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The Impact of Training on Productivity and Wages Evidence from Belgian Firm Level Panel Data

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The Impact of Training on Productivity and Wages

Evidence from Belgian Firm Level Panel Data

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Abstract

This paper uses longitudinal data of more than 13,000 firms to analyze the effects of on-the-job training on firm level productivity and wages. Workers receiving training are on average more productive than workers not receiving training. This makes firms more productive. On-the-job training increases firm level measured productivity between 1 and 2%, compared to firms that do not provide training. The effect of training on wages is also positive, but much lower than the effect on productivity. Average wages increase only by 0.5%. Sectoral spillovers between firms that train workers are found, but only in firms active in the manufacturing sector. In non-manufacturing no spillovers seem to take place. The results are consistent with recent theories that explain on-the-job training, related to imperfect competition in the labor market, such as monopsony and union bargaining.

JEL: J01, J24, J42, M53

Key words: on-the-job-training, productivity, firm level data, monopsony

I. Introduction

In recent years trade unions, employers and policy makers in developed economies emphasize the importance of skill upgrading of workers and life long learning in order to cope with increased pressures induced by technological change and globalization. The OECD jobs strategy emphasizes the importance of not only general training through the education system, but also of on the job training for labor productivity growth and enhancing employment security. Apart from the benefits for individual earnings and employment prospects, human capital externalities to other workers may arise. In his influential paper, Lucas (1988) argues that such externalities can explain the long-run income differences between rich and poor countries. However, as shown by Becker (1964) and Mincer (1974) standard economic theory suggests that firms should not pay for on-the-job training if the prospects for the worker could be improved outside the firm. This prediction does not seem to fit the facts as pointed out by Acemoglu and Pischke (1998, 1999). They argue that wage compression at the firm level, explained by for instance imperfect information or union bargaining power is one of the reasons why firms provide on the-job-training. On-the-job training increases labor productivity more than wages, which creates ex post monopsony power. This encourages employers to provide on the job training, even if these skills are general. Since the firm is able to obtain part of the marginal product of the worker it also has an incentive to increase this marginal product by providing on-the-job training.

While there exists substantial evidence that general training increases wages¹ and productivity of workers, there is hardly any work that studies the impact of on the job training on firm level productivity and wages. Moretti (2004) focuses on plant level productivity gains from general training, but he has no data on firm specific training. He finds that plants operating in cities that experience a large increase in the share of college graduates have higher productivity gains than in cities that have a lower increase in college graduates, but these productivity gains are offset by wage increases. Bartel (1995) studies how on-the-job training affect wage profiles of workers and job performance scores in one large firm and finds that training has a

¹ Card (1999), for instance, summarizes various studies and concludes that the impact of a year of schooling on wages is about 10%.

positive effect. Dearden, Reed and Van Reenen (2005) analyze the link between training, wages and productivity at the sector level using a panel of British industries. They find that raising the proportion of workers in an industry who receive training by one percentage point increases value added per worker in the industry by 0.6% and average wages by 0.3%.

An important innovation of the present paper is the use of firm level longitudinal data with information at the firm level of on-the-job training. This gives a number of advantages over previous studies. First, not only the direct impact of on the job training on firm level productivity and wages can be analyzed, but also training spillovers between firms operating in the same sector can be estimated. Such spillovers could matter if there is mobility of workers between firms, either voluntarily or forced mobility due to e.g. restructuring. Second, there is a lot of heterogeneity between firms in terms of productivity, which can be taken into account in the analysis. In particular, I analyze whether the impact of firm-specific training is different for laggard firms than for frontier ones, the latter operating close to the efficiency frontier. Third, an analysis at the firm level allows me to control for the endogeneity of on-the-job training or selection effects, which is harder at the industry or aggregate level.

The data that I use are based on the income and loss statements of Belgian firms that are traced between 1997 and 2006. The data include information on total employment, value added, wages and other financial data, but more importantly, Belgian firms have to submit by law a supplement to the financial statements, which is called the social account. This includes information on various elements of firm specific training, such as the proportion of workers that took training, the number of hours that training was taken, the cost price for the firm, etc..

I find evidence that on the job training has a positive impact on labor productivity and wages workers earn. Furthermore, I find that the impact of on the job training on productivity is larger than on wages, which is consistent with imperfect competition, such as monopsony, on the labor market.

The next section provides an empirical model. In section III I discuss the data and some basic facts that emerge from them. Section IV gives the results of the econometric analysis and section V concludes the paper.

II. Empirical Model

I follow Dearden et al (2005) and assume that a firm produces output, Q , according to a Cobb-Douglas production technology, or

$$Q = AL^\alpha K^\beta \quad (1)$$

With L the total labor force, K the capital stock in the firm. Furthermore, the total labor force in the firm consists of trained (L^T) and non-trained workers (L^{NT}), or

$$L = L^T + \gamma L^{NT} \quad (2)$$

If non-trained workers are more productive than trained than the parameter γ will be larger than 1. Substituting (2) in (1) gives,

$$Q = A(1+(\gamma-1)TRAIN)^\alpha L^\alpha K^\beta \quad (3)$$

With $TRAIN=L^T/L$.

Using small case letters to indicate logarithms I get

$$q = a + \alpha \ln(1+(\gamma-1)TRAIN) + \alpha l + \beta k \quad (4)$$

Which approximately equivalent to²

$$q = a + \alpha(\gamma-1)TRAIN + \alpha l + \beta k \quad (5)$$

Assuming constant returns to scale, adding a white noise error term, time and firm specific subscripts gives

² This holds when $(\gamma-1)TRAIN$ is small, as $\ln(1+x) = x$

$$q_{it}-l_{it} = a_i + \alpha(\gamma-1)TRAIN_{it} + \beta(k_{it}-l_{it}) + \varepsilon_{it} \quad (6)$$

Note that in (6) firm specific effects are included. These proxy for firm specific unobservables that stay constant over time within the same firm. In particular, such firm specific effects can control for potential selection effects that stay constant over time. For instance, if the most productivity firms engage in training programs, such firm specific effects can capture the inverse Mills ratio that would be associated with first estimating a selection model. Also differences between different types of firms, such as high-tech versus low-tech firms, are captured by these unobserved firm specific effects. Obviously if selection effects do not stay constant over time, such firm specific effects will not capture this entirely. Therefore I will also report estimates where the potential endogeneity of the labor input and the proportion of workers that receive training are taken into account. Rather than trying to pin down instrumental variables, which are often very difficult to find, I use the approach proposed by Olley-Pakes (1996) to control for the simultaneity between productivity shocks and the choice of input factors.

Equation (6) will be augmented with a number of variables. In particular, I will analyze whether apart from the direct effect of training there exist training spillover effects from other firms. To this end, I construct a measure of training intensity at the 2-digit NACE sector level. In addition, I will analyze whether firms that are operating far from the efficiency frontier benefit more from training than firms that operate close to the efficiency frontier. Recent work by Aghion et al (2006) shows that modeling firm heterogeneity is important for understanding different responses of firms to economic shocks. It can be expected that laggard firms have a higher incentive to set up training programs as laggard firms gain most from imitating technologies and best practices from frontier firms.

III. Data

I use a unique panel data set of about 13,000 Belgian firms operating in both the manufacturing and the non-manufacturing sectors that are traced between 1997 and 2005. The data include in addition to the financial statements of firms also information on training programs that the firm is engaged in. In particular, there is

information on the number and gender of workers that are being trained, the average duration of training and the cost price of training. In table 1 summary statistics are given. About one third of all firms in the sample provide training on the job. Of those, on average 42% of all workers receive such training, which lasts on average 33 hours or about one working week. From table 1 it is also clear that firms that provide training are typically larger in terms of sales and employment and have on average higher labor costs and higher value added. It is remarkable that the capital intensity, proxied by tangible fixed assets relative to employment in the firm, is higher in firms that provide no training. However, this is due to the fact that more firms operating in the non-manufacturing sector provide training relative to firms in manufacturing, 43% respectively 27%, and in non-manufacturing firms are typically less capital intensive.

In our empirical analysis I also want to assess the importance of sectoral training spillovers between firms. Typically, if there is worker and firm turnover, training gained in one firm should have beneficial effects on the performance of other firms. I measure training spillovers between firms operating in the same sector by the proportion of workers receiving training in a particular 2-digit sector relative to total employment. Similar measures have been used to trace the effects of technological spillovers from foreign direct investment to domestic firms, where the fraction of total employment in foreign firms in a particular sector relative to total employment of that sector has been used as a measure of horizontal spillovers³.

Finally, as I want to analyze the impact of training taking into account firm heterogeneity I define heterogeneity in terms of differences in labor productivity between firms. In particular, I define the average technological distance to the frontier firm in the beginning of the sample period, t_0 , in our case 1997, as the labor productivity of a firm relative to the maximum observed labor productivity in the sample or

$$Distance = labor\ productivity_{it} / \max(labor\ productivity_{t0}) \quad (7)$$

The average distance in the sample is 60%, which means that the average firms is 40% less efficient than the best firm.

³ e.g. Aitkin and Harrison (1999)

Table 1: Summary Statistics

| | Firms with training (32% of the sample) | Firms with no training (68% of the sample) |
|---|--|---|
| Sales | 45,006 (245,736) | 16,298 (54,348) |
| Value Added | 10,203 (20,886) | 2,945 (9,504) |
| Employment | 126 (291) | 42 (161) |
| Labor cost per worker | 51 (20) | 49 (19) |
| Capital-Labor Ratio | 97 (455) | 159 (866) |
| | Sample of firms with training | |
| Proportion of workers who receive Training | 0.42 | |
| Of which men | 0.31 | |
| Of which women | 0.11 | |
| Training sector spillover | 0.26 (0.09) | |
| Cost of training per worker receiving training | 1.25 (2.65) | |
| Number of hours per worker receiving training | 33 (65) | |

Note: nominal values in 1000 of Euros, standard deviations in brackets.

IV. Results

IV.1. Effects on Productivity and Wages

Table 2 shows the results of estimating (6). All equations include year effects and firm level fixed effects. Fixed effects estimation is equivalent to a difference-in-difference estimator, which controls for potential selection effects. Year effects control for aggregate shocks, such as aggregate price movements and the business cycle. In the final column also a full set of year sector interaction dummies is included to control for different unobserved sector effects over the business cycle, these can capture for instance sector specific price movements. The first three columns of table 2 show the effect of training without taking into account firm heterogeneity in terms of distance to the frontier firm. Based on column (2) where the capital labor ratio is taken into account, I find a positive and significant effect of training on labor productivity. The coefficient of 0.023 implies that the parameter γ in equation(2) is equal to 1.024^4 and since it is larger than 1 it implies that workers who received

⁴ This is found by dividing 0.023 by (1-0.076), which is equal to $\gamma-1$.

training or on average more productive than workers who did not so, in particular, they are 2.4% more productive on average. In column (3) I test whether there exist sector spillovers from training. The coefficient is estimated positive, but not statistically significant. Finally, in column (4) I analyze whether firms operating far from the efficiency frontier benefit more from firm-specific training than frontier firms. The negative and significant coefficient estimated on the interaction term of distance with training indicates that training pays off more for laggard firms than for frontier firms.

These results are consistent with the model of Acemoglu and Pischke (1998,1999), who explain the provision of firm specific training by the monopsony power it generates from providing this training. Training increases productivity of workers and gives the firm monopsony rents, i.e. workers earn less than their actual productivity. If the monopsony explanation of providing firm specific training holds, then the average wage increase due to training should be lower than the average productivity increase. Also, when there is wage compression in firms, the average wage increase will be limited.

Table 2: Effect of Training on Labor Productivity

| | (1) | (2) | (3) | (4) |
|---------------------------|---------------------|---------------------|---------------------|----------------------|
| Proportion Training | 0.027*** (0.007) | 0.023*** (0.007) | 0.023*** (0.087) | 0.235*** (0.087) |
| ln(K/L) | - | 0.076*** (0.003) | 0.076*** (0.004) | 0.076*** (0.004) |
| Training Sector spillover | - | - | 0.006 (0.035) | -0.102 (0.331) |
| DistanceXTraining | - | - | - | -0.343*** (0.147) |
| Year dummies | Yes | Yes | Yes | Yes |
| Year X Sector dummies | No | No | No | Yes |
| Firm Fixed effects | Yes | Yes | Yes | Yes |
| Adjusted R ² | 0.76 | 0.77 | 0.77 | 0.77 |
| # observations | 92,863 | 92,863 | 92,863 | 70,074 |

Note: robust standard errors in brackets, ***/** means significant at the 1%/ 5% level or below.

In table 3, I report results of estimating a wage equation as a function of the proportion of workers receiving training. While the effect of training on average wages at the firm level is estimated positive and statistically significant, its effect is much lower than the effect on average productivity. This is consistent the monopsony

explanation of providing firm training. Under perfect competition the increase in labor productivity should be passed on the workers by the same amount. In table 3 the coefficient on the proportion of training is 0.01, while in table 2 – the estimates of productivity – it is double this number, 0.02. Suppose that a firm that switches from no training at all to a level of training that the average firm offers its employees, or based on the summary statistics 40% of all employees receive training in the average firm that offers training. In this case average productivity increases by 1%, while average wages only by half a percent.

In terms of sector spillovers from training (column 3), wages seems to respond negatively on average. This means that in sectors where there is more training average wages are lower than in sectors where there is less training. Again, this can be explained by monopsony. In particular, if sectors that provide training are characterized by monopsony, while sectors that do not provide training are characterized by perfect competition, average wages should be lower in the former than in the latter. In the final column of table 3 the effect of laggard versus frontier firms on wages is reported. While a similar result is found as for the productivity effects, the effect of firm heterogeneity is estimated lower on wages than on productivity. This also indicates that wages are more compressed than productivity.

Table 3: Effect of Training on Log Wages

| | (1) | (2) | (3) | (4) |
|---------------------------|---------------------|---------------------|---------------------|---------------------|
| Proportion Training | 0.010*** (0.003) | 0.008*** (0.003) | 0.009*** (0.003) | 0.068** (0.030) |
| ln(K/L) | - | 0.020*** (0.001) | 0.020*** (0.001) | 0.018*** (0.001) |
| Training Sector spillover | - | - | -0.03** (0.016) | -0.054 (0.15) |
| DistanceXTraining | - | - | - | -0.098** (0.05) |
| Year dummies | Yes | Yes | Yes | Yes |
| Year X Sector dummies | No | No | No | Yes |
| Firm Fixed effects | Yes | Yes | Yes | Yes |
| Adjusted R ² | 0.85 | 0.85 | 0.85 | 0.86 |
| # observations | 92,863 | 92,863 | 92,863 | 70,074 |

Notes: as in table 2

Table 4 reports both effects on labor productivity and on wages for the manufacturing sector and the non-manufacturing sector separately. The type of training and the production process is arguably different in manufacturing firms

compared to non-manufacturing firms and so could be the effect of training. It is clear that the effect of training on measured labor productivity is larger in the non-manufacturing than in the manufacturing sector. However, there are positive sector spillovers from training to other firms in the manufacturing sector, while no statistically significant effect can be found in firms operating in the non-manufacturing sector. The pattern reported earlier for wages also persists for the manufacturing and non-manufacturing sector separately, i.e. the effect on wages is estimated much lower, which suggests imperfect competition in the labor market.

Table 4: Effect of Training on Labor Productivity and Wages
Manufacturing versus Non-Manufacturing

| | Labor Productivity | | Wages | |
|---------------------------|---------------------|---------------------|---------------------|---------------------|
| | Manufacturing | Non-Manufacturing | Manufacturing | Non-Manufacturing |
| Proportion Training | 0.020** (0.009) | 0.034*** (0.012) | 0.012*** (0.004) | 0.004 (0.005) |
| ln(K/L) | 0.074*** (0.004) | 0.086*** (0.008) | 0.019*** (0.001) | 0.021*** (0.003) |
| Training Sector spillover | 0.090** (0.044) | 0.059 (0.059) | -0.030 (0.021) | -0.025 (0.027) |
| Year dummies | Yes | Yes | Yes | Yes |
| Firm Fixed effects | Yes | Yes | Yes | Yes |
| Adjusted R ² | 0.77 | 0.74 | 0.85 | 0.81 |
| # observations | 67,949 | 24,914 | 67,949 | 24,914 |

Notes: as in table 2

IV.2. Robustness

In this section I report a number of robustness checks. In particular, both labor productivity and wages may be characterized by serial correlation. Furthermore, the measure of labor productivity does not take into account the productivity of other input factors used in the production process. In addition, there is also a potential simultaneity bias between the choice of the input factors and the productivity shock, which can result in biased estimates. I will therefore report results based on estimating total factor productivity. In doing so, I also drop the assumption of constant returns to scale which was an extra restriction imposed earlier.

In the first two columns of table 5 I report results where an AR(1) process is taken into account in estimating the effects on labor productivity and wages. Because

of the AR(1) process there is a direct (short run) effect of training and a long run effect of training. The direct effect of training on productivity and wages is estimated positive and statistically significant and is equal to 0.01 and 0.009 respectively. The long run effect is very close to the results reported earlier, 0.02 and 0.01 respectively⁵. In the three last columns of table 5 I report the results of estimating total factor productivity (TFP) using the Olley-Pakes (1996) approach for estimating TFP. As we now know, this approach uses the investment function, which is monotonically increasing in the productivity shock and therefore it can be inverted to write productivity as a function of investment and the capital stock. This allows me to substitute the productivity shock by the investment function, which allows consistent estimation of the labor input, including the proportion of training. The appendix provides in detail the Olley-Pakes (OP) algorithm. It is clear that the effect of training in both the manufacturing and the non-manufacturing sector is estimated positive and statistically significant. In fact, the effect is now estimated at 3.5% for the overall sample, which is larger than the effect when just labor productivity is used. For the manufacturing sector positive spillover effects are found, but for non-manufacturing no statistically significant effect is found, although the effect of spillovers is estimated positive.

Table 5: Different estimation methods

| | Labor productivity AR(1) | Wages AR(1) | TFP O-P | TFP O-P Manufacturing | TFP O-P Non-Manufacturing |
|---------------------------|--------------------------|----------------------|---------------------|-----------------------|---------------------------|
| Proportion Training | 0.012** (0.006) | 0.009*** (0.002) | 0.034*** (0.008) | 0.035*** (0.010) | 0.038*** (0.014) |
| ln(K/L) | 0.074*** (0.002) | 0.025*** (0.0009) | - | - | - |
| Training sector spillover | 0.053* (0.029) | 0.017 (0.013) | -0.012 (0.041) | 0.11** (0.05) | 0.056 (0.062) |
| Year dummies | Yes | Yes | Yes | Yes | Yes |
| Fixed effects | Yes | Yes | Yes | Yes | Yes |
| Rho | 0.44 | 0.41 | - | - | - |
| # obs. | 79,986 | 79,986 | 55,882 | 40,056 | 15,826 |

Notes: as in table 2

⁵ 0.012/(1-0.44) and 0.009/(1-0.41)

V. Conclusions

This paper uses firm level longitudinal data to analyze the effects of on-the-job training on firm level productivity and wages. In particular, information of the income and loss statements of 13,000 Belgian firms is combined with information of the proportion on workers in the firm receiving training to assess whether workers receiving training have higher productivity and wages. I find evidence for both.

About a third of the firms in the sample provide on-the-job training. Of those on average 42% of the workers receive training. The average training is 33 hours and the average cost of training is 1,250 Euro. Workers receiving training are on average 2.4% more productivity than workers not receiving training in the same firm. This yields a higher average productivity of the firm which varies between 1 and 2%, depending on the sector the firm is operating in and the methodology used. Since the average labor productivity or value added per worker in firms that provide training is 80,000 Euro in the sample, this implies a gain of between 800 and 1,600, which is comparable to the average cost of training of 1,250 Euro. The effect of training on wages is also positive, but much lower than the effect on productivity. Average wages increase only by 0.5%. Furthermore, firm heterogeneity seems to matter. Laggard firms benefit more from on-the-job training than frontier firms.

The results are consistent with imperfect competition in the labor market, such as a monopsony explanation for on-the-job training as proposed by Acemoglu and Pischke (1998). Training provides monopsony power to the firm, it increases the labor productivity of workers, but the latter cannot extract all the rents. So, the wage increase must be lower than the productivity increase.

Finally, the results in this paper suggest that on-the-job training is both in the interest of firms and workers, which is beneficial for general welfare.

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Appendix: Olley-Pakes (1995) Algorithm

In the O-P estimation procedure the unobservable productivity shock ω can be identified using an observable investment function, $i_t = I_t(k_t, \omega_t)$ that is monotonically increasing in ω and the state variable capital k . By inverting the investment function an expression for productivity can be written as an unknown function h of investment and capital ($\omega_{it} = h_t(i_{it}, k_{it})$). This implies that the production function can be rewritten like this (y stands for log output, l for log employment)

$$y_{it} = \beta_l l_{it} + \phi_t(i_{it}, k_{it}) + \eta_{it}$$

and

$$\phi_t = \beta_0 + \beta_k k_{it} + h_t(i_{it}, k_{it})$$

The above expression can be estimated semi-parametrically to obtain a consistent estimate of the coefficient on labor.⁶

In the second step of the procedure, information is used on firm dynamics to obtain a consistent estimate of the capital coefficient. In particular, it is assumed that productivity ω , follows a first order Markov process g , i.e. $\omega_{t+1} = E(\omega_{t+1} | \omega_t) + \xi_{t+1}$ where ξ_{t+1} represents the news in the process and is assumed to be uncorrelated with the productivity shock and with the capital input at $t+1$ (k_{t+1}). Capital used in any given period $t+1$, is assumed to be known and fixed at the beginning of that period. News arriving at $t+1$ is therefore uncorrelated with capital $E(\xi_{t+1} k) = 0$. However, the news is not uncorrelated with the variable input (labor). For this reason the labor input is subtracted from the production and we consider the expectation of $E(y_{t+1} - \beta_l l_{t+1})$ conditional on the survival of the firm. A firm's probability of survival P_t (with $P_t = \Pr\{\chi_{t+1} = 1\}$) into the next period depends on whether its efficiency level exceeds a critical productivity level ($\chi_{t+1}=1$ if $\omega_{t+1} > \underline{\omega}_{t+1}$ and 0 otherwise). All this results in the following expression

$$\begin{aligned} E[y_{t+1} - \beta_l l_{t+1} | k_{t+1}, \chi_{t+1} = 1] &= \beta_0 + \beta_k k_{it+1} + E[\omega_{t+1} | \omega_t, \chi_{t+1} = 1] \\ &= \beta_k k_{it+1} + g(\underline{\omega}_{t+1}, \omega_t) \end{aligned}$$

Using the above and using the law of motion for the productivity shocks, we get

$$\begin{aligned} y_{it+1} - \beta_l l_{it+1} &= \beta_0 + \beta_k k_{it+1} + E(\omega_{it+1} | \omega_{it}, \chi_{t+1} = 1) + \xi_{it+1} + \eta_{it+1} \\ &= \beta_k k_{it+1} + g(\underline{\omega}_{it+1}, \omega_{it}) + \xi_{it+1} + \eta_{it+1} \\ &= \beta_k k_{it+1} + g(P_{t+1}, \phi_t - \beta_k k_{it}) + \xi_{it+1} + \eta_{it+1} \end{aligned}$$

The final step in the Olley and Pakes correction method, is to arrive at a consistent estimate of the capital coefficient. The coefficient on capital is obtained by minimizing the sum of squares of the residuals in the equation below, thereby taking the first stage estimates of β_l and ϕ_t and the estimated probability of survival P_t and

⁶ We proxy $\phi_t(i_{it}, k_{it})$ with a 5th order polynomial in investment and capital and included time dummies to control for aggregate shocks in investment.

substituting them for the true values.

$$y_{t+1} - \hat{\beta}_t l_{t+1} = c + \beta_k k_{t+1} + \sum_{j=0}^{s-m} \sum_{m=0}^s \beta_{mj} (\hat{\phi}_t - \beta_k k_t)^m \hat{P}_{t+1}^j + e_{t+1}$$

where s denotes the order of the polynomial used to estimate the coefficient on capital. We use bootstrapping methods to come up with the correct standard errors for the series estimator of the capital coefficient.