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# Underreporting and Minimum Wage

*Aljoša Feldina and Sašo Polanec*



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# Underreporting and Minimum Wage

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## Abstract

We propose a theory of firm that is consistent with empirically observed phenomenon of firms simultaneously engaging in both formal and informal activities. Firms face endogenous probability of auditing that depends on the scale of underreporting of both sales and wages. We characterize the conditions for a firm facing corporate, value added and payroll taxes to hide a part of its activities. Underreporting gives rise to employment levels above those expected for firms that report truthfully. We also show that an underreporting firm is less affected by the introduction of the minimum wage. It further decreases the level of reported sales to mitigate the adverse effect that the minimum wage causes and decreases optimal employment by relatively less than an honest firm. The latter hypotheses is tested empirically using a matched employer-employee data set for Slovenia during the period of several minimum wage hikes (2007-2010).

*Keywords:* informal economy, minimum wage, taxation, auditing

*JEL:* H26, J38, D21

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## 1. Introduction

The effects of public policy instruments on the size of informal economy have been studied extensively both theoretically and empirically.<sup>1</sup> Rauch (1991) pioneered this field by developing a theoretical model to analyze the impact of the minimum wage on the size of informal sector. Rauch finds that when the government enforces a

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<sup>1</sup>To cite only a few theoretical contributions, Rauch (1991), Fugazza and Jacques (2004), and Albrecht et al. (2009) deal with labour market and social policies, de Mello (2009), Kopczuk and Slemrod (2006), and de Paula and Scheinkman (2010) consider value added taxes, while Ihrig and Moe (2004), and Prado (2011) model opposing roles of taxation and enforcement. Results are aligned in general, various taxes give rise to more informality in the economy, while better enforcement and benefits from engaging in formal sector work in opposite direction. See, e.g., Tybout (2000), for references on empirical evidence.

minimum wage that is above a market clearing wage only for firms above a certain size, firms smaller than this will form and will employ those that became unemployed due to instalment of minimum wage. These firms optimally operate in informal sector and pay lower wages. Further, Rauch shows that the smallest formal sector firms are larger than the largest informal sector firms, identifying a *missing middle* - a gap between the size distributions of firms in two respective sectors.<sup>2</sup> Moreover, he shows that the size of the formal sector varies inversely with its minimum wage. Finally, less enforcement, in the sense of increasing the minimum size of the firms that fall under government attention, increases informal sector wages.

The analysis of Rauch (1991), however, neglects an important part of the informal economy that is related to the activity of the firms participating in the formal economy, namely underreporting. There is a lot of day-to-day anecdotal evidence on smaller firms or sole proprietors offering to invoice only a small part of labour expenses that barely covers the necessary work embedded in the product or service sold to their customers. The remaining part of the firm's expenses and its profit margin are paid for in cash. In doing this, firms hide a part of actual sales and wage bill, thus decreasing their tax payments. Firms with a propensity to operate in such a manner do not operate in the informal economy completely, since they still report to tax authorities, employ labour within legal boundaries and have access to legal financing channels. In fact, for some countries, this is the most important form of informal economy. For example, Flajs (2012) estimates that in Slovenia, where the share of informal economy is estimated to be around 8.3 percent of GDP in 2007, the largest contribution may be attributed to underreporting of output (4.1 percent of GDP), followed by inflated costs of materials and services, which is also performed by firms operating in the formal sector.<sup>3</sup>

In this paper, we develop a theory of firm that operates both in formal and informal sector by engaging in underreporting sales and wages.<sup>4</sup> We contribute to

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<sup>2</sup>Endogenous formation of distinct formal and informal sectors with a gap between respective firm size distributions is studied in more recent contributions as well. See for example Fortin et al. (1997), Dabla-Norris et al. (2008), and de Paula and Scheinkman (2010).

<sup>3</sup>Underreporting sales and wages is also an important phenomenon in other countries. See, for example, evidence on underreporting in Mongolia in Zhou and Oostendorp (2010) and in Elek et al. (2012) for Hungary. In countries like the U.S., the extent of underreporting is likely to be smaller. Slemrod (2007) reports that only 1 percent of salaries in the U.S. are underreported to tax collection agency.

<sup>4</sup>Our approach is reminiscent of the Ihrig and Moe (2004) and Bigio and Zilberman (2011) in this respect as they endogenize the decision to operate in informal sector. They allow informality to be a part of equilibrium behaviour. A positive size of informal sector is a result of explicit modeling of a trade-off between lowering agent's tax contribution and risk of being caught and penalized. See also work of Allingham and Sandmo (1972), who were the first to consider such trade-off when analyzing individual taxpayer's incentives to evade taxes.

the theoretical literature on informal economy in three important ways. First, we study the behaviour of firms in a more realistic environment with three different tax instruments (corporate, value added and payroll tax), which interact in creating incentives for underreporting. Namely, by underreporting sales, firms decrease value added and corporate taxes, while by underreporting wages, firms reduce the burden of payroll tax. The second contribution is that once we understand firms' incentives to underreport sales and wages, we introduce a minimum wage instrument and study its effects on their optimal employment decisions. We compare an optimal response to the change of minimum wage by a law-abiding firm and a firm hiding a part of its activities, respectively.

Another novelty is the use of explicit form of firm's profit. We allow a firm's employment and output decisions to affect both product price and labour wage. Optimal underreporting schedules are a result of a firm maximizing its expected profits. Underreporting decreases firms' tax contributions, but increases the probability of being audited and penalized.<sup>5</sup> Firms caught hiding activities in our model pay taxes owed and a lump sum penalty. In modeling the auditing probability, we follow the approach by de Mello (2009) and Bigio and Zilberman (2011), who relate auditing probability to the extent of underreporting. In Bigio and Zilberman (2011), the latter depends on employment and reported income, while we use the relative amounts of reported sales and reported wages to determine a firm's chances of being audited.

The key results of our theoretical analysis are related to the response of a firm's optimal employment level to the instalment of the minimum wage and to the subsequent change in it. Evading a part of taxes lowers a firm's marginal costs and, consequently, it produces more and employs more than it does when honest. This result stands in contrast to models and empirical regularities exhibiting a gap between the largest informal and smallest formal firms. Even though the reported sales might be below those of a truthfully reporting firm, our firm engaging in informality is larger and generates a higher market surplus than its formal peer does.

Moreover, we identify circumstances under which an increase in the minimum wage reduces an underreporting firm's optimal employment by relatively less than it reduces an honest firm's optimal employment.

The final goal of our work is to test this theoretical prediction. For this purpose, we used a matched employer-employee data set for Slovenia, a country that imposes the minimum wage and exhibits a large informal economy with firms engaged in

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<sup>5</sup>Allingham and Sandmo (1972) originate models of this kind of a trade-off in tax evasion problems.

underreporting. In order to test our key hypothesis, we exploit increases in the minimum wage that took place during the early period of economic crisis (2007-2010). In this period, the economy faced both a negative foreign demand shock and restricted access to international financial markets, while the government continued to increase the minimum wage. In fact, in 2010, the minimum wage rose by 22.9 percent to account for almost 49 percent of the average wage (it represented 42 percent of the average wage prior to the increase). Our theoretical model predicts that when the minimum wage is set above the market-clearing wage and is thus binding, its increase should lead to a decline in job survival in both honest and dishonest firms. However, dishonest firms should decrease job survival to a lesser extent due to their ability to absorb the shock. Following the evidence provided by Flajs (2012), who estimated that 90 percent of Slovenian informal economy can be attributed to registered business units such as small firms (up to 3 employees) and sole proprietors, we use small sole proprietors as an identifying group of businesses with higher incidence of informal economy and find weaker job destruction in these businesses, which confirms the prediction of our theoretical model.

The remainder of this paper is organized in the following way. We present the setting of our model in Section 2, derive firm's optimal decisions when underreporting in Section 3, and develop results on minimum wage effects in Section 4. Section 5 gives some institutional background for Slovenia, while Section 6 presents our empirical findings. We conclude in Section 7.

## 2. Theoretical framework

A firm employs  $L$  units of labour and operates a single input technology  $f(L)$ . It faces a downward sloping demand for  $q$  units of its product in inverse form  $p = p(q)$ . It hires labour in a competitive market with non-decreasing labour supply function  $w = w(L)$ .<sup>6</sup> The firm's expense for a worker receiving wage  $w$  is  $(1 + \tau)w$ , where  $\tau > 0$  is a payroll tax. Production level  $q$  also brings material costs  $\mu(q)$ .<sup>7</sup> The added value the firm creates,  $p(q)q - \mu(q)$  is subject to value added taxation with a rate  $\nu > 0$ , while the firm's net profit is subject to corporate taxes with fixed tax

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<sup>6</sup>Effects of labour market competitiveness on the size of the informal sector are analyzed by Goto and Mano (2012).

<sup>7</sup>We implicitly assume that the firm employs a production function  $F(L, M) = \min(f(L), g(M))$ , where  $M$  represents a composite material input that it needs in its production and  $g$  is an increasing continuous function. Producing  $q$  units demands exactly  $M = g^{-1}(q)$  units of material input. Knowing the price schedule of the material input,  $p_M(M)$ , we can always derive material costs:  $\mu(q) = \int_0^q p_M(g^{-1}(\varrho))d\varrho$ . Using production function of this form enables us to concentrate only on employment decisions while allowing for value added taxation as well.

rate  $\psi > 0$  as well.

When tax authorities have limited resources for inspection, there is a positive probability that a firm underreporting sales and profits to lessen its tax burden might go unpunished. We decompose a firm's expected profit into three parts: *reported profit*  $\pi_R$  that the firm reports to tax authorities and is subject to all taxes, *shadow profit*  $\pi_S$  that the firm hides from tax authorities to evade tax payments, and *amended profit*  $\pi_A$  that the firm reports after inspection reveals it was underreporting its numbers. The latter is done by understating either sales or wages or both. Instead of reporting actual revenues (sales, henceforth)  $p(q)q \equiv p(f(L))f(L)$ , only fraction  $\rho \leq 1$  is presented to tax authorities to avoid a part of value added and corporate taxes. To avoid a part of a payroll tax, the firm understates its wage bill by reporting only fraction  $\omega \leq 1$  of wages.

We assume that the manager of an underreporting firm is risk neutral in that she maximizes the firm's expected profit. When we consider the truthfully reporting firm below, the distinction between the two is in that the manager of the honest firm is infinitely risk averse. The three parts of underreporting firm's expected profit can be written as:

$$\begin{aligned}\pi_R(L, \rho, \omega) &= (1 - \nu) [\rho p(q(L))q(L) - \mu(q(L))] - (1 + \tau) \omega w(L)L, \\ \pi_S(L, \rho, \omega) &= (1 - \rho) p(q(L))q(L) - (1 - \omega) w(L)L, \\ \pi_A(L, \rho, \omega) &= (1 - \nu) (1 - \rho) p(q(L))q(L) - (1 + \tau) (1 - \omega) w(L)L.\end{aligned}\tag{2.1}$$

$\pi_R$  reports full material costs, since they decrease both value added and corporate taxes.  $\pi_S$  accounts for unreported sales and understated wage bill, while  $\pi_A$  accounts for the fact that components of  $\pi_S$  will be taxed with respective rates if the firm's practices are disclosed. In case of being audited, the firm faces two consequences. First, the unreported profit is fully taxed, and second, a fixed fine to the amount of  $\phi > 0$  is levied on the firm. Suppose the probability of firm being inspected by tax authorities is  $\alpha \in [0, 1]$ . The firm's expected profit is then:<sup>8</sup>

$$\pi = (1 - \psi) \pi_R + (1 - \alpha) \pi_S + \alpha [(1 - \psi) \pi_A - \phi]\tag{2.2}$$

We assume that the firm believes that auditing probability depends on the respective ratios of reported sales to true sales and reported wages to true wages. We posit

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<sup>8</sup>To save space in already lengthy expressions, we omit denoting the dependence of firm's profit on employed labour and reported sales and wages. We explicitly show the dependence, where it is really necessary.

$\alpha$  to be a twice continuously differentiable function  $\alpha : [0, 1] \times [0, 1] \rightarrow [0, 1]$ , with  $\partial\alpha(\rho, \omega)/\partial x \leq 0$ ,  $x \in \{\rho, \omega\}$ ,  $\alpha(0, \omega) = 1$ ,  $\alpha(\rho, 0) = 1$ , and  $\alpha(1, 1) = 0$ . The derivative condition assumes that being more honest in the reported quantity does not increase the probability of an audit. The first two equalities say that complete underreporting of one of the quantities results in inspection, while honest reporting of both means no audit as assumed by the third equality.<sup>9</sup> We further assume  $\partial^2\alpha(\rho, \omega)/\partial x^2 \geq 0$ ,  $x \in \{\rho, \omega\}$  and  $\partial^2\alpha(\rho, \omega)/\partial\rho\partial\omega \leq 0$ . The first inequality says that audit probability is non-concave in  $\rho$  and  $\omega$ , meaning that it decreases faster when reported sales (wages) are increasing at low values and slower when they are increasing at values closer to actual sales (actual wages). The cross derivative condition says that auditing probability becomes more responsive to reported sales (wages) when reported wages (sales) increase.

### 3. Underreporting, employment and missing middle

When a firm bears the costs of posting vacancies and hiring and/or firing costs, small changes in policy instruments are unlikely to change that firm's level of employment.<sup>10</sup> We therefore assume that, at the time decisions about reported sales and wages are made, employment is a state variable and we model the firm's decision-making process as a two stage problem. The firm, knowing all the policy instruments, first seeks optimal employment and only after that sets optimal values of reported sales and wages. Given the sequence of decisions, the firm's profit is of the form  $\pi(L, \rho(L), \omega(L))$  and all relevant quantities depend on employment level as well. We omit showing these dependencies unless it is necessary to avoid ambiguity.

#### 3.1. Wage Underreporting

We start with a firm's incentive to underreport wages. A firm employing  $L$  units of labour is setting the reported wage bill  $\omega wL$ . Derivative of expected profit (2.2) with respect to the fraction of reported wage bill is:

$$\begin{aligned} \frac{\partial\pi}{\partial\omega} = & (1 - \psi) \frac{\partial\pi_R}{\partial\omega} + (1 - \alpha) \frac{\partial\pi_S}{\partial\omega} + \alpha(1 - \psi) \frac{\partial\pi_A}{\partial\omega} \\ & + \frac{\partial\alpha}{\partial\omega} [(1 - \psi) \pi_A - \pi_S - \phi]. \end{aligned} \quad (3.1)$$

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<sup>9</sup>Firm could actually even be audited, but since no underreporting takes place, no additional taxes or fines would be levied onto the firm.

<sup>10</sup>These types of costs are considered in labour models of matching with search frictions in line with Mortensen and Pissarides (1994). An explicit extension of their model to economy with an informal sector is in Albrecht et al. (2009), who analyze the effects of labour market policies on respective outcomes in formal and informal sectors of economy.



Three profit derivatives on the right-hand side are obtained from (2.1):

$$\frac{\partial \pi_R}{\partial \omega} = -(1 + \tau) wL, \quad \frac{\partial \pi_S}{\partial \omega} = wL, \quad \frac{\partial \pi_A}{\partial \omega} = (1 + \tau) wL. \quad (3.2)$$

Combining (2.1), (3.1) and (3.2) yields:

$$\begin{aligned} \frac{\partial \pi}{\partial \omega} &= (1 - \alpha) (\psi - \tau + \psi \tau) wL \\ &+ \frac{\partial \alpha}{\partial \omega} [(-\psi - \nu + \psi \nu) (1 - \rho) pq + (\psi - \tau + \psi \tau) (1 - \omega) wL - \phi]. \end{aligned} \quad (3.3)$$

The first term on the right hand-side represents the direct effect of increasing  $\omega$  on the firm's expected profit. If the corporate tax rate,  $\psi$ , is not too large compared to the payroll tax rate,  $\tau$ , or  $\psi < \tau / (1 + \tau)$ , the firm wants to underreport wages. The second term on the right-hand side is the indirect effect of  $\omega$  on profit. Higher reported wages decrease the probability of an audit and consequently the expected marginal penalties from being prosecuted as well. The first term in the brackets shows the potential marginal loss from corporate and value added taxes on unreported sales and the second term quantifies potential gain or loss from interplay of corporate and payroll taxes on the unreported part of the wage bill. These two, in addition to fixed fine  $\phi$ , are now marginally avoided due to lower auditing probability. Our first result follows.

**Proposition 1.** *A firm that truthfully reports both sales and wages, i.e.,  $\rho = 1$  and  $\omega = 1$ , has an incentive to decrease reported wage fraction  $\omega$ , i.e.,  $\partial \pi(\rho = 1, \omega = 1) / \partial \omega < 0$ , when  $(\tau - \psi - \psi \tau) wL > -\phi \partial \alpha(1, 1) / \partial \omega$ .*

*Proof.* When  $\rho = 1$  and  $\omega = 1$ , the first two terms in the brackets in (3.3) become zero. Term  $-\phi \partial \alpha / \partial \omega$  is positive, therefore  $\tau - \psi - \psi \tau > 0$  is necessary for  $\partial \pi / \partial \omega < 0$ . Since  $\alpha(1, 1) = 0$ , sufficient condition for the latter is that the direct effect overweighs the indirect one, or  $(\tau - \psi - \psi \tau) wL > -\phi \partial \alpha(1, 1) / \partial \omega$ .  $\square$

When payroll tax rate as compared to corporate tax rate and lump sum fine  $\phi$  is large enough, the firm will trim down the reported wage.<sup>11</sup> This is due to payroll

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<sup>11</sup>The taxonomy of conditions under which there exists an interior solution  $\omega^* \in (0, 1)$  to  $\partial \pi(\rho, \omega^*) / \partial \omega = 0$  is not our goal, but note the following. With  $\omega = 0$ , we have  $\alpha = 1$ , and the firm is fully taxed and fine  $\phi$  is paid. Hence, as long as  $\rho > 0$ , it must be  $\pi(L, \rho, \omega = 1) = \pi(L, \rho, \omega = 0) + \phi$ . Given Proposition 1 and some continuity, there are circumstances under which  $\partial \pi(L, \rho, \omega = 1) / \partial \omega < 0$ . With additional quasi-concavity of profit with respect to  $\omega$ , we can therefore guarantee the interior solution we seek. Since we explicitly show strict concavity of profit with respect to  $\rho$  in subsection 3.2 (see footnote 12) and similar argument can be readily made for  $\omega$ , quasi-concavity in question follows.

taxes hurting it more than corporate taxes combined with a fixed fine in expected terms.

Since we are interested in instances of the market in which a firm is motivated to underreport wages, we henceforth assume that the condition from Proposition 1 is fulfilled.

**Assumption 1:**  $\tau - \psi - \psi\tau > -\phi\partial\alpha(1, 1)/\partial\omega$ .

### 3.2. Sales Underreporting

We follow steps similar to those in the last section to analyze the derivative of a firm's expected profit with respect to the reported fraction of sales  $\rho$ . Differentiating (2.2) gives:

$$\begin{aligned}\frac{\partial\pi}{\partial\rho} = & (1 - \psi) \frac{\partial\pi_R}{\partial\rho} + (1 - \alpha) \frac{\partial\pi_S}{\partial\rho} + \alpha(1 - \psi) \frac{\partial\pi_A}{\partial\rho} \\ & + \frac{\partial\alpha}{\partial\rho} [(1 - \psi) \pi_A - \pi_S - \phi],\end{aligned}$$

where derivatives of profit with respect to fraction of reported sales on the right-hand side are obtained from (2.1):

$$\frac{\partial\pi_R}{\partial\rho} = (1 - \nu) pq, \quad \frac{\partial\pi_S}{\partial\rho} = -pq, \quad \frac{\partial\pi_A}{\partial\rho} = -(1 - \nu) pq.$$

Combining all relevant expressions, we get:

$$\begin{aligned}\frac{\partial\pi}{\partial\rho} = & (1 - \alpha) (-\psi - \nu + \psi\nu) pq \\ & + \frac{\partial\alpha}{\partial\rho} [(-\psi - \nu + \psi\nu) (1 - \rho) pq + (\psi - \tau + \psi\tau) (1 - \omega) wL - \phi]\end{aligned}\tag{3.4}$$

The first term on the right-hand side of (3.4) accounts for the direct effect of  $\rho$  on expected profit and is clearly negative. Both corporate and value added taxes drive profits down when reported sales increase. The second term on the right-hand side again shows the indirect effect of  $\rho$  on a firm's profit. Higher reported sales marginally decrease the probability of inspection and consequently the expected penalties from tax evasion. Our second result follows.

**Proposition 2.** *A firm that truthfully reports both sales and wages, i.e.,  $\rho = 1$  and  $\omega = 1$ , has an incentive to decrease reported sales fraction  $\rho$ , i.e.,  $\partial\pi(\rho = 1, \omega = 1)/\partial\rho < 0$ , when  $(\psi + \nu - \psi\nu)pq > -\phi\partial\alpha(1, 1)/\partial\rho$ .*

*Proof.* When  $\rho = 1$  and  $\omega = 1$ , the first two terms in the brackets in (3.4) become zero,  $(-\psi - \nu + \psi\nu)pq$  is negative and the remaining term  $-\phi \frac{\partial \alpha(1,1)}{\partial \rho}$  is positive. If the latter does not exceed the absolute value of the former, result follows.  $\square$

Under conditions demanded for Propositions 1 and 2, a firm will search for optimal underreporting fractions of sales and wages. In general, firms employing  $L$  units of labour solve the following optimization program for  $\rho^*(L)$  and  $\omega^*(L)$ :

$$\max_{\rho, \omega} \pi(L, \rho(L), \omega(L)) \quad \text{s.t.} \quad 0 \leq \rho \leq 1, 0 \leq \omega \leq 1. \quad (3.5)$$

Since we are interested in how the minimum wage, which might bind the reported wage from below, affects firm's decisions, we explore sales underreporting further.

Profit maximization with respect to  $\rho$  can be carried out regardless of reported wage fraction  $\omega$  being optimal or not. We equate (3.4) with zero and examine the total differential of resulting necessary condition  $\partial \pi(\rho^*, \omega)/\partial \rho = 0$  with respect to  $\rho$  and  $\omega$  to get:

$$\frac{d\rho^*}{d\omega} = - \frac{\partial^2 \pi}{\partial \omega \partial \rho} \bigg/ \frac{\partial^2 \pi}{\partial \rho^2}. \quad (3.6)$$

The two right-hand side derivatives are obtained from (3.4). The denominator is

$$\begin{aligned} \frac{\partial^2 \pi}{\partial \rho^2} = & -2 \frac{\partial \alpha}{\partial \rho} (-\psi - \nu + \psi\nu) pq \\ & + \frac{\partial^2 \alpha}{\partial \rho^2} [(-\psi - \nu + \psi\nu)(1 - \rho^*)pq + (\psi - \tau + \psi\tau)(1 - \omega)wL - \phi]. \end{aligned} \quad (3.7)$$

The first term on the right-hand side is clearly negative and since the derivative in the second line of (3.7) is non-negative, the whole expression is unambiguously negative.<sup>12</sup>

The numerator of (3.6) is:

$$\begin{aligned} - \frac{\partial^2 \pi}{\partial \omega \partial \rho} = & \frac{\partial \alpha}{\partial \omega} (-\psi - \nu + \psi\nu) pq + \frac{\partial \alpha}{\partial \rho} (\psi - \tau + \psi\tau) wL - \\ & \frac{\partial^2 \alpha}{\partial \rho \partial \omega} [(-\psi - \nu + \psi\nu)(1 - \rho^*)pq + (\psi - \tau + \psi\tau)(1 - \omega)wL - \phi], \end{aligned}$$

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<sup>12</sup>This result is independent of  $\rho \in [0, 1]$  and shows that our interior solution  $\rho^*$  in fact maximizes firm's expected profit.

and when we rearrange it:

$$-\frac{\partial^2 \pi}{\partial w_m \partial \rho} = \left( \frac{\partial \alpha}{\partial \omega} - \frac{\partial^2 \alpha}{\partial \rho \partial \omega} (1 - \rho^*) \right) (-\psi - \nu + \psi \nu) pq + \left( \frac{\partial \alpha}{\partial \rho} - \frac{\partial^2 \alpha}{\partial \rho \partial \omega} (1 - \omega) \right) (\psi - \tau + \psi \tau) wL - \phi \frac{\partial^2 \alpha}{\partial \rho \partial \omega}. \quad (3.8)$$

Under our assumptions on  $\alpha$ , the three derivatives are all non-positive. Note that when  $\rho^* = \omega = 1$ , derivative (3.8) is clearly positive. Derivative (3.6) is then negative, saying that a rise in reported fraction of wages motivates a firm to cut the reported fraction of sales. For the following proposition to hold, we need (3.8) to be positive. If the contents of respective first parentheses of first two summands are negative, the result is established. If not, the first two summands must not exceed the last one.

**Proposition 3.** *When  $\psi - \tau + \psi \tau < 0$ , i.e., firm underreports its wage bill, an increase in fraction of reported wage  $\omega$  decreases optimally reported fraction of sales  $\rho^*$ , i.e.,  $d\rho^*/d\omega < 0$ , if:*

$$\phi \frac{\partial^2 \alpha}{\partial \rho \partial \omega} < \left( \frac{\partial \alpha}{\partial \omega} - \frac{\partial^2 \alpha}{\partial \rho \partial \omega} (1 - \rho^*) \right) (-\psi - \nu + \psi \nu) pq + \left( \frac{\partial \alpha}{\partial \rho} - \frac{\partial^2 \alpha}{\partial \rho \partial \omega} (1 - \omega) \right) (\psi - \tau + \psi \tau) wL. \quad (3.9)$$

The first condition from Proposition 3 is satisfied when a firm is underreporting its wages (Proposition 1 and Assumption 1). In the example we present in what follows, the inequality (3.9) is satisfied.

Intuitively, when a firm is forced to report higher wages, for example, through an increase in the minimum wage, and consequently pays higher payroll taxes, its incentive to offset the latter by trimming reported sales becomes stronger. Less wage underreporting also results in a lower probability of being audited, which reinforces this incentive further.

### 3.3. Employment level

For known equilibrium reported fractions  $\rho^*(L)$  and  $\omega^*(L)$ , a firm determines optimal employment level. We decompose the derivative of a firm's profit with

respect to labour into three terms:

$$\frac{d\pi(L, \rho^*(L), \omega^*(L))}{dL} = \frac{\partial\pi(L, \rho^*(L), \omega^*(L))}{\partial L} + \frac{\frac{\partial\pi(L, \rho^*(L), \omega^*(L))}{\partial \rho} d\rho^*(L)}{dL} + \frac{\frac{\partial\pi(L, \rho^*(L), \omega^*(L))}{\partial \omega} d\omega^*(L)}{dL}. \quad (3.10)$$

When a firm optimally underreports both quantities, it must be  $\partial\pi(L, \rho^*, \omega)/\partial\rho = 0$  and  $\partial\pi(L, \rho, \omega^*)/\partial\omega = 0$  in the firm's second stage decision making. Recalling (2.2), the necessary condition for deriving optimal employment  $L^*$  becomes:

$$(1 - \psi) \frac{\partial\pi_R(L^*)}{\partial L} + (1 - \alpha) \frac{\partial\pi_S(L^*)}{\partial L} + \alpha(1 - \psi) \frac{\partial\pi_A(L^*)}{\partial L} = 0, \quad (3.11)$$

where from (2.1):

$$\begin{aligned} \frac{\partial\pi_R}{\partial L} &= (1 - \nu) \left[ \rho^* \left( p + \frac{\partial p}{\partial q} q \right) - \frac{\partial\mu}{\partial q} \right] \frac{\partial q}{\partial L} - (1 + \tau) \omega^* \left( w + \frac{\partial w}{\partial L} L \right), \\ \frac{\partial\pi_S}{\partial L} &= (1 - \rho^*) \left( p + \frac{\partial p}{\partial q} q \right) \frac{\partial q}{\partial L} - (1 - \omega^*) \left( w + \frac{\partial w}{\partial L} L \right), \\ \frac{\partial\pi_A}{\partial L} &= (1 - \nu) (1 - \rho^*) \left( p + \frac{\partial p}{\partial q} q \right) \frac{\partial q}{\partial L} - (1 + \tau) (1 - \omega^*) \left( w + \frac{\partial w}{\partial L} L \right). \end{aligned}$$

The necessary condition (3.11) can be rewritten as a firm's after tax expected value analogue to the classic *marginal revenue product of last unit of labour*,  $MRP_L$ , equals *marginal expenditures*,  $ME_L$ , that it brings:

$$\left\{ [1 - (\alpha + (1 - \alpha) \rho^*) (\psi + \nu - \psi\nu)] \left( p + q \frac{\partial p}{\partial q} \right) - (1 - \psi) (1 - \nu) \frac{\partial\mu}{\partial q} \right\} \frac{\partial q}{\partial L} = [1 + (\alpha + (1 - \alpha) \omega^*) (\tau - \psi - \psi\tau)] \left( w + L^* \frac{\partial w}{\partial L} \right). \quad (3.12)$$

The term in curly braces on the left-hand side of (3.12) represents the expected value of marginal revenue obtained by the last unit of labour's marginal product that is either taxed fully with probability  $\alpha$  or just the reported portion  $\rho^*$  is taxed with probability  $1 - \alpha$ . Marginal revenue is also reduced by the marginal material costs that marginal product brings. Similarly, the right-hand side is a product of a classic marginal expenditures term with a correction factor that includes response to taxes and underreporting. We further explore expression (3.12) to compare optimal employment levels under truthful reporting and underreporting.

### 3.3.1. Truthful reporting

When a firm reports truthfully, we have  $\rho = 1$ ,  $\omega = 1$ ,  $\alpha = 0$ . (3.12) therefore simplifies to:

$$(1 - \psi)(1 - \nu) \left( p + q \frac{\partial p}{\partial q} - \frac{\partial \mu}{\partial q} \right) \frac{\partial q}{\partial L} = (1 - \psi)(1 + \tau) \left( w + \bar{L} \frac{\partial w}{\partial L} \right), \quad (3.13)$$

where  $\bar{L}$  stands for optimal employment in this case. We point out an obvious and intuitive result. Value added tax ( $\nu$ ) pushes the  $MRP_L$  curve down and to the left, while payroll tax ( $\tau$ ) pushes  $ME_L$  up and to the left. Optimal employment  $\bar{L}$  determined from the intersection of the two is therefore lower than it would have been without the two taxes.

### 3.3.2. Underreporting and the missing middle

Equations (3.12) and (3.13) differ only in two terms standing for expectations with respect to underreporting and auditing probability. We next show that when a firm underreports at least one of the two quantities, it employs more labour ( $L^*$ ) than an honest firm ( $\bar{L}$ ).

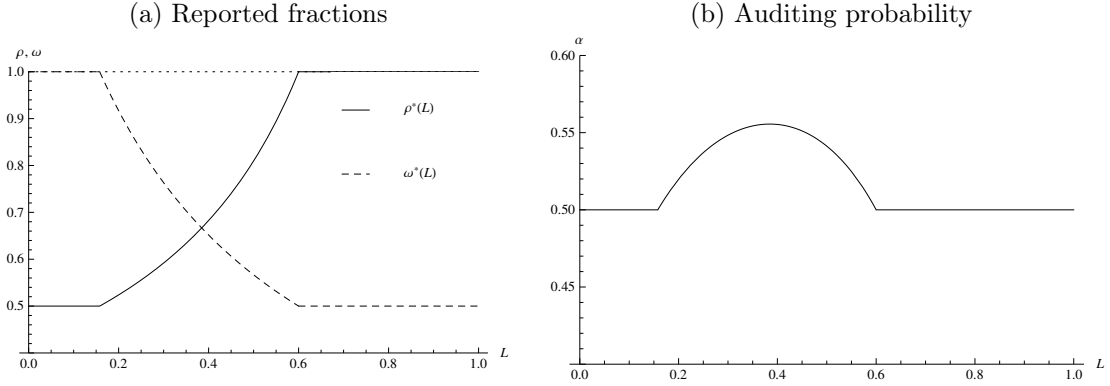
**Proposition 4.** *When a firm finds it optimal to underreport either sales or wages or both, it employs more labour than it would under truthful reporting of the two, i.e.,  $L^* > \bar{L}$*

*Proof.* When a firm is underreporting (see Propositions 1 and 2), at least one of the inequalities  $0 < \rho^* < 1$  and  $0 < \omega^* < 1$  holds. It follows that  $\alpha < 1$  and at least one of expectations factors  $\alpha + (1 - \alpha)\rho^*$  and  $\alpha + (1 - \alpha)\omega^*$  from (3.12) is less than one. It follows that when underreporting sales, the left-hand side of (3.12) lies above the left-hand side of (3.13). If the wage bill is underreported, the right-hand side of (3.12) lies below the right-hand side of (3.13). In case both  $0 < \rho^* < 1$  and  $0 < \omega^* < 1$ , both curves are affected as described. The resulting employment level with underreporting,  $L^*$ , therefore lies to the right of the optimal employment level with truthful reporting,  $\bar{L}$ .  $\square$

By underreporting, the firm reduces the costs brought by taxes and is stimulated to increase its production and consequently the amount of labour employed. Underreporting increases the expected costs of being caught, of course, but this effect is outweighed by the first one. Net effect increases employment.

This result gives a novel perspective to the missing middle result. The latter is usually a consequence of heterogeneity in managerial ability. Managers that lack ability self-select into the informal sector since they cannot cope with the higher

Figure 3.1: Optimal underreporting



costs of operating in a formal sector. A small amount of such heterogeneity would not change our result. The manager of a truthfully reporting firm may be more competent than a manager of a firm that underreports but, if she is extremely risk averse in face of positive fine  $\phi$ , she will employ less than her dishonest colleague. As we show in our example (subsection 3.4), an underreporting firm may be larger than a truthfully reporting one when measured in employment and actual sales, but smaller when measured in reported sales. The missing middle is therefore not supported in our model.

Higher production necessarily means higher welfare that underreporting firm brings to the market.

**Corollary.** *Underreporting increases firm's production and the welfare it brings to the market.*

#### 3.4. Example: linear production, product demand and labour supply

We illustrate our results in a simple setting. A firm faces a linear product demand function  $p(q) = 1 - q$ , which is served by a linear production  $q \equiv f(L) = L$ . Material costs brought by production level  $q$  are  $\mu(q) = 0.05q$  and labour is supplied according to increasing schedule  $w(L) = 0.1 + 0.3L$ . Auditing probability that satisfies the assumptions that we use is  $\alpha = 1 - \rho\omega$ .

Tax rates are: corporate rate  $\psi = 0.1$ , value added rate  $\nu = 0.2$ , and payroll rate  $\tau = 1.0$ , while we assume no lump sum fine for underreporting firm being caught ( $\phi = 0$ ). Under these values, conditions from both Proposition 1 and Proposition 2 are satisfied and the firm is going to underreport.

##### 3.4.1. Optimal underreporting

We solve the optimization program (3.5) but omit showing rather tedious analytical forms of  $\rho^*(L)$  and  $\omega^*(L)$ , and present their graphical forms instead.

Table 1: Truthful reporting vs. underreporting

	$\bar{L}, L^*$	$w$	$\rho$	$\omega$	$\alpha$	$pq$	$\rho pq$	$wL$	$\omega wL$	$\pi$
TR	0.2	0.16	1	1	0	0.16	0.16	0.032	0.032	0.0504
UR	0.2178	0.1653	0.5346	0.8853	0.5267	0.1704	0.0911	0.0360	0.0319	0.0621

Source: Own calculations.

Note: TR and UR stand for a truthful reporting firm and an underreporting firm, respectively.

Figure 3.1(a) shows that for low levels of employment and low wages, a firm does not underreport wages. It actually has an incentive to overreport them, which we rule out. Its motivation is to reduce reported profits and corporate taxes, which hurt them more than payroll taxes in this region (for  $L < 0.1579$ ).

Quantities sold are low and therefore the market price is relatively high when compared to constant per unit marginal costs, which makes added value per unit high as well. This is why a firm's underreporting of sales is most pronounced in this region. The fact that  $\rho^*(L) = 0.5$ , for  $L \in [0, 0.1579]$ , is explained by expression (3.4). Specifically, one of our assumptions is  $\phi = 0$  and since  $\omega^*(L) = 1$ , for  $L \in [0, 0.1579]$ , the second term in brackets in (3.4) also equals zero. When searching for interior solution  $\rho^*$  the remainder is set to zero and the whole equation is divided by  $(-\psi - \nu + \psi\nu)pq$ . Noting that  $\alpha = 1 - \rho$  and  $\partial\alpha/\partial\rho = -1$  brings the result  $\rho^* = 0.5$ .

For large  $L$ , the roles of underreporting the two quantities reverse. Lower product prices render it unnecessary to underreport the sales, while underreporting wages is at its maximum. The latter is  $\omega^* = 0.5$  for  $L \in [0.6, 1]$ , for reasons similar to those presented in explaining  $\rho^* = 0.5$  for low values of  $L$ .

Figure 3.1(b) presents the firm's probability of being audited under optimal underreporting policies. Due to high tax rates and the absence of a lump sum fine, the firm is willing to bear substantial risk of being audited. The probability we derive does not fall below 0.5 for any employment level.

### 3.4.2. Employment

Solving expressions (3.13) and (3.12) for optimal employment under truthful reporting and underreporting, respectively, yields  $\bar{L} = 0.2$  and  $L^* = 0.2178$ ; results that are in line with Proposition 4 - underreporting firm employs more.

Table 1 compares all relevant variables under the two regimes. The underreporting firm employs almost 9 percent more labour, pays higher actual wages and makes 23 percent higher profits than a truthfully reporting firm does.



#### 4. Minimum wage and employment

Suppose that the government imposes a minimum wage rate  $w_m$  that is perfectly enforceable. If  $w_m < \omega^*(L)w(L)$ , the minimum wage is not binding and does not affect the firm's decisions. Therefore, we analyze a firm that finds minimum wage  $w_m$  to be binding in its reporting since without it the firm would optimally report  $\omega^*(L)w(L) < w_m$ . The minimum wage is hence not only an instrument used in reducing the poverty of low wage labour, but also cuts incentives to hide a part of a wage bill and workers' personal incomes from tax authorities. We are interested in how underreporting firms react to this policy instrument when staying in formal economy.

##### 4.1. Truthful reporting

We denote a firm's competitive employment level and wage rate obtained by equating firm's after tax marginal revenue product of labour to labour supply  $(1 - \psi)(1 + \tau)w(L)$ , by  $\bar{L}_c$  and  $\bar{w}_c$ , respectively. Minimum wage employment with truthful reporting,  $\bar{L}_m$  is determined by the relation between  $w_m$  and  $\bar{w}_c$ . If a minimum wage is above the competitive wage ( $w_m > \bar{w}_c$ ), then  $\bar{L}_m$  solves

$$(1 - \psi)(1 - \nu) \left[ p + \frac{\partial p}{\partial q} q - \frac{\partial \mu}{\partial q} \right] \cdot \frac{\partial q}{\partial L} = (1 - \psi)(1 + \tau)w_m, \quad (4.1)$$

as it follows from (3.13). If the opposite is the case ( $w_m \leq \bar{w}_c$ ), then

$$w_m = w(\bar{L}_m). \quad (4.2)$$

If the minimum wage is set below marginal expenses from optimal employment in the no minimum wage environment, the optimal employment level with minimum wage necessarily exceeds the one without it, i.e.  $\bar{L}_m > \bar{L}$ . This result has been known at least since Robinson (1933) and was recently revised by Danziger (2010), who shows that under imperfect enforcement of minimum wage, small firms will decrease employment.

Suppose that minimum wage  $w_m$  is set above the competitive wage  $\bar{w}_c$ . When minimum wage increases, the right-hand side of (4.1) does as well and optimal employment level decreases. The following rather obvious result is stated as a reference to use in our later example and empirical tests.

**Proposition 5.** *Suppose  $w_m \geq \bar{w}_c$ . Then, for truthfully reporting firm,  $d\bar{L}_m/dw_m < 0$ .*

#### 4.2. Underreporting

Suppose that an underreporting firm finds that  $\omega^*(L)w(L) < w_m$ , for some amount of  $L$  it employs. Since we assume the minimum wage is perfectly enforceable, the firm's reported fraction of wages is revised to be  $\omega_m(L) \equiv \max(w_m/w(L), \omega^*(L))$ . For employment values where  $\omega_m(L) > \omega^*(L)$ , we expect the firm to readjust the reported fraction of sales. The second argument of a maximum function takes into account that at higher levels of  $L$ , reported wage  $\omega^*(L)w(L)$  exceeds  $w_m$ . In addition, since for low employment levels  $w_m$  exceeds  $w(L)$ , we have  $\omega_m > 1$ ; we further define:

$$\bar{\omega}_m(L) \equiv \min(\omega_m(L), 1) = \begin{cases} 1 & \text{if } w(L) < w_m \\ \frac{w_m}{w(L)} & \text{if } \omega^*(L)w(L) < w_m \leq w(L) \\ \omega^*(L) & \text{if } w_m \leq \omega^*(L)w(L) \end{cases} \quad (4.3)$$

##### 4.2.1. Minimum wage and sales underreporting

When a firm faces minimum wage  $w_m$  that binds its reported wage, optimal sales underreporting fraction,  $\rho^*$ , changes. We denote the optimal fraction of reported sales in the presence of minimum wage by  $\rho_m^*$ . We then modify (3.7) and (3.8) by changing  $\omega$  into  $\bar{\omega}_m$  and  $\rho^*$  into  $\rho_m^*$ , and examine the total differential of  $\partial\pi(\rho_m^*, \bar{\omega}_m)/\partial\rho = 0$  with respect to  $\rho_m^*$  and  $w_m$ . Since  $w_m$  enters the expression only through  $\bar{\omega}_m$ , we can write:

$$\frac{d\rho_m^*}{dw_m} = \frac{d\rho^*}{d\bar{\omega}_m} \frac{\partial\bar{\omega}_m}{\partial w_m}, \quad (4.4)$$

where  $\partial\rho^*/\partial\bar{\omega}_m$  was already studied in (3.6) and Proposition 3.

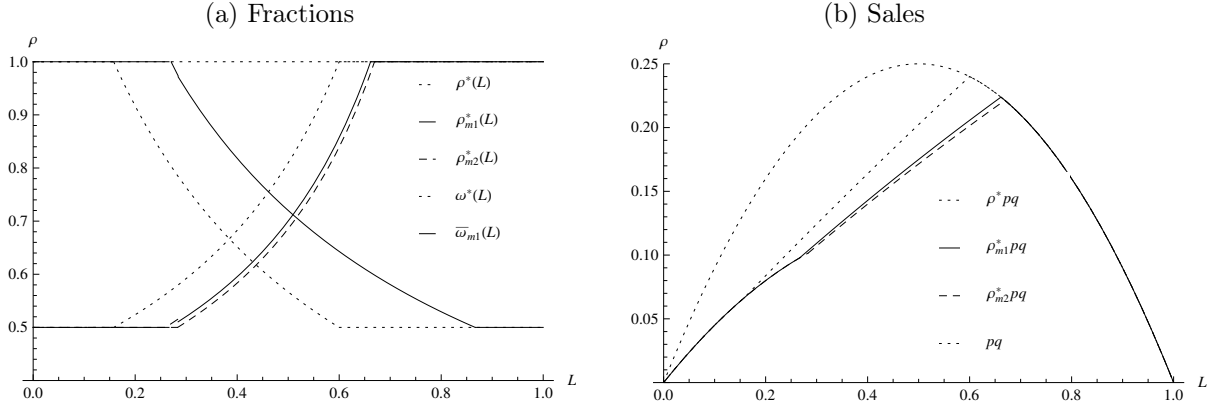
We divide behaviour of (4.4) into two types according to the relationship between  $w_m$  and  $w(L)$ . First, when  $w_m \geq w(L)$ , we have  $\bar{\omega}_m = 1$ . Since an increase in minimum wage does not affect the reported wage fraction, i.e.  $\partial\bar{\omega}_m/\partial w_m = 0$ , the derivative (4.4) equals zero. A change in minimum wage does not change a reported fraction of sales.<sup>13</sup>

Second, when  $\omega^*(L)w(L) < w_m < w(L)$ , we have  $\bar{\omega}_m = w_m/w(L) < 1$  and  $\partial\bar{\omega}_m/\partial w_m = 1/w(L) > 0$ . This means that derivative  $\partial\rho_m^*/\partial w_m$  moves in the same direction as  $\partial\rho^*/\partial\bar{\omega}_m$ , which was already studied in subsection 3.2 (Proposition 3).

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<sup>13</sup>Result of this kind was already seen in Figure 3.1 (a) where for low  $L$  we have  $\omega^* = 1$  and  $\rho^* = 0.5$ .

Figure 4.1: Optimal response in sales underreporting to minimum wage



**Proposition 6.** *When  $w_m < w(L)$ , optimal reported fraction of sales reacts to an increase in a minimum wage by a shift in the same direction as it does when a reported fraction of wage bill increases regardless of minimum wage being installed or not, i.e.,  $\text{sgn}(\partial \rho_m^* / \partial w_m) = \text{sgn}(\partial \rho^* / \partial \bar{\omega}_m)$ . When  $w_m \geq w(L)$ ,  $\partial \rho_m^* / \partial w_m = 0$ .*

*Example.* Suppose that in the setting of Section 3.4, the government initially imposes a minimum wage  $w_{m1} = 0.18$  and eventually increases it to  $w_{m2} = 0.185$ . Optimal response in sales underreporting is shown in Figure 4.1. The two leftmost dotted lines in part (a) are original reporting schedules with no minimum wage, while the full lines represent reporting schedules under  $w_{m1}$ . The non-increasing curve left of  $L = 0.8667$  is the wage reporting fraction exogenously forced onto the firm by a minimum wage, while the non-decreasing full line curve represents the response in sales underreporting. The latter lies below its no-minimum-wage analogue for  $0.1580 < L < 0.6616$ , while for  $L$  below this interval, reported fraction of sales stays the same. When minimum wage rises to  $w_{m2} = 0.185$ , the dashed line shows the same kind of a selective downward shift in sales underreporting. Figure 4.1 (b) shows corresponding changes in the absolute values of sales.

#### 4.2.2. Employment

When a firm faces minimum wage and seeks an optimal employment level, the derivative of its expected profit with respect to employment level (3.10) changes. It is still true that profit maximization in the second stage of the firm's decision process makes  $\partial \pi(L, \rho = \rho^*, \omega) / \partial \rho = 0$ ; whereas for levels of  $L$  for which  $w_m$  binds reported wage  $\omega^*(L) w(L)$ , we have  $\partial \pi(L, \rho, \omega = \bar{\omega}_m) / \partial \omega < 0$ . As long as employment is within the region in which the minimum wage binds the reported wage from below,

we rewrite (3.10):<sup>14</sup>

$$\begin{aligned} \frac{d\pi(L, \rho_m^*(L), w_m)}{dL} &= \frac{\partial\pi(L, \rho_m^*(L), w_m)}{\partial L} + \\ &\quad \frac{\partial\pi(L, \rho_m^*(L), w_m)}{\partial \rho} \frac{d\rho_m^*(L)}{dL} + \frac{\partial\pi(L, \rho_m^*(L), w_m)}{\partial w_m} \frac{dw_m}{dL}. \end{aligned} \quad (4.5)$$

Given the discussion above, the only non zero term on the right-hand side of derivative (4.5) is the first one. Nevertheless, auditing probability might change with employment level through the wage level. Specifically, auditing probability in this case is  $\alpha(\rho_m^*(L), \bar{\omega}_m(L))$ , where  $\bar{\omega}_m(L)$  is defined in (4.3). This introduces an additional term in optimality condition that is not present in (3.11). Optimal employment level in the presence of minimum wage,  $L_m^*$ , solves necessary condition:

$$\begin{aligned} (1 - \psi) \frac{\partial\pi_R(L_m^*)}{\partial L} + (1 - \alpha) \frac{\partial\pi_S(L_m^*)}{\partial L} + \alpha (1 - \psi) \frac{\partial\pi_A(L_m^*)}{\partial L} + \\ \frac{\partial\alpha(\rho_m^*, \bar{\omega}_m(L_m^*))}{\partial w} \frac{\partial w(L_m^*)}{\partial L} [(1 - \psi) \pi_A(L_m^*) - \pi_S(L_m^*) - \phi] = 0. \end{aligned} \quad (4.6)$$

We rewrite the three expected profit components from (2.1) that we need in (4.6) and add the derivative of  $\alpha$  with respect to  $L$ :

$$\begin{aligned} \pi_R &= (1 - \nu) [\rho pq - \mu] - (1 + \tau) w_m L, \\ \pi_S &= (1 - \rho) pq - (1 - \bar{\omega}_m) w L, \\ \pi_A &= (1 - \nu) (1 - \rho) pq - (1 + \tau) (1 - \bar{\omega}_m) w L, \\ \frac{\partial\alpha}{\partial w} \frac{\partial w}{\partial L} &= \frac{\partial\alpha}{\partial \omega} \frac{\partial \bar{\omega}_m}{\partial w} \frac{\partial w}{\partial L}. \end{aligned} \quad (4.7)$$

Based on the behaviour of fraction  $\bar{\omega}_m$ , we observe three types of solution to (4.6) for three intervals of  $L$ . In addition to its definition in (4.3), we need the derivative of  $\bar{\omega}_m$  with respect to  $L$ :

$$\frac{\partial \bar{\omega}_m}{\partial w} \frac{\partial w}{\partial L} = \begin{cases} 0 & \text{if } w(L) < w_m \\ -\frac{w_m}{w^2(L)} \frac{\partial w(L)}{\partial L} & \text{if } \omega^*(L) w(L) < w_m \leq w(L). \\ \frac{\partial \omega^*(L)}{\partial L} & \text{if } w_m \leq \omega^*(L) w(L) \end{cases} \quad (4.8)$$

In the last of the three cases, the minimum wage is not binding and we are in the solution from Section 3.3.2. In the second case, (4.6) includes all four terms from

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<sup>14</sup>If employment exceeds such values and reported wage grows above minimum wage, solution presented in subsection 3.3.2 applies.

(4.7). Since we believe that minimum wage  $w_m$  is usually set above market clearing wage  $w(L)$ , we do not present the necessary condition for  $L_m^*$  such that  $w_m \leq w(L)$ . It can readily be developed, but the results turn out not to be very instructive.

When the minimum wage not only binds the reported wage but the actual wage as well ( $w(L) < w_m$ ), necessary condition (4.6) simplifies considerably. The fourth term from the left-hand side equals zero due to derivative from (4.8). Since  $\bar{\omega}_m = 1$  (see (4.3)), the wage bill parts of  $\pi_S$  and  $\pi_A$  in (4.7) equal zero as well. (4.6) can then be written as:

$$\left\{ [1 - (\alpha + (1 - \alpha)\rho_m^*)(\psi + \nu - \psi\nu)] \left( p + q \frac{\partial p}{\partial q} \right) - (1 - \psi)(1 - \nu) \frac{\partial \mu}{\partial q} \right\} \frac{\partial q}{\partial L} = (1 - \psi)(1 + \tau)w_m. \quad (4.9)$$

In the final step, we compare the effect that a change in a minimum wage has on optimal employment of a truthfully reporting firm to the effect it has on optimal employment of an underreporting firm. We expect the latter to be able to absorb a part of the adverse effect of a higher minimum wage through sales underreporting and to decrease employment by relatively less. Since the two respective employment levels are determined by different marginal revenue product curves (left-hand sides of (4.1) and (4.9)), we take extra care when drawing the conclusions. We first rewrite necessary conditions for optimal employment (4.1) and (4.9) as (truthfully reporting firm):

$$(1 - \psi - \nu + \psi\nu) \left( p + q \frac{\partial p}{\partial q} \right) \frac{\partial q}{\partial L} = (1 - \psi) \left[ (1 + \tau)w_m + (1 - \nu) \frac{\partial \mu}{\partial q} \frac{\partial q}{\partial L} \right] \quad (4.10)$$

and (underreporting firm)

$$[1 - (\alpha + (1 - \alpha)\rho_m^*)(\psi + \nu - \psi\nu)] \left( p + q \frac{\partial p}{\partial q} \right) \frac{\partial q}{\partial L} = (1 - \psi) \left[ (1 + \tau)w_m + (1 - \nu) \frac{\partial \mu}{\partial q} \frac{\partial q}{\partial L} \right]. \quad (4.11)$$

Note that the right-hand sides of the two expressions are identical, i.e. respective sums of marginal expenses for labour and materials have identical shapes. We denote the left-hand side of (4.10) by  $MRP_L^T$  and the left-hand side of (4.11) by  $MRP_L^U$ . We have shown in Section 3.3.2 that  $MRP_L^U$  lies above  $MRP_L^T$ . Furthermore, in AppendixA (see Claim 1, proof of Proposition 7), we show that for  $L \leq L_m^*$ , we have  $MRP_L^U(L) = aMRP_L^T(L)$ , where  $a > 1$  is a constant. If the right-hand sides

of (4.10) and (4.11) were constant, we could present fairly tight comparisons of the optimal employment responses to minimum wage changes for the two types of firms' behaviour. The caveat is that the marginal material expense term,  $\frac{\partial \mu}{\partial q} \frac{\partial q}{\partial L}$ , is not constant in general. We would hence need to relate  $MRP_L^T$  and  $MRP_L^U$  curves at different vertical levels, which is out of reach. To derive a part of our exact results, we will need to assume  $\frac{\partial \mu}{\partial q} \frac{\partial q}{\partial L} = \text{const}$  and then try to relax the assumption in a discussion of the results.

We define the elasticity of the optimal employment level with respect to the minimum wage for an underreporting firm as  $\eta^U \equiv \frac{\partial L_m^*/L_m^*}{\partial w_m/w_m}$ , and for a truthfully reporting firm as  $\eta^T \equiv \frac{\partial \bar{L}_m/\bar{L}_m}{\partial w_m/w_m}$ . For one of the parts of the result, we also need to define a competitive wage,  $w_c^*$ , for an underreporting firm. It is determined by the intersection of the left-hand side of (4.9) and labour supply  $(1 - \psi)(1 + \tau)w(L)$ . Since  $L^* > \bar{L}$  (Proposition 4), it must be  $w_c^* > \bar{w}_c$  (see the definition of  $\bar{w}_c$  in the beginning of subsection 4.1).

Our last result shows circumstances under which an increase in minimum wage decreases the optimal employment level in an underreporting firm relatively less than it does in a truthfully reporting firm. We also show that there is an interval of minimum wages within which an underreporting firm increases its employment, while truthfully reporting firm decreases it. The proof is deferred to Appendix A.

**Proposition 7.** *Suppose that:*

1.  $w_m \geq w_c^*$  and that  $\left(p + q \frac{\partial p}{\partial q}\right) \frac{\partial q}{\partial L}$  is concave. Then  $\eta^T < \eta^U < 0$ .
  2.  $w_m \geq w_c^*$  and that  $\left(p + q \frac{\partial p}{\partial q}\right) \frac{\partial q}{\partial L}$  is convex of the form either:
    - (a)  $b(L + z)^{-c} - v$ , where  $b, c > 0$  and  $z, v \geq 0$ , with  $z + v > 0$ , or
    - (b)  $v - bL^c$ , where  $b, c, v > 0$ .
- If in addition (c)  $\frac{\partial \mu(q(L))}{\partial L} = \text{const}$ , then  $\eta^T < \eta^U < 0$ .
3.  $w_c^* > w_m \geq \bar{w}_c$  and  $w_m \geq w(L_m^*)$ . Then  $\eta^T < 0 < \eta^U$ .

We believe that the majority of real life firms operate in circumstances where  $w_m \geq w_c^*$ , described in Proposition 7, parts 1 and 2. These results say that when the minimum wage exceeds the competitive wage for both underreporting and truthfully reporting firms, the latter will decrease its optimal employment relatively more provided that its  $MRP_L^T$  curve is not too convex. For the second part, we also need a rather strong assumption on the shape of marginal material expenses, i.e.,  $\frac{\partial \mu(q(L))}{\partial L} = \text{const}$ . This part of a result is not of an *if and only if* nature. There certainly exist other classes of convex  $MRP_L^T(L)$  curves for which  $\eta^T < \eta^U < 0$  will hold, and, in addition, the  $\frac{\partial \mu(q(L))}{\partial L}$  curve might be (at least slightly) increasing or decreasing for the result still to be valid.

*Example.* In our example from 3.4,  $MRP_L^T(L) = (1 - \psi)(1 - \nu)(1 - 2L)$  is linear, hence concave, and the result from Proposition 7.1. applies.

## 5. Institutional Background and Data

### 5.1. Institutional Background

The theoretical model predicts that an increase in a binding minimum wage should reduce employment to a lesser extent in firms engaged in underreporting in comparison to truthfully reporting firms. In order to test whether this prediction is consistent with the actual behaviour of firms, we need to use data for a country with a sizeable informal economy that increased its binding minimum wage. Slovenia, for which we have access to linked employer-employee data set, meets these two conditions. Since we shall use these data in empirical assessment of theoretical model, it is useful to provide a brief overview of key features of the informal economy in Slovenia and its minimum wage legislation.

#### 5.1.1. Informal Economy

According to recently published official estimates by the Slovenian Statistical Office (Flajs, 2012), the share of informal economy in Slovenia was 8.3 percent of GDP in 2007.<sup>15</sup> It is significantly higher than the corresponding values for established market economies (e.g. 6.5 percent in neighboring Austria in 2004) and lower than the corresponding values for the majority of transition economies (e.g., 15.3 percent in Hungary in 2004; 9.5 percent in Estonia in 2000). A decomposition of the official estimate into separate types of informal economy shows that the largest contribution may be attributed to the underreporting of output (i.e. 4.1 percent of GDP), which is consistent with the underreporting considered in our theoretical model. An alternative way of evading taxes, as reported by SORS, is inflation of costs of materials and services. Underreported sales can be found primarily in final goods production, while inflated costs may be attributed to all types of goods. Thus, the informal economy in Slovenia is not a parallel activity, but an important part of economic activity of registered business units. In particular, Flajs estimates that 90 percent of the informal economy can be attributed to registered business units

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<sup>15</sup>This estimate is calculated according to the net principle of the European System of Accounts (1995) methodology and is included in the official estimates of GDP for Slovenia since 1995. It includes only value added that is omitted due to tax evasion. According to the gross principle, which includes consumption, the extent of the informal economy is estimated to be 12.2 percent. Both of these estimates are significantly lower (roughly half) than the unofficial estimates of informal economy.

such as small firms (up to 3 employees) and sole proprietors.<sup>16</sup> Furthermore, there are large differences in the presence of informal economy across industries. It is high in industries such as Real Estate, Renting and Business Services (33.5 percent of value added), Construction (30.3 percent), Other (Personal) Services (20.1 percent), Agriculture, Hunting and Forestry (15.7 percent), Fishing (15.1 percent) and Retail, Wholesale and Car Repair (12.7 percent), while it is low in Financial Intermediation (0.9 percent), Utilities (0.7 percent), Mining (3.3 percent) and Manufacturing (5.4 percent).

#### *5.1.2. Minimum Wage*

The law on the minimum wages in Slovenia was first passed in 1995 and was subject to change annually. It sets the minimum monthly pay for a full-time employee (i.e. 40-42 hours per week), excluding compensation of workers for work related reimbursements such as income-tax-free compensation for daily commuting and meals during work, business trips, overtime and holiday bonus. Actual gross wage may thus be lower than the minimum wage for part-time workers and workers on paid sick leave, but also higher if workers engage in overtime work.

Table 2 reports the minimum and the average wage in the entire economy in the period from August 2006 to December 2010. Although the majority of the changes aimed to preserve the ratio between the minimum and the average wage to around 42 percent, there is an important exception. The minimum wage increased by 22.9 percent to almost 49 percent of the average wage on March 1, 2010. The government allowed firms to introduce this new minimum wage gradually if they proved to be in financial distress and signed a written agreement with worker representatives. In those firms, minimum wage increased by 9.6 percent (to 43.7 percent of the average wage), which was still relatively high for a declining economy (see the description of aggregate dynamics of private sector in subsection 5.1.3). The annualized minimum wages in Table 2 are not directly applicable in our analysis as the personal income tax data are based on calendar time. Hence we also report the annual minimum wages for calendar years in Table 3.<sup>17</sup> It is evident that significant increases in the minimum wage took place in 2008 with a 7.52 percent increase and in 2010 with an 18.63 percent increase for financially viable firms and 8.18 percent for firms in financial distress.

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<sup>16</sup>Flajs (2012) estimates that informal economy cannot be detected in business units with 30 or more employees.

<sup>17</sup>The annual minimum wage is calculated as a weighted average of annualized monthly wages. For example, in 2007 the minimum wage is 6,328.8 EUR, which is obtained as a weighted average of 6,262 EUR and 6,462.4 with weights 8/12 and 4/12, respectively.



Table 2: Annualized Minimum and Average Wage in Slovenia, 2006-2010

Valid since	(1) Minimum gross wage	(2) Average gross wage	(3) Ratio (1)/(2)
August 1, 2006	6,262.0	14,529.2	43.10
August 1, 2007	6,462.4	15,352.8	42.09
March 1, 2008	6,798.4	16,234.4	41.88
August 1, 2008	7,070.3	16,860.0	41.94
August 1, 2009	7,169.2	16,981.0	42.22
March 1, 2010	8,809.8	17,990.8	48.97
March 1, 2010 <sup>a</sup>	7,856.3	17,990.8	43.67

Source: Official Gazette, various editions.

Notes: The minimum and the average gross wage are given in EUR (current price). Ratio between the minimum and the average gross wage (3) is given in percent.

<sup>a</sup> Minimum wage applicable in financially distressed firms.

Table 3: Annual Minimum Wage in Slovenia, 2007-2010

Year	Minimum wage	Growth rate
2007	6,328.8	
2008	6,805.0	7.52
2009	7,103.3	4.38
2010	8,399.6	18.63
2010 <sup>a</sup>	7,684.5	8.18

Source: Official Gazette, various editions.

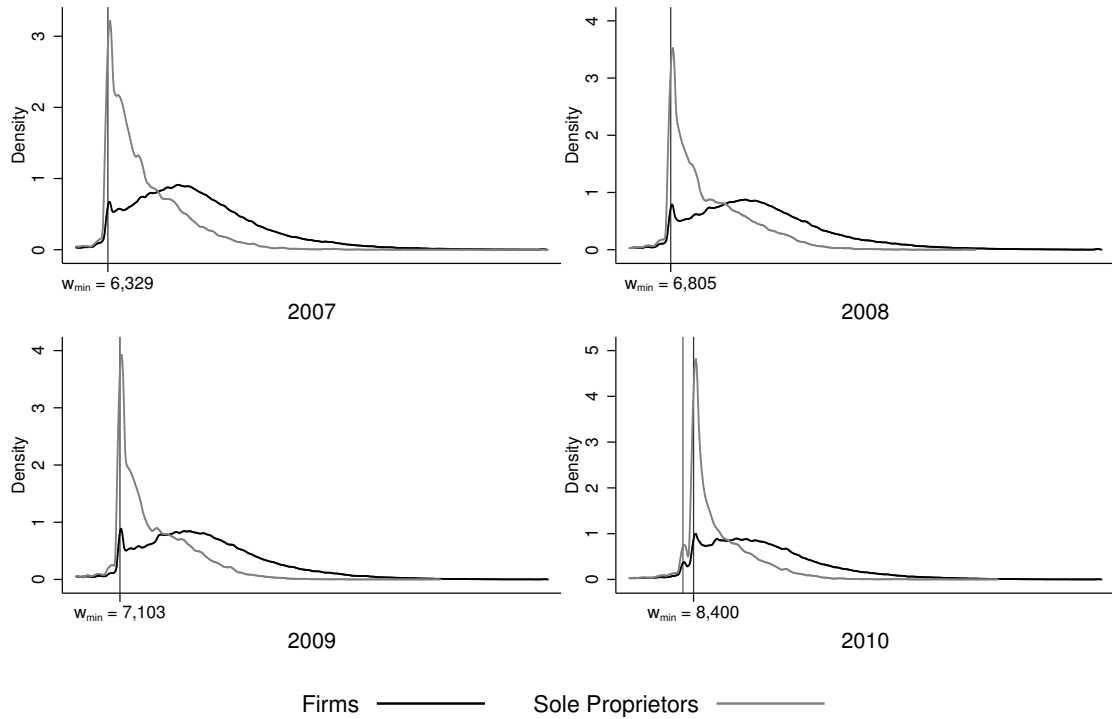
Notes: The annual minimum wage is calculated as a weighted average of annualized monthly wages.

<sup>a</sup> Minimum wage applicable in financially distressed firms.

In order to demonstrate the importance of minimum wages in Slovenia for determination of actual wages, we plot the distributions of gross wages (in logs) for all full time employees (FTE) that did not change employer, job description, hours worked and type of shift work, and did not improve educational attainment. Figure 5.1 shows that the minimum wage was indeed binding for some employment contracts, especially for workers who were employed by sole proprietors. Moreover, increases in the minimum wage during the period of economic crisis, which in Slovenia, started towards the end of 2008, led to an increase in the share of workers on minimum wage. It is also important to note that in 2010, minimum wages for both distressed and viable firms were frequent, although the higher minimum wage was paid more frequently.

Next, we provide evidence on the prevalence of minimum wage in firms of differ-

Figure 5.1: Distribution of Annual Gross Wage in Slovenian Private Sector and Annual Minimum Wage, 2007-2010



Source: AJPES, SORS, TARS and own calculations.

Notes: i) The non-parametric densities are estimated using the Stata module `kdens` that deals with bounded domain problem. ii) The densities for gross wage in current EUR (log scale).

ent size and industry affiliation by comparing the share of wages set in a  $\pm 5$  EUR interval around the minimum wage (Table 4). For 2010, we use only the top value of the minimum wage as firms were less likely to opt for the reduced minimum wage of financially distressed business units. For both firms and sole proprietors, we observe a negative correlation between the share of employees on minimum wage and firm size measured by the number of employees. In terms of industry affiliation, we focus on a comparison of minimum wage share between all Sole Proprietorships with Sole Proprietorship in cash-based Service Industries with allegedly higher informal economy such as Other Services (NACE Rev.1 2-digit code 93), Restaurants and Bars (NACE Rev.1 3-digit codes 553 and 554), and Retail Trade (52), and separately Construction (45).<sup>18</sup> It is evident that these industries exhibit higher shares of workers receiving minimum wages than all other industries. Moreover, in

<sup>18</sup>Other Services contains many cash-based services like laundry and chemical cleaning, hair-dressing and cosmetic saloons, funeral services, etc.

Construction, the share of FTE receiving minimum wage increases with firm size.<sup>19</sup>

Table 4: Share of Wages Set at the Minimum Wage, 2007-2010

Year	Number of Employees			
	1-3	4-9	10-29	30 or more
<i>Firms</i>				
2007	1.73	0.82	0.75	0.40
2008	1.54	0.70	0.57	0.44
2009	1.82	0.98	0.88	0.42
2010	2.01	1.13	0.82	0.14
<i>Sole Proprietorships (without Construction)</i>				
2007	6.31	2.53	1.45	3.32
2008	4.40	2.00	1.01	0.23
2009	5.24	2.02	1.01	0.48
2010	6.27	2.81	2.34	1.65
<i>Sole Proprietorships in Service Industries</i>				
2007	7.76	3.57	0.47	17.12
2008	5.75	2.36	0.34	0.88
2009	5.67	2.00	0.62	—
2010	7.44	3.6	2.57	—
<i>Sole Proprietorships in Construction</i>				
2007	6.78	6.83	11.06	26.29
2008	4.67	5.86	11.02	24.77
2009	7.92	7.28	11.31	24.32
2010	6.80	5.60	6.36	2.89

Source: Official Gazette, various editions.

Notes: Shares (in percent) are calculated as ratios between the number of FTE with annual wage set within a  $\pm 5$  EUR interval around the minimum wage and all FTE. In 2010 we consider the wage rate for non-distressed business units. Service Industries refer to Other Services, Restaurants and Bars and Retail Trade.

### 5.1.3. Aggregate Dynamics

We conclude this subsection with a brief overview of the dynamics of the Slovenian private sector for the period of our empirical analysis (2007-2010). The top panel of Table 5 shows the key statistics for firms in the private sector, while the bottom panel summarizes the data for sole proprietorships. The sample includes

<sup>19</sup>Economic crisis was particularly severe in Construction Industry, which is reflected in large exit and decline of firm size in this industry.

only active firms that employ at least one FTE. According to this definition, the total number of business units was around 46 thousand in 2007, out of which more than 29 thousand were firms and 17 thousand were sole proprietorships.

Table 5 reveals that during the period of large minimum wage hikes, the Slovenian economy faced a large contraction, which started in late 2008 and continued throughout 2009. The contraction affected all key aggregates such as aggregate sales, value added and employment in both firms and sole proprietorships, although firms started to recover in 2010 while sole proprietorships continued to contract. Another important difference between the two groups of business units is in the dynamics of their numbers. The number of firms continued to grow in 2009 and 2010, while the number of sole proprietorships declined in both years. The two groups of businesses also differ in terms of dynamics of employment and wages. The aggregate employment of firms declined more than the average, while the opposite happened in sole proprietorships. Moreover, the average wage did not decline between 2008 and 2010 in firms, while it grew in sole proprietorships in 2009 and declined in 2010. These numbers suggest an interpretation that is conducive to a wider presence of informal economy in sole proprietorships: those units with informal economy were more likely to survive and faced less need for adjustment in terms of employment.

## *5.2. Data*

### *5.2.1. Data Sources*

We constructed our employer-employee linked data set in a safe room at the Slovenian Statistical Office (SORS) from three distinct sources of data. The first is the Slovenian Tax Administration (TARS) that provides information on individual annual gross wages paid by each employer from personal income tax fillings. The next source of data is the Statistical Registry of Labour Force (SRLF), which is maintained by SORS and contains information on personal and employment characteristics of persons, such as age, gender, level and field of completed education, type of employment contract (definite and indefinite) and number of weekly working hours (without information on overtime). The third source of data is the Slovenian Agency for Public Legal Records and Related Services (AJPES), which contains firm-level accounting data (balance sheet and income statements). These include information on sales, costs of materials and services, and employment based on the number of working hours. From these, we can calculate a measure of firms' productivity. We merge these data sets using unique employee and employer identifiers.

Table 5: Aggregate Dynamics in Slovenia, 2007-2010

	Level	Growth rate		
	2007	2008	2009	2010
<i>Firms</i>				
Number of units	29,469	5.46	2.52	0.99
Aggregate				
Sales	71,336	9.55	-15.41	5.11
Value added	15,460	6.82	-6.42	0.49
Wage bill	7,113	10.78	-4.57	0.98
Employment	495,221	2.36	-6.03	-3.64
Average				
Sales	2,420,708	3.89	-17.49	4.08
Value added	524,606	1.29	-8.72	-0.50
Labour productivity	27,247	9.25	-7.13	1.74
Gross wage	12,725	9.06	0.93	3.85
Employment	16.80	-2.94	-8.33	-4.58
<i>Sole proprietorships</i>				
Number of units	16,802	2.84	-3.43	-8.24
Aggregate				
Sales	3,979	5.27	-17.11	-6.68
Value added	1,372	7.37	-13.98	-8.65
Wage bill	593	11.44	11.61	-32.81
Employment	56,978	3.98	-8.87	-17.92
Average				
Sales	236,790	2.36	-14.16	1.70
Value added	81,673	4.40	-10.93	-0.45
Labour productivity	26,630	3.28	-7.26	3.91
Gross wage	10,128	6.00	18.30	-20.76
Employment	3.37	1.67	-3.39	-0.68

Source: AJPES and own calculations

Notes: All nominal variables are given in current prices. Aggregate sales, value added and wage bill are given in million EUR. Average sales, value added, labour productivity and gross wage are given in EUR. The employment is based on actual number of hours worked. Gross wage is calculated as a ratio between wage bill and employment. Growth rates are given in percentages.

### 5.2.2. Summary Statistics

In empirical analysis, we investigate the effect of minimum wage increase on actual wage dynamics and job survival. In identifying those who were actually affected by these increases, we first restrict our sample to full-time employees who were working for the same employer for the entire calendar year due to annual frequency of data on wages. We further restrict the sample to those workers that were not facing retirement (less than 58 years of age and less than 40 years of work experience), with at the most 12 years of formal education (i.e., completed high school), and workers that were employed by active firms and sole proprietorships in the service industries listed above, which are likely to feature business units in the informal economy. Finally, we only consider workers whose annual wage in period  $t$ ,  $w_t$ , was between the minimum wage in periods  $t$  and  $t + 1$ , as for them, increases in minimum wage are more likely to be binding.

Individual and firm-level characteristics for our restricted sample of 20,693 individuals and 7,212 business units are summarized in Table 6. Due to restrictions on earned wages, the sample is not representative of the entire population of employees and business units. It contains a lower share of females, workers are younger, less experienced, less educated and clearly paid less. The sample also contains a greater share of sole proprietorships, small business units and less productive firms.

Table 6 also contains a variable called *expected wage growth* that reflects growth caused by an increase in a minimum wage. It is defined as:

$$\gamma_{w,it}^{Exp} = \frac{w_{m,i,t+1} - w_{it}}{w_{it}}, \quad (5.1)$$

where  $w_{it}$  denotes annual gross wage of individual  $i$  in period  $t$  and  $w_{m,i,t+1}$  denotes the annual minimum wage valid in period  $t + 1$ . This variable thus measures the relative increase of wage that is required in order to pay the minimum wage. The average actual wage growth rate exceeded that of the average expected wage growth; firms on average paid workers more than necessary to match the minimum wage.

## 6. Empirical Analysis

### 6.1. Empirical Strategy

Our main hypothesis states that an increase in the minimum wage should have differential effects across firms with different propensities to engage in informal economy. Since we do not know which firm is engaging in informal economy and to what extent, we need to make an identifying assumption. This relies on characteristics of firms that are more likely to underreport sales and wages. Flajs (2012) argues that

Table 6: Individual and Firm Characteristics for Estimation Sample, 2007-2010

Variable	Number	Mean	S.d.
Individuals	20,693		
Business units	7,212		
Observations	26,021		
<i>Personal characteristics</i>			
Female (share)		26.63	44.20
Age of worker (years)		35.63	9.61
Job duration (years)		3.82	4.21
Work week (hours )		40.00	0.05
Schooling (years)		9.70	1.79
Gross wage (EUR)		7,180	558.10
Actual wage growth (percent)		10.87	14.76
Expected wage growth (percent)		7.83	5.83
<i>Business units characteristics</i>			
Sole proprietorships (share)		41.42	49.26
Small business units (share)		27.34	44.57
Bars and Restaurants			
Number of employees		138.91	859.17
Labour productivity (EUR)		15,851	11,606

Source: AJPES and own calculations

Notes: Sample consists of all full time workers with maximum of 12 years of education that had a wage between current and next period wage, while all workers consists of a sample of all full-time workers. Personal and business units characteristics are calculated for all observations. The expected wage growth is defined as the expected relative wage increase caused by minimum wage increase.

informal economy is more frequently observed in service firms with an important share of cash transactions, in smaller business units and sole proprietorships due to greater control of owners. As already described above, we limit our attention to a subset of service industries in which informal economy is more likely to be present. This reduces the influence of heterogeneous demand shocks across industries during the economic crisis. Hence we do not contrast firms in services to firms in other industries, but instead exploit the fact that informal economy is more prevalent among small business units (1-3 employees) and sole proprietorships.

In order to test our key hypothesis using the linked employer-employee data, we estimate a job survival equation and test whether an interaction term between the expected wage growth and the dummy variable for small sole proprietorships is statistically significant. A complementary hypothesis is that firms that engage in informal economy are more likely to comply with minimum wage legislation, which can be tested by estimating a wage growth equation. Since the error terms of a wage growth equation and an equation for a latent variable underlying the job survival decision - joint surplus generated by occupied job - are positively correlated, the parameters of the wage growth equation are likely to be biased due to selection bias. To avoid this problem, we estimate the job survival and wage growth equations using the Heckman (1979) two-step sample selection model. The model without exclusion restrictions has the following specification:

$$\Pr[s_{it} = 1] = \Phi(\mathbf{x}'_{it-1}\beta^s + \sum_j \alpha_j^s I\{i \in j\} + \sum_\tau \alpha_\tau^s I\{\tau = t\}), \quad (6.1)$$

$$\gamma_{w,it} = \mathbf{x}'_{it-1}\beta^w + \sum_j \alpha_j^w I\{i \in j\} + \sum_\tau \alpha_\tau^w I\{\tau = t\} + \xi_{it}^w, \quad (6.2)$$

where  $\Pr[s_{it} = 1]$  denotes the probability of job  $i$  surviving in period  $t$ , while  $\gamma_{w,it}$  is the actual wage growth rate.  $\Phi$  is the cumulative distribution function of the normal distribution and  $\mathbf{x}$  denotes the vector of explanatory variables, which includes expected wage growth, small sole proprietorship dummy and their interaction terms and a set of control variables. Among the controls are age and its square, gender, job duration and its square, dummy variable assuming value 1 for individuals with completed high school degree and 0 for individuals with less than high school degree, and business-unit-level labour productivity (in logs). Coefficients  $\alpha_j$ 's and  $\alpha_\tau$ 's correspond to two sets of industry and time dummies, respectively.

## 6.2. Results

The estimates of equations 6.1 and 6.2 are summarized in Table 7. Models 1 and 2 show the estimates of parameters without and with the dummy variable for small



sole proprietorships and its interaction with expected wage growth. The coefficient for expected wage growth in the wage growth equation is statistically significantly greater than 0 and significantly lower than 1. This result suggests that businesses indeed adjust actual wages towards the minimum wage, but with imperfect compliance. The coefficient for expected wage growth in the job survival equation is negative, although not statistically significant. This suggests that an increase in minimum wage per se did not reduce job survival significantly. Next, the coefficient for small sole proprietorships is negative and significant in both equations, which suggests that wage growth is lower in small sole proprietorships and job survival less likely. However, the interaction effect between expected wage growth and small sole proprietorship is positive in both wage growth and job survival equations. The coefficient 0.261 in the wage growth equation suggests that compliance with expected wage growth is larger in small proprietorships. The coefficient for the product between expected wage growth and small sole proprietorship dummy is not capturing the sign of the true interaction effect for nonlinear job survival equation (see Ai and Norton, 2003). The full interaction effect is the following expression, which captures the fact that  $x_1$  is a continuous variable (expected wage growth) and variable  $x_2$  is a dummy variable – small sole proprietorship status<sup>20</sup>:

$$\frac{\Delta \frac{\partial \Phi(u)}{\partial x_1}}{\Delta x_2} = (\beta_1 + \beta_{12})\phi(\beta_1 + \beta_{12})x_1 + \beta_2 + \mathbf{X}'\beta - \beta_1\phi(\beta_1x_1 + \mathbf{X}'\beta), \quad (6.3)$$

where  $u$  is an index of cumulative normal distribution  $\Phi$  (and its density  $\phi$ ),  $\beta_1$  is a coefficient for expected wage growth,  $\beta_2$  is a coefficient for small sole proprietorship dummy,  $\beta_{12}$  is a coefficient for the product term and  $\mathbf{X}'\beta$  captures the contribution of the remaining variables in the model. Thus, the interaction term does not only depend on the coefficient for the product of the two variables, i.e., 1.137, but also on other variables and coefficients. Its value, evaluated at the mean values of variables in the model, is 0.441 (*s.e.* = 0.133), which is statistically significantly different from zero. Moreover, the interaction effect is positive for all values of predicted values of job survival, which confirms our main hypothesis that business units with higher presence of informal economy are less likely to destroy jobs when minimum wage hikes take place.

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<sup>20</sup>The dummy variable is 1 if business unit is a small sole proprietorship and 0 otherwise.

Table 7: Estimates of wage growth and job survival equations, 2007-2010

	Wage growth eq.		Job survival eq.	
	Model 1	Model 2	Model 1	Model 2
Expected wage growth	0.479** (0.03)	0.435** (0.04)	-0.0922 (0.18)	-0.174 (0.19)
Small sole proprietorship (dummy)		-0.0417* (0.02)		-0.301** (0.03)
Expected wage growth * Small sole proprietorship		0.261** (0.10)		1.137** (0.34)
Age	0.00209 (0.004)	0.00583* (0.0035)	0.0460** (0.006)	0.0450** (0.006)
Age Sq.	-0.0000303 (0.00005)	-0.0000767* (0.00004)	-0.000563** (0.00008)	-0.000558** (0.00008)
Female (dummy)	-0.00774 (0.01)	-0.0127** (0.01)	-0.0852** (0.02)	-0.0675** (0.02)
High school (dummy)	0.0173** (0.003)	0.0168** (0.003)	-0.0113 (0.02)	-0.00733 (0.02)
Job duration	0.00213 (0.003)	0.00613* (0.004)	0.0437** (0.005)	0.0489** (0.005)
Job duration Sq.	-0.0000857 (0.00008)	-0.000180** (0.00009)	-0.000932** (0.0003)	-0.00110** (0.0003)
Labour Productivity (log)	0.0219** (0.004)	0.0261** (0.004)	0.0293* (0.02)	0.0460** (0.02)
Observations	26,021	26,021	26,021	26,021
$\chi^2$			1115.5	875.3
$\lambda$			0.000809	0.141
$s.e.(\lambda)$			0.139	0.12
$\rho$			0.00568	0.791

Notes: Standard errors are given in parentheses. \*  $p < 0.10$ , \*\*  $p < 0.05$ .

All equations include 2-digit NACE industry and year dummies.  $\lambda$  denotes the inverse of Mills ratio.

## 7. Conclusion

The existing theoretical literature on the effects of the minimum wage on firms' decisions to engage in informal economy predicts that firms in informal economy are smaller than those in formal economy, thereby creating the missing middle in the firm-size distribution. In the light of empirical evidence on existence of firms that simultaneously participate in both formal and informal economy, we propose an alternative theoretical framework that allows for firms to engage in both formal and informal economy. We show that under plausible conditions regarding the tax rates on different types of taxes, it is optimal for firms to engage in underreporting of sales and wage bills. When faced with an increase in the minimum wage, such firms respond with lower reported sales and reduce employment to a lesser extent than firms that do not engage in informal economy. In order to test this prediction, we use actual data on individual-level wages and job survival for a set of Slovenian businesses in service industries with important share of cash transactions. We confirm our main prediction by finding that job destruction caused by minimum wage hikes was lower in small sole proprietorships which tend to be more involved in informal economy.

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## Appendix A. Building a proof of Proposition 7

We structure the proof in several steps.

Let  $X, Y \subseteq \mathbb{R}_{\geq 0}$  be intervals of non-negative real numbers and let  $f : X \rightarrow Y$  be a continuous function with  $f'(x) < 0$ . Set  $a > 1$  and form a function  $g(x) = a f(x)$ . If  $f(x)$  is concave, the following holds:

**Lemma 1.** *Let  $f''(x) \leq 0$ . Then for  $x_2 > x_1$ :*

$$f'(x_1) x_1 > g'(x_2) x_2. \quad (\text{A.1})$$

*Proof.* We know from construction that  $g'(x_2) = a f'(x_2) < f'(x_2) < 0$ , and due to concavity  $f'(x_2) < f'(x_1) < 0$ . Result follows.  $\square$

If  $f(x)$  was our  $MRP_L^T$  and  $g(x)$  was  $MRP_L^U$  (defined as the left-hand sides of (4.10) and (4.11), respectively), expression (A.1) would have an obvious extension to the two elasticities we wish to relate,  $\eta^T$  and  $\eta^U$ . We cannot readily extend the result from Lemma 1 to convex functions. In regions where the curvature of a convex function is too high, (A.1) does not hold even when we compare both sides of the inequality at the same height, as presented in Example 1.

**Example 1.** Let the two functions in question be defined as:

$$f(x) = \begin{cases} 10 - 9x & \text{if } 0 \leq x < 1 \\ 1.1 - 0.1x & \text{if } 1 \leq x \leq 11 \end{cases} \quad \text{and } g(x) = 2f(x).$$

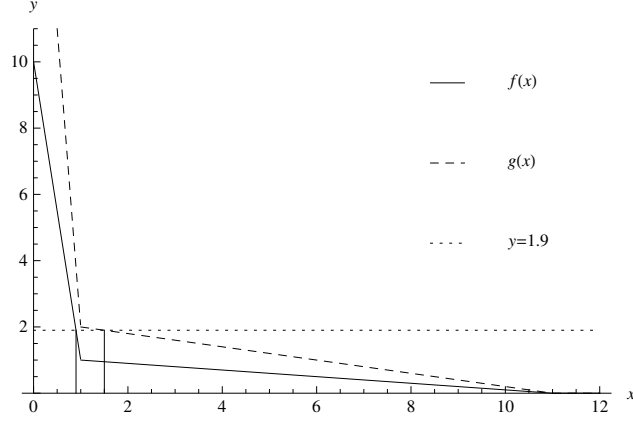
For  $y = 1.9$ , we obtain  $x_1 = f^{-1}(1.9) = 0.9$  and  $x_2 = g^{-1}(1.9) = 1.5$ . The two sides of the inequality (A.1) are then  $f'(x_1) x_1 = -9 \cdot 0.9 = -8.1$  and  $g'(x_2) x_2 = -0.2 \cdot 1.5 = -0.3$ ; the inequality does not hold. As obvious from Figure A.1, this is a consequence of extreme “curvature” of  $f(x)$  in  $x = 1$ .

For the purpose of following lemmas, we set  $y \in Y$  and denote by  $x_1, x_2 \in X$  solutions to  $f(x_1) = y$  and  $g(x_2) = y$ , respectively. We first show that shifting a convex function that (almost) satisfies (A.1) to the left, conforms to Lemma 1, when the expressions are calculated at the same height,  $y$ .

**Lemma 2.** *Let  $z > 0$ . Define  $\bar{f}(x) = f(x + z)$ ,  $\bar{g}(x) = a\bar{f}(x)$ , and  $\bar{x}_1$  and  $\bar{x}_2$  as respective solutions to  $\bar{f}(\bar{x}_1) = y$  and  $\bar{g}(\bar{x}_2) = y$ . If (a)  $f'(x_1) x_1 \geq g'(x_2) x_2$ , then (b)  $\bar{f}'(\bar{x}_1) \bar{x}_1 > \bar{g}'(\bar{x}_2) \bar{x}_2$ .*

*Proof.* We first note that (i)  $x_1 = \bar{x}_1 + z$  and  $x_2 = \bar{x}_2 + z$ , and (ii)  $\bar{f}'(\bar{x}_1) = f'(x_1)$  and  $\bar{g}'(\bar{x}_1) = g'(x_1)$ .

Figure A.1: Violation of Lemma 1



We next show that (iii)  $\frac{x_2-z}{x_1-z}$  is increasing in  $z$  for  $x_2 > x_1$ . We differentiate and obtain  $d\left(\frac{x_2-z}{x_1-z}\right)/dz = \frac{x_2-x_1}{(x_1-z)^2} > 0$ .

We rearrange (a) where we note that  $g'(x) < 0$  to get (iv)  $\frac{f'(x_1)}{g'(x_2)} \leq \frac{x_2}{x_1}$ . We use (i) and (ii) to rearrange (b) in the same manner:  $\frac{f'(x_1)}{g'(x_2)} < \frac{x_2-z}{x_1-z}$ . The last inequality holds due to (iii) and (iv), and confirms the result.  $\square$

We next identify two types of convex functions that satisfy (A.1). Hyperbolas that are not shifted by a constant term in any direction are almost one such class of functions.

**Lemma 3.** *Let  $f(x) = bx^{-c}$ , where  $b, c > 0$  are constants. Then*

$$f'(x_1)x_1 = g'(x_2)x_2. \quad (\text{A.2})$$

*Proof.* We invert  $f(x)$  and  $g(x)$  to get  $\varphi(y) \equiv f^{-1}(y) = \left(\frac{y}{b}\right)^{-\frac{1}{c}}$  and  $\gamma(y) \equiv g^{-1}(y) = \left(\frac{y}{ab}\right)^{-\frac{1}{c}}$ , respectively. Calculating  $\frac{\varphi(y)}{\varphi'(y)}$  and  $\frac{\gamma(y)}{\gamma'(y)}$ , which are equivalent to  $f'(x_1)x_1$  and  $g'(x_2)x_2$ , respectively, shows that each of these two equals  $-cy$ . This confirms the result.  $\square$

When we shift  $f(x) = bx^{-c}$  to the left or down, the (A.1) becomes satisfied, which is what we show next.

**Lemma 4.** *Let  $\bar{f}(x) = b(x+z)^{-c} - v$ , where  $z, v \geq 0$  with  $z+v > 0$ , and  $\bar{g}(x) = a\bar{f}(x)$ . Let  $\bar{x}_1$  and  $\bar{x}_2$  solve  $\bar{f}(\bar{x}_1) = y$  and  $\bar{g}(\bar{x}_2) = y$ , respectively. Then  $\bar{f}'(\bar{x}_1)\bar{x}_1 > \bar{g}'(\bar{x}_2)\bar{x}_2$ .*

*Proof.* Suppose for a moment that  $z = 0$  and denote all functions and variables with upper bars to have tilde marks above them instead. We invert  $\tilde{f}(x)$  and  $\tilde{g}(x)$  to get  $\tilde{\varphi}(y) \equiv \tilde{f}^{-1}(y) = \left(\frac{y+v}{b}\right)^{-\frac{1}{c}}$  and  $\tilde{\gamma}(y) \equiv \tilde{g}^{-1}(y) = \left(\frac{y+av}{ab}\right)^{-\frac{1}{c}}$ , respectively. We derive

$\frac{\tilde{\varphi}(y)}{\tilde{\varphi}'(y)} = -c(y+v)$  and  $\frac{\tilde{\gamma}(y)}{\tilde{\gamma}'(y)} = -c(y+av)$ . Since these two ratios are equivalent to  $\tilde{f}'(\tilde{x}_1)\tilde{x}_1$  and  $\tilde{g}'(\tilde{x}_2)\tilde{x}_2$ , respectively, we have confirmed the result in case of  $z = 0$ .

For  $v \geq 0$  and  $z = 0$ , we now know that (i)  $\tilde{f}'(\tilde{x}_1)\tilde{x}_1 \geq \tilde{g}'(\tilde{x}_2)\tilde{x}_2$ , where the equality part comes from (A.2) in case  $v = 0$ . Suppose now that  $z > 0$ , meaning that we are back at original functions and variables with upper bars. Then, with (i) and Lemma 2, we have the result.  $\square$

We look at another class of decreasing convex functions that take on positive values in the first quadrant.

**Lemma 5.** *Let  $f(x) = v - bx^c$  and  $g(x) = af(x)$ , where  $b, c, v > 0$ . Then  $f'(x_1)x_1 > g'(x_2)x_2$ .*

*Proof.* We invert  $f(x)$  and  $g(x)$  to get  $\varphi(y) \equiv f^{-1}(y) = \left(\frac{v-y}{b}\right)^{\frac{1}{c}}$  and  $\gamma(y) \equiv g^{-1}(y) = \left(\frac{av-y}{ab}\right)^{\frac{1}{c}}$ , respectively. We then derive  $\frac{\varphi(y)}{\varphi'(y)} = -c(v-y)$  and  $\frac{\gamma(y)}{\gamma'(y)} = -c(av-y)$ . These are equivalent to  $f'(x_1)x_1$  and  $g'(x_2)x_2$ , respectively. Since  $v$  is an intercept with  $y$  axes, we have  $v > y$  and our result follows.  $\square$

### **Proof of Proposition 7.**

*Claim 1.* *If  $L \leq L_m^*$  and  $w_m > w_c^*$ , then  $MRP_L^U(L) = aMRP_L^T(L)$ , where  $a > 1$  is a constant, and  $MRP_L^T(L)$  and  $MRP_L^U(L)$  are defined as left-hand sides of (4.10) and (4.11), respectively.*

*Proof.* Since  $w_m \geq w_c^*$ , we also have  $w_m > w(L_m^*)$  and  $\rho_m^*(L) = \text{const} < 1$  (see Proposition 6). The latter means that any increase in  $w_m$  leaves  $\rho_m^*$  unaffected and, consequently, auditing probability  $\alpha (= \text{const} < 1)$  does not change. This means that  $1 - (\alpha + (1 - \alpha)\rho_m^*)(\psi + \nu - \psi\nu) = \text{const} > 1 - \psi - \nu + \psi\nu$ , proving our claim.  $\square$

*Part 1 of Proposition 7.*  $MRP_L^T(L)$  and  $MRP_L^U(L)$  are both decreasing. Intersection of these two curves with the right-hand side of (4.11) yields respective optimal employment levels  $\bar{L}_m$  and  $L_m^*$ . A change in  $w_m$  traces out new respective optimal employment levels exactly from curves  $MRP_L^T(L)$  and  $MRP_L^U(L)$ . Due to Claim 1, we have  $L_m^* > \bar{L}_m$ . Using the latter and the assumption that  $MRP_L^T$  is concave, we use Lemma 1 to get  $\frac{\partial w_m}{\partial L_m^*} \bar{L}_m > \frac{\partial w_m}{\partial L_m^*} L_m^*$ . Divide the inequality by  $w_m$  to get  $\frac{1}{\eta^T} > \frac{1}{\eta^U}$ . Since  $MRP_L^T(L)$  is decreasing, both  $\frac{\partial L_m^*}{\partial w_m}$  and  $\frac{\partial \bar{L}_m}{\partial w_m}$  are negative and we obtain the result  $\eta^T < \eta^U < 0$ .

*Parts 2 (a) and (b) of Proposition 7.* In Lemmas 4 and 5, we compare expressions of the type  $f'(x)x$  at the same positive vertical level. To use those results, we need



to make the right-hand side of (4.11) constant which is achieved by  $\frac{\partial \mu(q(L))}{\partial L} = \text{const.}$  Using this constraint, we apply Lemma 4 for part 2 (a) and Lemma 5 for part 2 (b) to get  $\frac{\partial w_m}{\partial L_m} \bar{L}_m > \frac{\partial w_m}{\partial L_m^*} L_m^*$ . Again, divide the inequality by  $w_m$  to get  $\frac{1}{\eta^T} > \frac{1}{\eta^U}$  and rearrange to obtain  $\eta^T < \eta^U < 0$ .

*Part 3 of Proposition 7.* Since  $w_m \geq \bar{w}_c$ , the optimal employment for a truthfully reporting firm,  $\bar{L}_m$ , is determined by a decreasing  $MRP_L^T$ , hence  $\eta^T < 0$ . Since  $w(L^*) \leq w_m < w_c^*$ , there is no wage underreporting by an underreporting firm. A firm's optimal employment  $L_m^*$  is then determined from the labour supply with  $w_m = w(L_m^*)$ . Wage schedule  $w(L)$  is increasing, hence  $\eta^U > 0$ . This completes the proof of Proposition 7.