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Ecological decision making and optimal stopping rules

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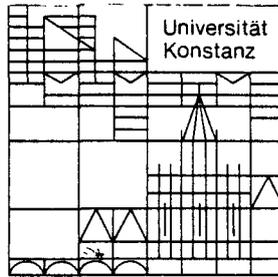
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**Fakultät für
Wirtschaftswissenschaften
und Statistik**

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**Ecological Decision Making
and Optimal Stopping Rules**

Diskussionsbeiträge

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ECOLOGICAL DECISION MAKING
AND OPTIMAL STOPPING RULES

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Serie A - Nr. 189

June 1984
Revised September 1984

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Under normal circumstances, the instincts make no mistakes.
It is reason which is fallible.

Ronald Coase quoting Jacob Viner
paraphrasing Adam Smith.

Serie A: Volkswirtschaftliche Beiträge
Serie B: Finanzwissenschaftliche Arbeitspapiere
Serie C: Betriebswirtschaftliche Beiträge

1. Preliminaries

In their three back-to-back papers Coase (1978), Ghiselin (1978) and Hirshleifer (1978) argue persuasively for recognizing "a body of knowledge called natural economy coordinate with political economy (economics), together forming a branch of knowledge which we may call general economy."¹ The unifying principles include competition, division of labor, cooperation and conflict.

A major goal of this paper is to draw attention to another unifying principle, the search and matching behavior that characterizes all organisms from the pre-biotic molecules through the hunter-gatherer societies (the primary application) to modern human societies. We argue that this search and matching behavior is nature's primary response to uncertainty and that it can be best described by the theory of optimal stopping rules. Indeed, we contend that search, matching, and the optimal stopping paradigm are self-similar² phenomena that occur at every level of being.³ For example, humans will search until they find an acceptable job, an acceptable location, an acceptable mate, an acceptable store, etc. It is as if the individual wrapped himself in a blanket of contracts to secure protection from uncertainty. Each

¹Ghiselin (1978).

²Roughly speaking a transformation is self-similar if its structure is invariant to magnification. See Mandelbrot (1983) and Good (1984).

³The details of this argument will be presented elsewhere. We will show that this view is consistent with the recent seminal work by Eigen and Schuster (1979), Prigogine (1980), Haken (1977), and Mandelbrot (1983) as well as with the more traditional and seminal models of MacArthur (1972), Charnov (1976), Stigler (1962), and others.

of these matches (contracts) is a fixed point,⁴ and it is the constellation of fixed points that determines the individual's welfare. The multi-dimensional distribution from which the individual searches is also a fixed point⁵ and is one of the main constituents of a society. The formation and dissolution of an individual's fixed points and the fluctuation of the fixed point distribution are inter-related and comprise the essence of the human condition causing both exhilaration and dejection.

The formation of matches and their dissolution are interdependent processes.⁶ As an illustration, consider a stable marriage (match 1) where each partner has achieved a "permanent job (match 2 and 3) in region r_0 (match 4). Now suppose the husband is unexpectedly dismissed. After a prolonged period of search U_1 in region r_0 he decides that he must move to r_1 and dissolve the marriage. This he does and after another long spell of unemployment U_2 eventually gets a permanent job in r_3 . Clearly, the cost of the unemployment entails not only its length, but also the attachments (matches) that were severed while it was endured. Thus reflection on the unraveling of joint matches sheds light on the cost of unemployment.

This optimal stopping approach to economic behavior has led to a reformulation of microeconomic theory. Workers and employers search for one another until a match is consummated. Consumers search for low price stores

⁴That is, the solution to the functional search equation is a unique fixed point. Recall that x is a fixed point of the function f if $x = f(x)$. An extensive discussion is contained in Lippman and McCall (forthcoming b).

⁵A proof is contained in Berninghaus (forthcoming).

⁶This is also true of animal behavior. The large literature on foraging, mating and migration does not contain a model that treats these activities jointly using optimal stopping rules. This could be a significant contribution. See Hamilton (1983).

and stores advertise prices until an implicit contract (match) is struck between stores and consumers. Firms seek the best locations and industries. Individuals search for marriage mates, and so on. Thus at any point in time, the economy is composed of numerous matches (fixed points), unattached searchers, and equilibrium distributions of wages and prices (fixed points). As the economy fluctuates, matches dissolve, equilibrium distributions shift and a business cycle unfolds. For a complete description of this process see Lippman and McCall (forthcoming a). The main point to emphasize here is that this stochastic view of economics has behavioral implications quite different from those of the deterministic economic model. We suspect that this is also true for the general economy.

2. Introduction To Ecological Decisionmaking

This paper was inspired by Smith's (1983) excellent review of the anthropological applications of optimal foraging theory and by the recent papers by Hill (1982) and Lovejoy (1981) on human evolution. As Smith (p. 625) notes,

Humans have spent the bulk of their evolutionary history in foraging economies. In appreciation of this fact, anthropologists have endeavored to establish the general rules that apply to this mode of organization in the hope of explaining observed differences and similarities in the hunter-gatherer societies and of accounting for both the persistence and transformation of this once universal way of life.

Smith then applied the optimal foraging theory that was developed in evolutionary ecology to the basic foraging problems confronting preagricultural economies.

The purpose of this paper is to extend Smith's analysis of foraging by applying probabilistic methods (primarily optimal stopping rules) used in analyzing decision-making under uncertainty in economics to the corresponding decisions in societies.⁷ In a series of footnotes, we will also indicate how these rules are applicable to the study of animal behavior.⁸ Thus the

⁷Jack Hirshleifer was the first to suggest that our models of economic search might be applicable to the foraging problem. Eric Charnov endorsed this position and gave us considerable encouragement. Indeed, were it not for Charnov's enthusiasm about this work, and the education provided by the many papers he has shared with us, this paper would never have been written. Our current plan is to compose a joint paper with Charnov on animal behavior under uncertainty. We are also indebted to Smith for his extensive and incisive comments on an earlier draft. The paper also benefited from comments made during a seminar at the University of Konstanz. Finally, we thank our colleagues Clower and Leijonhufvud for their valuable suggestions. Of course, all of these commentators are exonerated from responsibility for the errors and inanities that remain in the paper.

⁸We could also run a series of subfootnotes describing the search and matching and mismatching behavior of molecules. Eigen et al. (1981) find that molecules cooperate at the very beginning of life. Macromolecules form "binding" agreements until it is in their "self-interest" to split. Monod

"ecological" decisions in the title refer to those made by hunter-gatherer societies and also by animals.

Ecological decisions include (i) foraging (its modern counterparts are search for low price/high quality commodities and search for the "best" job), (ii) search for a suitable settlement, including the decisions when to stay, when to leave, and where to resettle (immigration and migration), (iii) mate choice (marriage and divorce), (iv) when to fight and when to retreat (warfare),⁹ and (v) the formation and dissolution of treaties.

Optimal stopping rules is the methodology that we will use to analyze the ecological decisions of hunter-gatherer societies.¹⁰ The theory of optimal stopping is well-developed, but only in the last 15 years has it been applied in economics. It is now a flourishing activity and has provided theoretical and empirical insights into most branches of economic theory.¹¹ Optimal stopping rules is only one of the probabilistic topics that is currently receiving much attention in the economics of information and uncertainty.¹² The most germane of these other topics are adverse selection, moral hazard,

(1971) also addresses the manner in which molecules "find" one another. Stopping rules describing these matches are analogous to the marriage agreement we will study in Section 4. Koppers (1983) presents an excellent discussion of molecular evolution. The application of stopping rules to the molecular theory of evolution will be presented in a subsequent paper.

⁹The corresponding animal decisions are (i) foraging, which includes the subsidiary decisions of diet breadth, patch choice, time allocation, and foraging group size (see Smith), (ii) nest choice, (iii) mate choice, and (iv) the decision to fight or run.

¹⁰The founder of optimal stopping rules was Wald (1941) with Cayley (1875) an early precursor. For the history of optimal stopping rules see Chapter 2 of Lippman and McCall (forthcoming, a).

¹¹These include labor economics, macroeconomics, industrial organization and econometrics. See Lippman and McCall (forthcoming, a).

¹²For surveys of these methods see Hey (1979), Hirshleifer and Riley (1979), and Lippman and McCall (1982).

consumption under uncertainty and production under uncertainty. We will refer to them when they seem appropriate.¹³

Before proceeding three methodological points are of considerable importance. First, there has been criticism of the application of foraging models, designed to study animal behavior, to humans.¹⁴ The success of the

¹³Renewed interest in insurance by economists has given rise to a vast literature on incentives and risk sharing that attend the principal-agent problem. The two major problems common to this literature are "moral hazard" and "adverse selection". See Hirshleifer and Riley (1979), Lippman and McCall (Ch. 2, forthcoming a), and Radner (1983). Recent applications of this theory to primitive societies are contained in Posner (1983), Lippman and McCall (forthcoming, b), and Hawkes et al. (1984). They show that when contracting transpires in an uncertain environment, two basic problems present themselves: moral hazard and adverse selection. Both are founded on imperfect information. The prototypical example of these problems is the insurance contract between an insurance company (the principal) and the insured (the agent). By paying a premium the agent transfers the risk associated with a particular activity to the principal. This risk transfer affects the incentives and behavior of the agent. It is these incentive effects that are commonly referred to as the moral hazard problem. It has its roots in the inability of the principal to costlessly observe the actions of the agent. Hence when the untoward event occurs, the principal is not sure whether it was caused by the agent's carelessness or by chance. Moral hazard can be reduced by requiring the agent to bear some of the costs of the contingency and/or by monitoring the agent's behavior. Adverse selection is similar to moral hazard in that the problem arises because the principal does not have costless access to information possessed by agents and vice versa. For example, purchasers of health insurance have much more information about their health status than insurance companies. Because the insurance company cannot discriminate between healthy and sickly agents the latter will pose as healthy agents and be "adversely selected" (insured) by the principal. Insurance companies can and do cope with these informational asymmetries by (a) experience rating, that is, continually adjusting premiums to reflect the size and incidence of each agent's claims and (b) designing policies that elicit the information necessary for partitioning agents into distinct categories.

¹⁴In his review of Mate Choice, Hamilton (1983) observes: Despite the absence, by and large, of man, the book certainly cannot be read without thought of him (and her). Or indeed without noticing the busy trade in terms and ideas between, on the one hand, the exciting, agonizing world we all know so well from the inside and, on the other, the parody world of the animals that we can observe a little more objectively -- a world of nest and plume and song and tiny combat. How seriously one takes the correspondence to human behaviour depends greatly on temperament, but it can be said that the more animals are studied the more human their problems and reactions seem.

former in no way guarantees the appropriateness of the latter. Our view on this is completely reversed. As will be elucidated, the stochastic models expositied here already have proven extremely useful in explaining human behavior, witness the extensive empirical literature indicating that the sophisticated sequential models of job search, mate choice and consumer search do indeed explain observed behavior.¹⁵ Consequently, one might expect a similar success in the application of foraging models.

Furthermore, we suspect that these same probabilistic stopping rule models, appropriately modified, can supplement what till now has been a primarily deterministic modelling of animal behavior.¹⁶ Indeed, we contend that a general optimal stopping theory of foraging, mating, and migration may illuminate the structure of modern society,¹⁷ animal societies, and primitive societies. The interchange among the empirical workers in these fields should be especially valuable. While difficult, it is much easier to conduct controlled experiments with animals than with humans -- For example, compare the Income Maintenance experiment with the animal experiments reported in

¹⁵For example, see Becker et. al. (1977), Marvel (1976), Hutchens (1979), Kiefer and Neumann (1981), Lancaster and Chesher (1983), and Gotz and McCall (1984). A critical appraisal of this enormous empirical search literature is contained in Chapter 11 of Lippman and McCall (forthcoming, a). Some of these tests are relatively naive, but others are based on the most sophisticated econometric methods available as developed in Flinn and Heckman (1982) and Heckman and Singer (1984a,b).

¹⁶Important exceptions include: Oaten (1977), Krebs et al. (1978), Green (1980, 1984), McNamara (1982) and Stephens and Charnov (1982). Also see Hamilton (1983) and Wittenberger (1983).

¹⁷As a consequence of these models we already have a much better understanding of phenomena that were inexplicable with purely deterministic models. This includes unemployment, quits, layoffs, divorce, advertising and brand switching. We suspect that the mating, migration and foraging strategies of animals and primitive man will be similarly enlightened by probabilistic considerations.

Mating Choice -- and, of course, empirical tests of primitive human behavior are the most difficult. Yet the study of primitive societies could have a decisive effect on our understanding of the genesis of religion and "custom" that in turn have profoundly influenced human behavior.¹⁸ And these animal experiments can be quite pertinent to human behavior: for example, it has been shown that animal males who are not dominant undergo a chemical transformation inducing lassitude, thereby reducing both their aggression and sex drive and perhaps saving their own lives¹⁹ (See Wilson (1975, Chapter 11) and Axelrod and Reisine (1984)).

The second preliminary remark concerns the use of optimization models to explain human and animal behavior.²⁰ With respect to human behavior, several prominent economists (see Hey (1981) and references therein) maintain that the search strategies devised by theorists are too complex to be implemented by average decisionmakers. However, as already indicated, the empirical tests of

¹⁸For example, how can we explain the persistence of incorrect ideas? See Lovejoy (1933).

¹⁹It could well be that this mechanism was not at work among primitive men. Grudges were held and the leader was attacked when he was most vulnerable. Violence must have had a strong selective value (see Monod (1971) and the current coverage of crime in your local newspaper) and it no doubt frequently spilled over within tribes and families. Somehow the selective value of violence and stifling of innovation must be explained. Did Cain (the farmer) kill Abel (the shepherd) because Cain was an innovator whose sacrifice to God was appreciated less than Abel's.

²⁰Our position is similar to Peirce's. "[I]f man's mind has been developed under the influence of those [natural] laws, it is to be expected that he should have a natural light, or light of nature, or instinctive insight, or genius, tending to make him guess those laws aright, or nearly aright. This conclusion is confirmed when we find that every species of animal is endowed with a similar genius." This position is similar to that developed in the recent book by Kithker (1984), who seems unaware of Peirce. Peirce is probably America's greatest philosopher. He, not James, invented pragmatism, understood the importance of uncertainty in all natural phenomena, and proposed a methodology of science that is superior to Popper's. See Rescher (1978) and Hartshorne, et al. (1958-1983).

these models suggest otherwise. While much more empirical work is required before we can be "sufficiently" confident, the work reported thus far supports our position.²¹

Finally, it should be noted that the long and productive two-way interchange between economists and biologists beginning with Smith, Malthus, and Darwin suggests that anthropology may benefit from not only biology but also economics and, what perhaps may be more significant, enrich both of these disciplines. The main problem confronting primitive man was the precariousness of his life. The response to this uncertainty depended to some extent on the way in which it was manifested. Awesome events like fires, floods, and famine were no doubt attributed to angry dieties who were placated by religious practices and magic.²² On the other hand, risks that were

²¹In his comment on Smith's paper Christenson notes that uncertainty models now may be unnecessary since man has adapted to uncertainty by holding buffer stocks, etc. The point is that these risk reducing institutions cannot be explained by deterministic models.

²² For primitive men had none of the elaborate arts of protection and use which we now enjoy and no confidence in his own powers when they were inforced by appliances of art. He lived under conditions in which he was extraordinarily exposed to peril, and at the same time he was without the means of defense which are today matters of course. Most of our simplest tools and utensils did not exist; there was no accurate foresight; men faced the forces of nature in a state of nakedness which was more than physical; save under unusually benign conditions he was beset with dangers that knew no remission. In consequence, mystery attended experiences of good and evil; they could not be traced to their natural causes and they seemed to be the dispensations, the gifts and the inflictions, of powers beyond possibility of control. The precarious crises of birth, puberty, illness, death, war, famine, plague, the uncertainties of the hunt, the vicissitudes of climate and the great seasonal changes, kept imagination occupied with the uncertain. Any scene or object that was implicated in any conspicuous tragedy or triumph, in no matter how accidental a way, got a peculiar significance. It was seized upon as a harbinger of good or as an omen of evil.

Dewey (1980, pp. 9-10)

systematic like internal tribal quarrels and invasion by outsiders gave rise to practical responses. Religious and practical response remain the two fundamental ways for coping with the inherent instability of life. The rituals and religious practices are for the most part ignored in the analysis presented here — they are constraints on rational behavior. Understanding the genesis and persistence of these "constraints" is one of the major contributions that anthropology can make to economics and biology.

The paper begins with the band's choice²³ of settlement or home base. We assume that for one reason or another the old settlement has become inadequate. As with most of these choice problems, we formulate it as a simple stopping rule problem. Because the reader may be unfamiliar with this technique, we give a fairly complete discussion of this first application.

Having chosen a settlement, the group's next problem is to design an optimal hunting strategy. The breadth of the diet will determine the techniques of the hunt, that is, the number of hunting parties and the size and duties of each participant. We assume that there is a leader who makes the critical decisions²⁴ — where to hunt and when to pursue an observed prey -- and monitors the performance of members of the hunting party. Presumably, the sharing rule will depend on performance although this may be predetermined by tribal custom. The where-to-hunt and when-to-pursue decisions are posed as optimal stopping rules.

²³The band may be composed of only one family or a single individual.

²⁴Smith points out (private correspondence) that in most human foraging societies, democracy prevails and there is no single decisionmaker. For simplicity, we do not study the voting behavior — only the decision.

We next turn to the mating decision. For simplicity, we assume first that the female chooses her mate,²⁵ and following Hill (1982), that the best providers are the best mates. Presumably a large variability in the provision of skills of males would give rise to polygamy, but custom may thwart this tendency. If it does we must explain why the custom arose in the first place.²⁶

In making her choice, the female sets some critical level of provisioning and all males who exceed that are possible mates. Males will not be completely passive in this process and perhaps none of the males above a woman's critical number will be interested in her, in which case she revises the number downward.²⁷ We then consider explicitly joint mate choice by male and female. Next migration and the warfare it might induce are briefly discussed. The formation and violation of treaties is then analyzed. The final section contains some provisos and suggestions for future work.

²⁵Once again we recognize the naivete of this view. Surely custom and religion will play a large role in mate choice. The point is that whoever does the matching can be assumed to follow optimal stopping rules. Furthermore, one would suppose that the religious practices which endure confirm the forces of natural selection.

²⁶See Lovejoy (1981) and Wilson (1975).

²⁷Needless to say, adaptation is crucial to the applicability of these stopping rules. To avoid clutter and enhance clarity, each rule formulated here will be primarily nonadaptive.

3. Settlement Choice²⁸

There are several characteristics that determine the suitability of a particular site: proximity to food and water, availability of building materials, desirability for child rearing, distance from hostile tribes, and the ease with which it can be defended. There are several ways of formulating this as a stopping rule problem. Since there are $n > 1$ characteristics, we could formulate it as a multi-dimensional stopping problem with the leader of the tribe determining when the process (a particular site) has entered the stopping set. The drawback of this approach is that calculation of the stopping set is quite difficult.²⁹ The second alternative is to assume that the different characteristics are not equally valued by all members of the tribe. For example, the men may be more concerned with defense, while for the women child rearing is paramount. These different rankings can also be formulated as a multi-dimensional stopping rule problem with, in the simplest version, the i^{th} subgroup only concerned with the i^{th} characteristic.³⁰

²⁸The same method described here can be used to explain nest choice by an animal, plant location by a firm and regional location and/or house choice by a modern family. For example, suppose a queen bumblebee has just emerged from hibernation and is searching for a nest. The belated information model is relevant here. In fact, there may be two pieces of belated information. The bee discovers a site that looks acceptable, that is, its immediately observed characteristics are greater than a critical number x_{α} . The bee checks it out and finds it occupied by another bee. It checks x'_{α} the critical number for occupied sites and the nest is still acceptable. Thus, it fights. If it wins it stays and if it loses it looks elsewhere. Suppose it wins. Now it gives the site a thorough look. If it remains acceptable it stays, if a defect is revealed it leaves and looks elsewhere. This can be formalized by slightly extending the belated model in Section 3. An excellent discussion of bumblebee behavior and economics is presented in Heinrich (1979).

²⁹See Brock and Rothschild (1983) and references therein.

³⁰See Kurano et al. (1980).

Finally, all characteristics may not be immediately observable. This is a problem of belated information which we will study later.³¹

To avoid these problems, we will assume that the leader is the sole decisionmaker, all n characteristics are immediately observable, and he wishes to choose that site with the largest expected utility u per unit time, where $u = u(x_1, x_2, \dots, x_n)$. This converts the n dimensional problem into a one dimensional problem to which the simplest stopping rule analysis applies.

Analysis

Let c be the cost of locating a potential settlement and let the value of the next site be a random variable U with density f . The optimal return from search when a site with utility value u has just been observed is denoted by $V(u)$. The leader has two options with returns $S(u)$ and $C(u)$: stop searching and receive u every period hereafter (assume infinite horizon), or he can continue searching in which case the optimal return is

$$C(u) = -c + \beta \int_0^{\infty} V(y) f(y) dy = -c + \beta EV(U),$$

where $\beta \equiv \frac{1}{1+r}$ and r is the appropriate discount rate. Thus,

$$\begin{aligned} V(u) &= \max \{S(u); C(u)\} & (1) \\ &= \max \{u/(1-\beta), -c + \beta \int_0^{\infty} V(y) f(y) dy\} \\ &\equiv \max \{u/(1-\beta), \xi\}. \end{aligned} \quad ^{32}$$

³¹See Lippman and McCall (1982) and Wilde (1979).

³²Using a contraction mapping argument, it is easy to show that the solution of this functional equation is a unique fixed point. See Lippman and McCall (forthcoming, b).

As shown in Figure 1 where both $S(u)$ and $C(u)$ are plotted against u , the function $C(u)$ is the constant ξ^* whereas $S(u)$ is a line through the origin. Clearly, for any $\frac{u}{1-\beta} < \xi$, $C(u) > S(u)$ and search continues; for any $\frac{u}{1-\beta} > \xi$, $S(u) > C(u)$ and search terminates.

Computing $\xi(1-\beta)$, the reservation utility, is not difficult:

$$\begin{aligned} \xi(1-\beta) &= (1-\beta) [-c + \beta EV(U)] \\ &= -(1-\beta)c + \beta(1-\beta) \left[\int_0^{\xi(1-\beta)} \xi f(u) du + \int_{\xi(1-\beta)}^{\infty} \frac{u}{1-\beta} f(u) du \right] \\ &= -(1-\beta)c + \beta(1-\beta)\xi \left[\int_0^{\xi(1-\beta)} f(u) du + \int_{\xi(1-\beta)}^{\infty} f(u) du - \int_{\xi(1-\beta)}^{\infty} f(u) du \right] \\ &\quad + \int_{\xi(1-\beta)}^{\infty} \beta u f(u) du \\ &= -(1-\beta)c + \beta(1-\beta)\xi + \beta \int_{\xi(1-\beta)}^{\infty} [u - (1-\beta)\xi] f(u) du \\ &\equiv -(1-\beta)c + \beta(1-\beta)\xi + \beta H(\xi(1-\beta)) \end{aligned}$$

so that

$$c = \frac{\beta}{1-\beta} H(\xi(1-\beta)) - (1-\beta)\xi^{33} \quad (2)$$

The H function is portrayed in Figure 2. It is a convex, non-negative, strictly decreasing function which approaches 0 and $E(U)$ as ξ approaches ∞ and 0, respectively. Clearly, $(1-\beta)\xi$ is the unique solution to (2).

Equation (2) has a simple economic interpretation: the critical value ξ associated with the optimal stopping rule is chosen to equate c , the marginal cost of finding one more settlement, with the expected marginal

³³Of course, in a finite-horizon model with n periods remaining ξ_n will decline with decreasing n .

Figure 1
The Optimal Policy

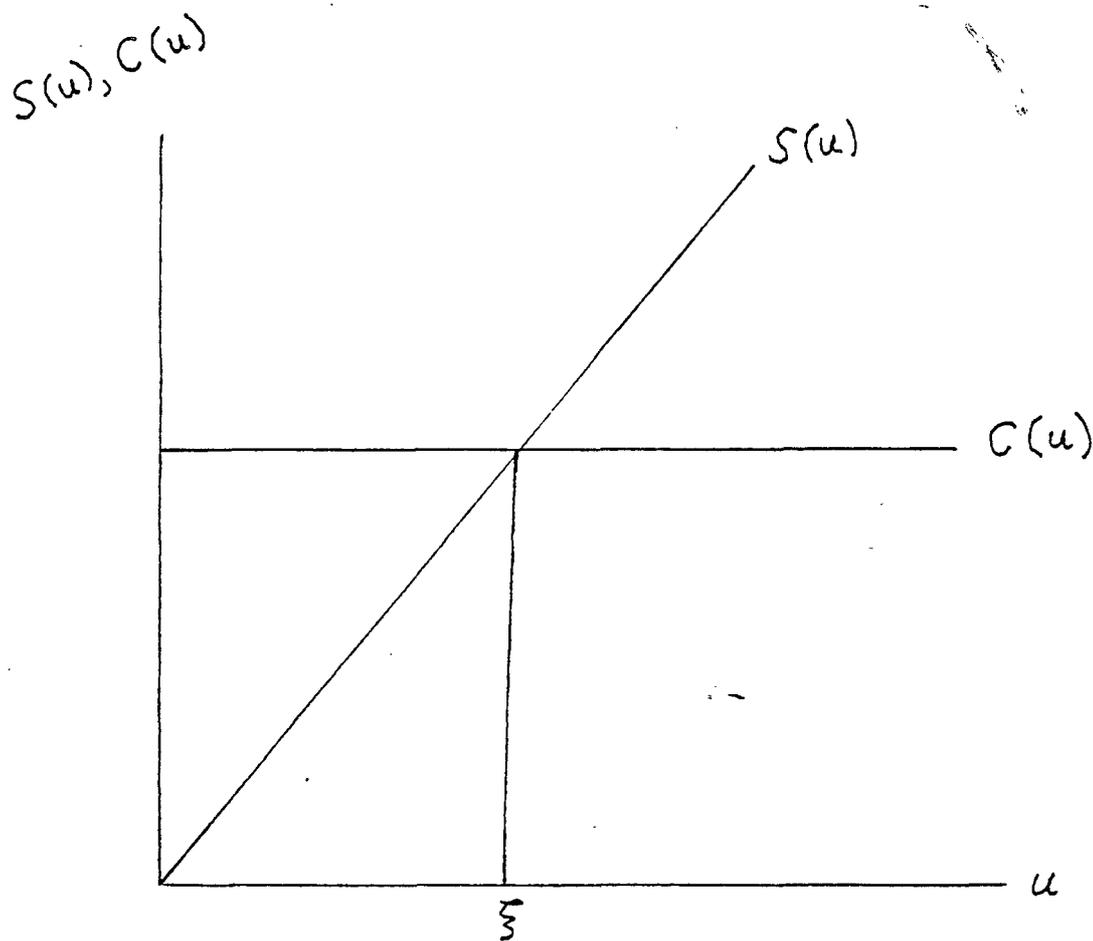
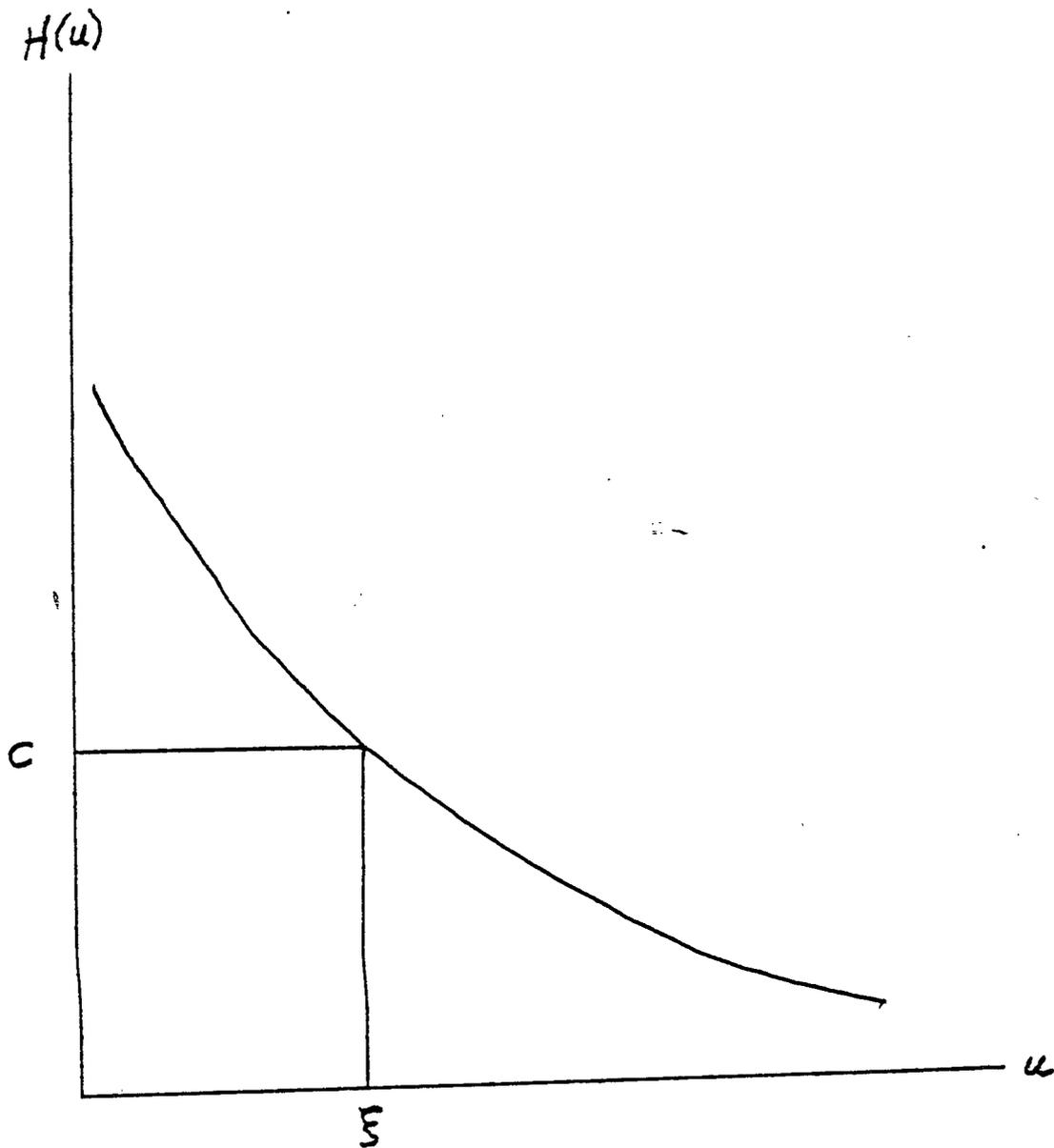


Figure 2
The H-Function



return from one more observation. To see that $H(\xi(1-\beta))\beta/(1-\beta) - (1-\beta)\xi$ is the expected marginal gain, note that $(1-\beta)\xi$ is directly foregone when the tribe searches one more period and $H(\xi(1-\beta))$ is the undiscounted expected marginal gain in utility in any given future period from one more observation. Thus, it suffices for the searcher to behave myopically; namely, he need only compare his return from accepting a settlement with the expected return from exactly one more observation.

It is important to distinguish between the reservation utility property and the myopic property: the former tells which settlements are acceptable, specifically those exceeding ξ , whereas the latter provides a simple method for calculating ξ . Summing up, the structure of the optimal policy is such that it is characterized by a single number, referred to as the reservation utility. In addition, this number is obtained by comparing the value of stopping not with the value of continuing on (perhaps for a long time) in an "optimal" manner but rather with the value of taking exactly one more observation. Furthermore, the expected return from following the optimal policy is precisely equal to the reservation utility.³⁴

In the above analysis we have only considered the case of sampling with recall wherein the optimal policy has the reservation utility property; in particular, the recall option is never utilized. Consequently, it is clear that it makes no difference whether or not recall is permitted. But if, for example, the marginal cost c of search were rising with the passage of time

³⁴The actual values of the x_i 's that correspond to the stopping boundary might be obtainable if u' is invertible.

It is also worth observing that a mean preserving increase in F yields a higher value of ξ . That is, our risk neutral agents prefer more risky distributions. For details, see Lippman and McCall (1982).

or the searcher were risk averse or the number of search opportunities were finite, then the recall option would play a part, with offers that were previously unacceptable possibly later becoming acceptable.

4. The Foraging Problem

The technology of the hunt will determine the size and number of hunting parties and also presumably influence the sharing rule. The breadth of the diet is simultaneously determined, that is, the hunting technology depends on the list of acceptable prey which in turn depends on the pattern of animal availability.

Consider two extreme cases. If the prey is large and ferocious and the location is known, a single large hunting party is to be expected. On the other hand, if the prey is randomly distributed and easily captured the party size will be small, with the law of large numbers perhaps dictating that one is the optimal size.³⁵ The flexibility of the hunt is the ease with which the party can switch to hunting x (first encounter) when y is the primary prey. Flexibility will increase when the hunting parties are able to communicate with one another. Of course, the arrival rate λ of acceptable animals and the probability of escape are determined by technology, flexibility, and breadth of diet. The leader of each hunting party corresponds to the manager of a firm whose tasks are to make key decisions, like deciding to search a specific patch and give pursuit to a particular animal, and to monitor the behavior of the members of his party. The sharing rule will depend on his evaluation of each member's performance.³⁶

This rather nontechnical discussion can be formalized using recent results from the economics of information and uncertainty.³⁷

³⁵See Smith (forthcoming).

³⁶In practice, Hill (1982) and Smith (forthcoming) finds that equal decision is the sharing rule among the Ache and Inuit.

³⁷See Hirshleifer and Riley (1979), Lippman and McCall Chapters 4 and 5 (forthcoming, b) and Radner (1983).

The size and number of hunting parties is limited by the size of the tribe. The tribe size depends on the dispersion of resources, the need for defense from outside marauders, and other benefits associated with group size like improved care of the young and a wide selection of mates.³⁸ There are no doubt diseconomies of scale as well, with disharmony and intratribal conflict, the weakening of kinship bonds, and the increased demand for food being the most conspicuous. However, at this stage of our education, such a modeling effort would probably be more presumptuous than insightful.

Patch Choice³⁹

The first decision is where to hunt. Suppose that the region can be subdivided into n hunting patches. Which patch should be chosen first and when should it be vacated? The answer to both questions is supplied by the

³⁸With respect to animal behavior, Macdonald (1983) presents two extreme examples to show

how the pattern of food availability influences the way in which carnivore groupings develop. In Portsmouth, female dockyard cats live in groups (mean 5.4 individuals) sharing large areas of their individual home ranges (mean 1.8 hectares). Each colony's range is centred on a 'skip' of refuse containing edible offal and from which other female cats are debarred. Similarly, golden jackals, *Canis aureus*, observed in Israel form stable social groups of as many as 25 adults around a feeding site. These jackals ring their territory with communal latrines, patrol the borders as a cohesive group, share food and repel intruders collaboratively — behavior very different from that found in other ecological circumstances. These cats and jackals fed largely from one or a few sites whose existence provided the possibility of group information and whose collaborative defence provided one obvious advantage of group living.

Similar points are made in Bekoff and Wells (1980) for coyotes in the wild.

³⁹The related literature in animal foraging includes: Charnov (1973, 1976), Charnov and Orians (1973), Dawkins and Smith (1971), Green (1984), Krebs et. al. (1974), Krebs et. al. (1978), MacArthur and Pianka (1966), McNamara (1982), and Stephens and Charnov (1982).

following Gittins stopping rule.⁴⁰ A Gittins index G_i $i = 1, 2, \dots, n$, is assigned to each patch based on the leader's subjective estimate of the amount and kind of prey present, that is, the capture rate θ . The optimal switch-point policy has the following structure.⁴¹ The party begins hunting in the patch with the largest index, say $G_m = \max_{1 \leq i \leq n} G_i$. As it searches the chosen patch it acquires more information about the capture rate parameter θ_m . [Another complication is that θ_m is a function of t and declines as the party hunts in the selected patch. Thus they may decide eventually to leave patch m even though the prior estimate of θ_m is equal to the posterior value at $t = 0$. This complication is treated later.] If on the basis of information acquired from s searches the subjective posterior value of ξ_m , say $\xi_m(s)$, drops below the critical number ξ_2 for what was initially the second best patch, then the party switches to this second patch. The critical number ξ_2 measures the opportunity cost of searching patch m . When $\xi_2 > \xi_m(s)$, it pays to switch from patch m to what is now the highest expected yield patch. The same process continues on patch 2, accumulating information and revising ξ_2 . After hunting on patch 2 for r periods, ξ_2 has been revised to the posterior value $\xi_2(r)$. If this value is less than $\xi_m(s)$ and ξ_3 is greater than $\xi_m(s)$, the party switches to patch 3. If $\xi_2(m)$ is less than $\xi_m(s)$ and ξ_3 is also less than $\xi_m(s)$, the party returns to patch m . The party continues in this fashion until hunting is completed. This multi-armed bandit policy tells the hunting party how to choose a patch, when to leave a patch, and finally which patch to occupy next. Several

⁴⁰This generalizes the 2-armed bandit model considered by Krebs et. al. (1978).

⁴¹See Gittins (1979) or Ross (1983) for a complete discussion. Ross' presentation is especially lucid.

simplifying assumptions have been made: first the hunters do not have to evade predators during their search and second, the characteristics of the $n-1$ patches are unaltered during hunting on the n^{th} patch.

Prey Choice

For a particular animal to be attacked it must be on our menu, i.e., the list of desirable prey. If it is, the next question is whether it should be attacked when observed. Let F be the cumulative distribution function of the random variable X , the net nutritional value (net of energy expended in capture) of an x animal. The cost of finding another desirable animal is k . Suppose further that the pursuit cost of this animal can only be assessed after attack commences. A costly animal reduces its net value by α , whereas an easily captured animal increases its net value by α . The total net value of an x animal is $X + \alpha Z$, where

$$Z = \begin{cases} +1 & \text{with probability } 1/2 \\ -1 & \text{with probability } 1/2. \end{cases}^{42}$$

Let $V_{\alpha}(x)$ be the maximal expected discounted benefits attainable over an infinite horizon when F is the offer distribution, αZ is the benefit observable only after the attack begins, and x is the initial value of the currently available animal. Similarly, let C_{α} be the optimal return when the currently available animal is rejected. Then

$$\begin{aligned} V_{\alpha}(x) &= \max\{C_{\alpha}; x + 1/2 \beta \left[\frac{x-\alpha}{1-\beta} \vee C_{\alpha} + \frac{x+\alpha}{1-\beta} \vee C_{\alpha} \right]\},^{43} \\ &\equiv \max\{C_{\alpha}; S_{\alpha}(x)\}. \end{aligned} \quad (3)$$

⁴²These assumptions may seem restrictive. In fact, they can be substantially relaxed without changing the structure. See Lippman and McCall (1982).

⁴³The notation $x \vee y$ is shorthand for $\max(x, y)$.

where

$$C_{\alpha} = -c + \beta \int_0^{\infty} V_{\alpha}(x) dF(x). \quad (4)$$

Clearly, the value of continued search is C_{α} whereas $S_{\alpha}(x)$ is the return to attacking the animal and then deciding (in an optimal manner), on the basis of the newly revealed benefits, whether to quit or to kill. It has been shown that the form of the optimal policy is $[y_{\alpha} \equiv (1-\beta)C_{\alpha} + \alpha]$.

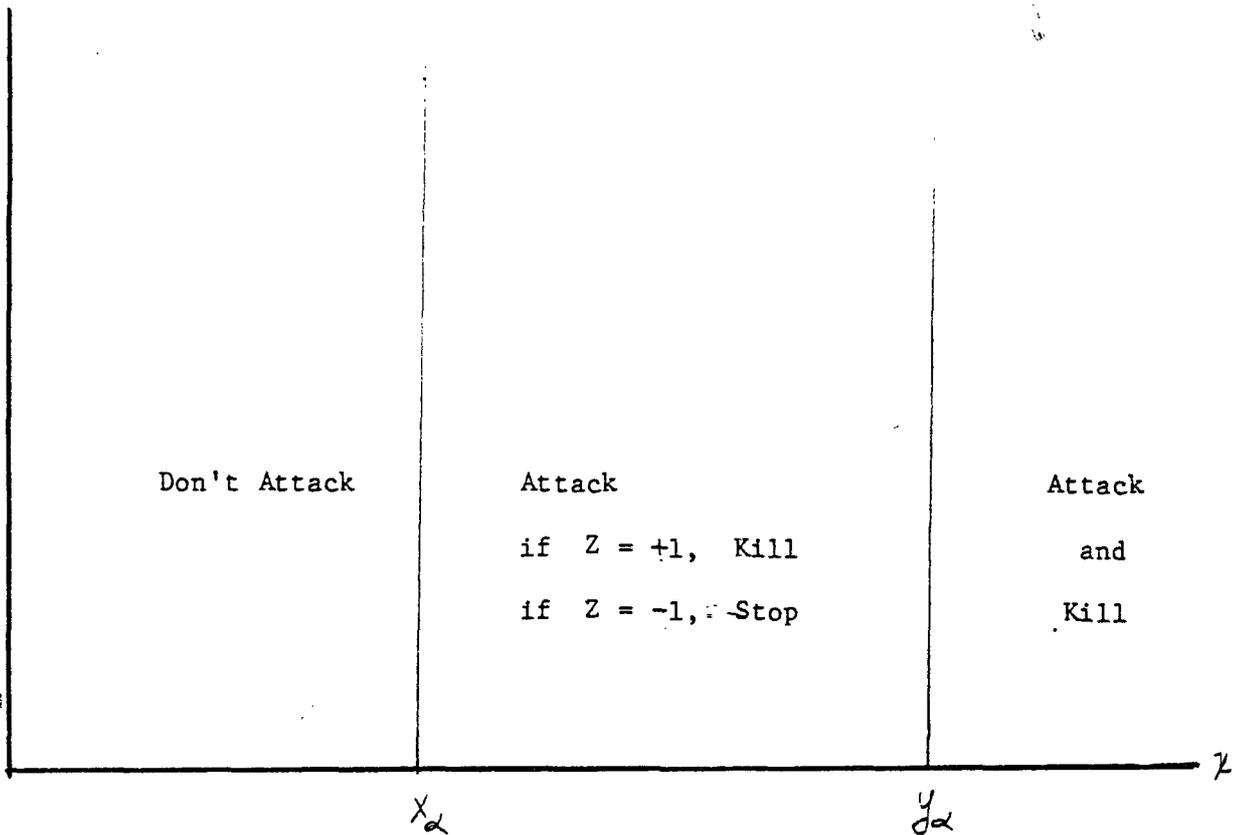
$$V_{\alpha}(x) = \begin{cases} C_{\alpha}, & x < x_{\alpha} \\ x - \frac{1}{2} \beta [C_{\alpha} + \frac{x+\alpha}{1-\beta}], & x_{\alpha} < x < y_{\alpha} \\ \frac{x}{1-\beta}, & x > y_{\alpha}, \end{cases}$$

where $x_{\alpha} = (1-\beta)C_{\alpha} - \frac{\beta}{2-\beta^2}$. (See Figure 3).

Thus the optimal (switchpoint) policy is attack the animal if and only if $x > x_{\alpha}$. If $x_{\alpha} < x < y_{\alpha}$, determine the cost of capture. If it is not too costly, continue the attack; otherwise, cease the attack and search for another animal. Finally, if $x > y_{\alpha}$, attack and kill the animal.

Figure 3

Structure of the Optimal Prey Policy



5. Mating Choice⁴⁴

There is a current debate as to whether monogamy or polygamy characterizes mating.⁴⁵ Wilson (1975) states three conditions that explain monogamy among animals: (1) Two adults are required for territorial defense; (2) Two adults are required to contend with the lean environment and (3) "early breeding is so advantageous that the head start allowed by monogamous pairing is decisive." Martin and May (1981) also note that there is a "well established rule that dimorphism is not found among monogamous mammals." Furthermore, monogamy is frequently practiced when large investments in the offspring are required for their survival.

With respect to early hominids Lovejoy (1981) argues that "conditions were prime for the establishment of male parental investment and a monogamous mating structure." Hill (1982) provocatively argues that promiscuity characterized hominid mating with males trading food for copulation privileges. Because females were not fully provisioned this accounts for sexual dimorphism. Care of offspring was conducted by mothers and grandmothers.⁴⁶

The analysis that we present here for monogamous mate choice is easily extended to polygamy, but the monogamous case can be presented with more

⁴⁴Partridge (1980) has demonstrated that "matings between randomly chosen pairs of flies produce offspring with lower survival between larvae and adult than matings where choice can occur."

⁴⁵From their study of 185 existing simple human societies Ford and Beach (1952) found that about half were monogamous. Smith notes that the larger samples reported in Murdock's Ethnographic Atlas show that about 79% are polygamous.

⁴⁶Hill's model is analogous to that of the brown hyenas described in Owens and Owens (1984). The clan was composed of one to five females and their offspring and a single dominant male. Thus, the relationship is polygamous with the dominant female actually giving birth. The other females (who are close relatives) help in the rearing of the pups. The helping behavior of close relatives was consistent with kin selection theory.

clarity. Indeed, if the female (or male) alone makes the choice, then the policy is exactly like that for site selection. Thus, there is a critical number ξ such that all males (females) below it are unacceptable and all males (females) above it are acceptable.⁴⁷

Suppose though that both are involved in the decisionmaking⁴⁸ and that there is belated information. Then the optimal stopping rule has the following structure.

Assume that the female samples x from the population of males at cost c_x and the male samples y from the population of females at cost c_y . Let α and γ be the belated information for females and males, respectively. (See Section 4).

This information is acquired only after a more intensive investigation of the potential mate has been conducted. If permissible this would include living together for a short period before marriage. The belated information is either good (+1) or bad (-1), so the total return from choosing male x is $X + \gamma Z_x$ and the total return from choosing male y is $Y + \alpha Z_y$, where

$$Z_i = \begin{cases} +1 & \text{with probability } 1/2 \\ -1 & \text{with probability } 1/2 \end{cases}, \quad i=x,y.$$

⁴⁷One would expect that the male competition for females would be conducted on the provisioning field and battle field rather than direct competition among tribal numbers.

⁴⁸Halliday (1983) observes that "when both sexes make substantial parental investment, the reproductive success of each will depend on the quality of parental care performed by their partner. We would thus expect both partners to exercise choice ... mating competition should be most intense in the lower-investing sex." Hence, Lovejoy's hominids should follow the joint choice mating strategy with cooperative hunting and perhaps equal sharing among the males, whereas Hill's hominid females should choose mates, while the males compete vigorously in their hunting and demand payment according to their marginal products in any joint hunting venture.

Thus the optimal mating policy has the following form when both male and female maximize the expected discounted benefits attainable over an infinite horizon, F is the distribution of X and G is the distribution of Y , there is belated information, and x and y are the current values of X and Y , respectively.⁴⁹

Male's Optimal Mating Policy

Do Not Mate, (Continue Search)	if $y < y_\alpha$
Acquire More Information,	if $y_\alpha < y < z_\alpha$;
Mate,	if $Z_y = +1$
Do Not Mate,	if $Z_y = -1$
Mate,	if $y > z_\alpha$.

Female's Optimal Mating Policy

Do Not Mate, (Continue Search)	if $x < x_\gamma$
Acquire More Information,	if $x_\gamma < x < z_\gamma$
Mate,	if $Z_x = +1$
Do Not Mate	if $Z_x = -1$
Mate,	if $x > z_\gamma$. ⁵⁰

⁴⁹The functional equations for the male and female, $V_\alpha(y)$ and $V_\gamma(x)$, respectively, are given by equations (3) and (4) appropriately modified.

⁵⁰In their study of mate choice among the Kittiwake Gulls, Coulson and Thomas (1983) found that the pair bond corresponded to the predictions of the belated information model. Most divorces occurred among the younger birds who failed to breed. The divorce rate then diminished with age. They note that they "do not envisage divorce as a deliberate decision on the part of the failed breeders but that it stems from failed breeders leaving the colony earlier — an returning at a less-consistent time in the following spring." This is an important methodological point in that the behavior that is consistent with our optimal stopping model is the behavior that nature selects. Alchian (1950) was the first to make this observation.

6. Migration and Conflict

The decision to leave a settlement and search for a new home site is caused primarily by inadequate resources to support the entire band.⁵¹ If the initial settlement can support some portion of the band, then only partial migration will occur. One would guess that the decision to separate is preceded by considerable intratribal conflict. If the group thinks that the food shortage is temporary, they may decide to invade neighbors' hunting territories rather than migrate. Of course, other bands may be having similar problems, thus making intertribal warfare inevitable.

Suppose the group decides to stay and invade a neighbor's territory. Which neighbor should be chosen? Optimal stopping rules can be used to characterize this decision. Let there be n neighbors and rank them by the Gittins index, where this index measures the expected net gain from an invasion.⁵² Initiate a probing action against the highest ranked neighbor. Based on his response (say, strong = +1 or weak = -1) revise the Gittins index and calculate the invasion threshold number. Invade if the response is weak and the threshold number is exceeded. Otherwise, probe the neighbor with the highest revised Gittins index. Continue this process until an invasion

⁵¹The introductory essay in Aidley's (1981) book Animal Migration is especially informative. He notes that many migrations are seasonal and are caused by the demand for food, shelter and breeding sites. It seems likely that migration is initiated by hormonal changes and/or environmental changes. For example, in some aphids crowding or a reduction in the nutrient value of the host plant produces winged individuals from a previously wingless population.

⁵²Neighbors may not be passive in which case this conflict model must be formulated as a game. Reinganum (1983) considers a model in which both parties follow optimal stopping rules.

decision is made. If none is optimal, either stay put or migrate.⁵³

Migration will proceed in the same fashion, that is, via these probing actions accumulate information about the desirability of alternative settlement sites. The settlement decision described in Section 2 is a special case of this procedure.

⁵³For a full discussion of this Gittins procedure see Chapter 8 of Lippman and McCall (forthcoming, a) or McCall and McCall (1981).

7. Agreements in Hunter-Gatherer Societies

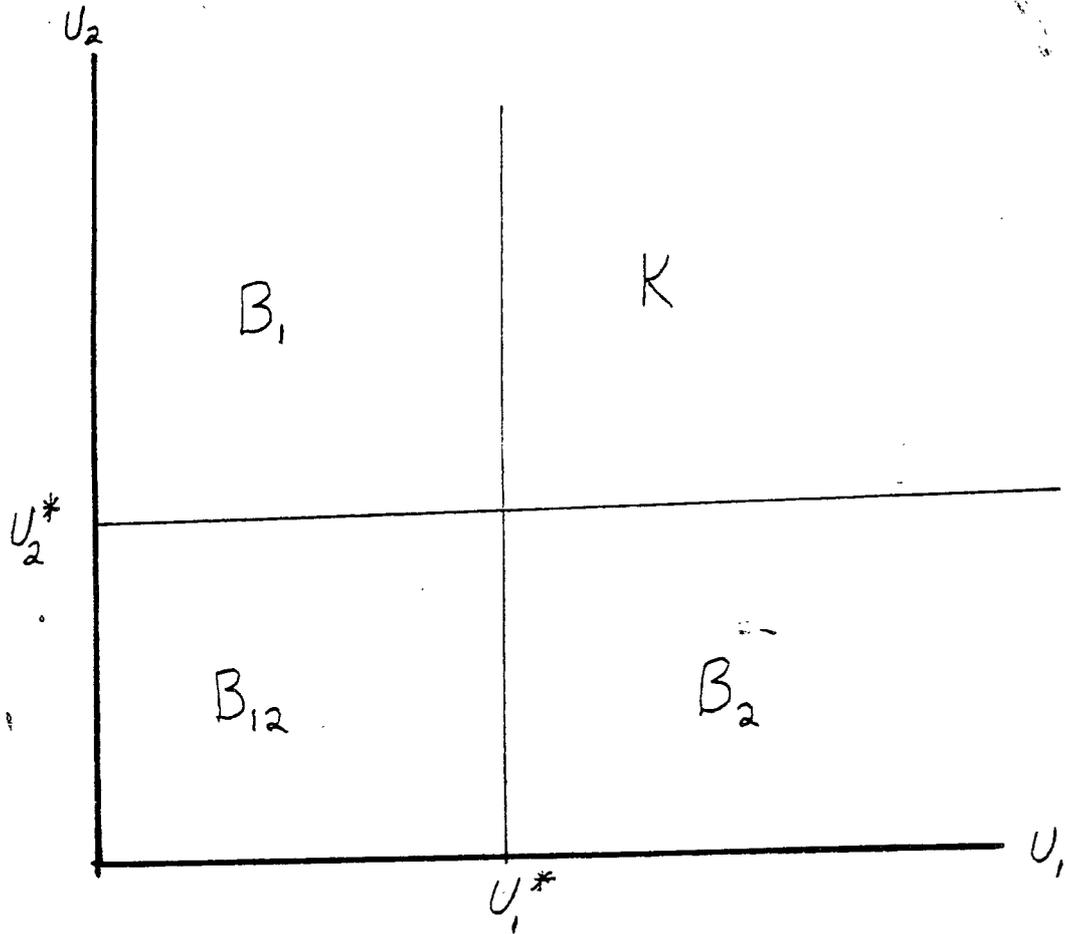
Agreements among bands and among members of the same band have essentially the same structure as the mating model. Thus agreements are made when the critical numbers of both parties are exceeded. These numbers may be functions of several variables, not necessarily the same for each party. For example, $U_1(x_1, x_2, \dots, x_n)$ and $U_2(y_1, y_2, \dots, y_m)$ may be the critical numbers for tribes 1 and 2, respectively. Furthermore, not all the y variables may be observable by 1 nor the x variables by 2. Thus, there may be cheating as these variables fluctuate. As an illustration, let X_3 be integer valued, e.g., the group's population size, and fluctuate according to a symmetric random walk:

$$P(X_{3,t} = j \mid X_{3,t-1} = i) = P_{ij} = \begin{cases} 1/2, & j = i+1 \\ 1/2, & j = i-1 \end{cases}$$

Assume U_1 is decreasing in x_3 and hold all other variables fixed at their initial values. Then with probability 1, U_1 will eventually enter the region where it pays to break the treaty by, for example, hunting in 2's territory. The B regions have been calculated to include all the precommitments that were made to reduce the probability of cheating like, for example, intermarriage among the group chiefs and/or their children, the holding of hostages, and finally warfare. Nevertheless, if B_1 B_2 B_{12} is not null, it will eventually be hit and the treaty will be broken. (See Figure 4.)

As another example of treaty breaking, let $X(t)$ be the net food available for the two bands, that is, $X(t) = X_1(t) - X_2(t)$, where X_1 and X_2 are total food for tribes 1 and 2, respectively. Similarly, let $Y(t)$ be the net population of the two bands, that is, $Y(t) = Y_1(t) - Y_2(t)$, where Y_1 and Y_2 are the populations of 1 and 2, respectively.

Figure 4
On Keeping Promises



Suppose that when the treaty was made $X(t) = Y(t) = 0$. Suppose further that group 1 will invade group 2 whenever its relative strength ($X_1 > X_2$ and $Y_1 > Y_2$) or its population relative to 2's food supply ($X_2 > X_1$ and $Y_1 > Y_2$) is so large that $R(t) = [(X_2(t) - X_1(t))^2 + (Y_1(t) - Y_2(t))^2]^{1/2}$ exceeds a critical number ξ . Similarly, 2 will invade 1 whenever its relative strength ($X_2 > X_1, Y_2 > Y_1$) or its population relative to 1's food supply ($X_1 > X_2, Y_2 > Y_1$) is so large that $R(t) = [(X_2(t) - X_1(t))^2 + (Y_2(t) - Y_1(t))^2]^{1/2}$ exceeds ξ . (See Figure 5).⁵⁴ Finally, assume that $R(t)$ fluctuates according to radial Brownian motion.

It is easy to show that $R(t)^2 - 2t$ is a martingale⁵⁵ and, hence the martingale optional sampling theorem can be applied to calculate $E(T)$, the expected time till the treaty is broken.⁵⁶ Here T is simply the smallest t such that $R(t)$ exceeds ξ , and $E(T) = \frac{1}{2} \xi^2$.

⁵⁴The radius of the circle in Figure 5 is ξ . In the figure the boundary of the circle is first hit when band 1 is large and hungry. Thus, band 1 attacks 2. The expected time from treaty till the attack is $\frac{\xi^2}{2}$. Note that $\frac{1}{2}$ the time the treaty is broken by group 1 and $\frac{1}{2}$ the time it is broken by group 2.

⁵⁵A stochastic process $\{X_n, n \geq 1\}$ is a martingale process if,

- $E(|X_n|) < \infty$, all n and
- $E(X_{n+1} | X_1, X_2, \dots, X_n) = X_n$.

⁵⁶A discussion of this theorem and an analogous application are contained in Lippman and McCall (1982).

Figure 5

Broken Treaties

$Y(t)$, net population ($Y_1 - Y_2$)

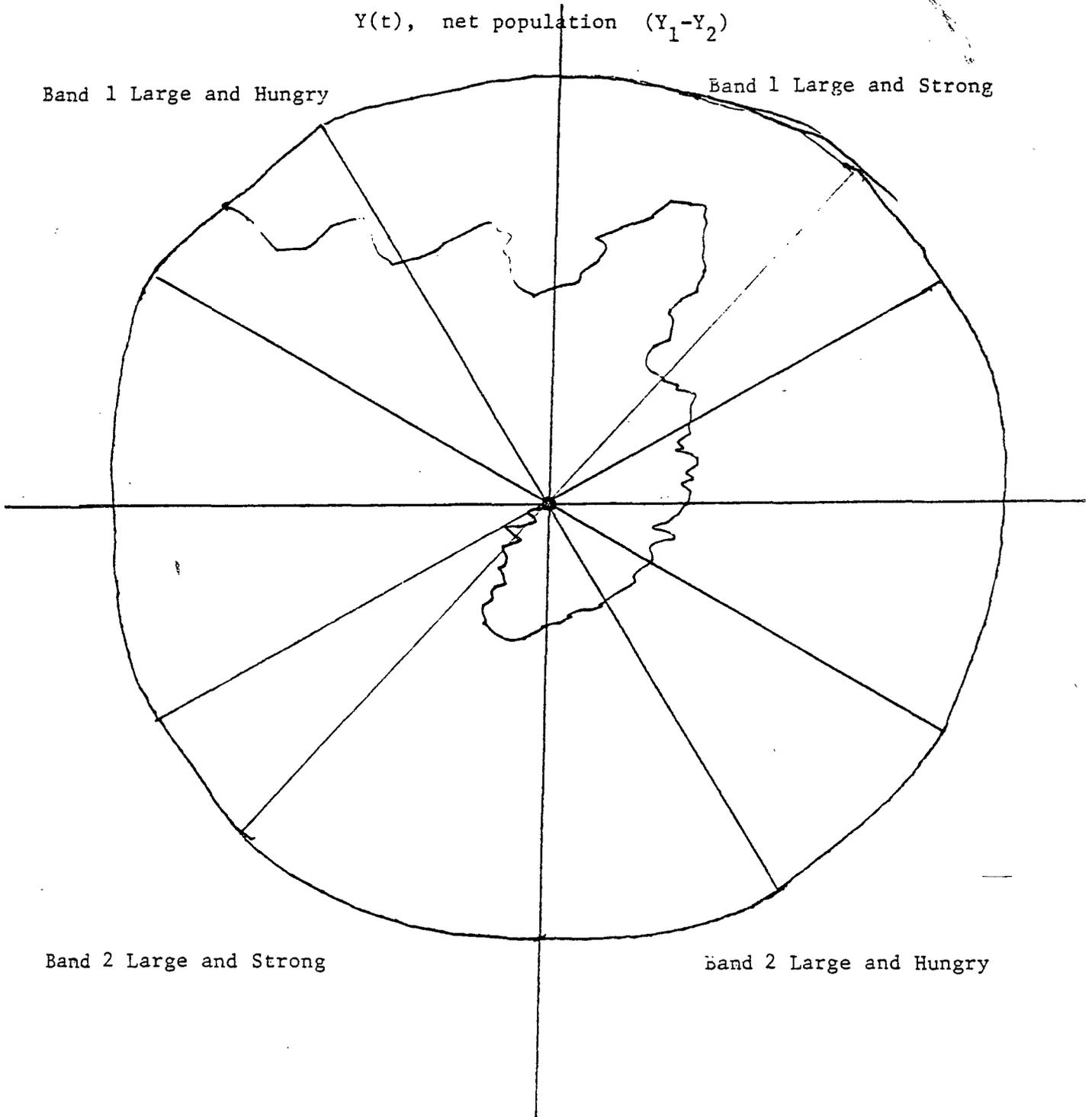
Band 1 Large and Hungry

Band 1 Large and Strong

$X(t)$, net food ($X_1 - X_2$)

Band 2 Large and Strong

Band 2 Large and Hungry



8. Conclusion

The models and methods presented here can be extended in a number of ways. It would seem premature, however, to perform these extensions until we are certain whether the models in their present form provide enlightenment and can be subjected to empirical testing. Thus, we conclude this paper by merely listing several of these extensions.

- (1) The models can be modified to include a fluctuating environment without nullifying the simple structure of the optimal policy, namely the critical value property.⁵⁷
- (2) The industrial organization of hunter-gatherer societies and animal societies can be derived. For example, the distribution of settlements over patches can be ascertained observing which patches are monopolized, which are multi-inhabited and which are empty.⁵⁸
- (3) Dynamic predator-prey models can be designed with optimal stopping on both sides. While no doubt quite difficult these switchpoint strategies could smooth out the system and eliminate equilibria that are not robust against changes in the system's parameters.⁵⁹
- (4) The role of magic and religion in hunter-gatherer cultures should definitely be studied.

⁵⁷Lippman and McCall (1976) analyze a model in which the economy fluctuates according to a Markov chain.

⁵⁸Tilman (1982) is a study of community structure.

⁵⁹For a discussion of this problem see May (1974) and Pimm (1984). The difficulties and challenges associated with nonlinear phenomena in biology are described in Enns et al. (1981).

(5) The various information problems that are crucial to the design of primitive organizations also merit research. These include moral hazard, adverse selection, the free-rider problem, and signalling.

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