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Government Expenditure Policies in Equilibrium and Disequilibrium

DISKUSSIONSBEITRÄGE

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Government Expenditure Policies in Equilibrium and Disequilibrium

by

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I. Introduction

A major problem with the standard treatment of government expenditure policies in the traditional macroeconomic literature of Keynesian persuasion is the complete neglect of the effect which government expenditures, or more precisely the goods and services acquired or produced with these expenditures, have on the well-being, or welfare of the private sector, and therefore on its expenditure decisions with respect to privately produced goods and services. Although this point has been emphasized a long time ago by Bailey,¹ it has been almost completely neglected in the traditional literature on the subject.² Maybe the reason for this is given by the fact that Bailey discusses the issue within the traditional Keynesian framework, where consumption expenditure functions are specified on an essentially ad hoc basis. A recent development in the macroeconomic literature has been an increased emphasis on the choice theoretic foundations of macroeconomic theory. In particular, the type of approach pioneered by Patinkin, Clower, and Leijonhufvud, and further developed by Barro and Grossman and others has permitted an interpretation of traditional Keynesian macrotheory as a price theoretically based theory applicable to certain disequilibrium situations.³ It seems that a careful examination of the macroeconomic effects of government expenditures should proceed along the lines suggested by this type of approach, i.e., should be based on a model with firm foundations in microeconomic analysis. Only then can we avoid an essentially arbitrary specification of consumption expenditure functions. Such an analysis is presented in this paper. It is demonstrated that the
The macroeconomic effect of government expenditure policies may depend crucially on the particular state of the economy, i.e., on the particular combination of excess supplies and demands existing in different markets of the economy.

We will concentrate on the case of tax financed government expenditures, that is the case which is usually summarized by the so-called balanced budget multiplier results. However, the analysis has relevance for other modes of financing, too. In particular, if issuing government bonds is just a way of replacing current taxes with future taxes (see, e.g., Bailey, and Barro), everything said in this paper applies to the case of debt financing, too.

II. Microeconomic Behavior

The typical treatment of a tax financed increase in government expenditures in the literature finds that it will result in a net increase in the total demand for commodities ("goods and services"), because the private sector is assumed to respond to the newly introduced taxes by partly reducing (current) commodity demand and partly reducing savings; i.e., private demand is reduced, but by an amount which is smaller than the simultaneously occurring increase in government demand. The major weakness of this approach is, as Bailey emphasizes, that the private sector's expenditure decisions are viewed as depending on the private sector's net income after deduction of taxes ("disposable income") only, that is, entirely independent of the quantity and nature of government provided goods and services. However, the latter contribute to the private sector's well-being, precisely like privately produced goods
and services, and therefore should be expected to have a bearing on how the private sector divides his disposable income between consumption and saving (i.e., current and future consumption).\

Bailey discusses this issue in the context of a traditional Keynesian consumption function, which views consumption expenditures as dependent on some income variable. We will start from a microeconomic basis where a representative household has, in any period, the choice between leisure (or sales of labor services), consumption of commodities ("goods and services"), and savings. Alternatively expressed, the household faces leisure, current consumption of commodities, and future consumption of commodities as his objects of choice. The analysis is based on a Fisherian, or "life cycle" model of choice over time, where a bond market allows the transfer of savings (wealth) from the current into future periods, or vice versa. We thus consider an economy with three "markets": labor, commodities, and bonds. All market participants are assumed to act as perfect competitors (price takers). As Clower, Leijonhufvud, and Barro-Grossman have emphasized, the type of behavior where demand functions for specific goods (e.g., current consumption) contain income as an independent argument follows from this framework as a special case, if we face a labor market disequilibrium (excess supply) situation, where the individual household's (labor) income is exogeneously determined by the state of the labor market (and the household's "queuing position" in this market).

Since most government expenditures—apart from pure transfer payments—are expenditures on factor services, especially labor, we will concentrate on this case. Government uses tax revenues to acquire labor
services, which in turn it employs in order to produce goods and services (e.g., schooling, highways, defense, recreational facilities, etc.).

Households are assumed to be utility maximizers. Their utility functions contain as arguments the consumption of leisure $L_i$, or, alternatively, the amounts of labor services sold $N_i$, the consumption of privately produced goods and services, or commodities, $y_i$, and the consumption of government produced goods and services $g_i$, each for all $n$ periods in the planning horizon:

$$U = U(N_i, y_i, g_i; i = 1, \ldots, n).$$

The utility function is assumed to have the usual properties (including positive first and negative second derivatives for $y_i$ and $g_i$, negative first and positive second derivatives for $N_i$). The government provides the goods and services it produces to the households in the private sector free of charge; i.e., they are entirely tax financed. We denote by $t_i$ the taxes paid in period $i$, expressed in real terms, i.e., in terms of commodity units. The output of government goods and services is related to the resource inputs acquired by government $N_g$ through a government production function $g = f(N_g)$, with positive but decreasing marginal product. The cost of producing these services $g$ (expressed in commodity units), which are financed via taxation $t$, are equal to $t = wN_g = w f^{-1}(g) = wt(g)$, where $w$ is the real wage rate (the price of labor services, in terms of output units). We assume for the present purpose that these taxes are raised in the form of a lump sum tax, in order to abstract from the relative price effects associated with different types of taxes (this permits better comparison with the results
of conventional macrotheory, where these relative price effects are completely neglected).

The representative household thus faces for each period a budget constraint of the following form:

\[(2) \ wN_i + \pi_i - wt(g_i) + c_i = y_i + s_i\]

where \(\pi_i\) denotes nonlabor income (the household's income from ownership in business firms, treated as predetermined by the household in his decisions), \(c_i\) represents the interest income (payment) on accumulated bonds (debt), and \(s_i\) denotes the savings in period \(i\), i.e., additions to accumulated wealth. Note that it is unimportant whether taxes \(t(g_i)\) are levied from the households directly, or are collected as business taxes (as long as we adhere to the above assumption that they are raised as lump sum taxes). In the latter case, the tax would reduce net non-labor income, instead of net labor income. We also assume, for simplicity, that taxes are shared by the public in proportion to the government services received, that is, we follow the usual macroeconomic convention to abstract from distribution effects. The level of government-produced goods and services for each period is not a choice variable of the households; it is determined by government in an autonomous manner, based on some calculus not endogeneous to the present model. Also, we assume that government produced goods and services cannot be resold.

For an \(n\)-period planning horizon, we can thus write the representative household's problem as follows: Maximize utility function \((1)\), subject to the constraint

\[(3) \ \sum_{i=1}^{n} v_i y_i = \sum_{i=1}^{n} v_i \left[ wN_i + \pi_i - wt(g_i) \right], \]
where \( v_i = \left( \frac{1}{1+r} \right)^{i-1} \), and \( r \) denotes the interest rate. (For simplicity, we assume expectation of constant prices \( w \) and \( r \) over the whole planning horizon.)

Note that if we want to allow the household to end up with a nonzero wealth in the terminal period—a "bequest"—we can simply interpret the last period \( n \) as the first period after the decision maker's death. If we consider "the family" as the decision unit, living generation after generation, we have \( n \to \infty \).

Private producers (firms) are assumed to behave as traditional profit maximizers. We assume that the capital stock is stationary, so that we can abstract from problems of capital accumulation and growth. The representative firm can then simply be viewed as maximizing its profits

\[ w_i = h(N^f_i) - wN^f_i \]

for each period \( i \), subject to the given wage rate \( w \) and the production function \( h(N^f) \), where \( N^f_i \) denotes labor services bought in period \( i \) by the firm. The production function \( h \) is assumed to exhibit positive but diminishing marginal productivity of labor. The profits of the firm flow to the household sector.\(^7\)

In equilibrium, households and firms determine all their demands and supplies (of labor, commodities, bonds) as a function of prices and resource endowments. In disequilibrium situations, certain quantity variables may become exogeneous to certain market participants, as emphasized by the disequilibrium approach referred to above.\(^8\) We will consider individual behavior without such disequilibrium-generated quantity constraints, as well as with such constraints.
Household Behavior. The household's problem is to maximize (1) subject to constraint (3). This yields the following set of optimality conditions:

\[(a) \quad u_i + \lambda v_i = 0 \quad (i = 1 \cdots n)\]
\[(b) \quad u_i - \lambda v_i w = 0 \quad (i = 1 \cdots n)\]
\[(c) \quad \sum_{i=1}^{n} v_i y_i - \sum_{i=1}^{n} [v_i w N_i + \pi_i - wt(g_i)] = 0\]

These 2n+1 equations implicitly determine the \(y_i\), \(N_i\), and the Lagrange multiplier \(\lambda\), for given \(r\) (and thus \(v_i\)), \(w\), \(\pi_i\) and \(g_i\). The commodity demand and labor supply functions implicit in (4) thus contain as arguments the relative prices \(r\) and \(w\), as well as the expected stream of government produced goods and services.  

In a situation of excess supply of labor (unemployment), the representative household is unable to sell the full amount of labor services he wishes to sell, and \(N_i\), and consequently his labor income \(w N_i\), becomes exogeneous to him—being determined by the extent of market excess supply and his individual "queuing" position.  

Comparative static effects of parameter changes on the endogeneous variables (i.e., the \(y_i\) levels, and the \(N_i\) levels, if they are permitted to adjust) are obtained by differentiation of the system of equations (4). A change in the stream of government provided goods and services (or the government expenditures necessary to produce this stream)
affects the household's decision in two ways: First, via the budget constraint, and thus the government production function \( t(g) \); second via the utility function. The weakness of the conventional treatment of government expenditures is that only the first of these effects is taken into account, while the second is neglected, i.e., the fact that an increase in government produced services in a period changes the marginal valuation of privately produced commodities and leisure in this period is disregarded. Obviously, the result of such a parameter change will, in general, depend on the substitution and complementarity relationships between private and government produced goods, as well as leisure. Also, the government production, or transformation function is usually not taken into account.

It is useful at this point to distinguish between a permanent increase in government expenditures, introduced in the current period (\( dg_i = dg_0, i = 1 \ldots n \)), and a temporary increase (\( dg_i = dg_0, dg_j = 0, j = 2 \ldots n \)). How will these changes affect current consumption expenditures \( y_i \) and savings (or bond accumulation), or, more generally, how do they affect the distribution of consumption over time? In particular, will current consumption of privately produced goods \( y_i \) be reduced by an amount smaller or larger than \( dg_i \)?

Consider first the case of a permanent increase in \( g \), introduced in period one. In this case, the budget constraint (4c) implies:

\[
(5a) \quad \frac{dy_i}{1} \left( v_i \frac{dN_i}{dG_0} - v_i \frac{dG_i}{dG_0} \right) = -wt_i \quad (i \in I)
\]

or, if the \( N_i \) cannot adjust (\( dN_i = 0 \))
It is quite clear on intuitive grounds, that the magnitude of the multipliers $\frac{dy_i}{d\gamma_0}$ (and $\frac{d\bar{W}_i}{d\gamma_0}$, if they are allowed to adjust) will depend on the complementarity/substitution relationships between the household's different choice objects, and on $t_g$, that is, the marginal productivity of labor in government production. An increase in the flow of government services $g_0$ can induce substitutions in two directions. First, contemporaneous substitution between leisure, government and privately produced goods and services is possible in each time period, and thus over the planning period as a whole. If the $N_i$ can adjust, the increase in $g_0$ may be compensated by reductions in the consumption of both leisure and privately produced commodities. However, this is not necessarily so. For instance, if government-provided goods and services are relatively "time intensive," i.e., are complementary to leisure (as might be the case, e.g., for recreational facilities, such as state parks) the consumption of leisure may actually increase, and the demand for privately produced goods consequently decline by more than $dg_0$. If the $N_i$ are not permitted to adjust, the increased consumption of government-provided services, over the planning period as a whole, has to be matched precisely by a corresponding reduction in the consumption of private commodities, as stated by (5b). Second, across-period substitution, i.e., a redistribution of "consumption" (in a broad sense, including all utility generating objects) over time is possible. Regardless of whether the $N_i$ can adjust or not, there is no a priori reason why the distribution of consumption (in the broad sense just mentioned) should be changed in favor
of either present or future periods. This is the same reasoning as was employed by Modigliani and Brumberg in their classic article on the life cycle hypothesis of consumption behavior when they stated, while discussing the effect of an increase in total "wealth" or resources, "...we are unable to think of any systematic factor that would tend to favor any particular period relative to any other." That means, for the case where the \( N_1 \) cannot adjust (which corresponds to the usual assumption in fiscal policy models) that the likelihood for the reduction in the level of \( y \) in each period to be larger or smaller than the cost (in terms of \( y \)) of the additional government services, \( w_t g dg_0 \), has to be considered the same. If the marginal product of labor in government production (whose inverse is given by \( t_g \)) is equal to the wage rate \( w \) (and thus to the marginal product of labor in private production), we have \( w t_g = 1 \), and thus equal likelihood for \( dy_1/dg_1 \), and specifically for \( dy_1/dg_1 \), to be larger or smaller than minus one.

For instance, for just two periods, and assuming (for simplicity) zero across period derivatives for the utility function, we would have (for \( dN_1 = 0 \)):

\[
\frac{dy_1}{dg_0} = \frac{wt_g (v_1 + v_2) v_1 U_{22} + v_2^2 U_{1g} - v_1 v_2^2 U_{2g}}{-v_2^2 U_{11} - v_1^2 U_{22}}
\]

(6)

\[
\frac{dy_2}{dg_0} = \frac{wt_g (v_1 + v_2) v_2 U_{11} - v_1 v_2^2 U_{1g} + v_1^2 U_{2g}}{-v_2^2 U_{11} - v_1^2 U_{22}}
\]

(\( U_{11} = \frac{\partial^2 U}{\partial y_1^2} \), \( U_{22} = \frac{\partial^2 U}{\partial y_2^2} \), \( U_{1g} = \frac{\partial^2 U}{\partial y_1 \partial g_1} \), \( U_{2g} = \frac{\partial^2 U}{\partial y_2 \partial g_2} \)).
This implies (as is clear from the budget constraint alone) that
\[
\frac{\partial y_1}{\partial g_0} + v_2 \frac{\partial y_2}{\partial g_0} = -w_t (v_1 + v_2),
\]
or, if government is equally productive, at the margin, as the private sector \((w_t = 1)\), that
\[
\frac{\partial y_1}{\partial g_0} + v_2 \frac{\partial y_2}{\partial g_0} = -(v_1 + v_2).
\]

This is satisfied, e.g., if both \(\frac{\partial y_1}{\partial g_0}\) and \(\frac{\partial y_2}{\partial g_0}\) have a value of minus one \((-w_t\), more generally speaking). It is quite possible, of course, for \(|\frac{\partial y_1}{\partial g_0}|\) to be less than one. But then, \(|\frac{\partial y_2}{\partial g_0}|\) must exceed one, and vice versa. Since the expressions for these two multipliers are perfectly symmetrical, the likelihood for them to be less or more than one appears to be the same.

Now consider the case of a temporary increase in \(g\) during period one. In this case, the budget constraint \((4c)\) implies
\[
(7a) \quad \sum_{i} \left( v_i \frac{\partial y_i}{\partial g_0} + v_i \frac{\partial N_i}{\partial g_0} \right) = -w_t g
\]
or, if the \(N_i\) cannot adjust \((\Delta N_i = 0)\)
\[
(7b) \quad \sum_{i} \frac{\partial y_i}{\partial g_0} = -w_t g
\]

If the marginal utility of privately produced goods and services were completely independent of government produced goods and services \((U_{yg} = 0)\), or if the current increase in the flow of government goods and services would affect the marginal utility of private goods and services in all periods in the same way, it would seem reasonable to expect that the
reduction in "disposable income" \(-w_{t_g}\) would affect the household's consumption of private goods in all periods in a similar way (as Keynesian fiscal policy models implicitly seem to assume). However, these assumptions are rather unlikely. It does seem much more likely that the current increase in the stream of government goods and services will mainly affect the marginal utility and therefore the consumption of private goods and services in the current period. On purely a priori grounds it does again seem unclear why the increase in government produced goods and services during the current period should induce the household to change his "overall consumption" (in the broad sense including all choice objects) in favor of either present or future periods. Clearly, the precise result will again depend on the degrees of substitutability of supplementarity between the (current) consumptions of government produced goods, the consumption of privately produced goods and services in the current (and future) periods, and—if the \(N_i\) can adjust—the consumption of leisure time in the current (and future) periods.

The condition for all \(\frac{dc_i}{dg_0} (i = 2 \ldots n)\) to be zero, and thus \(\frac{dc_1}{dg_0} = -w_{t_g}\), is found from the optimality conditions (4) to be

\[
(8) \quad U_{i_g} U_i - U_{i_g} U_{i_1} = (U_{i_1} U_i - U_{i_1} U_{i_1})w_{t_g} \quad (i = 2 \ldots n)
\]

(where \(U_i = \partial U/\partial y_i\), \(U_{i_g} = \partial^2 U/\partial y_i \partial g_0\), \(U_{i_1} = \partial^2 U/\partial y_i \partial y_1\)).

Firm Behavior. Firms are assumed to behave as regular profit maximizers. The representative firm maximizes its profits \(\pi = h(N) - wW\) for each period, subject to the given wage rate \(w\) and the production function \(h(N)\). The firm's optimal demand for labor, and its optimal supply of output, are derived in the traditional manner as functions of the wage
rate \( w \). In this regard, the present model does not differ from the traditional procedure. In disequilibrium situations, the representative firm will not be able to choose its output and labor input freely, as emphasized by Patinkin and Barro-Grossman. In a situation involving excess supply of commodities, the representative firm's effective demand for labor will be independent of the wage rate and depend only on the actual amount of output the firm finds it can sell. (In a situation involving excess demand for labor, on the other hand, the firm's effective supply of commodities will depend on the available amount of labor alone, independent of the wage rate.)

**Government Behavior.** The role of government in the model has been essentially described above in the context of our discussion of household behavior. Government is viewed as making its decisions in an autonomous manner, derived from some calculus not endogeneous to the present model (i.e., there is no feedback mechanism from the performance of the economy to government behavior, in accordance with the tradition in macroeconomic theory).

### III. General Equilibrium

In this section, we will consider an aggregative version of the model under conditions of general equilibrium (for the current period). That is, we will assume that every agent in the economy is able to sell or buy any amount of any good he wishes to sell or buy; nobody faces constraints on his sales and purchases. The demand and supply functions for the system under these conditions will look as follows.
Supply of goods and services by private firms:

\( y^s = y^s(w) \)

Household demand for privately produced goods and services (for the current period):

\( y^d = y^d(w, r, g_1) \)

The nature of the relationship between \( y^d \) and \( g \) was discussed in the previous section and does, as emphasized there, in general depend on the whole expected stream of \( g_1 \).

Supply of labor services by households (for the current period):

\( N^s = N^s(w, r, g_1) \)

Demand for labor by private firms:

\( N^d_f = N^d(w) \)

Demand for labor by government (for the current period):

\( N^d_g = f^{-1}(g) = t(g) \)

The supplies (and consumption) of government produced goods and services for the current period \( g \), as well as for all future periods \( (i = 2\ldots n) \) are treated as exogeneous policy parameters.

Market equilibrium requires that all markets are cleared:

\( y^s(w) - y^d(w, r, g_1) = 0 \)

\( N^s(w, r, g_1) - N^d(w) - t(g) = 0 \)

Equilibrium in the commodity and labor market implies, via the budget constraints in the system, equilibrium in the remaining market, i.e., the
market for bonds. Since we have assumed, for simplicity, a stationary economy with no capital accumulation, full equilibrium obviously requires that aggregate savings are zero: From the household's budget constraints (2) we have \( s = wN^s + \pi - wt(g) - y^d \), and from the firms profit function \( \pi = y^s - wN^d \), \( y^s = y^d \), and \( N^s = N^d + N^d \) thus imply \( s = w(N^s - N^d - N^d) = 0 \).

An increase in the amount of government produced goods and services \( g \) will affect the general equilibrium solution through both markets. First, it will directly raise labor demand. Second, it will affect the demand for privately produced goods and services (without directly affecting the supply of such goods and services, however). Finally, it may also affect the supply of labor. From (14a) and (14b) we find the effect of an increase in \( g \) on equilibrium \( w \)

\[
\frac{dw}{dg} = \frac{y^d N^d + y^d(t - N^s)}{(y^s - y^d)N^s + y^d(N^s - N^d)}
\]

and thus the effect on equilibrium employment

\[
\frac{dN}{dg} = N^d \frac{dw}{dg} + t \quad (where \quad \frac{dN}{dg}, \quad and \quad \frac{dw}{dg}, \quad \text{may depend on whether the increase in} \quad g \quad \text{is temporary or permanent, as discussed in the previous section}).
\]

Quite clearly, the signs of these multipliers cannot be evaluated without imposing numerous restrictions on the partial derivatives of (14a) and (14b). If changes in the interest rate affect mainly the distribution of consumption over time, but not the contemporaneous choice between leisure and commodities \( N^s \to 0 \), and if, furthermore, \( N^s \) is
zero or negligible, we have (15') \( \frac{dv}{dg} = \frac{t}{N^w_d - N^w_s} > 0 \) (if \( N^w_d - N^w_s > 0 \)), as is usually assumed), and thus (16') \( \frac{dw}{dg} = \frac{t}{N^w_d - N^w_s} > 0 \) (if \( N^s_w > 0 \)). For more general forms of the excess demand functions (14a) and (14b), however, the signs of these multipliers will be ambiguous. However, in the context of general equilibrium, policy multipliers such as those discussed here are not really of major interest, anyway, since economic stabilization policy essentially deals with problems of disequilibrium, rather than equilibrium. After all, we typically want to know whether, and how, government policy measures can help in eliminating certain disequilibrium situations, such as, e.g., unemployment of labor. The discussion of some disequilibrium constellations will be the subject of the following section.

IV. Disequilibrium Constellations

The demand and supply functions of the general equilibrium model of the previous section are derived on the basis of the assumption that every decision unit (household, firm) can, at the given prices, buy and sell any amount it wishes of any good, i.e., it experiences no quantity constraints (except on the resource endowments brought into the period). This cannot be the case (for every decision unit) in disequilibrium, as was pointed out by recent contributions to disequilibrium theory. Thus, the demand and supply functions of the preceding section cannot apply in such circumstances. Demand and supply functions for disequilibrium must take account of the quantity constraints on sales or purchases which the
disequilibrium imposes on at least some market participants. The precise nature of these constraints depends on the precise nature of the specific disequilibrium constellation in question, or course. In the following, we will consider two possible constellations:

a) Excess supply in both the labor and the goods and service markets;
b) Excess supply in the labor market, combined with excess demand in the goods and service market.

It will be shown, that the macroeconomic effects of government expenditure policies will be quite different in these two cases.

Excess Supply in both the Labor and the Goods and Service Markets (General Excess Supply).

Excess supply in the labor market means that the representative household is not able to satisfy his (notional) labor supply function (11), but has to accept a constraint on his sales of labor services which is exogenous to him. Consequently, following Clower, we have the effective demand for goods and services $\bar{y}^d$ depending, in the Keynesian fashion, on actual current employment $N$ (and thus on actual current income $w N + \pi$), since it enters as an exogeneous constraint into the household's decision process, in addition to the other exogeneous factors (i.e., the household is able to satisfy his optimality conditions (4a) and (4c) only, but not (4b)):

\[(17) \quad \bar{y}^d = \bar{y}(w, r, N, \xi) \quad \text{with} \quad \bar{y}_N > 0 , \quad \bar{y}_\xi < 0 .\]

Excess supply in the commodity market means that the representative firm cannot satisfy its desired (or notional) supply of output. Following
Patinkin and Barro-Grossman, the firm's effective labor demand $N^d_f$ then depends on the actual amount of output it can sell, $y$, independent of the wage rate:\textsuperscript{13}

\begin{equation}
N^d_f = h^{-1}(y)
\end{equation}

The total effective demand for labor is thus

\begin{equation}
N^d = N^d_f + N_g = h^{-1}(y) + t(g) = \bar{N}(y, g), \text{ with } \bar{N}_y > 0, \bar{N}_g = t > 0.
\end{equation}

According to (19), an increase in $g$ does, given $y$, increase labor demand and thus employment. But at the same time, according to (17), an increase in $g$ does, given $N$, lower the effective demand for privately produced goods and services $\bar{y}^d$, and thus the amount of output private firms can sell and—via (19)—their demand for labor $h^{-1}(y)$.

If the prices $w$ and $r$ are fixed, so that the existing excess supplies in the two markets cannot be eliminated through price adjustments, the two effective demand functions (17) and (19) together yield a Barro-Grossman, or Keynesian, model of simultaneous output and employment determination in the short run. That is, they allow us to simultaneously determine (short run) equilibrium levels of $y$ and $N$, such that $y = \bar{y}^d$ and $N = \bar{N}^d$:\textsuperscript{15}

\begin{equation}
y = \bar{y}(N, g; w_0, r_0)
\end{equation}

\begin{equation}
N = \bar{N}(y, g)
\end{equation}

Graphically, this can be represented by a "Keynesian cross" type diagram, with the $45^\circ$ line being replaced by the $\bar{N}(y, g)$ function (Figure 1).
A change in $g$ shifts both curves: The $y( )$ function will shift downwards, and the $N( )$ function to the right, as a result of an increase in $g$.

Algebraically, the effect of an increase in $g$ on the employment level $N$ is obtained from (20) as

$$\frac{dN}{dg} = \frac{\bar{N} \cdot g + \bar{N} \cdot \bar{y} \cdot g}{1 - \bar{N} \cdot \bar{y} \cdot \bar{N}}$$

Note that in Figure 1 the two curves have been drawn such that $\bar{y} \cdot \bar{N} < \frac{1}{\bar{y}}$, or $1 > \bar{y} \cdot \bar{N}$, which assures that the denominator in (21) is positive. This corresponds to the usual stability condition in Keynesian models that the marginal propensity to spend is less than unity. If this condition is satisfied, the sign of (21) still depends on the relative magnitudes of $\bar{N} \cdot g$ (which is positive) and $\bar{N} \cdot \bar{y} \cdot g$ (which is negative, because $\bar{y} \cdot g$ is negative, as discussed in section II).

If the increase in $g$ does not affect the distribution of consumption over time, its effect on the demand for privately produced
goods and services (here denoted as \( \bar{y}_g \)) is, according to the micro-
analysis in section II, equal to \(-\bar{w}_t\). Since \( \bar{N}_g \) is the inverse of the
marginal product of labor in private production, and thus, in a competitive
system, of the wage rate \( \bar{w} \), and \( \bar{N}_g \equiv t \), we would have in this case
\( \bar{y}_g = - (\bar{N}_g/\bar{N}_y) \), and thus \( dN/dg = 0 \) according to (21). Since, as empha-
sized in section II, there is no a priori reason why a change in \( g \) should
influence the intertemporal distribution of consumption in any systematic
way, there is no a priori reason to expect \( dN/dg \) to diverge from zero
in any systematic way, in the case under discussion here. Expressed
differently, the likelihood for \( dN/dg \) to be positive appears to be
neither larger nor smaller than the likelihood for it to be negative.

**Excess Supply in the Labor Market, combined with Excess Demand in the
Goods and Service Market.**

Such a situation requires a wage rate which is "too high" (relative
to the equilibrium wage rate), together with an interest rate which
favors too much current consumption at the expense of future consump-
tion. (In an economy with money, it may be generated, e.g., by a quickly
reduced rate of monetary expansion during an inflationary period
characterized by excess demand for goods and services. Such an action
may, via lags in the adjustment of expectations raise the wage rate,
relative to the equilibrium rate, and thus create unemployment, while
some excess demand for goods may still persist (a movement along a
"short run Phillips curve" to the right).

In such a situation, firms face no quantity constraints (no sales
constraints, no employment constraints). But the representative
household faces two simultaneous constraints here: an employment constraint as well as a purchase constraint. This leaves the household no choice at all with respect to his current decisions; \( N, y \) and \( g \) are all predetermined from it's point of view (by the extent of the market disequilibria, and the individual household's "queuing position" in the respective markets), and thus so is, via the budget constraint, it's savings for the current period. An increase in \( g \), ceteris paribus, can only affect savings here. Similarly, an increase in \( N \), given \( y \) and \( g \), must raise savings. An increase in \( y \), given \( N \) and \( g \), must reduce savings.

Movements of employment \( N \) and output \( y \) are then essentially firm-determined, with households adjusting passively. The macroeconomic effects of a change in \( g \) are thus determined by how it affects the behavior of firms. The firm's demand for labor and supply of output functions are given by (12) and (9), respectively. For fixed prices, we have then the following system of short run output and employment determination (in place of (20):

\[
N = N^d_f(v_0) + t(g) \tag{22}
\]

\[
y = y^s(v_0)
\]

In contrast to the case discussed first, an increase in \( g \) here will raise \( N \) regardless of the issues involved there. The fact that an increase in \( g \) reduces—via the budget constraint—the demand for privately produced goods \( (y^d) \) will not affect firm demand for labor in a situation where these goods are in excess demand. An increase in \( g \) will
not only unambiguously raise employment \( N \), but furthermore lower the extent of the excess demand in the goods market, here.

Formally we have, of course,

\[
(23) \quad \frac{dN}{dg} = t_g > 0,
\]

in this case. The short run response of the system to an increase in \( g \) is thus quite different in this case compared to the case considered first, because of the different nature of the constraints imposed on the economic units in the system by the two types of disequilibrium constellations.

**Concluding Remarks**

The preceding discussion of the effects of government expenditure policies did concentrate on situations involving excess supply in the labor market, i.e., involuntary unemployment. The analysis could, of course, be extended to other possible disequilibrium constellations, specifically those involving excess demand for labor.\(^{16}\) It was the basic purpose of this paper, to demonstrate that a satisfactory analysis of government expenditure policies with choice theoretic foundations is considerably more complicated than the standard fiscal policy models seem to suggest, and, specifically, that an analysis of their short run effects requires careful differentiation between various possible constellations of excess demand and excess supply in the different markets of the economy, and the constraints they impose on the economic agents in the system.
Footnotes

1 See Bailey (1962), Chapter 10, pp. 71-81.

2 One somewhat paradoxical implication of this neglect is the result that, according to the standard treatment, a tax financed government expenditure has a positive multiplier effect on income if it represents an expenditure on goods and services, but a zero effect if it is paid out by government as a simple transfer payment. From the consumer's point of view, is there such a substantial difference between the two situations?


4 See Bailey (1962, pp. 75-77), Barro (1974).

5 The traditional treatment assumes, in effect, that the private sector has a marginal propensity to spend of unity with respect to income provided to it by government in the form of specific goods and services, while it has a marginal propensity to spend of less than one with respect to all other types of income.

6 Which are not at issue here, since in their case the net effect of increasing government spending and taxes by equivalent amounts is clearly zero, even in the traditional analysis, because of an unchanged "disposable income."

7 But note that the household takes his $π_i$ for all periods as exogenous, since he is not allowed to invest his savings in the form of equity in firms during the planning period (no capital accumulation, predetermined ownership pattern for firms).

8 See note 3 above.

9 Appropriate second order conditions must be satisfied, too, of course.


12 See Patinkin (1965, Ch. 13), and Barro and Grossman (1971, p. 85).
In a model with just one variable input.

Which is, by assumption, taxed financed in this context.

The notional supply functions (9) and (11) cannot be satisfied here, by assumption, since the prices \( w \) and \( r \) are assumed to be fixed accordingly.

See Barro and Grossman (op. cit., pp. 90-92) for an analysis of a situation of general excess demand, in a system without government activities.
References


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