' \alpha E_{\alpha\theta} \left[ \frac{\partial E}{\partial K} \frac{\partial K}{\partial \theta} + \frac{\partial E}{\partial N} \frac{\partial N}{\partial \theta} + \frac{\partial E}{\partial z} \frac{\partial z}{\partial p} \frac{\partial p}{\partial \theta} \right] \\ & - \psi' \left[ \partial \left( \frac{\partial E}{\partial K} \right) / \partial \theta \cdot \frac{\partial K}{\partial \theta} + \partial \left( \frac{\partial E}{\partial N} \right) / \partial \theta \cdot \frac{\partial N}{\partial \theta} + \partial \left( \frac{\partial E}{\partial z} \right) / \partial \theta \cdot \frac{\partial z}{\partial p} \cdot \frac{\partial p}{\partial \theta} \right] \geq 0. \end{aligned} \quad (47)$$

Sufficient for this second-order condition to hold are the following signs of partial derivatives:<sup>25</sup>

(i)  $\partial K / \partial \theta, \partial N / \partial \theta, \partial p / \partial \theta \leq 0$ ;

(ii)  $\partial \left( \frac{\partial E}{\partial K} \right) / \partial \theta, \partial \left( \frac{\partial E}{\partial N} \right) / \partial \theta \geq 0, \partial \left( \frac{\partial E}{\partial z} \right) / \partial \theta \leq 0$ .

(Recall that we have assumed  $\psi' > 0, \psi'' > 0$ ; moreover,  $E_{\alpha\theta}, E_K, E_N < 0$ , and  $E_z > 0$ .)

### *Participation Constraint*

The management's reservation utility  $U^o$  is exogenously given; if the management's utility were to fall below this level, it would leave the firm. This must be avoided since the board needs the management for production. Hence, the board has to consider the following participation constraint of the management:

$$U \geq U^o. \quad (48)$$

Therefore, for the worst type of firm the regulator will set the management's information rent  $U - U^o$  equal to zero and therefore

$$U(\underline{\theta}) = U^o. \quad (49)$$

For all better types of firms the constraint must not be binding to ensure incentive compatibility unless the firm remains in full public ownership:

$$U(\theta) > U^o \quad \text{for all } \theta > \underline{\theta}, \quad \text{if } \alpha > 0, \quad (50)$$

$$U(\theta) = U^o \quad \text{for all } \theta > \underline{\theta}, \quad \text{if } \alpha = 0. \quad (51)$$

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<sup>24</sup>For this formulation of the second-order condition see Guesnerie and Laffont (1984), pp. 336-341. For the differentiation note that in the first-order condition (42)  $dt/d\hat{\theta}, dK/d\hat{\theta}, dz/dp$  and  $dp/d\hat{\theta}$  do not depend on  $\theta$ , but only on  $\hat{\theta}$ . This also holds for  $dN/d\hat{\theta}$  – if  $\theta$  changes, this is corrected by changing  $e(\theta, \hat{\theta})$ . However,  $\psi'(e)$  and the partial derivatives of  $E$  depend on the actual  $\theta$ , via  $E(\theta, \hat{\theta})$ , and are differentiated with respect to the actual  $\theta$ .

<sup>25</sup>Cfr. in a similar context Laffont and Tirole (1990:32).



## Appendix 2: The Main Optimization Approach and the Proofs of the Propositions

### *Objective, Constraints, Control and State Variables*

The optimization approach consists of maximizing the expected value of the objective function

$$\int_{\underline{\theta}}^{\bar{\theta}} [\log(V(\cdot) - V^o) + \log(W(\cdot) - W^o)] f(\theta) d\theta \quad (52)$$

subject to the following constraints:<sup>26</sup>

$$z(p(\theta)) = g(\alpha(\theta)\theta, e(\theta), K(\theta), N(\theta)), \quad (\lambda(\theta)), \quad (53)$$

$$s(\theta) \leq \phi(\theta), \quad (\nu(\theta)), \quad (54)$$

$$R^o \leq (1 - \alpha(\theta))\phi(\theta)\Pi(\theta) + s(\theta)\alpha(\theta)\Pi(\theta), \quad (\pi(\theta)), \quad (55)$$

$$\alpha(\theta) \leq 1, \quad (\xi(\theta)), \quad (56)$$

$$\phi(\theta) \leq 1, \quad (\tau(\theta)), \quad (57)$$

$$\dot{U}(\theta) = -\psi'(e(\theta))\alpha(\theta)E_{\alpha\theta}(\alpha(\theta)\theta, K(\theta), N(\theta), z(p(\theta))), \quad (\mu(\theta)). \quad (58)$$

This is an optimal control problem with

- state variable  $U$  and
- control variables  $\alpha, \phi, e, K, N, p$ , and  $s$ .

Initial and terminal condition of the state variable  $U(\theta)$  are as follows:

- initial condition  $U(\underline{\theta}) = U^o$ ,
- terminal condition  $U(\bar{\theta})$  free.

### *The Hamiltonian*

The generalized Hamiltonian is as follows

$$\begin{aligned} \mathcal{H} = & \log[V(\phi\Pi, \alpha) - V^o] f(\theta) + \log\left[W\left(\frac{(1-\phi)\Pi}{N_o}, N\right) - W^o\right] f(\theta) \\ & - \lambda[g(\alpha\theta, e, K, N) - z(p)] - \nu(\phi - s) \\ & - \pi[(1-\alpha)\phi\Pi + s\alpha\Pi - R^o] \\ & - \xi(1-\alpha) - \tau(1-\phi) \\ & - \mu[\psi'(e)\alpha E_{\alpha\theta}(\alpha\theta, K, N, z(p))]. \end{aligned} \quad (59)$$

For the following derivations note that it is appropriate to substitute  $t = U + \psi(e)$  in the above Hamiltonian. This substitution matters wherever the profit  $\Pi$  enters the generalized Hamiltonian.

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<sup>26</sup>The non-negativity of  $s$  and  $\alpha$  is not introduced as explicit structural constraints, but is taken care of in the necessary optimum conditions. See Panik (1976: 297). The management's participation constraint also is not introduced as explicit structural constraint, but considered by postulating the respective transversality condition for  $\theta = \underline{\theta}$ . For this procedure see Seierstad and Sydsæter (1987: 185-6).

### The Main Necessary Conditions

Applying the Maximum Principle for optimization problems with mixed constraints leads to the following necessary conditions for  $\{U^*(\theta); \alpha^*(\theta), \dots, s^*(\theta)\}$  to solve the problem:<sup>27</sup>

$$\mathcal{H}_s \leq 0; \quad \mathcal{H}_s s = 0; \quad \nu(\phi - s) = 0; \quad \nu \leq 0; \quad 0 \leq s \leq \phi, \quad (60)$$

$$\mathcal{H}_\alpha \leq 0; \quad \mathcal{H}_\alpha \alpha = 0; \quad \xi(1 - \alpha) = 0; \quad \xi \leq 0; \quad 0 \leq \alpha \leq 1, \quad (61)$$

$$\mathcal{H}_\phi = 0; \quad \tau(1 - \phi) = 0; \quad \tau \leq 0; \quad 0 < \phi \leq 1, \quad (62)$$

$$\mathcal{H}_N = 0; \quad \mathcal{H}_K = 0; \quad \mathcal{H}_p = 0, \quad (63)$$

$$\mathcal{H}_e \leq 0; \quad \mathcal{H}_e e = 0; \quad e \geq 0, \quad (64)$$

$$z = g(\alpha\theta, e, K, N), \quad (65)$$

$$[(1 - \alpha)\phi + s\alpha]\Pi \geq R^o; \quad \pi \{[(1 - \alpha)\phi + s\alpha]\Pi - R^o\} = 0; \quad \pi \leq 0, \quad (66)$$

$$- \mathcal{H}_U = \dot{\mu}(\theta). \quad (67)$$

The transversality conditions are

$$\mu(\underline{\theta}) \leq 0, \quad (68)$$

$$\mu(\bar{\theta}) = 0. \quad (69)$$

We are now in a position to determine the sign of the multiplier which is associated with the incentive-compatibility condition. We obtain  $\mu(\theta) < 0$  for  $\underline{\theta} \leq \theta < \bar{\theta}$ . Condition (67) implies

$$\dot{\mu}(\theta) = b_{V1}\phi + b_{W1} \frac{1 - \phi}{N_o} - \pi[(1 - \alpha)\phi + s\alpha], \quad (70)$$

where we have applied the following abbreviations:

$$b_{V_i} := \frac{V_i}{V - V^o} f; \quad i = 1, 2, \quad (71)$$

$$b_{W_i} := \frac{W_i}{W - W^o} f; \quad i = 1, 2. \quad (72)$$

On the right-hand side of (70), the first two terms are strictly positive, the third term is weakly positive. Therefore,  $\mu$  is strictly increasing in  $\theta$ . However, for the best type of firm the transversality condition (69) is  $\mu(\bar{\theta}) = 0$ . Hence,  $\mu(\theta) < 0$  for all  $\theta < \bar{\theta}$ . This is compatible with the transversality condition (68) which establishes  $\mu(\underline{\theta}) \leq 0$ .

### Stage 1 of the Game

The instruments  $s, \alpha$ , and  $\phi$  are determined on the basis of the stage-one game. Hence propositions 1-3 are based on the interpretation of (60)-(62) where

$$\mathcal{H}_s = \nu - \pi\alpha\Pi, \quad (73)$$

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<sup>27</sup>In equation (62) we have  $\mathcal{H}_\phi = 0$  instead of  $\mathcal{H}_\phi \leq 0$  because  $\phi > 0$  due to the government revenue requirement.

$$\mathcal{H}_\alpha = \Omega + \pi(\phi - s)\Pi + \xi, \quad (74)$$

$$\mathcal{H}_\phi = F\Pi - \nu + \tau, \quad (75)$$

where we have applied the following abbreviations:

$$\Omega := b_{V2} - \lambda g_{\alpha\theta}\theta - \mu\psi' \left[ E_{\alpha\theta} + \alpha\theta \frac{\partial E_{\alpha\theta}}{\partial(\alpha\theta)} \right], \quad (76)$$

$$F := \left[ b_{V1} - \frac{b_{W1}}{N^o} - \pi(1 - \alpha) \right]. \quad (77)$$

**Lemma** (Sign of  $\Omega$ )

L.1  $\lambda < 0$  if  $\partial E_{\alpha\theta}/\partial K \leq 0$  (sufficient condition);

L.2 if  $\lambda < 0$  and  $\partial E_{\alpha\theta}/\partial(\alpha\theta) \geq 0$ , then  $\Omega > 0$  (sufficient condition).

*Proof of the Lemma*

L.1 A simple transformation of  $\mathcal{H}_K$  yields  $-b_{V1}\phi - b_{W1}(1 - \phi)/N^o + \pi[(1 - \alpha)\phi + s\alpha] - (1/p_K)\mu\psi'\alpha(\partial E_{\alpha\theta}/\partial K) = \lambda g_K/p_K$ . Assume  $\partial E_{\alpha\theta}/\partial K \leq 0$ . Recall the further assumptions on partial derivatives made in this paper. Then  $\lambda < 0$  results.

L.2 Assume  $\lambda < 0$  and  $\partial E_{\alpha\theta}/\partial(\alpha\theta) \geq 0$ . Recall the further assumptions on partial derivatives. Then  $\Omega := b_{V2} - \lambda g_{\alpha\theta}\theta - \mu\psi' [E_{\alpha\theta} + \alpha\theta(\partial E_{\alpha\theta}/\partial(\alpha\theta))] > 0$  results.

**Proof of Proposition 1** (Degree of Privatization)

*Preliminary:* If  $\Omega > 0$  and  $\alpha > 0$ , we have  $s > 0$ .

Assume  $s = 0$ . Then  $\phi > s$ , because  $\phi = 0$  is excluded by the government revenue requirement. Therefore the issue price constraint is non-binding and  $\nu = 0$ . Then we must have  $0 \leq \pi\alpha\Pi$ . Since  $\alpha > 0$ , the inequality  $0 \leq \pi\alpha\Pi$  requires  $\pi = 0$ . For  $s = 0$  we have  $(1 - \alpha)\phi\Pi \geq R^o > 0$ . As  $\phi, \Pi > 0$ , this implies  $1 - \alpha > 0$  which, in turn, implies  $\xi = 0$ . Since  $\alpha > 0$ ,  $\mathcal{H}_\alpha\alpha$  requires  $\mathcal{H}_\alpha = 0$  or, equivalently  $\Omega = 0$ . Hence  $s$  can only be zero if  $\Omega = 0$ . It must be positive if  $\Omega > 0$ .

Let us now turn to the proof of proposition 1.

- 1.1 – Consider first  $\Omega > 0$ . For  $0 < \alpha < 1$  we have  $\xi = 0$  and therefore  $\Omega + \pi(\phi - s)\Pi = 0$ . Since  $\Omega > 0$ , this requires  $\pi(\phi - s)\Pi < 0$ . This is possible only if  $\pi < 0$  and at the same time  $\phi > s$ . However,  $\phi > s$  implies  $\nu = 0$ . Since  $\Omega > 0$  and  $\alpha > 0$  imply  $s > 0$ , we have  $\mathcal{H}_s = 0$ , implying  $\nu = \pi\alpha\Pi = 0$ . This cannot hold if  $\pi < 0$ , and  $0 < \alpha < 1$ . Hence, partial privatization is excluded. – Full public ownership is possible; the beginning of the proof is the same as above. However,  $s = 0$  leads to  $\nu \leq \pi\alpha\Pi$  where  $\pi\alpha\Pi = 0$  because of  $\alpha = 0$ . This is compatible with  $\nu = 0$ . – Full privatization is possible:  $\alpha = 1$  implies  $\xi \leq 0$  and  $\mathcal{H}_\alpha = 0$ .  $\Omega + \pi(\phi - s)\Pi + \xi = 0$  is compatible with  $\pi \leq 0$  and  $\phi \geq s$ .

- Consider second  $\Omega < 0$ . If  $0 < \alpha \leq 1$ , we have  $\Omega + \pi(\phi - s)\Pi + \xi = 0$ . This requires  $\pi(\phi - s)\Pi > 0$ . Since  $\pi \leq 0$  and  $\phi \geq s$ , this is impossible. However, if  $\alpha = 0$ , we have  $\Omega + \pi(\phi - s)\Pi + \xi \leq 0$  which is compatible with  $\pi \leq 0$  and  $\phi \geq s$ . Hence, the firm must always remain in full public ownership.

- Consider third  $\Omega = 0$ . Then  $\pi(\phi - s)\Pi + \xi \leq 0$ , which is compatible with either  $\pi < 0$  and  $\phi = s$  or  $\pi = 0$  and  $\phi \geq s$ . Therefore, all realizations of  $\alpha \in [0, 1]$  are possible.

1.2 If  $F < 0$ , we have  $\nu < 0$  as can be proved by contradiction. Assume  $\nu = 0$ . Then we must have  $F\Pi + \tau = 0$  which is impossible because of  $F\Pi < 0$  and  $\tau \leq 0$ . Hence  $\nu < 0$ , which implies  $\phi = s$ . Next we prove  $\alpha \neq 0$ . Since  $\phi > 0$ , and  $\phi = s$ , we have  $s > 0$  and, therefore,  $\mathcal{H}_s = 0$ , which implies  $\nu = \pi\alpha\Pi$ . If  $\alpha = 0$ , this is a contradiction to  $\nu < 0$ . So  $\alpha = 0$  is impossible. Moreover, because of  $\nu = \pi\alpha\Pi$ ,  $\nu < 0$  and  $\pi = 0$  cannot hold at the same time. Hence, we have  $\pi < 0$ , and the government revenue constraint is binding.

1.3 If  $s > 0$ , we have  $\mathcal{H}_s = 0$  or, equivalently  $\nu = \pi\alpha\Pi$ . Hence  $\pi = 0$  and  $\nu < 0$  cannot hold at the same time. The other three combinations are possible:

-  $\pi < 0, \nu = 0$ : it can be seen directly that this combination can only occur at  $\alpha = 0$ . Because of  $\pi < 0$ , in this optimum we always have  $(1 - \alpha)\phi\Pi + s\alpha\Pi = R^o$ .

-  $\pi < 0, \nu < 0$ : given these signs of  $\nu$  and  $\pi$ , the case of no privatization is excluded because  $\nu = \pi\alpha\Pi$ . Further, because of  $\nu < 0$ , we have  $\phi = s$ , and because of  $\pi < 0$ , we have  $(1 - \alpha)\phi\Pi + s\alpha\Pi = R^o$ .

-  $\pi = 0, \nu = 0$ : since  $\pi = 0$ , we have  $\pi(\phi - s)\Pi = 0$  and, therefore  $\mathcal{H}_\alpha = \Omega + \xi \leq 0$ . If  $\Omega > 0$ , we obtain  $\xi < 0$ : the firm is fully privatized. If  $\Omega \leq 0$ , we have  $\xi \leq 0$ , and no conclusion on the degree of privatization can be made. It is directly evident that the zero signs of the Lagrangean multipliers imply that any combination of binding and non-binding revenue constraint and issue price constraint is possible.

### Proof of Proposition 2 (Issue Price of Shares)

2.1 Has been proved as a preliminary to proof 1.

2.2 If  $s = \phi$ , we obtain  $\mathcal{H}_\alpha = \Omega + \xi \leq 0$ . If  $\Omega > 0$ , we have  $\xi < 0$  which implies  $\alpha = 1$ . If  $\Omega \leq 0$ , we have  $\xi \leq 0$  and, therefore,  $\alpha$  may deviate from unity.<sup>28</sup>

### Proof of Proposition 3 (Employee Shares)

3.1  $F\Pi - \nu + \tau = 0$  holds only if  $\tau < 0$  because  $F\Pi > 0$  and  $\nu \leq 0$ . Hence the optimal  $\phi$  is unity, which means we have no employee shares. Note that this holds for both  $\nu = 0$  and  $\nu < 0$ .

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<sup>28</sup>It is interesting to consider the government budget constraint. Let us distinguish two cases: (i) If  $s = \phi$ , we have  $\nu \leq 0$  and, given  $s > 0$  and assuming  $\alpha > 0$ , we cannot exclude  $\pi = 0$ . The government revenue constraint can be binding or not-binding. (ii)  $s < \phi$  implies  $\nu = 0$ . Since  $s > 0$ , we have  $0 = \pi\alpha\Pi$ . Once again, the case of  $\alpha = 0$  is of no economic interest. Therefore, let us assume  $\alpha > 0$ . Hence we must have  $\pi = 0$ . The government revenue constraint is not necessarily binding,  $[(1 - \alpha)\phi + s\alpha]\Pi \geq R^o$ .

3.2 The first sentence has been proved in proof 1.2 above. The second sentence ( $\phi \leq 1$ ) results trivially from  $F\Pi - \nu + \tau = 0$ .

3.3  $H_\phi$  is reduced to  $-\nu + \tau = 0$  which can be fulfilled for (a)  $\nu = 0$  and  $\tau = 0$ , i.e.  $s \leq \phi$  and  $\phi \leq 1$  or for (b)  $\nu < 0$  and  $\tau < 0$ , i.e.  $s = \phi$  and  $\phi = 1$ .

### Stages 2 and 3 of the Game

The instruments  $K, N$ , and  $p$  are determined at stage 2 of the game, based on the following marginal conditions:

$$\begin{aligned}\mathcal{H}_K &= -b_{V1}p_K\phi - \frac{b_{W1}}{N^o}p_K(1 - \phi) - \lambda g_K \\ &+ \pi p_K[(1 - \alpha)\phi + s\alpha] - p_K\mathcal{I}_K = 0,\end{aligned}\tag{78}$$

$$\begin{aligned}\mathcal{H}_N &= -b_{V1}p_N\phi - \frac{b_{W1}}{N^o}p_N(1 - \phi) + b_{W2} - \lambda g_N \\ &+ \pi p_N[(1 - \alpha)\phi + s\alpha] - p_N\mathcal{I}_N = 0,\end{aligned}\tag{79}$$

$$\begin{aligned}\mathcal{H}_p &= (z + pz_p) \left( b_{V1}\phi + \frac{b_{W1}}{N^o}(1 - \phi) - \pi[(1 - \alpha)\phi + s\alpha] \right) \\ &+ \lambda z_p - \mathcal{I}_p = 0,\end{aligned}\tag{80}$$

where the incomplete-information problem enters via the incentive-correction terms

$$\mathcal{I}_k = \frac{\mu\psi'\alpha}{p_k} \cdot \frac{\partial E_{\alpha\theta}}{\partial k}; \quad k = K, N,\tag{81}$$

$$\mathcal{I}_p = \mu\psi'\alpha \frac{\partial E_{\alpha\theta}}{\partial z} \frac{\partial z}{\partial p}.\tag{82}$$

### Proof of Proposition 4 (Efficiency)

*Preliminary:*

Simple algebraic manipulation of  $\mathcal{H}_N = 0$  and  $\mathcal{H}_K = 0$  yields

$$-b_{V1}\phi - \frac{b_{W1}}{N^o}(1 - \phi) + \pi[(1 - \alpha)\phi + s\alpha] - \mathcal{I}_K = \frac{\lambda g_K}{p_K}.\tag{83}$$

$$-b_{V1}\phi - \frac{b_{W1}}{N^o}(1 - \phi) + \pi[(1 - \alpha)\phi + s\alpha] - \mathcal{I}_N = \frac{\lambda g_N}{p_N} - \frac{b_{W2}}{p_N},\tag{84}$$

Subtracting (84) from (83) leads to

$$(\mathcal{I}_N - \mathcal{I}_K) + \lambda \left( \frac{g_N}{p_N} - \frac{g_K}{p_K} \right) = \frac{b_{W2}}{p_N}.\tag{85}$$

Let us now turn to the proof of proposition 4:

4.1 Set  $\mathcal{I}_K = \mathcal{I}_N = 0$  (because of  $\alpha = 0$ ). Then  $\lambda < 0$  follows directly from (83). We have

$$\lambda \left( \frac{g_N}{p_N} - \frac{g_K}{p_K} \right) = \frac{b_{W2}}{p_N} > 0, \quad (86)$$

since  $b_{W2} > 0, p_N > 0$ . Therefore, after some transformations, condition (85) requires

$$\frac{g_N}{g_K} < \frac{p_N}{p_K}, \quad (87)$$

which means too high labor inputs relative to the capital inputs.

4.2 Here we have

$$\lambda \left( \frac{g_N}{p_N} - \frac{g_K}{p_K} \right) = \frac{b_{W2}}{p_N} - (\mathcal{I}_N - \mathcal{I}_K). \quad (88)$$

Recall the proof 4.1 above: overmanning results because the right-hand side of equality (86) is positive. By analogy, we obtain overmanning in the general case (88) if  $(\mathcal{I}_N - \mathcal{I}_K) < b_{W2}/p_N$ . In contrast, overcapitalization results if  $(\mathcal{I}_N - \mathcal{I}_K) > b_{W2}/p_N$ .

### Proof of Proposition 5 (Pricing)

5.1 Substitute (83) into (80) and transform to obtain

$$\left[ p \left( \lambda + I_K \frac{p_K}{g_K} \right) - \lambda \frac{p_K}{g_K} \right] z_p = -z \left( \lambda + I_K \frac{p_K}{g_K} \right) - I_p \frac{p_K}{g_K}. \quad (89)$$

Now divide this equation by  $\lambda + I_K(p_K/g_K)$  to obtain (35).

5.2 Set  $\mathcal{I}_K = \mathcal{I}_p = 0$  (because of  $\alpha = 0$ ), to obtain (37).

### Proof of Proposition 6 (Effort)

6.1 At stage 3 of the game the management chooses its effort level, which can be characterized by the following marginal condition:

$$\mathcal{H}_e = -b_{V1}\phi\psi' - b_{W1}(1-\phi)\psi'/N_o - \lambda g_e + \pi\psi'[(1-\alpha)\phi + s\alpha] - \mu\psi''\alpha E_{\alpha\theta}(\cdot) = 0 \quad (90)$$

Now recall the condition  $\mathcal{H}_U$ :

$$\mathcal{H}_U = -b_{V1}\phi - b_{W1} \frac{1-\phi}{N_o} + \pi[(1-\alpha)\phi + s\alpha]. \quad (91)$$

We substitute this equation into (90) to obtain:

$$\lambda g_e + \mu\psi''\alpha E_{\alpha\theta} = \mathcal{H}_U\psi'. \quad (92)$$

Since  $U_\psi = -1$ , this equation can be transformed into:

$$-\lambda g_e - \mu\psi''\alpha E_{\alpha\theta} = \mathcal{H}_U U_\psi \psi', \quad (93)$$

or, equivalently,

$$-\lambda g_e = \mathcal{H}_U U_\psi \psi' - \mu \frac{\partial \dot{U}}{\partial e}. \quad (94)$$

Therefore, three terms are traded-off to determine the optimal effort level:

- $-\lambda g_e$ . Here  $g_e > 0$  measures the marginal productivity increase caused by the increase in effort. The weight  $-\lambda$  can best be interpreted as follows. The market-clearing constraint (4) can be rewritten as an inequality  $z \leq g(\cdot)$  or, equivalently  $z + g_o = g(\cdot)$ , which in our problem is evaluated at  $g_o = 0$ . The artificial variable  $g_o$  measures the amount of the produced good which has to be thrown away because it is not bought by the demand  $z(p)$ . Then we get  $\mathcal{H}_{g_o} = \lambda < 0$ . This shows that the weight  $-\lambda$  measures how the Hamiltonian is influenced by the marginal productivity increase caused by the marginal increase in effort.
- $\mathcal{H}_U U_\psi \psi'$ . The increase in effort reduces the managerial utility,  $U_\psi \psi' < 0$ . This is transformed into units of changes of the Hamiltonian by  $\mathcal{H}_U < 0$ : the stockholders and the trade union representatives want to reduce the managerial utility as far as possible, that is, they want to achieve highest possible effort at the lowest possible managerial income (considering of course the management's participation and IC-constraints).
- $-\mu \partial \dot{U} / \partial e$ . This term results from the differentiation of the management's IC-condition;  $-\mu \partial \dot{U} / \partial e = \mu \partial [\psi' \alpha E_{\alpha\theta}] / \partial e = \mu \psi'' \alpha E_{\alpha\theta}$ . Recall that  $\mu < 0, \psi'' > 0, \alpha \geq 0$  and  $E_{\alpha\theta} < 0$ . Therefore, we obtain  $\partial \dot{U} / \partial e \geq 0$ . This term measures how the increase in management's utility with respect to  $\theta$  has to be increased to compensate the management for the effort increase. The only way to achieve such an increase in the management's utility is a compensating increase in the managerial incentive income. The weight  $-\mu$  transforms the managerial utility change into units of changes of the Hamiltonian.

6.2 As the first benchmark we consider the case of *cost minimization under full information*. Here we have the following optimization problem:

$$\min_{e, K, U} p_N N(\alpha\theta, e, K, z(p)) + p_K K + U + \psi(e) - \beta(U - U^o), \quad (95)$$

where  $\beta$  is the Lagrangean multiplier associated with the management's participation constraint. We obtain the following marginal conditions:

$$p_N N_e = -\psi' \quad (96)$$

$$p_N N_K = -p_K \quad (97)$$

$$\beta = 1. \quad (98)$$

As the second benchmark consider the case *cost minimization under imperfect information* which requires minimization of the following objective function

$$[p_N N(\alpha\theta, e, K, z(p)) + p_K K + U + \psi(e)]f(\theta), \quad (99)$$

subject to the same constraints, initial and terminal conditions as in our regular problem (59).<sup>29</sup> It is immediately obvious that the solutions of our three-stage game do not minimize costs, regardless of whether we consider the case of full information or the case of imperfect information.

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<sup>29</sup>In the profit function the labor inputs  $N$  are always to be substituted by the labor-requirement function  $N(\alpha\theta, e, K, z(p))$ .



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