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EARMARKED TAXATION: WELFARE VERSUS POLITICAL SUPPORT

Dieter Bös

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Abstract

Compared with the traditional public-finance approach of a monolithic fully informed planner, earmarking of taxation is less likely to be optimal if a principal-agent setting is considered, where taxing and spending are performed by two separate agents which are monitored by the parliament. We assume that the parliament either maximizes welfare or expected votes. Vote maximizers are more inclined to choose earmarking, but at the price of inefficiently high costs.

Keywords: Earmarking, efficient taxation, asymmetric information

JEL-Classification: H21, D82

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1 Introduction

Parliamentary decisions on general taxes, like the income tax or the value-added tax, are independent of parliamentary decisions on public expenditures and the parliament is free to decide how the total sum of general-tax money is split to be spent for various public tasks. This principle has been introduced because it maximizes the degrees of freedom given to policymakers. In particular, it makes it easier to pursue distributional and stabilizational targets. Recently, however, it has often been claimed that this principle is one of the reasons for the increasing disaffection from the state. If particular taxes were levied to cover particular public expenditures, the population would better understand the rationale of taxation, and the public expenditures would be chosen in closer accordance with the individual citizens’ preferences. The recent trend, which has made efficiency the primary goal of fiscal policy, makes policy proposals more attractive, which implement combined tax-expenditure packages.\(^1\) This paper shows when and why such an earmarking package may be chosen as optimal policy by politicians of various type: vote maximizers or welfare maximizers, fully informed monolithic planners or decentralized asymmetrically informed government hierarchies.

In contrast to general taxation, earmarking provides a direct link between revenues of a particular tax and expenditures for a particular public task.\(^2\) This direct link refers to the aggregate revenues of the particular tax and the total expenditures for the public task. It does not refer to the individual tax burden and the individual satisfaction from the public expenditures. By way of an example, the individual consumer knows that the revenues from the gasoline tax are used to finance highways. However, he cannot choose the amount of highways he personally prefers: earmarking does not stipulate an individual benefit principle. In other words, earmarked taxation is not an application of a user-fee principle, where the individual consumer pays a price for some publicly provided good as he does for privately provided goods. Whereas the application of a user-fee principle increases welfare by allowing the individual consumers to choose their preferred quantity of publicly provided goods, this is not necessarily the case with earmarked taxation. A median voter – but only she – is the only person for whom earmarking and user-fee principle are identical.\(^3\)

\(^1\)For a discussion of recent political and economic arguments on earmarking see Wilkinson (1994).
\(^2\)Note that new taxes are often introduced to finance particular public expenditures without explicit earmarking. A present example is the German solidarity surcharge on the income tax which has been justified by the increased financial pressures from the German unification. A well-known historical example is the German champagne tax which was introduced in 1902 because the Reich wanted to expand its fleet. These general taxes often have been mistaken for earmarked taxes.
\(^3\)In several papers in the volume edited by Wagner (1991) user charges and earmarking are explicitly
The modern theory of earmarked taxation starts with Buchanan’s classic 1963 paper. He presents a model with two public goods and asks whether it is preferable to finance both goods by general-tax funds or by separate earmarked taxes. General-tax funding in his model implies two decisions. First, a budgetary authority decides on the relative shares of the public goods. Then a median voter decides on the budget size, taking these shares as given. Therefore, the median voter is constrained in his decision which is not the case if earmarking ‘unbundles’ the public goods and allows the median voter to choose freely which quantities of the public goods should be provided at which tax prices. Hence, earmarking leads to a more efficient allocation than general-tax funding. This is just the opposite of the traditional public-finance literature which assumes a monolithic planner who decides on both taxation and public goods. In this literature the planner is considered unconstrained in the case of general taxation, whereas earmarking consists in adding further budget constraints and therefore earmarking is undesirable.

The decisive innovation of Buchanan’s essay is the treatment of political processes as multi-agent approaches, in contrast to the monolithic planner of the traditional public-finance literature: in his model, general-tax funding implies that different agents decide on the budget mix and on the budget size. However, it is necessary to challenge Buchanan’s assumption that in the case of earmarking any single decision is made by one and only one agent, namely a median voter. This is not even appropriate for the type of median-voter decisions Buchanan had in mind: there must be an agency which organizes the voting process and, in contrast to Buchanan, this agency will have a personal interest in influencing the decision. Hence, there are two agents: the median voter and the agenda setter. In a country like Germany, where there are no direct-democracy elements in the federal constitution, earmarking also is characterized by an institutional design with several agents: one agent levies the earmarked tax and another spends the money. And in a coalition government it is quite possible that the finance minister belongs to the social-democratic party whereas the spending minister is a member of the green party: this seems to guarantee that each minister sets up his own realm and will not easily reveal private information to the other minister.

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4Several follower papers extended Buchanan’s theoretical analysis, in particular Goetz (1968), Browning (1975) and Athanassakos (1990).

5The agenda-setter approach started in the late seventies and early eighties with several papers by Romer and Rosenthal. For a brief overview see Mueller (1989), pp. 259-61, 273.

6Note that the separation between a taxing and a spending authority in Germany even holds for the old-age pension system, where the health-insurance funds serve as taxer and forward the money to the pension funds. See §§28h, k Sozialgesetzbuch (SGB).
If we want to find optimal decisions which result from various political actors’ combined activities, it is reasonable to ask for the correct incentives which must be given to these actors. Such incentive problems in the past twenty years have been modeled under the label of the principal-agent approach which confronts a fully informed agent and a principal who is only incompletely informed about an exogenous parameter and/or some personal activity of the agent [see, in particular, Laffont and Tirole (1993)]. The present paper applies this line of thought to earmarked taxation. The basic decisions on taxation and public expenditures are made by the parliament as the principal. They are executed by two agents, namely a taxing minister and a spending minister. To avoid clumsy terminology we introduce abbreviated terms and refer to the parliament as the ‘planner,’ to the finance minister as the ‘taxer,’ and to the spending minister as the ‘spender.’ We shall assume that the parliament’s decision is driven by the incumbent party’s intention to maximize expected votes; a welfare-maximizing incumbent is considered as a benchmark. Vote maximization is modeled according to Coughlin et al. (1990); the recent lobbying approach by Dixit et al. (1997) could have been used to attain similar results. The two ministers will be modeled as self-interested individuals who use their private information to get more income, to exert less effort, and to gain more bureaucratic power.

The distinction between a finance and a spending authority can also be found in various other papers, although none of these papers is devoted to earmarked taxation under asymmetric information. Gordon and Wilson (1997) deal with such a distinction in a full-information approach, where the sum of revenues of many indirect taxes is used to finance many public goods. This is not an earmarking budget constraint unless the number of taxes and public goods are reduced to one, which is not the main way to read the paper. Persson et al. (1997) present full-information models on general elections where ex post the voters evaluate the policies performed in the past by a tax-agenda setter, an expenditure-agenda setter and the Congress that votes on the decisions of the two agenda setters. In Tirole (1994, section 7) the ministers are two principals who monitor one agent, namely a public enterprise, in contrast to our paper, where the ministers are two agents which are monitored by one principal. Finally, Laffont and Martimort (1998) could be understood as if their specialized agencies were a taxer and a spender. In their model policy makers are captured by various interest groups which implies that ‘splitting powers among non-benevolent agencies reduces their discretion, increases the transaction costs of capture and improves social welfare.’ (p. 674)

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For regulation under asymmetric information see also Bös (1994).
The above papers show that separation of powers among governmental agents is an important research agenda. In practice, such a separation can very often be found in the case of earmarking. Therefore, it is a straightforward step to investigate a planner-taxer-spender approach of earmarking. This leaves for another paper those cases of earmarking where a single agent is given a tax base and a spending competence and has to satisfy a budget constraint. By way of example, one could think of a health minister who is entitled to raise a tax on tobacco and spend the money on cancer research. It might come as a surprise that the single-agent approach is much more complicated than the two-agent approach of the present paper if we realistically assume that there is separate private information with respect to taxes and with respect to public expenditures, for instance, if the parliament lacks precise knowledge of the tax morale and of the costs of providing the public good for which the tax revenues are designated. If two different agents are responsible for taxation and for public spending, then at different points of time these agents can be induced to reveal the truth about their private information. The one-dimensional revelation principle is applied twice. However, if at a single point of time a single agent must be induced to reveal his two-dimensional private information, this leads to a much more complicated model. Recall that the usual literature from Mirrlees’ income-tax model to Laffont-Tirole’s procurement and regulation models only deals with one private-information parameter. In fact, if there is more than one continuous private-information parameter, there is (yet) no fully satisfactory solution approach.\(^8\)

It is the aim of this paper to show whether and why earmarking can result endogenously in a planner-taxer-spender approach. For this purpose we set up a control-theoretic principal-agent model to explain to what extent the planner’s objectives, his lack of information, and the explicit separation of taxer and spender make earmarking likely to occur or not.\(^9\) The paper is organized as follows: in section 2 we define earmarking, present the institutional set-up, and display the time schedule of the multi-stage game. The sequencing of decisions on taxation and on public expenditures is driven by problems of asymmetric information. In section 3, it is shown how the taxer and the spender can

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\(^8\)Armstrong (1996) imposes strong assumptions on consumers’ utilities and on the distribution of the various private-information parameters. Rochet and Choné (1998) characterize the optimal solution of a multidimensional screening problem by adapting the notion of sweeping operator used in potential theory; however, their characterization result is not constructive: it does not tell how to find the optimal solution. – As usual in such a situation, several scientists have started to calculate numerical examples or to use simple discrete formulations that allow full solutions but retain basic multidimensional elements. See, for instance, Armstrong and Rochet (1999).

\(^9\)Since this is a theoretical paper, it has no relation to the empirical branch of the public-choice literature on earmarking which deals with the influence of earmarking on the size of the public sector [e.g. Dye and McGuire (1992)], or on the interplay between earmarked taxation, rent seeking and lobbying [e.g. Kimenyi et al. (1991), Wyrick and Arnold (1989)].
be enticed into truthfully revealing their private information. Finally, in section 4, we compare a welfare-maximizing planner and a vote-maximizing planner and look at the desirability of earmarking in these two cases. We shall show that in both cases earmarking is less likely to be optimal in an asymmetric-information principal-agent approach than in the traditional public-finance approach where a monolithic planner proceeds under full information. Vote maximizers are more inclined to choose earmarking, but at the price of inefficiently high costs. – Since we intentionally avoid being too technical in this paper, there exist several appendices which will be sent to the reader on request.

2 A theory of earmarked taxation

2.1 What is earmarking?

In its strictest sense, earmarking requires that the revenues of a particular tax are devoted to the provision of a public good and that the public good is only financed from this tax: revenues \( R = \text{costs} \ C \). A closer examination of institutional arrangements, however, shows that in practice the term ‘earmarking’ often is used although the revenues of the special tax full below the expenditures for the public good and have to be supplemented by general-tax funds.\(^{10}\) Some examples are as follows:

– in Germany, DM 3 bill. of the gasoline-tax revenues are earmarked for local transportation purposes, which does by no means cover all expenditures for that purpose;

– if ‘social security’ is taken as one general public good, in practically all European countries the ‘earmarked’ payroll taxes do not cover the expenditures for social security;\(^{11}\)

– for environmental purposes, the Austrian budget for 1998 reports earmarked revenues of AS 118 mill., but expenditures of AS 618 mill.

On the other hand, it violates the spirit of earmarking if the special-tax revenues are used to finance goods other than the respective public good. This becomes particularly clear in the Swiss law which explicitly states that any surplus from gasoline taxation which is not used for transportation purposes, is to be credited to a special account which can be used for excess expenditures on transportation in the following years. Another fitting example

\(^{10}\)As far as I see, in the literature there is only one short note which explicitly deals with this budgetary substitution between earmarked and general taxes, namely Oakland (1985).

\(^{11}\)A table is presented in appendix 1, sent to the reader on request. Wide definitions of public goods are appropriate to consider the problem of earmarking: if the German contributions to the public pension insurance are also used to cover early retirement benefits or pensions of former GDR employees, these remain to be pension expenditures, although the individual pensions may not be actuarially calculated. It is too demanding to include in the definition of earmarking the requirement of actuarially calculated pensions, as is sometimes argued in Germany.
is the German ‘waterpenny,’ a state tax which originally was earmarked for compensation payments to those farmers who had been damaged by environmental restrictions. These compensations were to be financed by taxes imposed on water users. When it became clear that the revenues of this tax exceeded the expenditures it had been designated for, the earmarking clause was given up.\(^\text{12}\)

The logic behind all of the above examples can be captured by the following definition:

**DEFINITION:** earmarking is given if the revenues from a particular tax are devoted to the provision of a public good and \( R \leq C \).

This definition is crucial for the topic of the present paper: can earmarking result endogenously in a planner-taxer-spender approach? To handle this question we impose on the planner the following artificial budget constraint:

\[
R \geq C. \tag{1}
\]

If this constraint is slack at the optimum, the special tax is not earmarked since it subsidizes other goods. If the constraint is binding, we have a case of earmarked taxation. Note that in the latter case the planner would do better to give up the constraint (1) since most likely it would be preferable to have the expenditures for the public good subsidized by general taxation. This is why I call the constraint ‘artificial.’ Note, however, that in the unconstrained optimum the tax would still be earmarked although it would not cover all of the costs of the public good for which it has been designated. Hence, a binding constraint (1) is really a proof of the desirability of earmarking.

Having now examined the particulars of earmarking, it remains to describe the interplay of the various taxes in our model. We consider two taxes: an *excise tax*, which may or may not be earmarked, and a *general tax* which is the aggregate of all other taxes of the economy. The general tax must be introduced in our model because it supplements the excise-tax funds if they are too low to cover the costs of the public-good provision and because the incomes of taxer and spender are financed by general-tax funds. This is usual in practice unless earmarking consists in the establishment of a public enterprise which administers the provision of some public good.

An explicit modeling of the general tax is outside the scope of this paper. We instead choose a partial approach, where the excise tax is explicitly modeled, whereas the general tax is reflected by its shadow costs \( \lambda \) which are exogenous in our analysis. In a general

\(^{12}\)For details on Baden-Württemberg see Karl-Bräuer-Institut (1990), pp. 91-106.
model, where all taxes are expanded to their optimal level, the shadow costs of every single tax will be identical, as is well-known from Mayshar (1990) or Ahmad and Stern (1984). In our model, this does not necessarily hold. If the excise tax has relatively low distortionary effects which, however, increase with increasing revenue, then the planner will expand the excise-tax revenues over the amount which is needed to finance the public good and at this optimum the shadow costs of the excise tax will just be equated to the shadow costs of the general tax. However, if the excise tax causes relatively high distortions, then the planner will keep the revenues from this tax as low as possible, that is \( R = C \), and this implies that the artificial budget constraint drives a wedge between the shadow costs of the excise tax and of the general tax. The details are much more complicated because of the many effects the excise tax brings about in our model. Consider an increase in the excise tax. It reduces the consumer demand for the private good. It influences the artificial budget constraint and, therefore, it also affects the public-good decision. It changes the taxer’s effort because he likes higher tax revenues. Finally, it influences the incentive-compatibility constraint which must hold to entice the taxer into true revelation of his private information.

2.2 The sequence of events

Taxation comes prior to spending: you must raise the money before you can spend it.\(^{13}\) This is crucial in a theoretical setting where the planner faces two principal-agent problems with asymmetric information. He has to solve the taxer’s problem before proceeding to the spender’s approach. We model this sequential setting by the following nine-stage game:

- at date 1, both taxer and spender have private information; only the taxer knows the actual value of a tax-morale parameter which influences the costs of tax collection; only the spender knows the actual value of a public-good parameter which is relevant for the costs of providing the public good; the density functions of these parameters are common knowledge;
- at date 2, the planner-tancer principal-agent problem is solved; the planner stipulates the taxer’s incentive income (direct mechanism);
- at date 3, the taxer announces the actual value of the tax-morale parameter;
- at date 4, the taxer chooses his effort; the planner informs the consumers about the tax rate and the true value of the tax-morale parameter;

\(^{13}\)This excludes interim financing by public debt. The same assumption is made in Gordon and Wilson (1997) and in Persson et al. (1997).
• at date 5, the consumers articulate their demand; the taxer raises the excise tax; he is paid his incentive income;
• at date 6, the planner-spender principal-agent problem is solved; the planner stipulates the spender’s incentive income (direct mechanism);
• at date 7, the spender announces the actual value of the public-good parameter;
• at date 8, the spender chooses his effort;
• at date 9, the spender gets the financial means which are necessary to cover his costs; he provides the public good; he is paid his incentive income.

Formally, the public-good parameter is denoted \( \theta \in [\bar{\theta}, \overline{\theta}] \). It is distributed with density \( f(\theta) \) and cumulative distribution \( F(\theta) \). The hazard rate \( f(\theta)/F(\theta) \) is (weakly) decreasing in \( \theta \). Analogous assumptions hold for the tax-morale parameter \( \beta \) which is distributed with density \( z(\beta) \), independent of the distribution of \( \theta \).

3 Spender, consumer, taxer

3.1 The spender’s decision

The spender has private information about the cost-increasing public-good parameter and about his own cost-reducing effort. Hence, we have both hidden information and hidden action: we deal with a problem which combines moral hazard and adverse selection. The cost function, therefore, is as follows:

\[
C(\theta, e, G); \quad C_\theta > 0, \quad C_e < 0, \quad C_G > 0,
\]

where subscripts denote partial derivatives. \( \theta \) is the public-good parameter, \( e \) is the spender’s effort, and \( G \) is the amount of the public good.

For his disutility from effort the spender is compensated by a monetary income. In contrast to traditional analyses, the spender also gains direct utility from the outcome of his activities, that is, from the public good. Behind this specification is an argumentation à la Niskanen (1971). In the spender’s opinion his influence is increasing in the quantity of the public good he produces or procures. This seems to be a realistic description of many spending authorities. As an alternative we could have modeled the spender as a pure Niskanen bureaucrat whose satisfaction depends on his expenditures, that is, on the costs of providing the public good. However, in our opinion, direct satisfaction from high costs is not too plausible a description of public spending authorities. Generals seem to be less interested in the nominal costs, than in the real extent, of national defense, as
measured, for instance, by the number of combat aircrafts and tanks. Hence, we chose to let the spender’s utility directly depend on the quantity of the public good and not on the costs of providing this good.\footnote{An extension at the end of section 4 deals with the case of an expenditure-oriented spender. This explicit inclusion of the pure Niskanen spender should satisfy those readers who prefer a symmetrical treatment of spender and taker so that both get direct satisfaction from their ‘budgets,’ that is, expenditures for the spender and tax revenues for the taker.}

The following utility function captures the essential features of each of the arguments described above:\footnote{The linearity of the utility function is a characteristic of the Laffont-Tirole approach. If a general utility function $U(I, e, G)$ is chosen, the problem at stake can be modeled analogously to the regulation problem treated in Börs (1994), pp. 323-4. However, the resulting marginal conditions escape any clear-cut economic interpretation; only numerical simulations could help.}

\begin{equation}
U(I, e, G) = I - \psi(e) + wG,
\end{equation}

where $I$ is the monetary income and $\psi(e)$ is a convex disutility function. The marginal utility which is derived from the public good is lower than the marginal utility of income, $w \in (0, 1)$. The spender is only willing to work for the planner if his utility does not fall below his reservation utility which we normalize to zero (participation constraint).

Since the spender is compensated by a monetary income and by direct enjoyment of the public good, in principle the monetary income could be negative if the spender is a real public-good fan with low disutility of effort. The only limit for this negative income would be the spender’s wealth constraint. Back in the European middle ages many noblemen, serving in the king’s army, did that at a negative income. Modern government employees will never do that. Their income must never be negative and this constraint has to be considered by the planner.

When stipulating the spender’s income, the planner has to take into account the spender’s interests. It would be futile to instruct the spender to work at some effort level which is optimal for the planner: since effort is unobservable, the spender would lie to the planner. Typically, the spender would overstate the public-good parameter: ‘the public-good parameter was so high that I had to work very hard to attain the quantity which the planner observes; hence my disutility from effort was so high that I must receive a high income...’ The revelation principle, however, allows to solve this problem by choosing an incentive income in such a way that the spender achieves highest utility when informing the planner of the true value of the public-good parameter. For the application of this direct mechanism it is decisive which variables are observed by the planner. We follow the Laffont-Tirole tradition in assuming that ex post the planner is able to observe the
quantity of the public good and the costs spent. Hence, the spender cannot announce any arbitrary value of the public-good parameter. The planner’s observations must always be consistent with the effects of effort, and the actual and announced values of the public-good parameter.

If the spender is caught announcing a false public-good parameter, he loses his job. This not only implies the total loss of his income, but also the total loss of his utility from the public good. The latter assumption can be justified as follows: the spender’s utility accrues at date 9 when he provides the public good and simultaneously gets his income. If the spender is fired at date 9, he has no time to enjoy the personal influence which he derives from the public good. This implies that the spender likes a high quantity of a public good not because he likes the public good itself, but because this high quantity gives him an important position. He is not the type of general who would argue: even if I am fired, I have set up a great army. He rather gets satisfaction only from commanding this army himself.

The above requirements are captured by a contract with the following properties:

- if the spender announces the correct public-good parameter, he gets an income which depends on the announced value of the parameter. Whether the announcement is correct, is checked by the planner: the planner’s observations must be equal to the public-good quantity and to the costs which are derived from the planner’s optimization, given the announced value of the public-good parameter;
- no income is paid if the spender is caught lying;
- the income is defined in such a way that truth-telling is the spender’s dominant strategy (incentive compatibility, IC). In a simplified version, this condition can be written as

\[ \frac{dU(\theta)}{d\theta} = h(\theta, e(\theta), G(\theta)), \]

(4)

with \( h < 0 \) because of our assumptions on the spender’s disutility from effort and on the public-good cost function. It is plausible that the incentive pay has to be chosen in such a way that the spender’s utility is decreasing in the public-good parameter. The spender is inclined to overstate the parameter. However, since this reduces his utility, he will not do so.

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16 The detailed derivation of the IC condition is presented in appendix 2, sent to the reader on request. This appendix also contains the exact definition of \( h(\cdot) \) and its properties, in particular \( h_x < 0 \), and the assumptions needed to satisfy the second-order IC condition, that is, to avoid bunching.
3.2 The consumers’ decision

Total demand for the taxed private good depends on the consumer price, but also on the quantity of the public good which will be provided by the spender. The private and the public good may be complements, for instance if a tax on gasoline is earmarked for highways. Alternatively, the goods may be substitutes, for instance if a tax on gasoline is earmarked for urban mass transit systems (bus, rail, subway). Of course, it is also possible that there is no link between the goods, for instance, to subsidize the ‘Luftbrücke’ to Berlin, in 1948 a 2 Pfennig surcharge on every letter was introduced in the Federal Republic of Germany. The present paper gives a general explanation of why earmarking occurs, regardless of whether the goods are complementary, substitutional or neutral.\footnote{This is in strong contrast to Breman and Buchanan's (1980) Leviathan approach, where earmarking is only meaningful in the case of complementarity: Leviathan is a monolithic, fully-informed planner who wants to maximize tax revenues and, therefore, is only interested in supplying public goods if such a supply increases a tax base, which happens if the tax base and the public good are complements.}

How do the consumers perceive the feedback of public-good supply on their demand for the private good? When articulating this demand at date 5, they are not informed about the exact amount of the public good which will be provided later on. They know the actual value of the tax-morale parameter and they know everything which is common knowledge, that is, the shape of all functions and the support of the public-good parameter. However, they are not informed about the actual value of this parameter which still is private information of the spender. They can only calculate an expected value of the public good:

$$g(\beta) := \mathcal{E}_\theta G^*(\beta, \theta),$$

(5)

where $\mathcal{E}_\theta$ is the expectation with respect to the public-good parameter, and $G^*(\beta, \theta)$ is the solution of the planner’s optimization at date 6, which is perfectly anticipated by the consumers. Accordingly, consumer demand is represented by the following demand function:

$$x = x(p, g(\beta)); \quad x_p < 0,$$

(6)

where $p$ is the consumer price. We speak of complementarity between the private and the public good if $x_g > 0$, well knowing that this complementarity is only defined in expectation. Substitutability is defined analogously.

3.3 The taxer’s decision

Since the taxer is just another minister, it is straightforward to model his decision analogously to the spender’s. Raising the excise tax is a costly activity just as producing a
public good. The taxer has private information about a cost-increasing tax-morale parameter and about his own cost-reducing effort. First, the lower the tax morale, the higher the costs of collecting a particular tax revenue. The planner only knows how the possible states of tax morale are distributed, so he can calculate expected tax revenues. The taxer, operating at the ‘tax-collection front’ is precisely informed about the present state of this morale. Second, the higher the taxer’s effort, the more efficient the tax administration and, consequently, the lower the tax-collection costs. Although the parliament passes the tax law, it is the taxer who regulates the details of the tax administration by internal decrees which may imply more or less efficient administration and it is here where his effort becomes decisive. Taking account of the tax-collection costs, the tax revenues are as follows:

$$R = (p - p^0) x(p, g) - T(\beta, a, x); \quad T_\beta > 0; \ T_a < 0; \ T_x > 0,$$

(7)

where $p$ is the consumer price, $p^0$ is the constant producer price, and $T$ is the tax-collection cost function. This function depends on the tax-morale parameter $\beta$, on the taxer’s effort $a$ and on the tax base $x$.

The taxer is a pure Niskanen bureaucrat. He draws direct utility from the tax revenues and this bureaucratic utility is traded-off with his monetary income and his disutility from effort. Accordingly, we impute to the taxer the following utility function:

$$V(J, a, R) = J - \phi(a) + mR; \quad m \in (0, 1),$$

(8)

where $V$ is the monetary income and $\phi(a)$ measures the convex disutility from effort.

The asymmetric information forces the planner to stipulate an incentive-compatible income. Note that both taxer and planner are equally well informed about the public-good parameter, and that at date 5 the planner can observe the tax revenues, the consumer price and the actual quantity of the private good. Moreover, these observations enable the planner to calculate the tax-collection costs. Hence, the following contract is written:

- if the taxer announces the correct tax-morale parameter, he gets an income which depends on the announced value of the parameter. Whether the announcement is correct, is checked by the planner on the basis of his observations at date 5. Note that this incentive pay is not conditioned on the expected quantity of the public good (and cannot be conditioned on it, since an expected quantity is not observable);
- no income is paid if the taxer is caught lying;
- the income is incentive compatible. The IC condition in this case is complicated because of the interplay between the announced and the actual values of the tax-morale parameter.
The reader may consider the tax-collection function $T(\beta, a, x)$. There is a double dependency on the actual value of the tax-morale parameter; first, the direct effect, second, an indirect effect because the private-good demand depends on the expected public-good quantity which, in turn, depends on the actual value of the tax-morale parameter. Such a second effect could not be found in the spender’s problem. Unfortunately, because of this second effect it cannot be guaranteed that the taxer’s utility is always decreasing in the tax-collection parameter.\footnote{For details see appendix 3. The taxer’s participation constraint $V(\beta) \geq 0$ has to be considered explicitly as a pure state constraint in the planner-taxer principal-agent approach.}

4 The planner: welfare versus political support

4.1 The two-stage optimization

The planner solves a two-stage control problem. Applying backward induction, he first solves the spender approach which refers to date 6. At this date, the true value of the tax-morale parameter is known to the planner and every single function in the optimization is defined for this true value. Moreover, consumer demand is already known and so are the taxer’s variables. Hence, the planner faces a constant amount of tax revenues on which he has no influence. Now consider the artificial budget constraint (1), which we introduce to get insight into the planner’s interest in earmarking. For a given amount of tax revenues this constraint requires

$$\overline{R} \geq C(\theta, e, G).$$

(9)

Further constraints of the planner’s optimization are the spender’s participation and incentive-compatibility conditions. In order to pay as little information rent as possible, for the worst realization of the public-good parameter the spender’s information rent will be depressed to zero. For all better realizations positive information rents will be paid, otherwise the incentive-compatibility condition could not hold. Hence, the planner postulates the initial condition $U(\overline{\theta}) = 0$, and imposes the IC constraint. The spender’s utility at the best realization of the public-good parameter is free (terminal condition).

– Given the constraints, the planner maximizes his objective function, using effort and public-good quantity as controls, and the spender’s utility as state variable.

To avoid clumsiness in the formulation of the problem we make the following simplifications: (i) We do not explicitly introduce non-negativity constraints with respect to the control variables, but assume interior solutions in what follows. (ii) We also do not explicitly introduce the non-negativity constraints with respect to the ministers’ incomes.
This implies that the parameters $w$ and $m$ are small. (iii) Moreover, we assume that at the optimum there is always taxation and not subsidization. – Explicitly introducing all these further constraints would only inflate the individual marginal conditions without changing anything in the economic results.

Finally, the planner considers the taxer approach which refers to date 2. In doing so, he anticipates the results of the spender approach, that is, the public-good quantity and the spender’s utility and effort. However, since at date 2 the planner does not know the actual realization of the public-good parameter, the anticipation refers to expected values only, for instance, $g(\beta) := \mathcal{E}_\theta G^*(\beta, \theta)$. This anticipation implicitly takes account of the artificial budget constraint which the planner knows to hold at date 6. Therefore, this constraint is not explicitly considered in the taxer approach and, accordingly, the various marginal conditions which result from this approach cannot contain any answer to the question when and why earmarking may be optimal. Hence, we shall not explicitly present the detailed taxer approach in the text of this paper, and shall only deal with the spender approach of date 6 which explicitly takes account of the artificial budget constraint we need for our analysis of the desirability of earmarking. (Of course, to find the subgame-perfect equilibrium of the game, the taxer approach of stage 2 must explicitly be solved. This is done in Appendix 3 which is sent to the reader on request.)

4.2 The welfare-maximizing planner

Let us first consider a planner who maximizes expected welfare à la Laffont-Tirole (1993):  
\[ \Omega^W = S - px - (1 + \lambda)(I + J) + (1 + \lambda)(R - C) + U + V. \]  

The welfare function consists of the sum of consumers’ and ministers’ utilities,\textsuperscript{19} that is,  
\begin{itemize}
  \item gross consumer surplus $S(G, x)$ with $\pi := \partial S/\partial G$ and $p = \partial S/\partial x$;
  \item consumers’ financial losses from paying for the public and the private goods and for the ministers’ incomes which are paid from general taxation, whence the shadow costs of general taxation $\lambda$ enter the objective function;
  \item consumers’ financial gains from a potential surplus of excise-tax revenues over the costs of the public good, where we assume that this surplus is used to reduce general taxation;
  \item the utilities of the ministers. This implies that the planner internalizes the spender’s and the taxer’s interests in a high quantity of the public good and in high tax revenues.
\end{itemize}

\textsuperscript{19}Producer surplus in our model is zero, because we have assumed a constant producer price $p^o$. 
At date 6, the planner maximizes the expected value of his objective function, that is, $\mathcal{E}_6 Q^W$. After substituting $I = U + \psi - wG$, the planner faces the following generalized Hamiltonian

$$\mathcal{H}^W = \left\{ S(G(\theta)) - \lambda U - (1 + \lambda) \left[ \psi(e(\theta)) - wG(\theta) + C(\theta, e(\theta), G(\theta)) \right] \right\} f(\theta)$$

$$+ \tau(\theta) \left[ R - C(\theta, e(\theta), G(\theta)) \right] + \mu(\theta) \left[ h(\theta, e(\theta), G(\theta)) \right] - \text{constant},$$

where $\tau$ and $\mu$ are the Lagrangean multipliers associated with the artificial budget constraint and with the spender's IC condition. constant is an abbreviation for constant terms which result from the taxer’s and the consumers’ decisions.

This optimization approach has an inherent tendency against earmarking, and it is the asymmetric-information setting which is responsible for that. Since the planner does not know the precise value of the public-good parameter, he has to take account of the artificial budget constraint for every single realization of the public-good parameter. Since the excise-tax revenues are given at date 6, the budget constraint $R \geq C$ is most likely to be binding at the worst public-good parameter. In all other cases, costs will tend to be lower, and hence will not exhaust $R$: this is a tendency against earmarking. Note that this tendency depends decisively on the sequencing of the game, which implies a strict separation of the taxer and the spender problem. This separation makes it impossible to reduce the tax revenues if the public-good characteristic turns out to be low and hence the costs are low. Therefore, it is not only the planner’s lack of information about the public-good characteristic, but also the explicit consideration of the internal organization of the government which creates the above-mentioned tendency against earmarking.

We solve the planner’s optimization approach to get an answer to the question of whether and why earmarking can result endogenously. Hence, it is not the objective of this paper to describe how effort and the public-good quantity are determined by the respective marginal conditions. For such analyses the reader may be referred to Atkinson and Stern (1974) and to Laffont and Tirole (1993, chapter 3). Hence, here and in the following models, I shall only pick one marginal condition, namely that which is best suited to deal with earmarking. It turns out that the public-good related marginal condition never is particularly illuminating with respect to our topic. Therefore, this condition is never presented in the text.

However, the effort-related marginal condition is well suited to discuss the planner’s decision on earmarking. This condition is as follows:\(^{20}\)

\(^{20}\)To obtain this condition we need the following control-theoretic procedure: differentiating the Hamil-
\[
\psi' = -f(1 + \lambda) + \tau C_e + \frac{\lambda}{1 + \lambda} \frac{F}{f} h_e.
\] (12)

Recall that earmarking is given if the artificial budget constraint is binding. Hence, if we can prove that \( \tau > 0 \), we have a proof that earmarking is the planner’s optimal policy. If we only know that \( \tau \geq 0 \), or if we know that \( \tau = 0 \), then the constraint is either binding or not: earmarking may result, but this is not necessarily the case.\(^{21}\) Unfortunately, on the basis of equation (12) it is impossible to show when earmarking is optimal. One would be inclined to argue that a low \( C_e \) or a high \( h_e \) (both in absolute terms) require a compensating increase in \( \tau \) whence earmarking becomes more likely in such a case. However, such an interpretation would imply ceteris-paribus assumptions on endogenous variables and, therefore, cannot help to find any general statement about when \( \tau > 0 \).\(^{22}\)

Nevertheless, equation (12) clearly shows what drives the planner’s decision to earmark or not to earmark the excise tax. He compares the shadow costs of the involved taxes. If the artificial budget constraint is slack (\( \tau = 0 \)), the excise tax has been expanded to its social optimum, and the shadow costs of general and of excise taxation are equal. In this case the weight attached to \(-C_e\) is minimal, namely equal to one. Otherwise, if \( \tau > 0 \) and we have earmarking, the weight will be larger, reflecting the fact that there is more tax distortion in the economy since the shadow costs of the excise tax are larger than the shadow costs of the general taxation. The planner’s decision on earmarking, moreover, depends on the incentive income which is paid to the spender: the far-right term in (12) results from the differentiation of the spender’s IC constraint. Since the spender’s income is paid from general taxation, this term is weighted by \( \lambda \).

To further motivate the planner’s decision, let us compare our principal-agent approach with a full-information benchmark model of the traditional public-finance type. There is a monolithic planner. The internal organization of the government is ignored; there are no agents who have to be paid an income. Tax collection is costless. However, shadow costs of public funds are taken into consideration. Since the planner is fully informed,
he does not face a multi-stage game, but at one point of time chooses both the public-good quantity and the excise tax. The artificial budget constraint is explicitly considered. Hence, the planner faces the following Lagrangean:

$$\mathcal{L}^W = S(G, x(p, G)) - px(p, G) + (1 + \lambda + \tau) \left[ (p - p^0) x(p, G) - C(G) \right].$$  \hspace{1cm} (13)

Can earmarking result endogenously from this optimization approach? Let us consider the tax-related marginal condition. As could be expected, the excise tax follows a Ramsey rule

$$\frac{p - p^0}{p} = -\frac{\lambda + \tau}{1 + \lambda + \tau} \cdot \frac{1}{\eta},$$  \hspace{1cm} (14)

where $\eta$ is the price elasticity of demand. Since we deal with excise taxation only, not with subsidization, the above condition requires $\lambda + \tau > 0$: earmarking may or may not be chosen by the planner. However, equation (14) reveals the driving forces behind the planner’s decision to earmark or not the earmark the excise tax. Gone are the asymmetric-information effects of separate budget constraints for every state of the world, gone is the spender’s IC problem, and the absence of these two effects makes it more probable to attain earmarking as a result of the planner’s optimization. The tax-related marginal condition shows that in the traditional public-finance approach it is only the interplay between the distortionary effects of the excise tax and the general tax that matters. Consider first an excise tax on a good with inelastic demand, so the distortion is relatively low although increasing in revenue. In this case the excise-tax revenue will be extended so as to equate the distortionary effects of the excise tax and the general tax. Hence, there will be no earmarking. Otherwise, if the excise tax is fairly distortionary, the artificial budget constraint will be binding, so the tax is earmarked (and in a next step the planner will repeat his optimization without the artificial budget constraint and equate shadow costs at an excise-tax revenue below the costs, financing the remainder by the general tax).

### 4.3 The politically-minded planner

We consider a planner who maximizes a political-support function à la Coughlin et al. (1990).\footnote{For a derivation of this function see appendix 4 of the present paper, sent to the reader on request. An intuitive introduction into this model can also be found in Mueller (1989), pp. 203-5. Since the Coughlin et al. approach is based on a general voting model, it is the planner (that is, the incumbent party in the parliament) who is influenced by the various interest groups. Alternatively, one could assume that spender and taxer are influenced by different interest groups, the spender by the firm which produces the public good and the taxer by a representative consumer of the taxed private good. This would imply a totally different model of earmarked taxation, following e.g. Lee and Tollison’s (1991) full-information rent-seeking approach or, preferably, Laffont and Martimort’s (1998) asymmetric-information model.} This function results from a probabilistic voting model and is a weighted sum of
the utility levels which the members of various interest groups derive from the planner’s policy. Let us assume that the planner considers the following interest groups: first, the group which has a vested interest in the provided public good and in the taxed private good; second, the general-tax payers who finance the ministers’ incomes; third, the beneficiaries of the budget surplus which results if the excise tax is not earmarked; fourth, the supporters of the spender and fifth, the supporters of the taxer. The political weights given to the individual members of these groups are $\sigma_{VI}$ for the vested-interest persons, $\sigma_{GT}$ for the general-tax payers, $\sigma_{BS}$ for the beneficiaries of the budget surplus and $\sigma_{S}$, $\sigma_{T}$ for the supporters of the ministers. These weights are larger the greater the homogeneity of the respective interest group, that is, the lower the planner’s uncertainty about the political support he gets from the members of this group.

The planner, therefore, employs the following political-support function:

$$\Omega^P = \sigma_{VI}(S(G, x) - px) - \sigma_{GT}(I + J) + \sigma_{BS}(R - C) + \sigma_{S} U + \sigma_{T} V + \mathcal{K},$$  

(15)

where $\mathcal{K}$ is a sum of terms which are constant for the choice of the planner’s instruments (resulting from the probabilistic voting approach). This political-support function has intentionally been formulated in such a way that it is as similar as possible to the objective function of the welfare-maximizing planner. Of course, when it comes to political support, the general-tax payers do not pay heed to the shadow costs of public funds, since this is a pure welfare concept.\footnote{We assume that the supporters of a particular minister are the many persons working in the ministry. They all have identical utility functions, $U/n_S$ and $V/n_T$, where $n_S$ and $n_T$ are the numbers of these supporters. Alternatively, we could have assumed that each of the ministers himself is an interest group, consisting of a single person.}

At date 6, the planner maximizes the expected value of the political-support function, that is, $\mathcal{E}_\theta \Omega^P$. This leads to the following generalized Hamiltonian:

$$\mathcal{H}^P = \left\{ \sigma_{VI} S(G(\theta)) + \sigma_{S} U(\theta) - \sigma_{GT} \left[ U(\theta) + \psi(e(\theta)) - wG(\theta) \right] - \sigma_{BS} C \left( \theta, e(\theta), G(\theta) \right) \right\} f(\theta) + \tau(\theta) \left[ \mathcal{P} - C(\theta, e(\theta), G(\theta)) \right] + \mu(\theta) \left[ h(\theta, e(\theta), G(\theta)) \right] - \text{constant},$$  

(16)

where $\text{constant}$ abbreviates the sum of all terms which are constant for the spender’s optimization, resulting both from the taxer’s and consumers’ decisions and from the probabilistic-voting approach. We once again know that $\tau \geq 0$. Fortunately, more infor-\footnote{One of the referees of this paper argued that rational taxpayer-voters should be concerned about the shadow costs of public funds since they measure a real cost incurred by the taxpayers. If the general-tax payers and the beneficiaries of the budget surplus thought like that, their payoffs would have to be multiplied with $(1 + \lambda)$. The qualitative results of the model would remain unchanged.}
mation can be derived from the effort-related marginal condition: \(26\)

\[
\sigma_{GT} \psi f = -\sigma_{BS} C_e f - \tau C_e + (\sigma_{GT} - \sigma_S) F h_e.
\]  

(17)

It can be seen directly that \(\tau > 0\) results from this marginal condition if simultaneously the political weight of the group of beneficiaries is zero and the far-right IC term is negative or zero. This double requirement in itself is surprising. Intuitively, one would have assumed that it is sufficient for earmarking if the beneficiaries have zero weight, because no policy maker would produce a budget surplus if this surplus has no effect on the votes. However, recall that the artificial budget constraint in our asymmetric-information setting is imposed for every single realization of the public-good parameter whence \(\bar{R} \geq C\) is most likely to be binding for the worst realization of the parameter. Therefore, to attain earmarking for favorable realizations of the parameter, the spender must be discouraged, so that he exerts less effort, which increases costs and leads to earmarking. This mechanism works if the IC term is negative or zero, that is, if the political weight given to the general-tax payers exceeds the weight given to the supporters of the spender \((\sigma_{GT} \geq \sigma_S)\), since \(h_e\) is negative. \(27\) As can be seen in the Hamiltonian, the general-tax payers want to keep the spender’s utility low, whereas the supporters of the spender want to keep it high. If the general-tax payers have the higher political weight, maximization of political support requires as low a spender’s utility as possible, which induces lower effort. This effect is reinforced by the general-tax payers’ interest in low spender’s disutility from effort, see once again the Hamiltonian. Lower effort, in turn, induces higher costs, which are increased until they reach the barrier \(\bar{R}\) and earmarking results.

Obviously, the above result depends on the planner’s lack of information and on the separation of the taxer and the spender problems. This shows clearly if we compare our approach with a political full-information benchmark model of the traditional public-finance type, where we have the following Lagrangean function:

\[
\mathcal{L}^P = \sigma_{VI} \left[ S(G, x(p, G)) - px(p, G) \right] + (\sigma_{BS} + \tau) \left[ (p - p^0) x(p, G) - C(G) \right] + \mathcal{K}.  
\]  

(18)

The tax-related marginal condition is:

\[
\frac{p - p^0}{p} = -\frac{\sigma_{BS} - \sigma_{VI} + \tau}{\sigma_{BS} + \tau} \cdot \frac{1}{\eta}. 
\]  

(19)

\(26\) Once again, we need some control theory to derive this condition. We have \(-\mathcal{H}^P U = \hat{\mu}(\theta) \iff \hat{\mu}(\theta) = (\sigma_{GT} - \sigma_S) f(\theta)\). Combining this equation with the transversality condition \(\mu(\theta) = 0\), we get \(\mu(\theta) = (\sigma_{GT} - \sigma_S) F(\theta)\). For all \(\theta > \theta_0\).

\(27\) Note the formal connection to the IC condition: \(\mu(\theta) = (\sigma_{GT} - \sigma_S) F(\theta)\). Compare footnote 26.

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Earmarking occurs more easily in the traditional public-finance approach. It is sufficient that the beneficiaries of the budget surplus have no political weight ($\sigma_{BS} = 0$). This is just the intuitive result which we did not obtain in the asymmetric-information setting. The traditional political planner really does not produce any budget surplus if this does not lead to any further votes. Now we recognize where our intuition came from: we were stuck in a traditional public-finance bias!

Alternatively, earmarking results if the vested interests dominate the beneficiaries in their political support of the planner ($\sigma_{VI} > \sigma_{BS}$). This result is quite plausible because the vested-interest voters are only interested in the quantities of the public and the private good and do not care for the costs of the public-good production. The beneficiaries of the budget surplus, on the other hand, want low costs. Accordingly, a high political weight of vested interests leads to costs which are equal to the excise tax revenues.

Considering the traditional public-finance approach, we clearly see what can be learned from the principal-agent approach of the present paper: the general-tax payers become a driving force in favor of earmarking and the vehicle to exert their influence is the spender’s incentive-compatibility condition. Since the spender’s income is paid from general taxation, the general-tax payers influence the spender’s incentive pay which is shaped according to his incentive-compatibility condition. Unfortunately, they use their influence so as to discourage the spender’s effort.

Extensions

There are two extensions which we would like to treat in the framework of the principal-agent approach:

(i) For a most general formulation of a political objective function we assumed that the beneficiaries and the general-tax payers are two separate groups. As an exact counterpart to the welfare assumption that the surplus allows reduction in general taxation, let us now assume that any budget surplus is given to the general-tax payers. Then we obtain an effort-related marginal condition which is similar to (17), but with $\sigma_{BS} = \sigma_{GT}$. Now it is impossible to combine a zero political weight of the beneficiaries and a positive weight of the general-tax payers. Earmarking is less likely to occur because the general-tax payers now have to trade-off the benefits from a budget surplus of the excise tax and the costs of paying the incentive income to the spender: the first works against earmarking, the

\footnote{The comparison with a benchmark model of a monolithic-planner approach is straightforward and, therefore, left to the reader.}
second favors it.

(ii) How do the political results change if the spender gets direct satisfaction from his expenditures and not from the public-good quantity? His utility function becomes  \( U = I - \psi + wC \). This leads to significant changes in the effort-related marginal condition. Earmarking is less likely to occur with an expenditure-oriented spender. The list of conditions which simultaneously must hold to attain earmarking must be extended by the condition that the spender must not be interested in his expenditures \((\bar{w} = 0)\). Pure Niskanen spenders may prevent the planner from choosing earmarking as an optimal vote-maximizing policy.

5 Summary and conclusion

This paper dealt with earmarked taxation in a framework, where the parliament monitors a taxing authority and a spending authority, but faces hidden action and hidden information of these two authorities. This approach was compared with the traditional public-finance approach of a monolithic fully informed planner. Let us summarize the main findings:

(i) An examination of institutional arrangements and of the terms of policy debate suggests that the expression ‘earmarking’ is conventionally understood as referring to a situation where the revenues of a special tax are equal to the expenditures for a particular public good or fall below these expenditures.

(ii) The planner’s lack of information and the explicit consideration of the internal organization of the government create a tendency against earmarking. The special-tax revenues must be planned in such a way that they are cost covering even in the worst state of the world. When the public expenditure is planned, the special-tax revenues are already given which makes it impossible to reduce the tax revenues because a favorable state of the world induces low costs. A welfare-maximizing planner, who is incompletely informed, will typically realize an involuntary budget surplus where the special-tax revenues exceed the costs, whence the tax cannot be considered as earmarked. In contrast, a full-information monolithic welfare maximizer of the traditional public-finance type will more readily employ earmarking: he only will compare the interplay between the shadow costs of the special tax and of general taxation: earmarking will result if the excise tax is fairly distortionary compared with general taxation.

(iii) If the parliament maximizes expected votes, earmarking also is more likely to occur in the traditional public-finance approach: it is sufficient that the beneficiaries of an earmarked-tax budget surplus have zero political weight. This is intuitive because it says
that no policy maker will produce a budget surplus if this surplus has no effect on votes. However, this traditional intuition does not hold if the planner’s lack of information and the separation of government powers is included in the model. Once again, the special-tax revenues must be cost covering even in the worst state of the world, and cannot be adjusted if low costs result from more favorable states of the world. Therefore, if the beneficiaries have zero political weight, this is still not sufficient for earmarking. Earmarking will only result if in addition the general-tax payers drive the planner to discourage the spender’s effort in such a way that the costs are increased so that they are equal to the tax revenues regardless of the state of the world.

Let us finally address the general implications of this paper. As mentioned in the very first paragraph of the paper, it has recently been claimed that earmarking would imply that the population would better understand the rationale of taxation, that public expenditures would be chosen in closer accordance with the individual citizen’s preferences, and that earmarking would imply pressure on the government to operate more efficiently. Are these arguments correct? Do they really distinguish earmarked taxation from general taxation? I am not convinced that this is true. Let us, therefore, evaluate these arguments in turn. First, does earmarking really lead to a better understanding of the rationale of taxation? As long as we deal with rational taxpayers, this cannot serve as a special argument for earmarking. A rational taxpayer would also anticipate the connection between the general taxes he pays and the various public expenditures for which they are used. Second, does earmarking really imply that public expenditures are chosen in closer accordance with individual preferences? This is not correct. Since money is spent after it has been raised, there is only a connection between the expected value of the public good and the tax paid. Moreover, earmarking is not a user-fee principle, the earmarked tax increases the price of a private good, it is not a price of the public good. The individual consumer cannot buy the quantity of the public good he wants to. The Niskanen interests of the involved ministries, furthermore, lead to a deviation from individual consumer preferences. Third, is there really a pressure on the government to operate more efficiently? It has become very clear through the analysis of the present paper that this is not the case due to the asymmetric-information problem and the consideration of the internal organization of the government. Hence, the particular advantages which have often been claimed for earmarked taxation cannot be shown to hold in a planner-taxer-spender model with asymmetric information.

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References


Appendices

Appendix 1  Social security in Europe 1992 in DM bill.

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Appendix 2  The spender’s incentive compatibility

This appendix explicitly derives the spender’s IC condition. For this purpose, let us first describe the spender’s utility function in more detail. We have

$$U(I, e, G) = I - \psi(e) + wG; \quad w \in (0, 1). \quad (20)$$

Here $\psi(e)$ is a convex disutility function, that is, $\psi' > 0$, $\psi'' > 0$. Moreover, the disutility function fulfills the Inada conditions and $\psi'' \geq 0$. The latter assumption guarantees that the optimal incentive scheme is not stochastic (see Laffont-Tirole (1993), pp. 119-20). We assume that it pays to work, that is, $U(0, 0, 0) < 0$, where the zero on the right-hand side is the spender’s reservation utility.

Let us next define the effort-requirement function $E$ which is obtained by inverting the cost function $C(\theta, e, G)$:

$$E = E(\theta, G, C); \quad E_\theta > 0, \ E_G > 0, \ E_C < 0. \quad (21)$$

This function measures the minimal effort which is necessary to attain a particular quantity $G$ at a particular value of $\theta$ and at costs $C$. 

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The planner applies a direct mechanism (revelation principle). Hence, at date 6 he stipulates the following incentive income in order to induce the spender to reveal the true value of the cost characteristic \( \theta \):

\[
\text{incentive pay} = \begin{cases} 
I(\hat{\theta}) & \text{if } G = G(\hat{\theta}), \text{ and } C = C(\hat{\theta}); \\
0 & \text{otherwise}, 
\end{cases} 
\]

where \( \hat{\theta} \) is that value of the characteristic which has been announced by the spender.

If the spender truthfully announces \( \theta \), his utility depends on \( \theta \), say \( \bar{U}(\theta) \). However, if he announces some arbitrary \( \hat{\theta} \), his utility is \( U(\theta, \hat{\theta}) \), since his effort depends on both \( \theta \) and \( \hat{\theta} \) because he must choose the effort-saving effect of his lying in such a way that the public-good quantity and the costs are just as observed by the planner. Any income schedule which induces truth-telling behavior of the spender requires that lying must never lead to higher utility than telling the truth,

\[
D^S = \bar{U}(\theta) - U(\theta, \hat{\theta}) \geq 0, 
\]

where

\[
U(\theta, \hat{\theta}) = I(\hat{\theta}) - \psi \left( E(\theta, G(\hat{\theta}), C(\hat{\theta})) \right) + wG(\hat{\theta}).
\]

Formally, the planner chooses the spender’s income in such a way that the distance function \( D^S \) is minimized with respect to \( \hat{\theta} \). As necessary first-order condition we obtain:

\[
I_{\hat{\theta}} - \psi' \left( E_G G_{\hat{\theta}} + E_C C_{\hat{\theta}} \right) + wG_{\hat{\theta}} = 0. 
\]

This first-order condition can be simplified as follows. Consider changes in the spender’s utility which result from changes in the actual \( \theta \),

\[
\dot{U}(\theta) := \frac{dU}{d\theta} = I_{\theta} - \psi' E_{\theta} - \psi' \left( E_G G_{\theta} + E_C C_{\theta} \right) + wG_{\theta}. 
\]

At \( \theta = \hat{\theta} \), all differential quotients are observable by the planner. Hence, the spender will choose his effort in such a way that all of these differential quotients are equal in (25) and (26). Therefore, these equations can be combined to obtain the spender’s first-order IC condition

\[
\dot{U}(\theta) = -\psi' E_{\theta}, 
\]

where, according to our assumptions on \( \psi \) and \( C \), we have \( -\psi' E_{\theta} < 0 \).

As is well known, the second-order IC condition \( U_{\theta\theta} \leq 0 \) can be rewritten as follows:\(^{29}\)

\[
U_{\theta\theta} \geq 0 \quad \text{at } \theta = \hat{\theta}. 
\]

\(^{29}\)See Guesnerie and Laffont (1984). For the differentiation note that in the first-order condition (25) \( C_{\hat{\theta}}, I_{\hat{\theta}} \) and \( G_{\hat{\theta}} \) are independent of \( \theta \); they depend only on \( \hat{\theta} \). On the other hand, \( \psi', E_C \) and \( E_G \) depend on the actual \( \theta \) because we have \( E(\theta, \hat{\theta}) \).
This condition requires
\[ -\psi''E_\theta \left( E_\theta G_\theta + E_C C_\theta \right) - \psi' \left( E_{\theta G} G_\theta + E_{C\theta} C_\theta \right) \geq 0. \] (29)

In our model we have assumed \( \psi' > 0 \) and \( \psi'' > 0 \). Moreover, we have \( E_\theta > 0, E_\theta > 0, E_C < 0, G_\theta < 0, C_\theta > 0 \). Hence, \( E_{\theta G} \geq 0 \) and \( E_{C\theta} \leq 0 \) are sufficient for the second-order IC condition to hold.

In the planner’s optimization approaches \( H^W \) and \( H^P \), the IC constraint can explicitly be written as
\[ \dot{U}(\theta) = -\psi'(e(\theta)) E_\theta \left( \theta, G(\theta), C(\theta, e(\theta), G(\theta)) \right). \] (30)

In the text we have abbreviated \( h(\theta, e(\theta), G(\theta)) := -\psi'(e(\theta)) E_\theta \left( \theta, G(\theta), C(\theta, e(\theta), G(\theta)) \right) \).

This allows to deduce the precise properties of the function \( h(\cdot) \):

- Let us first determine the sign of \( h_e \). We have \( d(\psi' E_\theta)/de = \psi' E_{\theta C} C_e + \psi'' E_\theta \). The first term on the right-hand side is non-negative, because \( E_{\theta C} \leq 0 \) follows from the second-order conditions of the spender’s incentive-compatibility problem as shown above. The second term is strictly positive by definition, \( \psi'' > 0 \) and \( E_\theta > 0 \). Accordingly, we obtain \( h_e < 0 \).

- Let us second deal with the sign of \( h_C \). This sign is indeterminate. We have \( d(\psi' E_\theta)/dG = \psi' dE_\theta/dG = \psi' \left[ E_{\theta G} + E_{C\theta} C_G \right] \). Here \( \psi' \) and \( C_G \) are positive by assumption. From the second-order conditions of the spender’s incentive-compatibility problem we have \( E_{\theta G} \geq 0, E_{C\theta} \leq 0 \). Hence \( h_C \) can be positive or negative. A special case is Laffont and Tirole’s (1993, pp. 178-9) incentive-pricing dichotomy. In our paper this dichotomy would refer to a case where \( h_C = 0 \) because \( dE_\theta/dG = 0 \), as derived from a cost function which is specified as \( C(\zeta(\theta, e), G) \).

**Appendix 3  The planner-taxer problem**

**A.3.1  The taxer’s incentive compatibility**

To derive the taxer’s IC condition, we rewrite the taxer’s utility function as\(^{30}\)
\[ V(J, A, R) = J - \phi(A) + mR; \quad m \in (0, 1), \] (31)
where \( A \) is an effort-requirement function which is obtained from inverting the tax-collection cost function \( T(\beta, a, x) \):
\[ A = A\left( \beta, T, x(p, g(\beta)) \right); \quad A_\beta > 0, \quad A_T < 0, \quad A_x > 0. \] (32)

\(^{30}\)The taxer’s utility function has analogous properties as the spender’s. For details see appendix 2 above.
The planner stipulates an incentive pay which induces the highest taxer utility at $\beta = \hat{\beta}$, whence the taxer will tell the truth about the actual value of $\beta$:

$$\text{incentive pay} = \begin{cases} 
J(\hat{\beta}) & \text{if } R = R(\hat{\beta}), \ p = p(\hat{\beta}), \text{ and } T = T(\hat{\beta}); \\
0 & \text{otherwise},
\end{cases} \tag{33}$$

where $\hat{\beta}$ is the announced value of $\beta$. The taxer’s income, the tax-collection costs, the consumer price, and the tax revenues depend on the announced $\hat{\beta}$, as stipulated in the incentive-pay contract (33). However, this incentive pay is not conditioned on the expected quantity of the public good (and cannot be conditioned on it, since an expected quantity is not observable). Therefore, $g(\beta)$ depends on the actual $\beta$ and not on the announcement.

On the basis of this incentive income, the taxer announces that $\hat{\beta}$ which minimizes the distance function

$$\mathcal{D}^T = \hat{V}(\beta) - V(\beta, \hat{\beta}), \tag{34}$$

where

$$V(\beta, \hat{\beta}) = J(\hat{\beta}) - \phi\left[A(\beta, T(\hat{\beta}), x(p(\hat{\beta}), g(\beta)))\right] + mR(\hat{\beta}). \tag{35}$$

We obtain the following necessary first-order condition:

$$J_{\beta} - \phi'(A_T T_{\beta} + A_x x_p p_{\beta}) + mR_{\beta} = 0. \tag{36}$$

This condition can be substituted into

$$\hat{V}(\beta) := dV/d\beta = J_\beta - \phi' A_\beta = \phi'(A_T T_{\beta} + A_x x_p p_{\beta} + A_x x_g g_{\beta}) + mR_{\beta}. \tag{37}$$

This leads to the IC condition

$$\hat{V}(\beta) = -\phi' \left(A_\beta + A_x x_g g_{\beta}\right). \tag{38}$$

For the most plausible case of $g_{\beta} < 0$, $\hat{V}(\beta)$ is always negative if the private and the public good are complements or neutral, that is, if $x_g \leq 0$. However, if they are substitutes, this property does not necessarily hold and we face a trade-off; the utility of the taxer may be increasing or decreasing in the tax-morale parameter, it may also be increasing for particular values of the parameter and decreasing for others.

The second-order IC condition is less likely to be fulfilled than in the case of asymmetric information on costs. In our model the condition $V_{\beta\beta} \geq 0$ at $\beta = \hat{\beta}$ requires\[^{31}\]

$$-\phi'' A_\beta (A_T T_{\beta} + A_x x_p p_{\beta}) - \phi' (A_T T_{\beta} + (A_x x_p + A_x x_p p_{\beta}) p_{\beta}) \geq 0. \tag{39}$$

\[^{31}\]This implies differentiating equation (36) with respect to $\beta$. Note that in (36) there is no term which relates to $g$. 

v
We only consider the case where this condition holds, that is, we exclude bunching. It is directly evident that the probability of bunching is much higher than in the case of the spender’s second-order condition in appendix 2.\footnote{Only a few assumptions made in the paper help evaluate the taxer’s second-order conditions, namely $A_0 > 0, A_T < 0, A_x > 0, \varphi > 0, \varphi'' > 0$. Moreover $p_0 > 0$ is plausible: the more inefficient the tax administration, the higher the consumer price.}

\subsection{Standard treatment}

The taxer’s IC condition reveals that the state variable $V(\beta)$ can be increasing as well as decreasing in $\beta$. To deal with that problem, in this subsection we assume that $V(\beta) = 0$ and $V(\beta) > 0$ for all $\beta < \beta$. Under this assumption, the usual combination of participation and incentive-compatibility constraints can be applied.\footnote{That is, the planner postulates the initial condition $V(0) = 0$ and imposes the IC constraint. The taxer’s utility at the best realization of the tax-morale parameter is free (terminal condition).} The original problem without this assumption is treated in subsection A.3.3.

The planner will anticipate the optimization of date 6 by substituting into the Lagrangean the solutions of this approach. These solutions are defined for each and any value of $\beta$, namely $U^*(\beta, \theta), G^*(\beta, \theta)$ and $e^*(\beta, \theta)$, and the planner considers the expectations of these variables. This consideration implicitly takes account (i) of the date-6 budget constraint,\footnote{This is equation (9) of the main text of the paper.} and (ii) of the spender’s participation and incentive-compatibility constraints. Hence, none of these constraints is explicitly included in the planner’s optimization at date 2.\footnote{However, another constraint must be added: recall that the demand for the private goods depends on $g(\beta)$ and that $G^*(\beta, \theta)$ enters in the planner’s objective function. Hence, the planner must explicitly make sure that for every $\beta$ the equality $g(\beta) := \epsilon_0 G^*(\beta, \theta)$ is guaranteed.}

If the planner maximizes welfare, he solves the following generalized Hamiltonian with respect to the state variable $V$ and the controls $a, g$ and $p$:\footnote{In the generalized Hamiltonian we have abbreviated $x(\cdot) = x(p(\beta), g(\beta))$ and not explicitly denoted all the functional dependencies in the taxer’s incentive-compatibility constraint.}

\begin{equation}
\mathcal{H}^T = \left[ \mathcal{E}_0 S (G^*(\beta, \theta), x(\cdot)) - p(\beta) x(\cdot) \right] z(\beta) \\
- \lambda \left[ \mathcal{E}_0 U^*(\beta, \theta) + V(\beta) \right] z(\beta) \\
- (1 + \lambda) \left[ \mathcal{E}_0 \left( \mathcal{E}_0 \left( e^*(\beta, \theta) - w G^*(\beta, \theta) \right) + \phi(a(\beta)) \right) \\
- m \left( p(\beta) - p^0(\cdot) \right) x(\cdot) - T(\beta, a, x(\cdot)) \right] z(\beta) \\
- \gamma(\beta) \left[ g(\beta) - \mathcal{E}_0 G^*(\beta, \theta) \right] \\
- \nu(\beta) \left\{ \phi'(a(\beta)) \left[ A_{\beta} + A_x x_{g\beta} \right] \right\}.
\end{equation}

\footnote{In the generalized Hamiltonian we have abbreviated $x(\cdot) = x(p(\beta), g(\beta))$ and not explicitly denoted all the functional dependencies in the taxer’s incentive-compatibility constraint.}
For a politically-minded planner the generalized Hamiltonian can be formulated analogously.

Since at date 2 there is no explicit consideration of earmarking, the various marginal conditions have no connection with the special topic of the paper. Hence, we shall not explicitly present them in this subsection. To forgo this presentation is an easy decision, because the various marginal conditions do not come as a surprise. Let us present the gist of these conditions for the welfare-maximizing planner:

- the tax is imposed according to a modified Ramsey rule which includes the taxer’s incentive-correction term;
- the planner takes account of Atkinson-Stern (1974) terms of demand interdependency between the private and the public good, the latter in expected terms. This results from differentiating the generalized Hamiltonian with respect to $g$;
- the taxer’s effort relates his disutility from work to the marginal cost savings in tax collection, and to an incentive-correction term.

A.3.3 The original problem (with pure state constraint)\(^{37}\)

Let us now drop the assumption that $V(\overline{\beta}) = 0$ and $V(\beta) > 0$ for all $\beta < \overline{\beta}$ and deal with the original problem. This implies that the taxer’s participation constraint must explicitly be considered as a pure state constraint of the optimization approach:

$$V(\beta) \geq 0; \quad \text{for all } \beta. \quad (41)$$

This pure state constraint a priori has neither an initial nor a terminal condition. However, the state variable $V$ cannot be free at both $\overline{\beta}$ and $\overline{\beta}$. If optimal values of the control variables $a, p$ and $g$ have been found, the IC constraint $\dot{V}^* = -\rho'(A_\beta + A_gg)$ is a function of these values,

$$\dot{V}^* = \dot{V}^* (\beta, a^*(\beta), p^*(\beta), g^*(\beta)). \quad (42)$$

Equation (42) links $V^*(\overline{\beta})$ and $V^*(\beta)$ to each other. Hence, they cannot both be free. In the following we fix

$$V^*(\overline{\beta}) = V(\overline{\beta}, a^*(\overline{\beta}), p^*(\overline{\beta}), g^*(\overline{\beta})) =: \overline{V}. \quad (43)$$

Accordingly,

$$V(\overline{\beta}) = \overline{V} \quad (44)$$

serves as an initial condition for the state variable $V$. The taxer’s utility at the best realization of $\beta$ is free.

\(^{37}\)I gratefully acknowledge helpful comments on this control-theoretic problem by Norbert Christopeit.
The Hamiltonian $\mathcal{H}^T$ has to be extended as follows: we define the pure state constraint by $Q(\beta, V(\beta)) := V(\beta) \geq 0$. Then the constraint is included in the optimization approach by the following Lagrangean:

$$\mathcal{Y}^T = \mathcal{H}^T - \xi(\beta) [Q_\beta + Q_V V_*]$$

$$= \mathcal{H}^T + \xi(\beta) \phi' \left( A_\beta + A_g g_\beta \right), \quad (45)$$

since $Q_\beta = 0, Q_V = 1$. Differentiating $\mathcal{Y}^T$ with respect to the state variable yields

$$\dot{\nu} = -\mathcal{Y}_V^T = \lambda \dot{z}(\beta). \quad (46)$$

Considering this and the transversality condition, we obtain:

$$\nu(\beta) = 0, \quad (47)$$

$$\nu(\beta) = \lambda Z(\beta) \quad \text{for all } \beta > \beta. \quad (48)$$

The multiplier $\xi(\beta)$ is piecewise continuous, non-increasing, constant on every interval on which $Q(\beta, V^*(\beta)) > 0$, that is $V^*(\beta) > 0$, and continuous at every point of continuity of $(a^*(\beta), p^*(\beta), g^*(\beta))$; moreover $\xi(\beta) \leq 0$. Therefore, $\xi \leq 0$ for $\beta > \beta$.

The above results imply $\nu(\beta) - \xi(\beta) > 0$ for all $\beta > \beta$. Now consider the first-order conditions which result from differentiating $\mathcal{Y}^T$ with respect to the control variables $a, p$ and $g$. They are formally identical to those which in the main text were derived by differentiating $\mathcal{H}^T$ with respect to the same controls. There is only one difference: in the incentive-correction terms $\nu(\beta)$ is now replaced by $\nu(\beta) - \xi(\beta)$. Accordingly, all IC terms look more complicated, however, the qualitative economic results remain unchanged.

**Appendix 4  The planner’s political-support function**

The formulation of this appendix refers to the planner’s decision at date 6; hence all expectations are taken over the cost characteristic $\theta$. The extension to the formulation of the objective at date 2 is straightforward. In the probabilistic voting model of Coughlin et al. (1990), an individual $i$ is characterized by its membership in an interest group $j$. Each individual is a member of one interest group and all members of an interest group are assumed identical. The interest groups add up to the total voting population. The individuals choose to support either the planner, that is, the incumbent, or the opposition. We use the indices $\Gamma$ for the incumbent and $\Psi$ for the opposition. The probability $prob_{\Gamma ij}$

---

38For this procedure see Hestenes (1966), pp. 352-74. $\mathcal{H}^T$ is as defined in equation (40) above.
that individual \( i \) of group \( j \) supports the incumbent instead of the opposition depends on the net benefit it gets from the incumbent’s (the planner’s) policy and on a ‘bias term’ \( b_{ij} \). Thus,

\[
prob_{ij} = \begin{cases} 
1 & \text{if } \mathcal{E}(\mathcal{U}_{\Gamma ij}) > \mathcal{E}(\mathcal{U}_{\Psi ij}) - b_{ij}, \\
0 & \text{if } \mathcal{E}(\mathcal{U}_{\Gamma ij}) \leq \mathcal{E}(\mathcal{U}_{\Psi ij}) - b_{ij}, 
\end{cases}
\]  

\( (49) \)

where \( \mathcal{E}(\mathcal{U}_{\Gamma ij}) \) is the individual’s expected utility if the incumbent’s announced policy is realized, \( \mathcal{E}(\mathcal{U}_{\Psi ij}) \) is the expected utility it would get from the opposition’s policy and \( b_{ij} \) is the bias term that includes all other reasons for which the individual favors the incumbent \( (b_{ij} > 0) \) or the opposition \( (b_{ij} < 0) \). Following Coughlin et al. (1990) we assume that within each interest group these bias terms are uniformly distributed on the range \([\ell_j, r_j]\). The distribution function \( D \) of \( b_{ij} \) therefore is

\[
D(b_{ij}) = \frac{b_{ij} - \ell_j}{r_j - \ell_j},
\]  

\( (50) \)

from which the incumbent can infer the probability that he will be supported by individual \( i \) of group \( j \), given the announced policy:

\[
Prob \{b_{ij} > \mathcal{E}(\mathcal{U}_{\Psi ij}) - \mathcal{E}(\mathcal{U}_{\Gamma ij})\} = 1 - D(\mathcal{E}(\mathcal{U}_{\Psi ij}) - \mathcal{E}(\mathcal{U}_{\Gamma ij})).
\]  

\( (51) \)

We consider five different interest groups:

- the vested-interest group which is interested in the provided public good and in the taxed private good, \( j = VI \),
- the general taxpayers, \( j = GT \),
- the recipients of the budget surplus, \( j = BS \),
- the supporters of the spender, \( j = S \), and
- the supporters of the taxer, \( j = T \).

Each interest group has \( n_j \) voters. Since the members of any group are identical, the individual utilities can simply be added up to obtain aggregate utilities of the five groups.

This procedure is usual for consumer surplus. With respect to the taxes, the procedure implies that the general tax is split equally among the taxpayers and that an excise-tax budget surplus is split equally among the beneficiaries. Similarly the spender and taxer utilities are equally split among the respective minister’s supporters. Let us denote by \( \mathcal{U}_{\Gamma j} \) the \( j \)’th group’s aggregate utility if the incumbent’s announced policy is realized.

Since we deal with the planner’s optimization at date 6, the taxer’s and consumers’ decisions have already been made. Therefore, various (overlined) variables are constant for the spender’s decision. We have:

\[
\mathcal{E}(\mathcal{U}_{\Gamma VI}) = \mathcal{E}_\theta \left[ S(G, \bar{x}) - \bar{p} \bar{x} \right],
\]

\[
\mathcal{E}(\mathcal{U}_{\Gamma GT}) = - \mathcal{E}_\theta \left[ \overline{t} + \overline{f} \right],
\]
\[ \mathcal{E}(\mathcal{U}_{BS}) = \mathcal{E}_\theta \left[ \overline{R} - C \right], \]
\[ \mathcal{E}(\mathcal{U}_{S}) = \mathcal{E}_\theta U, \]
\[ \mathcal{E}(\mathcal{U}_{T}) = \mathcal{E}_\theta V. \]

Hence, the planner’s objective function is as follows:
\[ \mathcal{E}_\theta \Omega^p = \mathcal{E}_\theta \left[ \sum_{j=1}^{5} \mathcal{U}^j(\cdot) \right], \]  
(53)

where
\[ \mathcal{E}_\theta \left[ \mathcal{U}^j(\cdot) \right] := 1 - \frac{1}{r_j - \ell_j} \left\{ \mathcal{E}(\mathcal{U}_{qj}) - \mathcal{E} (\mathcal{U}_{rj}) - \ell_j \right\}, \quad j = VI, GT, BS, S, T. \]  
(54)

We substitute from (52) to obtain
\[ \mathcal{E}_\theta \Omega^p = \mathcal{E}_\theta \left\{ \sigma_{VI} \left( S(G, \overline{x}) - \overline{p} \overline{x} \right) - \sigma_{GT} (I + \overline{J}) + \sigma_{BS} (\overline{R} - C') + \sigma_S U + \sigma_T V \right\} + \mathcal{K}, \]  
(55)

where we have defined \( \sigma_j := 1/(r_j - \ell_j) \) for \( j = VI, GT, BS, S, T \) and where \( \mathcal{K} \) abbreviates a sum of terms which are constant for the spender’s optimization, resulting from the probabilistic-voting approach. Of course, all overlined variables can be dropped when formulating the Hamiltonian.

References only used in the appendices


