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PRICE RESPONSIVENESS
AND MARKET CONDITIONS

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PRICE RESPONSIVENESS AND MARKET CONDITIONS

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PRICE RESPONSIVENESS AND MARKET CONDITIONS*

As is well known one can establish under certain appropriate conditions a systematic relationship between the intensity of the reactions of an optimizing agent vis-à-vis parameter changes and the restrictiveness of the environment in which it operates. This phenomenon, the so-called strong¹ LeChâtelier principle, is formally merely a property of bordered Hessians and a consequence of the fact that by subjecting an agent to additional 'just binding' constraints the curvature of the surface of its feasible set is being made progressively more concave at the chosen optimal point. Now, Edlefsen [3] has recently shown that essentially the same predictions as under the LeChâtelier principle follow if the increased concavity of the feasible set's curvature at the extremum point is brought about by the replacement of a given constraint through another more concave one rather than by addition of further constraints. This type of problem may appear to be somewhat artificial at first sight but it arises naturally in the context of hedonic price functions, and as Edlefsen demonstrated, its analysis leads to new and very interesting insights into the effects of nonlinearities in the constraints on certain aspects in the behaviour of households, for example.

As has already been indicated, and is also apparent from the type of proof Edlefsen uses, his approach is still very much in the LeChâtelier tradition: the feasible set is varied systematically

*This is a revised version of a short chapter in my thesis "Über die Wirksamkeit der Preispolitik in Entwicklungsländern" (Konstanz, mimeo, October 1981) which relates the analysis there to the interesting new comparative statics phenomenon that was recently established by Edlefsen [3].

¹The helpful distinction between the strong and the so-called weak LeChâtelier principle is due to Eichhorn and Oettli [4]. The basic reference is, of course, Samuelson [11]. For a more recent treatment see Silberberg [12], Kusumoto [8], Fujimoto [6] and Hatta [7].

whereas the objective function is held fixed. One may wonder whether the reverse, i.e. varying the objective function systematically while keeping the feasible set fixed, would not lead to a similar phenomenon -- and indeed, it does. It is the purpose of this paper to briefly develop this idea. Section 1 sets out with a short description of the type of static optimization problems which are to be compared and goes on to prove the main result in two versions. In section 2, the stronger version will then be applied to a simple problem in the theory of the firm: Our aim there is to establish a systematic relationship between the price responsiveness of a producer and the conditions prevailing in its markets.

1. The Effects of Nonlinearities in the Objective Function on the Bordered Hessian

In what follows, we will compare the comparative statics of the two optimization problems

$$(1) \quad \max_x f(x, \alpha) \quad \text{s.t.} \quad g(x, \alpha) = 0$$

and

$$(1') \quad \max_x \hat{f}(x, \alpha) \quad \text{s.t.} \quad g(x, \alpha) = 0$$

which differ only in that the scalar valued objective functions $f(x, \alpha)$ and $\hat{f}(x, \alpha)$ may differ. The vector of decision variables, $x = (x_1, \dots, x_n)'$, the vector of parameters, $\alpha = (\alpha_1, \dots, \alpha_m)'$, and the vector valued constraint function, $g = (g^1, \dots, g^r)'$, are the same in both problems. Define the associated Lagrangeans

$$(2) \quad L(x, \lambda, \alpha) = f(x, \alpha) - \lambda' g(x, \alpha)$$

and

$$(2') \quad \hat{L}(x, \hat{\lambda}, \alpha) = \hat{f}(x, \alpha) - \hat{\lambda}' g(x, \alpha)$$

where $\lambda = (\lambda_1, \dots, \lambda_r)'$ and $\hat{\lambda} = (\hat{\lambda}_1, \dots, \hat{\lambda}_r)'$ denote the respective Lagrange multipliers. Throughout it will be assumed that the two problems achieve a regular interior maximum for any given set of parameters, i.e. we assume that at these extremum points x^* and \hat{x}^* , the respective first and second order sufficient conditions hold. This allows us to conclude immediately that the associated bordered Hessians'

$$(3) \quad H(x, \lambda, \alpha) = \begin{bmatrix} f_{xx} - \lambda' g_{xx} & -g'_x \\ -g_x & 0 \end{bmatrix}$$

and

$$(3') \quad \hat{H}(x, \hat{\lambda}, \alpha) = \begin{bmatrix} \hat{f}_{xx} - \hat{\lambda}' g_{xx} & -g'_x \\ -g_x & 0 \end{bmatrix}$$

are regular at the points (x^*, λ^*, α) and $(\hat{x}^*, \hat{\lambda}^*, \alpha)$, respectively, and that the submatrices A and \hat{A} of their conformably partitioned inverses

$$(4) \quad H^{-1}(x^*, \lambda^*, \alpha) = \begin{bmatrix} A & B' \\ B & C \end{bmatrix}$$

and

$$(4') \quad \hat{H}^{-1}(\hat{x}^*, \hat{\lambda}^*, \alpha) = \begin{bmatrix} \hat{A} & \hat{B}' \\ \hat{B} & \hat{C} \end{bmatrix}$$

are negative semidefinite². We state the structure of these bordered Hessians and their inverses rather explicitly since the result below is expressed in terms of the relationship between these inverse bordered Hessians which play, as is well known, a crucial rôle in the comparative static analysis.

So far, we haven't specified in what sense the objective function $\hat{f}(x, \alpha)$ is to differ from the objective function $f(x, \alpha)$. In essence, what we will assume is that a level curve of the objective function

²See Debreu [2] or Quirk [10, pp. 23-24] who gives a simple proof.

$\hat{f}(x, \alpha)$ is tangent to the level curve of the objective function $f(x, \alpha)$ at the point x^* but that it bends away from this tangent hyperplane more quickly than the latter. More precisely, we will investigate the effect of letting $\hat{f}(x, \alpha)$ possess the same gradient as $f(x, \alpha)$ at the point x^* but making it relatively concave. This is clearly the reverse analogue to the case covered by Edlefsen or the classical strong LeChâtelier principle. The result then is:

(i) if the comparability condition

$$(5) \quad \hat{f}_x(x^*, \alpha) = f_x(x^*, \alpha)$$

holds, problems (1) and (1') lead to the same decision, i.e. $\hat{x}^* = x^*$. Furthermore, $\hat{\lambda}^* = \lambda^*$.

(ii) Given the comparability condition (5), if

$$(6) \quad h' \left(\hat{f}_{xx}(x^*, \alpha) - f_{xx}(x^*, \alpha) \right) h \leq 0 \quad \text{for all } h = (h_1, \dots, h_n)'$$

holds, the difference $H^{-1}(x^*, \lambda^*, \alpha) - \hat{H}^{-1}(\hat{x}^*, \hat{\lambda}^*, \alpha)$ will be negative semidefinite.

(iii) Given the comparability condition (5), if the weaker condition

$$(7) \quad h' \left(\hat{f}_{xx}(x^*, \alpha) - f_{xx}(x^*, \alpha) \right) h \leq 0 \quad \text{for all } h \text{ such that } g_x(x^*, \alpha)h = 0$$

holds, then still the difference $A - \hat{A}$ will be negative semidefinite.

Proof: Compare the first order conditions associated with (1) and (1'), which are

$$(8) \quad f_x(x^*, \alpha) - \lambda^* g_x(x^*, \alpha) = 0, \quad g(x^*, \alpha) = 0$$

and

$$(8') \quad \hat{f}_x(\hat{x}^*, \alpha) - \hat{\lambda}^* g_x(\hat{x}^*, \alpha) = 0, \quad g(\hat{x}^*, \alpha) = 0,$$

respectively, and observe that any solution (x^*, λ^*) solving (8) solves

in view of (5) likewise (8'). Thus we conclude $\hat{x}^*=x^*$ and $\hat{\lambda}^*=\lambda^*$ which proves (i) and means in particular that the bordered Hessians as well as their inverses are to be evaluated at the same point (x^*,λ^*,α) . Note that accordingly $\hat{H}(\hat{x}^*,\hat{\lambda}^*,\alpha)$ can be expressed as

$$(9) \quad \hat{H}(\hat{x}^*,\hat{\lambda}^*,\alpha) = H(x^*,\lambda^*,\alpha) + S \quad ,$$

where S has the simple structure

$$(10) \quad S = \begin{bmatrix} D & 0 \\ 0 & 0 \end{bmatrix} \quad \text{with} \quad D = \hat{f}_{xx}(x^*,\alpha) - f_{xx}(x^*,\alpha) \quad .$$

Using (9), we have obviously $\hat{H}\hat{H}^{-1}\hat{H} = H+2S+SH^{-1}S$ and $\hat{H}\hat{H}^{-1}\hat{H} = H+S$.

Subtracting these two equations and multiplying the result from both sides with \hat{H}^{-1} gives $\hat{H}^{-1}\hat{H}^{-1} = \hat{H}^{-1}S\hat{H}^{-1} + \hat{H}^{-1}SH^{-1}S\hat{H}^{-1}$. Set $v'=(u',w')$, where u and w are n- and r-vectors, respectively, and multiply the last equation from both sides with this vector. Multiplying through and using (4), (4') and (10) gives

$$(11) \quad v'(\hat{H}^{-1}\hat{H}^{-1})v = (u'\hat{A}+w'\hat{B})D(\hat{A}u+\hat{B}'w) + ([u'\hat{A}+w'\hat{B}']D)A(D[\hat{A}u+\hat{B}'w]) \quad .$$

Now, the second term on the RHS is clearly non-positive since A must be negative semidefinite by the second order sufficient conditions. If also D is unconditionally negative semidefinite, then the entire RHS is non-positive. Thus, in view of (6), (ii) is proven. Let $w=0$ and reconsider (11) which then collapses, using again (4) and (4'), to read

$$(12) \quad u'(A-\hat{A})u = (u'\hat{A})D(\hat{A}u) + (u'\hat{A}D)A(D\hat{A}u) \quad .$$

The second term on the RHS is again clearly non-positive. In order to establish the non-positivity of the entire RHS of (12) and thus to prove (iii), we must merely show that the n-vector $h=\hat{A}u$ occurring in the first term on the RHS has the property that is required in (7). Now observe that when multiplying out the equation $\hat{H}\hat{H}^{-1}=I$ in partitioned form using (3') and (4'), one of the resulting four equations is $g_x\hat{A}=0$. Hence, $g_x\hat{A}u=0$ holds which means that the vector

$\hat{h}=\hat{A}u$ indeed possesses the required property $g_x \hat{h}=0$. This then establishes (iii) and completes the proof.

Condition (7) is clearly much weaker than condition (6) because it requires the relative concavity of $\hat{f}(x,\alpha)$ to $f(x,\alpha)$ to hold only for those directions around x^* that are feasible given the constraints while (6) requires this to hold for all directions. However, unless the problem under consideration is of an exceedingly simple structure, the validity of condition (7) will in general be rather difficult to check if (6) is violated.

2. An Application to the Theory of the Firm

It should be evident that Edlefsen's analysis of the household choosing between quantity and quality [3, section 5] when facing parametric or hedonic prices can be directly replicated by considering this household if it were to minimize expenditures given a certain prescribed utility level and using (iii) above. On this basis one could likewise easily show that the compensated demand of a household which is entitled to progressive rebates on its consumer goods purchases will be more sensitive to a change in a conjugate parameter such as an indirect tax or a subsidy on a commodity than if it got no rebates.

However, in order to investigate a problem which cannot be solved that easily with Edlefsen's approach we will analyze here in some more detail the comparative static reactions of two producers who are exposed to identical technological as well as other constraints but face different market conditions. It is a well known fact that e.g. a profit maximizing monopolist will react qualitatively much the same way to a price change in a competitive factor market as a producer facing competitive conditions in his output market. But while the direction of these reactions to such price changes are identical, the intensity will in general differ. As a matter of fact, it can be shown

under fairly reasonable assumptions that the monopolist will respond less intensely to a price change in a factor market than a perfect competitor. One might be inclined to think that it is essentially the elasticity of demand or supply in the various relevant output or input markets which determines the intensity of the response. This is, however, not correct for it is the the speed with which marginal revenue or marginal expenditure in the various markets changes rather than the size of the demand or supply elasticities in these markets which is responsible for a more or less intense reaction to a change in a conjugate parameter. To put it more precisely and generally, we intend to show that the more rapidly marginal revenue declines or marginal expenditure rises in at least one output or input market the less intensely a profit maximizing producer tends to react in any market to a change in prices, indirect taxes, shifts in demand or supply or any other conjugate parameter. This finding will allow us to portray the general demand and supply behaviour of a producer with monopolistic or monopsonistic power in a certain market j as being bracketed between two extremes, namely the behaviour under perfect competition and the behaviour under quantity rationing in market j .

Letting $x=(x_1, \dots, x_n)'$ denote quantities, we will use the common convention to speak of an output or an input i according to whether the associated quantity x_i is positive or negative, respectively. Let p_i , the (positive) price prevailing in market i , be a function of x_i alone³ and define $z^i(x_i)=x_i p_i(x_i)$ as the gross revenue or, if negative, the gross expenditure in market i . Consider then a producer solving the profit maximization problem

$$(13) \quad \max_x \quad \beta'z(x) \quad \text{s.t.} \quad h(x)-\gamma \leq 0$$

³Letting the prices p_i be functions of x_i alone rather than of the entire vector x precludes the possibility to allow for interconnected markets and appears therefore to be unwarranted. However, dropping this assumption would substantially complicate the comparative static analysis below and lead to results that appear to resist a useful economic interpretation. This will become more apparent when viewing (14), (15) and (16) below and considering that without this assumption the relevant matrices z_x and $\beta'z_{xx}$ would cease to be diagonal.

where $z(x) = (z^1(x_1), \dots, z^n(x_n))'$ is the vector of gross revenues and expenditures, $\beta = (\beta_1, \dots, \beta_n)'$ is a vector of strictly positive shift parameters which may accommodate taxes, subsidies, exchange rates etc. if relevant, and where the product $\beta'z(x)$ denotes accordingly net profits. Writing the constraints here in the inequality form $h(x) - \gamma \leq 0$ where $h(x) = (h^1(x), \dots, h^r(x))'$ and $\gamma = (\gamma_1, \dots, \gamma_r)'$, one can obviously interpret $h^j(x)$ as the use of resource j , which must not exceed the available amount γ_j . We will assume, incidentally, that all inactive constraints have already been deleted so that at the optimum we will have $h(x^*) - \gamma = 0$. Writing $L = \beta'z(x) - \lambda'(h(x) - \gamma)$ for the associated Lagrangean, the complete comparative static system for this producer is given by

$$\begin{bmatrix} x_{\beta}^* & x_{\gamma}^* \\ \lambda_{\beta}^* & \lambda_{\gamma}^* \end{bmatrix} = - H^{-1}(x^*, \lambda^*, \beta, \gamma) \begin{bmatrix} z_x(x^*) & 0 \\ 0 & I \end{bmatrix},$$

and using the partitioning proposed in (4), the more promising parts of these comparative static reactions are

$$(14.1) \quad x_{\beta}^* = - A z_x(x^*)$$

$$(14.2) \quad \lambda_{\gamma}^* = - C$$

Consider now a second producer solving

$$(13') \quad \max_x \quad \beta' \hat{z}(x) \quad \text{s.t.} \quad h(x) - \gamma \leq 0$$

and the associated comparative static system, the relevant parts of which, in obvious notation, are

$$(14.1') \quad \hat{x}_{\beta}^* = - \hat{A} \hat{z}_x(\hat{x}^*)$$

$$(14.2') \quad \hat{\lambda}_{\gamma}^* = - \hat{C}$$

This second producer faces obviously the same constraints as the first producer, but is exposed to different market conditions as reflected by the differing profit function, or more precisely, by the differing gross revenue and expenditure functions. Assume now that marginal

profits are identical at the point x^* so that

$$(15) \quad \beta' \hat{z}_x(x^*) = \beta' z_x(x^*) \quad ,$$

and that the difference between the two profit functions is simply that the profit function of the second producer is relatively concave to that of the first producer at this point, i.e.

$$(16) \quad h' \left(\beta' \hat{z}_{xx}(x^*) - \beta' z_{xx}(x^*) \right) h \leq 0 \quad \text{for all } h \quad .$$

Since z_x and likewise \hat{z}_x are diagonal by assumption, (15) is tantamount to requiring $\partial \hat{z}_1^i(x_1^*) / \partial x_1 = \partial z_1^i(x_1^*) / \partial x_1$ for $i=1, \dots, n$ which means that marginal (gross) revenues and expenditures in all markets have to be identical for both producers at the point x^* . And since also $\beta' \hat{z}_{xx}$ and $\beta' z_{xx}$ are diagonal matrices, (16) requires in effect $\partial^2 \hat{z}_1^i(x_1^*) / \partial x_1^2 \leq \partial^2 z_1^i(x_1^*) / \partial x_1^2$ for all $i=1, \dots, n$ which means that for the second producer marginal gross revenue has to fall or marginal expenditure has to rise in any market at least as rapidly as for the first producer. Now, (15) is nothing but the comparability condition (5) so that we have immediately by (i)

$$(17) \quad \hat{x}^*(\beta, \gamma) = x^*(\beta, \gamma) \quad \text{and} \quad \hat{\lambda}^*(\beta, \gamma) = \lambda^*(\beta, \gamma)$$

which says that both producers will make the same decision and face identical shadow prices. And since (16) is in turn equivalent to condition (6), we conclude furthermore on the basis of (ii) above that the difference $H^{-1}(x^*, \lambda^*, \beta, \gamma) - \hat{H}^{-1}(\hat{x}^*, \hat{\lambda}^*, \beta, \gamma)$ and consequently in particular

$$(18) \quad A - \hat{A} \quad \text{and} \quad C - \hat{C} \quad \text{are negative semidefinite} \quad .$$

Using (14), (14'), (15) and (17) we have $\hat{x}_\beta^* - x_\beta^* = (A - \hat{A}) z_x(x^*)$ and $\hat{\lambda}_\gamma^* - \lambda_\gamma^* = C - \hat{C}$ which in view of (18) implies in particular⁴

⁴Note that $\partial x_i^* / \partial \beta_i = - a_{ii} \partial z_1^i(x_1^*) / \partial x_1$ where a_{ii} denotes the i -th diagonal element of A which must be negative semidefinite. The reaction $\partial x_i^* / \partial \beta_i$ has therefore the same sign as $\partial z_1^i / \partial x_1$. Since that applies also for the reaction $\partial \hat{x}_i^* / \partial \beta_i$, both reactions will consequently have the same sign and differ merely in magnitude.

$$(19.1) \quad \left| \frac{\partial \hat{x}_i^*}{\partial \beta_i} \right| \leq \left| \frac{\partial x_i^*}{\partial \beta_i} \right| \quad i=1, \dots, n$$

and

$$(19.2) \quad \frac{\partial \hat{\lambda}_j^*}{\partial \gamma_j} \leq \frac{\partial \lambda_j^*}{\partial \gamma_j} \quad j=1, \dots, r$$

Thus, (19.1) indeed shows, the steeper the decline in marginal revenue or the rise in marginal expenditure in at least one market, the less intense the reaction of a profit maximizing producer in any market to a change in a conjugate parameter. This phenomenon is entirely plausible. Consider for example the simple case of a producer facing competitive conditions in all markets and compare his behaviour with that of a producer in exactly the same situation except that marginal expenditure in market j does not stay constant but rises with increasing demand. He is thus monopsonist in this factor market. Let the price of an output i rise. This is clearly for both producers an incentive to expand production of output i . But the incentive for the second producer will erode more quickly since in the process of expanding output the use of factor j will normally⁵ also expand and thus become more expensive while for the first producer its price stays the same. Let us stick to this simple setup one more moment. When starting from perfectly competitive conditions in factor market j and allowing these market conditions to become increasingly more 'monopsonistic' so that the second derivative of the expenditure function $z^j(x_j)$ goes from first zero to (minus) infinity, one may establish on the basis of (ii) a chain of increasingly weaker reactions $\partial x_i^*/\partial \beta_i$ ($i=1, \dots, n$) the end of which simulates the situation of quantity

⁵Alternatively, let factor j be inferior with respect to output i in the sense that demand for that factor falls when the price of output i rises (compare Bear [1] or Ferguson [5] for a symmetric but slightly different definition of an inferior input). In this case the marginal expenditure saved because of the reduction in demand for that factor is absolutely smaller for the second producer which again leads to the conclusion that his output expansion will be comparatively less pronounced.

rationing in factor market j . It is in this sense that the demand and supply behaviour of an ordinary monopsonist in factor market j or, more precisely, his immediate reactions $\partial x_i^*/\partial \beta_i$ ($i=1, \dots, n$) can be considered to be bracketed between those of a comparable producer facing perfect competition in market j on the one hand and those of a producer facing quantity rationing in this market on the other.

(19.2) has also an interesting interpretation. In view of the inequality constraints considered here, the Lagrange multipliers must be non-negative. One would furthermore expect that these shadow prices λ_j^* and the allocations of the respective resources γ_j tend to move in opposite directions, and indeed, it can be shown under fairly general conditions that $\partial \lambda_j^*/\partial \gamma_j \leq 0$ ($j=1, \dots, r$) will hold⁶. With this case in mind and interpreting the shadow prices as a measure of the incentive to overcome a given resource constraint by a marginal step, (19.2) says the quicker marginal revenue in a product market falls or marginal expenditure in a factor market rises, the more rapidly the incentive to overcome any resource bottleneck will fade away. (19.2) thus points at just another aspect of how monopolistic or monopsonistic power and even more so quantity rationing in any market tends to dampen the level of activity of a producer.

Summary

Edlefsen [3] has shown that a phenomenon of great similarity to the strong LeChâtelier principle can be established when altering the feasible set of an optimizing agent by suitably replacing existing constraints rather than adding new ones. Here it was demonstrated that essentially the same phenomenon occurs when altering the objective function in a systematic manner rather than the feasible set. The result obtained is general enough to allow a replication of Edlefsen's

⁶See Leblanc and Van Moeseke [9, Proposition 1].

analysis of a household choosing between quantity and quality when facing hedonic prices, and served here furthermore in an application to the theory of the firm where it was demonstrated that a systematic relationship exists between the intensity of the reactions of a producer to changes in a conjugate parameter and the conditions prevailing in his markets.

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