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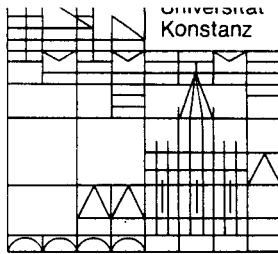
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**Fakultät für
Wirtschaftswissenschaften
und Statistik**

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**The Use of Associated Gas
in a Petroleum Exporting Country:
An Optimal Control Model**

Diskussionsbeiträge

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THE USE OF ASSOCIATED GAS
IN A PETROLEUM EXPORTING COUNTRY:
AN OPTIMAL CONTROL MODEL

Michael Rauscher
Serie A - Nr. 200

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Serie A: Volkswirtschaftliche Beiträge

Serie B: Finanzwissenschaftliche Arbeitspapiere

Serie C: Betriebswirtschaftliche Beiträge

Abstract

Currently, the world petroleum market is facing attempts by petroleum exporting countries to extend their influence further downstream. The process of vertical integration is based on natural gas which accrues as a by-product of crude oil recovery. Gas can be used for downstream production, for reinjection into the oilfield in order to enhance future recovery, or it can simply be flared. The model investigates how to optimally use the associated gas in an intertemporal framework. The price and extraction paths, associated with the optimal use of gas are examined, compared with the relevant time paths of the "standard" model.

The results presented in this paper originate from my work in the Sonderforschungsbereich 5 "Staatliche Allokationspolitik im marktwirtschaftlichen System", University of Mannheim. I have benefitted from helpful comments by E. Mohr and H. Siebert.

1. Introduction

During the last fifteen years, the world petroleum market has experienced significant structural changes. After having seized the property rights over their resources, many oil exporting countries are now extending their influence further downstream. They begin to become involved in refining, production of petrochemicals, and transportation of crude oil and products (See tables 1 and 2).¹⁾

Table 1: Refinery capacity in petroleum exporting countries (1000 barrels/calendarday)

	1977	1982	1987*
Algeria	116	471	471
Iran	995	560	560
Iraq	184	306	556
Kuwait	594	594	799
Libya	130	130	350
Nigeria	57	247	247
Saudi-Arabia	703	1028	2023
Venezuela	1445	1323	1323
total OPEC	4822	5416	7916

* planned (data on planned expansions from: Fesharaki, Isaak (1984), data on existing capacities: OPEC Annual Statistical Bulletin 1982)

¹⁾ The process of vertical integration has been described in detail by Fesharaki and Isaak (1983 and 1984).

Table 2: Capacity for ethylene production
in petroleum exporting countries
(1000 tons/year)

	1978	1982	1987*
Algeria	120	120	120
Iran	-	26	326
Iraq	-	-	-
Kuwait	-	-	350
Libya	-	-	330
Nigeria	-	-	300
Saudi-Arabia	-	-	1606
Venezuela	150	150	150
total OPEC	270	576	3602

* planned (all data from: OPEC Annual Statistical Bulletin 1982)

All these activities require large amounts of capital and know-how. At present, the success of vertical integration is far from clear since the oil exporting countries expand into shrinking markets. Since the beginning of the eighties, the western world faces overcapacities both in the refining and in the petrochemicals sectors.²⁾ While there are good political reasons for downstream integration, the attainability of the economic goals remains questionable. According to the theory of vertical integration a monopolist can extend his or her power and increase profits by vertically integrating downstream markets. OPEC, however, is no upstream monopolist. But as a swing producer on the world petroleum market, OPEC has some influence on prices (which, however, has considerably been weakened during the last five years). Secondly, if the downstream industry is governed by a linear-limitational technology, no additional profits can be

²⁾ The situation of the downstream markets has recently been described by Quinlan (1983), Baum (1985) and Vielvoye (1985).

derived from vertical integration.³⁾ And most of the petroleum processing requires a linear-limitational technology. For instance, the input-output ratio in refining as well as petrochemicals production is constant. Capital or labour cannot be substituted for crude oil. Nevertheless, petroleum exporting countries may profitably operate downstream activities. They do have the advantage of owning an almost free good which can be used as a feedstock for petrochemicals production or as an energy input in the refining industry: So-called associated gas accrues as a by-product of petroleum extraction. Most of this associated gas has been flared until recently, since there existed almost no demand for it. While some of the gas could be used for seawater desalination and domestic energy needs, most of it had to be wasted. The flaring-total-recovery ratio of OPEC gas lies in a range of about 60% or more in the middle of the seventies and 30% recently (See table 3 for more detailed data). The decrease has been caused by the halving of demand for OPEC oil during the last five years. It should be expected that, with increasing demand, the amount of available gas will increase. In addition to domestic energy generation, refining, and production of petrochemicals, two further alternative utilisations for associated gas exist: It either can be liquefied and exported, or else be reinjected into the oil field in order to sustain the pressure and increase the amount of recoverable oil. Gas liquefaction is also a kind of downstream production, for it is a production process in which a raw material is transformed into an intermediate product. There remain four possibilities of gas utilisation:

³⁾ This is due to the fact that a monopolist as well as a competing firm cannot change factor intensities in order to adjust production to demand conditions. See Blair, Kaserman (1983), ch. 3, for further details.

- domestic use,
- reinjection,
- export of downstream products and
- flaring.

Table 3: Utilisation of natural gas 1982

	world		OPEC		non-OPEC	
	Bill m ³	%	Bill m ³	%	Bill m ³	%
total recovery	1857	100	263	100	1594	100
net production	1600	86	108	41	1491	94
reinjection	144	8	80	30	64	4
flared	120	6	75	28	45	3

Source: OPEC Annual Statistical Bulletin 1982

The decision how to optimally use associated gas will be examined in an optimal control framework. Three of the abovementioned alternatives will be considered in the model. The possibility of domestic use, however, will not be taken into account. Both vertical integration and reinjection have intertemporal consequences. Downstream production requires capital, and capital utilisation has a positive effect on future consumption. Reinjection, on the other hand, increases the resource base future generations can avail themselves of. Thus both activities are costly to the present generation, but benefitting future ones.

In the following section the basic assumptions of the model will be exposed. The model is in the Hotelling (1931) tradition. Here, however, the resource owner is not an enterprise which intends to maximise profits but a resource exporting country maximising welfare. Along these lines, there has been recent research, e.g. by Dasgupta, Eastwood and Heal (1978), Aarrestad (1979), Seierstad and Sydsæter (1983) and Siebert (1985). In some respects, the model is similar to the ones which have been examined by Dasgupta, Eastwood and Heal (1978), Aarrestad (1979) and Siebert (1985, ch. 6). In

Aarrestad's and Siebert's models the resource exporting country imports capital goods which can be used in the domestic industry. However, the resource itself is not used in the downstream process. Dasgupta, Eastwood and Heal even consider the possibility of a downstream production process. In their model the resource can be used as an input into the production of a consumption good, the production function being of the neoclassical, well-behaved Cobb-Douglas type. This assumption cannot be used for examining the problem of gas utilisation since, as far as intermediate products are concerned, the input-output ratio is fixed.

In the present model as well as in the other models the resource exporting country has the possibility of borrowing abroad and buying foreign assets. It is assumed that neither borrowing nor lending constraints exist, except that debt has to be repaid at the end of the programme. The petroleum exporting country can export crude oil or an intermediate good, which is produced in the downstream sector by means of associated gas and capital and which can be sold in a competitive downstream market. Associated gas can also be used for reinjection to enhance future oil recovery or else simply be flared. A multi-purpose good is imported. It can be used for consumption, downstream investment, or for reinjecting associated gas. The petroleum exporting country is assumed to be a monopolist.⁴⁾ This assumption does not seem to be realistic, since OPEC's power on the world petroleum market has shrunk considerably during the last five years and, recently, problems of cartel stability have been arising. But OPEC as a swing producer can still influence the petroleum

⁴⁾ The case of a resource exporting monopolist has been examined by Kemp and Long (1979), whose primary concern has been the interaction of resource-rich and resource-poor countries. The question of optimal taxation as a defense against monopolistic pricing will not be dealt with here.

price by means of its production policy.⁵⁾ In the long run a substantial increase in the demand for OPEC oil should be expected, leading to a strong position of OPEC countries on the world petroleum market again. Another reason for assuming the upstream monopoly is computational simplicity. For, otherwise, differential game theory would have been to be employed.

In the following section the model will be presented. Section 3 contains the formal treatment of the optimisation problem. Section 4 is devoted to the long-run properties and the existence of optimal policies. In sections 5, 6 and 7, conditions for the optimal pricing of petroleum and the optimal use of associated gas are derived. A critical evaluation of the basic assumptions of the model concludes the paper.

2. The Model⁶⁾

The petroleum exporting country intends to maximise the present value of welfare u , which is assumed to be an increasing and strictly concave function of consumption C , $u(\cdot)$ be continuous for $C \geq 0$ and twice differentiable for $C > 0$, $u(0) = 0$, $u'(0) = \infty$ and $u'(\infty) = 0$.

$$(1) \quad \max \int_0^T u(C) e^{-\delta t} dt$$

⁵⁾ This question has been dealt with in an article by Ruggeri (1983) who has investigated the interdependency of OPEC's pricing policy and its market share.

⁶⁾ For the sake of a simpler notation, time indices will be omitted. All the upper case letters denote time dependent variables, the lower case letters parameters or functions. The model is formulated such that all parameters are positive.

subject to the constraints

- (2a) $\dot{S} = \sigma G_1 - R$ S=S₀ for t=0
- (2b) $\lim_{t \rightarrow \tau} S \geq 0$
- (3) $gR \geq G_1 + G_2$
- (4a) $Z = aG_2$
- (4b) $aG_2 \leq bK$
- (5) $\dot{K} = I - mK$ K=0 for t=0
- (6) $p(R)R + \pi Z + rV = \dot{V} + I + C + wG_1$ V=V₀ for t=0
- (7a) $G_1 \geq 0$
- (7b) $G_2 \geq 0$

List of variables:

- S stock of recoverable oil
- K capital stock
- V stock of foreign assets
- C consumption
- I gross investment
- R production of crude oil
- gR recovery of associated gas
- G₁ reinjected gas
- G₂ gas used in the downstream industry
- Z output of the downstream industry
- σ additional units of recoverable oil per unit of reinjected gas
- a,b marginal (and average) productivities of the factors which are used in the downstream industry
- p price of oil measured in units of the consumption good
- π constant price of the intermediate good measured in

	units of the consumption good
w	imported goods to be used for reinjecting a unit of gas
m	rate of depreciation of the capital stock
δ	discount rate
r	interest rate
t	time
T	free terminal time of the programme

Equation (2a) denotes the changes of the resource stock, due to extraction of oil and reinjection of gas. In order to simplify the following analysis, zero extraction cost and constant additions to the stock per unit of reinjected gas are assumed.⁷⁾ If equality holds in (3) all the associated gas is used, either in the downstream industry or for reinjection. Otherwise, gas is flared. Obviously,

$$\sigma g < 1$$

for, otherwise, reinjection would create an infinite resource stock. σg is the percentage by which the resource base can be extended by reinjection.⁸⁾ Realistic values of σg are in the range of 0.2 to 0.3. (4a) and (4b) is the production function of the downstream industry. Marginal and average productivities are assumed to be constant. Natural gas is the limiting factor. If inequality holds in (4b), there is excess capacity. The interpretation of (5) is straightforward: net investment equals gross investment minus capital depreciation. (6) is the balance of payments equation where the LHS denotes exports plus returns on foreign assets. The RHS denotes imports (consumption goods C, capital goods I, and goods

⁷⁾ In a more realistic formulation, the additions should also depend on the size of the reservoir and cumulative extraction.

⁸⁾ In this model, σg also is the percentage of reinjected gas which can be re-recovered.

needed for reinjection wG_1) plus capital exports. Negative values of V denote debt. In the long run, V should be positive:

$$(8) \quad \lim_{t \rightarrow T} V \geq 0.$$

In the model, there exists only one imported good which can be used for consumption, downstream investment, reinjection and saving. Therefore, foreign assets as well as capital are assumed to be completely shiftable. They can be transformed into each other or into consumption goods without additional costs in an infinitesimal length of time. The prices of exported goods are measured in units of the imported good. The price of the downstream intermediate product is assumed to be constant and independent of the crude oil price. The inverse demand function $p(R)$ be continuous for $R \geq 0$, twice differentiable for $R > 0$, monotonously decreasing and demand be zero at a finite price p^* . p^* might be interpreted as the price of a substitute which is produced by means of a backstop technology.

$$(9a) \quad p'(R) < 0$$

$$(9b) \quad p(0) = p^*$$

(7a) and (7b) are the non-negativity constraints. A non-negativity constraint for petroleum extraction R is not needed, because the combination of (7a), (7b) and (3) assures that R is positive.

3. Conditions of optimality

The present value Lagrangian is⁹⁾

$$(10) \quad L = u(C) + \Gamma(\sigma G_1 - R) + \Phi(p(R)R + \pi a G_2 - w G_1 - C - I + rV) \\ + \Omega(I - mK) + \theta_2(bK - aG_2) + \theta_G(gR - G_1 - G_2) \\ + \theta_1 G_1 + \theta_2 G_2,$$

where the multipliers Γ , Φ , Ω , θ_2 , θ_G , θ_1 and θ_2 satisfy the following conditions:

$$(11a) \quad \theta_2(bK - aG_2) = 0 \quad \theta_2 \geq 0$$

$$(11b) \quad \theta_G(gR - G_1 - G_2) = 0 \quad \theta_G \geq 0$$

$$(11c) \quad \theta_1 G_1 = 0 \quad \theta_1 \geq 0$$

$$(11d) \quad \theta_2 G_2 = 0 \quad \theta_2 \geq 0$$

$$(12) \quad \dot{\Gamma} = \delta \Gamma$$

$$(13) \quad \dot{\Omega} = (\delta + m)\Omega - b\theta_2$$

$$(14) \quad \dot{\Phi} = (\delta - r)\Phi$$

$$(15a) \quad \lim_{t \rightarrow T} e^{-\delta t} L = 0$$

$$(15b) \quad \lim_{t \rightarrow T} e^{-\delta t} \Gamma S = 0$$

$$(15c) \quad \lim_{t \rightarrow T} e^{-\delta t} \Phi V = 0.$$

Γ , Φ and Ω are the shadow prices of petroleum in situ, foreign assets, and capital, respectively. They denote the effects of marginal changes of the resource stock, the foreign assets stock or the capital stock on the welfare integral. Equations (12) to (14) imply that the present values of these shadow prices are constant. (15a) is the transversality condition

⁹⁾ The optimality conditions are derived by application of Pontryagin's maximum principle. See Takayama (1974) and Long, Voursden (1977) .

determining the end of the programme. (15b) and (15c) are the transversality conditions for the resource stock and the stock of foreign assets, respectively.¹⁰⁾

Maximisation with respect to the control variables C, I, R, G₁, and G₂ yields

$$(16) \quad u' = \Phi \quad (\text{wrt } C)$$

$$(17) \quad \Phi = \Omega \quad (\text{wrt } I)$$

$$(18) \quad \Gamma = \Phi p(R) [1 + \epsilon(p(R))] + g\theta_G \quad (11) \quad (\text{wrt } R)$$

$$(19) \quad \sigma\Gamma = w\Phi + \theta_G - \theta_1 \quad (\text{wrt } G_1)$$

$$(20) \quad \Phi a\pi = a\theta_2 + \theta_G - \theta_2 \quad (\text{wrt } G_2)$$

From equations (16), (17), and (18) it follows that the costate variables Φ , Γ and Ω are positive. Petroleum in situ as well as capital and foreign assets have a positive effect on future consumption and, therefore, on future welfare, too. Note that a positive value of Γ implies that the resource is depleted on the optimal path (at least approximately if the extraction period extends to infinity). A further condition for optimality is that the matrix of the second derivatives be negative semidefinite. It can easily be shown that this is true for

$$(21) \quad Rp'' + 2p' < 0$$

Marginal revenue should be a decreasing function of R. $p(R)$ is assumed to satisfy the second order condition. E.g., this holds for isoelastic demand functions whose price elasticity of demand is larger than -1 and for linear demand functions.

¹⁰⁾ If the optimal time horizon extends to infinity, (15b) and (15c) cannot be applied. Then, only (15a) is a necessary condition for an optimal path. See Michel (1982).

¹¹⁾ $\epsilon(p(R))$ is the price elasticity of oil demand: $\epsilon(p(R)) \equiv p'(R)R/p(R)$. Obviously, it is negative.

Establishing growth rates in (16) and inserting (14) yields the consumption profile on the optimal path:

$$(22) \quad \hat{u}'(C) \equiv \epsilon[u'(C)] \cdot \hat{C} = \delta - r$$

This result is the well known Ramsey rule.¹²⁾ If the Ramsey rule holds, the planner is indifferent whether to save or consume the last unit of the consumption good.

4. The length of the optimal programme

In order to determine the optimal end point of the programme, the transversality condition (15a) has to be examined. Before this can be done, a further result is needed:

From (17) it follows that the growth rates of the shadow prices of capital and foreign assets are equal. Therefore, the RHSs of (13) and (14) can be equalised:

$$(23) \quad \Phi(r+m) = b\theta_z.$$

Φ , the shadow price for foreign assets is positive, since on an optimal path it must equal marginal utility, which by definition is larger than 0. Therefore, θ_z has to be positive, in order for equation (23) to hold. A positive value of θ_z means that, on an optimal path, the capital stock has to be fully utilised. There will be no excess capacity. This is a result of the underlying assumptions of the model: There is no non-negativity constraint on gross investments, which means that capital is completely shiftable. Excess capacity can be reduced without additional costs by transforming capital into consumption goods or foreign assets. This will be done if on an optimal path downstream production has to be reduced.

¹²⁾ This result has first been derived in Ramsey's seminal paper. See Ramsey (1928) p. 554.

Inserting (23) and the optimality conditions (16) to (20) into (10) the value of the Lagrangian for $t=T$ can be determined:

$$(24) \quad \lim_{t \rightarrow T} e^{-\delta t} L = \lim_{t \rightarrow T} e^{-\delta t} [u - u' C + \Phi (-\epsilon(p)pR + rK + rV)] = 0$$

Φ , $-\epsilon(p)pR$, rK , and rV are non-negative. Therefore, it depends on $u - u' C$, whether the optimal programme ends within finite time. From the concavity of the utility function and $u(0)=0$ it follows that $u - u' C > 0$ for $C > 0$ and $u - u' C \rightarrow 0$ for $C \rightarrow 0$. A necessary condition for a finite endpoint is that $C(T)=0$ or, equivalently, $u'[C(T)]=\infty$. Since the growth rate of marginal utility is constant, there can be no optimal consumption path leading to infinite marginal utility within finite time. Therefore, the time horizon has to be infinite.

If the interest rate exceeds the discount rate convergence problems arise, since consumption increases during the whole planning period. In order to ensure that the welfare function (1) is finite, a constraint on r , δ and the shape of the utility function has to be established.¹³⁾ It can easily be shown that this constraint is

$$(25) \quad \delta \cdot [\epsilon(u) - \epsilon(u')] - \epsilon(u) \cdot r > 0$$

where $\epsilon(u)$ and $\epsilon(u')$ are the elasticities of utility and marginal utility, respectively.

For an infinite time horizon, (24) has to be rewritten:

$$(26) \quad \lim_{t \rightarrow \infty} e^{-\delta t} L = \lim_{t \rightarrow \infty} e^{-\delta t} [u - u' C + \Phi (-\epsilon(p)pR + rK + rV)] = 0$$

¹³⁾ If the welfare integral were infinite, the overtaking criterion would have to be applied.

As t goes to infinity, oil extraction has to go to zero, since crude oil is an exhaustible resource.¹⁴⁾ The production of associated gas as a by-product of oil recovery has to go to zero, too. The capital-gas ratio in the downstream sector has been assumed to be constant, and above it has been established that excess capacity cannot be optimal. Therefore, the capital stock must go to zero for $t \rightarrow \infty$. Using these results and (14), (26) can be rewritten:

$$(27) \quad \lim_{t \rightarrow \infty} e^{-\delta t} [u(C) - C \cdot u'(C)] + e^{-rt} \Phi_0 rV = 0.$$

This result has been derived by Toussaint (1984) in the context of a somewhat simpler model. From $u - u' \cdot C > 0$ it follows that

$$\lim_{t \rightarrow \infty} e^{-\delta t} [u(C) - C \cdot u'(C)] = \lim_{t \rightarrow \infty} e^{-\delta t} u(C) = 0.$$

This simply is the condition for the convergence of the welfare integral which has been assumed to be satisfied. Therefore, (27) can be simplified:

$$(28) \quad \lim_{t \rightarrow \infty} e^{-rt} \Phi_0 rV = 0.$$

This implies that, in the long run, the growth rate of the foreign assets stock must be less than the interest rate.

A larger growth rate could only occur if V were negative. This follows from the balance of payments equation (6) which, as all other variables go to zero, becomes

$$\hat{V} = r - C/V.$$

Only if foreign debt is not repaid (i.e. $V < 0$) can the growth rate be larger than the interest rate. As figures 1A and 1B

¹⁴⁾ It will be shown later that oil extraction comes to an end within finite time.

Figure 1A: Increasing consumption

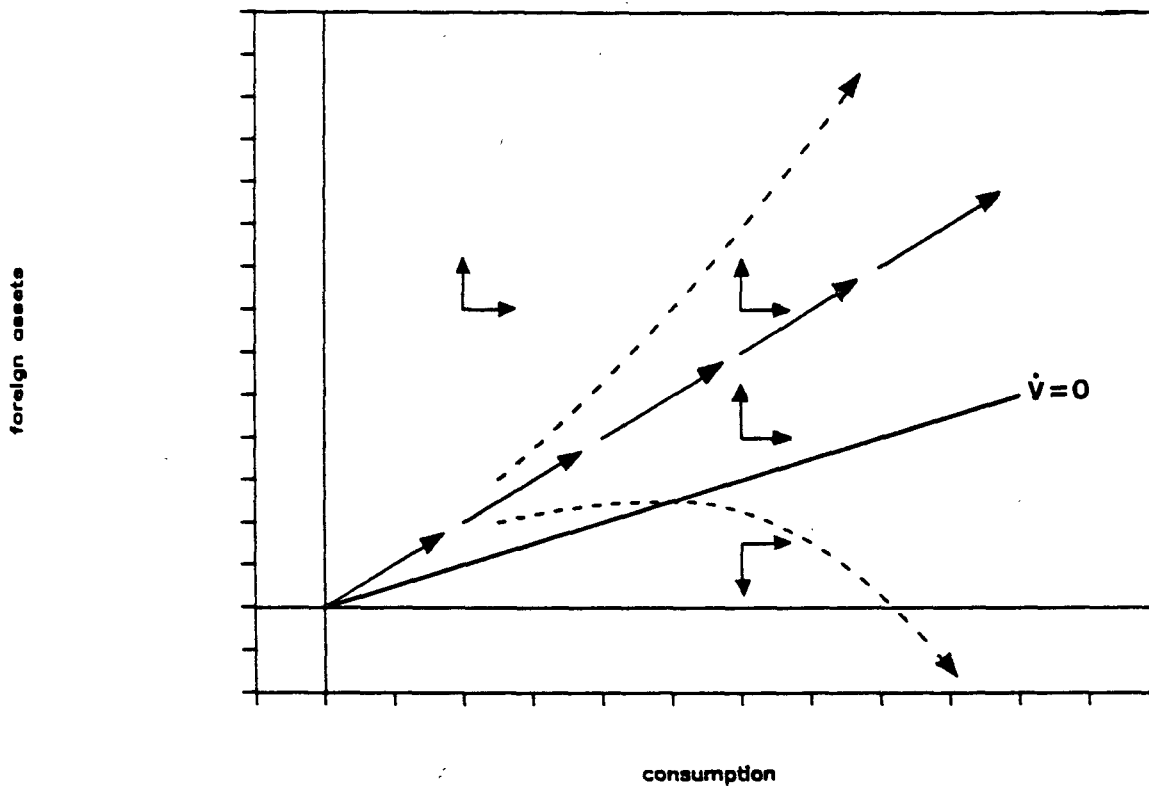
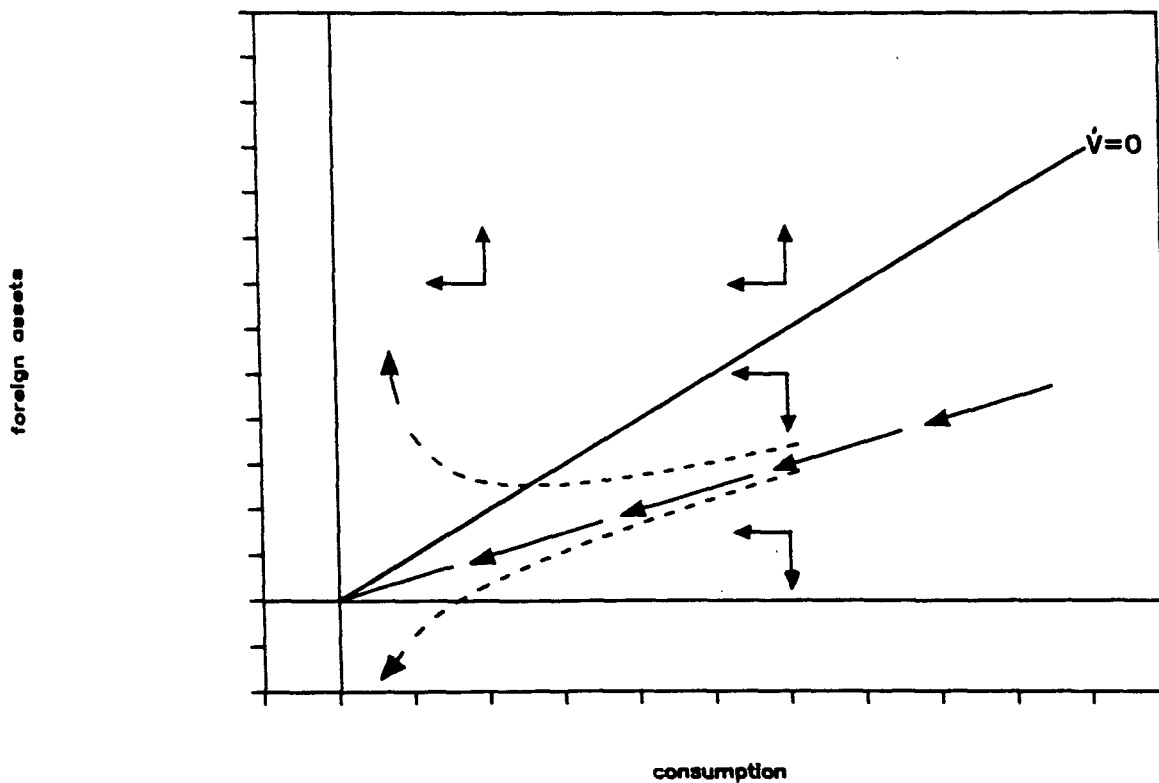


Figure 1B: Decreasing consumption



show, there exists a steady-state expansion path for foreign assets and consumption. If consumption is too low foreign assets are over-accumulated. If consumption is too high the resource exporting country is in a situation where it has to borrow in order to sustain the consumption path. If borrowing at the beginning of the planning period is necessary for building up the capital stock in the downstream sector, foreign debt should be repaid before the resource is exhausted. Moreover, the country should accumulate foreign assets; it should switch from a borrower's position to a lender's position during the interval of petroleum exports. Now, a (rather qualitative) condition for the existence of an optimal programme can be derived. It depends on the initial endowments V_0 and S_0 . If the country has not borrowed in the past, i.e. if $V_0 \geq 0$, there are no problems. If V_0 is negative and the petroleum fields are relatively small, it might be possible that the country is unable to repay foreign debt and to become a lender. In this case the transversality condition is violated and an optimal programme does not exist.

5. The optimal use of associated gas

The question to be answered by this model is, what the petroleum exporting country will do with its associated gas. Is reinjection better than industrialisation, and under which circumstances can it be optimal to flare gas? First, conditions for the optimality of reinjection will be derived. Combining (18) and (19) yields

$$(29) \quad \Phi (\sigma p[1+\epsilon(p)] - w) = (1-\sigma g)\theta_G - \theta_1.$$

$\sigma p[1+\epsilon(p)]$ is the marginal return of selling the petroleum, which has been gained from the reinjection of one unit of gas, while w are the costs of reinjecting a unit. So, if the LHS of (29) is negative (i.e. the costs of reinjection exceed the returns), θ_1 must be positive, so that no gas will be reinjected. On the other hand, if reinjection costs are low

and the LHS of (29) is positive, θ_G has to be positive - meaning that it cannot be optimal to flare gas. However, reinjection might not occur if investment in the downstream industry turns out to be more profitable.

In order to derive a condition for the optimality of downstream production θ_2 has to be inserted from (23) into (20):

$$(30) \quad \Phi (b\pi - m - r) = (b/a)\theta_G - (b/a)\theta_2$$

The shadow prices on the RHS can either be positive or zero. $(b\pi - m)$ is the net capital productivity of the downstream industry. If it is less than the interest rate, θ_2 becomes positive; none of the associated gas is to be used in the downstream industry, because foreign assets yield a higher rate of return than capital in the downstream industry. On the other hand, if the interest rate is less than the net productivity of capital, θ_G is positive and gas should not be flared. If net capital productivity equals the rate of interest, flaring and vertical integration are equivalent. A fourth case, with both θ_2 and θ_G being positive, might occur if reinjection is optimal. Then, associated gas is neither flared nor used in the downstream industry. - If, in reality, investments in the downstream sector are made, although foreign assets yield a higher rate of return, it should not be interpreted as an irrational behaviour. There might be other benefits of keeping capital at home, e.g. the possibility of forward or backward linkages. Furthermore, petroleum exporting countries have experienced the risk of forfeiting foreign assets since the freezing of Iranian assets in the USA.

In order to analyse the decision between the choice of reinjection and vertical integration, (29) and (30) have to be combined. It is possible to distinguish eight different policies. They are listed in table 4.

TABLE 4: OPTIMAL POLICIES

Policy	Flaring	Reinjection	Downstream
1	yes	no	no
2	yes	no	yes
3	yes	yes	no
4	yes	yes	yes
5	no	no	no
6	no	no	yes
7	no	yes	no
8	no	yes	yes

Policies 1 to 4 may be analysed without further formal treatment:

Policy 1: $\theta_G=0$, $\theta_1>0$, $\theta_2>0$. On the one hand, reinjection costs exceed the returns on enhanced oil recovery. On the other hand, capital productivity of the downstream industry is less than the rate of interest. Therefore, it is optimal to flare all of the associated gas.

Policy 2: $\theta_G=0$, $\theta_1>0$, $\theta_2=0$. Reinjection costs are too high so that reinjection cannot be optimal. The net productivity of capital in the downstream industry just equals the interest rate and, therefore, the planner is indifferent whether to invest downstream or hold foreign assets.

Policy 3: $\theta_G=0$, $\theta_1=0$, $\theta_2>0$. Reinjection costs just equal returns. Downstream investment is less profitable than holding foreign assets. Therefore, associated gas might be flared or reinjected, but it will not be used in the industry.

Policy 4: $\theta_G=0$, $\theta_1=0$, $\theta_2=0$. The right hand sides of (29) and (30) are zero. Reinjection, downstream production, and flaring are equivalent.

In order to derive a condition for the optimal use of associated gas, in case flaring does not occur, θ_G in (30) has to be substituted by θ_G from (29):

$$(31) \quad \Phi [\sigma p[1+\epsilon(p)]-w - (a/b)(1-\sigma g)(b\pi-m-r)] \\ = (1-\sigma g)\theta_2 - \theta_1.$$

The interpretation of the LHS is not straightforward. $\sigma p[1+\epsilon(p)]-w$ is the net return of additional petroleum sales per unit of reinjected gas. (a/b) is the gas-capital ratio in the downstream industry. So, $(a/b)(b\pi-m-r)$ is the additional profit that can be made from using a unit of gas in the downstream industry, instead of transforming the capital into foreign assets. $(1-\sigma g)$ is the percentage of reinjected gas which can not be re-recovered. Multiplying $(1-\sigma g)$ by $(a/b)(b\pi-m-r)$ we obtain the downstream profits which cannot be made, because a part of the associated gas vanishes. These are the opportunity costs of reinjection. The LHS of equation (31) now can be interpreted in the following way: Φ is the shadow price of foreign assets (measured in utility units), and the terms in brackets are the benefits of reinjection minus its opportunity costs. So the LHS of (31) denotes the net effect of reinjecting a unit of associated gas on future utility. If it is positive, θ_2 has to be positive, i.e. none of the gas will be used in the downstream industry. If, on the other hand, it is negative all the associated gas should be reinjected.

Policy 5: $\theta_6 > 0$, $\theta_1 > 0$, $\theta_2 > 0$. In this case, the resource is exhausted. No associated gas is available for any kind of use.

Policy 6: $\theta_6 > 0$, $\theta_1 > 0$, $\theta_2 = 0$. Vertical integration yields a larger return than reinjection and is, therefore, preferred.

Policy 7: $\theta_6 > 0$, $\theta_1 = 0$, $\theta_2 > 0$. Reinjection yields a larger return than the downstream activity and is, therefore, preferred.

Policy 8: $\theta_6 > 0$, $\theta_1 = 0$, $\theta_2 = 0$. The RHS of (31) is zero. The planner is indifferent between reinjecting associated gas and holding foreign assets or investing in the downstream sector.

The following summarising conclusions can be drawn from equations (29), (30), and (31):

Proposition 1

If

$$\sigma p[1+\epsilon(p)] - w > 0$$

reinjection is preferred to flaring.

Proposition 2

If

$$b\pi - m > r$$

downstream production is preferred to flaring and holding foreign assets.

Proposition 3

If

$$\sigma p[1+\epsilon(p)] - w - (a/b)(1-\sigma g)(b\pi - m - r) > 0$$

reinjection is preferred to downstream production.

In order to determine the sequence of the different policies, one first has to derive the optimal path of the petroleum price, which determines the decision whether or not to reinject.

6. The price path

The petroleum price on the optimal path is determined by equation (18)

$$(18) \quad \Gamma = \Phi p(R) [1+\epsilon(p)] + g\theta_G.$$

Matters are simple if it is optimal to flare associated gas. In this case $\theta_G=0$ and, by establishing growth rates and

inserting (12) and (14) into (18), the standard Hotelling result for a resource extracting monopolist can be derived:

$$(32) \quad \hat{p} + [1+\epsilon(p)] = r$$

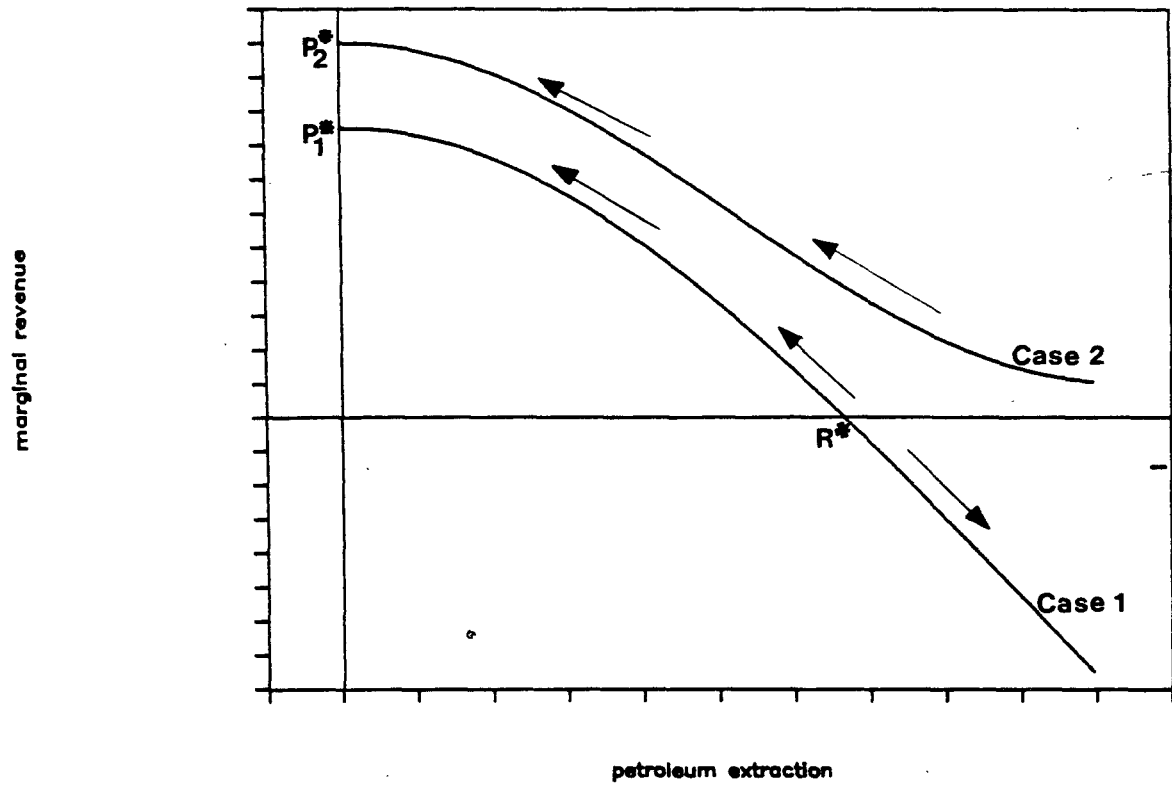
The marginal revenue from petroleum sales grows at the rate of interest. This is the condition for intertemporal efficiency of resource extraction. If the growth rate were lower it would be better to extract the entire resource stock at the beginning of the planning period. If the growth rate were larger than the interest rate it would be optimal to delay extraction until the end of the planning period (in this model to infinity), since the present value of the resource in situ increases over time. Condition (32) assures that the return on extracting a marginal unit of petroleum equals the return on storing a marginal unit. From (32) the optimal extraction path can easily be obtained:

$$(33) \quad \dot{R} = r \frac{p + p'R}{2p' + p''R}$$

The denominator is negative because of the second order condition (21). It can now be shown that for all classes of demand functions satisfying (21) R is a decreasing function of time. This is illustrated by Figure 2: R^* is the locus where the marginal revenue MR is 0. To the left of R^* , MR is positive and, because of (33), R decreases. To the right of R^* , resource extraction increases. R^* is an unstable equilibrium point.¹⁵⁾ Due to the finiteness of the resource stock, extraction cannot increase forever. Therefore $R_0 < R^*$ and

¹⁵⁾ Algebraically, this can be shown by differentiating the RHS of (33) with respect to R at R^* . The derivative is r , i.e. it is positive and the equilibrium is not locally asymptotically stable.

Figure 2: The optimal extraction path



R is decreasing during the entire planning period.¹⁶⁾ This implies an increasing resource price.

Note that the extraction period is finite. According to (32), marginal revenue increases by a positive rate. For $t \rightarrow \infty$, marginal revenue would go to infinity, but the maximum possible revenue, as determined by the shape of the demand function, is the finite choke price p^* .¹⁷⁾ This means that the resource will be exhausted within finite time.

These are standard results of the Hotelling model. Matters become more complicated if gas is not flared and θ_G is positive. If the associated gas is to be used in the downstream industry, it follows from (30) that

$$(34) \quad \theta_G = \Phi(a/b)(b\pi - m - r)$$

so that the optimal path is

$$(35) \quad \mu\{ p[1+\epsilon(p)] + g(a/b)(b\pi - m - r) \} = r,$$

where $\mu\{\cdot\}$ is the growth rate. Again, marginal revenue grows at the interest rate, but in this case marginal revenue is the total marginal revenue from petroleum and intermediate goods sales. $g(a/b)(b\pi - m - r)$ is the additional profit derived from selling the downstream product, which has been produced by means of gas associated to a unit of recovered oil.

¹⁶⁾ If an equilibrium does not exist, i.e. if $p+p'R > 0$ for all $R > 0$, \dot{R} is negative for all R . This is denoted as CASE 2 in Figure 2.

¹⁷⁾ Proof:

$$\lim_{R \rightarrow 0} (p + p'R) = p^* + \lim_{R \rightarrow 0} \frac{p(0) - p(R)}{R} \cdot R = p^*$$

If associated gas is reinjected, combining (12), (14), (18) and (19) yields

$$(36) \quad \mu \{ p[1+\epsilon(p)] - wg \} = r.$$

wg are the costs of reinjection per unit of recovered petroleum. ReInjection costs have the same effect on the optimal price path as extraction costs in the standard model, the main difference being that extraction costs have to be paid. The existence of extraction costs restricts the set of feasible decisions, while the costs of reinjection do not. The planner is free to choose whether to incur the costs of reinjecting associated gas or not.

The optimal extraction path in the case gas is used downstream is

$$(37) \quad R = r \frac{p + p'R + g(a/b)(b\pi - m - r)}{2p' + p''R}.$$

R is again decreasing because the denominator is negative and $g(a/b)(b\pi - m - r)$ is positive. If reinjection takes place, the extraction path changes according to

$$(38) \quad R = r \frac{p + p'R - wg}{2p' + p''R}.$$

R again is decreasing.¹⁸⁾ In both the case of reinjection and downstream production the extraction period is finite. Otherwise marginal revenue would increase to infinity which is not compatible with the shape of the demand function.

¹⁸⁾ This can be proved by showing that the numerator $p[1+\epsilon(p)] - cg$ is positive: From (29) and (31) it follows that

$$\sigma g \cdot p[1+\epsilon(p)] \geq \max \{ wg ; wg + g \cdot (1-\sigma g) \cdot (a/b) \cdot (b\pi - m - r) \}$$

This implies that $\sigma g \cdot p[1+\epsilon(p)] - wg \geq 0$. Furthermore, $\sigma g < 1$. Therefore, $p[1+\epsilon(p)] - wg > 0$.

The speeds of extraction can now be compared for the three options the planner has:

$$(39a) \quad \dot{R}_{reinject} > \dot{R}_{flare} > \dot{R}_{downstr.}$$

and

$$(39b) \quad \dot{p}_{reinject} < \dot{p}_{flare} < \dot{p}_{downstr.}$$

for $R_{reinject} = R_{flare} = R_{downstr.}$ Compared to the standard case, the oil price increases relatively fast in case of downstream production and relatively slowly if associated gas is reinjected. This implies an earlier exhaustion of the resource if gas is used in the downstream sector, and a longer depletion time in case gas is reinjected. This is due to the fact that additional downstream profits act like a premium on extraction, while reinjection is costly. The initial extraction rate should be high in the case of downstream production. If associated gas is reinjected two effects have to be taken into account: The cost effect lowers the initial extraction rate while the effect of an increased resource base enables a higher extraction rate in every period. The relative position of the extraction path, as compared to the standard case, therefore is indetermined.¹⁹⁾

It can be seen from (33), (37), and (38) that the optimal extraction path does not depend on other activities of the economy. The size of the oil field, some technical parameters, and the demand function determine crude oil production and the oil price for the whole programme. The paths of the other variables depend on the extraction path and can be computed, once the extraction path is known. Note that jumps in foreign assets/debt, V , and capital, K , can occur. This is due to the assumption of malleability. If downstream production is superior to flaring a jump occurs at the beginning of the programme. Foreign assets have to be

¹⁹⁾ See also the appendix.

transformed into a capital stock which allows for processing all the associated gas. If there were restrictions on borrowing or transforming foreign assets into physical capital, it might be optimal to flare a decreasing percentage of gas during an initial interval.

7. Changing gas utilisation

In the foregoing sections, conditions for the optimality of different utilisations of associated gas, and the behaviour of prices have been derived. It has been shown that, independently of the way in which gas is used, the petroleum price increases along an optimal path. Now the sequence of the abovementioned policies 1 to 8 will be examined. First, it has to be proved that, along an optimal path, jumps in the price cannot occur, if switches of gas utilisation from flaring to reinjection or from downstream production to reinjection occur. From (29) and (31), it can easily be seen that:

$$(40a) \quad \lim_{\substack{\theta_G \rightarrow 0 \\ \theta_1 = 0}} p[l+\epsilon(p)] = \lim_{\substack{\theta_1 \rightarrow 0 \\ \theta_G = 0}} p[l+\epsilon(p)] = w/\sigma$$

$$(40b) \quad \lim_{\substack{\theta_2 \rightarrow 0 \\ \theta_1 = 0}} p[l+\epsilon(p)] = \lim_{\substack{\theta_1 \rightarrow 0 \\ \theta_2 = 0}} p[l+\epsilon(p)] = \frac{1}{\sigma} (w+(1-\sigma g)(b\pi-m-r)).$$

Jumps in p and R do not occur along an optimal path. But note that, according to (33), (37) and (38), there will be jumps in the growth rates of oil price and oil extraction. Since the price is an increasing function of time, a phase of reinjection cannot be followed by a phase of flaring or downstream utilisation of associated gas. Five scenarios are imaginable:

TABLE 5: SCENARIOS

Scenario	Flaring	Downstream	Reinjection
A	1st	-	-
B	1st	-	2nd
C	-	1st	-
D	-	1st	2nd
E	-	-	1st

In scenario A neither reinjection nor downstream utilisation of associated gas are profitable. The net productivity in the downstream sector is smaller than the rate of interest and the reinjection costs are larger than the maximum attainable price at which the additional petroleum can be sold. Reinjection costs are lower in scenario B where, after some time of flaring, it becomes optimal to reinject associated gas. Scenarios C and D are similar to A and B, except the fact that now the use of gas in the downstream sector yields a higher rate of return than holding foreign assets and is, therefore, preferred. The reinjection costs have to be compared to the marginal revenue from selling additionally recoverable oil plus the net profit from the use of the associated gas in the downstream industry. Scenario E is imaginable if reinjection costs are low and/or if the oil fields are small so that the optimal programme starts with a high initial price.

8. Conclusions and final remarks

The model enables the identification of parameters which are critical to a petroleum exporting country's decision whether to develop a downstream industry based on associated gas. Furthermore, conditions for the optimality of reinjecting

associated gas into the oil field have been derived. It has been shown that along an optimal path the relative profitability of reinjecting associated gas increases as the oil price increases. The possibility of making additional profits in a downstream sector implies a faster depletion of petroleum, while in the case of reinjection the depletion date is postponed.

If, however, the model is compared to the real world, significant differences arise. For instance, the model tells us that the petroleum exporting country should first develop the downstream industry and later reinject associated gas. In reality, however, gas has been reinjected long before the possibility of using it for the production of intermediate goods was taken into account. The reason, why the model's fit to the real world is rather poor, are its simplifying assumptions, e.g. the possibility of unconstrained borrowing, the perfect shiftability of the capital good and the constancy of capital productivity in the downstream sector. As has been mentioned above, if a constraint on borrowing were introduced, a scenario starting with flaring would become imaginable, even if vertical integration were profitable. Furthermore, capital productivity in the downstream sector should be an increasing function of the size of the capital stock, due to learning effects. The second argument for an increasing returns to scale function is the need for infrastructure investments which are to be treated like fixed costs of vertical integration.

While the consequences of abandoning these assumptions can be discussed without further formal treatment, another extension would change the whole context of the model. Here it has been assumed that the price of the downstream product is independent of the oil price. In a recent paper (Razavi, Fesharaki (1984)) it has been shown that there is a strong correlation between the two, and that OPEC might undermine its own crude price by selling petroleum-related middle products. Countries not owning natural gas use

petroleum products as a feedstock in the chemical industry and as an energy input in refining. If petroleum exporting countries have a cost advantage in these industries by using an almost free good, associated gas, as an input they are able to sell their products at a relatively low price on the world market. This causes a decline in petroleum processing in the rest of the world, a decline in crude oil demand and, finally, a decline in the oil price. The investigation of these interactions between petroleum exporting and importing countries remains an area for further research.

Appendix: An Example

The effects of reinjection and downstream utilisation of associated gas on the shape of the extraction path will be shown in a simple example. In this example the extraction path can be calculated explicitly. Assume

$$(A1) \quad p = p^* - \frac{1}{2}R,$$

$$(A2) \quad R = p^* - M,$$

where M is marginal revenue. Define

$$(A3) \quad x \equiv \begin{cases} 0 & \text{for flaring} \\ -wg & \text{for reinjection} \\ g(a/b)(b\pi - m - r) & \text{for downstream production.} \end{cases}$$

The condition for an optimal path is

$$(A4) \quad \widehat{(M+x)} = r,$$

which can be integrated such that

$$(A5) \quad M = (M_0 + x) e^{rt} - x.$$

At the end of the extraction period τ , marginal revenue equals the choke price p^* . This in turn implies

$$(A6) \quad M = (p^* - x) e^{r(t-\tau)} - x.$$

In order to calculate the unknown variables τ and M_0 , and to determine R_0 , the resource stock equation has to be integrated:

$$(A7) \quad \dot{S} = -yR, \quad y = \begin{cases} (1-\sigma g) & \text{for reinjection} \\ 1 & \text{otherwise.} \end{cases}$$

Using (A2) gives

$$(A8) \quad \dot{S} = y(M - p^*).$$

Inserting (A6) and integrating yields:

$$(A9) \quad e^{-r\tau} + \tau = \frac{rS_0}{y(p^*+x)} .$$

The LHS increases in τ for $\tau > 0$. If downstream production takes place x is larger than it were in the standard case. Therefore, the depletion time is shorter. On the other hand, if associated gas is reinjected, both x and y are less than in the standard case, which implies a longer extraction period. Using (A2), (A6) and (A9), the initial extraction rate R_0 can be determined:

$$(A10) \quad R_0 = (p^*+x)(1-r\tau) + rS_0/y .$$

The first term on the RHS, $(p^*+x)(1-r\tau)$, is a kind of a (inverse) cost effect. It is relatively large in the case of downstream production, since downstream activities yield additional profits (negative costs). It is relatively small if gas is reinjected, since reinjection is costly. The initial extraction rate is a decreasing function of the costs of associated gas use. rS_0/y is the resource effect. If downstream utilisation of associated gas is optimal, the resource effect is not different from the standard case. On the other hand, if gas is reinjected, the resource base is increased and a higher initial extraction rate is possible. Since in the reinjection case the two effects work into opposite directions, R_0 depends on the strength of these effects, and therefore on the parameters. In the case gas is used for downstream production, R_0 always is smaller than in the standard case.

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