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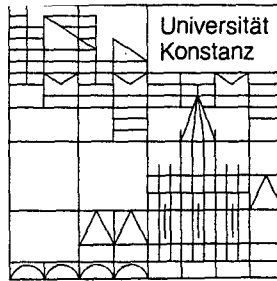
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**Monopoly and
Lagged Demand Adjustments**

Diskussionsbeiträge

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MONOPOLY AND
LAGGED DEMAND ADJUSTMENTS

Michael Rauscher
Serie A - Nr. 220

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The results presented in this paper originate from my work in the Sonderforschungsbereich 178 "Internationalisierung der Wirtschaft". I thank Anke Meyer, Ernst Mohr, and Horst Siebert for their comments on an earlier version of the paper. All remaining errors are my own responsibility. Proofs of equations (11) to (15) are available from the author on request.

Serie A: Volkswirtschaftliche Beiträge

Serie B: Finanzwissenschaftliche Arbeitspapiere

Serie C: Betriebswirtschaftliche Beiträge

Abstract

In a dynamic optimisation model the profit maximising behaviour of a monopolist facing lagged adjustments of demand is investigated. It is shown that the long run equilibrium price differs from the static Cournot price. The monopolist sacrifices some of the long run profits in order to exploit the short run inelasticity of demand. If applied to OPEC and the world petroleum market, the model is able to explain the ups and downs of the oil price during the seventies and eighties.

1. Introduction

In the framework of the conventional Cournot model of monopolistic competition demand is usually assumed to be static. This assumption does not seem to be very realistic in most cases. Several reasons for a dynamic specification of demand functions are conceivable. First, there is the possibility of changing preferences. If the changes are exogenous, the analysis can be carried out within the framework of the static model, and the dynamic model is nothing else but a sequence of several static models. If, however, preferences depend on past experience with the good, the monopolist has to solve an intertemporal optimisation problem. A similar situation arises, if adjustment processes on the demand side have finite speed. Then, current demand does not only depend on the current price but also on past demand which contains information on the history of prices. This approach to modeling dynamic demand behaviour has widely been used in econometric demand models, especially for the energy sector. For a good survey see Berndt, Morrison and Watkins (1981). The underlying assumption is a putty-clay vintage structure of energy using capital. Since it is impossible to change the energy intensity of the whole capital stock at once, but only of new

investments, demand reactions are lagged. The object of this paper is to analyse the behaviour of a monopolist facing lagged demand reactions on his or her pricing policy.

The intertemporal behaviour of a monopolist has been analysed by Jaquemin (1972) and Jaquemin and Thisse (1973). The generality of their models allows for applying them to a wide range of economic questions, e.g. both the interdependent preferences and the lagged demand models. Their basic assumption is that there exists a vector of state variables, changing endogeneously and affecting the level of profits. The main results are: First, under certain conditions, there is an equilibrium which is a saddle point and, second, the monopolist should exert more of his or her power in the initial period and reduce the effort in the long run, near the equilibrium. In order to derive the properties of the equilibrium, however, Jaquemin and Thisse used particular types of profit and adjustment functions. They assumed a concave profit function and a linear adjustment process on the demand side. As will be seen later, the assumption of concavity does not hold for the lagged demand model. In addition, the analysis will be extended to a more general class of adjustment functions.

Another model with a similar structure has been investigated by Evans (1924). The main difference is, that in the present model, we specify a process by which realised demand adjusts to desired demand while in Evans' model demand is a function of the price and its change:

$$q_t = a p_t + b_t + h dp_t/dt ,$$

where q_t is demand, p_t the price and a , b , and h are constant parameters. By rearranging this equation, it can be seen that it describes an adjustment process during which a difference between current demand q_t and desired demand $ap_t + b_t$ causes price changes with adjustment speed h^{-1} . His results are that the Cournot monopoly price is a particular solution of the maximisation problem and that, for an infinite time horizon, the only solutions which remain finite are those which approach the Cournot monopoly price asymptotically. The second result will also be derived in the present paper if we let the discount rate go to zero.

The article is organised as follows: In the next two sections the basic model will be presented, and the conditions for profit maximisation will be derived. In the fourth part of the paper, the long run equilibrium will be analysed. The following section contains an application of the model to OPEC behaviour and the development of the oil price. It will be shown that the model can be used to explain some aspects of the rise and fall of oil prices during the seventies and eighties. Some final remarks conclude the paper.

2. The Model

We consider a monopolist who intends to maximise the present value of future profits which he or she makes by producing and selling a homogenous good. δ is a positive rate of discount, q_t the quantity of the good (its positive initial value q_0 given exogenously), p_t its price, and $c(q_t)$ the cost of producing q_t (the cost function $c(q_t)$ being positive and strictly monotonously increasing). The problem the monopolist has to solve is

$$(1) \quad \max_{p_t} \int_0^{\infty} e^{-\delta t} (p_t q_t - c(q_t)) dt .$$

Usually it is assumed that demand reactions to price changes are completed instantaneously. Then q_t is a function of p_t only. In some cases, however, this assumption is not very realistic, e.g. if the demand for a factor, being used in a downstream production process, depends on the technology which is embodied in the current capital stock. Changing the factor intensity of production requires investments which are costly and take time. Therefore, the adjustment of demand from the current to the desired level takes time either. The adjustment process can be described as a function of price and quantity.

¹ Note that the profit function is not concave, if q_t is not a function of p_t , since the determinant of the matrix of second derivatives is negative.

$$(2) \quad \dot{q}_t = \beta G(p_t, q_t) \quad 2$$

It is assumed that the higher the price at a given level of demand or the higher demand at a given price level the stronger is the effort to reduce demand. This means that the adjustment function is strictly monotonously decreasing in both variables. β is a positive shift parameter for the adjustment speed which will be used for comparative dynamics in later sections. There exist combinations of prices and quantities for which demand is constant. This locus of equilibria can be described by a function $q_t^* = D(p_t)$. $D(\cdot)$ be non-negative, continuous, and monotonously decreasing. q_t^* can be interpreted as the desired demand to which realised demand adjusts. We allow for a finite choke price above which desired demand is zero. In equilibrium the partial derivatives of $G(\cdot)$ satisfy the following condition:

$$(3) \quad G_p(p_t, D(p_t)) = -D'(p_t) G_q(p_t, D(p_t)) .$$

The current value Hamiltonian for the above problem is:

$$(4) \quad H_t = p_t q_t - c(q_t) + \Phi_t \beta G(p_t, q_t) . \quad 3$$

Φ_t is the shadow price of demand. It is the present value of future profits which can be made, if the current demand is increased by a marginal unit. Since today's demand is positively correlated to future demand, the shadow

² Let a dot above a variable denote its derivative with respect to time and let a subscript (except t) denote the partial derivative of a function with respect to a variable.

³ For Pontryagin's maximum principle, see Takayama (1974). Non-negativity constraints have been neglected here. Because of the shape of the adjustment function negative quantities cannot occur. Negative prices are not excluded, since the monopolist might find it optimal to "sell" the product at a negative price in order to make consumers addictive to it.

price should be positive. Omitting the time subscript, on an optimal path Φ should change according to

$$(5) \quad \dot{\Phi} = (\delta - \beta G_q) \Phi - (p - c') .$$

In the optimum, $H_p = 0$:

$$(6) \quad -\beta \Phi G_p = q .$$

The effect of a marginal price change on current profits should equal the present value of the decline in future profits, caused by the same price variation. By the shape of the demand and adjustment functions, this implies that the shadow price of demand is positive. It equals the ratio of actual demand to the (negative) change in demand caused by a marginal price variation. The larger β the speedier the adjustment process and the lower the shadow price. The shadow price, therefore, indicates the value of the monopolist's ability to exploit the demand rigidity.

Finally, a transversality condition for $t \rightarrow \infty$ is required. According to Michel (1982), the discounted Hamiltonian has to go to zero. It is shown in the Appendix that this implies

$$(7) \quad p q > c \quad \text{for } t \rightarrow \infty .$$

In the long run, gross revenue should exceed costs along an optimal path. For $t \rightarrow \infty$, boundary solutions or negative prices are not admissible. Negative future profits cannot be optimal, although it might be optimal to sacrifice some future profits, if the rate of time preference is high.

3. Conditions for Profit Maximisation

The second order conditions of optimality are fulfilled, if the Hamiltonian is strictly quasiconcave in p and q . Therefore the Hessian matrix of second derivatives should have negative diagonal elements and a positive determinant.

$$\begin{aligned} H_{pp} &= \beta \Phi G_{pp} \\ H_{qq} &= -c'' + \beta \Phi G_{qq} \\ H_{pq} &= 1 + \beta \Phi G_{pq} \end{aligned}$$

Using (6) the second derivatives along the optimal path can be obtained:

$$(8) \quad -H_{pp} = \frac{G_{pp}}{G_p} q > 0$$

$$(9) \quad -H_{qq} = c'' + \frac{qG_{qq}}{G_p} > 0$$

$$(10) \quad H_{pq} = H_{qp} = 1 - \frac{qG_{pq}}{G_p}$$

If H_{pq} is sufficiently small, the determinant of the Hessian is positive. In the following, it is assumed that the second order conditions for a maximum hold.

Establishing growth rates in (6) yields

$$\hat{\Phi} + \hat{G}_p = \hat{q}.$$

By inserting (5) and rearranging terms, we get an equation which determines the price along an optimal path:

$$(11) \quad \frac{G_{pp}}{\beta G_p} \dot{p} = -\frac{\delta}{\beta} + G_q + \frac{G}{q} - \frac{G_{pq}G}{G_p} - \frac{G_p}{q} (p-c')$$

From the second order condition (8), we know that G_{pp}/G_p is positive, and the direction of the price changes along the optimal path is given by the RHS of (11). Since the terms on the RHS are related to each other in a complex manner, the direction of the price change cannot be determined. If demand is in equilibrium, implying $G=0$, (11) can be rewritten:

$$(11a) \quad \frac{G_{pp}}{\beta G_p} \dot{p} = -\frac{\delta}{\beta} + G_q - \frac{G_p}{q} (p-c') \quad \text{if } \dot{q} = 0$$

While the first two terms on the RHS are negative anyway, the sign of the last term depends on the price level. If the price is small compared to marginal cost, the price change is negative. This result will be needed in the following section.

4. The Long Run Equilibrium

The usual procedure would now be to set $\dot{p}=0$ and analyse the shapes of the loci of constant prices and quantities in the p - q -plane. Since the derivatives of \dot{p} with respect to p and q contain several terms whose signs are not known, the analysis will be restricted to the neighbourhood of an equilibrium. We therefore assume that there exists a single equilibrium point where changes in prices and quantities do not take place. This implies $G=0$, $q=D(p)$, and $G_q = -G_p/D'$. The equilibrium is then given by

$$p^* = c' - \frac{D}{D'} - \frac{\delta D}{\beta G_p}$$

or

$$(12) \quad p^* = \frac{c' - \frac{\delta D}{\beta G_p}}{1 + \frac{1}{\epsilon}}$$

where $\epsilon = pD'/D$ is the elasticity of desired demand ($\epsilon < 0$). This result resembles the conventional static result of monopolistic competition. There is an additional effect which is caused by the rigidity of demand. For high values of β or a discount rate close to zero, p^* approaches the static monopoly price:

$$\lim_{\beta \rightarrow \infty} p^* = \lim_{\delta \rightarrow 0} p^* = \frac{c'}{1 + \frac{1}{\epsilon}}$$

Since the equilibrium as given by (12) differs from the Cournot solution, if the adjustment speed and the discount rate are positive and finite, profits are lower than in the static case.

In order to derive conditions for the stability of the equilibrium, it is necessary to analyse the partial derivatives of \dot{p} and \dot{q} with respect to p and q . Obviously, the locus of constant quantities is given by $q^* = D(p)$. Partial derivation of \dot{p} and noting that in equilibrium $G = 0$, $D(p) = q$ and $G_q = -G_p/D'$ yields

$$(13) \quad \left. \frac{d\dot{p}}{dp} \right|_{\dot{p}=0, \dot{q}=0} = \beta G_{pp}/D' + \delta$$

$$(14) \quad \left. \frac{G_{pp}}{\beta G_p} \frac{d\dot{p}}{dq} \right|_{\dot{p}=0, \dot{q}=0} = \underbrace{G_{qq}}_{<0} + \underbrace{c'' \frac{G_p}{q}}_{>0} - \underbrace{\left(2 \frac{G_p}{qD'} + \frac{\delta}{\beta q} \right) \left(1 - \frac{qG_{pq}}{G_p} \right)}_{?}$$

From (13), it can be seen that, along the optimal path, the price change is an increasing function of the price. The signs, as indicated below the RHS of (14), follow from the shapes of the adjustment and demand function and from the second order condition (9). The last term in brackets is the cross partial derivative of the Hamiltonian as specified in equation (10). Since it has been assumed to be close to zero, the normal case is that $d\dot{p}/dq$ is negative. Then, the $\dot{p}=0$ -curve is increasing close to the equilibrium in the p - q -plane, and there exists a saddle path leading to the equilibrium as shown in Figure 1. The saddle path is the optimal path. Other paths cannot be optimal, since they either imply negative prices or demand going to zero in the long run. In both cases the transversality condition (7) would be violated. If the $\dot{p}=0$ -line were decreasing and flatter than the equilibrium demand curve, exploding cycles would occur. This, however, would imply a increasing price left of the equilibrium. But, using equation (11a), it has been shown that \dot{p} is negative

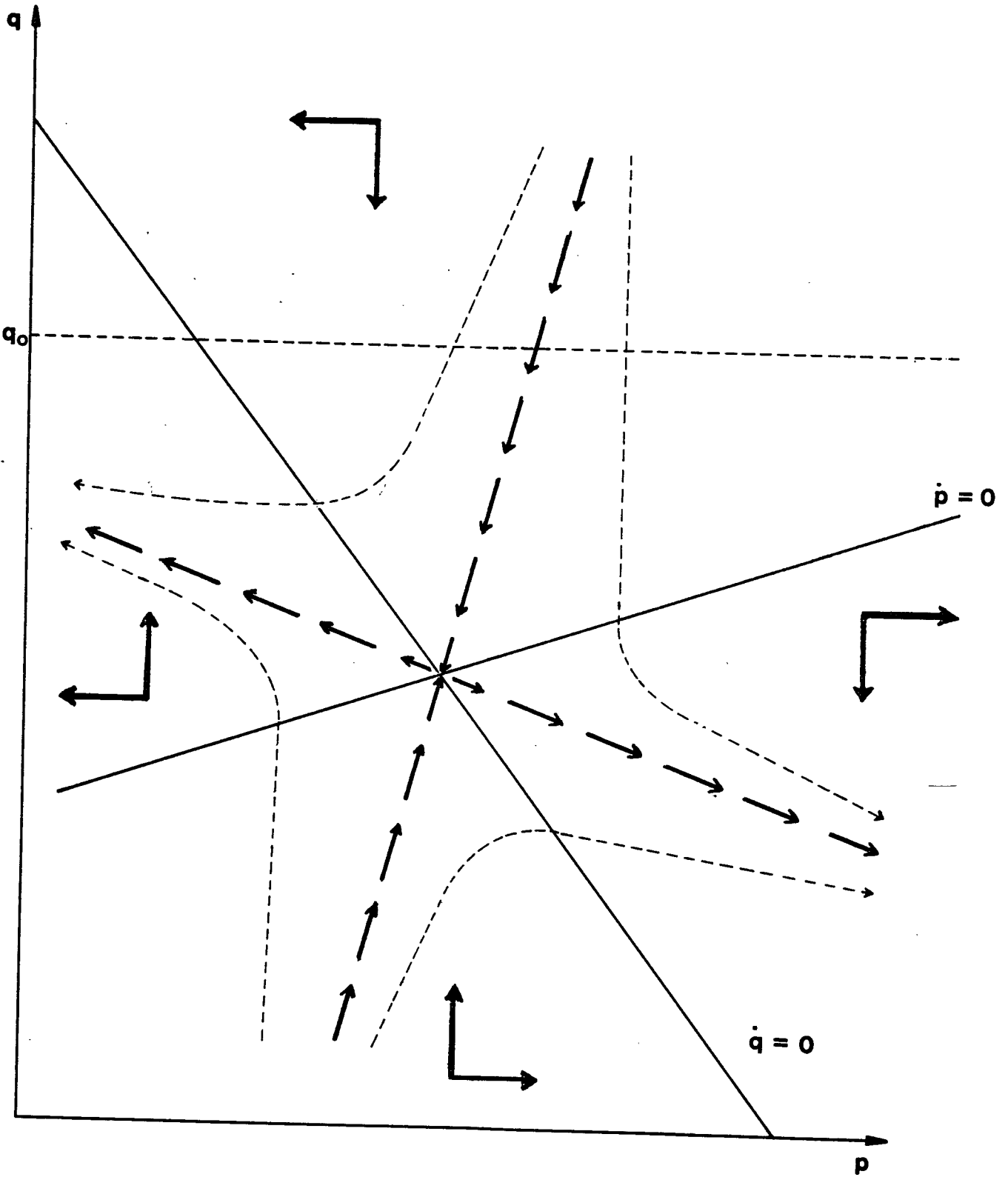


Figure 1

in this area of the p - q -plane. Such paths can occur only if there are more than one equilibrium points.⁴

THEOREM 1

If the Hamiltonian (4) is strictly quasiconcave in p and q and there exists a single equilibrium in p and q , the equilibrium is a saddle point and the saddle path is the optimal solution of the monopolist's optimisation problem.

The transversality condition implies a second theorem:

THEOREM 2

An optimal solution to the above optimisation problem exists, iff there exists an equilibrium

$$p^* = \frac{c' - \frac{\delta D}{\beta G_p}}{1 + \frac{1}{\epsilon}}$$

in which the price exceeds average cost.

Behaviour near the equilibrium can also be studied by computing the eigenvalues of the Jacobian matrix of partial derivatives. These are z_1 and z_2 :

$$(15) \quad 2 z_{1,2} = \delta \pm \sqrt{\delta^2 + 4 \beta G_p \left(\frac{\dot{d}p}{D' dp} + \frac{\dot{d}p}{dq} \right)}$$

and

$$(16) \quad z_1 > 0, \quad z_2 < 0, \quad \text{if } \frac{\dot{d}p}{dp} + D' \frac{\dot{d}p}{dq} > 0.$$

⁴ The occurrence of multiple equilibria cannot be excluded but this case shall not be dealt with here.

Otherwise, both eigenvalues would be positive or complex with a positive real parts, and the equilibrium would be unstable, which is not possible if there exists a single equilibrium. So the normal case is the saddle path towards a long run equilibrium. If the initial demand q_0 exceeds the equilibrium demand, its rigidity leads to overshooting of the price. There is a price shock and the new price is located on the saddle path in the p - q -plane. As demand adjusts, the monopolist reduces the price until the long run equilibrium is reached. This equilibrium differs from the static Cournot solution, if the monopolist has a positive discount rate. Such a scenario can be caused by cartelisation. If, on the other hand, initial demand is lower than the equilibrium demand the monopolist cuts the price in order to make his or her customers sort of addictive to the product. In the extreme, this might imply a negative price in an initial period, i.e. the customers do not only get the product as a free lunch but also are given additional incentives to use the product.

How the long run equilibrium is affected by variations of the adjustment speed and the discount rate, can be seen by differentiating (11) with respect to these parameters. Leaving the price and quantity constant, this operation yields

$$(17) \quad \dot{dp} = \frac{\delta + G_{pp}/G_p p}{\beta G_{pp}/G_p} d\beta - \frac{1}{G_{pp}/G_p} d\delta .$$

p is decreasing in δ and, for p not being too negative, increasing in β . A higher discount rate means that, for combinations of p and q lying on the former $\dot{p}=0$ -line, \dot{p} now becomes negative. Therefore, the new $\dot{p}=0$ -line must lie in the area where before \dot{p} was positive. A similar argument can be used for a variation in β . The result is shown in Figure 2. It can be seen, that a higher rate of discount and slower reactions on the demand side lead to a higher equilibrium price. Furthermore, the higher the speed of adjustment, the steeper is the saddle path: The price changes due to profit maximizing behaviour of the monopolist are relatively small compared to those due to the adjustments of demand.

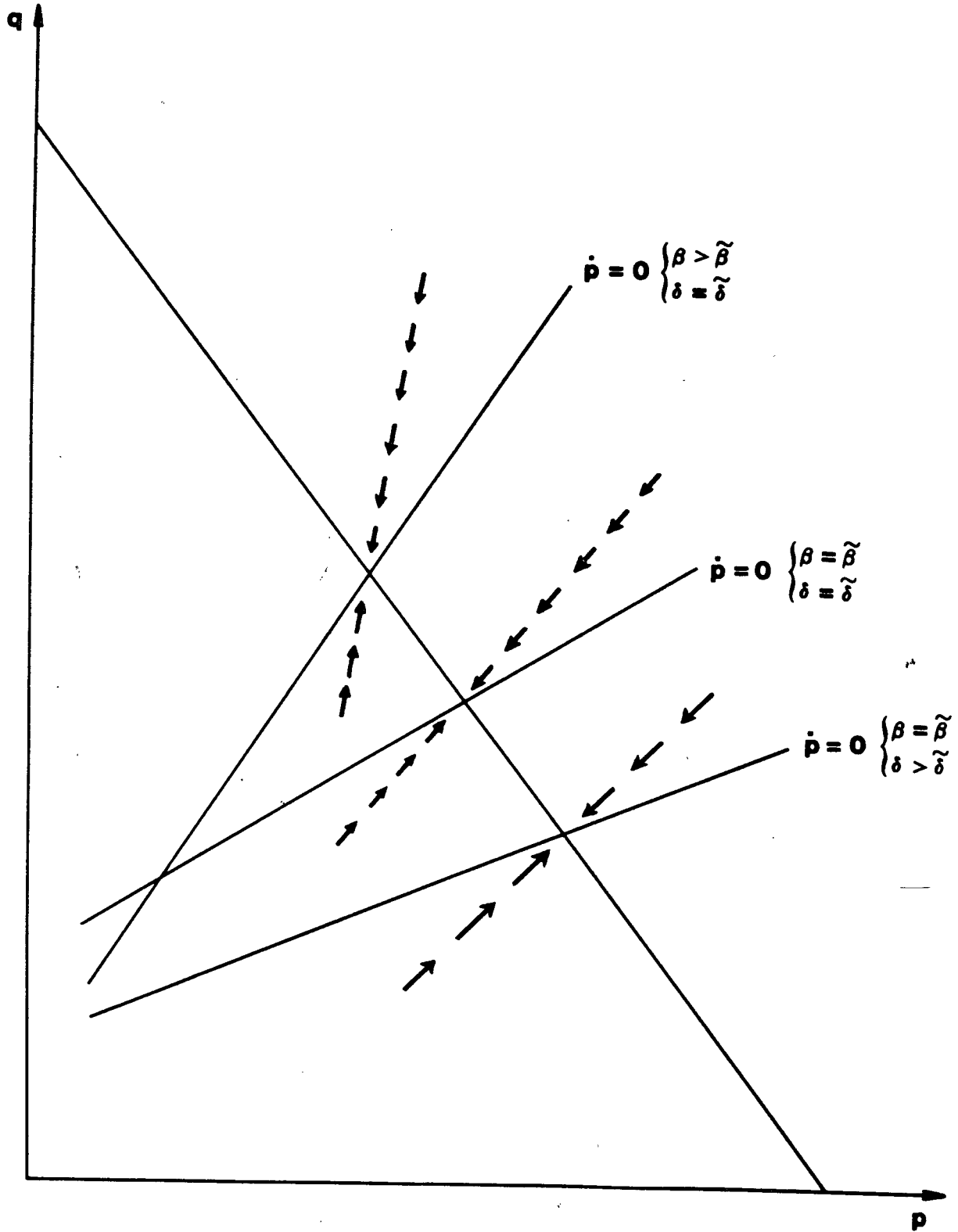


Figure 2

5. An Example: OPEC Behaviour and the Price of Oil

If we apply the above model to the world petroleum market, we claim that OPEC is the dominant firm in the market. Demand adjustments are lagged, since the development and installation of substitution technologies as well as the mobilisation of petroleum reserves in non-OPEC countries takes time. A model, similar to the one which has been examined above, has been used by Pindyck (1978) in order to analyse the effects of cartelisation. Pindyck's model, however, is not explicitly solved. Optimal paths have been computed by means of numerical simulation. His findings are that there is a price shock, followed by a period of declining prices and then increasing prices. The period of falling prices is caused by monopolistic behaviour, i.e. the appropriation of short run rents accruing from the rigidity of demand. The long run price increase is the scarcity effect or Hotelling rent. So, Pindyck's model explains an oil price shock and what OPEC should do in order to maximise its welfare. But it fails to explain some of the real world events, for instance that there were three price shocks instead of one: in 1973/74, 1979/80, and finally 1986 in the opposite direction.

Optimisation models usually claim that the planner has perfect information about the relevant parameters. Since this assumption seldom holds in reality, it is a fruitful task to analyse what happens, if plans are made on the basis of false assumptions about the state of the world. First, recall the historical events on the petroleum market. Since there was almost no price variation prior to the first oil crisis, OPEC could not have much information on how demand would react on a massive price increase. After the first oil price shock it turned out that demand was surprisingly inelastic and the adjustment process went very slow. Consumption of petroleum products reached a new peak in 1979. OPEC had not used its whole potential of increasing the price of petroleum. An interpretation of what has happened then is given in Figure 3.

Point I denotes the situation before the second oil crisis. OPEC had not used its whole potential to raise prices. Having experienced the slow reaction of demand after the first oil crises, it was widely believed that the speed of adjustment was low. It seemed to be optimal to increase the price to II in

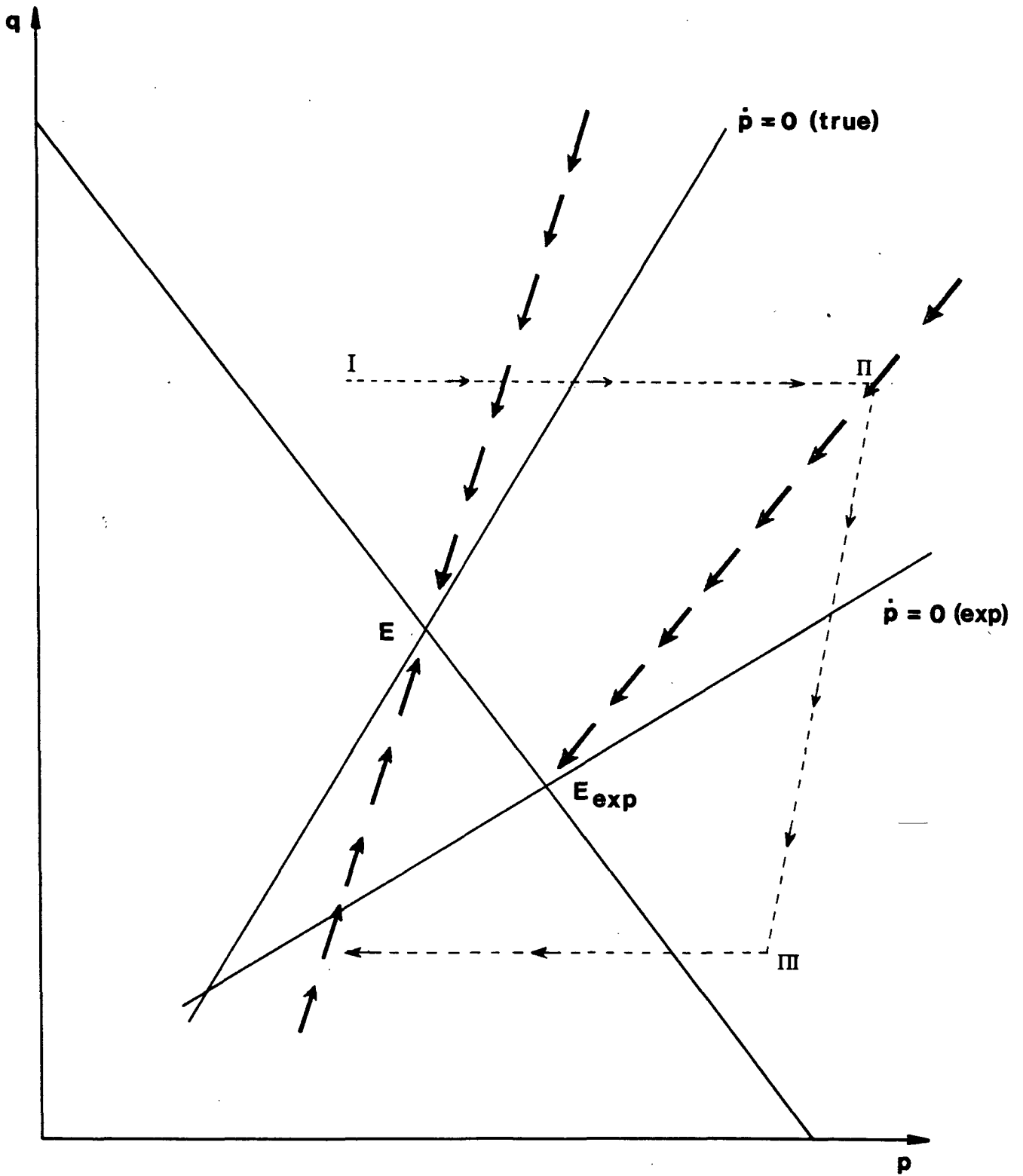


Figure 3

order to reach the expected long run equilibrium E_{exp} . The true equilibrium E , however, would have demanded a much lower initial price. The decrease in demand turned out to be much stronger than it had been expected. In Figure 3, this implies that the realised path is much steeper than the path which has been expected to be optimal. Somewhen (point III) it has become clear that the oil price was still too high while demand for OPEC oil had sunk far below the desired level. This was the reason for cutting the price substantially in early 1986.

Of course, the above interpretation does not tell the whole story. First, the exhaustibility of petroleum is not taken into account. Second, the dependence of demand on GDP growth has been neglected. Third, there have been false expectations not only with respect to the adjustment process, but also with respect to the demand function itself, e.g. the price at which synthetic substitutes become competitive. Finally, by applying the leading firm model to the world petroleum market and assuming that OPEC is a rational profit maximizing monolithic block, political motives for setting prices and questions of cartel stability have been neglected. Despite these apparent shortcomings, however, it is suggestive how a model of monopolistic behaviour in a world of lagged demand reactions can explain some important economic aspects of OPEC pricing policy.

6. Summary and Conclusions

The intertemporal behaviour of a monopolist facing lagged demand reactions has been examined. It was shown that, if there exists an equilibrium in which both sides of the market do not intend to adjust prices or quantities, this equilibrium is a long run solution to the monopolist's optimisation problem. It differs from the static Cournot solution, if the adjustment speed of demand is finite and the monopolist discounts future profits. Finally, the case of a monopolist who has false expectations on the behaviour of demand was investigated. He or she chooses a suboptimal path which has to be corrected when the real behaviour of demand becomes obvious. It was shown that this model of monopolistic behaviour under imperfect

information can be used to explain some of the economic aspects of recent OPEC pricing policy.

The model could be extended by the introduction of adjustment costs for capacity variations. For the application to the world petroleum market, the exhaustibility of the resource and the behaviour of the fringe of the cartel should be modeled. While the former extension would only alter the cost function by introducing the user cost of the resource, the latter might yield new insights going beyond the frame of the model presented here.

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Appendix

Proof of (7):

$e^{-\delta t} H \rightarrow 0$ for $t \rightarrow \infty$. This implies that the growth rate of the discounted Hamiltonian must be negative. Therefore,

$$\dot{H} < \delta H$$

$$H_p \dot{p} + H_\Phi \dot{\Phi} + H_q \dot{q} < \delta H$$

Along an optimal path, $H_p = 0$, $\dot{q} = H_\Phi$, and $\dot{\Phi} = \delta\Phi - H_q$. Therefore,

$$(\delta\Phi - H_q) \dot{q} + H_q \dot{q} < \delta H$$

$$\delta\Phi \dot{q} < \delta(pq - c + \Phi\dot{q})$$

$$c < pq$$

■