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An experimental study on the incentives of the probabilistic serial mechanism

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An Experimental Study on the Incentives of the Probabilistic Serial Mechanism

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Abstract

An Experimental Study on the Incentives of the Probabilistic Serial Mechanism

by David Hugh-Jones, Morimitsu Kurino and Christoph Vanberg*

We report an experiment on the Probabilistic Serial (PS) mechanism for allocating indivisible goods. The PS mechanism, a recently discovered alternative to the widely used Random Serial Dictatorship mechanism, has attractive fairness and efficiency properties if people report their preferences truthfully. However, the mechanism is not strategy-proof, so participants may not truthfully report their preferences. We investigate misreporting in a set of simple applications of the PS mechanism. We confront subjects with situations in which theory suggests that there is an incentive or no incentive to misreport. We find little misreporting in situations where misreporting is a Nash equilibrium. However, we also find a significant degree of misreporting in situations where there is actually no benefit in doing so. These findings suggest that the PS mechanism may have problems in terms of truthful elicitation.

Keywords: Probabilistic serial mechanism, incentives

JEL classification: C78, C91, C92

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1. Introduction

In the last decade, market designers have explored real-life applications of indivisible good resource allocation problems when monetary transfers are not allowed. Examples include on-campus housing for college students [2], the assignment of seats in public schools to pupils [3], kidney transplantation for patients [16], and course seats to college students [18, 6].

In such problems, it is often wasteful to require fairness *ex post*, since divisions are impossible.⁴ Thus, randomization is commonly used to ensure fairness *ex ante*. The simplest problem is the so-called *random assignment problem* [12, 5] where n objects are to be allocated to n agents and each agent consumes exactly one object. An (ordinal) mechanism is a systematic way of assigning objects to agents in either a deterministic or random way for each reported ordinal preference profile.⁵ The most popular mechanism in theory and practice is the *random serial dictatorship (RSD)* mechanism, which randomly orders agents and then lets them successively choose their favorite objects among those available. Bogomolnaia and Moulin [5] pointed out that the RSD mechanism may be inefficient from an *ex ante* perspective. Specifically, they show that the RSD mechanism induces a distribution over final allocations that is not *ordinally efficient*. This means that there is another way of randomly allocating the objects which would be unanimously preferred by all participants.

Bogomolnaia and Moulin proposed the *probabilistic serial (PS)* mechanism as an alternative to RSD. The PS mechanism achieves ordinal efficiency. In addition, the PS mechanism has the property that it is envy-free.⁶ The RSD is not envy free. These considerations suggest that RSD, which is widely used in real-world markets, could be replaced by the PS mechanism.

However, the attractive fairness properties of the PS mechanism identified by Bogomolnaia and Moulin assume that participants truthfully reveal their preferences, i.e. their rankings over the objects to be allocated. If participants misreport their rankings, these properties may not hold. And indeed, a problematic feature of the PS mechanism is that it may induce incentives for participants to misreport. Even where such incentives are not present, the mechanism is less transparent than the RSD, and thus it is possible that human participants may falsely perceive incentives to misreport. In technical terms, these concerns are about the *strategy-proofness* of an allocation mechanism. A *strategy-proof* mechanism does not allow any agent to benefit from misrepresenting her preference regardless of others' reporting. The PS mechanism is not fully strategy-proof

⁴For example, see Kesten and Yazici [14].

⁵In this paper we focus on ordinal mechanisms rather than cardinal mechanisms that elicit cardinal utilities. There are three common justifications for the ordinal approach as explained in Hashimoto, Hirata, Kesten, Kurino, and Ünver [11]: First, since agents are boundedly rational, cardinal preferences are difficult to elicit. Second, ordinal mechanisms are relatively simpler and more practical than cardinal ones. Third, real-life matching markets function mostly through elicitation of ordinal preferences.

⁶A random assignment is *envy-free* if no agent prefers anybody else's assignment regardless of her cardinal utilities.

because in some situations some agents may benefit from misreporting their preferences.⁷

In this paper, we report a laboratory experiment investigating the extent to which agents misrepresent their preferences under the PS mechanism. We implement a situation where subjects have an incentive to misrepresent their preferences. We also implement situations where subjects do not benefit from misrepresenting their preferences, but may mistakenly believe that they do. We find little strategic misrepresentation in the former situation, but high levels of misrepresentation in the latter situations. The phenomenon is robust to experience and feedback.

2. The Probabilistic Serial (PS) Mechanism

We first introduce the model and then describe the random serial dictatorship mechanism (RSD) and the probabilistic serial mechanism (PS). The random assignment problem, also called a house allocation problem, is the allocation problem where n (indivisible) objects are to be allocated to n agents without monetary transfers, and each agent is to receive exactly one object [12, 5]. Let $I = \{1, \dots, n\}$ be the set of agents and A the set of objects. Each agent $i \in I$ has a strict preference relation \succ_i on A . We denote by \mathbf{P} the set of strict preferences on A . Let $\succ = (\succ_i)_{i \in I} \in \mathbf{P}^I$ denote the strict preference profile. We fix I and A throughout the paper.

We assume that each agent has a utility function over objects. The utility function of agent i associates a real number with each object, i.e., a function $u_i : A \rightarrow \mathbb{R}$. We say that u_i is **consistent** with \succ_i when for all $a, b \in A$, $u_i(a) > u_i(b)$ if and only if $a \succ_i b$.

We consider the situation where probabilistic assignments are possible. A **random allotment for some agent** i is a probability distribution on objects, $P_i := (p_{i,a})_{a \in A}$, where $p_{i,a}$ denotes the probability that agent i receives object a . Thus, for each $a \in A$, $p_{i,a} \in [0, 1]$ and $\sum_{a \in A} p_{i,a} = 1$. A **random assignment** is a collection of random allotments for all agents, denoted by $P = (P_i)_{i \in A}$, such that for each object $a \in A$, $\sum_{i \in I} p_{i,a} = 1$. In other words, a random assignment is a bistochastic matrix. Let \mathcal{R} be the set of random assignments. A **deterministic assignment** specifies which object each agent is assigned such that no two distinct objects are assigned to an agent. That is, it is a random assignment each of whose entries is 0 or 1. Let \mathcal{D} be the set of deterministic assignments.

Note that to implement a random assignment, we need to have a probability distribution over deterministic assignments – a lottery. To describe this relation, we introduce some notation and results. A **lottery** is a probability distribution w on the set \mathcal{D} of deterministic assignments. Given

⁷Note that the mechanism we consider is ordinal. A mechanism is strongly strategy-proof if for each agent the induced random assignment under truthful reporting (first-order) stochastically dominates the one under misreporting: equivalently, for all cardinal utilities consistent with truthful preferences, the expected payoff under truthful reporting is greater than or equal to the one under misreporting. The PS mechanism is weakly strategy-proof in the following sense: for each agent the induced random assignment under truthful reporting is not stochastically dominated by the one under misreporting. However, as in our experiment, agents with certain cardinal utilities may gain from misreporting their preferences. A recent paper by Kojima and Manea [15] shows that in a large but finite economy where there is a sufficiently large supply of each kind of object, PS becomes strategy-proof.

a lottery w on \mathcal{D} , where $w(\mu)$ is the probability that an assignment $D \in \mathcal{D}$ is chosen, we have its induced random assignment as $\sum_{D \in \mathcal{D}} w(D)D$. Perhaps surprisingly, the converse is also true : any random assignment can be implemented by a lottery over deterministic assignments [4, 19]. Thus, we focus on random assignments.

We extend preferences \succ_i defined over A to the set \mathcal{D} of deterministic assignments in the obvious way. We extend agent i 's utility function u_i , initially defined over A , to the set \mathcal{R} of random assignments by taking expected utilities: for each $P \in \mathcal{R}$, $u_i(P) = \sum_{a \in A} p_{i,a} u_i(a)$.

We focus on ordinal mechanisms. An **(ordinal) mechanism** is a systematic way of selecting a random assignment for each preference profile instead of each cardinal utility profile. Formally, a mechanism is a function from \mathbf{P}^I to \mathcal{R} . Here we assume that there is a central authority who uses a mechanism φ . The central authority asks each agent $i \in I$ to report her ordinal preference \succ_i , and based on the reported preference profile $\succ = (\succ_i)_{i \in I}$, the random assignment $\varphi(\succ)$ is determined.

The random serial dictatorship mechanism is defined as follows. Put the agents in some arbitrary order, from first to last, denoted by $f = (i_1, \dots, i_n)$. The **serial dictatorship mechanism induced by f** , denoted by SD^f , is a deterministic mechanism such that given a preference profile \succ , the first agent i_1 is assigned her favorite (most preferred) object according to \succ_{i_1} , the second agent i_2 is assigned her favorite object among the remaining ones according to \succ_{i_2} , and so on. Let \mathcal{F} be the set of orderings of agents. The **random serial dictatorship mechanism (RSD)** picks an ordering at random from the uniform distribution on \mathcal{F} and implements it. Thus:

$$RSD(\succ) = \frac{1}{n!} \sum_{f \in \mathcal{F}} SD^f(\succ) \quad \text{for each } \succ \in \mathbf{P}^I.$$

Example 1. We illustrate how to compute the RSD outcome. This example is used in our experiment. There are three agents (1,2,3) and three objects (a , b and c) with preferences as given in the following table.

$$\begin{array}{l|ccc} \succ_1 & a & c & b \\ \succ_2 & a & b & c \\ \succ_3 & b & a & c \end{array}$$

For example, the ordering (2, 3, 1) is drawn with probability of 1/6. Under this ordering, the first agent 2 gets her favorite object a , the second agent 3 gets her favorite object b among the remaining objects b, c , and the last agent 1 gets the remaining object c . The outcome is shown below.

$$\begin{aligned} RSD(\succ) &= \frac{1}{6} \underbrace{\begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix}}_{\text{ordering is (1,2,3)}} + \frac{1}{6} \underbrace{\begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix}}_{\text{ordering is (1,3,2)}} + \frac{1}{6} \underbrace{\begin{pmatrix} & & 1 \\ 1 & & \\ & 1 & \end{pmatrix}}_{\text{ordering is (2,1,3)}} + \frac{1}{6} \underbrace{\begin{pmatrix} & & 1 \\ 1 & & \\ & 1 & \end{pmatrix}}_{\text{ordering is (2,3,1)}} \\ &+ \frac{1}{6} \underbrace{\begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix}}_{\text{ordering is (3,1,2)}} + \frac{1}{6} \underbrace{\begin{pmatrix} & & 1 \\ 1 & & \\ & 1 & \end{pmatrix}}_{\text{ordering is (3,2,1)}} = \frac{1}{6} \begin{pmatrix} 3 & 0 & 3 \\ 3 & 1 & 2 \\ 0 & 5 & 1 \end{pmatrix}. \\ &\qquad\qquad\qquad \qquad\qquad\qquad \qquad\qquad\qquad \begin{matrix} a & b & c \end{matrix} \end{aligned}$$

◇

The **probabilistic serial mechanism (PS)** [5] selects the random assignment by the following **simultaneous eating algorithm**: Given a preference profile \succ , think of each object as an infinitely divisible unit of probability that all agents “eat” from time 0 to 1.

Step 1: Each agent “eats away” from her favorite object at the same unit speed. Proceed to the next step when an object is completely exhausted.

Step s : Each agent eats away from her remaining favorite object at the same unit speed. Proceed to the next step when an object is completely exhausted.

The algorithm stops when each agent has eaten exactly 1 total unit of objects (i.e., at time 1). The random allotment of an agent i by PS is then given by the amount of each object that she ate before the algorithm stopped.

Example 2. We illustrate how to compute the PS outcome. Consider the same preference profile as in Example 1. In Step 1, agents 1 and 2 eat away from their favorite object a , while agent 3 eats away from her favorite object b . At time $1/2$, a is exhausted and the amount of $1/2$ of b is available. Then, in Step 2, all agents turn to her next favorite among the remaining objects b, c . That is, agent 1 eats away from her object c , while agents 2 and 3 eat away from object b . At time $t = 1/2 + 1/4 = 3/4$, b is exhausted and the amount of $3/4$ of c is available. In the last step, all agents eat away from the remaining object c . Thus,

$$PS(\succ) = \underbrace{\begin{pmatrix} \frac{1}{2} & & \\ \frac{1}{2} & & \\ & & \frac{1}{2} \end{pmatrix}}_{\text{amounts eaten in Step 1}} + \underbrace{\begin{pmatrix} & \frac{1}{4} & \\ \frac{1}{4} & & \\ \frac{1}{4} & & \end{pmatrix}}_{\text{amounts eaten in Step 2}} + \underbrace{\begin{pmatrix} & & \frac{1}{4} \\ & & \frac{1}{4} \\ & & \frac{1}{4} \end{pmatrix}}_{\text{amounts eaten in Step 1}} = \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{3}{4} & \frac{1}{4} \end{pmatrix} \begin{matrix} a \\ b \\ c \end{matrix}.$$

◇

Let $PS(\succ)$ be the random assignment of PS for problem \succ .

2.1. Efficiency

We turn to efficiency notions that rely only on ordinal preferences. A deterministic assignment D is **Pareto efficient** at \succ if there is no deterministic assignment $D' \in \mathcal{D}$ such that for each $i \in I$, $D' \succeq_i D$ and for some $i \in I$, $D' \succ_i D$. A random assignment $P \in \mathcal{R}$ is **ex post efficient** at \succ if it assigns positive probability only to Pareto efficient deterministic assignments.

Ex post efficiency is a weak condition, since a large set of deterministic assignments are Pareto efficient. Bogomolnaia and Moulin [5] introduce an appealing *ex ante* notion of ordinal efficiency based on first-order stochastic dominance: Let $i \in I$, and consider two random allotments P_i and Q_i . Then, P_i (**first-order**) **stochastically dominates** Q_i at \succ_i , denoted by $P_i \succeq_i^{sd} Q_i$ if for all $a \in A$, $\sum_{b \in A: b \succeq_i a} p_{i,b} \geq \sum_{b \in A: b \succeq_i a} q_{i,b}$. In other words, for any set of i 's “most preferred” alternatives, P_i assigns at least as great total probability to that set as Q_i does. If the inequality is strict for at least one $a \in A$, we say that P_i **strictly stochastically dominates** Q_i at \succ_i , denoted

by \succ_i^{sd} . Note that \succeq_i^{sd} is not complete as a binary relation. Also, \succeq_i^{sd} has a characterization in terms of cardinal utilities: $P_i \succeq_i^{sd} Q_i$ if and only if $u_i(P_i) \geq u_i(Q_i)$ for any utility function u_i consistent with \succ_i . Thus, stochastic dominance is a sufficient (but not necessary) condition for agent i to prefer the allotment P_i to Q_i .

Now we are ready to formalize ordinal efficiency. A random assignment P **stochastically dominates** Q **at** \succ if for each $i \in I$, $P_i \succeq_i^{sd} Q_i$. A random assignment is **ordinally efficient at** \succ_i if it is not stochastically dominated by another random assignment. Finally a mechanism is **ordinally efficient (ex post efficient)** if it selects an ordinally efficient (*ex post* efficient) random assignment for each preference profile.

Although it is *ex post* efficient, RSD is not ordinally efficient, as the following example shows.

Example 3. ${}^8I = \{1, 2, 3, 4\}$ and $A = \{a, b, \dots\}$.

$$\begin{array}{l|lll} \gamma_1 & a & b & \dots \\ \gamma_2 & a & b & \dots \\ \gamma_3 & b & a & \dots \\ \gamma_4 & b & a & \dots \end{array} \quad RSD(\succ) = \frac{1}{24} \begin{pmatrix} 10 & 2 & \dots \\ 10 & 2 & \dots \\ 2 & 10 & \dots \\ 2 & 10 & \dots \\ a & b & \end{pmatrix} \quad PS(\succ) = \frac{1}{24} \begin{pmatrix} 12 & 0 & \dots \\ 12 & 0 & \dots \\ 0 & 12 & \dots \\ 0 & 12 & \dots \\ a & b & \end{pmatrix}.$$

Here, under RSD, agent 1 gets alternative b if agent 2 is chosen first and she is chosen second, with probability 1 in 12. Under the PS mechanism, agents 1 and 2 each eat half of alternative a while agents 3 and 4 each eat half of alternative b . The random assignment $RSD(\succ)$ is not ordinally efficient, as it is stochastically dominated by $PS(\succ)$ at \succ . Note that for each $\succ' \in \mathbf{P}^I$, while $PS(\succ')$ is ordinally efficient, in general $PS(\succ')$ does not always stochastically dominate $RSD(\succ')$ at \succ' [5]. \diamond

To understand the substance of these properties, it may be helpful to consider their implications in this example. The fact that random assignment $PS(\succ)$ stochastically dominates random assignment $RSD(\succ)$ means that all four agents in this example would prefer $PS(\succ)$ to $RSD(\succ)$ *ex ante*, regardless of their cardinal utilities. Although no mutually advantageous trades would be possible after either assignment is “realized,” all agents would therefore agree *ex ante* to a shift from RSD to PS as a mechanism in this case.⁹

As our mechanisms are ordinal, ordinal efficiency is the most appealing efficiency property in our environment. However, given a specific utility profile, we cannot conclude that there is no other random assignment which all agents prefer to PS, since ordinal efficiency is a sufficient, but not necessary, condition for this. In fact, it is conceivable that all agents prefer a random assignment to one which is ordinally efficient.¹⁰ Thus ordinal efficiency is a property quite akin to Pareto efficiency, and likewise constitutes an arguably minimalist notion of desirability from a ‘social’ perspective.

⁸This example is from Bogomolnaia and Moulin [5].

⁹Kesten [13] shows that we can always achieve a PS assignment after allowing agents to engage in specific trades (based on Gale’s top trading cycles [17]) starting from the equal division of probabilistic shares.

¹⁰*Ex ante* efficiency, which is Pareto efficiency in terms of cardinal utilities, is stronger than ordinal efficiency [5].

2.2. Incentive compatibility

A mechanism φ induces a preference revelation game where each agent $i \in I$ is equipped with her strategy set \mathbf{P} . That is, a strategy is a reported preference which may or may not coincide with the agent's true preference. For a given profile of reported preferences \succ , the outcome of the game is $\varphi(\succ)$. We emphasize that the ordinal efficiency of a mechanism is defined for reported preference profiles. Even if a mechanism provides agents with an ordinally efficient random assignment at a reported preference profile, the random assignment induced by the mechanism might not be ordinally efficient at a truthful preference profile. For this reason, it is important for a mechanism to elicit truthful preferences from agents. For expositional purposes, we reserve the notation \succ to denote a truthful preference profile, and \succ'_i to denote an arbitrary preference of agent i . Similarly we use u to denote a truthful utility profile, and u'_i an arbitrary utility function of agent i . Now we are ready to introduce our notion of incentive compatibility.

Under a mechanism φ , a preference profile \succ' is a **Nash equilibrium under u** if for each $i \in I$ and each $\succ''_i \in \mathbf{P}$, $u_i[\varphi(\succ')] \geq u_i[\varphi(\succ''_i, \succ'_{-i})]$.

A mechanism φ is **strategy-proof** if for each $i \in I$, each $\succ \in \mathbf{P}^I$, and each $\succ'_i \in \mathbf{P}$, $\varphi_i(\succ) \succeq_i^{sd} \varphi_i(\succ'_i, \succ_{-i})$. . It is straightforward to see that under a non-strategy-proof mechanism, it is not always a Nash equilibrium under some utility profile for some agent to report a truthful preference. We will examine this case for the PS mechanism in the next section.

2.3. Related literature

To our knowledge, there is no experimental study on either RSD or the PS mechanism. A slightly different problem, called *a house allocation problem with existing tenants* [2], has been studied experimentally by Chen and Sönmez [7], Chen and Sönmez [8], and Guillen and Kesten [10]. The informational setting of Chen and Sönmez [8] and Guillen and Kesten [10] is incomplete information, while that of Chen and Sönmez [7] is complete information like our experiment.

3. Experimental Design

We implement the PS mechanism in a laboratory experiment. Our aim is to assess the extent to which individuals truthfully report their preferences. For this purpose, we design the smallest market which captures the incentive issues of the PS.¹¹ Subjects play in groups of three, and three objects (a, b, c) are allocated within each group. Utility profiles are induced through monetary payments. Subjects participate in four games involving different payoff profiles, given in the four tables below. Two of these games are symmetric (Sym-H and Sym-L), and two are asymmetric (Asym-H and Asym-L). In ordinal terms, the two symmetric and the two asymmetric games are identical. As we will see, however, incentives differ between these games because the value of

¹¹In a large but finite problem where there is a sufficiently large amount of each object, the PS becomes strategy-proof [15].

the second best option differs. The rationale behind the games' construction is as follows. The Sym-H and Sym-L games create no incentive to misreport one's preferences. The Asym-H game, by contrast, gives one agent an incentive to misreport his or her preferences, assuming the other agents report truthfully. Asym-H may also create a false perception by another agent that s/he should misreport his/her preferences: see below. The Asym-L game is a control, allowing us to check that behavior in Asym-H is indeed related to the incentives created by the valuable second best option.

Agents can be in one of eight situations in total (two in the symmetric games, six in the asymmetric games). We give these situations labels SH1 *et cetera*, as in the table, and refer to the agent in the relevant situation as "agent SH1" *et cetera*.

Sym-H				Sym-L			
Situation	a	b	c	Situation	a	b	c
SH1	100	95	0	SL1	100	30	0
SH1	100	95	0	SL1	100	30	0
SH1	100	95	0	SL1	100	30	0

Asym-H				Asym-L			
Situation	a	b	c	Situation	a	b	c
AH1	100	0	95	AL1	100	0	30
AH2	100	95	0	AL2	100	30	0
AH3	95	100	0	AL3	30	100	0

3.1. Benchmark predictions

We derive benchmark predictions for the PS mechanism by identifying Nash equilibria in the four games under consideration. Let us denote the utility profiles under the tables Sym-H, Sym-L, Asym-H, and Asym-L by u^{SH} , u^{SL} , u^{AH} , and u^{AL} . The utility profiles u^{SH} and u^{SL} of Sym-H and Sym-L are consistent with the ordinal preference profile \succ^S , where for each agent $i \in I$, $a \succ_i^S b \succ_i^S c$. Under this preference profile, truthful reporting induces the same random assignment under both PS and RSD mechanisms. That is,

$$PS(\succ^S) = RSD(\succ^S) = \begin{pmatrix} a & b & c \\ 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \end{pmatrix}.$$

For both of these symmetric utility profiles, truthful reporting by all agents constitutes the unique Nash equilibrium under the PS mechanism.¹²

¹²This result is verified by computation.

Next, consider the asymmetric utility profiles u^{AH} and u^{AL} , both of which are consistent with the ordinal preference profile in Example 1, denoted by \succ^{AS} . Under this preference profile, truthful reporting under the PS mechanism induces the following random assignment whose calculation is given in Example 3.

$$PS(\succ^{AS}) = \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{3}{4} & \frac{1}{4} \end{pmatrix}.$$

Suppose instead that agent 3 reports a false preference as $a \succ'_3 b \succ'_3 c$ (while agents 1 and 2 continue to report truthfully). This would induce the random assignment

$$PS(\succ'_3, \succ_{-3}^{AS}) = \begin{pmatrix} \frac{1}{3} & 0 & \frac{2}{3} \\ \frac{1}{3} & \frac{1}{2} & \frac{1}{6} \\ \frac{1}{3} & \frac{1}{2} & \frac{1}{6} \end{pmatrix}.$$

Note that $PS_3(\succ^{AS})$ does not stochastically dominate $PS_3(\succ'_3, \succ_{-3}^{AS})$ at \succ_3^{AS} as $PS_{3,b}(\succ^{AS}) + PS_{3,a}(\succ^{AS}) = 3/4 < 5/6 = PS_{3,b}(\succ'_3, \succ_{-3}^{AS}) + PS_{3,b}(\succ'_3, \succ_{-3}^{AS})$. By misreporting his preference, agent 3 decreases the probability of obtaining either his worst (c) or his best (b) outcome, in favor of his middle outcome (a). In expected utility terms, this deviation is attractive if his middle outcome is sufficiently close in value to his most preferred, as is the case under utility profile u^{AH} . Indeed, the following calculation demonstrates that lying is a best response for agent AH3 if the others report truthfully:

$$u_3^{AH}[PS_3(\succ^{AS})] = 0 \cdot 95 + \frac{3}{4} \cdot 100 + \frac{1}{4} \cdot 0 = 75 < \frac{1}{3} \cdot 95 + \frac{1}{2} \cdot 100 + \frac{1}{6} \cdot 0 = 81.66 = u_3^{AH}[PS_3(\succ'_3, \succ_{-3}^{AS})].$$

As a consequence, truthful reporting by all three agents is not a Nash equilibrium under utility profile u^{AH} . Closer analysis reveals that the outcome where only agent AH3 misreports by switching his top two choices constitutes the unique Nash Equilibrium under profile u^{AH} . Under u^{AL} , truthful reporting by all three agents is a Nash equilibrium, since in this case agent AL3 does not value his middle option as highly.¹³ We summarize these theoretical results in the following proposition.

Proposition 1. *Under the probabilistic serial (PS) mechanism,*

1. *in the two symmetric games (Sym-H and Sym-L), and in the asymmetric game where the middle options have less value (Asym-L), truthful reporting by all three agents constitutes the unique undominated Nash equilibrium.*
2. *in the asymmetric game in which the middle options are valued highly (Asym-H)*

¹³There exists a further equilibrium in which agent AL3 deviates by swapping his worst two options. However, this strategy is weakly dominated.

- (a) *truthful reporting by all agents is not a Nash equilibrium, since agent AH3 can profitably deviate by switching her top two choices.*
- (b) *the preference profile where only agent AH3 lies by switching her top objects constitutes the unique Nash equilibrium*

Proof. We have already verified Part 2(b), whose proof is from Bogomolnaia and Moulin [5]. The other parts are verified by computation. \square

The following result is well known and apparent. Thus no proof will be provided.

Proposition 2. *Under the random serial dictatorship (RSD) mechanism, truthful reporting constitutes a weakly dominant strategy for all agents in all games.*

Based on Propositions 1 and 2, we formulate the following benchmark predictions.

HYPOTHESIS 1: Under the PS mechanism,

1. Subjects will report their true preferences in all situations except situation AH3.
2. In situation AH3, subjects will misreport their preferences by swapping the ranking of their top two objects.

HYPOTHESIS 2: Under the RSD mechanism, subjects will report truthfully in all situations.

3.2. Alternative predictions

We have seen that theory suggests subjects should benefit from misreporting only in situation AH3 (if others report truthfully) under the PS mechanism. Despite this, it is conceivable that subjects might falsely perceive an incentive to deviate. As an example, consider an agent in situation AH1. This agent gets his top payoff from object a , but also gets a high payoff from object c . Object a is ranked first or second by both of the other agents. If agent AH1 expects the other two agents to report sincerely, it may therefore appear sensible for agents in situation AH1 to (falsely) rank object c first, avoiding competition for object a . The intuition underlying this perception is that object c is a “bird in the hand”, and that one risks losing it if one goes for option a “in the bush.” In fact, this intuition is mistaken: in the unique Nash equilibrium, where agent AH2 is sincere and agent AH3 is insincere, agent AH1 ought to be sincere. Agent AH1 should also be sincere if both other agents are sincere. These facts can be verified by computation. The correct intuition is that one increases one’s total probability of winning either a or c by going for object a first.

A similar intuition applies to agents in situation SH1. Here, any agent might consider ranking object b first in order to avoid competition for a . However, it is easy to verify that this would not be a good idea given sincere reporting by the other agents. In fact, sincerity by agent 1 is always optimal unless one of the other agents declared (b, c, a) , ranking her best object last.

Although these intuitions are false in the context of the PS mechanism, we suspect that they are quite compelling, as they do apply in other situations where people compete to obtain scarce

objects. In many such situations, there is indeed an advantage to avoid competition for the most popular option in order to obtain a highly valued alternative. In the school choice context, when the non-strategy-proof Boston mechanism was used for the Boston Public Schools system before 2004, experts often advised parents to place less popular schools high on their list in order to improve their chances of getting into these schools [3, 1]. This strategy is beneficial in so many contexts that one might conjecture the existence of a ‘don’t chase the popular girl’ heuristic. If such a heuristic exists, individuals may employ it in the PS mechanism even when this is not beneficial. If so, this would be bad news for the PS mechanism. In contrast, it is quite apparent that the intuition does not apply in the context of the RSD mechanism, as there is no ‘competition’ when agents sequentially choose. These considerations lead us to formulate the following hypothesis.

HYPOTHESIS 3a: Under the PS mechanism, subjects in situations SH1 and AH1 will misreport by switching their top two objects. This behavior is most likely to occur in situation AH1, since there the middle option is both highly valued and not desired by either of the remaining agents.

Note that the behavior predicted in Hypothesis 3a is inconsistent with the theoretical equilibrium predictions. If this behavior occurs within the experiment, the best responses for other agents in the relevant games may be different from those predicted in equilibrium. It is therefore interesting to ask what the best responses to these deviations from equilibrium play would be.

First, consider situation SH1. As explained above, sincerity is a best response in this situation as long as neither of the other agents declare (b, c, a) , ranking the best object (a) worst. In this sense, sincerity in situation SH1 is robust to deviations from equilibrium play. Next consider game Asym-H. If agent AH1 reports (c, a, b) , as Hypothesis 3a predicts, then truth-telling is always a best response for agents AH2 and AH3. Thus, the benchmark prediction concerning insincerity in situation AH3 is sensitive to the deviation from equilibrium play predicted in Hypothesis 3a. This will be important to keep in mind if we should find substantial amounts of misreporting in situation AH1.

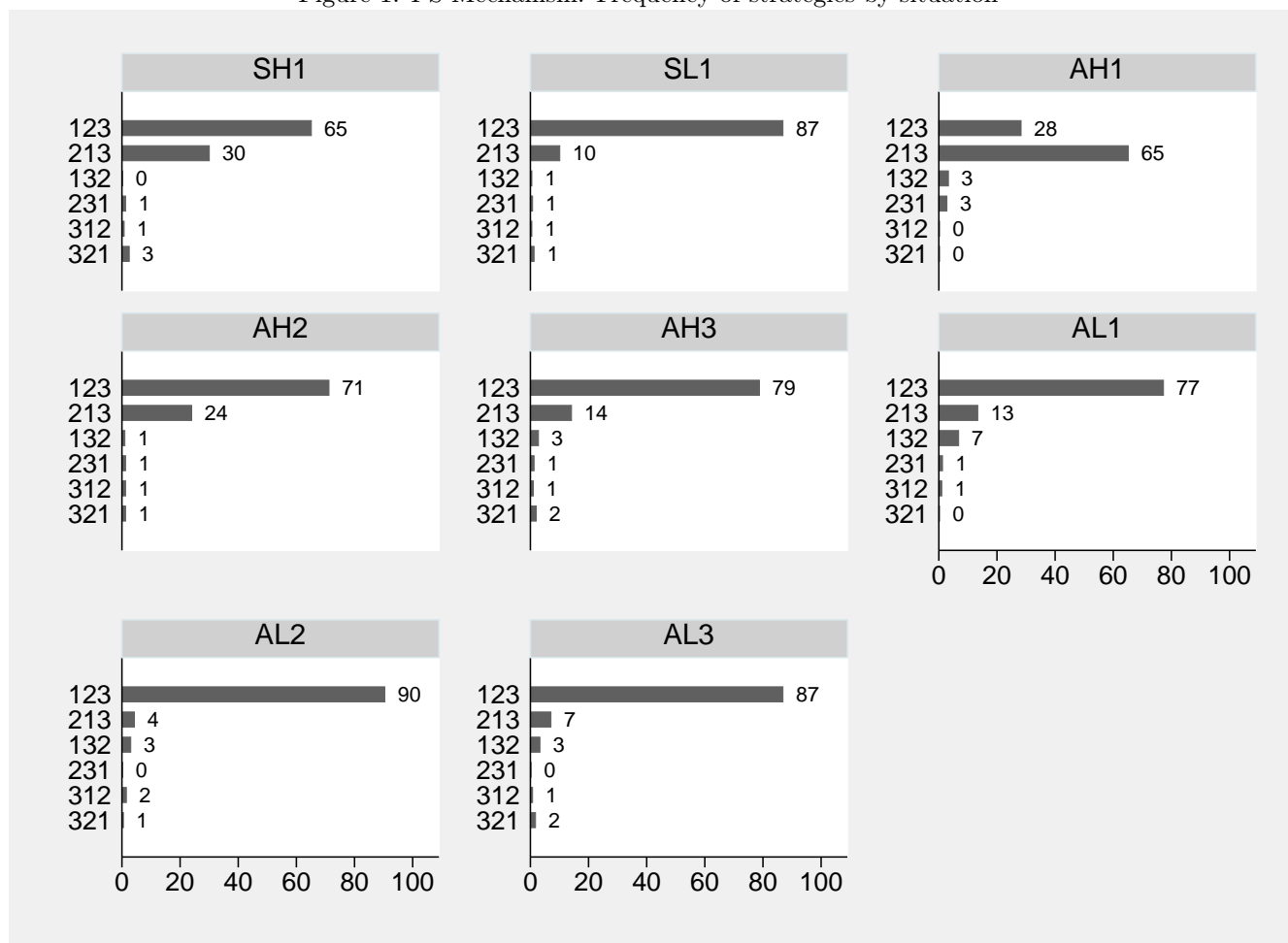
HYPOTHESIS 3b: Under the PS mechanism, if substantial misreporting occurs in situation AH1, we expect truthful reporting in situation AH3.

4. Results

We conducted experiments on the PS mechanism at the University of Warwick in January 2010. 66 subjects took part in the PS experiments in four sessions. As a benchmark, in May 2011 we also ran two sessions, with 36 subjects, using the RSD mechanism.

Experiments were programmed in z-Tree [9]. Each subject participated in 48 repetitions, experiencing each of the eight situations six times. The groups of three agents which played each game were rematched every round, as follows: within each session, subjects were placed in “matching

Figure 1: PS Mechanism: Frequency of strategies by situation[†]



[†]Each three-digit code represents the placement of the options truly ranked 1st, 2nd, and 3rd. For example, strategy 123 is “sincere” while strategy 213 means switching the top two options.

groups” of six people and rematched within this group alone.¹⁴ This prevents contamination of behavior between matching groups, and allows us to treat the groups of 6 as fully independent observations. One repetition was chosen at random for payment, and subjects were paid privately.

4.1. Aggregate statistics pooling all observations under PS

Figure 1 shows the different patterns of behavior within each of the eight distinct situations that subjects face in the experiment on the PS mechanism. Although these numbers involve repeated observations from the same subjects and matching groups, they will give us a good rough picture of important patterns in our data. Table 4.1 reports the overall frequency of sincere reporting within each situation.

Overall, subjects report sincere rankings 73% of the time. Two patterns are immediately

¹⁴Subjects were not aware of the exact design of matching groups; they were simply told that they would be rematched “at random.”

Table 1: PS Mechanism: Frequency of sincere behavior across situations

Situation	Agent	Object values			Sincere	N
		A	B	C		
Game 1						
SH1	1	100	95	0	65.15 %	396
	2	100	95	0		
	3	100	95	0		
Game 2						
SL1	1	100	30	0	86.87 %	396
	2	100	30	0		
	3	100	30	0		
Game 3						
AH1	1	100	0	95	28.28 %	396
AH2	2	100	95	0	71.21 %	396
AH3	3	95	100	0	78.79 %	396
Game 4						
AL1	1	100	0	30	77.27 %	396
AL2	2	100	30	0	90.40 %	396
AL3	3	30	100	0	86.87 %	396
Total					73.11 %	3,168

Table 2: PS Mechanism: % of non-sincere reports formed by switching top two options

	Total	situation							
		1	2	3	4	5	6	7	8
top switches	78%	86%	77%	91%	83%	67%	59%	45%	54%
N non-sincere	852	138	52	284	114	84	90	38	52

discernible. The first is that situation AH1 involves by far the least sincere reporting, with only 28%. Second, non-sincere behavior appears to be more common in situations where the payoff difference between the top two choices is small. Thus, sincere reporting is more common in situation SL1 (87%) than it is in situation SH1 (65%), and likewise for situations AL1 , AL2 and AL3 vs. situations AH1, AH2 and AH3 respectively.

Another pattern observable at this aggregate level is that subjects who report non-sincere rankings most often reverse the ranking of their top two choices. Quite sensibly, subjects rarely try to game the mechanism by falsely ranking their *least* preferred option.¹⁵ This is shown in Table 2. Especially in those situations where insincere behavior is the most common (SH1, AH1, AH2, AL1), the vast majority of false rankings are formed by swapping the two top choices.

These data appear to support Hypothesis 3: insincere behavior occurs most often in situation AH1, followed by situations SH1 and AH2. Using matching groups as units of observation

¹⁵The fact that we observe this behavior at all may be attributable to experimentation or confusion.

Figure 2: PS Mechanism: Frequencies of sincere reporting by situation and matching group[†]



[†] Each bar in these graphs represents 36 observations from a single matching group of 6 individuals. (Each individual is observed 6 times in each situation.)

In order to assess statistical significance, we now turn to the next level of aggregation, the matching group. We will be able to do some significance testing here because the groups yield statistically independent observations.

Figure 2 shows the overall frequencies of sincere reporting in each situation under the PS mechanism, separately for each matching group. For any given pair of situations, say SH1 and SL1, we have 11 independently measured pairs of frequencies. These data can be used to compare sincerity between situations using a Wilcoxon signed-rank test.¹⁶

First, we can reject both parts of hypothesis 1. Subjects do not act sincere in all situations but

¹⁶This is a nonparametric paired-sample test. In our case, the members of each matching group are observed in two conditions. The test is based on counting the number of groups for whom the fraction of sincere reports is larger in one condition than in the other. The central assumption is that the measurements are statistically independent between the groups.

situation AH3. Furthermore, situation AH3 elicits *more* sincere behavior than any other treatment in which the middle option is worth 95.

Result 1. *Non-sincere behavior is not more common in Situation AH3 than in situations SH1, AH1 or AH2.*

Next, consider the claim made above that situation AH1 involves less sincere reporting than all other situations. Signed rank tests show that this difference is highly significant, with $p < 0.01$ on all seven comparisons. In fact, situation AH1 involves the least sincere reporting across all situations within each and every matching group.

Result 2. *Non-sincere behavior is significantly more common in Situation AH1 than in any other situation.*

Next, consider the idea that strategic behavior is more common when the top options are similar in value. The relevant comparisons are situations SL1 vs. SH1, AL1 vs. AH1, AL2 vs. AH2 (all of which yield $p < 0.01$ due to the fact that the latter situation involves more sincerity than the former in every single matching group), and AL3 vs. AH3 ($p=0.02$).

Result 3. *Non-sincere behavior is significantly more common in situations where the top two choices are similar in value.*

Together these results lend support to Hypothesis 3. We now turn to exploring why and how this behavior occurs.

4.2. Can insincerity in situation AH1 be rationalized?

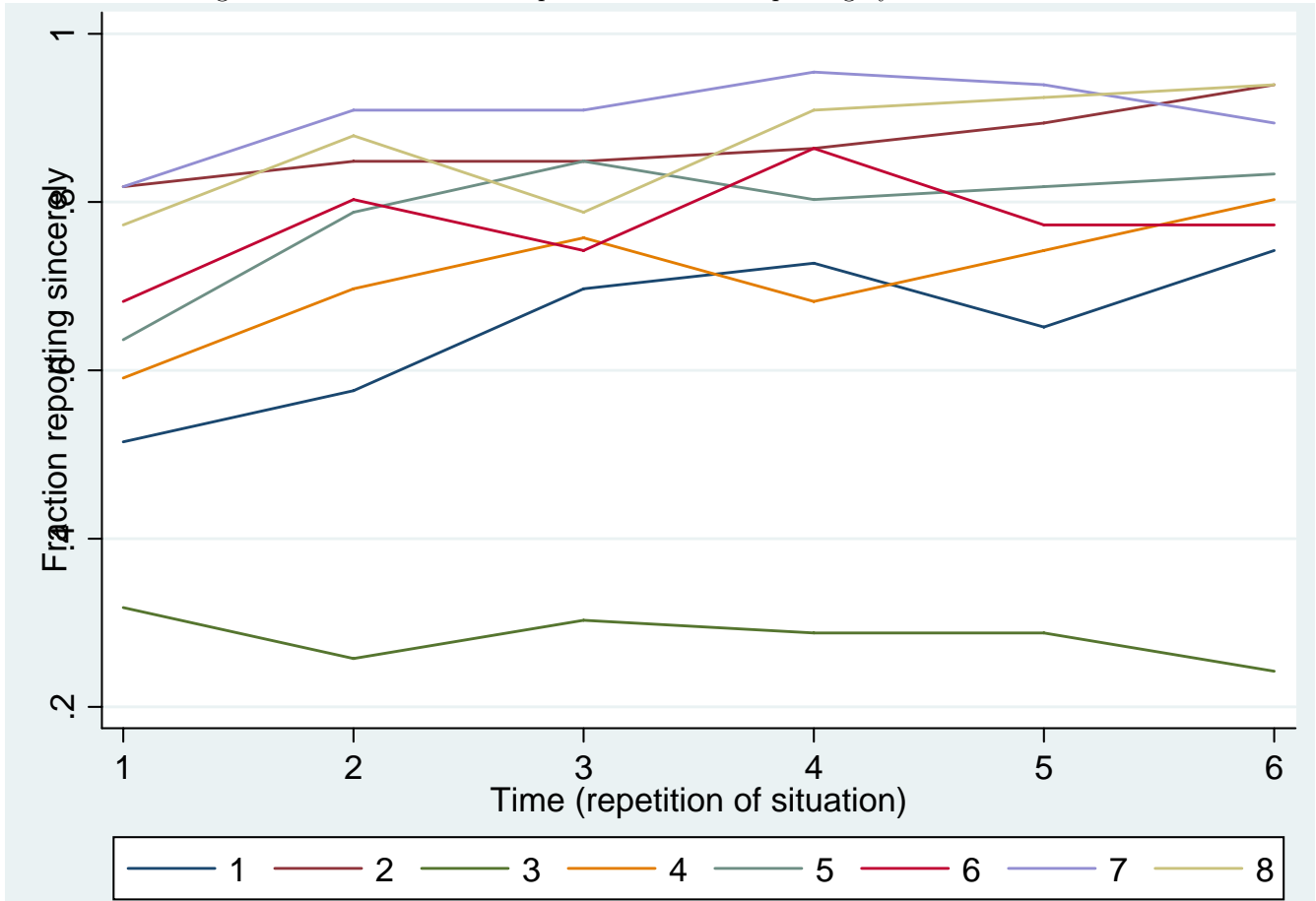
One possibility is that insincerity in situation AH1 is a best response to observed behavior. However, we can discount this at once. It can be verified that it is optimal for agent AH1 to swap her top two preferences, only if one of the other agents declared (c, b, a) , putting their third preference first. But agent AH2 or AH3 declared (c, b, a) in only 1% of cases (9 out of 792). By contrast, because in a majority of cases agent AH1 reported insincerely, we can rationalize agent AH3's high levels of sincerity as a best response (see Hypothesis 3b).

Since AH1's insincerity is not a best response, it is natural to ask whether it disappears over time. Figure 3 takes an initial look at this question by graphing the frequency of sincere behavior in each situation, over the 6 repetitions of the situation which each subject goes through. While in other situations, insincerity appears to decrease, it does not do so in situation AH1. Individual-level regressions confirm this first impression: insincerity is not significantly predicted either by repetition from 1 to 6, or by time period from 1 to 48.¹⁷

A potentially important alternative hypothesis is that subjects are intentionally "settling for" their second best option in situation AH1, in order to allow the other subjects to obtain their first

¹⁷Details in the appendix.

Figure 3: PS Mechanism: Frequencies of sincere reporting by situation over time[†]



[†] The time index refers to the number of repetitions of each situation. Each subject sees each situation 6 times. The frequency reported is the fraction of subjects reporting truthfully the n-th time they are in that situation. (thus it combines decisions that are not actually made simultaneously, as when Tina is in situation AH1 for the third time in period 20, and Tom in period 25)

choices for sure. If subjects in roles AH2 and AH3 report sincerely, then the subject in role AH1 can choose to switch her top two choices, in which case with 100% probability the result is that she gets her second choice (worth 95) and the other subjects each get their first choices (worth 100). If instead she reports sincerely, the resulting lottery gives her an equal chance of obtaining her top two choices and each of the other subjects has a 25% chance of obtaining their worst option. Thus, observed behavior might be explained by social preferences, rather than by the behavioral heuristic that we posit.

If this were so, then under the RSD mechanism, there would also be a strong incentive for this subject to switch her top two preferences. For again, by doing so she would ensure her second choice and ensure that other subjects each get their first choices. By reporting sincerely she would again get an equal chance of her top two choices, while agent AH2 would get her worst option with probability 50% and agent AH3 would get her worst option with probability one in six.

To test this explanation, we examine behaviour in the two sessions which implemented the RSD mechanism. Table 3 shows the frequency of sincere behavior under RSD. As in the PS mechanism, there is less sincere behavior in situation AH1 than in all other situations. However, the difference is much smaller, with a majority of agents behaving sincerely. In fact, there was significantly more sincere behavior in the RSD sessions than in the PS sessions (Wilcoxon rank-sum test, $p < 0.01$).

There were six matching groups in the RSD sessions. Wilcoxon signed-rank tests at matching group level showed some evidence for less sincere behavior in situation AH1 compared to situations SL1, AH3, AL1, AL2 and AL3 ($p < 0.1$ in each case) but not compared to situations SH1 or AH2 ($p = 0.344$ in both cases).

5. Conclusion

We report the first laboratory experiment on the PS mechanism. While the PS mechanism is not strategy-proof, we were not able to observe strategic misrepresentation of preferences in situations where it would have benefited agents. However, we did find high levels of misrepresentation in a situation where this was not appropriate. This behavior did not become less prevalent with subjects' experience.

We conclude that the design of matching mechanisms, like auction design, needs to be sensitive to behavioral anomalies as well as to optimal strategic behavior. This could be a concern for the use of the mechanism in real-world situations, particularly because, while PS becomes strategy-proof in large enough economies, this does not protect against mistaken strategic behaviour.

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Table 3: RSD Mechanism: Frequency of sincere behavior across situations

Situation	Agent	Object values			Sincere	N
		A	B	C		
Game 1						
SH1	1	100	95	0	67.13 %	216
	2	100	95	0		
	3	100	95	0		
Game 2						
SL1	1	100	30	0	83.80 %	216
	2	100	30	0		
	3	100	30	0		
Game 3						
AH1	1	100	0	95	62.96 %	216
AH2	2	100	95	0	73.16 %	216
AH3	3	95	100	0	85.19 %	216
Game 4						
AL1	1	100	0	30	90.74 %	216
AL2	2	100	30	0	89.35 %	216
AL3	3	30	100	0	88.89 %	216
Total					80.15 %	1,728

- [1] ABDULKADIROĞLU, A., P. A. PATHAK, A. E. ROTH, AND T. SÖNMEZ (2005): “The Boston Public School Match,” *American Economic Review Papers and Proceedings*, 95, 368–372.
- [2] ABDULKADIROĞLU, A., AND T. SÖNMEZ (1999): “House Allocation with Existing Tenants,” *Journal of Economic Theory*, 88, 233–260.
- [3] ——— (2003): “School Choice: A Mechanism Design Approach,” *American Economic Review*, 93, 729–747.
- [4] BIRKHOFF, G. (1946): “Three Observations on Linear Algebra,” *Revi. Univ. Nac. Tucuman, ser A*, 5, 147–151.
- [5] BOGOMOLNAIA, A., AND H. MOULIN (2001): “A New Solution to the Random Assignment Problem,” *Journal of Economic Theory*, 100, 295–328.
- [6] BUDISH, E., AND E. CANTILLON (2012): “The Multi-unit Assignment Problem: Theory and Evidence from Course Allocation at Harvard,” *American Economic Review*, 102(5), 2237–2271.
- [7] CHEN, Y., AND T. SÖNMEZ (2002): “Improving Efficiency of On-campus Housing: An Experimental Study,” *American Economic Review*, 92, 1669–1686.
- [8] ——— (2004): “An Experimental Study of House Allocation Mechanisms,” *Economics Letters*, 83, 137–140.

- [9] FISCHBACHER, U. (2007): “z-Tree: Zurich Toolbox for Ready-made Economic Experiments,” *Experimental Economics*, 10(2), 171–178.
- [10] GUILLEN, P., AND O. KESTEN (2012): “Matching Markets with Mixed Ownership: The Case for a Real-life Mechanism,” *International Economic Review*, 53(3), 1027–1046.
- [11] HASHIMOTO, T., D. HIRATA, O. KESTEN, M. KURINO, AND M. U. ÜNVER (2012): “Two Axiomatic Approaches to the Probabilistic Serial Mechanism,” Forthcoming, *Theoretical Economics*.
- [12] HYLLAND, A., AND R. ZECKHAUSER (1979): “The Efficient Allocation of Individuals to Positions,” *Journal of Political Economy*, 87, 293–314.
- [13] KESTEN, O. (2009): “Why Do Popular Mechanisms Lack Efficiency in Random Environments?,” *Journal of Economic Theory*, 144, 2209–2226.
- [14] KESTEN, O., AND A. YAZICI (2012): “The Pareto Dominant Strategy-Proof and Fair Rule for Problems with Indivisible Goods,” *Economic Theory*, 50 (2).
- [15] KOJIMA, F., AND M. MANEA (2010): “Incentives in the Probabilistic Serial Mechanism,” *Journal of Economic Theory*, 144, 106–123.
- [16] ROTH, A. E., T. SÖNMEZ, AND M. U. ÜNVER (2004): “Kidney Exchange,” *Quarterly Journal of Economics*, 119, 457–488.
- [17] SHAPLEY, L., AND H. SCARF (1974): “On Cores and Indivisibility,” *Journal of Mathematical Economics*, 1, 23–37.
- [18] SÖNMEZ, T., AND M. U. ÜNVER (2010): “Course Bidding at Business Schools,” *International Economic Review*, 51, 99–123.
- [19] VON NEUMANN, J. (1953): “A Certain Zero-sum Two-person Game Equivalent to the Optimal Assignment Problem,” in *Contributions to the theory of games, Vol. 2*, ed. by H. W. Kuhn, and A. W. Tucker. Princeton University Press, Princeton, New Jersey.

Appendix

Regressions of insincerity on time. The dependent variable is a dummy taking the value 1 if the subject switched her top two options. The regressions show a marginally significant increase in insincerity over time. If we use a dummy for any kind of insincerity as a dependent variable, significance vanishes.

	(1)	(2)
Constant	0.1628 (0.2885)	0.2087 (0.2463)
Repetition	0.1345 + (0.0760)	
Period		0.0163 + (0.0090)
N	396	396
N indiv.	66	66
Chi-sq	4.69	5.00

Standard errors clustered by individual in parentheses. + $p < 0.10$

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