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Skating on Thin Ice:
Rule Changes and Team Strategies in the NHL. ¹

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Abstract

In an effort to stimulate a more exciting and entertaining style of play, the National Hockey Association (NHL) changed the rewards associated with the results of overtime games. Under the new rules, teams tied at the end of regulation both receive a single point regardless of the outcome in overtime. A team scoring in the sudden-death 5-minute overtime period would earn an additional point. Prior to the rule change in the 1999-2000 season, the team losing in overtime would receive no points while the winning team earned 2 points. This paper presents a theoretical model to explain the effect of the rule change on the strategy of play during both the overtime period and the regulation time game. The results suggest that under the new overtime format equally powerful teams will play more offensively in overtime resulting in more games decided by a sudden-death goal. The results also suggest that while increasing the likelihood of attacking in overtime, the rule change would have a perverse effect on the style of play during regulation by causing them to play conservatively for the tie. Empirical data confirm the theoretical results. The paper also shows that increasing the rewards to a win in regulation time would prevent teams from playing defensively during regular time.

JEL Classification C72, L83

Keywords: Ice Hockey, Game Theory, NHL Overtime Rule.
1 Introduction

Economic studies on the influence of rules, institutions and incentive structures on behavior are complicated by the fact that incentives and observations on behavior are affected by a variety of factors outside the relationship of interest. For this reason, environments which eliminate some of these outside influences are of particular interest. Some recent studies have used observed data from sporting competitions to assess the effects of incentives. Rule changes in sports provide a unique natural experiment to test the consequences of theoretical payoff structures. For example, the performance of individual competitors has been shown to increase directly with the level of rewards in the sports ranging from bowling (Ehrenberg and Bognanno 1990) to auto racing (Maloney and Terkun, 2002). In terms of team sports, Banerjee and Swinnen (2003) use a game theoretic model along with empirical evidence to show that the introduction of a sudden death rule in soccer has not changed the style of play despite FIFA’s reasoning of introducing a "golden goal" for more aggressive play during extra time. Similar rule changes were recently introduced in professional ice hockey.

Despite the inherent attraction associated with the speed and action of ice hockey, the popularity in North America of the game’s premier professional league, the National Hockey League (NHL), lags behind that of the other three major professional sports leagues, the National Football League (NFL), Major League Baseball (MLB), and the National Basketball Association (NBA). The NHL has attempted to increase the general appeal of the game by introducing several rule changes ranging from tougher fighting penalties to increasing the size of the area behind the nets. One of the most significant changes was the introduction in the 1983-84 season of a 5 minute, sudden-death overtime period to settle any regular season games that had ended in a tie. A team scoring in overtime would receive 2 points for the win while the losing team got 0 points for the loss. Each team received a single point if the game remained tied at the end of overtime. In an effort to combat the perceived conservative play in overtime, the NHL implemented a new point structure in the 1999-2000 season. Teams tied at the end of regulation would both receive a single point regardless of the outcome in overtime. A team scoring in the sudden-death overtime would earn an additional point. While the intent of the rule change was to increase the excitement of the game through a more attacking style of play in the overtime, the change in the reward system could have a perverse and opposite effect on the style of play.

The purpose of this paper is to determine the effect of changes in overtime rules on the play by NHL teams during regulation time and in overtime. The paper begins with a game theoretic model that determines optimal team strategies under alternative overtime point systems. The theoretical results suggest that rule changes should result in more overtime games being decided within the extra-time period but that more games will end
up tied after the normal 60 minute regulation time. Empirical results are then presented confirming the hypotheses in section 3. Implications of other rule changes are assessed in the conclusions.

2 Payoff Structures

The time points are defined such that teams can score a maximum of one goal during each time point.\(^1\) We begin by breaking the game involving teams A and B into an arbitrary set of \(T\) discrete time points \(N = \{1, \ldots, t, \ldots, T\}\). For example, \(t\) may represent a minute of play in a hockey game.

Consider the game from the point of view of team A. At every \(t\), the state of the game for team A is described by the random state vector \(X_t\), where

\[
X_t = \begin{cases} 
1, & \text{if team A scores a goal, at time } t, \\
0, & \text{if neither team scores a goal, at time } t, \\
-1, & \text{if team B scores a goal, at time } t.
\end{cases}
\]

The probabilities of this random variable are given by

\[
\Pr(X_t = 1) = p, \quad \Pr(X_t = -1) = q \quad \text{and} \quad \Pr(X_t = 0) = r = 1 - p - q.
\]

The probability of team A scoring a goal is the same as the probability of team B conceding a goal. This implies that the state vector of team B is \(-X_t\). At every instant \(t\) there is a zero sum game between A and B implying that both cannot score during the same time point. The probabilities \(p\) and \(q\) are functions of the actions of the teams.

Each team decides before the start of each time point on their playing strategy. We assume the strategies are defined in terms of two actions, defensive play, denoted by \(L\), and offensive play, denoted by \(H\). We define the actions sets of the two teams at a given moment \(t\) as \(S^A_t\) and \(S^B_t\), where \(S^i_t = \{H, L\}, i = A, B\).

The probability functions \(p\) and \(q\) are defined as

\[
p : S^A_t \times S^B_t \rightarrow [0,1] \quad \text{and} \quad q : S^A_t \times S^B_t \rightarrow [0,1].
\]

We assume that the probability of scoring is higher with offensive play; formally:

\[
p(H, .) > p(L, .) \quad \text{and} \quad q(., H) > q(., L) \quad (2.1)
\]

Further throughout the paper, we shall assume that the teams involved are equally likely to score under identical situations and similar styles of play. This is a reasonable assumption given the fact that a game is more likely to go to extra times if the teams are of similar qualities (a similar assumption is made by Palomino et al. (1999) and Banerjee and Swinnen (2004)). Formally:

\(^1\)In hockey, there are three stop-time periods of 20 minutes in a regulation game and a 5 minute sudden-death overtime period if the game is tied after 60 minutes of regulation play.
Definition 1  Teams $A$ and $B$ are defined to be equally powerful if and only if
\[ p(s^A, s^B) = q(s^B, s^A) \]  
for all $(s^A, s^B) \in S^A \times S^B$.

Throughout our paper we shall assume that the teams are equally powerful.

3  Optimal Overtime Strategies

The framework defined above is applied initially to the overtime period to determine how a changing reward system affects play in overtime. Overtime is the 5 minute additional period NHL teams play at the end of a regular season game when the score is tied. The game may end before completion of the overtime period if a team scores a goal. The team that scores first in overtime wins the game. Since its introduction to the NHL in 1982, the length of overtime and its sudden-death format have remained unchanged. However, the rewards associated with overtime play were changed in the 1999-2000 season as described further below.

The sudden-death nature of overtime can be modeled as a stochastic process $W_t$ defined in terms of a goal difference at time $t$. Initially, the goal difference is zero, $W_0 = 0$. If team A (B) scores during the next time interval, then $W_t = 1$ ($W_t = -1$) and team A (B) wins the game with no chance for the other team to come back and score. The team with a positive goal difference is the winner. Thus, $W_t$ is similar to a random walk with absorbing barriers at 1 and $-1$. The transition probabilities associated with this stochastic process are given by:

\[ \Pr(W_t = d' | W_{t-1} = d) = \begin{cases} 
1 & \text{if } d' = d \neq 0, \\
p & \text{if } d' = +1 \text{ and } d = 0, \\
r & \text{if } d' = d = 0, \\
q & \text{if } d' = -1 \text{ and } d = 0, \\
0 & \text{otherwise.} 
\end{cases} \]

with the initial condition of $\Pr(W_0 = 0) = 1$.

A strategy of a team (A or B) at period $t$ depends on the history of the game upto period $t - 1$. Note that the history of the game at period $t$ is the outcome of the random variable $W_{t-1} = d$.

3.1  The old overtime rule

From the introduction of overtime to the NHL in 1982-1983 to the completion of the 1998-1999 season, the team scoring in overtime received two points for the win while the
losing team received none. Both teams earned a single point if nobody scored and the game was still tied after the overtime period. We refer to this payoff system as the "old rule".

We can define the incentive scheme $U^{\text{old}}_A$ for team A as,

$$U^{\text{old}}_A = I \{ W_T > 0 \} - I \{ W_T < 0 \}$$ (3.3)

where $I \{ . \}$ is the indicator function. Team B's end of play payoff will be

$$U^{\text{old}}_B = -U^{\text{old}}_A.$$

If team A at the end of the game has more goals than team B, then team A wins (receives 1) and otherwise team B wins (team A gets -1). The teams get nothing when the goal difference is zero. Since the strategies will only depend on the differences in payoff, this normalisation does not matter.

We derive the equilibrium strategy sequences by solving the tree of subgames backwards for every $t$. Assume for a moment that there is only one period (This is equivalent to assuming that there is a draw at the beginning of the last period). Let team A chose strategy, $s^A$ and team B $s^B$. Then with the assumptions of equality of teams the expected payoff matrix (of Team A) under the old rule is:

<table>
<thead>
<tr>
<th>$s^B \setminus s^A$</th>
<th>$H$</th>
<th>$L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H$</td>
<td>$0$</td>
<td>$-(p(H, L) - p(L, H))$</td>
</tr>
<tr>
<td>$L$</td>
<td>$p(H, L) - p(L, H)$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

(3.4)

where the columns (rows) represent the possible strategy of team A (B).

From the above matrix it is clear that if $p(H, L) > p(L, H)$, then team A will not be better off deviating from $H$ if team B plays $H$. Therefore $(H, H)$ is the equilibrium. If $p(H, L) < p(L, H)$, then the last row and the last column dominates and $(L, L)$ is the equilibrium.

Denote $p(H, L) - p(L, H) = \alpha$. The equilibrium therefore depends on the sign of $\alpha$. We refer to the value of $\alpha$ as the comparative advantage of team A.

**Definition 2** Team A has a comparative advantage in playing offensive hockey if and only if

$$p(H, L) > p(L, H)$$ (3.5)

A team has a comparative advantage in playing offensive hockey if the team is more likely to score playing an offensive strategy against a defending team of equal quality compared to when it plays defensively against an equal quality team playing offensively. The previous analysis implies that if team A has a comparative advantage in playing offensive ($\alpha > 0$), then so would team B because both are equal. Thus, playing offensive
is the optimal strategy for both teams in all time periods. This also holds for general
$T$ – period strategies of the overtime game.

Now let us extend this to more periods. Let

$$V_{old_{i,d}, t}^i, i = A \text{ or } B$$

be the value of the game at period $t$ such that the history $W_{t-1} = d$. Notice that the only
history which is relevant for our analysis is when the teams are tied, i.e. $d = 0$, otherwise
the game ends. We can recursively write the value function as

$$V_{old_{t-1,0}, t}^A = \max_{s^A} r(s^A_t, s^B_t) V_{old_{t,0}}^A + p(s^A_t, s^B_t) - q(s^A_t, s^B_t)$$

$$V_{old_{t-1,1}, t}^A = 1 = -V_{old_{t-1,1}}^A$$

and

$$V_{old_{t-1,0}, t}^B = \max_{s^B} r(s^A_t, s^B_t) V_{old_{t,0}}^B + q(s^A_t, s^B_t) - p(s^A_t, s^B_t)$$

$$V_{old_{t-1,1}, t}^B = -1 = -V_{old_{t-1,1}}^B$$

Finally $V_{old_{T+1,d}, T}^i = U_{old_{T}^i}$.

Clearly from (3.4) at period $T$, if $\alpha > 0$ the teams will play $(H, H)$, therefore $V_{old_{T,0}}^A = 0$.
Now solving backwards, assume that at period $t$ whenever the game is drawn ($W_{t-1} = 0$)
the teams play their comparative advantage that is $(H, H)$, and therefore $V_{old_{t,0}}^A = 0 =
V_{old_{t,0}}^B$. So the game at period $t - 1$ can be represented by the same matrix as in (3.4).
Hence it follows that at period $t - 1$ the teams also play their comparative advantage, that
is $(H, H)$, when the score is tied at period $t - 2$ i.e. $W_{t-2} = 0$. Notice that the value of
the game at start of the overtime is given by $V_{old_{0,0}}^A = 0$. Therefore the teams will always
play their comparative advantage.

Similarly if the comparative advantage of the teams are to play defensively ($\alpha < 0$)
they will always play defensively. This implies that even if the teams are given the freedom
to change their style of play as many times as they like, they will always chose the style
which is consistent with their comparative advantage.

These conclusions are summarised in the theorem below.

**Theorem 1** Under the old rule and assuming the teams are of equal quality,

a) if teams have a comparative advantage in playing offensive hockey ($\alpha > 0$) then
$(H, H)$ is the only equilibrium;

b) if teams have a comparative advantage in playing defensive hockey ($\alpha < 0$), then
$(L, L)$ is the only equilibrium,

where $H$ is a $T$-vector of $H$’s and $L$ is a $T$-vector of $L$’s.

c) The value of the game is zero, $V_{old_{0,0}}^A = 0$. 

5
3.2 The new overtime rule

In the 1999-2000 season, the NHL introduced a new reward system for overtime games. Under the new rule (also called Rule 89), both teams receive one point if they draw in regulation regardless of the overtime result. A team that wins the game in overtime gets an additional point, hence two in total, as in the old rule. However, the team that loses in overtime still keeps its one point—unlike the old overtime rule.

The new incentive scheme is no longer a zero sum game. If team A wins in overtime, team B receives the same reward as if the game was tied. In terms of the notation defined above, team A receives 1 point and team B earns 0 points which is what both teams would get if the game remained tied after overtime. Again, since the strategies will only depend on the differences in payoff, this normalisation does not matter.

Define the end of play payoff $U_{new}^A$ of team A under the new rule as,

$$U_{new}^A = I \{W_T > 0\}$$

where $I \{\cdot\}$ is the indicator function.

Team B’s end of play payoff will be,

$$U_{new}^B = I \{W_T < 0\}$$

The intended effect of the change in the overtime incentive scheme was to encourage offensive play. To determine the effects, we evaluate it from the point of view of Team A. Since Team B’s utility function is symmetric, it will behave the same way as team A.

With the payoff structures associated with the overtime point systems established above, we can now define the strategies of play that will maximize a team’s expected payoff. To derive the equilibrium strategy sequences, we assume as in the old overtime game, that the teams involved are equally likely to score under similar situations and similar styles of play (2.2). Note that this is not a zero sum game anymore: for both teams the payoffs are always positive.

Let $V_{new,t,d}, (i = A \text{ or } B)$ be the value of the game at period $t$ such that the history $W_{t-1} = d$. As before the only history which is relevant is when the teams are tied, i.e. $W_{t-1} = d = 0$, otherwise the game ends. We can recursively write the value function of Team A as

$$V_{new,t-1,0}^A = \max_{s_t^A} r(s_t^A, s_t^B) V_{new,t,0}^A + p(s_t^A, s_t^B)$$

$$V_{new,t-1,1}^A = 1$$

$$V_{new,t-1,-1}^A = 0.$$
Similarly, for Team B as

\[ V_{new}^{B, t-1, 0} = \max_{s_t^B} r(s_t^A, s_t^B) V_{new}^{B, t, 0} + q(s_t^A, s_t^B) \]

\[ V_{new}^{A, t-1, 1} = 1 \]

\[ V_{new}^{A, t-1, -1} = 0 \]

Finally \( V_{new}^{i, T+1, 0} = U_{new}^{i} \).

As before, consider first that there is only one period (i.e. the situation at the start of the last period) in a game. Let team A chose strategy, \( s^A \) and team B \( s^B \). Then with the assumption of equality of teams, the expected payoff matrix (Team A and Team B) under the new rule is:

<table>
<thead>
<tr>
<th>( s^B )</th>
<th>( H )</th>
<th>( L )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H )</td>
<td>((p(H, H), p(H, H)))</td>
<td>((p(L, H), p(H, L)))</td>
</tr>
<tr>
<td>( L )</td>
<td>((p(H, L), p(L, H)))</td>
<td>((p(L, L), p(L, L)))</td>
</tr>
</tbody>
</table>

(3.8)

where the columns (rows) represent the possible strategy of team A (B). From the above matrix since \( p(H, H) > p(L, H) \) and \( p(H, H) > p(H, L) \) by (2.1), then team A will not be better off deviating from \( H \) if team B plays \( H \). Therefore \((H, H)\) is the equilibrium, this hold irrespective of the comparative advantage of the teams, i.e. the sign of \( \alpha \).

This also holds for general \( T \) - period strategies of the overtime game when \( \alpha > 0 \).

**Theorem 2** Under the new rule and assuming the teams are of equal quality,

a) if teams have a comparative advantage in playing offensive hockey (\( \alpha > 0 \)) then \((H, H)\) is the equilibrium;

b) and \( \lim_{T \to \infty} V_{new}^{i, 0} = \frac{1}{2}, i=A, B. \)

where \( H \) is a \( T \)-vector of \( H \)'s.

c) if teams have a comparative advantage in playing defensive hockey (\( \alpha < 0 \)) then there exists a time \( t_0 \), when the teams will deviate and play \((H, H)\) for all \( t > t_0 \).

Therefore even though the team has a comparative advantage in playing defensively, it will not always play defensively. At some point during the game \( (t_0) \) it will start playing offensively since this increases the expected payoff to the team. This implies that even teams with a comparative advantage of playing defensive hockey (\( \alpha < 0 \)) will not play defensively throughout the overtime game.

Thus, our model shows that the rule change will lead to an increase in offensive play in overtime, as even the defensive teams would start to attack towards the end of the game.

## 4 Impact on Regular Time Game Strategies

Changing the points awarded in overtime will not only affect the style of play in overtime but may also have an effect on the play during the 60 minute regular time game. To
show this, we define a random variable $Z_t$, as the goal difference between teams A and B from the perspective of team A at time $t$. This stochastic process is a Markov chain (more precisely a random walk on integers) with $Z_t = Z_{t-1} + X_t, t = 1, \ldots, T$ and $Z_0 = 0$. The transition probabilities of this stochastic process are

$$
\Pr (Z_t = d'|Z_{t-1} = d) = \begin{cases} 
p & \text{if } d' = d + 1 
q & \text{if } d' = d 
r & \text{if } d' = d - 1 
0 & \text{otherwise}
\end{cases}.
$$

(4.9)

with the initial condition of $\Pr (Z_0 = 0) = 1$.

A winner is determined after the regular time period of $T$, if there is a positive goal difference. If there is no goal difference, the game goes to overtime with the payoff structures defined in the previous section. Therefore, we define the end of play payoff of team A as,

$$
U_{\text{reg}}^A (\lambda) = I \{Z_T > 0\} - I \{Z_T < 0\} + \lambda I \{Z_T = 0\}
$$

where $I \{.\}$ is the indicator function and $\lambda$ is the expected payoff from the extra time game. Since the overtime game under the old rule was zero sum with a skew symmetric payoff matrix, the value of the game is zero (Theorem 1 part c)). Hence, under the old rule, $\lambda = 0$. Under the new rule, the game is no longer a zero sum game, and the expected value of overtime is always positive, but the exact value depends on the number of periods, $T$ the strategy has been considered. But as a limiting case, i.e. when $T$ is large enough, $\lambda = \frac{1}{2}$ (Theorem 2 part b)).

Team B’s end of play payoff will be

$$
U_{\text{reg}}^B (\lambda) = I \{Z_T < 0\} - I \{Z_T > 0\} + \lambda I \{Z_T = 0\}
$$

As before, a strategy of a team (A or B) at period $t$ depends on the history of the game upto period $t - 1$. Note that the history of the game at period $t$ is the outcome of the random variable $Z_{t-1} = d$, the goal difference of the previous period. Note that the only relevant history is the case when the goal difference is less than the time left for play, i.e. $|d| < T-t$, otherwise one of the teams has already won.

Let $V_{\text{reg}}^i_{t,d}, (i = A \text{ or } B)$ be the value of the game at period $t$ such that the history $Z_{t-1} = d$. We can recursively write the value function as

$$
V_{\text{reg}}^A_{t-1,d} = \max_{s_t^A} \left( p \left(s_t^A, s_t^B\right) V_{\text{reg}}^A_{t,d+1} + q \left(s_t^A, s_t^B\right) V_{\text{reg}}^A_{t,d-1} + r \left(s_t^A, s_t^B\right) V_{\text{reg}}^A_{t,d} \right)
$$

(4.10)

and

$$
V_{\text{reg}}^B_{t-1,d} = \max_{s_t^B} \left( p \left(s_t^A, s_t^B\right) V_{\text{reg}}^B_{t,d+1} + q \left(s_t^A, s_t^B\right) V_{\text{reg}}^B_{t,d-1} + r \left(s_t^A, s_t^B\right) V_{\text{reg}}^B_{t,d} \right).
$$

(4.11)

Finally $V_{\text{reg}}^i_{T+1,d} = U_{\text{reg}}^i (\lambda)$. 

8
To demonstrate our argument that the new rule will make the teams play more defensively during regulation time, all we need to show is that there will be a change of team strategy under the new rules (for offensive teams) during regulation time when the teams are tied.

4.1 Under the old overtime rule

The overtime game under the old rule was zero sum with a skew symmetric payoff matrix. The value of the game is zero (Theorem 1 part c)). Hence under the old rule $\lambda = 0$. Also note that $U_{reg}^A(0) = -U_{reg}^B(0)$.

Let us look at only the last period of the regulation game when at the start of this period the goal difference is zero. This period is similar to the first period the overtime game under the old rule. If team A chooses strategy, $s^A$ and team B $s^B$, the expected payoff matrix in regular time for Team A is:

\[
\begin{array}{c|cc}
     & H & L \\
\hline
H & 0 & -\alpha \\
L & \alpha & 0
\end{array}
\]

Assuming there is comparative advantage to offensive play by the teams ($\alpha > 0$), team A will not be better off deviating from $H$ if team B plays $H$ during regulation time. Therefore, $(H, H)$ is the equilibrium when the old overtime ($\lambda = 0$) incentive rule prevailed. If $\alpha < 0$, then the last row and the last column dominates and $(L, L)$ is the equilibrium. The value of the game at period $T$ is then $V_{reg{T,0}} = 0$. Also note that

We shall show that whenever the game is tied the teams will play their comparative advantage

**Theorem 3** Under the old overtime rule and assuming the teams are of equal quality,

1) the value functions satisfies

\[
\begin{align*}
V_{reg}^{i,t,d} &= -V_{reg}^{i,t,d}, \\
V_{reg}^{i,t,d} &= -V_{reg}^{i,t,-d}, i = A, B
\end{align*}
\]

for all $d$ and all $t$.

2) When the game is drawn ($d=0$) at time $t$,

a) the teams will play $(H, H)$ if have a comparative advantage in playing offensive hockey ($\alpha > 0$),

b) the teams will play $(L, L)$ if have a comparative advantage in playing offensive hockey ($\alpha < 0$),

Theorem 3 (proof: see appendix) implies that under the old overtime rule, the optimal regular time strategy chosen by the teams depends on whether the teams have a “comparative advantage” in playing offensively or defensively, when the game is still drawn.
4.2 Under the new overtime rule

The new overtime rule may have a perverse effect of the style of play during the end of the regular time game. Note that under the new rule, the game is no longer zero sum and the expected value of the overtime is always positive. The exact value depends on the number of periods ($T$) but as a limiting case, i.e. when $T$ is large enough, $\lambda = \frac{1}{2}$ (Theorem 2 part b)), i.e. the teams expects to get half a point from the overtime game.

An important implication is that teams may no longer play offensive hockey even if they have a comparative advantage in doing so when the game is drawn. Formally, if the teams are equal (2.2) and have a comparative advantage in playing offensive hockey $\alpha > 0$ (3.5) $(H, H)$ will not be the dominant strategy when $d = 0$, for some teams unlike under the old incentive regime.

Again, let us look at only the last period of the regulation game when at the start of the period the goal difference is zero ($d = 0$). If team A chooses strategy, $s^A$ and team B $s^B$, the expected payoff in regulation time for Team A,

$$V_{reg}^{A}_{T,0} = \max_{s^A} p(s^A_T, s^B_T) - q(s^A_T, s^B_T) + \frac{1}{2} r_s(s^A_T, s^B_T)$$

and for Team B

$$V_{reg}^{B}_{T,0} = \max_{s^B} q(s^A_T, s^B_T) - p(s^A_T, s^B_T) + \frac{1}{2} r_s(s^A_T, s^B_T).$$

In matrix form,

<table>
<thead>
<tr>
<th>$s^B \setminus s^A$</th>
<th>$H$</th>
<th>$L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H$</td>
<td>$\left( \frac{1}{2} r (H, H), \frac{1}{2} r (H, H) \right)$</td>
<td>$\left( -\alpha + \frac{1}{2} r (L, H), \alpha + \frac{1}{2} r (L, H) \right)$</td>
</tr>
<tr>
<td>$L$</td>
<td>$\left( \alpha + \frac{1}{2} r (H, L), -\alpha + \frac{1}{2} r (H, L) \right)$</td>
<td>$\left( \frac{1}{2} r (L, L), \frac{1}{2} r (L, L) \right)$</td>
</tr>
</tbody>
</table>

It follows that Team A will not play $H$ as deviating to $L$ yields a better payoff. When $p(H, H) - p(H, L) > \frac{1}{2} \alpha$. The difference in payoff for Team A deviating from $H$ to $L$ is given by,

$$V_{reg}^{A}_{T,0}(L, H) - V_{reg}^{A}_{T,0}(H, H)$$

$$= -\alpha + \frac{1}{2} (r (L, H) - r (H, H))$$

$$= -\alpha + p(H, H) - \frac{p(H, L) + p(L, H)}{2} \tag{4.13}$$

Team A will not play $H$ as deviating to $L$ yields a better payoff if, if the team is characterised by the following condition:

$$p(H, H) - \frac{p(H, L) + p(L, H)}{2} > \alpha \tag{4.14}$$

$$p(H, H) - p(H, L) > \frac{1}{2} \alpha.$$
Similarly, Team B deviates to $L$ under the same conditions if the game is tied close to the end.

This shows that under the new rule some teams (teams which satisfy condition (4.14)) at some point $t_0$ will shift to defensive play even if their comparative advantage lies in playing offensively. Thus, the change in incentive to create more attractive play in overtime has the perverse effect of resulting in more conservative, defensive play in regulation time (especially towards the end).

This is rational behavior for players and coaches, because why risk losing a point by aggressively pursuing a win in regulation if the safer route is to get a regulation tie and go for the extra point in overtime? In short, with the current rules, you can lose a game but still get a point in the standings.

5 Empirical evidence

Data on the results of individual games for the last nine NHL seasons were collected from the website http://www.hockeynut.com/archive.html. The number of regulation and overtime games played and the results in the overtime and regulation games are summarized in Table 1. The first four seasons (from the 1995-1996 season through the 1998-1999 season) represent the “old rule period”, i.e. the seasons when the losing team in overtime game received no points while the last five seasons are the “new rule period” i.e. when each team tied at the end of regulation received a single point regardless of the outcome in overtime.

5.1 Model specification

In addition to the statistical summaries in Table 1, we estimated two separate logit models to measure the effects of the rule change on the (conditional) probability of a tie in overtime and a tie in regulation, respectively. The dependent variable in both models is defined as whether there a win or a tie in the relevant period (1=win, 0=tie) – for overtime this is identical to whether a goal was scored or not. The main explanatory variable of concern in our study was the rule change. This variable is proxied by a dummy variable equal to 1 for seasons with the new rule and 0 for years under the old rule.

In addition we included some variables to capture potentially important team-specific or game-specific effects. Total team payrolls and differences in team salaries were used to proxy the effect of absolute and relative offensive abilities by teams. Player salary information was obtained from http://www.rodneyfort.com/PHSportsEcon/Common/OtherData/NHLSalaries/NHLSalaries.html for the majority of the seasons and http://www.lcsnhockey.com/extra/1997/salary.asp for the 1997-98 years and http://www.lcsnhockey.com/extra/1998/salary.asp for the 1998-99 season. The sum of the team payrolls is assumed to increase the likelihood
of a goal scored and win since player salary is generally directly related to offensive skills. The absolute value of the difference is also assumed to increase the chance of the game being settled in overtime since there is a relative difference in the comparative ability of the teams to play offensive hockey.

Although the theoretical model did not explicitly consider the conference of the opponent, there may be implications of conference status on team strategy. Consequently, the conference effect is included as a control variable in the empirical model. Specifically, a dummy variable was included to capture the effects of games played between teams within the same or different conferences. The present 30-team NHL is divided into two conferences (East and West). The top 8 teams within each conference make the playoffs at the end of the 82 game regular season. A team’s overall profitability is determined in large part by the revenue earned from ticket sales of home playoff games. Thus, making the playoffs generates income but so does finishing further up the league standings as the higher placed team earns home-ice advantage in any playoff series. Thus, incentives and correspondingly team strategy in overtime may vary depending upon whether the opponent is from the same conference. Since relative positions determine playoff ranking and potential home-ice advantage, teams within the same conference may wish to avoid giving an additional point in overtime to their opponents. While teams from within the same conference could gain from an additional point in overtime, the consequences of giving up a goal are significant. In contrast, both teams have nothing to lose by attempting to score in overtime if they are in different conferences.

5.2 Regression results

The averages in Table 1 and the logit regression results in Table 2 are consistent with the predictions from our model on overtime team behavior, i.e. that teams take a more offensive approach in overtime if both teams are guaranteed a single point going into the overtime: (Theorem 1 and 2). If a team could lose a point by giving up a goal in the 5 minute overtime, teams would play a conservative strategy to avoid losing what they had gained over the 60 minutes of regulation play. Table 1 shows how under the old rule period, on average 71% of the games tied in regulation remained tied during overtime. In contrast, under the new rule, only 53% of the games tied after regulation remained tied in overtime. Moreover, the numbers are fairly constant for the various seasons within both the old rule and the new rule period, suggesting a clear structural effect on overtime team strategies.

However, there is a potential causality problem with this interpretation of the empirical evidence on overtime strategies. The modification to the NHL overtime rule at the beginning of the 1999-2000 season not only changed the point system but also the number of skaters on the ice. In order to further increase the possibility of scoring, the
NHL went from the 5-on-5 play used in regulation time to 4-on-4. Since both the point system and number of players for overtime were changed at the same time, the effect of each on the significant increase in overtime wins cannot be determined. However, Abrevaya (2002) gathered overtime results for the American Hockey League (AHL) which is a minor league affiliate of the NHL. The two overtime rule changes were implemented separately by the AHL. The data indicate that the awarding a single point in overtime was primarily responsible for the increased number of overtime wins. Prior to the rule change, 68% of overtime games remained tied and this dropped to 59% with the change in point system. The percentage dropped slightly further to 55% with the introduction of 4-on-4 play. Thus, the scoring system change has increased the likelihood of offensive strategy in overtime as desired.

The other explanatory variables on the probability of a win in overtime were statistically insignificant but were of the expected signs (column 2 of Table 2). Teams from within the same conference are less likely to score in overtime. In addition, games involving high payroll teams and ones with a large spread in team salaries are more likely to be involved in overtime game that results in a win for one team.

While the overtime rule change had the desired positive effect on play in overtime, the empirical results confirm the unintended consequences of the point change on regular time play. Theorem 3 predicts that the overtime rule changes would lead to more defensive play during regulation. The hypothesized effect is consistent with summary measures presented in Table 1. The percentage of games ending in a tie after the 60 minute regulation time increased from an average 20.2% to 23.1% with the change in the overtime payoff structure.

To measure whether the overtime rule change affected regular time play, we focus specifically on play in the third period. The logit regression in Table 2 measures the likelihood that the game remained tied, given that the game was tied with one period remaining. Our theory would suggest conservative play would dominate in the 3rd period leading to a greater chance of a tie game at the end. The regression results in the third column of Table 2 show that the change in the overtime rule indeed significantly reduced the likelihood of one team winning at the end of regulation given the game was tied with one period remaining. The other variables have the expected effects: conservative play in the 3rd period is more likely between teams of the same conference and teams with lower payrolls (both total and absolute difference). However, none of these effects are statistically significant.

The effect of the overtime rule change on play during regulation time was also assessed in terms of the likelihood of the goal spread increasing rather than staying the same. A third logit regression (column 4 in Table 2) estimated how the rule change affected the likelihood of the goal spread changing in the last period. Across the nine seasons, teams were separated by one goal at the end of the 2nd period in 38% of the games and there was no significant difference before and after the overtime rule change. Given the one goal
difference, the spread increased to two or more goals in approximately 40% of the games.

The logit regression results in Table 2 show that there was no significant impact of the rule change on the goal spread. The overtime rule change has no impact on scoring in games with a one goal spread at the end of the 2nd period since both before and after the rule change the team with the lead will attempt to protect the lead and play defensively while the team behind will attempt to close the gap and play offensively. This result is not inconsistent with the previous finding that teams in a close game are more likely to remain tied at the end of regulation play under the new overtime point scheme. In a one-goal game, the team ahead will likely to decide to play defensively and wait to counter-attack against a team pressing to score the equalizer. However, an offensive team seeking to increase its one goal lead may also expose itself and be more vulnerable to letting in a tying goal. Thus, the hypothesized conservative regulation play during regulation under the overtime rule change is likely to lead to games tied near the end remaining tight but have an ambiguous effect on games separated by one goal at the end of the 2nd period.

\[ U_{newreg}^A = 2I \{ Z_T > 0 \} - I \{ Z_T < 0 \} + \frac{1}{2} I \{ Z_T = 0 \} \]  \hspace{1cm} (6.15)

where \( I \{ . \} \) is the indicator function and the expected payoff from tying a regulation time game is \( \frac{1}{2} \).

Team B’s end of play payoff will be

\[ U_{newreg}^B = 2I \{ Z_T < 0 \} - I \{ Z_T > 0 \} + \frac{1}{2} I \{ Z_T = 0 \} \]  \hspace{1cm} (6.16)

6 Increasing rewards for a win in the regular time

Another contemplated change to the incentives facing NHL teams is to give a winning team three points for a regulation win as opposed to two. Such a change in the point structure has been implemented in most professional soccer leagues. One version of the reward system would give a team three points for a win in regulation, two for a win in overtime, one for a tie or an overtime loss, and zero for a loss in regulation. The intent of such a change is to create a more attacking style of game since teams would be rewarded for doing better and trying to win in regulation rather than play conservatively and settle for a guaranteed one point arising from a tie in regulation (Mullin, J (2003)).

To analyze how such a proposed rule change would affect team strategies, we will use the model developed above. We define the end of play payoff \( U_{reg}^A \) of team A as,
Notice that we can rewrite the utilities as

\[
U_{\text{newreg}}^A = \frac{3}{2} (I \{Z_T > 0\} - I \{Z_T < 0\}) + \frac{1}{2}
\]
\[
= \frac{3}{2} U_{\text{reg}}^A (0) + \frac{1}{2}
\]

and

\[
U_{\text{newreg}}^B = \frac{3}{2} (I \{Z_T > 0\} - I \{Z_T < 0\}) + \frac{1}{2}
\]
\[
= \frac{3}{2} U_{\text{reg}}^B (0) + \frac{1}{2}.
\]

Therefore the payoff matrix under this new regular time rule will be an affine transformation on the payoff matrix of the regular game under the old overtime rule. This implies that the optimal strategies of the teams A and B will be the same during the original regime and will be given by theorem (3). Hence the teams with comparative advantage in offensive hockey \((\alpha > 0)\) will play offensively \((H, H)\) when the game is drawn.

This shows that the change in the rewards to a regulation time win will be sufficient to stimulate more offensive play back into the game.

7 Concluding comments

Institutional reforms affect behavior, but not always in the way intended. Recent studies analyzing the effect of rule changes on strategic behavior in sporting competitions yield interesting insights. They are particularly interesting because due to the very nature of the sporting competitions many external factors which typically influence behavior are controlled for, or excluded.

In this paper we have analyzed the impact of the recent introduction of a rule change in the National Hockey League on the strategies of hockey teams. The NHL decreed that as of the 1999-2000 season, in case of a tie after the regulation time, both teams would get one point each; and that the winner in overtime would get an additional point. This change rewarded the teams which tied in regulation time but lost in overtime with a point which they did not get under the old rules. The rule change was intended to enhance the general appeal of the game by stimulating more offensive play in regulation time. Our analysis shows that this was a correct assumption. We demonstrate with our theoretical model that teams are more likely to play offensively in overtime under the new rule. The empirical evidence we present confirms these conclusions.

These findings appear to provide strong support for the NHL’s decision to introduce the 1999 rule change. However, such conclusion is not justified, since it is based on an incomplete analysis. The rule change also has another, unintended, impact. Our theoretical analysis shows how the rule change has a perverse effect on team strategies
in regulation time, causing more defensive play during the main part of the game. The empirical evidence provides support for this conclusion. Hence the conclusions based on the full effects of the rule change should be less positive, and more nuanced.

A series of additional rule changes to offset the perverse regulation time effect are being considered. The most prominent proposal is to raise the reward for the winning team to 3 points and keep the rest the same. In the last section of the paper we show that this proposal, if implemented, will mitigate the perverse effect in regulation time.

References


Appendix:

Proof of theorem (2):

a) From (3.8) it follows that at period $T$, the teams will play $(H, H)$: therefore $V_{old}^{i,T,0} = p(H, H), i = A$ or $B$. Now solving backwards, assume that whenever the game
is tied \( (d = 0) \) the teams play \((H, H)\), when \( \alpha > 0 \). Therefore

\[
V_{old}^A_{t, 0} = V_{old}^B_{t, 0} = \sum_{\tau = t}^{T} r(H, H)^{T - \tau} p(H, H)
\]

\[
= \frac{1}{2} \left(1 - r(H, H)^{T - t}\right) < \frac{1}{2}
\]

At period \( t - 1 \), if team A deviates to \( L \), then the difference in payoff is

\[
r(L, H) V_{new}^A_{t, 0} + p(H, H) - r(L, H) V_{new}^A_{t, 0} - p(L, H)
\]

\[
\left[r(L, H) - r(H, H)\right] V_{new}^A_{t, 0} + p(H, H) - p(L, H)
\]

\[
\left[p(L, H) + p(H, L) - 2p(H, H)\right] V_{new}^A_{t, 0} + p(H, H) - p(L, H)
\]

\[
> \left[p(L, H) + p(H, L) - 2p(H, H)\right] \frac{1}{2} + p(H, H) - p(L, H)
\]

\[
> \frac{p(H, L) - p(L, H)}{2} > 0.
\]

(Similarly for team B). Hence the teams play their comparative advantage, that is \((H, H)\).

b) Note that 

\[
V_{new}^A_{0, 0} = \frac{1}{2} \left(1 - r(H, H)^{T}\right)
\]

and taking \( T \to \infty \), we get the result.

c) Consider the case that the teams have a comparative advantage in playing defensive hockey \((\alpha < 0)\). From (3.8) it follows that at period \( T \), the teams will play \((H, H)\). Therefore \( V_{old}^i_{T, 0} = p(H, H), i = A \) or \( B \). Now solving backwards, assume that whenever the game is tied \( W_{t-1} = 0 \) the teams play \((H, H)\), therefore

\[
V_{old}^A_{t, 0} = V_{old}^B_{t, 0} = p(H, H) \cdot \sum_{\tau = t}^{T} r(H, H)^{T - \tau}
\]

\[
= \frac{1}{2} \left(1 - r(H, H)^{T - t}\right).
\]

At period \( t - 1 \) if team A deviates to \( L \), then the difference in payoff is

\[
r(L, H) V_{new}^A_{t, 0} + p(L, H) - r(H, H) V_{new}^A_{t, 0} - p(H, H)
\]

\[
\left[r(L, H) - r(H, H)\right] V_{new}^A_{t, 0} + p(L, H) - p(H, H)
\]

\[
\left[2p(H, H) - (p(L, H) + p(H, L))\right] V_{new}^A_{t, 0} + p(L, H) - p(H, H).
\]

Therefore team A deviates if

\[
\frac{1}{2} \left(1 - r(H, H)^{T - t}\right) = V_{new}^A_{t, 0} > \frac{p(H, H) - p(L, H)}{2p(H, H) - (p(L, H) + p(H, L))}.
\]

(7.17)

otherwise it continues to play \( H \). (Similarly for team B.)

Proof of Theorem (3):
1) Consider the last period \( t = T \). Then the value functions are

\[
\begin{align*}
V_{reg_{T-1,0}}^A &= \max_{s_t^A} p \left( s_t^A, s_t^B \right) - q \left( s_t^A, s_t^B \right) \\
V_{reg_{T-1,-1}}^A &= \max_{s_t^A} - \left( 1 - p \left( s_t^A, s_t^B \right) \right) \\
V_{reg_{T-1,1}}^A &= \max_{s_t^A} \left( 1 - p \left( s_t^A, s_t^B \right) \right) \\
V_{reg_{T-1,d>1}}^A &= 1 = -V_{reg_{T,d<1}}^A
\end{align*}
\]

and

\[
\begin{align*}
V_{reg_{T-1,0}}^B &= \max_{s_t^B} q \left( s_t^A, s_t^B \right) - p \left( s_t^A, s_t^B \right) \\
V_{reg_{T-1,-1}}^B &= \max_{s_t^B} \left( 1 - p \left( s_t^A, s_t^B \right) \right) \\
V_{reg_{T-1,1}}^B &= \max_{s_t^B} - \left( 1 - p \left( s_t^A, s_t^B \right) \right) \\
V_{reg_{T-1,d>1}}^B &= -1 = -V_{reg_{T,d<1}}^B.
\end{align*}
\]

The dominant strategy of the teams when they are trailing by at least one goal is to play \( L \) and the dominant strategy when they are leading is to play \( H \). From (4.12) it follows that, when the game is goalless the teams play \((H, H)\) when \( \alpha > 0 \) and play \((L, L)\) when \( \alpha < 0 \). Therefore

\[
\begin{align*}
V_{reg_{T-1,0}}^A &= V_{reg_{T-1,0}}^B = 0 \\
V_{reg_{T-1,d}}^A &= -V_{reg_{T-1,d}}^B = (1 - p(H, L)), \ d = 1, -1.
\end{align*}
\]

So the payoffs are symmetric across teams for all \( d \) at period \( T - 1 \).

Recursively solving backwards at period \( t - 1 \), assume that the goal difference is \( Z_{t-1} = d \), and \( V_{reg_{t,d}}^A = -V_{reg_{t,d}}^B \). Then from (4.10) and (4.11) we have

\[
\begin{align*}
V_{reg_{t-1,d}}^A + V_{reg_{t-1,d}}^B &= \left( p \left( s_{t-1}^A, s_{t-1}^B \right) V_{reg_{t,d+1}}^A + q \left( s_{t-1}^A, s_{t-1}^B \right) V_{reg_{t,d+1}}^B + r \left( s_{t-1}^A, s_{t-1}^B \right) V_{reg_{t,d+1}}^A + q \left( s_{t-1}^A, s_{t-1}^B \right) V_{reg_{t,d+1}}^B \right)
\end{align*}
\]

where \( (s_{t-1}^A, s_{t-1}^B) \) are the optimal strategies at period \( t \). Given that \( V_{reg_{t,d}}^A = -V_{reg_{t,d}}^B \), for all \( d \). We get \( V_{reg_{t-1,d}}^A + V_{reg_{t-1,d}}^B = 0 \).

Now assume \( V_{reg_{t,d}}^A + V_{reg_{t,-d}}^A = 0 \), for all \( d \), then

\[
\begin{align*}
V_{reg_{t-1,d}}^A + V_{reg_{t-1,-d}}^A &= \left( p \left( s_{t-1}^A, s_{t-1}^B \right) V_{reg_{t,d+1}}^A + q \left( s_{t-1}^A, s_{t-1}^B \right) V_{reg_{t,d+1}}^B + r \left( s_{t-1}^A, s_{t-1}^B \right) V_{reg_{t,d+1}}^A + q \left( s_{t-1}^A, s_{t-1}^B \right) V_{reg_{t,d+1}}^B \right)
\end{align*}
\]

18
By assumption of equality (2.2), we have $V_{reg^{A, t-1, d}} + V_{reg^{A, t-1, -d}} = 0$. Similarly $V_{reg^{B, t-1, d}} + V_{reg^{B, t-1, -d}} = 0$.

2) Let $V_{reg^{i, t, 1}} = -V_{reg^{i, t, -1}}$, and since $V_{reg^{i, 0}} = 0$, $i = A, B$, then

$$V_{reg^{A, t-1, 0}} = \max_{s^A} (p(s^A_t, s^B_t) - q(s^A_t, s^B_t)) V$$

$$V_{reg^{B, t-1, 0}} = \max_{s^B} (q(s^A_t, s^B_t) - p(s^A_t, s^B_t)) V$$

This is similar to the last period problem. Therefore when $\alpha > 0$, then $(s^*_A, s^*_B) = (H, H)$ is the dominant strategies; else if $\alpha < 0$ then the teams play $(s^*_A, s^*_B) = (L, L)$. 

19
<table>
<thead>
<tr>
<th></th>
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<td></td>
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<tr>
<td>% of overtime games won with different conference</td>
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* - Change of overtime rule.

Table 1: Number and results of overtime games in the NHL, 1995-2004.
<table>
<thead>
<tr>
<th>Explanatory Variables</th>
<th>Dependent Variable</th>
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<td></td>
<td>Goal scored during the overtime game</td>
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<tr>
<td>Overtime Rule (1=new)</td>
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<td>Conference Dummy (1=same)</td>
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<td></td>
<td>(0.097)</td>
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<tr>
<td>Sum of Team Salaries ($)</td>
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</tr>
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<td></td>
<td>(0.003)</td>
</tr>
<tr>
<td>Difference in Team Salaries ($)</td>
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<td></td>
<td>(0.005)</td>
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<td>% Predicted Correctly</td>
<td>61</td>
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<td>-2 Log likelihood</td>
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*- statistically significant at 5%
standard errors are in parentheses

Table 2: Logit regression results on effect of overtime rule change on regular and overtime play.