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WELFARE EFFECTS OF IMMIGRATION IN A DUAL LABOR MARKET

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IN A DUAL LABOR MARKET

Abstract

The paper analyses the welfare effects of immigration when some sectors of the economy are characterized by wage bargaining between unions and employers. We show that immigration is unambiguously beneficial if the wage elasticity of labor demand in the competitive sectors is smaller than in the unionised sectors. In the opposite case, the welfare effect of immigration is ambiguous; little immigration then reduces the native population’s welfare, whereas large scale immigration tends to enhance welfare.

Keywords: Immigration policy, trade unions, welfare

JEL Classification: J5, J61

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1 INTRODUCTION

In many labor markets, trade unions have a significant influence on the wage setting process. Surprisingly, the literature on labor migration has taken little notice of this fact. Most of the literature deals exclusively with competitive labor markets, e.g. the seminal paper by Berry and Soligo (1969). There, immigration increases the welfare measured by total income of the native population.

Things, however, are less obvious when frictions in the labor market are introduced. The unions’ bargaining power raises the wages in the unionized sectors above the competitive wage, thus distorting the allocation of labor. Workers who are unable to find a job in the unionized sectors either become unemployed or have to seek jobs at lower wages in sectors with competitive wage setting. Such distortions in the labor market also have an impact on the welfare consequences of immigration. Additional immigrant labor is no longer a pure benefit to the native population. With some probability, immigrant workers will find employment in the unionized sectors and will drive out some of the native workers. Instead of the high union wage, the natives will then receive either unemployment benefits or the lower competitive wage.\footnote{The case where the unions’ outside option is determined by unemployment benefits is analyzed in Schmidt (1994). We will solely focus on the alternative approach where the unions outside option is given by the wage rate in competitive labor market segments.}

On the other hand, there are also beneficial effects from immigration. Immigration ultimately reduces the wage rate in the competitive sectors of the economy and thus forces the trade unions to lower the wage demands in their own sectors. For the natives, there is a twofold benefit from this adjustment. First, the lower union wage leads to a more efficient allocation of labor across sectors. Second, what is lost in labor income due to the lower wages is gained by capital owners. As the losses partially accrue to foreign workers the natives’ welfare increases (provided they own the domestic capital stock).

In our analysis, the net impact of positive and negative effects on the natives’ welfare crucially depends on the elasticities of labor demand in the unionized and competitive sectors. It turns out that the positive and the negative impacts of immigration under unionization neutralize each other if the elasticities are the same across sectors. As in competitive labor
markets, immigration is strictly welfare enhancing. The positive welfare effect is even stronger if labor demand in the competitive sector is less elastic than in the unionized sector because then immigration leads to a significant reallocation of labor from the competitive to the unionized sector. In the opposite case, where the labor demand in the unionized sector is more rigid, the welfare consequences of immigration are ambiguous. Little immigration makes the native population worse off. Above a threshold level, immigration becomes welfare increasing and ultimately will lead to higher welfare level than in autarky. The U-shaped welfare effect emerges because, due to unionization, the migrants’ expected income exceeds their marginal product. The positive effect of immigration - the wage reduction for intramarginal migrants – can only become dominant if the immigrant labor force is sufficiently large.

Section 2 will set up the framework for our analysis. Section 3 discusses the welfare effects of immigration. In Section 4, we report a few simulation results for the cases where the welfare effect is ambiguous. In Section 5, we consider an extension of the model. Section 6 concludes.

2 The Model

2.1. Firms

We consider a small open economy which is divided into a unionized sector and a competitive sector. The wage rate in the unionized sector \((w)\) is determined by bargaining between unions and firms, whereas the wage rate in the competitive sector \((b)\) adjusts to equate supply and demand for labor. Firms within each sector are identical but may differ across sectors. The production technology used in the unionized sector is

\[
Y_U = L_U^\alpha \cdot K_U^\beta
\]

where \(0 < \alpha, \beta < 1\) and \(\alpha + \beta < 1\). Production per firm in the competitive sector is

\[
Y_C = L_C^\gamma \cdot K_C^\delta
\]

with \(0 < \gamma, \delta < 1\) and \(\gamma + \delta < 1\). \(L_i\) and \(K_i\) \((i = U, C)\) denote employment and capital input per firm in each sector. Note that the production technologies in (1) and (2) exhibit decreasing
returns to scale in $K$ and $L$; this formulation implicitly assumes that there is a third immobile factor which is trapped in each sector.\footnote{With constant returns to scale in $K$ and $L$ and international capital mobility, the wage rate would be determined by the world market interest rate. In this case, there would be no room for wage bargaining; see e.g. Bruno and Sachs (1985).} We normalize the overall number of firms in the economy to unity. The number of firms in the unionized sector is denoted by $\lambda$; accordingly, there are $1-\lambda$ firms in the competitive sector. All firms sell their output in competitive goods markets, where the price is also normalized to unity.

2.2 WORKERS, UNIONS, AND WAGE BARGAINING

There are $N+M$ workers in the economy, where $N$ denotes natives and $M$ is the number of immigrants. Each worker inelastically supplies one unit of labor. As in Schmidt et al. (1994), we assume that neither unions nor firms discriminate between immigrants and natives, and that immigrants have the same chances of achieving a job in the unionized sector as natives.\footnote{In Section 5, we will relax this assumption.} We further assume that wage bargaining takes place at the firm level. Both unions and firms take the wage rate in the competitive sector ($b$) and the number of immigrants ($M$) as given. The wage rate ($w$) is determined by union-firm bargaining whereas employment is set by the firms.\footnote{We thus use the right-to-manage approach to union-firm bargaining. This is in line with empirical findings [Oswald (1993)] according to which unions and firms explicitly bargain over wages only but not over employment. Of course, this does not exclude that there is implicit bargaining over both wages and employment as it is assumed in the efficient bargaining model.} We also assume that all agents in the economy are risk neutral.

The representative union maximizes the rents accruing to the workers in the respective firm. The union’s objective function is thus $U = (w - b) \cdot L_U$. The representative firm in the unionized sector maximizes profits ($\Pi$), which are

$$\Pi = L_U^\alpha \cdot K_U^\beta - w \cdot L_U - r \cdot K_U$$

where $r$ is the exogenously given interest rate in the international capital market. Capital and labor input in the unionized sector are determined by the marginal productivity conditions

$$w = \alpha \cdot L_U^{\alpha-1} \cdot K_U^\beta$$

and

$$r = \beta \cdot L_U^\alpha \cdot K_U^{\beta-1}. \hspace{1cm} (3)$$

$$r = \beta \cdot L_U^\alpha \cdot K_U^{\beta-1}. \hspace{1cm} (4)$$
Equations (3) and (4) yield the capital and labor demand functions $K_U(w, r)$ and $L_U(w, r)$.

We can now determine the outcome of the wage negotiations using the generalized Nash bargaining solution [Nash (1950), Binmore et al. (1986)]. In the case of disagreement, profits are zero and the workers have to seek employment in the competitive sector. Hence, the firm’s and the union’s threat points are zero. The Nash maximand can be written as

$$
\Omega = \theta \cdot \log[L_U(w, r) \cdot (w - b)] + (1 - \theta) \cdot \log \Pi(w, r)
$$

where $\theta$ denotes the union’s relative bargaining power. With $\theta = 1$, we have the case of a monopoly union; $\theta = 0$ describes the opposite case of competitive labor markets.\(^5\) Maximizing (5) over $w$ and making some rearrangements yields the wage rate in the unionized sector as

$$
w = \phi \cdot b
$$

where

$$\phi \equiv 1 + \frac{\theta \cdot (1 - \alpha - \beta)}{\alpha}.$$

Equation (6) implies that, if the union has any bargaining power ($\theta > 0$), the wage rate $w$ and, hence, the marginal productivity of labor in the unionized sector will be higher than in the competitive sector.

Workers who do not find jobs in unionized firms will be employed in the competitive sector. Since workers are remunerated their marginal product, the wage paid in the competitive sector amounts to

$$
b = \gamma \cdot L_C^{\gamma - 1} \cdot K_C^{\delta}.
$$

As the wage in the unionized sector is linked to the competitive wage ($w = \phi \cdot b$), the following condition has to hold in equilibrium

$$
\alpha \cdot L_U^{\alpha - 1} \cdot K_U^{\beta} = \phi \cdot \gamma \cdot L_C^{\gamma - 1} \cdot K_C^{\delta}.
$$

Clearing of the labor market implies that the workforce per firm in the competitive sector can be written as

\(^5\) For a textbook treatment of the right-to-manage approach to wage bargaining and its properties, see Booth (1995, Chapter 5).
\[ L_C = \frac{N + M - \lambda \cdot L_U}{1 - \lambda}. \]  

Equations (7) and (8) determine equilibrium employment in the two sectors as a function of the number of immigrants \((M)\). Substituting (8) into (7) and using the marginal productivity conditions for the capital input in both sectors allows us to derive the impact of immigration on employment in the unionized sector: 

\[ \frac{dL_U}{dM} = \frac{\varepsilon_U \cdot L_U}{(1 - \alpha - \beta) \cdot N + M - \lambda \cdot L_U} > 0 \]  

where \(\varepsilon_i\) is the wage elasticity of labor demand in sector \(i\): 

\[ \varepsilon_U = -\frac{\partial L_U}{\partial w} \cdot w = \frac{1 - \beta}{1 - \alpha - \beta} \quad \text{and} \quad \varepsilon_C = -\frac{\partial L_C}{\partial b} \cdot b = \frac{1 - \delta}{1 - \gamma - \delta}. \]

Immigration raises employment in the unionized sector because it raises the overall number of workers in the economy and thus reduces the wage rate in the competitive sector \((b)\). This reduces the reservation utility of the unions in the wage bargain with the firms. Consequently, the wage rate in the unionized sector also declines and employment increases.

### 3 How Does Immigration Affect the Welfare of the Natives?

The main interest of our analysis is the effect of immigration on the welfare of the natives, given that the labor market is distorted by unionization. Following Schmidt et al. (1994), we assume that all firms are owned by natives. Then the welfare of the natives \((V)\) is simply overall output minus capital costs minus the wages earned by immigrants, that is

\[ V = \lambda \cdot Y_U + (1 - \lambda) \cdot Y_C - r \cdot [\lambda \cdot K_U + (1 - \lambda) \cdot K_C] - \frac{M}{N + M} \cdot \lambda \cdot w \cdot L_U + (1 - \lambda) \cdot b \cdot L_C. \]  

The welfare effect of immigration can now be derived by differentiating (10) with respect to \(M\). This yields, after some rearrangements\(^6\)

\[ \frac{dV}{dM} = \Psi_1 \cdot (\varepsilon_U - \varepsilon_C) + \Psi_2 \cdot M \]

with

\(^6\) A detailed derivation is given in the Appendix.
\[ \Psi_1 \equiv \frac{\lambda \cdot w \cdot (1 - \frac{1}{\phi}) \cdot N \cdot L_U}{(N + M)^2} \cdot \frac{N + M - \lambda \cdot L_U}{\lambda \cdot L_U \cdot \varepsilon_U + (N + M - \lambda \cdot L_U) \cdot \varepsilon_U} > 0 \quad \text{and} \]

\[ \Psi_2 \equiv \left[ 1 - \frac{1}{\phi} \right] \frac{\lambda \cdot L_U}{N + M} + \frac{1}{\phi} \right] \cdot \frac{1}{\varepsilon_U} \cdot \frac{w}{L_U} \cdot \frac{dL_U}{dM} > 0. \]

Equation (11) allows us to analyze the welfare effects of immigration for a number of relevant cases. As a benchmark, consider the case of a competitive labor market. In this case, we have zero union bargaining power \((\theta = 0)\) and, hence, \(\phi = 1\). Equation (11) then collapses to

\[ \frac{dV}{dM} = \frac{1}{\varepsilon_U} \cdot \frac{w}{L_U} \cdot \frac{dL_U}{dM} \cdot M, \quad (11a) \]

which yields our

**Result 1:** If labor markets are competitive \((\theta = 0)\), we have

\[ \frac{dV}{dM} \begin{cases} = 0 & \text{if} & M = 0 \\ > 0 & \text{if} & M > 0 \end{cases}. \]

Result 1 is known from Berry and Soligo (1969) and can be explained as follows. Departing from an equilibrium with \(M = 0\), the arrival of a marginal immigrant does not affect the welfare of the natives since she exactly earns her marginal product. Additional immigration then benefits the natives because further immigrants also receive their marginal product but drive down the wage rate of intramarginal immigrants. This raises the profit income accruing to the natives.

Consider now the case of positive union bargaining power \((\theta > 0)\). In this case, the effect of immigration on the welfare of the natives is more complicated. For the following analysis, it is crucial to keep in mind that immigration under unionization has two countervailing effects on the welfare of the natives. On the one hand, for a given level of employment in the unionized sector, a marginal increase in the number of immigrants raises overall output by \(b\), that is the marginal productivity of labor in the competitive sector. (Note that if immigrants find a job in the unionized sector, they only replace other workers who then have to work in the competitive sector.) Because there is a positive probability of immigrants
finding a job in the unionized sector, their expected wage exceeds \( b \). This constitutes a negative effect on the natives’ welfare. On the other hand, immigration raises employment in the unionized sector, which improves the allocation of labor in the economy and raises the welfare of the natives.

The relative magnitude of these two effects and hence the sign of the overall impact of immigration on native welfare depends crucially on the wage elasticities of labor demand in the two sectors \( \varepsilon_U \) and \( \varepsilon_C \). Consider first the case where \( \varepsilon_U = \varepsilon_C \). Equation (11) then becomes

\[
\frac{dV}{dM} = \Psi_2 \cdot M
\]  

(11b)

**Result 2:** If the unions have some bargaining power (\( \theta > 0 \)) and \( \varepsilon_U = \varepsilon_C \), we have

\[
\frac{dV}{dM} = \begin{cases} 
0 & \text{if } M = 0 \\
> 0 & \text{if } M > 0
\end{cases}
\]

Surprisingly, for the case of \( \varepsilon_U = \varepsilon_C \), it turns out that the positive and the negative impacts of unionization neutralize each other and the qualitative result is identical to that of competitive labor markets. Consider next the case where \( \varepsilon_U > \varepsilon_C \). In this case, the term on the right-hand side of equation (11) is strictly positive for both \( M = 0 \) and \( M > 0 \). This yields

**Result 3:** If the unions have some bargaining power (\( \theta > 0 \)) and \( \varepsilon_U > \varepsilon_C \), we have

\[
\frac{dV}{dM} > 0.
\]

Result 3 has the following intuition. The main beneficial effect of immigration in a unionized labor market is that it leads to a reallocation of labor from the competitive to the unionized sector. The increase of employment in the unionized sector is stronger, the higher \( \varepsilon_U \) and the lower \( \varepsilon_C \). This explains why a strictly positive welfare effect emerges if \( \varepsilon_U > \varepsilon_C \).

Accordingly, immigration is less beneficial if \( \varepsilon_U < \varepsilon_C \). For this case, we can state
Result 4: If the unions have some bargaining power ($\theta > 0$) and $\epsilon_u < \epsilon_c$, we have

$$\frac{dV}{dM} \begin{cases} < 0 & \text{if } M = 0 \\ \geq 0 & \text{if } M > 0 \end{cases}$$

Departing from an equilibrium with $M = 0$, marginal immigration reduces the welfare of the natives in this case. Since labor demand in the competitive sector is relatively elastic, immigration only leads to a small reduction in $b$ and, hence, $w$. Consequently, the increase of employment in the unionized sector induced by immigration is too small to compensate for the fact that the marginal immigrant’s expected wage rate exceeds the marginal product of labor in the competitive sector.

For larger numbers of immigrants, however, the welfare effect of immigration will again become positive. This is due to the fact that immigration reduces the wages of the intramarginal migrants and thus leads to redistribution from immigrant workers to native firm owners. With a sufficient number of immigrants, welfare will finally exceed the autarky level.

Result 5: If the unions have some bargaining power ($\theta > 0$) and $\epsilon_u < \epsilon_c$, the natives gain from large scale immigration $[V(M \to \infty) > V(M = 0)]$.

To prove this result, we have to show that the welfare level for $M \to \infty$ exceeds welfare in autarky ($M = 0$). As welfare in autarky is finite, it will be sufficient to demonstrate that $\lim_{M \to \infty} V \to \infty$. Note first that employment in both sectors will go to infinity when the number of immigrants becomes infinitely large. This can be seen from equilibrium condition (7). If all immigrant labor went into one sector only, the marginal productivity of labor in this sector would converge to zero whereas the marginal productivity in the other sector would remain constant. This would clearly violate condition (7). Hence, equilibrium condition (7) can only hold when labor input in both sectors goes to infinity. Using the information that the marginal productivity of capital equals the world market interest rate, we can rewrite the welfare function (10) as
\[ V = \lambda \cdot \left( \frac{B}{r} \right)^{\beta/\alpha - \beta} \cdot L_u^{\alpha/\alpha - \beta} \cdot \left[ 1 - \frac{M}{N + M} \cdot \alpha - \beta \right] + \\
+ (1 - \lambda) \cdot \left( \frac{\delta}{r} \right)^{\beta/\gamma - \delta} \cdot L_c^{\gamma/\gamma - \delta} \cdot \left[ 1 - \frac{M}{N + M} \cdot \gamma - \delta \right]. \] (10a)

As \( L_u \) and \( L_c \) go to infinity for \( M \to \infty \), we have the desired result \( \lim_{M \to \infty} V \to \infty \). Hence, if immigration becomes sufficiently large, the natives are better off than in autarky. In the next section, we will report a few simulation results to give an impression of which levels of immigration are welfare enhancing and which are welfare reducing.

### 4 Simulation

The previous section showed that the effect of immigration on welfare is ambiguous if the labor demand in the competitive sector is more elastic than that of the unionized sector. Initially (at \( M = 0 \)), marginal immigration is unambiguously detrimental to the natives’ welfare but further immigration can either increase or decrease the total income of the native population. In this section, we will present some simulation results in order to assess how much immigration is needed in these cases to increase the natives’ welfare above the autarky level. Figure 1 reports the welfare effect of immigration for some selected parameter values in the monopoly union case (\( \theta = 1 \)). The horizontal axis measures the number of immigrants relative to the domestic labor force (\( M/N \)). Welfare, i.e. the sum of the natives’ income, is measured on the vertical axis. The welfare level in autarky (without any immigration) is normalized to unity.\(^7\)

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\(^7\) For the simulation, we have set \( N = 1 \). Changes in \( N \) will not affect the quantitative results as long as the endowment with the immobile factor per native remains constant.
Given that labor demand in the competitive market is more elastic than in the unionised sector, welfare becomes U-shaped. If there are only few foreign workers in the labor force, each immigrant’s marginal product falls short of his expected income – because with some probability she will find a job in the unionized high-wage sector. With a large number of immigrants already in the country, additional immigrants reduce the wage of the intramarginal migrants and thus increase the rents of the native capital owners.8

Figure 1 shows how the immigration needed to reach the welfare level in autarky is affected by the production elasticity of mobile capital and by the degree of unionization in the labor market. In our example with $\lambda = 0.5$ (half of the firms is in the unionized sector) and $\beta = 0.2$, an immigrant workforce of roughly 10 percent on top of the domestic workers is needed to reach a welfare above the autarky level. Note that for a given $\alpha$, the partial production elasticity $\beta$ implicitly determines the residual income of the immobile, sector specific factor. Overall, changes in $\beta$ have little effect on the critical level of immigration. When the unionization of the labor market proceeds, much higher levels of immigration will be needed to be welfare improving for the native population. In our example where unionization reaches 90 percent of the domestic firms, immigration has to be above 28 percent of the native population to be welfare improving.

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8 This also confirms an earlier result in Fuest and Thum (1998) where the unionized labor market is characterized by efficient bargaining.
5 Inferior Labor market Prospects for Immigrants

So far we have assumed that immigrants and natives have equal access to jobs in the unionized sector. However, it is natural to argue that immigrants are likely to have inferior prospects in the labor market, that is they may have greater difficulties in finding well paid jobs in unionized firms. In our model, this feature can be incorporated by assuming that only a fraction of the immigrants competes with the natives for unionized jobs. Let \( \rho \) be the share of immigrants who have the same labor market opportunities as natives. A share \( 1 - \rho \) of the immigrants has only access to the competitive sector earning the wage rate \( b \). Note that this approach is equivalent to assuming that each individual immigrant faces inferior labor market prospects relative to natives. Total expected income of immigrants \( EI \) is then given by

\[
EI \equiv \rho \left( \frac{\lambda \cdot L_w}{N + M} \cdot w + \frac{N + M - \lambda \cdot L_w}{N + M} \cdot b \right) + (1 - \rho) \cdot b \cdot M.
\]

This allows us write the welfare of the natives as

\[
V = \lambda \cdot Y_v + (1 - \lambda) \cdot Y_c - r \cdot \left[ \lambda \cdot K_v + (1 - \lambda) \cdot K_c \right] - EI
\]

Comparing the welfare functions in (10b) and (10) shows immediately that (10) is a special case of welfare in the extended model (10b) with \( \rho = 1 \). As unions do not discriminate against foreign workers, wage setting is not affected by the labor market prospects. Hence, the parameter \( \rho \) only influences the expected income of immigrants \( EI \). As \( EI \) is strictly increasing in \( \rho \), the natives’ welfare goes up when the immigrants’ labor market prospects decline. Hence, compared to our baseline scenario in Sections 3 and 4, immigration is always more favorable when immigrants have inferior chances of finding jobs in the unionized sector.

6 Conclusions

This paper gives a tentative answer to the question of how a country’s welfare reacts to immigration if trade unions have a say in the domestic labor market. Depending on the sectoral elasticities of labor demand, immigration can either be strictly beneficial to the natives’ welfare or have ambiguous effects. In the latter case, only sufficiently high levels of immigration can ensure that the natives are better off than in autarky.
To reach these conclusions the paper employs a simple model that entails the benefits of tractability and intuitive appeal. There are several directions in which the approach can be extended for future research. To name but a few: the production function could be generalized, the unions’ bargaining power may differ across sectors, and the relevant labor demand elasticities could be subject to an empirical investigation.

APPENDIX

The appendix shows that first derivative of the welfare function (10) can be written as in (11). Differentiating (10) with respect to \( M \) yields

\[
\frac{\partial V}{\partial M} = \lambda \cdot (w - b) \cdot \frac{dL_u}{dM} + b - \frac{\partial}{\partial M} \left\{ \frac{M}{N + M} \left[ \lambda \cdot w \cdot L_u + (1 - \lambda) \cdot b \cdot L_c \right] \right\}.
\]

(A1)

Note that the induced changes in \( K_U \) and \( K_C \) cancel out since \( \partial Y_U / \partial K_U = \partial Y_C / \partial K_C = r \). We rewrite the term in brackets as

\[
\Delta \equiv \left( 1 - \frac{1}{\phi} \right) \cdot \frac{M}{N + M} \cdot \lambda \cdot w \cdot L_u + \frac{1}{\phi} \cdot w \cdot M.
\]

and differentiate with respect to \( M \)

\[
\frac{\partial \Delta}{\partial M} = \frac{N}{(N + M)^2} \cdot \lambda \cdot w \cdot L_u \cdot \left( 1 - \frac{1}{\phi} \right) + \frac{w}{\phi} + \frac{M}{N + M} \cdot \lambda \cdot w \cdot \left( 1 - \frac{1}{\phi} \right) \cdot \frac{dL_u}{dM} + \left[ \left( 1 - \frac{1}{\phi} \right) \cdot \frac{M}{N + M} \cdot \lambda \cdot L_u + \frac{1}{\phi} \cdot M \right] \cdot \frac{\partial w}{\partial M}.
\]

(A2)

Substituting (A2) into (A1) and using \( w = \phi \cdot b \) yields

\[
\frac{\partial V}{\partial M} = \lambda \cdot w \cdot \left( 1 - \frac{1}{\phi} \right) \cdot \frac{dL_u}{dM} - \frac{N \cdot \lambda \cdot w \cdot L_u}{(N + M)^2} \cdot \left( 1 - \frac{1}{\phi} \right) - \frac{M \cdot \lambda \cdot w \cdot \left( 1 - \frac{1}{\phi} \right) \cdot dL_u}{dM} - \left[ \left( 1 - \frac{1}{\phi} \right) \cdot \frac{M}{N + M} \cdot \lambda \cdot L_u + \frac{1}{\phi} \cdot M \right] \cdot \frac{\partial w}{\partial M}.
\]

which simplifies to

\[
\frac{\partial V}{\partial M} = \lambda \cdot w \cdot \left( 1 - \frac{1}{\phi} \right) \cdot \frac{N}{N + M} \cdot \left\{ \frac{dL_u}{dM} - \frac{L_u}{N + M} \right\} - \left[ \left( 1 - \frac{1}{\phi} \right) \cdot \frac{\lambda \cdot L_u}{N + M} + \frac{1}{\phi} \right] \cdot M \cdot \frac{\partial w}{\partial M}.
\]
Now we make use of the information on $dL_u/dM$ from (9) and replace the immigrants’ effect on union wages by

$$\frac{\partial w}{\partial M} = -\frac{1}{\varepsilon_u} \cdot \frac{w}{L_u} \cdot \frac{dL_u}{dM} < 0.$$ \hspace{1cm}

This allows us to write

$$\frac{dV}{dM} = \lambda \cdot w \cdot \left(1 - \frac{1}{\phi} \right) \cdot \frac{N \cdot L_u}{(N + M)^2} \cdot \frac{N + M - \lambda \cdot L_u}{\lambda \cdot L_u \cdot \varepsilon_u + (N + M - \lambda \cdot L_u) \cdot \varepsilon_c} \cdot (\varepsilon_u - \varepsilon_c) +$$

$$+ \left[ 1 - \frac{1}{\phi} \right] \cdot \frac{\lambda \cdot L_u}{N + M} + \frac{1}{\phi} \cdot \frac{1}{\varepsilon_u} \cdot \frac{w}{L_u} \cdot \frac{dL_u}{dM} \cdot M$$

as in (11).

**REFERENCES**


