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## Growth and Convergence in a Two-Region Model of Unified Germany

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# CESifo *Working Paper Series*

## GROWTH AND CONVERGENCE IN A TWO-REGION MODEL OF UNIFIED GERMANY

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## GROWTH AND CONVERGENCE IN A TWO-REGION MODEL OF UNIFIED GERMANY

### Abstract

The paper sets up a two-region endogenous growth model to discuss growth and regional convergence of unified Germany. It emphasises the role of private and public capital accumulation during the developing process. The theoretical part derives fiscal policy rules which establish convergence of regional output per capita and convergence of regional human wealth. To assess the speed of convergence the model is calibrated with German data. Given a fiscal policy rule that is consistent with the data on government spending in East and West Germany after unification the model suggests that East Germany will reach eighty percent of West Germany's income per capita between 20 and 30 years after unification and that actual transfers are approximately sufficient to equalise regional human wealth. The results are compared with an extension of the model that includes wage setting behaviour and unemployment in the eastern region.

Keywords: German unification, regional convergence, government spending

JEL Classification: O41, O52, H31, H40

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# 1 Introduction

On 2 July 1990, East and West Germany became economically united through a common currency. Ten years after, the effects of German reunification continue to reverberate in Germany. The abrupt exposure of the structurally-weak East German economy to competition from the world markets in 1990 resulted in a strong decline of economic activity in the five new federal states (*Bundesländer*). The industrial depression that followed the shock of unification ended in 1991 but on the other hand there is no *Wirtschaftswunder*. West Germany's post-war recovery was buttressed by an undervalued Deutschmark and by low wages, both of which boosted exports. Eastern Germany has neither advantage; instead it is suffering from two policy blunders. Unification saddled the region with a currency whose value reflected western Germany's highly productive industry. And employers worsened that handicap by agreeing to raise eastern German wages to western levels far more rapidly than the productivity gap could be closed. Given this situation both state governments and the Treuhandanstalt, the agency charged with privatising eastern German industry, have rescued "industrial cores" on the theory that new businesses will spring up only where some industry still survives.<sup>1</sup> But the price of such rescues has been very high. After all, more than DM 1 trillion of public money has been pumped into the East since unity. The annual bill is still running at close to a net DM 150 billion, in part financed by a "solidarity" income-tax levy. Despite all that, the eastern jobless rate is still twice that of the West, productivity is a good third lower, exports remain weak. This is in sharp contrast to Helmut Kohl's now notorious claim in 1990 that there would be "flourishing landscapes" in the East within five years. Given these developments, the paper deals with the long-term prospects of the East German economy. We consider the impact of fiscal policy in a two-region endogenous growth model which emphasizes the role of private and public capital accumulation during the transition process.

The theoretical model can be understood as an extension of Ono and Shibata's (1992) two-country model with public capital accumulation and interregional fiscal transfers. Emphasising the role of factor accumulation during the catching-up process, economic growth is driven by the accumulation of private and public capital. Therefore, the model can also be understood as a two-country (region) version of Barro's (1990) model on government spending and economic growth. In contrast to Barro (1990), however, we will regard productive government expenditure as a flow variable that extends the stock of available public capital. Therefore, the model exhibits transitional dynamics and fiscal policy crucially determines the speed of (regional) convergence. The analysis of regional differences in financial and human wealth uses ideas recently developed in Caselli and Ventura (1999), and the solution technique employs the method of backward integration developed in Brunner and Strulik (1998).

The remainder is structured in six sections. Section 2 lays out the basic theoretical framework. Section 3 discusses interregional and economy-wide dynamics. Section 4 deals with fiscal redistribution. Section 5 presents numerical solutions for a calibrated model while section 6 analyses an extended model allowing for unemployment. Section 7 provides some conclusions and discusses limitations of the approach followed in the paper.

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<sup>1</sup>The importance of 'Industrial Cores' for regional growth has recently been modelled in Englmann und Walz (1995).

## 2 The Model

### 2.1 Firms

In each region there exists a large number of identical firms. They operate under perfect competition and employ private capital  $k_i$  and labour  $l_i$  to produce an output  $y_i$  with constant returns to scale in privately provided factors. Malleable output can be either used for consumption, private investment, or public expenditure and the production function has the Cobb-Douglas form:

$$y_i = A_i k_i^\alpha l_i^{1-\alpha}, \quad i = W, E, \quad 0 < \alpha < 1. \quad (1)$$

The productivity parameter  $A_i$  is exogenous to the firm but partly determined by productive government spending. The price of goods is normalised to one. Firms control labour inputs  $l_i$  and investment  $I_i$ . With depreciation rate  $\delta > 0$  the capital stock develops according to

$$\dot{k}_i = I_i - \delta k_i. \quad (2)$$

Firms have to pay a corporate tax on cash flow with constant rate  $\tau$ . They maximise the present value of their intertemporal net profit flow  $V_i(0) = \int_0^\infty \exp[-\bar{r}_i(t)t] \{(1-\tau)[y_i - w_i l_i] - I_i\} dt$  subject to (2), where  $w_i$  denotes the wage rate and

$$\bar{r}_i(t) = \frac{1}{t} \int_0^t r_i(s) ds \quad (3)$$

is the average interest rate between times 0 and  $t$ . According to the first order conditions, factor prices are given by

$$w_i = (1 - \alpha) A_i (K_i / L_i)^\alpha, \quad \text{and} \quad (4)$$

$$r_i = (1 - \tau) \alpha A_i (K_i / L_i)^{\alpha-1} - \delta. \quad (5)$$

In writing (4) and (5) we have used the fact that all firms are identical and hence choose a capital labour ratio that equals the capital labour ratio of their region,  $k_i / l_i = K_i / L_i$ .

### 2.2 Government Behaviour

The government taxes corporate income and individual income with a single rate. To avoid double taxation individual interest earnings are tax exempt. Tax earnings are at least partly spent on the accumulation of productive public capital  $G_i$ . The remainder is spent on transfer payments within a region,  $Z_i$ , and intra-regional transfers. The government runs a balanced budget which can be written as

$$\tau(Y_W + Y_E) = \dot{G}_W - \delta G_W + \dot{G}_E - \delta G_E + Z_W + Z_E, \quad (6)$$

where  $\delta$  is the depreciation rate of public capital. The analysis simplifies greatly if we assume a two-step procedure in government budgeting. In the first step the government decides separately for each region how to spend regional tax earnings. In the second step it performs interregional redistribution according to a system of fiscal equalisation (*Länderfinanzausgleich*). With  $0 \leq q_i \leq 1$  denoting the share spent on productive services and  $x$  denoting

the fraction of Western tax earning transferred to the East, government behaviour can be summarised as

$$\begin{aligned}\dot{G}_i &= q_i \tau Y_i - \delta G_i , \\ Z_W &= (1 - q_W - x) \tau Y_W , \\ Z_E &= (1 - q_E) \tau Y_E + x \tau Y_W .\end{aligned}\tag{7}$$

We follow Barro (1990) in assuming decreasing returns of infrastructure so that the macro-economy exhibits constant returns to scale in private and public capital. Since the regions differ in size their infrastructure has to be scaled by population size to eliminate unwanted scale effects. The regional factor productivity  $A_i$  is then given by

$$A_i = A(G_i/L_i)^{1-\alpha} , A > 0 ,\tag{8}$$

where  $A$  denotes a general productivity parameter which is assumed to be identical in both regions.<sup>2</sup>

### 2.3 Households

Each region is populated by a large number of households, numbered  $j = 1, 2, \dots, L_W$  and  $j = 1, 2, \dots, L_E$ , respectively. Each household supplies one unit of labour inelastically and maximises utility from intertemporal consumption

$$U_i^j = \int_0^\infty \frac{c_i^{j1-\sigma} - 1}{1-\sigma} e^{-\rho t} dt ,\tag{9}$$

subject to his budget constraint

$$c_i^j + \dot{a}_i^j = r_i a_i^j + (1 - \tau) w_i^j + z_i^j ,\tag{10}$$

where  $\rho$  denotes the time preference rate, and  $\sigma^{-1} < 1$  is the intertemporal elasticity of substitution. Individuals may differ in their initial endowment of financial wealth  $a_i^j(0)$  and in their received transfers,  $z_i^j$ . The heterogeneity in wealth across both region may explain the existence of intra- regional transfers. From the first order conditions of the corresponding current value Hamiltonian we obtain the Ramsey rule

$$\dot{c}_i/c_i = [r_i - \rho] / \sigma ,\tag{11}$$

which applies to all consumers independently from wealth or provenance.

### 2.4 A Fiscal Policy Rules for Economic Convergence

Unification induces a spontaneous equalisation of regional interest rates through capital movements towards the region with the higher net marginal product of private capital. Applying the interest parity on (5) and using (8) yields

$$\theta \equiv y_E/y_W = \frac{K_E/L_E}{K_W/L_W} = \frac{G_E}{G_W} \frac{1}{\lambda} , \text{ where}\tag{12}$$

$$\lambda \equiv L_E/L_W .\tag{13}$$

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<sup>2</sup>Equation (8) introduces a mechanism through which infrastructure provided by the government affects the productivity of privately owned factors. The empirical paper by Duggal et al. (1999) provides a rationale for the chosen specification.

Equations (12) and (13) introduce two measures of regional differences. The first one,  $\theta$ , measures the relative backwardness of the eastern region in terms of eastern income per capita relative to western income per capita. The second one,  $\lambda$ , is a scale variable that controls for the size of the regional work force.

Equation (12) displays the basic mechanism of the model: Interregional mobile private capital ties down the East's relative income per capita to its relative stock of infrastructure per capita. If, for example, the pre-unification levels of private and public capital per capita were  $K_E/L_E = 0.4K_W/L_W$  and  $G_E/L_E = 0.5G_W/L_W$ , then unification would lead to a spontaneous reallocation of private capital from the West to the East so that  $K_E/L_E = 0.5K_W/L_W$  and hence  $\theta = 0.5$ . For simplicity we assume that any possible difference in the region's relative endowment with private and public capital are equalised by private capital flows at unification time so that the starting point of the analysis is uniquely determined by the regional distribution of infrastructure.<sup>3</sup>

Consider now a government that simply adopts the "successful" western policy in the eastern part of the country, i.e.  $q_E = q_W$ . After insertion of (8) and (12) into (7) we obtain

$$\gamma_\theta \equiv \frac{\dot{\theta}}{\theta} = (q_E - q_W)\tau A \left( \frac{G_W}{K_W} \right)^{-\alpha} . \quad (14)$$

Hence, there will never be convergence if the richer region's fiscal policy is imposed upon the poorer region. In conclusion infrastructure spending in the poorer region must be temporarily higher to attract private capital. The following proposition shows the policy rule that establishes income convergence between the East and the West.

**Proposition 1** *Consider the two-region endogenous growth model as described above. Let the eastern region be initially backward in terms of per capita income relative to the western region. Then the unique set of fiscal policy functions that establishes economic convergence is determined by*

$$q_E = [f(\theta) + 1]q_W, \quad f' < 0, \quad f(1) = 0 . \quad (15)$$

The proof inserts (15) into (14). In the remainder of the paper we assume that the government has the objective to realise regional convergence and, therefore, adopts a policy rule from the set of feasible functions (15). Note that the policy does not depend on time but on the state of the system, namely the distance of the East from its western counterpart so that it constitutes a time consistent credible policy.

### 3 Economic Convergence

#### 3.1 Interregional and Economy-Wide Dynamics

From the country-wide perspective any income which is not spent on infrastructure accumulation is spend on either consumption,  $C$ , or private capital accumulation,  $\dot{K}$ . Hence the economy-wide private capital stock

$$K = K_W + K_E = (1 + \theta\lambda)K_W \quad (16)$$

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<sup>3</sup>The instantaneous relocation of private capital occurs because the model does not contain any adjustment costs.

develops according to

$$\dot{K} = (1 - \tau q_W)A \left( \frac{G_W}{K_W} \right)^{1-\alpha} K_W + (1 - \tau q_E)A \left( \frac{G_E}{K_E} \right)^{1-\alpha} K_E - C - \delta K \quad (17)$$

Let the economy-wide consumption-capital ratio be defined as  $\chi \equiv C/K$  and western infrastructure per unit of total private capital as  $g_W \equiv G_W/K$ . Using (12), (15), and (16) equation (17) can be rewritten as

$$\gamma_K \equiv \dot{K}/K = \{1 + \theta\lambda - \tau q_W [1 + \lambda(f(\theta) + 1)]\} A g_W^{1-\alpha} (1 + \lambda\theta)^{-\alpha} - \chi - \delta . \quad (18)$$

Employing (5) and (10) we obtain in the same way

$$\gamma_\chi \equiv \dot{C}/C - \gamma_K = \frac{1}{\sigma} [(1 - \tau)\alpha A g_W^{1-\alpha} (1 + \lambda\theta)^{1-\alpha} - (\delta + \rho)] - \gamma_K , \quad (19)$$

and (7) and (14) can be rewritten as

$$\gamma_{g_W} \equiv \dot{G}_W/G_W - \gamma_K = q_W \tau A g_W^{-\alpha} (1 + \lambda\theta)^{-\alpha} - \delta - \gamma_K , \quad (20)$$

$$\gamma_\theta = f(\theta) q_W \tau A g_W^{-\alpha} (1 + \lambda\theta)^{-\alpha} . \quad (21)$$

With equations (18) – (21) the economy is described by a three dimensional differential equation system in  $\theta$ ,  $g_W$  and  $\chi$ .

### 3.2 Equilibrium Analysis

At the equilibrium we have  $\theta^* = 1$  from (21). After insertion of (20) into (19) the equilibrium value of  $g_W$  is determined by the implicit function

$$0 = F(g_W) = [(1 - \tau)\alpha A g_W^{1-\alpha} (1 + \lambda)^{1-\alpha} - (\delta + \rho)] / \sigma - q_W \tau A g_W^{-\alpha} (1 + \lambda)^{-\alpha} + \delta . \quad (22)$$

Since  $F' > 0$  for all positive  $g_W$  and  $\lim_{g_W \rightarrow 0} F(g_W) = -\infty$  and  $\lim_{g_W \rightarrow \infty} F(g_W) = \infty$  a unique equilibrium  $g_W^*$  exists. At this equilibrium  $\chi^*$  is obtained as

$$\chi^* = (1 + \lambda)^{1-\alpha} A g_W^{*1-\alpha} [1 - \tau q_W - (1 - \tau)\alpha / \sigma] + (\delta + \rho) / \sigma - \delta \quad (23)$$

from (18) and (19). Generally, (22) can only be solved numerically. It is, however, useful to consider for a moment the special parameterisation  $\rho = (\sigma - 1)\delta$  for which  $g_W^*$  can be obtained analytically as

$$g_W^* = \frac{\sigma q_W \tau}{(1 - \tau)\alpha(1 + \lambda)} . \quad (24)$$

After insertion into (19) the equilibrium growth rate of the economy is calculated as

$$\gamma_C = [\alpha(1 - \tau) / \sigma]^\alpha A (q_W \tau)^{1-\alpha} - \delta . \quad (25)$$

By derivation with respect to  $\tau$  it can be verified that the long-run growth maximising tax rate is  $\tau = 1 - \alpha$ .<sup>4</sup> Hence, Barro's (1990) finding that the optimal income tax rate equals

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<sup>4</sup>The empirical literature on economic growth, dominated by cross-country regressions and tests of income convergence across countries, has recently been enriched by studies that provide direct tests of endogenous growth models utilising time series data. Kocherlakota and Yi (1997) have recently provided empirical evidence for the U.S. and the U.K. that permanent changes in fiscal policy can have permanent effects on growth rates even though the growth rate itself appears stable over time.

the production elasticity of infrastructure is replicated in the two-region growth model with inter- and intra-regional transfers.

The Jacobian determinant at the steady-state is computed as  $\det J = \partial\gamma_\theta/\partial\theta[\partial\gamma_{gW}/\partial gW - \partial\gamma_\chi/\partial gW]$  with

$$\begin{aligned}\frac{\partial\gamma_\theta}{\partial\theta} &= f'(\theta)(1+\lambda)^{-\alpha}q_W\tau Ag_W^{-\alpha} < 0, \\ \frac{\partial\gamma_{gW}}{\partial gW} - \frac{\partial\gamma_\chi}{\partial gW} &= -\alpha(1+\lambda)^{-\alpha}q_W\tau Ag_W^{-\alpha-1} - (1-\alpha)(1-\tau)(1+\lambda)^{1-\alpha}\alpha Ag_W^{-\alpha}/\sigma < 0\end{aligned}$$

so that the equilibrium is a saddlepoint. The eigenvalues are

$$\begin{aligned}\lambda_1 &= \frac{\partial\gamma_\theta}{\partial\theta} < 0 \text{ and} \\ \lambda_{2,3} &= \left(1 + \frac{\partial\gamma_{gW}}{\partial gW} \pm \sqrt{(1 + \partial\gamma_{gW}/\partial gW)^2 - 4(\partial\gamma_{gW}/\partial gW - \partial\gamma_\chi/\partial gW)}\right) / 2.\end{aligned}$$

Since  $\partial\gamma_{gW}/\partial gW - \partial\gamma_\chi/\partial gW < 0$  all eigenvalues are real so that the adjustment path towards the equilibrium is monotonous. For reasonable parameterisations of the model it turns out that two eigenvalues are negative, so that the stable manifold is two-dimensional.

At the steady-state consumption, public capital, and private capital grow with identical rates economy-wide, which can be seen from (19) and (20), as well as in its regional components, which can be seen from (11) and (12).

## 4 Interregional Income Redistribution: The Solidarity Pact

The problem of regional convergence has been solved independently from the existence of interregional income redistribution. So far, however, we have only discussed the convergence of regional income *produced* but not the problem of convergence of income *earned*, i.e. the convergence of regional wealth and regional consumption. For that purpose let us now compare the average individual in the East with its counterpart in the West. From (7) we see that the average western individual receives income transfers  $z_W = Z_W/L_W = (1 - q_W - x)\tau Y_W/L_W$  and the average eastern individual receives  $z_E = (1 - q_E)\tau Y_E/L_E + x\tau Y_W/L_E$ . With use of (4) and the definitions of  $\theta$  and  $\lambda$  their budget constraints (10) can be written as

$$\dot{a}_W = [1 - \alpha(1 - \tau) - q_W\tau - x\tau]y_W - c_W + ra_W, \quad (26)$$

$$\dot{a}_E = \left[(1 - \alpha(1 - \tau))\theta - q_E\tau\theta + \frac{x\tau}{\lambda}\right]y_W - c_E + ra_E. \quad (27)$$

After integrating (26) and (27) and substituting the integrated version of (11) we arrive after some amount of algebra at

$$\frac{c_E(t)}{c_W(t)} = \frac{c_E(0)}{c_W(0)} = \frac{a_E(0) + \int_0^\infty [(1 - \alpha(1 - \tau))\theta - q_E\tau\theta + \frac{x\tau}{\lambda}]y_W e^{-\int_0^t \bar{r} dv} dt}{a_W(0) + \int_0^\infty [1 - \alpha(1 - \tau) - q_W\tau - x\tau]y_W e^{-\int_0^t \bar{r} dv} dt} \quad (28)$$

The first sign of equality originates from the fact that all individuals choose the same intertemporal allocation of consumption. Regional levels of consumption converge if the right hand side of the equation equals one.

Let us first consider a government that attempts to realise convergence of consumption and the hypothetical case of spontaneous economic convergence at unification time. In this case the integrals in (28) are identical for  $x = 0$  but even then the eastern individual would be worse

off if he brings along a lower initial endowment of financial wealth. The only possibility to produce convergence of consumption is to partly expropriate the western individual. Suppose, for example, that  $K_E/L_E$  equals  $0.5K_W/L_E$  before unification and that all eastern capital is owned by the eastern population so that the average eastern individual is half as rich in terms of financial wealth as his western counterpart. If the westerner donates half of his wealth to the easterner than  $a_E(0) = a_W(0)$  and  $c_E = c_W$  for all  $t$ .

This leads to the following conclusion. While equating the  $a_i$ 's means to compensate the eastern individual for the bad performance of his economy *before* unification, equating the integrals means to compensate the eastern individual for the relatively bad performance of his native region *after* unification. We have emphasised this distinction to motivate the following assumption about government behaviour. The government does not take responsibility for the bad performance of the eastern region *before* unification but takes responsibility for the well being of eastern individuals *after* unification. Formally this means that the government tries to equalize consumption levels of individuals with initially equal financial wealth, i.e. it tries to equalize consumption levels *as if* the eastern individuals are initially equally equipped with financial wealth.

Assuming that the government compensates only disadvantages which originate from living in the poorer region after unification is of course a highly normative judgement but it has a very useful implication.<sup>5</sup> Since equating the integrals means equating human wealth or intertemporal non-financial income the income transfer policy prevents regional migration. Although, this applies in a strict sense only to the average eastern individual it is generally possible to construct a deliberate spending policy that realises the result for all individuals once their specific individual economic situation is known. In turn, any policy that tries to compensate for different initial  $a_i$  may lead to migration of western individuals to the East in order to receive transfers that make them at least partly off for their initial loss in financial wealth.<sup>6</sup>

After equating the integrals and inserting the income convergence policy (15) we obtain the income transfer policy as

$$x = \{[1 - \alpha(1 - \tau) - q_W\tau](1 - \theta) + f(\theta)q_W\tau\theta\} \left[ \frac{\lambda}{\tau(\lambda + 1)} \right] . \quad (29)$$

The magnitude of transfers depends on the chosen policy to achieve economic convergence. The policy rule implies that transfers will have an end when income per worker has converged,  $\theta = 1 \Leftrightarrow x = 0$ . Transfers decline with increasing degree of economic convergence,  $\partial x / \partial \theta < 0$ , and will be the lower the smaller the poorer region.

## 5 Model Calibration and Solution Technique

The model is calibrated so that its steady-state solution matches West Germany's pre-unification's performance. Unless otherwise specified the data is taken from Statistisches Bundesamt (1997). From the Eastern and Western population size we calculate  $\lambda = 0.25$ . We set  $\tau = 0.5$  from Western Germany's government share and  $q_W = 0.1$  from the average

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<sup>5</sup>The assumption seems reasonable since the German mainstream parties are overwhelming dominated by western politicians.

<sup>6</sup>Equating the integrals equates the intertemporal stream of wages and governmental transfers in the East and West. Equal human wealth, however, does not necessarily rule out the possibility of wage setting behaviour since this may be motivated by initial differences in financial wealth or any other reason beyond the scope of the model.

pre-unification value of West Germany's infrastructure share in government spending. From numerous other calibration studies we adopt  $\rho = 0.02$ .

Because  $\alpha$  simultaneously determines productivity of labour, private capital, and infrastructure, any numerical specification entails a shortcoming. Since the regional distribution of labour is fixed, and private capital is allowed to flow freely from region to region, differences in regional growth are determined by the distribution of infrastructure and the production elasticity of infrastructure is identified as the most decisive one in our model. We therefore decided to calibrate  $\alpha$  to match the infrastructure elasticity. Although there is some confusion in the empirical literature about the true value of the infrastructure elasticity, the comprehensive compilation of empirical studies in Sturm et al. (1998) finds that most researchers estimate an elasticity between 0.1 and 0.3. This leads us to the definition of a basic scenario with an infrastructure elasticity of 0.2 ( $\alpha = 0.8$ ) and the introduction of an alternative scenario with  $\alpha = 0.7$  in order to analyse the sensitivity of results with respect to the choice of the infrastructure elasticity.

The remaining parameters,  $A$ ,  $\delta$  and  $\sigma$  are specified so that the model meets West Germany's pre-unification capital output ratio, which was on average about 2.7 over the last decades, and its per capita income growth rate. West Germany, however, was itself far away from its long-run equilibrium path for most of the time after second World War and the economy was catching up with comparatively high growth rates. Hence, we select the average growth rate during the eighties, which was 1.75 percent, to be met by the model's steady-state growth rate. This leads to the specification of  $\sigma = 2.5$ ,  $\delta = 0.08$  and  $A = 0.504$  which results in  $g_y^* = 0.175$  and  $(K/Y)^* = 2.78$ . In the alternative scenario a value of  $A = 0.655$  results in the same long-run growth rate and  $(K/Y)^* = 2.45$ . While the second scenario underestimates the empirical capital output ratio it provides a better result for the steady-state ratio of public to private capital, which was around 0.28 in the eighties in West Germany (DIW, 1994, p. 458). Table 1 summarises the alternative parameterisations.

Table 1: MODEL PARAMETERISATIONS

$\alpha$	$\lambda$	$\tau$	$q_W$	$\delta$	$\rho$	$\sigma$	$A$	$\gamma_y^*$	$(K/Y)^*$	$(G_i/K_i)^*$
0.8	0.25	0.50	0.1	0.08	0.02	2.5	0.504	1.75%	2.78	18.4%
0.7	0.25	0.50	0.1	0.08	0.02	2.5	0.655	1.75%	2.45	21.1%

Our analysis begins after the collapse of the former GDR, i.e. after the initial slump of the eastern economy, so that  $t = 0$  in model time corresponds approximately to 1992 in real time, and our model is therefore now situated in year 7 after unification. In 1992 the private capital stock as well as the public capital stock per capita in East Germany was approximately 40 percent of the corresponding western level, i.e. our analysis starts at  $\theta = 0.4$ .<sup>7</sup>

The fiscal policy rule is specified as

$$f(\theta) = a \left[ \frac{1 - \theta}{\theta} \right]^\epsilon, \quad (30)$$

where broadly speaking, the parameter  $a$ , controls the absolute weight given to infrastructure expenditure and  $\epsilon > 0$ , controls the policy reaction on relative income improvements. A

<sup>7</sup>Data for private capital are from DIW (1995, p. 540), data for public capital are available in DIW (1994, p. 461).

higher  $\epsilon$  specifies a more reluctant fiscal policy at higher  $\theta$ 's, i.e when the eastern economy has already caught up parts of its initial backwardness. We set  $\epsilon = 1$  in the basic scenario and consider an alternative policy with  $\epsilon = 1.5$ . The choice of  $\epsilon$  and the actual government spending policy at  $t = 0$  determine the value of  $a$ , so that (30) meets the actual fiscal policy after unification.

Relating the data on government investment in DIW (1995, p. 537) to eastern GDP in the corresponding years we calculate infrastructure investment in eastern Germany of between 9 and 10 percent of East Germany's GDP in the early nineties, so that  $0.1 = \tau q_E$  implies  $q_E(0) = 0.2$  and  $a = 2/3$  for the basic fiscal policy rule and  $a = 0.5443$  for  $\epsilon = 1.5$ .

After inserting (30) and the parameters from Table 1 in (29) the initial share of transfers to the East in western tax earnings is obtained as

$$x(0) = \{[1 - 0.8 \cdot (1 - 0.5) \cdot 0.1 \cdot 0.4][1 - 0.4] + 0.8 \cdot 0.1 \cdot 0.4 \cdot 1\} \left[ \frac{0.25}{1.25 \cdot 0.5} \right] = 13.7\% .$$

The existence of a two-dimensional stable manifold provides us with a further advantage in modelling unification. It enables us to specify a second initial condition, which identifies a unique adjustment path from the feasible set of trajectories on the manifold. Under the assumption that West Germany was approximately developing on its steady-state growth path its public to private capital ratio *before unification* is implicitly determined by the steady-state *after unification* through  $G_W(0)/K_W(0) = g_W^*(1 + \lambda)$  and hence  $g_W(0) = g_W^*(1 + \lambda)/(1 + \theta(0)\lambda)$ .

We solve the numerical problem with the method of backward integration. The general method is described in Brunner and Strulik (1998) and the appendix of this paper shows its application to the two-region model. The main feature of the method is that it reveals – besides of arbitrarily small discretisation errors – the exact adjustment path rather than approximations. After having obtained the adjustment path we use (1), (8), (13), and (16) to recalculate regional growth rates as

$$\begin{aligned} \gamma_{K_W} &= \gamma_K - \gamma_\theta \frac{\theta\lambda}{1 + \lambda\theta} , \\ \gamma_{K_E} &= \gamma_{K_W} + \gamma_\theta , \\ \gamma_{g_E} &= \gamma_\theta + \gamma_{g_W} , \\ \gamma_{Y_W} &= (1 - \alpha)\gamma_{g_W} + \gamma_{K_W} , \\ \gamma_{Y_E} &= (1 - \alpha)\gamma_{g_E} + \gamma_{K_E} . \end{aligned} \tag{31}$$

## 5.1 Results

Eastern and western Germany's development and convergence in the basic scenario is described by the straight lines in Figure 1. The first panel on the left shows the fiscal policy  $q_E(\theta)$  as implied by (30). The other panels show the resulting time paths for several interesting variables. The main variable of interest, the degree of backwardness of the eastern region is shown in the first panel on the right hand side. In the basic scenario half of the initial gap between East and West is closed after about 10 years after unification and after about 30 years the eastern region shows 90 percent of the productivity of its western counterpart.<sup>8</sup>

Bearing in mind that the starting point in real time was 1992 the eastern economy has now, 1999, reached 63 percent of West Germany's productivity level.

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<sup>8</sup>This prediction is similar to the simulation results in Burda and Funke (1995). On the contrary Barro (1991), Barro and Sala-i-Martin (1991), Dornbusch and Wolf (1992), and Hughes Hallet and Ma (1993) have predicted that convergence of living standards will take much longer.

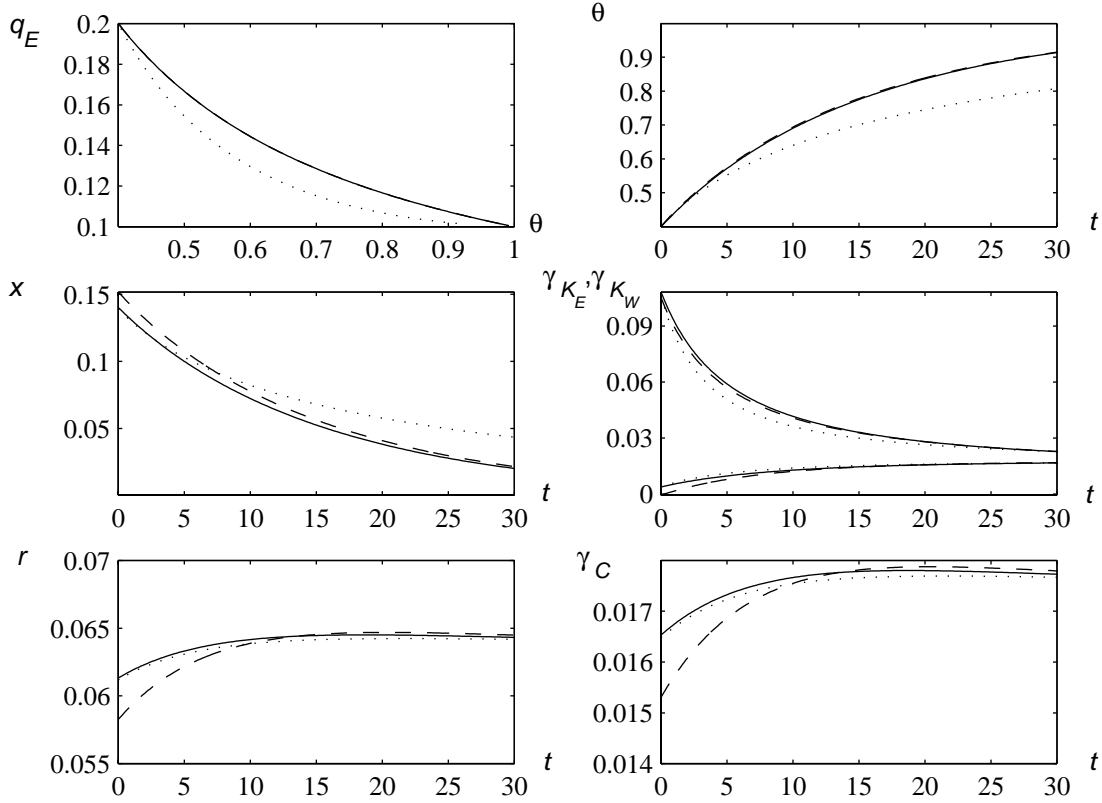


Figure 1: Economic Development after German Unification

Straight Lines: Basic Scenario, Dashed Lines:  $\alpha = 0.7$ , Dotted Lines: Reluctant Fiscal Policy

The third panel shows the implied income transfer policy. In the years shortly after unification average transfers approximately met the historical data with a tax share of about 13.7 percent initially and of about 8.5 percent after 7 years. However, while average transfers are compatible with the empirical data this is not true for the development of transfers over time.<sup>9</sup> Nevertheless, even under the model's successful convergence policy income transfers are significantly positive for a long time. For example, 15 (25) years after unification the eastern region still receives approximately 5 (2.7) percent of western tax earnings.

The next panel shows the development of regional private capital. The improvement in eastern infrastructure productivity goes hand in hand with large private investments in this region. The capital growth rate is above 10 percent shortly after unification and is still almost twice as high as its equilibrium value after fifteen years. While this sentence makes sense for every economist who is familiar with the law of diminishing returns the emphasis should maybe be different when the result is presented to the public: Although the eastern economy has started with extraordinary high growth rates, the growth rate will decrease to a third of its initial value 15 years after unification.

Capital in the eastern region is accumulated by eastern and western individuals. Realising the increasing productivity in the eastern region, westerners allocate large parts of their

<sup>9</sup>The figures in Deutsche Bundesbank (1996, p. 19) show that the total amount of transfers from western to eastern Germany sums up to approximately 5 percent of West Germany's GDP per year over the first 6 years after unification. Using  $\tau = 0.5$  this implies an  $x$  of 10 percent during this period.

investment to the East. This leads to a capital growth rate below one percent in the western region during the first five years after unification. Compared to the East, however, unification has only a relatively small impact on the western region. This outcome reflects the fact that the eastern region is much smaller in absolute size and can best be seen in the small effect on the economy-wide interest rate which falls only about 0.3 percentage points. To understand why the interest rate approaches its steady-state from below and not from above one has to recall that private capital is regionally mobile. Hence, productivity of private capital is bounded by the regional productivity of infrastructure. Eastern infrastructure, however, falls short of its equilibrium value at the moment of unification and then increases during the convergence process.

The relatively small deviation of the interest rate from its equilibrium value is reflected by a relatively small deviation of consumption growth from its equilibrium value as shown in the final panel on the right.

The dashed lines in Figure 1 show the development under the  $\alpha = 0.7$  assumption. The main result is depicted in the  $\theta$ -path which virtually coincides with the basic scenario. Because the change in  $\alpha$  affects both economies in exactly the same way relative regional deviation remains unchanged. A lower  $\alpha$  increases the importance of infrastructure accumulation in the development process. Since the speed of accumulation is determined by the policy rule, and the policy rule remains unchanged *both* regions develop with a slightly slower pace as compared to the basic scenario. The increased importance of infrastructure is also reflected by a higher initial decrease of the interest rate on private capital and hence by a higher decrease of growth rate of consumption. In the years after unification, both, Easterners and Westerners consume more and invest less as compared to the basic scenario.<sup>10</sup>

The dotted lines in Figure 1 represent the outcome of the basic parameterisation under the reluctant fiscal policy with  $\epsilon = 1.5$ . As shown in the  $q_E$  panel the government puts less weight on infrastructure development after the economy has already caught up parts of the initial gap. As a result the convergence process is similar during the jumpstart periods but slows down more quickly. The date is guesswork, of course, but according to the model, 80 percent of western productivity are now will be reached after 30 years (instead of 20 years). Due to the slower speed of convergence, transfer payments will be almost twice as high 20 years after unification.

## 6 Wage Setting and Unemployment

An extension of the model with wage setting behaviour is an interesting task for several reasons. First, in the jumpstart years Germany's unions successfully carried out a wage policy that moved eastern wages far out of line with productivity.<sup>11</sup> Hence, the inclusion of wage setting behaviour and unemployment provides a more realistic adjustment scenario. Furthermore, it is an interesting question in itself to what extent the introduction of unemployment alters the pace of development compared to the market equilibrium solution. Finally, it is interesting to identify the winners and losers of eastern wage setting behaviour. One frequently cited proposition is that the union wage policy was advised (if not imposed) by western umbrella organisations for the sake of its western members. The  $\gamma_{KE}, \gamma_{KW}$ -panel

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<sup>10</sup>The  $x$ -panel also suggests that a lower  $\alpha$  implies a slight increase of transfers. Since  $(1 - \alpha)$  also determines labour productivity its change also influences the regional difference in human wealth and hence income transfers according to (28) and (29). This shortcoming would not occur if we would have specified the production elasticity of labour independently from the production elasticity of infrastructure.

<sup>11</sup>A microeconomic foundation for such a wage policy is given in Burda and Funke (1993).

of Figure 1 seems to support this proposition. The western worker is clearly a double loser of unification. He has not only to suffer the loss of parts of his social transfers which are now used to compensate his eastern neighbour for his relatively bad initial position, but he also loses potential wage increases. Because firms invest in western Germany with a rate below the steady-state value, western productivity and wages grow with a rate below steady-state growth during the convergence period. Since we have assumed that West Germany was developing approximately on its steady-state before unification, this finding implies that western wages grow slower than they would have grown without unification. If lower investment in East Germany implies more investment in West Germany, western unions may have reason to prefer high wage growth in eastern Germany to protect their own workers.

When introducing wage setting behaviour we have to distinguish between population size and work force, and income per capita and income per worker. Let  $\bar{L}_E$  define the size of the eastern population,  $L_E$  the size of the actually employed work force,  $\lambda \equiv \bar{L}_E/L_W$  the East's relative population size, and  $\tilde{\lambda} = L_E/L_W$  the relative size of the eastern employed work force. While the East's relative income per worker is, as before, denoted by  $\theta$  we now have a second measure of regional disparity, which is the East's relative income per capita, measured by  $\theta_2$  and calculated as

$$\theta_2 \equiv \frac{Y_E/\bar{L}_E}{Y_W/L_W} = \theta \frac{\tilde{\lambda}}{\lambda} , \quad (32)$$

which is eastern productivity times the employment rate.

In order to maintain an analytically tractable model we cannot integrate wage bargaining into the general equilibrium context but have to impose a behavioural function describing the eastern wage development. This function can be thought of as a compound of a fixed and a flexible part. Let us suppose that eastern unions and employers have agreed on a wage path that follows  $w_E = \theta^\beta w_W$ ,  $0 < \beta < 1$ , so that the standard wage is above productivity level. If, for example,  $\beta = 0.4$ , eastern standard wages are 70 percent of western wages at unification time and would reach 81.5 percent when  $\theta$  reaches 0.6.

Private capital mobility, however, fixes East Germany's relative labour productivity to the value of its relative stock of infrastructure per capita. In consequence a relative wage of  $\theta^\beta$  would imply that nobody finds employment in East Germany. We, therefore, introduce a second, flexible part into the wage equation which comprises the assumption that at least some Easterners are willing to work for wages below the collectively agreed-upon level.<sup>12</sup> It seems reasonable to assume that the discount that these workers are willing to accept increases with the unemployment rate,  $u$ . This leads to the specification of the wage function as

$$w_E = (1 - bu)\theta^\beta w_W, \quad b > 0, \quad 0 < \beta < 1 . \quad (33)$$

It follows from the definition of  $\theta$  that  $u = 0$  at  $\theta = 1$ , so that Easterners demand and receive western wages when the eastern economy has completely converged. Hence, the long-run equilibrium remains the same as in the full employment scenario. By noting that

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<sup>12</sup>Over recent years, however, a gradual erosion of industry-wide wage bargaining has occurred; its coverage has fallen from 54% of West German companies and 72% of West German employees in 1995 to 48% and 68% respectively in 1998. Only half of East Germany's employees are now covered by sector bargaining. Even the IG Metall hinted in May 1999 that some element of profit-related pay might not be anathema after all; and IG Metall is following, not leading, other unions in this regard.

$1 - u = \tilde{\lambda}/\lambda$  and applying the interest parity (12) and (4) and (5) it can be seen that the eastern employment rate is unequivocally tied to the regional productivity differential:

$$\frac{\tilde{\lambda}}{\lambda} = \frac{1}{b} \left[ \theta^{1-\beta} + b - 1 \right] . \quad (34)$$

By the assumption that congestion is measured with respect to population size rather than with respect to the size of the work force, equation(14) remains valid, and therefore, the model can still be described by a three differential equations, and large parts of the analysis can be taken over from the previous section. We simply have to substitute  $\tilde{\lambda}$  from (34) for  $\lambda$  in the set of dynamic equations. Since  $\partial \tilde{\lambda} / \partial \theta > 0$  the whole stability analysis holds still true and the economy converges along the two-dimensional stable manifold towards the equilibrium of income convergence and full employment.

Although the structure of the dynamic system does not change some of our calculations derived from the dynamic paths change quite dramatically. This especially applies to the calculation of income transfers to the East. Since only  $\tilde{\lambda}/\lambda$  workers are employed and receive wages the budget constraint of the average eastern individual (27) changes to

$$\dot{a}_E = (1 - \tau)(1 - b + b + \tilde{\lambda}/\lambda)\theta^\beta w_W + r a_E + (1 - q_E)\tau Y_E / \bar{L}_E + x\tau Y_W / \bar{L}_E . \quad (35)$$

The compensation for the bad performance of the eastern economy now additionally includes the damage done by wage setting behaviour. The additional transfer can be understood as unemployment benefits. After inserting (34) and proceeding as in Section 4 we arrive at

$$x = \left\{ [1 - \alpha(1 - \tau) - q_W\tau] (1 - \theta \frac{\tilde{\lambda}}{\lambda}) + f(\theta)q_W\tau\theta \frac{\tilde{\lambda}}{\lambda} \right\} \left[ \frac{\lambda}{\tau(\lambda + 1)} \right] . \quad (36)$$

Full compensation transfers exceed the value obtained under the full employment scenario (29) and are the higher the higher the unemployment rate  $u = \tilde{\lambda}/\lambda$  is. Furthermore, the equations for regional capital development in (31) have to be rewritten as

$$\begin{aligned} \gamma_{K_W} &= \gamma_K - (\gamma_{\tilde{\lambda}} + \gamma_\theta) \frac{\tilde{\lambda}\theta}{1 + \tilde{\lambda}\theta} , \text{ and} \\ \gamma_{K_E} &= \gamma_{K_W} + \gamma_\theta + \gamma_{\tilde{\lambda}} , \end{aligned} \quad (37)$$

where  $\gamma_{\tilde{\lambda}}$  is the growth rate of the East's relative size of the work force:

$$\gamma_{\tilde{\lambda}} \equiv \frac{\dot{\tilde{\lambda}}}{\tilde{\lambda}} = (1 - \beta) \frac{\gamma_\theta \theta^{1-\beta}}{\theta^{1-\beta} + b - 1} . \quad (38)$$

These additional equations fully describe the extension of the model.

For the computation of the convergence path we take all parameters from our basic scenario.<sup>13</sup> Additionally, we have to specify the parameters of the eastern wage equation, which we do by selecting  $b = 1.5$  and  $\beta = 0.4$ . This implies an initial unemployment rate of about 21 percent. When the eastern economy has managed to catch up half of its initial gap the unemployment rate is approximately 9.5 percent.<sup>14</sup>

<sup>13</sup>The initial condition of the West being on its steady-state growth path before unification is now calculated as  $g_W(0) = g_W^*(1 + \lambda)/(1 + \theta\tilde{\lambda})$ .

<sup>14</sup>Since we assume that western workers are paid according to their productivity and neglect unemployment in the western region as well as short-time work and employment in job creation schemes, the correct interpretation of the unemployment rate is that it shows the level of which the effective Eastern unemployment rate exceeds the Western unemployment rate. At the end of 1991, i.e. at the initial date of our analysis, the effective Eastern unemployment rate was almost 30 percent (Sinn (1992, p. 30) and the Western unemployment rate was approximately 8 percent.

Figure 2 shows the results. Straight lines reproduce the outcome of the basic scenario from Figure 1 and dashed lines show the corresponding wage setting solution. Compared to Figure 1 we have inserted two new panels showing the development of relative income per capita and the unemployment rate. For that purpose we have skipped the  $q_E$ -curve which is the same as in Figure 1 and the  $r$ -curve since its shape is also reflected by the  $\gamma_C$ -curve which is still available.

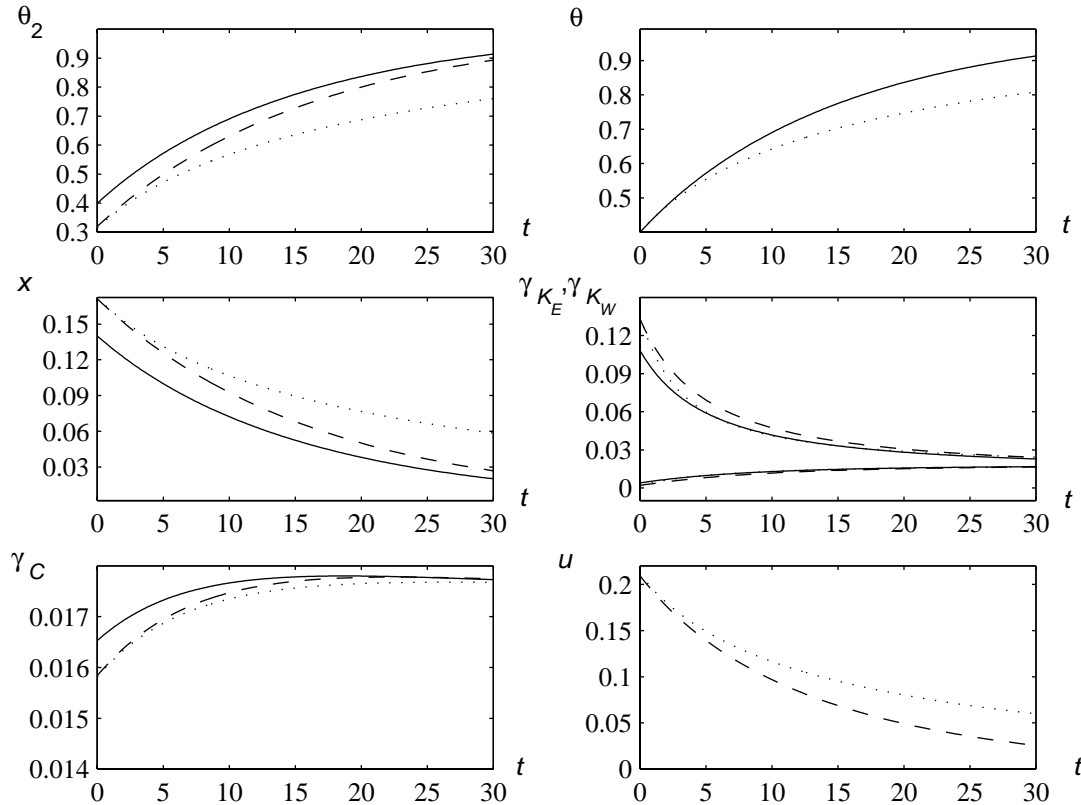


Figure 2: German Unification: Wage Setting Behaviour

Straight Lines: Basic Scenario, Dashed Lines: Wage Setting, Dotted Lines: Wage Setting & Reluctant Fiscal Policy

We have placed the  $\theta_2$  panel in the dominant upper left corner because this curve is essential to understand the remainder of Figure 2. Relative GDP per capita approaches the steady-state from the much lower initial value of about 0.32 if unemployment exists. Turning towards the regional growth rates, we see that the eastern capital growth rate under unemployment exceeds the corresponding full employment value. This is because the eastern economy starts from a much more severe initial situation than before. West Germany's capital growth rate, however, is only insignificantly lower in the unemployment scenario. To understand the result it may help to reconsider equation (12). Because of free private capital movements  $\theta$  is fixed by regional infrastructure policy. Lower employment in East Germany,  $L_E < \bar{L}_E$ , may then have two effects: a lower initial share of economy-wide private capital allocated in eastern Germany and second, a higher capital labour share in each existing firm in both regions. While the first effect results in a higher eastern capital growth rate along the convergence path the second effect additionally results in a lower western capital growth rate. Figure 2 shows that mainly the first effect takes place in our model economy.

Hence, worsening investment conditions in East Germany mainly provoke a substitution from investment towards consumption rather than a substitution of investment in the East with investment in the West. This can also be seen in the  $g_C$ -panel. The initial growth rate of consumption is significantly lower than under perfect market conditions implying a higher initial level of consumption and a lower initial value of the interest rate.

The employed *eastern* worker can be identified as the winner of wage setting behaviour. He starts with a higher wage rate as compared to perfect market conditions and experiences higher wage growth along the adjustment path. The question remains open whether the western worker benefits from eastern wage setting behaviour. Due to the initially higher capital labour ratio he indeed starts with a higher initial wage as compared to perfect market conditions. Since the wage rate eventually grows with the steady-state rate, it grows with a (slightly) smaller rate on the convergence path as compared to the perfect market scenario. Hence, eastern wage setting behaviour provides a *temporary* gain in wages for the western worker. As the  $x$ -panel shows, the western worker pays for his temporary gain in wages with a permanent loss of social transfers that are now received by the unemployed eastern workers.

Transfers required for full compensation for eastern backwardness are now initially more than 3.5 percentage points higher than under perfect market conditions and lie significantly above the compensation rate under perfect market conditions for most of the convergence process. Hence, the model does not support the argument that eastern wage setting behaviour has been the outcome of deliberate western union policy which takes the burden of additional transfers paid by western tax payers into account

If we assume that western tax payers successfully resist full compensation for eastern unemployment, the true  $x$ -curve would lie somewhere between the straight and the dashed line in Figure 2. This would in turn have the implication that the unemployed eastern population also suffers from wage setting behaviour and that there is only one group that benefits from wage setting which consists of the employed eastern population.

Let us now finally consider the worst case scenario of our parameterised model. This comprises the assumptions of wage setting behaviour and a reluctant infrastructure spending policy ( $\epsilon = 1.5$ ) and is represented by the dotted lines in Figure 2. It can be seen that both assumptions together slow down the convergence process considerably. Half of the initial gap in income per capita is now closed 15 years after unification time (instead of 10 years) and after 30 years (instead of 20 years) eastern income per capita has reached 80 percent of the western level. In addition, full compensation for a bad initial position implies that the East still receives large amounts of transfers long after unification time. For example, 20 years after unification the East still receives more than 7 percent of western tax earnings. Persistent high transfers are necessary because the initially high unemployment rate decreases only slowly under the reluctant infrastructure spending scheme. For example, 10 (20) years after unification the eastern unemployment rate is still above 11 (7) percent. In other words, the bill for the East will stay huge into the next decade.

## 7 Conclusion

The paper has analysed economic growth and regional convergence of unified Germany within a two-region endogenous growth model. The model has emphasised the importance of public and private capital accumulation in the initially backward region and on the fiscal interdependence of both regions and its implications on regional convergence of income produced as

well as income earned per inhabitant.<sup>15</sup> To keep the analysis tractable several other factors which certainly also influence the convergence process have been neglected. Some of these factors, like e.g. international capital movements and migration may enhance the speed of convergence while other factors, like initially inappropriate human capital the work force may slow down the adjustment pace.<sup>16</sup>

The theoretical part of the paper has developed a set of feasible fiscal policy rules which establish income per capita convergence. A verbal translation of the feasible policy is: Whenever infrastructure per capita in one region is lower as in the other region spend a higher share of tax earnings on infrastructure in this region. If this is not the case spend regionally identical shares. Looking back to the huge injection of public cash shows that Germany's government has followed this rule. The general fiscal policy rule, however, neither provides information about the speed of convergence nor does it specify the involved costs for citizens of the initially richer region. For that purpose we have calibrated the model with German data. Assuming a government that has the objective to compensate the eastern population for the relatively bad performance of its economy during the adjustment period, a major finding was that actual transfers paid by Western tax payers would have been approximately sufficient for full compensation, i.e. the equalisation of human wealth in both parts of Germany in a perfect market scenario. After introducing wage setting and unemployment in the eastern region we found that actual transfers are no longer sufficient for full compensation for initial backwardness.

The speed of convergence depends on the future effort in infrastructure accumulation in the East.<sup>17</sup> We have introduced alternative fiscal policy rules which are approximately consistent with the past but generate different patterns of future development. In all cases the eastern economy converges quite fast. In the most optimistic scenario it will reach 80 percent of West Germany's GDP per capita after 20 years. If future governments follow a reluctant infrastructure expenditure policy and persistent high unemployment exists, 80 percent will be reached about 10 years later.

Why do we think that the results obtained indicate a quite fast adjustment of the Eastern economy? For a better understanding it may help to compare East Germany's growth performance with West Germany's *Wirtschaftswunder* after World War II. In real international dollars West Germany's GDP per capita was 40 percent of the corresponding U.S. level in the initial period 1950-1955 (data from Summers and Heston, 1995). During this time the private investment/GDP ratio reached values between 20 and 25 percent. For the East German economy that started at a similar initial position we calculate a private investment ratio between 38 and 20 percent during the first five years in our unemployment scenario, a value approximately consistent with the actually attained empirical values. In 1990, 40 years after the onset of the German *Wirtschaftswunder*, West Germany had reached 80 percent of the U.S. GDP per capita. In comparison, our analysis suggests that eastern Germany may converge much faster and may have caught up 80 percent of its initial gap to West Germany's GDP per capita after 20 years.

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<sup>15</sup>See e.g. Young (1994) for the importance of factor accumulation for catching-up processes.

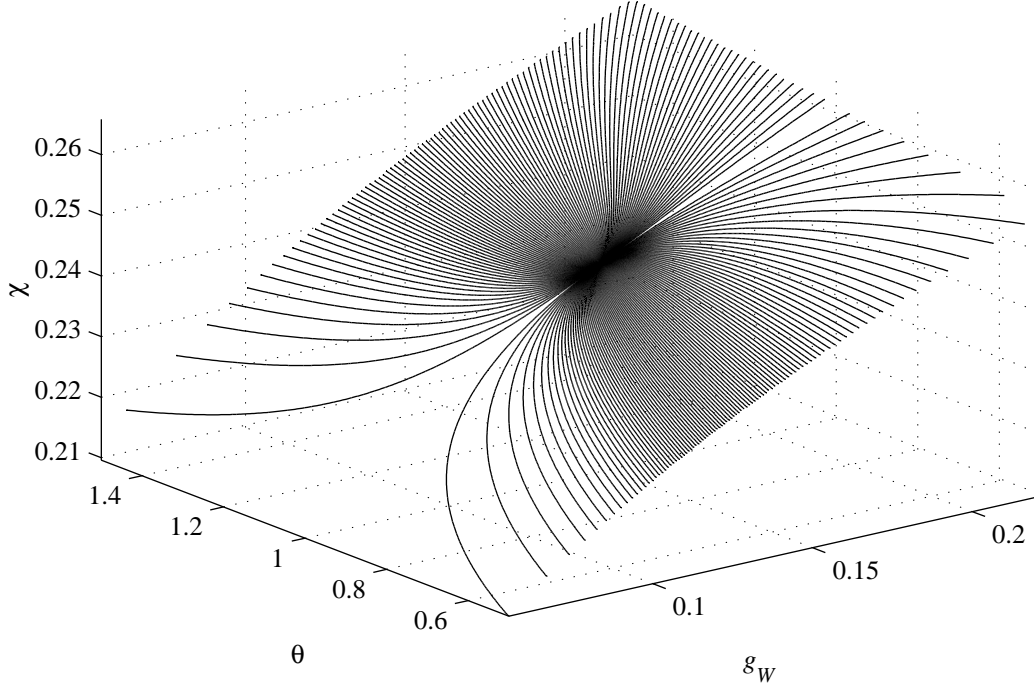
<sup>16</sup>In some ways, under direct pressure, the East is setting the West a positive example. Most eastern companies have now broken out of the straitjacket of nationwide wage bargaining first foisted on them by the West; easterners work long hours and are more flexible than western workers; real wage has been slashed.

<sup>17</sup>The importance of fiscal policy in the model does, of course, not imply that there is no need to streamline the bewildering array of investment promotion schemes since a significant part of the early tax-driven investment in the East went, in effect, down a black hole.

## Appendix

### A Brief Introduction to Backward Integration

All solution trajectories converge along the stable manifold towards the steady state  $(g_W^*, \chi^*, 1)$ . A reversion of time,  $t \mapsto -t$ , i.e. a multiplication of the right hand side of the dynamic system with  $(-1)$  reverses the stability characteristics. Visually this can be thought of as a reversion of the phase diagram. The originally inherently unstable boundary value problem is represented as a stable initial value problem which can be solved by standard methods. We use a fourth order Runge-Kutta-Fehlberg algorithm with stopping criterion  $\theta \approx \theta(0)$ .



The Stable Manifold: Basic Scenario

As starting values for backward integration, we iteratively use the values of a cycle in the  $g_W$ - $\theta$ -plane around the steady-state:

for  $i=0$  to 2 step 0.00001;  $g_W = g_W^* + \eta \sin(i\pi)$ ;  $\theta = \theta^* + \eta \cos(i\pi)$ ;  $\chi = \chi^* - \eta$ ,

where the parameter  $\eta$  specifies the initial deviation from the steady-state, which is set to  $10^{-12}$ . Since the cycle intersects every solution trajectory exactly once this procedure generates a set of possible solution trajectories on the stable manifold as shown in the Figure . From the subset of solution trajectories that end at the 'right' side of  $\theta$ , i.e. at  $\theta(0) = 0.4$  we select the one that matches most closely  $g_W(0) = g_W^*(1 + \lambda)/(1 + \theta(0)\lambda)$ . Once the trajectory is obtained time is reversed again so that the sequence of solution values provides the forward looking time path for the original system. This system starts  $\theta = 0.4$  and in a situation where the western region was close to its steady-state before unification. Forward looking the system then converges towards the steady-state, stopping at a distance of  $10^{-12}$  from the steady-state.

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