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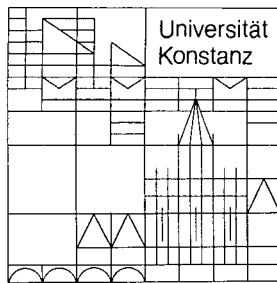
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**Fakultät für
Wirtschaftswissenschaften
und Statistik**

Jack Wahl

**An Information Paradox
with HARA Preferences**

Diskussionsbeiträge

AN INFORMATION PARADOX WITH HARA PREFERENCES

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AN INFORMATION PARADOX WITH HARA PREFERENCES

By Jack Wahl*

Abstract. *The incentive to trade implies that all investors in the market choose to be informed although the whole of them cannot profit from the information signal. This result is due to an insurance property of information.*

1. Introduction and Conclusions

In a complete and perfect financial market with exogenously given aggregate consumption new information will lead to a redistribution of consumption claims only. Hence all investors as a group cannot make a profit out of receiving the information.¹⁾ Does this imply that no investor will buy the information before the financial market opens? The paper argues that this is not the case.

The major results of the paper are found in Theorems 1 and 2. These Theorems imply a dominant action: It is always optimal for each investor to buy information before the financial market opens and, hence, to be informed when trading in the market. This result does not depend on how informed "the market" is, i.e., how many other investors received the signal.²⁾ Therefore, an acquisition of information is an acquisition of *insurance* protecting the acquirer from exploitation by informed traders in

the market. So information possesses an insurance property. This ex ante viewpoint is the result of all investors having rational expectations.

If each investor is informed, then the expected utility gains from information are zero or negative, depending on the form of wealth endowment. Hence only utility gains from non-speculative trading matter. Although the value of information before trade is non-positive, the insurance property of information produces the paradoxical result that investors are still better off buying the information than not buying it when they want to participate in the financial market.

2. The Hyperbolic Absolute Risk Aversion Model

Let us consider the following decision problem:

$$(1) \quad \text{Max}_{\{c\}} E [u_i(c_i)] = \frac{\gamma}{1-\gamma} \sum_{s=1}^S \pi_i(s) \left(\alpha_i + \frac{1}{\gamma} c_{is} \right)^{1-\gamma}$$

$$\text{s.t.} \quad \sum_{s=1}^S p_s c_{is} = w_i.$$

The variables entering the model are:

- $u_i(\cdot)$ - HARA-utility function of investor i ; ³⁾
- α_i, γ - non-zero parameters of the utility function; ⁴⁾
- c_{is} - consumption expenditure of investor i in future state of nature s ($s=1,2,\dots,S$);
- $\pi_i(s)$ - subjective probability belief of investor i about the occurrence of future state s ($s=1,2,\dots,S$);
- E - expectation operator;
- p_s - present price of a state s -contingent claim of 1\$;
- w_i - initial wealth of investor i .

We assume a complete financial market, that is the number of linearly independent securities is equal to the number of states of nature. Furthermore we consider a perfect financial market in which there are neither transactions nor information costs, no taxes, no restrictions on short sales, and in which all securities are perfectly divisible.

First we derive the investor's optimal portfolio of claims.

Lemma 1. *Let the financial market be complete and perfect. Then the demand of a price taking investor i for state contingent consumption claims is \Rightarrow*

$$(2) \quad \alpha_i + \frac{1}{\gamma} c_{is} = \frac{h_i(s)}{p_s} \left(\frac{\alpha_i}{r} + \frac{1}{\gamma} w_i \right),$$

where
$$h_i(s) \equiv \frac{\pi_i(s) (\alpha_i + \frac{1}{\gamma} c_{is})^{1-\gamma}}{\sum_{z=1}^S \pi_i(z) (\alpha_i + \frac{1}{\gamma} c_{iz})^{1-\gamma}} \quad (s=1,2,\dots,S);$$

$r \equiv 1 + \text{risk-free rate of interest.}$

Proof. The first order conditions of an interior solution of the decision problem are

$$(3) \quad \pi_i(s) (\alpha_i + \frac{1}{\gamma} c_{is})^{-\gamma} = \lambda_i p_s \quad \forall s,$$

where λ_i is investor i 's marginal utility of wealth. First multiply both sides of (3) by $\alpha_i + (1/\gamma)c_{is} \neq 0$, sum the result up over all states s , and solve this equation for λ_i . Second insert λ_i into equation (3). We then get (2) by rearranging terms, using the budget constraint and the definitions of $h_i(s)$ and r .***

Lemma 1 shows that the demand, c_{is} , is a linear function of a preference-adjusted probability belief, $h_i(s)$ ($s=1,2,\dots,S$). $h_i(\cdot)$ depends on the parameters of the utility function. Note that with logarithmic utility ($\gamma=1$), $h_i(\cdot) = \pi_i(\cdot)$.

Next consider investor i 's equilibrium demand of state contingent consumption claims, i.e., the sharing rule.

Lemma 2. *Let aggregate consumption be exogenously given. Then the sharing rule for state contingent consumption claims is*

$$(4) \quad \alpha_i + \frac{1}{\gamma} c_{is} = z_i \frac{h_i(s)}{H(s)} \left(A + \frac{1}{\gamma} C_s \right) \quad (s=1,2,\dots,S; i=1,2,\dots,I),$$

where $A \equiv \sum_{i=1}^I \alpha_i$;

$$z_i \equiv (\alpha_i/r + w_i/\gamma) / \sum_{j=1}^I (\alpha_j/r + w_j/\gamma);$$

$$H(s) \equiv \sum_{i=1}^I z_i h_i(s);$$

$$C_s \equiv \sum_{i=1}^I c_{is}.$$

Proof. In the following we make direct use of above definitions.

Lemma 1 implies

$$A + \frac{1}{\gamma} C_s = \sum_{j=1}^I \frac{h_j(s)}{p_s} (\alpha_j/r + w_j/\gamma) = \frac{H(s)}{p_s} \sum_{j=1}^I (\alpha_j/r + w_j/\gamma).$$

Inserting this into (2) such that p_s cancels out, we obtain

$$\frac{\alpha_i + \frac{1}{\gamma} c_{is}}{A + \frac{1}{\gamma} C_s} = \frac{h_i(s)}{H(s)} z_i$$

and, hence, equation (4).***

We notice from Lemma 2 that individual consumption is a state-dependent linear function of aggregate consumption. The state-dependence is due to the heterogeneity of probability be-

liefs. Nevertheless, allocational efficiency is achieved, for the financial market has been assumed complete.

The information value from the point of view of investor i , Δ_i , is determined from the expected change in expected utility by adjusting the optimal portfolio of claims for the signal, i.e., $\Delta_i \equiv E_i E_1(u_i | e) - E_1(u_i)$. This definition does *not* presume that investor i belongs to the group of the information acquirer. It only presumes that the information is available in the market. The signal itself may be interpreted as a draw from the sample of information events e with probability belief $\pi_i(e)$ ($e=1,2,\dots,E$).

Lemma 3. *Assume homogeneous prior beliefs. Then \Leftrightarrow*

$$(5) \quad \Delta_i \propto \frac{\gamma}{\gamma-1} \sum_{e=1}^E h_i(e) \left\{ \sum_{s=1}^S h_i(s|e) \left[\frac{z_i e h_i(s|e)}{z_i H(s|e)} \right]^{\gamma-1} - 1 \right\}.$$

Proof. (i) The fraction of aggregate consumption prior to the information asymmetry, z_i , depends on the signal, because the price change may induce a change in the risk-free interest rate and/or in wealth.

(ii) From the separation property of HARA-preferences⁷ it follows that with homogeneous beliefs each investor holds a state-independent fraction of the market portfolio. Hence $h_i(s) = H(s) \forall i$ from Lemma 2.

(iii) From (i) and (ii) and Lemma 2 it follows that

$$E E_i(u_i | e) = \frac{\gamma}{1-\gamma} \sum_{e=1}^E \pi(e) \sum_{s=1}^S \pi_i(s|e) \left[z_i e \frac{h_i(s|e)}{H(s|e)} \left[A + \frac{1}{\gamma} C \right] \right]^{1-\gamma} \text{ and}$$

$$E(u_i) = \frac{\gamma}{1-\gamma} \sum_{s=1}^S \pi(s) \left(z_i \left[A + \frac{1}{\gamma} C \right] \right)^{1-\gamma}.$$

Subtracting the second from the first equation and noting that $\pi(s) = \sum_{e \in E} \pi(e) \pi_i(s|e)$ ($s=1,2,\dots,S$) yields

$$\Delta_i = \frac{\gamma}{\gamma-1} \sum_{e \in E} \pi(e) \sum_{s \in S} \pi_i(s|e) \left[z_{ie} \frac{h_i(s|e)}{H(s|e)} \left[A + \frac{1}{\gamma} C_s \right] \right]^{1-\gamma} \left[\left(\frac{z_{ie} h_i(s|e)}{z_i H(s|e)} \right)^{\gamma-1} - 1 \right].$$

Since $h_i(s|e) = \pi_i(s|e) \left[z_{ie} \frac{h_i(s|e)}{H(s|e)} \right]^{1-\gamma} / \sum_{s \in S} \pi_i(s|e) \left[z_{ie} \frac{h_i(s|e)}{H(s|e)} \right]^{1-\gamma}$ it follows that

$$\Delta_i = k_i \frac{\gamma}{\gamma-1} \sum_{e \in E} h_i(e) \sum_{s \in S} h_i(s|e) \left[\left(\frac{z_{ie} h_i(s|e)}{z_i H(s|e)} \right)^{\gamma-1} - 1 \right],$$

where $h_i(e) \equiv \frac{\pi(e) k_{ie}}{\sum_{e \in E} \pi(e) k_{ie}}$, $k_{ie} \equiv \sum_{s \in S} \pi_i(s|e) \left[z_{ie} \frac{h_i(s|e)}{H(s|e)} \left[A + \frac{1}{\gamma} C_s \right] \right]^{1-\gamma}$

> 0 , and hence $k_i \equiv \sum_{e \in E} \pi(e) k_{ie}$ is a positive proportionality factor. The last equation implies the result, neglecting k_i .***

Lemma 3 implies that the information value depends on how informed "the market" is via a preference-adjusted "market belief", $H(\cdot)$. Let us therefore define the following scenarios and the related information values. Denote the proportion of wealth of all informed investors in the market by $\lambda \in [0,1]$. Let $\Delta_i^{(i)}(\lambda)$ be the value of information if investor i is informed, and $\Delta_i^{(u)}(\lambda)$ be the value of information if investor i is uninformed. Of course, both values are functions of the relative wealth of the informed group of investors. Remember that in each case the value of information is identical with the expected utility change.

We are now able to derive our main results. The following Theorems differ with respect to the assumption on the form of investors' initial endowments. The assumption of identical

"opinion" implies that investors with the same information will deduce the same probability adjustment.

Theorem 1. *With costless information, no differences of opinion, a fixed risk-free rate of interest, and initial wealth in the form of cash, we have*

(i) $\Delta_{i^{1'}}(\lambda) \geq 0$ with equality if and only if $\lambda=1$;

(ii) $\Delta_{i^{u'}}(\lambda) \leq 0$ with equality if and only if $\lambda=0$.

Hence $\Delta_{i^{1'}}(\lambda) > \Delta_{i^{u'}}(\lambda)$ for all λ and i . Therefore each investor will be informed.

Proof. Note that $h_i(s|e) > 0$ and $H(s|e) > 0 \forall s, e$. Let $I^{1'}$ denote the number of informed investors. We deduce $H(s|e) = \lambda h(s|e) +$

$(1-\lambda)h(s)$, where $\lambda = \frac{I^{1'}}{\sum_{i=1}^I z_i}$. Furthermore, z_i is independent of

the signal, for r is exogenously given and w_i is cash.

Consider (i). Define $\Delta_{1e} = \frac{\gamma}{\gamma-1} \{ \sum_{s=1}^S h_i(s|e)^\gamma H(s|e)^{1-\gamma} - 1 \}$.

First note that $\text{sgn}(\Delta_{i^{1'}}) = \text{sgn}(\Delta_{1e})$ for $h_i(e) > 0$ ($e=1, 2, \dots, E$).

1° Suppose $\lambda < 1$. So $h_i(s|e)$ cannot be proportional to $H(s|e)$. Then $\sum_{s=1}^S h_i(s|e)^\gamma H(s|e)^{1-\gamma} < 1$ if $\gamma < 0$ or $\gamma > 1$ ($0 < \gamma < 1$) by HÖLDER'S inequality. \Rightarrow This implies $\Delta_{1e} > 0$ if $\gamma \neq 1$. Let $\gamma=1$; then

$\lim_{\gamma \rightarrow 1} \Delta_{1e} = \sum_{s=1}^S h_i(s|e) \ln \frac{h_i(s|e)}{H(s|e)} > 0$ by JENSEN'S inequality. Hence

$\Delta_{1e} > 0$ if $\gamma=1$. It follows that $\Delta_{i^{1'}} > 0$.

2° Suppose $\lambda=1$. (Sufficiency) From Lemma 3 it follows immediately that $\Delta_{1e}=0$, because posterior beliefs are homogeneous.

(Necessity) By HÖLDER'S inequality $\Delta_{1e}=0$ implies that $h_i(s|e) \propto H(s|e)$. This in turn implies $\lambda=1$.

Consider (ii). Define

$\Delta_{1e} = \frac{\gamma}{\gamma-1} (\sum_{s=1}^S \pi_1(s|e) \left[\frac{h(s)}{H(s|e)} \right]^{\gamma-1} - 1)$. Note that $\text{sgn}(\Delta_{1e}) = \text{sgn}(\Delta_{1e})$ ($e=1,2,\dots,E$), since Lemma 3 implies Δ_{1e} if we allow for the fact that investor i is uninformed.

1° Suppose $\lambda > 0$. It follows from the definition of $h_1(s|e)$ that in equilibrium

$$(6) \quad h_1(s|e) \left[\frac{h(s)}{H(s|e)} \right]^{\gamma-1} = \pi_1(s|e) / \sum_{s=1}^S \pi_1(s|e) \left[\frac{h(s)}{H(s|e)} \right]^{1-\gamma}.$$

Inserting (6) into Δ_{1e} yields

$$(7) \quad \text{sgn}(\Delta_{1e}) = \text{sgn} \left(\frac{\gamma}{\gamma-1} \left(1 - \sum_{s=1}^S \pi_1(s|e) \left[\frac{h(s)}{H(s|e)} \right]^{1-\gamma} \right) \right).$$

Furthermore, HÖLDER'S inequality implies

$$(8) \quad \sum_{s=1}^S \pi_1(s|e) \left[\frac{h(s)}{H(s|e)} \right]^{1-\gamma} = \sum_{s=1}^S \pi_1(s|e)^\gamma \left[\sum_{s=1}^S \pi_1(s|e) \frac{h(s)}{H(s|e)} \right]^{1-\gamma} \\ > (<) \left[\sum_{s=1}^S \pi_1(s|e) \frac{h(s)}{H(s|e)} \right]^{1-\gamma} \quad \text{if } \gamma < 0 \text{ or } \gamma > 1 \quad (0 < \gamma < 1).$$

In equilibrium the first order conditions imply that $\text{sgn}[h_1(s|e) - H(s|e)] = +(-)\text{sgn}[\pi_1(s|e) - H(s|e)]$ if $\gamma > (<) 0$. Combining this with the fact that $\text{sgn}[h_1(s|e) - H(s|e)] = -\text{sgn}[h(s) - H(s|e)]$ we deduce

$$\sum_{s=1}^S \pi_1(s|e) \frac{h(s)}{H(s|e)} - 1 = \sum_{s=1}^S \frac{[\pi_1(s|e) - H(s|e)][h(s) - H(s|e)]}{H(s|e)} \leq (>) 0$$

if $\gamma > (<) 0$ with equality holding if $\lambda = 1$. Hence

$$\left[\sum_{s=1}^S \pi_1(s|e) \frac{h(s)}{H(s|e)} \right]^{1-\gamma} > (<) 1 \quad \text{if } \gamma < 0 \text{ or } \gamma > 1 \quad (0 < \gamma < 1).$$

We therefore find from inequality (8) that

$$\sum_{s=1}^S \pi_1(s|e) \left[\frac{h(s)}{H(s|e)} \right]^{1-\gamma} > (<) 1 \quad \text{if } \gamma < 0 \text{ or } \gamma > 1 \quad (0 < \gamma < 1).$$

This result implies from (7) $\Delta_{i_e} < 0$ if $\gamma \neq 1$. Let $\gamma = 1$; then noting that $|h_i(s|e) - H(s|e)| < |h_i(s|e) - h(s)|$ for all s we obtain

$$\lim_{\gamma \rightarrow 1} \Delta_{i_e} = \sum_{s=1}^S h_i(s|e) \ln \frac{h(s)}{H(s|e)} < 0. \text{ Hence } \Delta_{i_e} < 0 \text{ if } \gamma = 1. \text{ Therefore } \Delta_{i^{(u)}} < 0.$$

2° Suppose $\lambda = 0$. Then $\Delta_{i^{(u)}} = 0$ by definition, because if nobody is informed there is no change in expected utility.***

Theorem 2. *If investors have non-optimal endowments in the form of state contingent consumption claims and if, in other respects, the assumptions of Theorem 1 hold, then*

$$\Delta_{i^{(u)}}(\lambda) > \Delta_{i^{(u)}}(\lambda) \text{ for all } \lambda \text{ and } i.$$

Hence each investor will be informed.

Proof. 1° Let $\gamma \neq 1$. Then Theorem 1 implies for all e

$$\sum_{s=1}^S h_i(s|e) \left[\frac{h_i(s|e)}{H(s|e)} \right]^{\gamma-1} > 1 > \sum_{s=1}^S h_i(s|e) \left[\frac{h(s)}{H(s|e)} \right]^{\gamma-1} \text{ if } \gamma < 0 \text{ or } \gamma > 1.$$

If $0 < \gamma < 1$ then the inequalities are reversed.

Since $(z_{i_e}/z_i)^{\gamma-1} > 0$ we find

$$\frac{\gamma}{\gamma-1} \left[\sum_{s=1}^S h_i(s|e) \left(\frac{z_{i_e} h_i(s|e)}{z_i H(s|e)} \right)^{\gamma-1} - 1 \right] > \frac{\gamma}{\gamma-1} \left[\sum_{s=1}^S h_i(s|e) \left(\frac{z_{i_e} h(s)}{z_i H(s|e)} \right)^{\gamma-1} - 1 \right].$$

Multiplying the inequality by $h_i(e) > 0$ and summing up over all e yields the result, applying Lemma 3.

2° Let $\gamma = 1$. Theorem 1 implies for all e

$$\sum_{s=1}^S h_i(s|e) \ln \frac{h_i(s|e)}{H(s|e)} > 0 > \sum_{s=1}^S h_i(s|e) \ln \frac{h(s)}{H(s|e)}. \text{ Add } \ln \frac{z_{i_e}}{z_i} \text{ and}$$

$$\text{obtain } \sum_{s=1}^S h_i(s|e) \ln \frac{z_{i_e} h_i(s|e)}{z_i H(s|e)} > \sum_{s=1}^S h_i(s|e) \ln \frac{z_{i_e} h(s)}{z_i H(s|e)}.$$

Applying the final step of 1° yields the result.***

Finally we state the special case of homogeneous posterior beliefs for the scenario of Theorem 2 in the following

Corollary. *If the assumption of Theorem 2 hold, then $\Delta_i^{(1)} < 0$ for all i .*

Proof. Define $D_i \equiv \frac{1}{\gamma-1} (\sum_{\omega \in \Omega} \pi(\omega) h_i(\omega) (z_{i\omega}/z_i)^{\gamma-1} - 1)$. Note that we have homogeneous prior and posterior beliefs. Hence $h_i(\omega) =$

$$\frac{\pi(\omega) z_{i\omega}^{1-\gamma}}{\sum_{\omega \in \Omega} \pi(\omega) z_{i\omega}^{1-\gamma}} \quad \text{and} \quad z_i = \sum_{\omega \in \Omega} \pi(\omega) z_{i\omega}.$$

Inserting these

in D_i we get $\text{sgn}(D_i) = \text{sgn}(\frac{1}{\gamma-1} [z_i^{1-\gamma} - \sum_{\omega \in \Omega} \pi(\omega) z_{i\omega}^{1-\gamma}])$.

1° Let $\gamma \neq 1$. Then $\sum_{\omega \in \Omega} \pi(\omega) z_{i\omega}^{1-\gamma} > (<) z_i^{1-\gamma}$ if $\gamma < 0$ or $\gamma > 1$ ($0 < \gamma < 1$)

by JENSEN's inequality which yields $D_i < 0$.

2° Let $\gamma = 1$. Then $\lim_{\gamma \rightarrow 1} D_i = \sum_{\omega \in \Omega} \pi(\omega) \ln \frac{z_{i\omega}}{z_i} < 0$ by JENSEN's inequality.

From Lemma 3 we have $\text{sgn}[\Delta_i^{(1)}] = \text{sgn}(D_i)$. ***

Given that all investors act identically and buy the information before trading the corollary shows that there is only a redistribution of claims to be expected and, hence, the value of information is negative for *each* investor.

Notes

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- 1) See Hakansson/Kunkel/Dhlson (1982).
- 2) Of course, the dominance implies a Nash equilibrium.
- 3) See Hakansson (1970), Mossin (1973, p.113), Franke (1984, p.427). The HARA-utility function explicitly reads $u(c) = \gamma(\alpha + c/\gamma)^{1-\gamma}/(1-\gamma)$, $\alpha \neq 0$, $\gamma \neq 0$. $\gamma=1$ represents the generalized logarithmic utility function because $u(c) \sim u^*(c) = \gamma[(\alpha + c/\gamma)^{1-\gamma} - 1]/(1-\gamma)$ and $\lim_{\gamma \rightarrow 1} u^*(c) = \ln(\alpha + c)$. $\gamma = \infty$ represents the exponential utility function, because $u(c) \sim u^+(c) = \gamma[1 + c/(\gamma\alpha)]^{1-\gamma}/(1-\gamma)$ and $\lim_{\gamma \rightarrow \infty} u^+(c) = -\exp(-c/\alpha)$.
- 4) To simplify notation the proofs throughout the paper do not repeat that γ has to be different from zero.
- 5) Note that equation (2) is also appropriate for $\gamma = \infty$ (exponential utility), because $\lim_{\gamma \rightarrow \infty} h_1(s) = \pi_1(s) \exp(-c_{1s}/\alpha_1) / \sum_{z=1}^{\infty} \pi_1(z) \exp(-c_{1z}/\alpha_1)$. Hence a separate discussion of the exponential utility case is not necessary for our purpose.
- 6) $a \propto b$ means that a is proportional to b .
- 7) See Cass/Stiglitz (1970).
- 8) See Mitrinović (1970).

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