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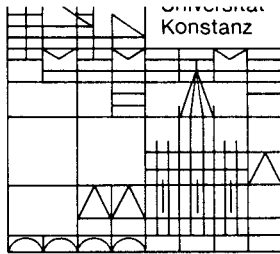
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**Incredibility of Perfect Threats
in Repeated Games:
The Dilemma of a Rational Player**

Diskussionsbeiträge

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INCREDIBILITY OF PERFECT THREATS IN REPEATED GAMES:
THE DILEMMA OF A RATIONAL PLAYER

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Abstract

The implicit assumption made when deterrence solutions to repeated games are constructed, is that perfectness is a sufficient condition for the credibility of threats. In this paper it is shown that the validity of this assumption hinges on another assumption which proves to be unsatisfactory – namely that players exhibit a different rationality at different stages of the game. A consequence of making rationality independent of time is that sufficient conditions for the credibility of threats fail to exist.

Serie A: Volkswirtschaftliche Beiträge

Serie B: Finanzwissenschaftliche Arbeitspapiere

Serie C: Betriebswirtschaftliche Beiträge

1. Introduction

Originally proposed by James Friedman, (partial) cooperative equilibria are by now widely accepted solution concepts for noncooperative repeated games.² The idea behind this concept is quite simple. If players meet one another repeatedly they may be deterred from deviating from cooperation by threats of retaliation: 'If you take the deviating action x , we will retaliate with y , which makes you regret x '. In noncooperative games, however, binding agreements to punish defectors cannot be stipulated; hence a threat must be credible in order to be an effective deterrence. Perfectness in the sense of Selten [1975] is considered a necessary and sufficient condition for the credibility of a threat.

Solutions for supergames ensuring cooperation through perfect threats are derived in Friedman [1971a,b], [1975] and Rubinstein [1979]. A partial cooperative solution for finite games with firms being content to 'almost' achieve their optimal responses to other firms' strategies is given in Radner [1980]. A partially collusive solution incorporating perfect threats for finitely repeated games where players have to post a small bond which is forfeited in case of defection is presented in Eswaran and Lewis [1983]. And optimal perfect threat solutions in the sense of ensuring the greatest possible degree of collusion are given in Abreu [1983].

The purpose of this paper is to show that credible deterrence solutions do not exist for finitely and infinitely repeated games. The sufficiency of perfectness for the credibility of threats hinges on the implicit assumption that individuals exhibit rationality of one kind at one point in time but rationality of another kind

at other times. Dropping this unsatisfactory rationality concept and making rationality independent of time has severe consequences for the credibility of threats. Now perfectness, though still necessary for credibility, is also a sufficient condition for the incredibility of threats. Sufficient conditions therefore do not exist.

In section 2, perfect deterrence equilibria are defined. Then in section 3, a concept of individual rationality is presented; and its consequences for the decision problem of a player in a repeated game is analysed. Section 4 concludes the paper with some remarks on the credibility of threats in differential games.

2. Perfect Deterrence Equilibria in Repeated Games

2.1 Repeated Games

Let G be an ordinary simultaneous game in normal form, $G = \langle \{S^i\}_{i=1}^n, \{\pi^i\}_{i=1}^n \rangle$, where n is the number of players, and where S^i and π^i are player i 's strategy set and payoff function respectively. An outcome ϑ , $\vartheta \in S$, is an n -tuple of ordinary game strategies $\langle \vartheta^{-i}, \vartheta^i \rangle$, where ϑ^{-i} denotes the $n-1$ -tuple $\langle \vartheta^1, \dots, \vartheta^{i-1}, \vartheta^{i+1}, \dots, \vartheta^n \rangle$ and where $S = S^1 \times \dots \times S^n$. Let $\pi(\vartheta) = \langle \pi^1(\vartheta), \dots, \pi^n(\vartheta) \rangle$ be the payoff vector associated with ϑ .

A repeated game $G^T = \langle \{S^i\}_{i=1}^n, \{\pi^i\}_{i=1}^n, \{>^i\}_{i=1}^n, T \rangle$ is a sequence of the ordinary game G played T times (T any integer in $[2, \infty)$), where ' $>^i$ ' is an evaluation relation on $\{\pi^i(t)\}_{t=1}^T$. If $a >^i b$, then player i strictly prefers a over b , and if $a \geq^i b$, then a is weakly preferred over b .

Since G^T is a repetition of G the set of possible outcomes at time τ , $S(\tau)$, is S . Let $q^i = \{q^i(t)\}_{t=1}^T$ be a strategy for player i in G^T , where $q^i(1) \in S^i$, and where $q^i(t): \prod_{\tau=1}^{t-1} S(\tau) \rightarrow S^i$ for $t = 2, 3, \dots, T$. A strategy for player i for the repeated game is therefore a choice at each stage of the game from the set of admissible ordinary game strategies S^i , possibly dependent on the past history of the game.

Let q be the n -tuple $\langle q^{-i}, q^i \rangle$, let Q^i be i 's set of repeated game strategies and let $Q = Q^1 \times \dots \times Q^n$. For $q \in Q$ a particular outcome at time t is given by $\vartheta(q)(t)$, where

$$\theta(q)(t) = (\dots, q^i(t)(\theta(q)(1), \dots, \theta(q)(t-1)), \dots).$$

Thus a particular outcome at t may depend via the n -tuple q on the outcomes at preceding stages of the game.

Definition 1: An n -tuple q is a Nash equilibrium point for the repeated game G^T if and only if $q \in Q$ and

$$\{\pi^i(\theta(q^{-i}(t), q^i(t)))\}_{t=1}^T \geq^i \{\pi^i(\theta(q^{-i}(t), p^i(t)))\}_{t=1}^T$$

for all i and all $p^i \in Q^i$.

In a Nash equilibrium of G^T the payoff sequence accruing to each player i from the T repetitions of G associated with the strategy q^i is (weakly) preferred to the sequence associated with any other admissible strategy, given no other player j , $j \neq i = 1, \dots, n$, chooses a strategy different from q^j . Note that the equilibrium strategies are required to be optimal with respect to the entire time horizon. A Nash equilibrium does not require strategies to be optimal with respect to any remaining repetitions of the game.

2.2 Subgames and Perfectness of Games

A history $H(t) = \{\theta(q)(\tau)\}_{\tau=1}^{t-1}$ is a $t-1$ period sequence of outcomes. $H(t) \in \prod_{\tau=1}^{t-1} S(\tau)$. A subgame $G_{H(t)}^T$ of the repeated game G^T is a game starting after a permissible history $H(t)$ for the remaining $T-(t-1)$ repetitions of the game. Let $r_{H(t)}^i = \{r_{H(t)}^i(\tau)\}_{\tau=t}^T$ be a strategy for i in the subgame $G_{H(t)}^T$, where $r_{H(t)}^i(\tau): \prod_{z=1}^{\tau-1} S(z) \rightarrow S^i$

for $\tau = t, t+1, \dots, T$, and where $S(z) = \vartheta(q)(z)$ for $z = 1, \dots, t-1$. Denote $\langle r_{H(t)}^{-i}, r_{H(t)}^i \rangle$ by $r_{H(t)}$ and let $R_{H(t)}^i$ be the set of feasible strategies for i in the subgame $G_{H(t)}^T$. Let $R_{H(t)} = R_{H(t)}^1 \times \dots \times R_{H(t)}^n$.

Definition 2: An n -tuple $r_{H(t)}$ is a *Nash equilibrium point* for the subgame $G_{H(t)}^T$ if and only if $r_{H(t)} \in R_{H(t)}$ and

$$\{\pi^i(\vartheta(r_{H(t)}^{-i}(\tau), r_{H(t)}^i(\tau)))\}_{\tau=t}^T \geq^i \{\pi^i(\vartheta(r_{H(t)}^{-i}(\tau), s_{H(t)}^i(\tau)))\}_{\tau=t}^T$$

for all i and all $s_{H(t)}^i \in R_{H(t)}^i$.

Definition 3: The equilibrium $r_{H(t)}$ is said to *jointly dominate* the equilibrium $v_{H(t)}$ if and only if

$$\{\pi^i(\vartheta(r_{H(t)}(\tau)))\}_{\tau=t}^T >^i \{\pi^i(\vartheta(v_{H(t)}(\tau)))\}_{\tau=t}^T$$

for all i .

We can now define a perfect equilibrium (in the sense of Selten [1975]).

Definition 4: An n -tuple q is a *perfect equilibrium* for G^T if and only if it is a Nash equilibrium point for each subgame $G_{H(t)}^T$, $t = 1, \dots, T$, and for all $H(t) \in \prod_{z=1}^{t-1} S(z)$.

A perfect strategy for player i therefore is an optimal strategy for the remainder of the game at all stages and for all permissible histories of the game. For the subsequent analysis it is important to note that a perfect strategy q^i is only optimal for player i at any stage and for any history $H(t)$, given that other players play

at each stage their perfect strategies $q_{H(t)}^{-i}$. That is, although $q_{H(t)}$ is stable against *unilateral* deviations of player i whatever the stage and state of the game, perfectness does not ensure optimality of $q_{H(t)}^i$ if other players deviate from $q_{H(t)}^{-i}$.

2.3. Perfect Deterrence Equilibria

A perfect deterrence equilibrium of G^T is a perfect equilibrium with special properties. Let q be a Nash equilibrium. If $y = \langle y^{-i}, y^i \rangle$ is a deterrence equilibrium, then it has the following properties:

- (i) $y \in Q$.
- (ii) The deterrence equilibrium y jointly dominates the Nash equilibrium q .
- (iii) There are at least some t for which there exists a cheat strategy d^i for some i such that

$$\pi^i(\vartheta(y_{H^*(t)}^{-i}(t), d^i)) > \pi^i(\vartheta(y_{H^*(t)}^{-i}(t), y_{H^*(t)}^i(t))),$$

where $H^*(t) = \{\vartheta(y)(\tau)\}_{\tau=1}^{t-1}$.

- (iv) d^i is observable.
- (v) y is perfect.

A player will only participate in (partial) cooperation if he prefers his payoff sequence associated with cooperation, over the sequence associated with strict noncooperation on his part. Hence in a deterrence equilibrium all players must prefer their respective equilibrium payoffs over their respective payoffs in the strictly noncooperative Nash equilibrium (property ii).

A deterrence equilibrium must not necessarily imply cooperation for the entire length of the game (see Radner [1980]). But at each repetition of the ordinary game for which the deterrence solution prescribes cooperation, there exist noncooperative cheat strategies, d^i , for some i such that the payoff to player i from cheating in that single play is higher than the payoff from abiding by the rules of cooperation (property iii).

A punishment in response to cheating can only be executed if cheating is observable (property iv). Therefore y must be a vector of pure strategies since randomized choices cannot be sensibly regarded as observable.

Each player i , enjoying single period gains from cheating, must be deterred from doing so. That is, for all t and all i $\{y_{H^*(t)}^i(\tau)\}_{\tau=t}^T$ must be player i 's optimal strategy for the subgame $G_{H^*(t)}^T$ given the threat $y_{H^*(t)}^{-i} = \{y_{H^*(t)}^{-i}(\tau)\}_{\tau=t}^T$, where $H^*(t) = \langle H^*(t-2), \vartheta(y_{H^*(t-1)}^{-i}(t-1), d^i) \rangle$. The long-run losses from cheating caused by the punishment $y_{H^*(t)}^{-i}$ must outweigh at each stage of the game the short-run gains from doing so. This must be so because in a noncooperative game binding agreements to stick to y cannot be made. Therefore a deterrence equilibrium must be subgame perfect. It is shown in Selten [1975] that a perfect equilibrium is subgame perfect.

Optimality of $y_{H^*(t)}^j$, $j = 1, \dots, n$, for all subgames $G_{H^*(t)}^T$, $t = 1, \dots, T$, does not suffice. A strategy y^j must be optimal for all possible histories of the game and not only for $H^*(t)$, because $y_{H^*(t)}^{-i}$ is only an effective deterrence if it can be trusted that $y_{H^*(t)}^{-i}$ will be executed once $H^*(t)$ has occurred. Since d^i may be

any element from S^i , a *necessary* condition for the credibility of $y_{H'(t)}^{-i}$, $i = 1, \dots, n$, is that a player j , $j \neq i = 1, \dots, n$, does not want to *unilaterally* deviate from the punishment $y_{H'(t)}^j$ for all permissible $H'(t)$. Once a deviation from cooperation has occurred it must be in each player's selfish interest to participate in the punishment, *given* that all other players participate (property v).

Perfectness has been taken to be a necessary and sufficient condition for the credibility of a threat. In the next section it will be shown, however, that perfectness is not a sufficient condition for credibility in a repeated game, where the players' rationality is invariant over time.

3. The Dilemma of a Rational Player

3.1. A Concept of Individual Rationality

All decision makers are assumed to be rational in accordance with the following postulates.

Postulate 1: A player i makes his decision $r_{H(t)}^i$, where $r_{H(t)}^i \in R_{H(t)}^i$, for $t = 1, \dots, T$, on the basis of, and *only* on the basis of information contained in $G_{H(t)}^T = \langle \{S^i\}_{i=1}^n, \{\pi^i\}_{i=1}^n, \{>^i\}_{i=1}^n, T, H(t) \rangle$.

Postulate 2: In choosing his own decision, a player assumes that the other players are rational in the same way as he himself is rational.

Postulate 3: If some decision is the rational decision to make for an individual player, then this decision can be correctly predicted by other players.

Postulate 4: Being able to predict the actions to be taken by other players, a player's own decision entails a payoff sequence which he prefers at least as much as any other payoff sequence, corresponding to the predicted actions of other players.

For a lucid interpretation and defense of these postulates — see Johansen [1982].³ It is shown there that a solution to non-cooperative games of individually rational players must be an equilibrium point. It is easy to see that this concept also implies that a deterrence equilibrium must be perfect. Because by postulate 3, if the execution of a threat $y_{H'(t)}^j$ is not player j 's optimal decision once $H'(t)$ has occurred, this can be predicted by others.

As far as deterrence *solutions* are concerned, one further rationality concept is needed.

Postulate 5: Comparing any two strictly noncooperative equilibria y and q , a player i chooses y^i if y jointly dominates q . If y is a perfect deterrence equilibrium and q is a strictly noncooperative equilibrium, then player i makes the decision y^i if y jointly dominates q and if he considers perfectness sufficient for the credibility of threats.

A deterrence equilibrium can only be considered a solution to a repeated game if all players, being confronted with both a strictly noncooperative equilibrium and a deterrence equilibrium, actually start out cooperatively at the beginning of the game. With a multiplicity of equilibria a player must be able to predict which of the alternative equilibrium strategies other players will choose in order to be able to make a sensible decision himself. Since a player's deterrence strategy is only optimal given that others also will choose their respective deterrence strategies, a player will only start out cooperatively if he trusts that all others will do the same.

The rationale behind postulate 5 then is this: If a strictly noncooperative equilibrium y jointly dominates a strictly noncooperative equilibrium q , there is unanimous consent among players as to which is the ex ante preferred solution. Therefore, in accordance with a tacit agreement, each player i can trustfully choose his preferred strategy y^i because he can expect others to do the same. (The equilibrium y is called a 'solution in the strict sense' (Luce and Raiffa [1966]; y^i is also called a 'rational joint strategy' (Harsanyi [1977]).) Consider now the choice between a deterrence equilibrium y and a

strictly noncooperative equilibrium q . If a player considers perfectness to be sufficient for the credibility of a threat, then, by postulate 2, he must think that all others do the same. If y jointly dominates q he then can expect others to choose their respective rational joint strategies so that he himself can choose his rational joint strategy y^i .

Postulate 5, though necessary to ensure that the deterrence equilibrium and not the strictly noncooperative equilibrium will be the solution to G^T , is not satisfactory. It merely requires players to exhibit joint rationality at the beginning of the game. It does not, however, require players to exhibit this rationality during the entire course of the game. By postulate 1, learning is not possible since all relevant information is already known at the beginning of the game. It therefore seems unreasonable to assume players to be rational in a certain way at one time, but rational in some other way at other times. I therefore shall assume that joint rationality holds at each stage of the game.

Postulate 5': Comparing any two strictly noncooperative equilibria $y_{H(t)}$ and $q_{H(t)}$, a player i chooses $y_{H(t)}^i$ if $y_{H(t)}$ jointly dominates $q_{H(t)}$. If $y_{H(t)}$ is a perfect deterrence equilibrium and $q_{H(t)}$ is a strictly noncooperative equilibrium, then player i makes the decision $y_{H(t)}^i$ if $y_{H(t)}$ jointly dominates $q_{H(t)}$ and if he considers perfectness sufficient for the credibility of threats.

Surprisingly this seemingly insignificant generalization has severe consequences for perfect deterrence equilibria. Perfectness then merely is necessary but not sufficient for credibility. Furthermore

it implies that perfectness is a sufficient condition for the *incredibility* of threats incorporated in deterrence equilibria. That is, it implies that sufficient conditions for the credibility of a threat do not exist. A deterrence equilibrium therefore cannot qualify as a solution to G^T .

3.2. Nonexistence of Credible Threats

Suppose there exists a perfect deterrence equilibrium y for G^T , implying (partial) cooperation up to and including some period T' , T' any integer in $[2, T]$. Suppose $H'(t+1) = \langle H^*(t-1), \vartheta(y_{H^*(t)}^{-i}(t)), d^i \rangle$ has occurred.

Lemma: In the subgame $G_{H'(t+1)}^T$ each player j will choose the strategy $y_{H^*(t+1)}^j$ if he considers perfect threats credible.

Proof: The proof will be given by demonstrating that for the history $H'(t+1)$, $y_{H^*(t+1)}$ is a perfect equilibrium. It must be true that $y \in Q$ and because of (v) that $y_{H^*(t)} \in R_{H^*(t)}$ for all t . Since $G_{H(t)}^T$ is an identical repetition of G for all $H(t)$ and all t , $S(z) = S$, $z = t, \dots, T$, for all t and all $H(t)$. Since $\prod_{z=1}^T S(z) \subset \prod_{\tau=1}^T S(\tau)$ for any $H(t) = \{\vartheta(q)(z)\}_{z=1}^{t-1} \in \prod_{z=1}^{t-1} S(z)$, t any integer in $[2, T]$, it must be true that $Q \subseteq R_{H(t)}$ for all permissible $H(t)$ and all t . By the repetitive character of the game

$$\{\pi^i(\vartheta(y(z)))\}_{z=t+1}^T = \{\pi^i(\vartheta(y_{H^*(t+1)}(z)))\}_{z=t+1}^T,$$

all i , all t , all y and all $H(t+1)$, since $H^*(t+1) = \{\vartheta(y)(\tau)\}_{\tau=1}^t$.

By (v) and feasibility of $y_{H^*(t+1)}$ for the subgame $G_{H^*(t+1)}^T$, $y_{H^*(t+1)}$ then must be a perfect equilibrium for the subgame $G_{H^*(t+1)}^T$. But then, by postulate 5', a player j will choose the strategy $y_{H^*(t+1)}^j$ for the subgame $G_{H^*(t+1)}^T$ if he considers perfect threats credible. Q.E.D.

Theorem: Credible deterrence equilibria do not exist for repeated games if the players' individual rationality is consistent with the rationality postulates 1-4 and 5'.

Proof: By (v), a deterrence strategy must be perfect. But then, by the Lemma, a player j will choose $y_{H^*(t+1)}^j$ if $H^*(t+1)$ has occurred. But then, by (iii), a rational player i will choose d^i . That is, if postulates 1-4 and 5' describe individual rationality, then perfectness is a sufficient condition that a threat is incredible. Since perfectness is necessary for credibility, sufficient conditions for the credibility of threats fail to exist. Q.E.D.

3.3 Interpretation

Consider the problem of a player j whether or not to believe in the credibility of a perfect threat. If he believes in the credibility of a perfect threat, then he will choose his cooperative strategy at the beginning of the game because he is convinced that cooperation benefits all and because he considers others to be rational (in the same way as he himself is). If he has observed a defection of player i , it would be in j 's own interest to execute the threat

$y_{H'}^j(t+1)$ only if others would do the same. However, a defection in the past does not make cooperation in the future impossible. Since future cooperation is rational precisely because the threats incorporated in the deterrence strategies are in his view credible, he must expect other players to be willing to jointly 'forget the sins of the past'. That is, to tacitly shelve punishment and jointly switch to the equilibrium strategies which are *ex ante* preferred by all. But now player *j* *being* rational is on the horns of a dilemma. Because now player *i* can confidently cheat, precisely because player *j* is convinced that the threat is credible. Player *j* therefore will not consider perfect threats credible and therefore will not participate in cooperation.

That threats of retaliation are incredible if 'bye-gones are bye-gones' is self-evident. But here a different and stronger point is made. It is, that conditions which are necessary for threat equilibria to be attained by players suffice that the same players let bye-gones be bye-gones.

4. Final Remarks

The nonexistence of credible threats does not pertain in full generality to differential games. In these games, the set of feasible strategies for a subgame depends on the history of the game. Therefore a defection from cooperation at some time may influence the state of the system to develop in a way such that a cooperative equilibrium does not exist for the remainder of time. A potential traitor then will have no doubt that the punishment would be executed. For a perfect deterrence equilibrium to be a solution, it would be necessary that at all stages of the game a defection would destroy the cooperative equilibrium. Generally, these strong conditions will not be fulfilled. There are even differential games where cooperation is always possible (see for example the cooperative equilibrium to a fishing game given in Lewis and Cowans [1983]). Therefore one is bound to expect that in most differential games players face the same dilemma as in repeated games.

Notes

1. I would like to thank David de Meza for very helpful comments on this topic.
2. Throughout this paper the terms (partial) cooperative equilibrium, deterrence equilibrium, and threat equilibrium will be used equivalently.
3. In Johansen [1982] these postulates are given for an ordinary game. Postulates 1 and 4 are stated somewhat differently to take account of the repetitive nature of the game.

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