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Transfers to Families with Children as a Principal-Agent Problem

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TRANSFERS TO FAMILIES WITH CHILDREN AS A PRINCIPAL-AGENT PROBLEM

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Working Paper No. 351

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Abstract

The relationship between government and parents is modelled as a principal-agent problem, with the former in the role of principal and the latter in the role of agents. We make three major points. The first is that, if the well-being of the child depends not only on luck, but also on parental actions that the government cannot readily observe, the latter can influence parental behaviour indirectly, by conditioning transfers on performance. The second point is that, if there are market inputs into the making of a happy or successful child, which the government can observe, but cannot ascribe to any particular parent or child because they are bought anonymously, an income transfer policy can be usefully complemented by an indirect tax policy that systematically distorts prices in favour of these inputs. The third is that, if parents care about their children, insurance and incentive considerations must be tempered by the need to compensate parents who have the misfortune of getting a child with low ability or, more generally, less well equipped to make the most of life. Ways of making these findings operative are discussed in some detail.

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Transfers to families with children as a principal-agent problem*

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Abstract

The relationship between government and parents is modelled as a principal-agent problem, with the former in the role of principal and the latter in the role of agents. We make three major points. The first is that, if the well-being of the child depends not only on luck, but also on parental actions that the government cannot readily observe, the latter can influence parental behaviour indirectly, by conditioning transfers on performance. The second point is that, if there are market inputs into the making of a happy or successful child, which the government can observe, but cannot ascribe to any particular parent or child because they are bought anonymously, an income transfer policy can be usefully complemented by an indirect tax policy that systematically distorts prices in favour of these inputs. The third is that, if parents care about their children, insurance and incentive considerations must be tempered by the need to compensate parents who have the misfortune of getting a child with low ability or, more generally, less well equipped to make the most of life. Ways of making these findings operative are discussed in some detail.

1 Introduction

Assuming that the future success or happiness of a child depends, to some extent, on actions taken by his or her parents, government policies aimed at improving the welfare of children must modify the behaviour of parents. In a broad sense, the relationship between government and parents may thus be seen as a principal-agent problem, with the government in the role of principal and parents in the role of agents. If the government cannot observe the actions taken by parents, but can observe or infer the outcome of those actions, the problem has the typical structure of an agency problem in the technical sense. On the other hand, the government-parents problem does not permit a straightforward application of standard results, because the objective functions of government...

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and parents need not be antithetic. Typically, one assumes that the government maximizes some convex combination of the utilities of citizens, or at any rate that it takes into account in some way the objectives of the latter. If parents care about their children's welfare, what parents want is thus unlikely to be diametrically opposed to what the government wants, but there may be reasons for which the latter attributes higher weight to the welfare of future generations than parents do.

The structure of the problem is as follows. We assume that the welfare of children depends on a random factor, and on the level of an action taken by parents, which can be time allocated to the care of children, or income allocated to their consumption. Parents choose the level of this action, taking government policy as given, under conditions of uncertainty about the outcome. The government chooses the policy so as to incentivize parents to take the desirable level of action. We assume that there exists some observable indicator of child welfare or degree of success. We consider the possible advantage of a policy consisting of making transfers to parents conditional on children's performance, under various alternative assumptions about parental preferences, parental ability to control fertility, and timing and observability of actions and outcomes. We also consider the possible advantage of supplementing or replacing an income transfer policy with price subsidies on child-specific goods.

We find that, if parents do not care about their children, and fertility is exogenous, the optimal (second-best) policy is to offer parents a subsidy per child, increasing in child performance. That is not necessarily true if parents care about their children, because the government has to compensate parents for an unfavorable outcome. The optimal payment schedule may then be U-shaped. This is a major departure from standard principal-agent, and also from public finance results. If fertility is endogenous, it is optimal to condition transfers to families not only on performance, but also on number of children. If there is a market input into the making of a happy or successful child, that the government can observe (and, therefore, tax or subsidize), it can attribute to any particular parent or child, because it is bought anonymously, a price subsidy on this input is generally welfare-improving. Possible ways of implementing these findings in practice are discussed in the concluding section.

2 Parents as agents

Parents (agents) derive utility from their own consumption, \( C \). Possibly, they also derive utility from the number of children they have, \( n \), and from the welfare of each of their children, \( x \). We assume that \( x \) depends on some action, \( a \), taken by the parents (attention or expenditure) and on a random variable, \( \theta \) (fortune, genetic inheritance) with given density function. Following Mirrlees (1974), we treat \( x \) itself as a random variable, with density function \( f(x,a) \).

\[ f(x,a) \text{ can be derived from the production function } x = x(a, \theta) \text{ and the density function of } \theta. \text{ In section 4, we directly use } x = x(a, \theta) \text{ and the density of } \theta. \]
The standard monotone likelihood ratio (MLR) condition, \( \frac{f_a}{f} \) increasing in \( x \),\(^2\) and the convex distribution function (CDF) condition,\(^3\) are assumed to hold.

The agent’s expected utility is given by

\[
E(U) = \int U[C + z(x)n]f(x, a)dx,
\]

where \( U(\cdot) \) is the ex-post utility function, and \( z(x) \) the income equivalent of \( x \). The household budget constraint is

\[
C = m + [y(x) - w(a)]n - c(n),
\]

where \( m \) is household income, \( y \) a per-child government transfer (possibly conditional on \( x \)), \( c(n) \) the fixed cost of having \( n \) children, and \( w(a) \) the cost per child (over and above \( c \)) of the action \( a \). To ensure concavity of \( E(U) \), we assume that both \( c \) and \( w \) are increasing and convex (non-decreasing marginal costs) in their respective arguments.

If fertility is exogenous, parents choose \( a \) to maximize (1), where \( C \) is given by (2). The first-order condition,

\[
-w'(a)n \int f'(x, a)dx + \int f(x, a)dx = 0.
\]

tells us that parents equalize the expected marginal cost in terms of \( a \) to the expected marginal benefit in terms of \( x \).

If fertility is endogenous, parents choose also \( n \). There is then the additional first-order condition,

\[
\int [y(x) + z(x) - w(a) - c'(n)]U'(x, a)dx = 0,
\]

which says that parents procreate to the point where the cost of an additional child equals the expected benefit.

Notice that, if parents do not care about their offspring, the benefit of having children and expending resources on them can only come from the government transfer, \( y \). Therefore, if \( z = 0 \), parents will undertake \( a \) only if \( y \) increases with \( x \). With endogenous fertility, children will be born only if \( y \) is positive, and large enough to at least cover \( c \).

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\(^2\)Since \( a \) is a continuous variable, the likelihood ratio is increasing in \( x \) if \( \frac{f_a}{f} \) is increasing in \( a \) (Milgrom, 1981).

\(^3\)Equivalently, we may assume, instead of CDF, that the following conditions are satisfied:

i) \( \int_{-\infty}^\infty F(x, a)dx \) is nonincreasing convex in \( a \) for each value of \( y \);

ii) \( f \geq 0 \) and \( F(x, a) \) is nondecreasing concave in \( a \);

iii) \( -U''/U' \) is nondecreasing (see Jewitt, 1988).

\(^4\)If \( a \) is time, \( w(a) \) includes an opportunity cost, and \( m \) is then to be interpreted as full income.
3 The government as principal

Let us now look at matters from the point of view of society, or of the government (the principal). To make the objectives of the principal diverge from those of the agent, or agents, we could simply assume that society values children, or the well-being of each child, more than parents do. We could, in other words, treat \( n \) or \( x \) as merit goods.\(^5\) It is more instructive, however, to let the difference between the objectives of principal and agents arise from the very structure of the problem.

A possible assumption we can make is that the utility of the principal is the sum of those of the agents — in other words, that the social welfare function is Benthamite. This would imply, among other things, that society only cares about its current members (the government is only interested in voters). According to this assumption, the well-being of children is taken into account in the design of policy only insofar as it directly or indirectly affects the utility of current adults. Assuming that all agents are the same\(^6\), and measuring \( x \) as the present value of the lifetime tax contributions expected from each future adult, the principal’s aim will then be to maximize (1), subject to the government budget constraint. Assuming that the number of agents and, thus, of future taxpayers is “large”, we may write this constraint in terms of expected tax revenue,

\[
n \int [x - y(x)] f(x, a) dx = 0. \tag{5}
\]

Equivalently, we may say that the government aims at maximizing

\[
\int U f(x, a) dx + \lambda n \int [x - y(x)] f(x, a) dx, \tag{6}
\]

where \( \lambda \) is a Lagrange-multiplier, measuring the marginal social utility of tax revenue. An alternative assumption we can make is that social welfare is a weighted sum of the utilities of present and future adults. Given that each future adult will get, one way or another, what is left of \( x \) after that each of the present adults has received \( y \), we can then interpret (6) as the social welfare function (the utility of the principal), and treat \( \lambda \) as a constant, representing the given social weight of future adults.

Whatever the interpretation, comparing (6) with (1) - (2) makes it clear that, if \( y \) does not depend on \( x \), a child will always count more for society than for the representative parent. That is because the (atomistic) parent has no way and no reason to take into account the effect of her choice of \( a \) and, if fertility is endogenous, \( n \) on the government budget constraint. There is thus an externality, that the principal will attempt to cure by inducing agents to choose the socially desirable level of \((a, n)\). Assuming that \( x \) is observable at the relevant time (more about this later), this can be accomplished by making

\(^5\)That is what effectively happens in Cigno (1983).

\(^6\)Abstracting, in other words, from issues of intra-generational distribution.
y conditional on x. In so doing, however, the principal will have to take into account the agent’s response. If fertility is exogenous, the principal is then restricted by the marginal condition (3). If fertility is endogenous, it is restricted by (3) − (4), but also by the fact that the agent could choose not to participate in the transfer scheme by having no children at all.7

3.1 Exogenous fertility

If n is given, the principal may be said to be choosing the function y( ), or a y(x) for each x, so as to maximize (1), subject to the government budget constraint (5), and to the incentive-compatibility constraint (3). Alternatively, if we take (6) as the welfare function, the principal chooses y(x) to maximize (6), subject only to (3). There is no need for a participation constraint because, with the number of children given, the agent cannot escape the government scheme by deciding not to have children.8

The first-order condition is that y must satisfy, for every x,

\[ U''f - \lambda f + \mu (-nwU''f + U'f) = 0, \]  

where \( \mu \) is the Lagrange-multiplier associated with (3).

Consider, first, the case where the principal can either observe or infer the level of the action \( a \) undertaken by the agent. As the incentive-compatibility constraint is not binding \( (\mu = 0) \), (7) reduces to \( U'' = \lambda \). A first best can then be implemented by choosing the payment schedule \( y(\cdot) \) so that

\[ y(x) + z(x) = \text{const}. \]  

If parents do not care about the well-being of their children \( z(x) \equiv 0 \), (8) tells us that the optimal policy is to pay parents a fixed sum, whatever the realization of \( x \). This is the familiar full-insurance result, associated with first-best solutions in principal-agent problems in the case where the principal is risk-neutral, and the agent risk-averse. It is interesting that, in our case, the result comes about as a consequence of the government budget constraint, and not of different attitudes to risk. If the government’s maximand is interpreted as a Lagrangean, the Principal is in fact as risk-averse (has the same utility function) as the Agents, but, given the large number of future tax payers, faces no financial Revenue risk. If the utility of parents directly depends on their children’s well-being, \( z(x) \neq 0 \), (8) tells us that the principal must assure the agent not a given level of income, but a given level of utility. In other words, the government must compensate parents for the misfortune of getting a low-ability child.

Consider, next, the case where \( a \) is not observable. Since the government must then depart from the full-insurance principle to satisfy the incentive-compatibility constraint, a first best is not achievable. With \( \mu \) positive, it is

---

7(4) applies only to interior solutions.

8Agents may also be able to vote against the policy. However, the presence of a positive externality ensures that the representative agent will have higher utility with the policy than without, and that the participation constraint is thus not binding.
convenient to re-write (7) as
\[
\frac{\lambda}{U'} = 1 + \mu (nrw' + \phi),
\]
(9)
where \( r = -\frac{U''}{U'} \) is the Arrow-Pratt measure of absolute risk aversion, and \( \phi = \left( \frac{f_2}{f} \right) \). Since \( r \) is a function of \( x \) and \( y \), and \( \phi \) a function of \( x \), total differentiation of (9) yields
\[
\frac{dy}{dx} = \frac{\lambda r}{\mu U'' - nrw'} - z'.
\]
(10)

Recall that both \( \phi' \) and \( w' \) are positive. Assume, plausibly, that aversion to risk does not increase with income (\( r' \leq 0 \)). If parents do not care about their children (\( z(x) = 0 \)), (9) then tells us that the second-best payment schedule is increasing in \( x \), \( y'(x) > 0 \); parents must be rewarded for getting a high \( x \), even though that is partly the result of chance. That may not be true if parents care about children. If \( z(\cdot) \) is increasing and concave (diminishing marginal utility of \( x \)), the r.h.s. of (10) may have any sign. Since \( z' \) is positive, and decreasing in \( x \), the optimal \( y'(x) \) is then more likely to be negative if \( x \) turns out small, positive if it turns out large. Therefore, if parents care about children, the second-best payment schedule may be \( U \)-shaped: decreasing in the child’s ability for low \( x \), increasing for high \( x \). The first-best principle, that parents should be compensated against the misfortune of getting a low-ability child, thus carries over (albeit, only as partial insurance) into the second-best solution.

3.2 Endogenous fertility

Let us now consider the case where parents can choose how many children to have. Since \( n \), unlike \( u \), is observable, the government can make the payment of the transfer conditional on the agent producing the required number of children, \( n^* \). Parents then get a transfer
\[
\begin{align*}
g_1 &= y^* \quad \text{if } n = n^* \\
g_2 &= y^* \quad \text{if } n \neq n^* ,
\end{align*}
\]
(11)
where \( g_2 \) is small enough (zero or negative) to deter anyone from deviating from the optimal (first or second best, as the case may be) value of \( n \). In theory of contracts language, that is called a "forcing contract". In the existing literature, however, the agent’s actions are either observable, or not observable. We extend the traditional framework by admitting the possibility of a non-observable action, to deal with which the government has to use the customary (distortionary) incentive mechanisms, and one that is observable, which the government can fix using a forcing contract. That way, the government is able
to separate the incentive problem concerning the observable action, $n$, from that concerning the non-observable action, $a$.\footnote{\text{Notice that, if there literally were only one agent, the revenue from the penalty for producing an $n$ different from $n^*$ could not enter the budget constraint as a resource for the government, otherwise the agent would get back what she paid, and the incentive to choose $n^*$ would then be lost. The problem does not arise if the agents are "many", as we are assuming, because the amount that the disobedient atomistic parent would get back is negligible. At an optimum, of course, everyone has $n^*$ children, and nobody pays any penalty.}}

The optimal policy is found maximizing (1) with respect to $(y(x), a, n)$, subject to (5) $- (3)$, or maximizing (6) subject to (3), without having to worry about (4).\footnote{\text{For the externality argument already discussed, participation is guaranteed by the fact that the agent will have higher utility with than without the policy.}} The first-order conditions determine, together with the optimal level of the action and the optimal payment to parents schedule, also the optimal number of children to be the object of the "forcing contract". The policy implications are qualitatively the same as in the exogenous-fertility case. If $a$ is observable, a first best is implemented assuring parents a given level of utility. The payment must again be constant if parents do not derive direct utility from their children's well-being, conditional on $x$ if they do. If $a$ is not observable, the second-best payment schedule is increasing in $x$ if parents do not derive direct utility from their children's well-being, possibly $U$-shaped if they do. The only difference is that $n$ is now fixed at its optimal (first or second-best) level, rather than at an arbitrary level as in the exogenous fertility case.

### 3.3 Intergenerational distribution

We remarked earlier that the qualitative properties of the (first or second-best) optimal payment schedule do not depend on whether we interpret (6) as a Lagrangean, or as a social welfare function with $\lambda$ as the fixed social weight of the future generation. The distributional implications, however, are quite different. According to the first interpretation, the government is maximizing the utility of the present generation. Since (5) will be satisfied as an equation, the entire surplus will then go to the parent generation. By contrast, if the second interpretation is followed, and $0 < \lambda \leq 1$, some or all the surplus will go to the child generation: the government has a positive budget imbalance in the current period, that it will be able to use for the benefit of future adults in the next period.

The two different interpretations of (6) may also have different allocative implications. If there are wealth effects in the utility function of the parents, the socially desirable level of $a$ is in fact not independent of how the surplus is divided between generations. In Section 5, we shall use a utility function that does not give rise to such effects.
4 Price subsidies

We now examine the possible advantages of adding another policy instrument to the government’s armoury: an indirect tax. We want to find out whether, in the case where \(a\) is not observable, the government can get closer to a first best by distorting prices. Interpret \(a\) as the quantity of a certain commodity that the agent buys anonymously on the child’s behalf. In a home production perspective, the cost of the action, \(w(a)\), need not be proportional to \(a\), because it typically includes, in addition to the expenditure for the commodity in question, the imputed cost of items that are in fixed supply, the use of which can only be increased at rising marginal cost. For example, if \(a\) is child books, the parent must spend time diverted from work or leisure to read the books to or with the young child. If \(a\) is ingredients for a meal, time and cooking facilities must be diverted from other uses to prepare the meal. If it is medicines, or services available outside the home, time and the family car must be diverted from other uses to fetch the medicines, or to deliver the child to medical surgery, school, gym, etc.

Let \(t\) denote a subsidy (a negative excise tax) on commodity \(a\). Since the tax on \(C\) is normalized to zero, a positive value of \(t\) indicates that the goods consumed by children are taxed less than those consumed by adults, a negative value that they are taxed more. The household budget constraint becomes

\[
C = m + [y(x) - w(a)] + t a n - c(n). \tag{12}
\]

Taking fertility to be either exogenous, or fixed at the socially optimal level by a forcing contract, the first-order condition on the agent’s choice of \(a\) is now

\[
- [w'(a) - t] n \int U' f(x, a) dx + \int U f_a(x, a) dx = 0. \tag{13}
\]

Let \(k\) denote the cost of administering the subsidy on \(a\) (or, more generally, of using indirect taxation). As we did not attach a cost to the delivery of \(y\), a positive value of \(k\) then means that indirect taxation is more expensive to administer than a direct transfer to families; a negative value that it is less expensive. The government maximizes

\[
\int U [m + [(y(x) + z(x) - w(a) + ta)n - c(n)] f(x, a) dx
\]

\[
+ \lambda n \int [x - (1 + k) ta - y(x)] f(x, a) dx, \tag{14}
\]

subject to the incentive-compatibility constraint (13).

The first-order conditions on the government’s choice of \(y(x)\) and \(t\) yield

\[\text{[11] See, for example, Cigno (1991).}\]
\[
\frac{\lambda}{U'} = 1 + \mu [n (ra' - t) + \phi]
\]  
(15)

and, using (15) to simplify,

\[
\alpha k = \frac{\mu}{\lambda} \int U' f(x, a) dx,
\]  
(16)

where \(\mu\) is again the Lagrange-multiplier associated with the incentive-compatibility constraint. The first of these conditions tells us the usual story, that the optimal \(y\) may be increasing or decreasing in \(x\). The difference is that, in the case where parents care about their children’s welfare \((z > 0)\), the optimal income transfer is more likely to be decreasing in \(x\) if \(t\) is positive \((i.e., \) child-specific commodities are subsidized, or taxed less than adult-specific commodities) than if it is negative or zero. That is because, if parents are induced by price distortions to spend more on children, there is less need to further incentivize them with a payment by results. The second condition tells us that the price distortion in favour of \(a\) must be increased to the point where the expected marginal benefit (expressed in income terms) of parental consumption is equal to a constant, proportional to the cost of administering the price subsidy, \(k\). Since \(U'\) is decreasing in income, the lower \(k\), the higher \(t\) (see Figure 1).

The reason why it may be optimal to use \(t\) alongside, or in place of, \(y\) is that, while the latter is, in general, uncertain, the former is certain. Incentives to risk-averse agents given through the price system are thus more cost-effective, from the government’s point of view, than incentives given through an uncertain transfer payment. If \(k\) is sufficiently small, or even negative \((i.e., \) the indirect tax or subsidy is less costly to administer than the transfer\), it may be optimal to use only \(t\). If \(k\) is sufficiently high, it may be optimal to use only \(y\). The role of uncertainty, and of parental attitudes towards it, in the government’s choice of policy mix will be further clarified in the next section.

5 A special case

We now consider a special case, with explicit functional forms, which is of considerable interest for a number of reasons. Indeed, while restrictive in one direction, the assumptions made in this section are much less restrictive in another. Suppose that the utility function of the representative agent is characterized by constant absolute risk aversion,

\[
U = -e^{-r' C(x) n},
\]  
(17)

where \(r\) is now a positive constant. Let

\[
x = a + \theta,
\]  
(18)
Assume that $\theta$ is normally distributed with zero mean, and variance equal to $\sigma^2$.

Holmström and Milgrom (1987) show that this can be considered as the reduced form of a dynamic model, where the agent chooses $a$ sequentially over a finite time interval. This is particularly desirable for our purposes because, in the real world, parental actions (expenditure, or attention expended on children) are sequential. Indeed, children also are generally born sequentially. Our assumption, that $\alpha$ and, where applicable, $n$ are chosen once and for all, thus makes sense only insofar as it captures the essence of what is actually a dynamic process. Holmström and Milgrom also show that, in this case, the optimal payment schedule is linear in $x$,

$$y(x) = \alpha + \beta x,$$

where $\alpha$ and $\beta$ are constants to be determined.

Assume that the income-equivalent of the utility from $x$ is also linear, $z(x) = \gamma x$, $0 < \gamma < 1$. With an exponential utility function, and a normally distributed random effect, maximizing expected utility is the same as maximizing its certainty equivalent. Treating fertility as endogenous (the exogenous fertility case is easily dealt with as a sub-case), and using (12), we can then say that parents choose $(a, n)$ to maximize the certainty-equivalentootnote{In the full dynamic version of the model, $a$ represents the drift rate of a stationary stochastic process. The agent observes $x$ as it accumulates, and adjusts her choice of $a$ over time. The principal, on the other hand, can only observe the final outcome. As a consequence, the compensation can only depend on the final level of $x$.} of (17),

$$m + [\alpha + (\beta + \gamma) a] n - w(a)n - c(n) + an - \frac{r}{2}(\beta + \gamma)^2 \sigma^2 n^2.$$  \hspace{1cm} (20)

The first-order conditionsootnote{That, incidentally, is true of most decision models, including the standard consumer choice model.} on the agent’s choice of $a$ and $n$ are

$$\beta + \gamma = w'(a) - t,$$  \hspace{1cm} (21)

$$\alpha + (\beta + \gamma) a - w(a) - r(\beta + \gamma)^2 \sigma^2 n = c'(n) - ta.$$  \hspace{1cm} (22)

### 5.1 Optimal transfers

Suppose that indirect taxation is not available ($t = 0$). Given (19), the government budget constraint can be written as

$$E \left[ -e^{-rX} \right] = -e^{-E(X) - \frac{r^2}{2} \text{Var}(x)}.$$  

Assume $x = a + \theta$, with $E(\theta) = 0$. Given $X = C$, $E(C) = m + (\alpha + \beta a)n - w(a)n - c(n)$, and $\text{Var}(C) = \sigma^2 a^2 n^2$.

The first order conditions are both necessary and sufficient since this special case guarantees concavity in $a$ of both the agent’s and the government's problem.
\( \alpha = (1 - \beta) a. \) \hspace{1cm} (23)

Suppose that the government is only interested in the expected utility of current adults. Since, in this case, the entire surplus generated by public intervention goes to the agent (see sub-section 3.3), and \( n \) does not enter the incentive-compatibility constraint (21), the agent's first-order condition with respect to this variable is identical to the principal's. As a consequence, there is no need to "force" parents to choose the optimal \( n \) using \( (11)^{16} \). The principal's problem is then choosing \((\alpha, \beta)\) to maximize (20), subject to (23) and (21).

Substituting (21) and (23) into (20), and differentiating with respect to \( a \) and \( n \), we get the first-order conditions characterizing a socially optimal choice of these variables,

\[
(1 + \gamma) n - w'(a)n - r w'' \sigma^2 n^2 = 0, \hspace{1cm} (24)
\]
\[
a - w(a) - r w'(a) \sigma^2 n - c'(n) = 0. \hspace{1cm} (25)
\]

Simplifying with the help of (21) - (22), we can solve (24) - (25) for the second-best values of \( \alpha \) and \( \beta \),

\[
\alpha^* = (1 - \beta^*) a^*, \hspace{1cm} (26)
\]
\[
\beta^* = \frac{1 - \gamma rw'' \sigma^2 n^*}{1 + rw'' \sigma^2 n^*}. \hspace{1cm} (27)
\]

The government must thus use the constant part of the transfer to induce parents to have the socially desirable number of children, and the variable part to induce them to spend the socially desirable amount of time or income on each of their children. Notice that, in the case where parents care for their children \((\gamma > 0)\), if \( \sigma^2 \) or \( r \) is high, the optimal \( \beta \) could be negative, and the optimal \( \alpha \) consequently larger than it would otherwise be. In other words, if uncertainty is high, or parents are very risk-averse, it may be optimal (even in second best) to give the latter a fixed payment, calculated on the basis of the child's expected degree of success, \( a \), rather than on the actual outcome, \( x \).

## 5.2 Optimal transfers and indirect taxation

Let us now introduce indirect taxation at the rate \( t \). The government budget constraint becomes

\[
\alpha = [1 - \beta - (1 + k) t] a. \hspace{1cm} (28)
\]

\(^{16}\)If we make the alternative assumption that the government aims to maximize a weighted sum of the benefit to current adults, and the benefit to future adults, \( (11) \) is needed to get the agent to choose the desired \( n \).
Substituting (21)–(28) in (20), and differentiating with respect to \(a\), \(t\) and \(n\), we get the first-order conditions on the optimal choice of these variables,

\[
(1 + \gamma) - w' - tk + (t - w')r w'' \sigma^2 n = 0, \tag{29}
\]

\[
(t - w')r \sigma^2 n + ka = 0, \tag{30}
\]

\[
(1 + \gamma) a - w(a) - t k a - c' + (t - w')r \sigma^2 n = 0. \tag{31}
\]

Using (21), we can solve (29)–(31) for the second-best values of \(a\), \(\beta\) and \(t\),

\[
\alpha^* = 1 + (1 + k \gamma^* a^*, \tag{32}
\]

\[
\beta^* = \frac{ka}{r \sigma^2 a^*} - \gamma, \tag{33}
\]

\[
t^* = \frac{1 + \gamma - ka \left( \frac{1}{\sigma^2 a^*} + w'' \right)}{1 + k} = \frac{1 - \beta^* - ka w''}{1 + k}. \tag{34}
\]

Of course, these formulae are valid only if it is optimal to use both \(t\) and \(y\) as instruments of policy (in other words, if the cost of administering the tax is not so high, relative to that of administering the transfer, that it is not worth using the tax at all, or so low that it is not worth using the transfer at all). Provided that the administrative cost of the tax is not "too high", the optimal \(t\) is positive: child-specific commodities should be subsidized. The greater the uncertainty faced by parents, however, and the more averse to risk they are, the higher the price subsidy, and the less dependent on actual performance should the income transfer be. The reason, as already pointed out, is that the price subsidy is certain, while transfers contingent on the child's success are uncertain. Other properties are quite obvious. The higher the cost of administering \(t\), the lower the price subsidy, and the higher the income transfer. The greater the utility that parents derive from their children (the higher \(\gamma\)), the smaller the contingent part of the transfer, and the higher the price subsidy.

6 Discussion

We have seen how transfers to parents, and price subsidies on child-specific commodities, should be optimally designed if the social optimum differs from
the laissez-faire solution. We have done this on the assumption that the
divergence arises from a pecuniary externality, but little of substance changes if
the externality takes a different form, or if children are treated as merit goods.
The essential points are three. The first is that, if the well-being of the child
depends not only on fortune, but also on parental actions that the government
cannot readily observe, the latter can influence parental behaviour indirectly, by
conditioning transfers on actual results. Policies towards the family may thus
be seen in a principal-agent perspective. The second point is that, if there are
market inputs into the making of a happy or successful child, which the govern-
ment can observe but not attribute to any particular parent or child, because
they are bought anonymously, the government income transfers can be usefully
supplemented (in extreme cases even replaced entirely) by an indirect tax pol-
icy that systematically distorts prices in favour of these inputs. That is because
incentives to risk-averse individuals are more cost-effective if they are certain,
than if they are conditional on uncertain events. The third point is that, if
parents care about their children, insurance and incentive considerations must
be tempered by the need to compensate parents who have the misfortune of
getting a child with low ability or, more generally, less well equipped to make
the most of life.

Remains the question of how the theoretical results on performance-related
transfers can be made operational. One of the central issues, in this respect,
is the observability and time of observation of the variable, \( x \) in our model,
that represents the well-being of the child. One possibility is to use indicators
of ability or success such as IQ or school performance, both of which reflect –
as hypothesized in the theory – the effects of both nature and nurture. Since
these indicators are observable fairly early in the life of the child, they have
the advantage that the reward to the parent can come soon after the action,
\( a \) in our model, that contributed to the outcome. An obvious form of transfer
conditional on these early measures of success are scholarships (in the case of a
minor, a scholarship constitutes a transfer not to the child, but to the parents,
who would otherwise have to bear the full cost of the child's maintenance and
education). Providing educational or other services beneficial to the child, and
maybe making fruition compulsory, could be effective too, but that is irrelevant
to the present discussion. Our concern, here, is with parental actions that are
not easily observable by the public authority (whether the child finds someone
at home when he or she comes back from school, whether proper supervision
is exercised over the child's use of time outside school, etc.) and, therefore,
difficult to enforce. School attendance, by contrast, is easily observable and
directly enforceable.

The disadvantage of measuring success early in the child's life-cycle is that
the full consequences of parental action may not be revealed until much later in
life. In the theoretical analysis, we found it analytically convenient to identify
the measure of success, \( x \), with the expected present value of the child's future
tax contributions. The full extent of these contributions cannot be properly
assessed until the subject is in middle life, and the parent who contributed to
the outcome on the point of retirement. A natural way to reward the parent
could then be through his or her pension, making it in other words possible to qualify for a pension by raising future taxpayers, as an alternative or in addition to paying social security contributions. That would have the advantage of alleviating one of the main problems of pay-as-you-go pension systems, namely that they tend to erode their own tax bases, because their very existence removes the incentive for working-age people to have children (Cigno and Rosati, 1996) and invest in their children’s human capital. Making it possible for a person to contribute to his or her own pension by raising future taxpayers would restore the incentive. Some element of that already exists in some pension systems.  

Finally, there is the problem of compensating parents who are happy if their children do well, unhappy if they do not. That is obviously difficult to calibrate without an estimate of the extent to which the utility of parents is affected by that of children. It is clear, however, that this consideration points towards a reduction in the conditional part of the transfer, and an increase in the price subsidy (in the special case of Section 5, we found that, the higher γ, the lower the optimal β, and the higher the optimal τ).

In conclusion, selective policies now gone out of fashion, and policies still very much in fashion but subject to critical scrutiny, could be re-vitalized and re-shaped in a principal-agent perspective. Scholarships, once used to allow bright poor children to attend fee-paying schools, came to be thought irrelevant, even divisive, with the advent of free public education, and of re-distribution in favour of families with children. They could become useful again, alongside free public (or subsidized private) education, as a means of inducing parents to invest in their children in ways that the government cannot readily observe, and to have more children. Social security, born as a form of compulsory saving, and currently in need of reform for reasons of sheer financial sustainability, could be re-designed in a way that would stop it eroding its own tax base, and make it an inducement to fertility and human capital investment.

7 References

——— and F. C. Rosati, 1996, Jointly Determined Saving and Fertility Be-

17For a proposal in this sense, see Werding (1997).

18Since 1996, for example, the German Rentenversicherung credits a person, that has withdrawn from the labour force in order to raise a child, with a notional pension contribution related to his or her former labour income. That, however, is a way of relating γ to α, not x. Furthermore, in more recent years, the notional contribution has ceased to be contingent on the parent actually withdrawing from the labour market, thus becoming, effectively, a fixed subsidy like our α. In the spirit of our result, by contrast, the parent’s pension entitlement should be related to the child’s actual tax payments. In addition to restoring incentives to have children and invest in their education, this would also have the desirable effect of making it in the interest of parents to discourage their children from tax evasion!


