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CONTROLLING INVESTMENT DECISIONS: HURDLE RATES AND INTERTEMPORAL COST ALLOCATION

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Abstract

We examine alternative performance measures for a manager who has superior information about the profitability of an investment project and who contributes to periodic operating cash flows through his efforts. We find that residual income based on a suitably chosen depreciation schedule is an optimal performance measure. To address the underlying asymmetric information problem, the capital charge rate in the calculation of residual income should be equal to the firm's hurdle rate, which is the critical internal rate of return below which the principal would not want to fund the project. This hurdle rate includes the compensation cost for the better informed manager and therefore exceeds the principal's cost of capital. We also show that residual income remains an optimal performance measure in settings where multiple divisions compete for scarce investment funds. In order to solve the resource allocation problem, the capital charge rate must then be increased to a competitive hurdle rate.

JEL Classification: D82, D92, M41
1 Introduction

Divisions within a firm are frequently responsible for both operating and investment decisions. Accordingly, the most common divisional performance measures are based on both operating results and asset values, e.g., Return on Investment, Residual Income, and Cash Flow Return on Investment. There has been considerable debate in recent years about the relative effectiveness of these alternative performance measures. Which performance measures are more effective in aligning the interests of owners and managers, and what impact does the adoption of these performance measures have on firms’ stock prices?\(^1\)

It appears that the residual income measure (and several variants thereof) has received particular attention in recent years.\(^2\) Residual income is calculated as accounting income minus an interest charge on the capital used by a division. To implement this performance measure, a firm have to decide on the capital charge rates that should apply to individual divisions. In addition, firms have to choose asset valuation rules, i.e., the rules for depreciating the assets acquired by a division. While such asset valuation rules are generally viewed as given for external financial reporting purposes, they are commonly treated as design variables for the purpose of managerial performance evaluation.\(^3\)

This paper examines the issues of depreciation and capital charge rates in a multiperiod principal-agent model. Agents are assumed to manage the divisions of a firm. In this capacity, managers have superior information regarding the profitability of investment projects available to their divisions. In addition, a manager’s effort choice is assumed to affect the periodic operating results of his division. Our results show that residual income is capable of generating both optimal investment and effort incentives, provided assets are depreciated so as to match periodic project cash flows with a share of the initial investment cost. At the same time, the capital charge rate imposed on a division must reflect not only the principal’s

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\(^1\)See, for example, Stewart (1994), Biddle, Bowen and Wallace (1997), and Balachandran (2000).

\(^2\)In particular, Stern Stewart and Co. has successfully advocated the Economic Value Added concept, see, for example, Stewart (1991, 1994), Ehrbar (1998), and Ehrbar and Stewart (1999). Very similar concepts have been promoted by other management consulting firms. Ittner and Larcker (1998) provide an overview of different “value based” performance measures used in practice.

\(^3\)For instance, Stern-Stewart propose up to 164 “adjustments” to generally accepted accounting rules.
cost of capital but also the underlying agency problem.

Our analysis builds on earlier work which has studied managerial incentives in the capital budgeting process. For instance, Antle and Eppen (1985) argue that an optimal incentive mechanism will induce underinvestment, i.e., the principal will forego marginally profitable investments, if the agent has better information about the investment opportunity. The argument for underinvestment, which is familiar from the adverse selection literature, is that the better informed agent earns informational rents. In order to balance the return from investment with the required compensation payments to the agent, the principal finds it desirable to curtail investment.4

In a different branch of the literature on investment incentives, Ramakrishnan (1988), Rogerson (1997) and Reichelstein (1997) have examined how a principal can create goal congruence, that is, induce a better informed agent to accept all projects with positive net present value, and only those.5 In contrast to the one-period model of Antle and Eppen (1985), the later papers on goal congruence allow for the project cash flows to be received over several periods. Goal congruence can be obtained by an intertemporal cost allocation scheme which subtracts from the periodic operating cash flow a suitable "share" of the initial investment cost. The resulting performance measure is identical to residual income for a particular depreciation schedule, which Rogerson (1997) refers to as the relative benefit depreciation schedule. To create first-best investment incentives, it is furthermore essential that the capital charge rate be equal to the principal's cost of capital.

One drawback of the above mentioned papers on goal congruence is the absence of a hidden action problem. Effectively, there is no conflict of interest between principal and agent, as evidenced by the fact that the compensation payments to the agent remain indeterminate. An immediate concern then is whether the insights about goal congruent performance mea-

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4See also Bernardo, Honglin, and Luo (2000). In contrast, Harris and Raviv (1996, 1998) demonstrate that an optimal capital budgeting mechanism can result in either under- or overinvestment. The possibility of overinvestment derives from the fact that in their model the manager has an intrinsic preference for investing in the project.

5Extending the earlier analysis of Antle and Eppen (1985), Antle and Fellingham (1990) consider repeated investment decisions. They find that even if projects are independent, it becomes desirable to connect them through a long-term contract with the agent.
sures and intertemporal cost allocations remain viable once the model is extended to include hidden action, and the desired incentives are derived from a unified optimization program.

Our results integrate the earlier results on capital budgeting and on goal congruence. In our model, the periodic operating cash flows reflect the agent's unobservable effort as well as the profitability of the investment. Since the principal cannot separate these two components, the hidden information and hidden action problems become intertwined. As a consequence, the optimal investment policy entails underinvestment, like the ones in the capital budgeting literature. Nonetheless, residual income based on the relative benefit depreciation schedule remains an optimal performance measure in this setting, provided the principal imposes a suitable capital charge rate. Specifically, the remaining book value of the asset should be burdened with an interest rate that is equal to the firm’s hurdle rate, the critical internal rate of return below which the principal would not want to fund the project. This hurdle rate incorporates the compensation cost for the better informed agent and therefore exceeds the principal's cost of capital.

Our results suggest that the earlier characterizations of goal congruent performance measures apply to a significant extent also in a second-best contract setting. By allocating an appropriate share of the initial investment in each subsequent period, the principal ensures that a profitable project makes a positive contribution to the agent’s performance measure in every period. As a consequence, the underlying moral hazard problem and the required bonus coefficients in each period have no bearing on the agent’s investment decision. Furthermore, the performance measure, i.e., residual income, and the depreciation rules depend on the underlying agency problem only through the optimal capital charge rate.

A common characteristic of the capital budgeting process in many firms appears to be that divisions compete for scarce investment funds.\(^6\) We find that our solution obtained in a single agent setting carries over to competitive situations with minor modifications. Specifically, residual income based on the relative benefit depreciation schedules remains an optimal performance measure. To ensure efficient resource allocation, however, the capital charge rate needs to be adjusted upward to a competitive hurdle rate, which is the lowest

\(^6\)See, for example, Taggart (1987), and Harris and Raviv (1996, 1998).
internal rate of return at which a division’s project would still be funded, given the rates of return available to other divisions. We show that managers will have a dominant strategy incentive to report their internal rates of return truthfully, if they anticipate that their division’s investment will be subsequently charged at the competitive hurdle rate.

While most of our analysis focuses on risk neutral parties, we do address the impact of risky projects and managerial risk aversion. For analytical tractability, we confine attention to a multiperiod version of the so-called LEN model: Linear contracts, Exponential Utility and Normally distributed noise. This framework is convenient for identifying the agency cost associated with the adoption of risky projects. This additional agency cost increases the principal’s hurdle rate beyond that in the risk neutral case. While residual income based on the relative benefit depreciation rules continues to be an optimal performance measure in the risk-averse setting, we find that the capital charge rate must be set below the risk adjusted hurdle rate. Otherwise, a risk averse manager would have a tendency to underinvest in risky projects.

The remainder of the paper is organized as follows. Section 2 presents the model. The optimal revelation mechanism is derived in Section 3. In Section 4, we establish the optimality of delegation schemes for which the agent’s compensation is based on residual income. Section 5 analyzes a setting in which multiple divisions are competing for funding of their own projects. Uncertain cash flows and the adoption of risky projects by a risk averse manager are analyzed in Section 6. We conclude in Section 7.

2 Model Description

We study a multiperiod agency problem with both hidden action and hidden information. Initially, we consider a setting with one agent (manager) who contributes unobservable productive effort in each of $T$ periods. In addition, the agent is assumed to have superior information regarding the profitability of an investment project. Though the agent is intrinsically indifferent about accepting or rejecting this project, he must be given appropriate investment incentives due to lack of separability in the periodic operating results. Speci-
cally, the principal is assumed to observe only the total operating cash flow in each period without being able to identify the components related to the project and those related to the agent’s periodic effort.

The investment project will be represented by the \((T + 2)\)-tuple:

\[
P = (b, x_1, \ldots, x_T, y).\]

Here, \(b\) denotes the initial cash investment at date 0. The project generates operating cash flow in the amount of \(x_t \cdot y\) at \(T\) subsequent dates. We interpret \(y\) as a “profitability” parameter which represents the agent’s superior information. In contrast, both parties are assumed to know the intertemporal distribution of the project’s operating cash flows, as represented by the distributional parameters \(\bar{x} = (x_1, \ldots, x_T)\), with \(\sum_{t=1}^{T} x_t = 1\).\(^7\) For any given \(y\), the project’s net present value will be denoted by:

\[
NPV(y) \equiv \sum_{t=1}^{T} y \cdot x_t \cdot \gamma^t - b,
\]

where \(\gamma \equiv (1+r)^{-1}\) and \(r\) denotes the cost of capital. For contracting purposes, the principal can rely on the operating cash flows received in periods 1 through \(T\) as well as the investment cash flow in period 0, provided the project is undertaken. Specifically, the operating cash flows are given by

\[
c_t = \lambda_t \cdot a_t + x_t \cdot y \cdot I
\]

for \(1 \leq t \leq T\), where \(a_t\) denotes the agent’s productive effort and \(I \in \{0, 1\}\) is an indicator variable which reflects whether the project was accepted at date 0.

While the structure of the operating cash flows in (1) suggests that the hidden information and the hidden action problem are separable, these two problems are in fact intertwined.

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\(^7\)The principal may be able to rely on a corporate controller in order to verify the intertemporal distribution of project cash flows, \((x_1, \ldots, x_T)\). For instance, if the project concerns a new factory, \(x_t\) may refer to the physical capacity available in period \(t\), while \(y\) represents the contribution margin attainable per unit of capacity. Capacity may vary over time possibly because of a “ramp-up” phase in earlier periods and increased time for maintenance and repairs in later periods. The premise then is that the controller, who acts as a steward for the principal, can assess capacity available in future years, but the manager is better informed about the contribution margin attainable.
Without hidden action, the investment problem would have a trivial solution since a fixed compensation payment would provide the manager with (weak) incentives to make first-best investment decisions. Conversely, with a risk neutral manager the moral hazard problem would have a straightforward solution in the absence of a hidden information. For the structure of operating cash flows in (1), however, the basic issue is that for a relatively profitable project, i.e., when $y$ assumes a high value, the agent will be able to achieve given operating results with less productive effort. Thus, the availability of the project enables the manager to earn informational rents. As explained in Section 4 below, our results can be extended in several ways beyond the structure of operating cash flows specified in (1).

When the parties enter into the contract at date 0, the agent is assumed to know the profitability parameter $y$, i.e., the agent has pre-contract private information. The timeline in Figure 1 summarizes the sequence of events in our model:

![Figure 1](image)

The agent chooses his effort $a_t$ from the interval $[0,1]$ at a personal cost of $v_t(a_t)$. Initially, we assume that the agent is risk neutral, and that his utility payoff is given by:

$$U = \sum_{t=1}^{T} [s_t - v_t(a_t)] \cdot \gamma^t,$$

where $s_t$ denotes the agent’s compensation payment in period $t$. Clearly, it is only the present value of the compensation payments that matters provided both parties can commit to a $T$-period contract. By the Revelation Principle we may focus on revelation mechanisms

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8Harris and Raviv (1996, 1998), Arya, Baldenius and Glover (1999) and Lambert (2000) consider settings in which the manager has an intrinsic preference for undertaking the project, i.e., the manager has a preference for “empire building”. In contrast, investment is assumed to be personally costly in Wagenhofer (1999).

9Alternatively, the agent may learn the parameter $y$ after entering the contract with the principal, but he cannot be prevented from quitting his job if his participation constraint is not satisfied.
which induce the agent to reveal his information truthfully. In our setting, a revelation mechanism specifies an investment decision rule \( I(\bar{y}) \in \{0, 1\} \), as well as required cash flows \( \bar{c}(\bar{y}) \equiv (c_1(\bar{y}), \ldots, c_T(\bar{y})) \) and compensation payments \( \bar{s}(\bar{y}) \equiv (s_1(\bar{y}), \ldots, s_T(\bar{y})) \), contingent on the agent’s report \( \bar{y} \). For any such mechanism, \( \{\bar{c}(\bar{y}), \bar{s}(\bar{y}), I(\bar{y})\} \), the agent’s utility payoff contingent on the true profitability parameter \( y \) and the reported \( \bar{y} \) becomes:

\[
U(\bar{y}, y) \equiv \sum_{t=1}^{T} [s_t(\bar{y}) - v_t(\alpha_t(\bar{y}, y))] \cdot \gamma^t,
\]

where

\[
\alpha_t(\bar{y}, y) \equiv \min\{a_t \mid \lambda_t \cdot a_t + x_t \cdot y \cdot I(\bar{y}) \geq c_t(\bar{y})\}
\]

The principal’s beliefs regarding \( y \) are represented by a density function \( f(y) \) on the interval \([y, \bar{y}]\). The principal certainly does not want to accept the project if \( y \) is below the first best cut-off level \( y^0 \), given by the condition \( NPV(y^0) = 0 \). To avoid a trivial investment problem we assume that \( y^0 \in (y, \bar{y}) \). The principal’s optimization program then becomes:

\[
P_0: \max_{(s, c, I)} \int_{y}^{\bar{y}} \{ \sum_{t=1}^{T} [c_t(y) - s_t(y)] \cdot \gamma^t - b \cdot I(y) \} f(y) dy
\]

subject to:

(i) \( U(y, y) \geq U(\bar{y}, y) \) for all \( y \) and \( \bar{y} \),

(ii) \( U(y, y) \geq 0 \) for all \( y \).

The optimization program in (2) is a standard adverse selection problem. The first constraint ensures that truthful reporting is in the manager’s best interest, while the second constraint ensures that the manager has no reason to quit (without loss of generality his external market alternative is normalized to zero). \(^{10}\)

\(^{10}\)Consistent with the interpretation in Antle and Eppen (1985), one may think of \( s_t \) as a “budget” that the manager receives in period \( t \) for covering the costs of certain tasks. The unverifiable cost associated with these tasks is \( v_t(\alpha_t(\bar{y}, \bar{y})) \), and the manager gets to spend the “slack”, i.e., \( s_t - v_t(\alpha_t(\cdot)) \), on managerial perks.
3 Optimal Incentive Schemes

The manager will generally earn informational rents on account of his private information since he can underreport $y$ and at the same time reduce his effort. Therefore his utility payoff $U(y, y)$ will generally exceed the minimum payoff of zero. The basic tradeoff for the principal is that the agent’s informational rent will be increasing both in the induced effort level of effort, $a_t$, as well as in the set of states in which the project is undertaken. It is well known from the adverse selection literature that the agent’s informational rent is determined by the derivative of his cost of effort function $v_t(\cdot)$ with respect to the unknown parameter $y$. Since $a_t = \frac{1}{\gamma} \cdot (c_t - x_t \cdot y \cdot I)$, the informational rent becomes:

$$U(y, y) = \int_{y}^{\bar{y}} \sum_{t=1}^{T} \frac{v_t(a_t(u, u))}{\lambda_t} \cdot x_t \cdot \gamma^t \cdot I(u) \, du,$$

for all $y \in [y, \bar{y}]$. The characterization in (3) is based on the “local” incentive compatibility conditions, i.e., the agent’s utility payoff must be at a stationary point if he reports truthfully.$^{11}$ The expression in (3) illustrates the basic trade-off mentioned above: the principal can reduce the agent’s informational rent either by creating “lower powered” action incentives (there will be no rents if $a_t = 0$) or, alternatively, by curtailing the set of states $y$ in which the project is undertaken.

In our analysis below, we focus on a setting in which the marginal cost of effort is sufficiently small so that the principal always finds it worthwhile to induce maximum effort, i.e., $a_t = 1$, in every period. The cost of effort $v_t(\cdot)$ as well as the marginal cost of effort $v_t'(\cdot)$ are assumed to be increasing and convex, with $v_t'(0) = 0$. Inducing the maximum effort level will indeed be optimal provided $v_t'(1)$ is sufficiently small relative to the other parameters of the model. For brevity, we denote $v_t' \equiv v_t'(1)$. We use the expression in (3), evaluated at $a_t = 1$, to solve the compensation payments $s_t(y)$ in $P_0$. The principal’s problem can then be restated as:

$^{11}$Standard references include Myerson (1981), Baron and Myerson (1982) and Laffont and Tirole (1986).
\[
P_1 : \max_{I(y)} \int_{y_0}^y \left\{ \sum_{t=1}^T \left[ \lambda_t - v_t(1) \right] \cdot \gamma^t + \left[ (NPV(y) - k \cdot H(y)) \cdot I(y) \right] f(y) dy \right\},
\]

where
\[
k = \sum_{t=1}^T \frac{v_t'}{\lambda_t} \cdot x_t \cdot \gamma^t,
\]
and \(H(y)\) denotes the inverse hazard rate, i.e., \(H(y) = \frac{1 - F(y)}{f(y)}\). The reduced objective function in \(P_1\) reflects that the expected value of the agent’s informational rent (as given by (3)) is equal to the expected value of \(k \cdot H(y) \cdot I(y)\); for details the reader is referred to the proof of Lemma 1 in the Appendix.\(^{12}\)

The following condition formalizes the notion that the marginal cost of managerial effort is sufficiently small (relative to the benefit \(\lambda_t\)) so as to make it desirable to induce maximum effort \(a_t = 1\) in every period:
\[
\lambda_t - v_t'(1) - H(y^0) \cdot \frac{v_t'(1)}{\lambda_t} \cdot x_t > 0.
\]

We recall that \(y^0\) is the level of \(y\) at which the project breaks even, i.e., \(NPV(y^0) = 0\). Condition (5) says that even if the project were to be accepted for all \(y \geq y^0\), the marginal return from effort, \(\lambda_t\), is still sufficiently large for the principal to prefer high effort despite the corresponding increase in the agent’s expected informational rent.

**Lemma 1** Given (5), the agent will be given an incentive to choose high effort, i.e., \(a_t = 1\), in each period. Furthermore, the optimal incentive scheme results in underinvestment, that is, the project is undertaken if and only if \(y\) exceeds some cut-off level \(y^*\) which satisfies \(y^0 < y^* < \bar{y}\) and solves the equation:
\[
NPV(y^*) - H(y^*) \cdot k = 0.
\]

\(^{12}\)Like in most of the adverse selection literature, we make the assumption that the inverse hazard rate, \(H(y)\), is decreasing in \(y\).
The equation for the optimal cut-off level \( y^* \) reflects that the principal is willing to forego marginally profitable projects in order to reduce the agent’s informational rent. In the terminology of Myerson (1981) and Baron and Myerson (1982), a project is profitable only if it yields a positive *virtual* NPV, which equals the actual NPV minus the (expected) incremental agency cost \( H(y) \cdot k \).\(^{13}\)

To develop the results of this paper, it will be convenient to consider an indirect revelation mechanism in which the agent is asked to report the project’s internal rate of return \( r(y) \); that is, the interest rate \( r(y) \) for which \( NPV(y \mid r(y)) \equiv 0 \). The project receives funding if and only if \( r(y) \) exceeds some critical level \( r^* \). Clearly, such a mechanism becomes equivalent to the optimal direct revelation mechanism if \( r^* \) is set equal to the internal rate of return corresponding to \( y^* \) in equation (6). We refer to \( r^* \) as the *hurdle rate*. It is the minimal internal rate of return required to make the project attractive for the principal given the agency costs imposed by asymmetric information. We note that \( r^* \) exceeds the principal’s cost of capital \( r \). At the same time, \( r^* \leq r(\overline{y}) \) since \( H(\overline{y}) = 0 \) and therefore the difference between the virtual and actual NPV goes to zero as \( y \) approaches \( \overline{y} \).

4 Hurdle Rates and Intertemporal Cost Allocations

We now consider incentive mechanisms in which the investment decision is delegated to the manager and investment incentives are created through an intertemporal allocation of the initial investment cost. This perspective is motivated by the widespread practice of charging organizational units with allocated costs that correspond to investment expenditures incurred in the past. Given our characterization in Lemma 1, we ask whether the following class of

\(^{13}\)Lemma 1 can readily be extended to settings in which the optimal effort levels are interior in the interval \([0,1]\). The induced levels of effort will depend on the agent’s report, since the principal can achieve a further separation of types by making \( a_t(y) \) increasing in \( y \). The resulting investment policy will again entail underinvestment, with the optimal cut-off level given by (6) after substituting \( v_t'(a_t(y)) \) for \( v_t'(1) \) in the expression for \( k \).
performance measures is compatible with optimal incentive contracting:

$$\pi_t = c_t - z_t \cdot b . \quad (7)$$

Here, $z_t$ is the "share" of the initial investment expenditure that is charged against the operating cash flow in period $t$. We refer to $(z_1, \ldots, z_T)$ as an \textit{intertemporal cost allocation rule}. In principle, the cost charges $z_t$ may depend on all the parameters of the underlying project and the underlying agency problem. An intertemporal cost allocation rule is said to generate an optimal performance measure if there exist coefficients $\{h_t, k_t\}_{t=1}^T$ such that the linear compensation scheme:

$$s_t(\pi_t) = h_t + k_t \cdot \pi_t , \quad (8)$$

for $1 \leq t \leq T$, achieves the same expected payoff for the principal as the optimal revelation mechanism identified in Lemma 1.

Rogerson (1997) suggests the following intertemporal cost allocation rule:

$$z_t(\bar{x}, \hat{\gamma}) = \frac{x_t}{\sum_{i=1}^T x_i \cdot \hat{\gamma}_i} , \quad (9)$$

which he terms the \textit{relative benefit cost allocation rule}, since the cost charge in period $t$ is proportional to the relative magnitude of the benefit parameter $x_t$. We note that for any generic interest $\hat{\gamma}$ (with $\hat{\gamma} \equiv (1 + \hat{\gamma})^{-1}$), the present value of the cost allocation charges in (9) is equal to one. To illustrate the implications of this cost allocation rule, it is useful to note that the project’s NPV relative to the generic interest $\hat{\gamma}$ is:

$$NPV(y \mid \hat{\gamma}) \equiv \sum_{t=1}^T x_t \cdot y \cdot \hat{\gamma}^t - b .$$

Under the relative benefit rule, a project then makes the following contribution to the agent’s performance measure in period $t$:

$$x_t \cdot y - \frac{x_t}{\sum_{i=1}^T x_i \cdot \hat{\gamma}_i} \cdot b = \frac{x_t}{\sum_{i=1}^T x_i \cdot \hat{\gamma}_i} \cdot NPV(y \mid \hat{\gamma}) . \quad (10)$$

Thus, the relative benefit cost allocation rule effectively "annuitizes" the project in the sense that the performance measure increases by a share of the project’s NPV (relative to $\hat{\gamma}$) in
every period. If the principal were to set \( \hat{r} = r \) and impose the relative benefit cost allocation rule, the agent would invest if and only if the project has a positive NPV, i.e., whenever \( y \geq y^0 \). By increasing \( \hat{r} \) to \( r^* \), the principal effectively calibrates the agent’s investment incentives, since the latter will only accept projects with rates of return greater than the hurdle rate \( r^* \).

**Proposition 1** The relative benefit cost allocation rule corresponding to the hurdle rate \( r^* \) generates an optimal performance measure.

**Proof:** The principal chooses the compensation coefficients \( \{k_t, h_t\}_{t=1}^T \) such that \( k_t = \frac{v_t'}{\lambda_t} \) and \( h_t = v_t - v_t' \) for all \( 1 \leq t \leq T \). The choice of bonus coefficients ensures that \( a_t = 1 \), and the fixed payments, \( h_t \), ensure that the participation constraint is satisfied. Since \( r^* \) is chosen such that \( \text{NPV}(y^* \mid r^*) = 0 \), the agent will accept the project if and only if \( y \geq y^* \). Thus, the delegation mechanism induces the same actions and investment decisions as the optimal revelation mechanism. The Revenue Equivalence Theorem then implies that the payments to the agent must coincide with those of the optimal revelation mechanism.\(^\text{14}\)

\[ \square \]

The intertemporal cost allocation charges of the form in (9) cannot be interpreted as depreciation charges because conventional accrual accounting requires the *undiscounted* sum of the depreciation charges to be equal to the cost of the investment. However, any performance measure generated by an intertemporal cost allocation rule is identical to *residual income* for a corresponding depreciation schedule. Denoting the depreciation charges by \( \{d_t\}_{t=1}^T \), residual income is defined as accounting income minus an interest charge on the remaining

\[^{14}\text{The Revenue Equivalence Theorem says that any two incentive mechanism which yield the same allocations and satisfy both incentive compatibility and the participation constraints must also yield the same expected payoff for the principal; see, for instance, Myerson (1981). To check this claim explicitly, we note that the agent’s informational rent under the delegation mechanism becomes:} \]

\[ \sum_{t=1}^T \frac{v_t'}{\lambda_t} \cdot \gamma^t \cdot \left[ \frac{x_t}{\sum_{i=1}^T x_i \cdot (1 + r^* - t)} \cdot \text{NPV}(y \mid r^*) - \text{NPV}(y^* \mid r^*) \right] = \sum_{t=1}^T \frac{v_t'}{\lambda_t} \cdot x_t \cdot \gamma^t \cdot [y - y^*] = k \cdot (y - y^*) \]

which is the informational rent given in (3).
book value of the original investment cost. Thus:

\[ RI_t = (c_t - d_t \cdot b) - \hat{r} \cdot B_{t-1}, \]  

(11)

where \( B_t = (1 - \sum_{i=1}^{t} d_i) \cdot b \) denotes book value at the end of period \( t \). For the residual income performance measure the relationship between intertemporal cost allocation rules and depreciation schedules is therefore given by:

\[ z_t = d_t + \hat{r} \cdot (1 - \sum_{i=1}^{t-1} d_i). \]  

(12)

It is well known that for any capital charge rate, \( \hat{r} \), the mapping given by (12) generates a one-to-one correspondence between intertemporal cost allocations, for which the present value of the cost charges is one, and depreciation schedules, for which the sum of the depreciation charges is equal to one.\(^{15}\) We shall refer to the relative benefit depreciation schedule as the one that maps to the relative benefit cost allocation rule in (12). Earlier literature has observed that when the project cash flows are uniform, i.e. \( x_1 = x_2 = \cdots = x_T \), the relative benefit depreciation rule coincides with the familiar annuity depreciation method.\(^{16}\) Stewart (1994) and Ehrbar and Stewart (1999) refer to the annuity depreciation method as the "sinking fund" depreciation method and advocate it as an alternative to straight line depreciation.

**Corollary to Proposition 1:** Residual income based on the hurdle rate \( r^* \) and the relative benefit depreciation rule is an optimal performance measure.

This result speaks to the question as to what capital charge rate should be imposed on a division if its performance is measured by residual income. With risk neutral parties and no constraints on investment capital, the capital charge rate can be set equal to the critical internal rate of return below which the principal would not want to undertake the project. Furthermore, if the principal relies on residual income and the relative benefit depreciation

\(^{15}\)See Preinreich (1938) and also Auctil, Jordan, and Mukherji (1996) and Rogerson (1997).

\(^{16}\)See, for example, Solomons (1966) and Ramakrishnan (1988).

13
rule, then the capital charge rate must be equal to the hurdle rate \( r^* \). Any other capital charge rate would necessarily distort the agent’s investment incentives.

We note, however, that even within the class of delegation schemes given by (7)–(8), the solution identified in Proposition 1 and its corollary is not unique.\(^{17}\) To illustrate, suppose the principal relies on straight line depreciation which is commonly used for external financial reporting purposes. The intertemporal cost charges are then given by:

\[
z^0_t(\hat{r}) = \frac{1}{T} + \hat{r} \cdot (1 - \frac{t - 1}{T}) ,
\]

and the capital charge rate \( \hat{r} \) must be chosen so that the agent invests if and only if \( y \) exceeds \( y^* \). Specifically, this requires:

\[
\sum_{t=1}^{T} k_t \cdot [x_t \cdot y^* - z^0_t(\hat{r}) \cdot b] \cdot \gamma^t = 0 .
\]

Since the parameters \( k_t \) are uniquely determined by the moral hazard problem, i.e., \( k_t = \frac{v_t}{x_t} \), one needs to find a charge rate \( \hat{r} \) so that (14) holds. We note, however, that \( \hat{r} \) is merely a “plug” variable which generally cannot be interpreted as an interest rate. As \( k_t \) and \( x_t \) vary, the resulting \( \hat{r} \), which solves (14), may well become negative.

In contrast, the solution identified in Proposition 1 and its corollary rely on the notion of matching revenues and expenses in each period. As argued in (10), for any capital charge rate \( \hat{r} \), and in particular for the hurdle rate \( r^* \), a positive (negative) NPV project makes a positive (negative) contribution to the manager’s performance measures in every period.\(^{18}\) We now argue that this form of matching is indeed necessary if one insists that the capital charge rate be equal to the hurdle rate \( r^* \). In the following analysis, we treat the parameters \( \{u_t\}_{t=0}^T \) and \( f(\cdot) \) of the problem as fixed, but allow the productivity parameters \( \tilde{\lambda} \equiv (\lambda_1, \ldots, \lambda_T) \) to vary in some open set \( \Lambda \) in \( \mathbb{R}_+^T \). As \( \tilde{\lambda} \) varies, the compensation parameters \( k_t \) and \( h_t \) are

---

\(^{17}\)Mishra and Vaysman (2000) examine a model in which there is no scope for intertemporal matching of project cash flows and investment cost. In the context of their model, they conclude that a variety of accounting and cash flow based incentive schemes can generate optimal contracts.

\(^{18}\)The analysis in Rogerson (1997) and Reichelstein (1997) formalizes the argument that if the agent discounts future payoffs at a rate possibly higher than \( \gamma \), the relative benefit cost allocation becomes essentially unique in order to align investment incentives.
allowed to vary correspondingly, and we denote them by $k_t(\lambda)$ and $h_t(\lambda)$. At the same time, the hurdle rate $r^*$, as given by (6) will vary as well.

We say that an intertemporal cost allocation rule \textit{robustly generates} an optimal performance measure of the form in (7) if this performance measure is optimal for all $\lambda$ in a neighborhood of $\lambda^0$, holding the capital charge rate fixed at the hurdle rate $r^*(\bar{\lambda})$, i.e.,

\[
\{s_t = h_t(\lambda) + k_t(\lambda) \cdot [c_t - z_t(\bar{x}, r^*(\lambda)) \cdot b]\}_{t=1}^T
\]

is an optimal incentive scheme.

\textbf{Proposition 2} \textit{If $r = r^*(\bar{\lambda})$, the relative benefit cost allocation rule is the only intertemporal cost allocation rule which robustly generates an optimal performance measure.}

The robustness criterion introduced in Proposition 2 requires that variations in the productivity parameters $\bar{\lambda} = (\lambda_1, \ldots, \lambda_T)$ must leave the performance measure unchanged whenever the optimal hurdle rate $r^*$ is unaffected by changes in $\bar{\lambda}$. At the same time, however, the bonus coefficients $(k_1, \ldots, k_T)$ must be adjusted to any changes in the productivity parameters of $\bar{\lambda}$. The only way to preserve optimal investment incentives then is to ensure that the relative magnitude of the bonus coefficients has no bearing on the investment decision. The relative benefit allocation rule has precisely this property because a desirable (undesirable) project makes a positive (negative) contribution to the performance measure in every period, and therefore the incentive to invest is independent of the bonus parameters.\textsuperscript{19}

Aside from robustness considerations, the relative benefit cost allocation rule also emerges as the unique solution if one imposes the additional requirement that the agent’s utility be non-negative in every period. Again, the moral hazard problem and the possibility that the project may not be undertaken uniquely determine $k_t$ and $h_t$ at the values identified in Proposition 2. The non-negative utility constraint then becomes:

\[
k_t \cdot [x_t \cdot y^* - z_t(\bar{r}) \cdot b] \geq 0
\]

\textsuperscript{19}If performance measures are restricted to be linear combinations of current accounting variables, i.e., income and book value, one might expect that Proposition 2 can be extended to identify residual income based on the relative benefit depreciation rule as the only performance measure which generates optimal incentives in a robust fashion. This can indeed be shown provided one imposes a somewhat more demanding robustness requirement. See Dutta and Reichstein (1999b) for details.
for all $1 \leq t \leq T$. In order for the incentive scheme to be optimal, (15) must hold as an equality in each period. Furthermore, since $y^*$ satisfies $NPV(y^* \mid r^*) = 0$, we have $\sum_{t=1}^{T} x_t \cdot (1 + r^*)^{-t} \cdot y^* = b$, and thus $z_t$ must be the relative benefit cost allocation rule in (9) with $\bar{r} = r^*$.

We conclude this section by noting that our model can be extended in several directions without affecting the validity of Proposition 1 and its corollary. First, the linearity of the compensation scheme together with the assumed risk neutrality immediately imply that Proposition 1 would be unaffected if the periodic operating cash flows were subject to additive noise terms. Second, we have assumed that the agent provides “general purpose” effort, but that his effort does not contribute specifically to the success of the project. Alternatively, suppose the agent provides two-dimensional effort $a_t = (a_1^t, a_2^t) \in [0, 1] \times [0, 1]$ such that:

$$c_t = \lambda_1^t \cdot a_1^t + (\lambda_2^t \cdot a_2^t + x_t \cdot y) \cdot I$$

and the total cost of effort is given by $v_t(a_1^t, a_2^t) = v_1^t(a_1^t) + v_2^t(a_2^t)$. If condition (5) continues to hold with respect to $a_1^t$ and $v_2^t(1) \leq v_2^*(1)$, it follows immediately that the above incentive scheme will remain optimal once the fixed payments $\{h_t\}$ are adjusted to account for the additional cost of effort.

Third, as noted in connection with Lemma 1, a more general formulation of our model will involve interior effort assignments, $a_t(y)$, which depend on the reported $y$. It is clear that in such settings the delegation mechanisms considered in (7)–(8) can no longer replicate the optimal revelation mechanism. However, the principal can still create optimal incentives through delegation schemes mechanism in which the agent first chooses from a menu of linear compensation schemes $\{k_t(y), h_t(y)\}_{t=1}^{T}$ and subsequently makes the investment decision. Furthermore, the capital charge rate can still be set equal to the hurdle rate $r^*$. Our focus on boundary solutions for the agent’s effort has allowed us to bypass the need for communication based mechanisms.

20With interior effort choices $a_t(y) < 1$, the characterization of the optimal $y^*$ and $r^*$ needs to be modified from that in (6).
5 Capital Budgeting for Competing Projects

This section examines a setting in which two divisions (agents) are vying for scarce investment capital. Specifically, we suppose that the firm can fund at most one of two available projects. One possible interpretation of this constraint is that in order to fund both projects the firm would have to borrow funds externally at a rate which is known to exceed the rate of return available from either of the two potential projects.

With competing divisions, the principal could use an auction mechanism to allocate the scarce capital resources. Consider, for instance, the following variant of a second-price auction mechanism, similar to the one suggested by Laffont and Tirole (1987) in a procurement setting. Each agent is asked to report his internal rate of return $r_i(y_i)$. Agent $i$’s project receives funding if and only if:

$$r_i(y_i) > \max\{r_j(y_j), r_i^*\},$$

where $r_i^*$ is agent $i$’s hurdle rate in a single agent setting. If agent $i$’s project is funded, his performance is measured by residual income combined with the relative benefit depreciation rule and a capital charge rate equal to $\max\{r_j(y_j), r_i^*\}$. Since neither agent’s report affects his own capital charge rate under this mechanism, each agent has a dominant strategy incentive to report his internal rate of return truthfully. While this mechanism ensures the selection of the project with the highest NPV, the principal’s objective is to maximize the overall virtual NPV that reflects the underlying information asymmetry and the resulting agency problem.\(^{21}\)

In order to maximize the virtual NPV, the principal can employ a suitable “calibration” function so as to make the two internal rates of return comparable.

To describe formally the incentive and resource allocation problem with two competing agents, suppose that the environment for each agent is exactly the one described in Section 2. We use the subscript $i$, $1 \leq i \leq 2$, to refer to agents, while the index $t$ again refers to time periods. Thus, $c_{it}$ denotes the cash flow delivered by Agent $i$ in period $t$. The realization of $y_i$ is agent $i$’s private information. Both the principal and agent $i$ initially view $y_i$ as a

\(^{21}\)In other resource allocation contexts, this argument has been made in the earlier work of Myerson (1981) and Harris, Kriebel and Raviv (1982).
random variable with probability density $f_i(y_i)$. In direct extension of program $P_0$ in Section 3, the principal’s optimization problem can now be stated as a direct revelation mechanism in which agent $i$ is asked to report $y_i$. We again use the vector notation $\vec{y} \equiv (y_1, y_2)$.

$$\mathbf{P}_2 : \max_{c(\vec{y}), s(\vec{y}), I(\vec{y})} \int_{\vec{y}_1}^{\vec{y}_2} \left\{ \sum_{i=1}^{2} \sum_{t=1}^{T} [c_{it}(\vec{y}) - s_{it}(\vec{y})] \cdot \gamma^t - b_1 \cdot I_1(\vec{y}) - b_2 \cdot I_2(\vec{y}) \right\} f_2(y_2) f_1(y_1) dy_2 dy_1$$

subject to:

(i) $I_1(y_1, y_2) + I_2(y_1, y_2) \leq 1,$

(ii) $\int_{\vec{y}_j}^{\vec{y}_j} [U_i(y_i, y_j, y_i) - U_i(\vec{y}_i, y_j, y_i)] f_j(y_j) dy_j \geq 0,$ for all $y_i$ and $\vec{y}_i$,

(iii) $\int_{\vec{y}_i}^{\vec{y}_i} U_i(y_i, y_j, y_i) f_j(y_j) dy_j \geq 0,$ for all $y_i$.

Here $U_i(\vec{y}_i, y_j, y_i)$ represents agent $i$’s utility contingent on reports $(\vec{y}_i, y_j)$ and his true type $y_i$. Specifically,

$$U_i(\vec{y}_i, y_j, y_i) = \sum_{t=1}^{T} [s_{it}(\vec{y}_i, y_j) - v_{it}(a_{it}(\vec{y}_i, y_j, y_i))] \cdot \gamma^t$$

where $a_{it}(\vec{y}_i, y_j, y_i)$ represents the minimal effort agent $i$ needs to exert in period $t$, when the revelation mechanism requires him to deliver the cash flow $c_{it}(\vec{y}_i, y_j)$ and his true state is $y_i$.

The incentive compatibility constraint in $\mathbf{P}_2$ requires that truthful reporting constitute a Bayesian-Nash equilibrium. At the same time, the participation constraints are required to hold in an interim sense, i.e., given his own type each agent must break even in expectation over the other agent’s possible types. By the same reasoning as in the single agent case, one can use the local incentive compatibility conditions and the participation constraints to transform $\mathbf{P}_2$ into an unconstrained optimization problem analogous to $\mathbf{P}_1$. As in the single agent case, we simplify the moral hazard problem by assuming that inequality (5)
holds for each agent and therefore the principal wants to induce \( a_{it} = 1 \). It then follows that the principal wants to fund agent \( i \)'s project if its virtual NPV exceeds both zero and the other project's virtual NPV. It will be convenient to denote the virtual NPV of agent \( i \) by \( \phi_i(y_i) = NPV_i(y_i) - k_i \cdot H_i(y_i) \), where \( k_i = \sum_{t=1}^{T} x_{it} \cdot \gamma^t \).

**Lemma 2** Given (5), the optimal capital budgeting rule is given by \( I_i(y_i, y_j) = 1 \) if and only if:

\[
\phi_i(y_i) > \max\{\phi_j(y_j), 0\}.
\] (16)

The proof of this result is omitted since the arguments involved are similar to the ones used in the proof of Lemma 1.

As in the single-agent case, the optimal capital budgeting policy can be implemented through an indirect revelation mechanism in which each agent is asked to report his project's internal rate of return \( r_i(y_i) \) and the project receives funding if and only if \( r_i(y_i) \) exceeds some critical threshold. To identify the hurdle rates with competing projects, we denote by \( \bar{r}_i \equiv r_i(y_i) \) and \( \bar{r}_j \equiv r_j(y_j) \) the lowest and highest possible internal rates of return. The calibrating function \( \Gamma_i : [\bar{r}_i, \bar{r}_j] \rightarrow [\gamma_0, \gamma_T] \) is then defined by

\[
\Gamma_i(r_i(y_i)) = r_j(y_j)
\]

where \( y_j \) is such that \( \phi_j(y_j) = \phi_i(y_i) \).\(^{22}\) We note that \( \Gamma_i \) is well-defined since \( r_i(\cdot) \) and \( \phi_i(\cdot) \) are monotone functions. In particular, it follows that \( \Gamma_i(r_i^*) = r_j^* \), since, by definition, \( r_i(y_i^*) \equiv r_i^* \) and \( \phi_1(y_1^*) = 0 = \phi_2(y_2^*) \). The competitive hurdle rate for agent \( i \) can now be defined as:

\[
r_i^*(r_j) = \max\{\Gamma_j(r_j), r_i^*\}.
\] (17)

Figure 2 illustrates the function \( \Gamma_1 \) and the optimal capital budgeting rules in (16).\(^{23}\)

\(^{22}\)If for given \( r_i(y_i) \), \( \phi_j(y_j) < \phi_i(y_i) \) for all \( y_j \), then \( \Gamma_i(r_i(y_i))=\bar{r}_j \). Conversely, \( \Gamma_i(r_i(y_i))=\gamma_0 \) if \( \phi_j(y_j) > \phi_i(y_i) \) for all \( y_j \).

\(^{23}\)We note that \( \Gamma_2 \) is the inverse of \( \Gamma_1(\cdot) \) since for any \( \Gamma_1(r_1) \in (\gamma_0, \gamma_T) \), \( \Gamma_2(\Gamma_1(r_1)) = r_1 \).
Suppose now that each agent receives an incentive scheme of the form in (7)–(8) based on the relative benefit cost allocation rule, and that the capital charge rate is determined according to (17). It then becomes a dominant strategy for both agents to report their internal rates of return truthfully since a project makes a positive contribution to the performance measure if and only if \( r_i(y_i) > r_i^*(r_j) \). We also note that the participation constraints hold ex-post, i.e., for every realization of the other agent’s type. The following proposition shows that this mechanism is indeed optimal. Furthermore, even though the optimization program in \( P_2 \) is stated in terms of Bayesian incentive compatibility and interim participation constraints, the principal obtains dominant strategy incentives and ex-post satisfaction of the participation constraints for “free”.

**Proposition 3** With competing projects, the relative benefit cost allocation rule based on the competitive hurdle rate \( r_i^*(r_j) \) generates an optimal performance measure for each agent.
Proof: See Appendix.

This result shows that our finding in Proposition 1 extends naturally to a setting with competing projects. Since residual income based on the relative benefit depreciation rule continues to generate an optimal performance measure, the effect of competition is captured entirely by the use of the competitive hurdle rates $r^*_i(r_j)$ instead of the individual hurdle rates $r_i$. The principal can still effectively delegate the investment decision since it will be true that $r_i(y_i) > r^*_i(r_j)$ for one of the agents, and that agent will be better off by not investing. We also note that the finding in Proposition 3 is consistent with the empirical evidence in Poterba and Summers (1992), who document that firms tend to impose internal hurdle rates which significantly exceed their “true” cost of capital. In our analysis the higher competitive hurdle rates reflect both the objective of limiting the manager’s informational rents and the objective of efficient project selection.

To illustrate the impact of the capital investment constraint, suppose the two agents are ex-ante identical, so that $\phi_1(y_1) > \phi_2(y_2)$ is equivalent to both $y_1 > y_2$ and $NPV_1(y_1) > NPV_2(y_2)$. In this special case, it follows that $r^*_1 = r^*_2 = r^*$, and furthermore the functions $\Gamma_i$ reduce to the identity function. Competition between the two agents can now be shown to entail two distinct advantages for the principal. First, the winning agent’s informational rents are reduced. As argued in (3), with one division the agent’s informational rent is $k \cdot (y - y^*)$. When there are two (ex-ante identical) agents, and $y_i > \max\{y_j, y^*\}$, the winning agent’s informational rent is reduced to $k \cdot (y_i - \max\{y_j, y^*\})$. Second, the principal achieves a decrease in the probability of underinvestment. In the single agent setting, positive NPV projects are foregone whenever $y^0 < y \leq y^*$, where $NPV(y^0|\varepsilon) = 0$. Thus the probability of underinvestment with one agent is $F(y^*) - F(y^0)$, while the addition of the second agent lowers this probability to $[F(y^*) - F(y^0)]^2$.

We conclude this section by examining how the optimal capital budgeting policy varies with the relative severity of the two divisions’ agency problems. Intuitively, an agent’s competitive hurdle rate should increase as this agent’s moral hazard problem become more severe. Similarly, one would expect a higher competitive hurdle rate for an agent who is subject to greater information asymmetry. In our model, the severity of the moral hazard

21
problem is captured by the parameter

\[ k_i \equiv \sum_{t=1}^{T} \frac{v_{it}^t}{\lambda_{it}} \cdot x_{it} \cdot \gamma^t. \]

In the following analysis, it will be convenient to assume that each \( y_i \) is normally distributed with mean \( \bar{y}_i \) and variance \( \sigma_i^2 \).\(^{24}\) The variance \( \sigma_i^2 \) then represents the degree of information asymmetry since for a normally distributed random variable the inverse hazard rate, \( H_i(y_i) \), is increasing in \( \sigma_i^2 \) for any given \( y_i \). For comparative statics purposes, we view the competitive hurdle rate \( r_i^*(r_j|\cdot) \) as parameterized by \( k_1, k_2, \sigma_1^2 \) and \( \sigma_2^2 \).

**Proposition 4** The competitive hurdle rate \( r_i^*(r_j|k_1, k_2, \sigma_1^2, \sigma_2^2) \) is uniformly increasing in both \( k_i \) and \( \sigma_i^2 \), and uniformly decreasing in both \( k_j \) and \( \sigma_j^2 \).

**Proof:** See Appendix.

The calibrating function \( \Gamma_i \) allows the principal to “handicap” a division which is subject to a more severe agency problem. To illustrate the use of Proposition 4, suppose \( NPV_1(y) = NPV_2(y) \) for all \( y \). If \( k_1 = k_2 \) and \( \sigma_1^2 = \sigma_2^2 \), then the two agents are ex-ante identical and \( \Gamma_i \) is the identity function. For \( k_1 > k_2 \) and \( \sigma_1^2 > \sigma_2^2 \), however, the principal faces a more severe moral hazard problem and greater informational asymmetry with agent 1. It then follows that \( r_i^* > r_j^* \) and furthermore the function \( \Gamma_1(r_1) \) will always be below the diagonal in the space of internal rates of return; see Figure 2. In this sense, agent 1’s bid for the project will effectively be “handicapped”.

## 6 Risk Aversion and the Adoption of Risky Projects

This section examines optimal incentive schemes when projects are risky and managers are risk averse. To begin with, we focus on a single agent setting. The question then becomes whether residual income remains an optimal performance measure and whether the capital charge rate and the depreciation rules are affected by the agent’s attitude towards risk.

\(^{24}\)We note that the internal rate of return \( r_i(y_i) \) is not defined for negative values of \( y_i \). Without loss of generality, however, we may set \( r_i(y_i) = r_i(y_i^0) \) for all \( y_i < y_i^0 \), where \( y_i^0 \) is the first-best cut-off level, i.e., \( NPV(y_i^0|r) \equiv 0. \)
To include risk in the model, suppose that the investment project generates cash inflows in the amount of \( x_t \cdot y + \tilde{\theta}_t \) in period \( t \), where \( \tilde{\theta}_t \) is a zero-mean random variable. For contracting purposes, the principal can rely on the investment cash flow in period 0 as well as the operating cash flows

\[
c_t = \lambda_t \cdot a_t + (x_t \cdot y + \tilde{\theta}_t) \cdot I + \varepsilon_t
\]

where \( \varepsilon_t \) is an unbiased noise term. The random variables \( \{\tilde{\theta}_t\}_{t=1}^T \) can be interpreted as “project risk” since they affect the operating cash flows only if the investment project is undertaken.

For the purpose of analytical tractability, we confine attention to a multiperiod LEN model: Linear contracts, Exponential Utility and Normally distributed noise terms.\(^{25}\) Therefore both \( \varepsilon_t \) and \( \tilde{\theta}_t \) are assumed to be normally distributed with mean zero and respective variances of \( \sigma_t^2 \) and \( \mu_t^2 \). In our context, the linearity requirement restricts the agent’s compensation scheme to be linear in the observed operating cash flows:

\[
s_t(c_1, \ldots, c_t, \tilde{y}) = h_t(\tilde{y}) + k_t(\tilde{y}) \cdot c_1 + \cdots + k_t(\tilde{y}) \cdot c_t.
\]

Here, \( h_t(\tilde{y}) \) denotes a payment the agent is to receive at date \( t \) contingent on his report \( \tilde{y} \), but independent of the history of actual operating cash flows. The bonus coefficient \( k_t(\tilde{y}) \), determines the share the agent is to receive in period \( t \) for a dollar of cash flow delivered in period \( i \), where \( t \geq i \). Again, these bonus coefficients can depend on the agent’s report \( \tilde{y} \).

For the risk-averse agent, we assume that his intertemporal preferences can be described by an additively separable utility function of the form

\[
U_0 = -\sum_{t=1}^{\infty} \gamma^t \cdot \exp \{-\rho \cdot (w_t - v_t(a_t))\} \cdot \gamma^t.
\]

\(^{25}\)Earlier papers which have relied on the LEN framework in single period models include Holmstrom and Milgrom (1991) and Meyer and Vickers (1997).
The agent trades off monetary consumption $w_t$ against the cost of effort $v_t(a_t)$ in each period. The parameter $\rho$ represents the agent’s degree of risk aversion. We assume that the agent has access to third party banking at the common interest rate $r$.

If $W_t$ denotes the agent’s savings at the end of period $t$, his consumption is given by $w_t = s_t + (1 + r) \cdot W_{t-1} - W_t$.

At the time of contracting, the agent’s expected utility depends on the true profitability parameter $y$, the agent’s report $\tilde{y}$, and the future action choices $(a_1, \ldots, a_T)$. The parties are assumed to commit to a contract that extends over $T$ periods. The agent accepts some alternative employment after date $T$. For simplicity, his market alternative in each period is normalized to zero, i.e., in every period the agent could have earned a fixed wage of zero in return for low effort ($a_t = 0$), externally. Similarly, there is no loss of generality in normalizing the agent’s initial wealth to zero. The LEN framework permits the following characterization of the agent’s expected utility.

**Lemma 3** Given the LEN framework, the certainty equivalent of the agent’s expected utility at date 0 takes the following mean-variance form:

$$CE(\tilde{y}, y) = \sum_{t=1}^{T} \left\{ h_t(\tilde{y}) + k_t(\tilde{y}) \cdot (\lambda_t \cdot a_t + x_t \cdot y \cdot I(\tilde{y})) - v_t(a_t) \right\} \cdot \gamma^t$$

$$- \frac{1}{2} \cdot \rho \cdot (1 - \gamma) \cdot (k_t(\tilde{y}))^2 \cdot (\sigma^2 + \mu^2 \cdot I(\tilde{y})) \right\} \cdot \gamma^t$$

where $$k_t(\tilde{y}) = \sum_{i=t}^{T} \gamma^{t-i} \cdot k_{it}(\tilde{y})$$

The proof of Lemma 2 is omitted since its steps closely follow the arguments in Dutta and Reichelstein (1999a). According to (21), the certainty equivalent of the agent’s expected utility is given by the present value of the mean-variance expressions corresponding to the agent’s compensation scheme in each period.\(^{27}\) The coefficient $k_t(\tilde{y})$ in (21) is the effective

\(^{26}\)The significance of third party banking is that the compensation scheme does not need to reflect the agent’s desire to smooth consumption over time. For any sequence of compensation payments, the agent will adjust his savings decisions so as to achieve a smooth consumption stream. See Fudenberg, Holmstrom, and Milgrom (1990) for further discussion of this assumption.

\(^{27}\)We note, however, that the effective coefficient of risk aversion, $\rho \cdot (1 - \gamma)$, is less than the nominal coefficient $\rho$ because the agent can spread the effect of an income shock in any given period over an infinite horizon.
bonus coefficient associated with the operating cash flow, \( c_t \), since \( k_t(\tilde{y}) \) is equal to the present value of the payments the agent receives for a dollar of cash flow delivered in period \( t \).

Given Lemma 2 the agent’s preferences over linear incentive schemes can be represented by \( CE(\tilde{y}, y) \), and therefore the principal’s optimization problem can be written as follows:

\[
P_3: \max_{\{k_t(y), h_t(y)\}, I(y)} \int_0^T \left\{ \sum_{t=1}^T [\lambda_t \cdot a_t(k_t(y)) \cdot (1 - k_t(y)) - h_t(y)] \cdot \gamma^t + (NPV(y) - y \cdot k(y)) \cdot I(y)) \right\} f(y) dy
\]

subject to:

(i) \( a_t(k_t(y)) \in \arg\max_{a_t} \{\lambda_t \cdot k_t(y) \cdot a_t - v_t(a_t)\} \),

(ii) \( CE(y, y) \geq 0 \) for all \( y \),

(iii) \( CE(y, y) \geq CE(\tilde{y}, y) \) for all \( y, \tilde{y} \)

(iv) \( k(y) = \sum_{t=1}^T k_t(y) \cdot x_t \cdot \gamma^t \).

In solving \( P_3 \), the principal faces the same trade-off as in the risk neutral setting. Stronger incentive provisions, i.e., higher bonus coefficients, \( k_t(y) \), results in larger informational rents as the agent captures a larger share of the surplus in favorable states. Again, the expected value of the informational rent is given by the expected value of \( H(y) \cdot k(y) \). Analogous to condition (5), we consider parameter settings such that it is desirable to induce maximum effort \( a_t = 1 \) in every period:

\[
\lambda_t - v_t' - \frac{1}{2} \cdot \rho \cdot (1 - \gamma) \cdot \left( \frac{v_t'}{\lambda_t} \right)^2 (\sigma_t^2 + \mu_t^2) - H(y^0) \left( \frac{v_t'}{\lambda_t} \right) \cdot x_t > 0 . \tag{22}
\]

As before, (22) implies that the principal will optimally choose \( k_t(y) = \frac{v_t'}{\lambda_t} \) and invest if, and only if, the profitability parameter is above a threshold level \( y^{**} \), which is given by the solution to the equation

\[
NPV(y^{**}) - H(y^{**}) \cdot k - R = 0, \tag{23}
\]

with \( R = \sum_{t=1}^T \left[ \frac{1}{2} \cdot \rho \cdot (1 - \gamma) \cdot \left( \frac{v_t'}{\lambda_t} \right)^2 \cdot \mu_t^2 \right] \cdot \gamma^t \).
and $k$ is given by (4). The above condition states that the principal will optimally invest only if the project is sufficiently profitable to cover the direct investment cost as well as the associated agency cost which now has two components: the (expected) informational rent, $H(y^{**}) \cdot k$, and compensation for the incremental project risk, $R$. Put differently, a profitable project requires a positive virtual NPV, which now also includes the risk premium $R$. Therefore, risk aversion increases the principal’s required cut-off level $y^{**}$ beyond $y^*$.\(^{28}\)

We denote by $r^{**}$ the *risk-adjusted hurdle rate* which solves

$$NPV(y^{**}|r^{**}) = 0.$$ 

Clearly, this risk-adjusted hurdle rate $r^{**}$ exceeds $r^*$, the hurdle rate in the risk-neutral setting.

We note that the risk adjustment to the optimal investment policy depends only on the variances of $\{\theta_t\}$, but not on variances of the $\{\varepsilon_t\}$. If there were no incremental project risk (i.e., if $R = 0$) then the agent’s risk aversion would have no impact on the optimal investment policy. The principal could use the same hurdle rate as in the risk neutral setting. Furthermore, the relative benefit cost allocation with a capital charge rate set equal to the hurdle rate $r^*$ would generate optimal incentives.\(^{29}\)

With incremental project risk, i.e., when $R > 0$, the principal would have to pay the incremental risk premium even if she knew the profitability parameter $y$. We denote by $y^0_R$ the corresponding cut-off level, i.e., $y^0_R$ solves

$$NPV(y^0_R) - R = 0.$$ 

Clearly, $y^0_R > y^0$ for any $R > 0$. We assume that $y^0_R < \overline{y}$, to avoid the trivial case in which a project that would never be undertaken due to its high risk. Delegation mechanisms of the form in (8) will remain optimal in this setting, if the following condition on the magnitude

\(^{28}\)Lambert (1983) arrives at a similar conclusion in a one-period model in which the agent also must be compensated for project risk.

\(^{29}\)This observation suggests that the LEN framework is not overly restrictive, if the agent’s degree of risk aversion is sufficiently “small”. To see this, note that the above incentive scheme approaches the one derived in Proposition 1 for a risk neutral agent. Yet, for a risk-neutral agent the linearity of the compensation scheme was derived endogenously rather than being imposed exogenously.
of project risk is satisfied:

\[ y_{R}^{0} \geq b + \frac{R}{k}. \tag{24} \]

We note that (24) will be satisfied if the variances \( \mu_{t}^{2} \) of the noise terms \( \theta_{t} \) are “sufficiently small”. The inequality in (24) will then hold because \( y_{R}^{0} > b \).\(^{30}\)

**Proposition 5** Given (22) and (24), there exists a capital charge rate \( \hat{r} \) such that the relative benefit cost allocation rule corresponding to \( \hat{r} \) generates an optimal performance measure. This capital charge rate \( \hat{r} \) is smaller than the risk adjusted hurdle rate \( r^{**} \).

To see why the capital charge rate has to be set below the risk-adjusted hurdle rate, suppose the two rates were identical. By definition of the relative benefit cost allocation scheme, it then follows that at the cut-off level \( y^{**} \) the cost charge in each period would be exactly equal to the expected cash flow from investment in the project. Therefore, a risk-neutral agent would be indifferent between investing and not investing at \( y = y^{**} \), but a risk-averse agent would reject the project at \( y = y^{**} \), and hence the investment incentives would be biased. To restore the desired incentives, and compensate the agent adequately for project risk, the capital charge rate needs to be lowered below \( r^{**} \).\(^{31}\) Depending on the parameters of the problem, the capital charge rate may even have to be reduced below the principal’s true cost of capital \( r \).\(^{32}\)

\(^{30}\)This claim follows from the inequalities:

\[ 0 < NPV(y_{R}^{0}) \leq \sum_{t=1}^{T} x_{t} \cdot y_{R}^{0} - b = y_{R}^{0} - b. \]

\(^{31}\)Christensen, Feltham, and Wu (2000) also examine the capital charge to be imposed on a risk-averse agent. In a symmetric information setting, they show that the optimal capital charge rate must be set below the riskless interest rate in order to induce the agent to take risky projects. They also consider a setting in which the agent receives perfect information regarding the project payoffs after contracting, but before making the investment decision. In such a setting, the principal wants to induce underinvestment, and he does so by setting the capital charge rate above the riskless interest rate.

\(^{32}\)For sufficiently risky projects, i.e., when (24) fails to hold, there may not exist a positive capital charge rate which adequately compensates the agent for the investment risk. Yet, the delegation schemes can still
Within the LEN framework one can also address the effects of both competition and risk aversion on the nature of optimal incentive schemes. With two risk averse agents, it can be shown that the risk adjusted competitive hurdle rate for agent $i$ is given by $\max \{ \Gamma_j^R(r_j), r_i^{**} \}$. Here, $\Gamma_j^R(\cdot)$ is essentially the same calibrating function as in Section 5, except that the virtual NPV’s now also reflect the constant risk premia, $R_1$ and $R_2$. Like in the risk neutral setting, competition therefore has the effect of increasing an agent’s hurdle rate.

It is then straightforward to extend our implementation results in Proposition 3 and Proposition 5 to settings with competing risk-averse setting. Analogous to the finding in Proposition 5, it can be shown that there exists a capital charge rate for each agent such that residual income based on the relative benefit depreciation rule is an optimal performance measure, and each agent has a dominant strategy incentive to report his internal rate of return truthfully. As in Proposition 5, proper compensation for the incremental project risk requires that each agent be charged a cost of capital lower than his risk adjusted competitive hurdle rate, i.e., $\max \{ \Gamma_j^R(r_j), r_i^{**} \}$.

7 Conclusion

In the context of a multiperiod agency model, we have analyzed managerial incentives for investment decisions in a multiperiod agency setting. Due to asymmetric information the principal applies a hurdle rate which is higher than her true cost of capital. Linear incentive schemes based on residual income can generate optimal investment and effort incentives provided the capital charge rate is set equal to the hurdle rate and the firm uses a depreciation method that achieves proper matching of investment costs and project cash flow. The bonus coefficients attached to residual income can then be freely adjusted to address the underlying moral hazard problem without altering the investment incentives. Such flexibility is desirable in order for a performance measure to accommodate an entire range of agency problems.

The incentive schemes we identify can accommodate both competition for project funding
and managerial risk aversion towards uncertain project cash flows. In both scenarios, the optimal incentive scheme can remain unchanged except for the capital charge rate. With competing risk-neutral agents, the principal wants to adjust the capital charge rate upwards to the competitive hurdle rate, which exceeds the hurdle rate in world without investment constraints. Managerial risk aversion also has a tendency to increase the applicable hurdle rate, yet in order to provide proper incentives for a risk averse manager, the capital charge rate must be set below the risk adjusted hurdle rate.

Our analysis provides a rationale for the widespread practice of charging divisions with costs that correspond to investment expenditures incurred in the past. In particular, our results provide theoretical support for the recent wave of adoptions of residual income as a divisional performance measure. Furthermore, the results of this paper provide theoretical guidance regarding the appropriate capital charge rate and the choice of intertemporal cost allocation schemes for the purpose of managerial performance evaluation.
Appendix

Proof of Lemma 1:

For any \( c_t \) satisfying (1), the agent’s cost of delivering \( c_t \) in state \( y \) is:

\[ D_t(c_t, y, I) = v_t(\lambda_t^{-1} \cdot (c_t - x_t \cdot y). \]

The participation constraint \( U(y, y) \geq 0 \) will hold with equality for the lowest type \( y \). This boundary condition combined with the “local” incentive compatibility conditions implies that for any policy \( \{I(y), \bar{c}(y)\} \) the agent will earn the following informational rents:

\[ U(y, y) = \sum_{t=1}^{T} \gamma^t \cdot \int_{y}^{y} -\frac{\partial}{\partial y} D_t(c_t(u), u, I(u)) du, \quad (25) \]

where \( U(y, y) \equiv \sum_{t=1}^{T} [s_t(y) - D_t(c_t(y), y, I(y))] \cdot \gamma^t \). The expected informational rent then becomes:

\[ \int_{y}^{y} U(y, y) \cdot f(y) dy = \sum_{t=1}^{T} \gamma^t \cdot \int_{y}^{y} \{-H(y) \cdot \frac{\partial}{\partial y} D_t(c_t(y), y, I(y))\} f(y) dy \quad (26) \]

Since

\[ \frac{\partial}{\partial y} D_t(c_t(y), y, I(y)) = -v_t(\lambda_t^{-1} \cdot (c_t(y) - x_t \cdot y \cdot I(y)) \cdot x_t \cdot \lambda_t \cdot I(y). \]

we can substitute (25) and (26) into the principal’s objective in \( P_0 \) to obtain:

\[
\max_{\{I(y), \bar{c}(y)\}} \int_{y}^{y} \left\{ \sum_{t=1}^{T} (c_t(y) - y \cdot I(y) \cdot x_t - v_t(\lambda_t^{-1} \cdot (c_t(y) - x_t \cdot y \cdot I(y)) \cdot \gamma^t
\right.

\[ + [\text{NPV}(y) - \sum_{t=1}^{T} H(y) - v_t(\lambda_t^{-1} \cdot (c_t(y) - x_t \cdot y \cdot I(y)) \cdot x_t \cdot \lambda_t \cdot I(y)] I(y) \right\} f(y) dy. \quad (27) \]

The objective in (25) can be maximized pointwise, subject to the boundary conditions \( x_t \cdot y \cdot I(y) \leq c_t(y) \leq x_t \cdot y \cdot I(y) + \lambda_t. \) Clearly, \( I(y) = 0 \) if \( y \leq y^0. \) For any \( y > y^0 \) for which \( I(y) = 1, \) the principal seeks to maximize:

\[ \lambda_t \cdot a_t - v_t(a_t) + \text{NPV}(y) - H(y) \cdot \frac{v_t(a_t)}{\lambda_t} \cdot x_t \cdot \gamma^t, \]
with regard to $a_t$. Since $H(y)$ is increasing in $y$, condition (5) implies that it is optimal to choose $a_t = 1$, or $c_t(y) = \lambda_t + x_t \cdot y$ whenever $y \geq y^0$ and $I(y) = 1$. As a consequence, the optimal cut-off, $y^*$, is given by (6).

To complete the proof, we note that the resulting scheme is indeed globally incentive compatible. As shown by Mirrlees (1986), a mechanism is incentive compatible provided it is locally incentive compatible, and $\frac{\partial}{\partial y} U(\tilde{y}, y)$ is (weakly) increasing in $\tilde{y}$.

For the above mechanism:

$$
\frac{\partial}{\partial y} U(\tilde{y}, y) = \sum_{t=1}^{T} v'_t(a_t(\tilde{y}, y)) \cdot \gamma^t \cdot I(\tilde{y}),
$$

which is increasing in $\tilde{y}$ since $v'_t(\cdot)$ is increasing, $a_t(\tilde{y}, y)$ is increasing in $\tilde{y}$ and the optimal $I(\cdot)$ is an upper-tail investment policy.

**Proof of Proposition 2:** Optimality requires that $k_t = \frac{v^t}{\lambda_t}$. With smaller bonus coefficients the agent would not have an incentive to exert the optimal effort $a_t = 1$. Conversely, $k_t > \frac{v^t}{\lambda_t}$ would give the agent additional and unnecessary rents.

Given $\bar{\lambda}^0$, we define

$$
\Lambda^0 = \{ \bar{\lambda} \mid \sum_{t=1}^{T} x_t \cdot \frac{v^t}{\lambda_t} \cdot \gamma^t = \sum_{t=1}^{T} x_t \cdot \frac{v^t}{\lambda^0_t} \cdot \gamma^t \}.
$$

(28)

It follows directly from (28) that $y^*(\bar{\lambda}) = y^*(\bar{\lambda}^0)$, and hence $r^*(\bar{\lambda}) = r^*(\bar{\lambda}^0)$, for all $\bar{\lambda} \in \Lambda^0$. In order for the agent to have the optimal investment incentives, it must be that

$$
\sum_{t=1}^{T} \gamma^t \cdot \frac{v^t}{\lambda_t} \cdot [x_t \cdot y^*(\bar{\lambda}) - z_t(x_t, r^*(\bar{\lambda})) \cdot b] = 0.
$$

(29)

By definition of $\Lambda^0$, the terms:

$$
\Delta_t \equiv x_t \cdot y^*(\bar{\lambda}) - z_t(x_t, r^*(\bar{\lambda})) \cdot b
$$

are constant for all $\bar{\lambda} \in \Lambda^0$. The requirement in (29) then becomes:

$$
\sum_{t=1}^{T} \frac{v^t}{\lambda_t} \gamma^t \cdot \Delta_t = 0.
$$

(30)
for all $\bar{\lambda} \in \Lambda^0$. Assuming that $x_0 \neq 0$ (without loss of generality), we solve for the variable $\lambda_0$ in (28) to obtain:

$$\Lambda^0 = \left\{ \bar{\lambda} \mid \frac{v'_0}{\lambda_0} \gamma = \frac{v'_0}{\lambda_0} \cdot \lambda_0 \cdot \sum_{t=1}^{T} \frac{x_t}{x_1} \cdot \left( \frac{v_t}{\lambda_t} - \frac{v_0}{\lambda_0} \right) \cdot \gamma^t \right\}.$$  

Substitution into (30) yields

$$\left[ \frac{v'_0}{\lambda_0} \cdot \gamma + \sum_{t=1}^{T} \frac{x_t}{x_1} \cdot \left( \frac{v_t}{\lambda_t} - \frac{v_0}{\lambda_0} \right) \cdot \gamma^t \right] \cdot \Delta_1 + \sum_{t=1}^{T} \frac{v'_0}{\lambda_t} \cdot \gamma^t \cdot \Delta_t = 0$$

for all $(\lambda_1, \ldots, \lambda_T)$ in a neighborhood of $(\lambda_0^1, \ldots, \lambda_0^T)$. The only way to satisfy this requirement is to have $\Delta_t = 0$ for $1 \leq t \leq T$. We recall that the hurdle rate is given by

$$NPV(y^* | r^*) = \sum_{t=1}^{T} x_t \cdot (1 + r^*)^{-t} \cdot y^* - b = 0.$$  

Therefore $\Delta_t = 0$ implies:

$$z_t(\bar{x}, r^*) = \frac{x_t}{\sum_{i=1}^{T} x_i \cdot (1 + r^*)^{-i}}.$$  

**Proof of Proposition 3:** We recall that $\phi_i(y_i) \equiv NPV_i(y_i) - k_i \cdot H_i(y_i)$ denotes the virtual NPV of the project at $y_i$. Since $\phi_i(\cdot)$ is strictly increasing, it follows that the conditions $\phi_2(y_2) \geq \max\{\phi_1(y_1), 0\}$ and $\phi_1(y_1) \geq \max\{\phi_2(y_2), 0\}$ are equivalent to $r_2 \geq \max\{\Gamma_1(r_1), r^*_1\}$ and $r_1 \geq \max\{\Gamma_2(r_2), r^*_1\}$, respectively. Thus, the competitive hurdle rate mechanism implements the same investment decisions as the optimal revelation mechanism. The choice of bonus coefficients $\{k_i\}$ ensures that $a_{d1} = 1$.

If we let $U_i(\bar{r}_i, r_i, r_j)$ denote the agent $i$'s utility contingent on his report $\bar{r}_i$ and true IRR’s $(r_i, r_j)$, then

$$U_i(\bar{r}_i, r_i, r_j) = k_i \cdot (y_i - \max\{y_i^*, y_j\}) \cdot I_i^*(\bar{y}_i, y_j)$$

where $I_i^*(\cdot, \cdot)$ denotes the optimal investment decision rules.
It is clear from (31) that the agent i’s participation constraint holds for each $y_j$, and each agent has (dominant strategy) incentives to report his IRR truthfully. Furthermore, the payments resulting from (31) coincide with those of the optimal revelation mechanism.

\[ \square \]

**Proof of Proposition 4:** If $y_i \sim N(\bar{y}_i, \sigma_i^2)$, then

\[
H_i(y_i \mid \sigma_i^2) = \int_{y_i}^{\infty} \frac{e^{-\left(u-\bar{y}_i\right)^2/2\sigma_i^2}}{e^{-\left(y_i-\bar{y}_i\right)^2/2\sigma_i^2}} \cdot du.
\]

Differentiating with respect to $\sigma_i^2$ reveals that the inverse hazard rate function $H_i(y_i \mid \sigma_i^2)$ is uniformly increasing in $\sigma_i^2$. Since the virtual NPV is given by

\[
\phi_i(y_i \mid k_i, \sigma_i^2) = NPV(y_i) - k_i \cdot H_i(y_i \mid \sigma_i^2),
\]

it follows that the virtual NPV is uniformly decreasing in $k_i$ and $\sigma_i^2$. This immediately implies that $\Gamma_i(r_i \mid k_1, k_2, \sigma_1^2, \sigma_2^2)$ is uniformly decreasing in $k_i$ and $\sigma_i^2$ and uniformly increasing in $k_j$ and $\sigma_j^2$. The result follows since $r_i^*$ is increasing in $k_i$ and $\sigma_i^2$. \[ \square \]

**Proof of Proposition 5.** Suppose the principal chooses the compensation coefficients \( \{h_t, k_t\} \) such that $k_t = \frac{\nu_t}{\lambda_t}$ and

\[
\sum_{t=1}^{T} \gamma^t \cdot [h_t - \frac{1}{2} \cdot \rho \cdot (1 - \gamma) \cdot \left( \frac{\nu_t}{\lambda_t} \right)^2 \cdot \sigma_i^2] = 0.
\]

This ensures that $a_t = 1$ and the participation constraint is satisfied. The agent will accept the project if and only if $y \geq y^{**}$ provided:

\[
\sum_{t=1}^{T} \left( x_t \cdot y^{**} - z_t \cdot b \right) \cdot \left( \frac{\nu_t}{\lambda_t} \right) \cdot \gamma^t - R = 0. \tag{32}
\]

Let $\hat{\gamma}$ denote the solution to the following equation:

\[
[y^{**} - \frac{R}{k}] \cdot \sum_{t=1}^{T} \hat{\gamma}^t \cdot x_t - b = 0. \tag{33}
\]
If condition (24) holds, i.e., \( y_R^0 \geq b + \frac{R}{k} \), there exists a unique \( \hat{\gamma} \in [0, 1] \), which solves (33). To see this, note that \( y^{**} - \frac{R}{k} > y_R^0 - \frac{R}{k} \geq b \). Therefore, the left hand side of (33) is positive when evaluated at \( \hat{\gamma} = 1 \) and negative at \( \hat{\gamma} = 0 \).

It is readily verified that (32) holds if \( z_t \) is chosen to be the relative cost allocation rule corresponding to \( \hat{r} \), i.e.,

\[
z_t(x, \hat{r}) = \frac{x_t}{\sum_{i=1}^{T} \hat{\gamma}^i \cdot x_i}.
\]

Following the argument in the proof of Proposition 1, the Revenue Equivalence Theorem then implies that the payments to the agent coincide with those of the optimal revelation mechanism. \( \square \)
References
Harris, M., H. Kriebel, and A. Raviv (1982), “Asymmetric Information, Incentives, and


