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Working Paper No. 119

**Lectures on Fertility, Savings,
Inter-Generational Transfers and Gender**

by

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Lectures on Fertility, Savings, Inter-Generational Transfers and Gender¹

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”Let us assume that men enter the labor market at about the age of twenty. They work for forty-five years or so and then live for fifteen years in retirement. Naturally, ... men will want to consume less than they produce in their working years so that they can consume something in the years when they produce nothing. ... If there were only Robinson Crusoe, he would hope to put by some durable goods which could be drawn on in his old age. He would, so to speak, want to trade with Mother Nature current consumption goods in return for future consumption goods. ... For the present purpose, I shall make the extreme assumption that nothing will keep at all. Thus no intertemporal trade with Nature is possible. If Crusoe were alone, he would obviously die at the beginning of his retirement years. But we live in a world where new generations are always coming along. ... [C]annot men during their productive years give up some of their product to bribe other men to support them in their retirement years?” (Samuelson, 1958)

1 Introduction²

The answer to Paul Samuelson’s question is "yes, if there are ways of ensuring that the bribed person will deliver his side of the deal when the time comes". Samuelson’s own solution to this enforcement problem is what he calls "social contrivances": contract law and its associated legal enforcement apparatus, money that "gives workers of one epoch a claim on workers of a later epoch" (Samuelson, 1958). But what about the very young? They need support too, indeed more than the old because, unlike them, they have not had an earlier phase of life in which to put by durable goods. Therefore, if anyone is willing to be "bribed", it is precisely them. The problem is that Samuelson’s contrivances are not much help here. In most legal systems, minors are not allowed to enter into binding commercial agreements (and babies could not anyway). Why is there no mention of them in Samuelson’s analysis then? A possible explanation is that Samuelson, and many other authors after him, assume that parents do

¹Prepared for the Max Planck Institute Population Economics Summer School, Rostock 2011.

²This section draws on Cigno (2006b).

not need any material incentive to take care of their own children, because successful animal species are genetically programmed to do so. But is that enough? The existence of laws and social norms deputed to ensure that children get adequate support suggests that it may not. This does not necessarily mean that parents do not care about their children, but does imply that externalities, or some other kind of coordination failure, could be responsible for at least some of the parents giving their children less than is socially desirable.

The aim of these lectures is to examine the scope for mutually beneficial intergenerational cooperation, and explain the emergence of certain norms and institutions as a rational response to the coordination problems we have just outlined. For simplicity, I will assume that parents can deterministically choose how many children to have, but the analysis may be extended to the more realistic case where parents can only choose the probability distribution of the number of births.³ Whichever is the case, the endogenization of fertility raises questions about the meaning of "efficiency". The ordinary Pareto criterion allows us to compare different allocations of goods to the same collection of individuals, and cannot thus be applied in a context where the existence of future adults depends on decisions taken by the present ones. We can use, however, the extended Pareto criterion proposed by Baland and Robinson (2002). According to this extension, an allocation **A** Pareto-dominates an allocation **B** if both the utility of the parent, *and* the average utility of the children, are higher in **A** than in **B**. The reference to an *average* level of utility leaves the door open for the possibility that the number of children associated with **A** is different from the one associated with **B**. Combined with the assumption that all members of the same family are the same, it then allows us to characterize an efficient allocation of family resources as one which equalizes the marginal rates of substitution of all family members, and the parent's marginal valuation of fertility to the marginal cost.

2 Parental altruism

Consider the following extension of the model of parental choice with descending altruism in Becker and Barro (1988). People live three periods, labelled $t = 0, 1, 2$. In period 0, they depend on their parents. In period 1, they work, and can choose to have children. In period 2, they can neither work nor have children. The economic environment is known with certainty and constant over time. The utility function is

$$U = u_0(a_0) + u_1(a_1) + u_2(a_2) + \beta n U^*(c, m + w), \quad 0 \leq \beta < 1, \quad (1)$$

where a_t is parental consumption in period t , β a measure of parental altruism, n the number of children, $U^*(c, m + w)$ a child's lifetime utility conditional on $(c, m + w)$, c the child's consumption in period 1, m the transfer this child receives from her parent in period 2, and w the wage rate. We assume that $u_t(\cdot)$ and $U^*(\cdot)$ are concave.

³See Cigno and Luporini (2011).

The budget constraints are

$$a_1 + (p + c)n + s = y \quad (2)$$

in period 1, and

$$a_2 + mn = sr \quad (3)$$

in period 2, where s is saving, r the interest factor, y parental income in period 1 and p the fixed cost of a birth (the total cost is $p + c$). This fixed cost includes the minimum income that the mother must forgo to give birth to this child, and the child's subsistence consumption. The maximum a person can borrow (without pledging collateral) in period 1 is b ,

$$-b \leq s. \quad (4)$$

Transfers to children cannot be negative,

$$0 \leq mn, \quad (5)$$

because parents cannot take money from their grown-up children in period 2, and a contract committing the latter to support the former in old age in return for goods and services received a period earlier, would not be legally enforceable.

2.1 Parental choice under conditions of certainty⁴

In period 1, a_0 is a by-gone. A person then chooses (a_1, a_2, s, m, n) to maximize (1), subject to (2) – (5). Form the Lagrangian

$$\mathcal{L} = u_1(y - (p + c)n - s) + u_2(sr - mn) + \beta n U^*(c, m + w) + \mu(b + s) + \nu mn,$$

where μ and ν are the Lagrange-multipliers of, respectively, (4) and (5). The first-order conditions for the maximization of this expression w.r.t. s , c , m and n are, respectively,

$$-u'_1 + ru'_2 + \mu = 0, \quad (6)$$

$$-nu'_1 + n\beta U^*_c = 0, \quad (7)$$

$$-nu'_2 + n\beta U^*_m + \nu n = 0 \quad (8)$$

and

$$-(p + c)u'_1 - mu'_2 + \beta U^* + \nu m = 0. \quad (9)$$

Therefore, parents save or borrow to the point where

$$MRS_{a_1, a_2} \equiv \frac{u'_1}{u'_2} = r + \frac{\mu}{u'_2},$$

⁴The analysis that follows draws on Baland and Robinson (2000, 2002), and Ch. 1 of Cigno and Rosati (2005).

give their grown-up children money to the point where

$$MRS_{c, m+w} \equiv \frac{U_1^*}{U_2^*} = \frac{u_1'}{u_2' - \nu},$$

and have children to the point where

$$MRS_{a_1, n} \equiv \beta \frac{U^*}{u_{a_1}} = p + c + \frac{m}{MRS_{a_1, a_2}} - \nu m.$$

If neither (4) nor (5) is binding at the optimum,

$$\mu = \nu = 0.$$

The marginal rates of substitution of present for future consumption are then equalized across generations, and their common value equated to the interest factor,

$$MRS_{a_1, a_2} = r = MRS_{c, m+w}.$$

parents have children to the point where the value of the last child is equal to the cost,

$$MRS_{a_1, n} = c + \frac{m}{r}.$$

Therefore, the allocation is a Pareto optimum.

If the borrowing constraint is binding,

$$\mu > 0 \text{ and } \nu = 0,$$

the children's MRS of present for future consumption is equated to the interest factor, but that of the parents is higher,

$$MRS_{a_1, a_2} > r = MRS_{c, m+w}.$$

If the transfers constraint is binding,

$$\mu = 0 \text{ and } \nu > 0,$$

the MRS of present for future consumption of the parents is equated to the interest factor, but the children's is higher,

$$MRS_{a_1, a_2} = r < MRS_{c, m+w}.$$

This case is illustrated in Figure 1. At point A, the children's MRS of present for future consumption is equated to the parent's, and to r . At B, the children's MRS is higher than r . B is located on an indifference curve lower than the one through A. If both constraints are binding,

$$\mu > 0 \text{ and } \nu > 0,$$

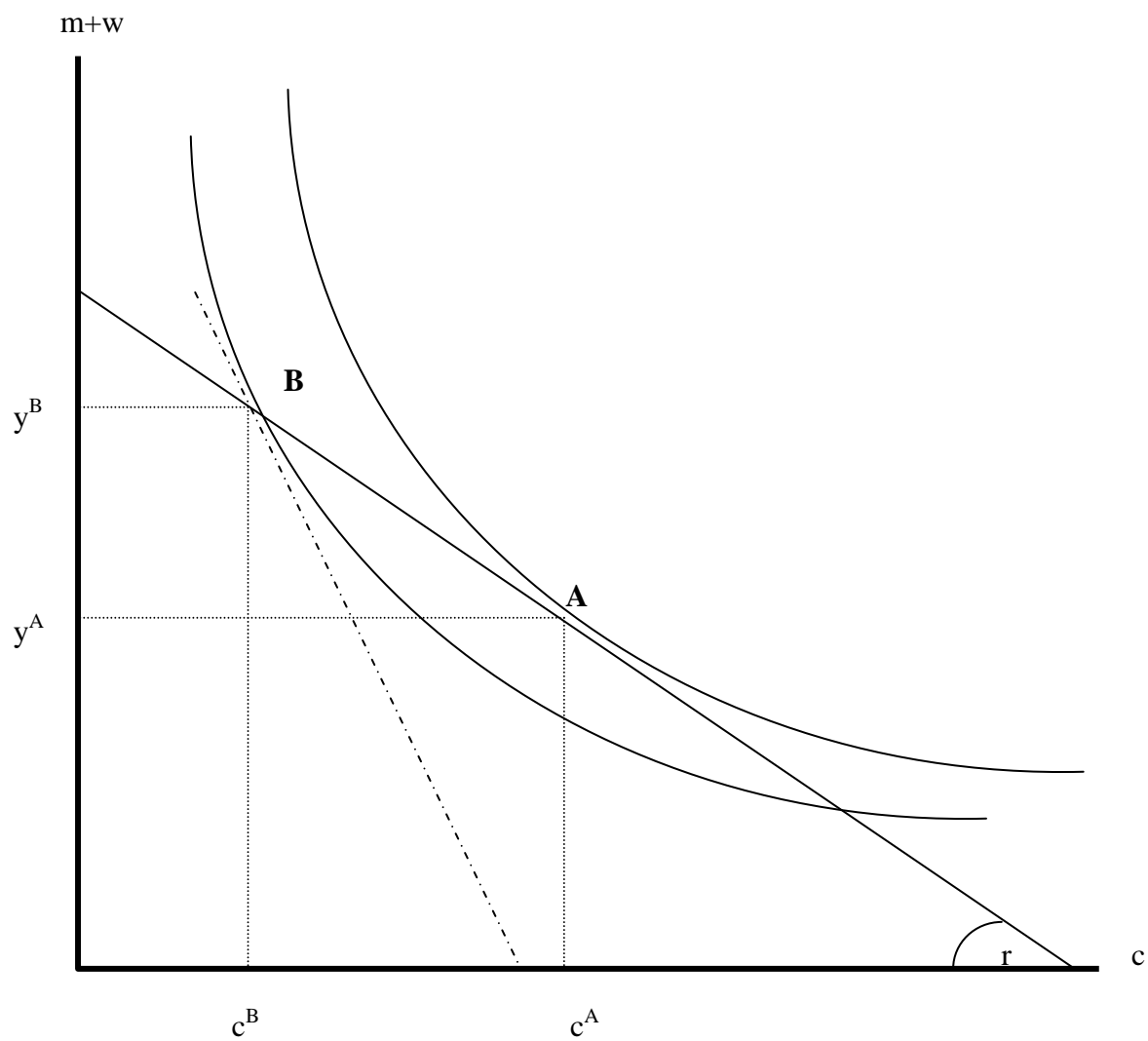


Figure 1: The effect of a nonnegative-transfer constraint.

the children's MRS is higher than the parent's, and both are higher than the interest factor,

$$r < MRS_{a_1, a_2} < MRS_{c, m+w}.$$

If either or both constraints are binding, the allocation will be inefficient because either or both intertemporal trade-offs are distorted, and the marginal cost of an extra child is inefficiently low,

$$MRS_{a_1, n} > p + c + \frac{m}{r}.$$

Therefore, fertility will be inefficiently high.

2.2 Parental choice under conditions of uncertainty⁵

If some period-2 parameter is uncertain, and parents cannot buy insurance, the allocation will be inefficient even if (4) and (5) are slack. Let us reformulate the model slightly. Let $n = 1$. Replace above-subsistence consumption, c , by educational expenditure, e , so that

$$w = e\omega,$$

where ω is the return to education. As a child's period-0 consumption is now a constant, we can re-write (1) as

$$U = u_0(a_0) + u_1(a_1) + u_2(a_2) + \beta U^*(m + e\omega), \quad 0 \leq \beta < 1, \quad (10)$$

If ω were certain, parents would be indifferent between (a) investing in the child's education (and possibly borrowing) in period 1, and (b) saving (and possibly giving the child money in period 2). If they chose (a), they would then raise e to the point where the opportunity-cost equals the return,

$$\frac{u_{a_1}}{\beta U^{*'}} = \omega. \quad (11)$$

Suppose, however, that the return to e is uncertain because a child's learning ability, or period-2 labour market conditions, are not known in advance. Education is then a risky investment. As the concavity of $U^*(\cdot)$ now implies risk aversion, parents may now buy education *and* save (buy conventional assets) at the same time. Assume, for simplicity, that ω is a random variable, with binomial distribution:

$$\begin{aligned} \omega &= 1 \text{ with probability } \pi, \\ \omega &= 0 \text{ with probability } (1 - \pi). \end{aligned}$$

If the parents can buy insurance i at the price (premium) ϕ , they will maximize

$$E(U) = u_1(y - p - e - s - \phi i) + u_2(sr - m) + \beta [\pi U^*(m + e) + (1 - \pi) U^*(m + i)]$$

⁵This subsection is inspired by Pouliot (2006).

The first-order conditions are the

$$-u'_1(y - p - e - s - \phi i) + \pi \beta U^{*'}(m + e) = 0,$$

$$-u'_1(y - p - e - s - \phi i) + r u'_2(sr - m) = 0,$$

$$-\phi u'_1(y - p - e - s - \phi i) + (1 - \pi) \beta U^{*'}(m + i) = 0$$

and

$$-u'_2(sr - m) + \pi \beta U^{*'}(m + e) + (1 - \pi) \beta U^{*'}(m + i) = 0.$$

Therefore, parents will borrow or save to the point where

$$MRS_{a_1, a_2} \equiv \frac{u_{a_1}(y - p - e - s - \phi i)}{u_{a_2}(sr - m)} = r,$$

and buy e and m to the point where the opportunity-cost equals the expected return,

$$\frac{u'_1(y - p - e - s - \phi i)}{\beta U^{*'}(m + e)} = \pi \quad (12)$$

and

$$u'_2(sr - m) = \beta [\pi U^{*'}(m + e) + (1 - \pi) U^{*'}(m + i)].$$

They also buy i to the point where the expected return equals the premium,

$$(1 - \pi) \frac{\beta U^{*'}(m + i)}{u'_1(y - p - e - s - \phi i)} = \phi.$$

If the insurance premium is "fair",

$$\phi = \frac{1 - \pi}{\pi},$$

the parents will insure in full against the risk of an unfavourable outcome.

If the market does not offer insurance against the risk of a low return to educational investment (e.g., because of the moral hazard), however, the parents will maximize

$$E(U) = u_1(y - p - e - s) + u_2(sr - m) + \beta [\pi U^*(m + e) + (1 - \pi) U^*(m)].$$

The first-order conditions are then

$$-u'_1(y - p - e - s - \phi i) + \pi \beta U^{*'}(m + e) = 0,$$

$$-u'_1(y - p - e - s - \phi i) + r u'_2(sr - m) = 0,$$

and

$$-u'_2(sr - m) + \pi \beta U^{*'}(m + e) + (1 - \pi) \beta U^{*'}(m + i) = 0.$$

Parents will now buy e to the point where

$$\frac{u'_1(y - p - e - s)}{\beta U^{*'}(m + e)} = \pi. \quad (13)$$

As $u'_1(\cdot)$ is a decreasing function,

$$\frac{u'_1(y - p - e - s)}{\beta U^{*t}(m + e)} < \frac{u'_1(y - p - e - s - \phi i)}{\beta U^{*t}(m + e)}.$$

Therefore, as the RHS of (13) is the same as that of (12), e is lower than it would be if parents could insure against the risk of a low return to education (low learning ability, poor labour market conditions or individual bad luck). Without insurance, educational investment is inefficiently low even if saving and transfers are interior.

3 Family rules⁶

We have seen that parental altruism does not guarantee efficiency, even if credit and insurance markets are perfect, because certain mutually beneficial contracts between parents and children are not legally enforceable. Could a set of family rules, a "family constitution", surrogate for a legally binding contract? In the absence of an external enforcement agency, it must be shown that such a constitution is self-enforcing, in the sense that it is in every family member's interest to obey it, and in the stronger one that it is in no generation's interest to change it.

A family constitution typically consists of a set of unwritten, typically unspoken, rules dictating (i) the minimum amount of money (or personal services yielding the same utility), z , that an "adult" (a person in period 1 of life) must transfer to each of her children, if she has any, when the latter are "young" (i.e., in period 0 of the children's lives), and (ii) the minimum amount of money (or personal services yielding the same utility), x , that the same adult must transfer to her parent when the latter is "old" (i.e., in period 2 of the parent's life), conditional on the parent having obeyed the rules in the previous period. The conditionality attached to (ii) makes it in every adult's interest to punish transgressors. That is important, because only an adult can punish another adult. Neither the young nor the old have the means to do so. Each adult has then a choice of two strategies: *comply* (cooperate), or *go it alone* in the market (defect). Consider first the simple case where people are completely selfish,

$$\beta = 0,$$

the personal services an adult might give her young children and old parent have perfect market substitutes (so that z and x are just money), and there is no uncertainty.

The pay-off of going it alone is

$$v(r, y) = \max_s u_1(y - s) + u_2(rs), \text{ s.t. } (4).$$

Since saving is the only means of providing for old-age consumption, (4) will never be binding. As in standard life-cycle theory, go-it-aloners will then equate

⁶The analysis that follows draws on Cigno (1993, 2006a).

the marginal rate of substitution of present for future consumption to the interest factor,

$$MRS_{a_1, a_2} \equiv \frac{u'_1(y-s)}{u'_2(rs)} = r. \quad (14)$$

The effects of small changes in (r, y) on the pay-off of this strategy are

$$v_r(r, y) \equiv su'_2(rs), \quad v_y(r, y) \equiv u'_1(y-s), \quad (15)$$

both positive.

Complying has a fixed cost, x , and sets a floor on the amount that can be transferred to young children.

$$nz \leq nc. \quad (16)$$

On the other hand, however, it relaxes the nonnegativity constraint on the amount that can be transferred to grown-up children, (5), which becomes

$$-nx \leq nm. \quad (17)$$

Having set β equal to zero, both these constraints will be satisfied as equations. As we will see in a moment, in equilibrium without uncertainty, the marginal return to money spent on children is strictly larger than the interest factor,

$$\frac{x}{p+z} > r. \quad (18)$$

Therefore, compliers will not lend, but will borrow their full credit ration and invest it in children. This tells that (4) will be satisfied as an equation.

Normalizing b to zero, we can write the pay-off of complying as

$$v^*(p, x, y, z) = \max_n u_1(y-x-(p+z)n) + u_2(xn).$$

A complier's choice of n will satisfy

$$MRS_{a_1, a_2} \equiv \frac{u'_1(y-x-(p+z)n)}{u'_2(xn)} = \frac{x}{p+z} \quad (19)$$

The effects of small changes in (p, x, y, z) on the pay-off of complying are

$$v_x^*(p, x, y, z) = -u'_1(y-x-(p+z)n) + nu'_2(xn), \quad (20)$$

$$v_y^*(p, x, y, z) = u'_1(y-x-(p+z)n), \quad (21)$$

$$v_z^*(p, x, y, z) = -nu'_2(xn), \quad (22)$$

$$v_p^*(p, x, y, z) = -nu'_1(y-x-(p+z)n). \quad (23)$$

The effect of y is unambiguously positive. Those of p and z are unambiguously negative. That of x may be negative or positive depending on whether MRS_{a_1, a_2} is higher or lower than n .

3.1 Self-enforcing family constitutions

Suppose that all (present and future) members of the same family are identical. For

$$v(r, y) \leq v^*(p, x, y, z), \quad (24)$$

complying with the family constitution (x, z) is the best individual response to every other (present or future) member of the family doing the same. This inequality is then the necessary and sufficient condition for the comply strategy to be a Nash equilibrium. Since complying implies threatening one's own parent of punishment if she does not comply too, and given that the threat is credible because carrying it out is in the interest of the person making it, this equilibrium will be sub-game perfect. In equilibrium, the threat will never be carried out because everybody complies.

Since a complier must pay her parent a fixed amount whatever her own choice of n , (24) implies (18). If (18) were not true, there would in fact be no way to recover the fixed cost of complying, x . Taken together with (18), this implies that a complier's marginal rate of substitution of present for future consumption is higher than the interest factor,

$$MRS_{a_1, a_2} > r.$$

Figure 2 illustrates the properties of the set of constitutions that can be supported by a sub-game perfect Nash equilibrium. All the (z, x) pairs that lie on or inside the frontier satisfy (24) given (p, r, y) . Notice that the frontier intersects the x axis, but not the z one. Since young children cannot make transfers, z cannot in fact be negative. It can be zero, however, because adults would be happy to subscribe to a constitution that did not oblige them to spend more than p for each of their children. On the other hand, adults would not countenance a constitution that did not entitle them to receive transfers from the same children in the next period. Therefore,

$$z \geq 0, \quad x > 0.$$

The slope of the line segment joining the point $(-p, 0)$ to any point (z, x) in the set of self-enforcing family constitutions is the marginal return to children implicit in the constitution represented by that point. The constitution with the highest marginal return is the one represented by point $(0, x^m)$.

In view of (20) – (22), the slope of the frontier is given by

$$\frac{dz}{dx} = \frac{u'_2(xn)}{u'_1(y - x - (p + z)n)} - \frac{1}{n}. \quad (25)$$

Since

$$\frac{d^2z}{d(x)^2} = \frac{nu'_1(y - x - (p + z)n)u''_2(xn) + u''_1(y - x - (p + z)n)u'_2(xn)}{u'_1(y - x - (p + z)n)^2} \quad (26)$$

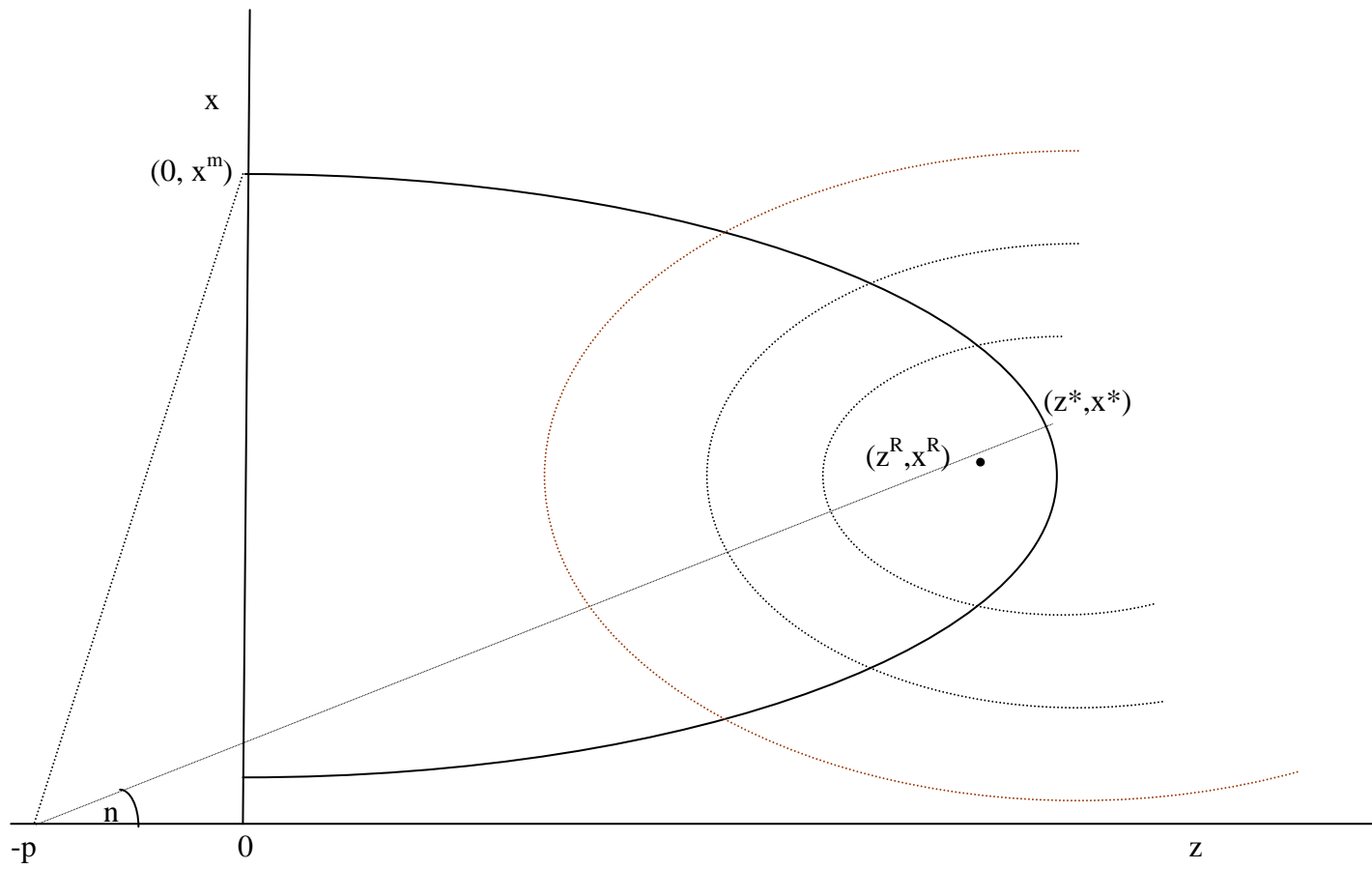


Figure 2: The Nash-frontier and the renegotiation-proof family constitution.

is clearly negative, z is maximized at point (z^*, x^*) , where (25) is equal to zero. In view of (19), this implies

$$n = \frac{x}{p+z}. \quad (27)$$

In view of (14), (19) and (18), MRS_{a_1, a_2} is higher for compliers than for go-it-aloners. Although v_y and v_y^* are both positive in view of (15) and (21), a rise in y would thus shift the frontier outwards, By contrast, a rise in p or r would shift it inwards, because v_p^* is negative in view of (23), and v_r positive in view of (15). The probability that a self-enforcing family constitution exists is thus increasing in y , and decreasing in (p, r) .

3.2 Renegotiation-proof family constitutions

A constitution worth its name must not be amendable unless unforeseen circumstances make it inviable (if a circumstance is foreseen, it will be incorporated in the constitution). The question is then, what is there to stop a generation setting itself up as a constitutional assembly, and re-writing the constitution to its own advantage? To answer it, Cigno (2006a) adapts the renegotiation-proofness concept developed in Bernheim and Ray (1989), and Maskin and Farrell (1989). In those papers, the players are always the same, and the arrangement is thus to do with the way an individual behaves towards his contemporaries. Here, by contrast, the players change at each round, and the arrangement concerns the way present adults behave towards their predecessors and descendants.

Any generation can proclaim a new constitution. Will subsequent ones take any notice? Not if the old constitution (i) satisfies (24), *and* (ii) is not Pareto-dominated by any other constitution also satisfying (24). If the existing constitution is undominated, the only way a person can offer her children a better deal, and not loose in the bargain, is in fact to pay her parent less than the existing constitution requires – in other words, to default on the existing constitution. But that makes her liable to punishment at the hands of her own children, who will be better-off upholding the existing constitution, which entitles them to pay nothing to her, than acquiescing to the proposed new one. Once established, a constitution satisfying the double requirement of being a sub-game perfect Nash equilibrium, and undominated by any other constitution that is itself a sub-game perfect Nash equilibrium, is thus unamendable. Let us look at its properties.

Selfish adults are interested only in their own adult and old-age consumption. As their children are interested in their entire lifetime consumption stream, however, a family constitution is renegotiation-proof if the transfers (x, z) it prescribes, and the n its adult members will choose in response to those prescriptions, as such that

$$V(n, x, z) \equiv u_0(z) + u_1(y - x - (p+z)n) + u_2(xn),$$

is at a maximum subject to (24). The first-order conditions for the maximization

of

$$\mathcal{L} = V(n, x, z) - \lambda [v(r, y) - v^*(p, x, y, z)]$$

are

$$u'_0(z) - nu'_1(y - x - (p + z)n) + \lambda v_z^* = 0, \quad (28)$$

,

$$-u'_1(y - x - (p + z)n) + nu'_2(xn) + \lambda v_x^* = 0 \quad (29)$$

and

$$-(p + z)u'_1(y - x - (p + z)n) + xu'_2(xn) = 0, \quad (30)$$

where λ is the Lagrange-multiplier of (24).

In view of (22), (28) implies

$$MRS_{a_0, a_1} \equiv \frac{u'_0(z)}{u'_1(y - x - (p + z)n)} \equiv (1 + \lambda)n.$$

In view of (20), (29) implies

$$MRS_{a_1, a_2} \equiv \frac{u'_1(y - x - (p + z)n)}{u'_2(xn)} \equiv n.$$

It is then clear that, if (24) is not binding ($\lambda = 0$), the marginal rates of substitution of present for future consumption of parents and children will be equalized, and the common value equated to the marginal return of money spent on children, in turn equal to the number of children,

$$MRS_{a_0, a_1} = MRS_{a_1, a_2} = \frac{x}{p + z} = n.$$

Since children are the only worthwhile form of investment, we can interpret the last of these equations as a kind of domestic *golden rule*. The domestic allocation is then dynamically efficient. If (24) is binding ($\lambda > 0$), by contrast,

$$MRS_{a_0, a_1} > MRS_{a_1, a_2} = \frac{x}{p + z} = n.$$

In that case, we have again the Baland-Robinson problem that children value their present consumption, at the margin, more than their parents do. In conclusion, the existence of a family constitution reduces the probability that parents will allocate resources inefficiently, but not to zero.

If (24) is not binding, the self-enforcing, renegotiation-proof constitution will lie inside the Nash-frontier. If (24) is binding, the constitution in question will be the one which maximizes parental support for young children, (x^*, z^*) , because that is the only point of the frontier where (27) is satisfied. In Figure 2, the broken curves, with slope $(u_0 - u_1n) / (-u_1 + u_2n)$, are the contours of $V(\cdot)$. The picture is drawn so that $V(\cdot)$ reaches a maximum inside the Nash frontier, at point (z^R, x^R) . That is not always true. If the unconstrained maximum violates the Nash constraint, the renegotiation-proof constitution will be at

a point of tangency between the frontier and a contour of $V(\cdot)$. Since the frontier shifts inwards as the interest rate rises relative to y , if a self-enforcing, renegotiation-proof constitution exists, the probability that this constitution is at point (x^*, z^*) , and thus maximizes transfers to the young, increases with r . Conversely, the higher is r , the lower is the probability that such a constitution exists.

Notice that the constitution which maximizes the marginal return to money spent on children, $(0, x^m)$, can never be renegotiation-proof. Browning (1975) makes the point that, since children do not vote, the pension system produced by a direct democracy will be larger than the one which maximizes the lifetime utility of the representative agent. Transferred from society to the family, this is the same as saying that, if family decisions were taken by majority voting, the adults of the day would always vote for setting z equal to zero, and x as high as possible. A family constitution prevents just that.

3.3 Extensions: uncertainty, personal attention and altruism

The first possible extension is to introduce uncertainty. Allowing exchange across states of nature, as well as across dates, extends the scope for cooperation within the family from the field of credit to that of mutual insurance (Di Tella and MacCulloch, 2002). Were it possible to make constitutional prescriptions fully contingent, uncertainty would make no difference of substance to the constitution story. Realistically assuming that doing this is prohibitively costly, however, uncertainty about a child's survival to adult age, and ability to pay her parent x if she does survive, makes complying a risky activity. But going-it-alone also is a risky activity, because the return to conventional assets is uncertain too. Assuming that the two kinds of risk are not positively correlated, a risk-averse complier may then find it optimal to save *and* have children (Rosati, 1996). This does away with the unrealistic prediction of the basic model (without altruism and uncertainty) that people will either save, or have children.

The second extension is to make utility depend not only on the consumption of market goods, but also on the consumption of personal services ("attention") without perfect market substitutes that the agent may receive from her parent when she is young, from her children when she is old. If we do that, c_t must be re-interpreted as the money equivalent of goods consumed and attention received in period t , constraint (16) as saying that a young person is entitled to receive from her parent a combination of money and attention yielding at least the same utility as z , and constraint (17) as saying that an old person is entitled to receive from each of her children a combination of money and attention yielding at least the same utility as x . As a complier will then give her elderly parent and young children the cost-minimizing combination of attention and market goods (Cigno and Rosati, 2000), this extension relaxes (24). Consequently, it raises the probability that a self-enforcing, renegotiation-proof family constitution exists, and that it will be efficient if it does exist. The intuition is that (a)

allowing working-age people to substitute attention for money reduces the cost of complying, and (b) as attention has no perfect market substitutes, and the only way to get it from one's own grown-up children is to comply, allowing for utility to depend on attention raises the benefit of complying.

The third extension is to allow for altruism Cigno (2006a).. The effect of setting $\beta > 0$ is similar to that of introducing attention in the utility function, because it relaxes (24). It is similar also to the effect of introducing uncertainty, because people may now save *and* have children (to leave bequests). In the uncertainty case, however, it is the compliers who may save and have children for risk-spreading reasons. Here, by contrast, it is the go-it-aloners who may do so for altruistic reasons (because they like children, and may want to make bequests). An altruistic go-it-aloner behaves in the way described by the Baland-Robinson model.

4 Behavioural and policy implications of family rules⁷

Granted that self-enforcing and renegotiation-proof family constitutions may exist, what difference does it make to the way individuals respond to a change in the economic environment? Recall that an agent may respond by either adjusting what variables she can under the present strategy, or by switching strategies. In this section, I will examine the effects of two environmental changes with a direct bearing on intergenerational transfers, the introduction of a mandatory pension system, and a tightening of the credit ration. Since an increase in y raises the pay-off of complying more than the pay-off of going it alone, there is a threshold level of this income, y_m , such that an agent will comply if, and only if,

$$y \geq y_m.$$

A small reduction in y will cause an adult at or just above y_m to switch from the comply, to go-it-alone strategy. A small increase in y will cause an adult just below y_m to switch the other way. I shall call an agent who switches strategies a *marginal player*.

4.1 Pensions

Consider the effects of introducing or expanding a pension system which requires each adult to pay τ , and entitles each old person to receive ψ . In order to isolate the effects of compulsory old-age insurance from those of a implicit pension tax or subsidy,⁸ I start by assuming that the scheme is actuarially fair. In our

⁷This section draws on Cigno (2006a), and Cigno et al. (2006).

⁸The implicit pension tax is defined as the difference, at the date of retirement, between the capitalized value of the contributions, and the expected value of the benefits; see Sinn (1990).

certainty framework, this simply means

$$\psi = r\tau. \quad (31)$$

I further assume that τ is no higher than the amount that a go-it-aloner would save without the policy. This implies that only compliers can be credit rationed. For simplicity, I will consider in detail only the case in which people are selfish ($\beta = 0$).

The pay-off of the go-it-alone strategy is now

$$v(r, y, \tau) = \max_s u_1(y - \tau - s) + u_2((\tau + s)r).$$

As the policy does not alter the wealth position of inframarginal go-it-aloners, these players have no reason to change their life plans. As it lowers their demand for old-age consumption, however, the policy induces them to reduce their savings by the amount of the extra pension contribution (total *crowding-out*),

$$\frac{ds}{d\tau} = -1. \quad (32)$$

Their fertility and transfers remain zero. The effect of y on the pay-off of going it alone,

$$v_y(r, y, \tau) = u'_1, \quad (33)$$

remains positive. That of τ ,

$$v_\tau(r, y, \tau) = -u'_1 + ru'_2, \quad (34)$$

is zero in view of (14).

The pay-off of the comply strategy is

$$v^*(p, x, y, z, \tau) = \max_n u_1(y - \tau - x - (p + z)n) + u'_2(nx + r\tau).$$

Although the policy reduces a complier's demand for old-age consumption, an inframarginal complier cannot respond by saving less, because she is saving nothing already. As her transfer behaviour is prescribed by the constitution, all that will change is then her utility. Using (27), the effect of y on the pay-off of complying,

$$v_y^*(p, x, y, z, \tau) = u'_1, \quad (35)$$

remains positive. That of τ ,

$$v_\tau^*(p, x, y, z, \tau) = -u'_1 + \left(r - \frac{x}{p + z}\right)u'_2, \quad (36)$$

is negative in view of (18).

The threshold level of income below which agents go it alone solves

$$v^*(p, x, y, z, \tau) = v(r, y, \tau).$$

Since $v_y^*(p, x^m, y^m, z^m, \tau)$ is larger than $v_y(r, y^m, \tau)$ in view of (14), (19) and (18), and having established that (36) is negative,

$$\frac{dy^m}{d\tau} = -\frac{v_\tau^*(p, x^m, y^m, z^m, \tau)}{v_y^*(p, x^m, y^m, z^m, \tau) - v_y(r, y^m, \tau)} \quad (37)$$

will be positive. The policy thus raises y^m , inducing marginal players to switch from the comply to the go-it-alone strategy. As a result, these agents will stop having children and making transfers, and will start saving.

It is thus clear that, in a selfish world, the policy has non-positive effects on the fertility and private transfers of all inframarginal players, and negative ones on those of marginal players. Aggregate fertility will consequently fall, and some of the old will be left without filial support. The effect on saving is non-positive where inframarginal players are concerned, positive where marginal ones are. If there are enough of the latter, the policy will then *raise* aggregate saving. This is in sharp contrast with the prediction of standard life-cycle theory. Altruism ($\beta > 0$) does not alter the proposition that an expansion in (actuarially fair) pension coverage will reduce aggregate fertility, and may raise aggregate saving. This proposition is in sharp contrast with the predictions of Becker and Barro (1988) that the policy will induce agents to save less, and will have no effect on their fertility and transfer behaviour (because it does not entail an intergenerational transfer).

If

$$\psi < r\tau, \quad (38)$$

players pay an *implicit pension tax*. This will reinforce the negative effect of participation on fertility, and make it more likely that the effect on aggregate saving will be negative too. If

$$\psi > r\tau, \quad (39)$$

players receive an *implicit pension subsidy* (e.g., inaugural gains for participants in a pay-as-you-go pension system). This will tend to offset the negative effect of participation on fertility, and make it more likely that the effect on aggregate saving will be positive.

Although all econometric (both time-series and cross-country) studies find that pensions have a negative effect on fertility, some⁹ estimate a negative, and others¹⁰ a positive effect on per-capita income growth. There is evidence, however, that an implicit pension tax encourages, and an implicit pension tax discourages saving.¹¹ Controlling for these implicit subsidies or taxes, the effect of pension coverage on saving is never negative. As per-capita income growth is increasing in saving (and more so if there is a negative quantity/quality of children trade-off), these findings are consistent with the hypothesis that a substantial number of agents optimizes subject to a family constitution. By contrast, they reject the hypothesis that all agents optimize subject only to the law of the land

⁹E.g., Ehrlich and Zhong (1998).

¹⁰E.G., Zhang and Zhang (2004).

¹¹See, e.g., Cigno and Rosati (1996, 1997), and Cigno *et al.* (2002).

(as in life-cycle theory, and in altruistic models *à la* Becker-Barro or Baland-Robinson).

4.2 Credit rationing

We now turn to the implications of a reduction in the credit ration. To do that, we must assume that the credit ration is positive in the first place and that the agent will have an income in period 2 (e.g., a pension) with which to pay back her debt. Let us then write y_t for the agent's income in period t ($y_t > 0$, $i = 1, 2$). For simplicity, we again assume that agents are selfish, but the results are qualitatively the same if we assume that they are altruistic. The pay-off of the go-it-alone strategy is now

$$v(b, r, y_1, y_2) = \max_s [u_1(y_1 - s) + u_2(y_2 + rs)], \text{ s.t. } (4).$$

If y_2 is sufficiently large, the agent would like to borrow. If (4) is binding, a reduction in b will then reduce the amount borrowed, and consequently the agent's utility. The effect on the pay-off of the go-it-alone strategy is

$$v_b(b, r, y_1, y_2) = \xi, \quad (40)$$

where ξ is the Lagrange-multiplier of (4), positive and equal to $u'_1(y_1 - s)$. That of a small change in current income (y_1 , formerly y),

$$v_{y_1}(b, r, y_1, y_2) = u'_1(y_1 - s), \quad (41)$$

is positive as usual.

Since a complier is always credit rationed, the pay-off of the comply strategy is

$$v^*(b, p, r, x, y_1, y_2, z) = \max_n [u_1(y_1 + b - x - (p + z)n) + u_2(y_2 - br + nx)].$$

A reduction in b will always reduce both the amount borrowed and the utility of inframarginal compliers. Their fertility and transfer behaviour will remain the same. The effect on the pay-off of the strategy,

$$v_b^*(b, p, r, x, y_1, y_2, z) = u'_1(y_1 + b - x - (p + z)n) - ru'_2(y_2 - br + nx), \quad (42)$$

is positive for the usual reason that the marginal rate of substitution is equated to n , greater than r . That of a small change in current income,

$$v_{y_1}^*(b, p, r, x, y_1, y_2, z) = u'_1(y_1 + b - x - (p + z)n), \quad (43)$$

is again positive.

The threshold level of current income, y_m , is now defined by

$$v^*(b, p, r, x, y_m, y_2, z) = v(b, r, y_m, y_2).$$

The effect of b on y^m is given by

$$\frac{dy^m}{db} = -\frac{v_b^*(b, p, r, x, y_m, y_2, z) - v_b(b, r, y_m, y_2)}{v_{y_1}^*(b, p, r, x, y_m, y_2, z) - v_{y_1}(b, r, y_m, y_2)}. \quad (44)$$

There are two possibilities. One is that the ration is binding only if the agent complies, in which case (44) is definitely negative. The other is that the ration is binding whichever the strategy, in which case (44) may take either sign. A reduction in b may thus induce agents just below the income threshold to switch from the go-it-alone to the comply strategy. These agents will start having children, and making transfers (to their own parents and children). In the presence of altruism, the first proposition remains true, but the second one becomes that agents just below the threshold may have more children, and make more transfers.

The constitution model thus predicts that tighter credit conditions may induce some agents to make more transfers despite the fact that they have become poorer. This contrasts with the prediction of conventional models, where people make transfers are either gifts or payment for services received, that a reduction in b will make the agent effectively poorer and, consequently, lead lead him to demand less of all normal goods. Controlling for possible endogeneity of the credit ration, Cigno *et al.* (2006) find survey data evidence that the probability of making a transfer does indeed increase if the agent is credit rationed. Since that cannot be explained by conventional models of either the altruistic or the exchange variety, the finding rejects the hypothesis that transfer behaviour is the outcome of unfettered individual optimization. Consistently with the constitution story, that paper also finds that the probability of making a transfer shows very low elasticity to the giver's own income and assets.

5 Gender¹²

So far, we have made no distinction between mother and father. We now introduce gender, and focus is on the union formed by a particular woman, f , and a particular man, m . The union produces local public goods that could not be procured in any other way. For simplicity, we assume that only one of these goods, children, uses resources that could otherwise be used in the production of private goods. We further assume that f and m are perfectly informed about each other's characteristics, and about the characteristics of all alternative partners. Behaviour is modelled as a two-person game. The choice of game is endogenous. If the game is non-cooperative, the number of children, denoted by n , is decided by f . Each party has the option of unilaterally withdrawing from the union. In real life, many unions break down while the children are still dependent on their parents, or even before the children are born. The cause of these early separations is imperfect information about the present partner, or about the availability of alternative ones. In our perfect-information framework,

¹²This section draws on Cigno (2009, 2011).

however, separation in period 1 makes no sense. Had either party had a better alternative to the present match, he or she would have taken it in the first instance. Separation may then make sense only in period 2, when the children are out of the way, and there are no more efficiency gains to be had.

As in Cigno (1991), we assume that a child requires at least t_0 units of specifically maternal time over the peri-natal period. Depending on school of pediatric thought, this minimum may be as short as three weeks, or as long as three years, but all our non-gender results survive if we set it equal to zero. Together with the fact that men cannot bear children (of which it is a reflection), this is the only natural asymmetry between the sexes to which we are going to admit. Any other asymmetry will be man-made. Above t_0 , the father's time is a substitute for the mother's. In most of the analysis, we will assume that it is a perfect substitute. That will permit us to derive analytical results in a fairly straightforward manner. Towards the end of the next section, we will argue that nothing of substance changes if an hour of extra attention by both parents makes a child happier than two hours of extra attention by only one parent would.

Let c denote the amount of goods money can buy ("money"), and t the amount of time ("attention") over and above t_0 , that a child receives from his or her parents in period 1. For the perfect-substitutability assumption,

$$t = t_f + t_m, \tag{45}$$

where t_i is the amount of attention provided by parent i , $i = f, m$. Plausibly assuming that the length of time for which the mother cannot be replaced by the father is relatively short,

$$t_0 < t.$$

The child's lifetime utility maximized conditionally on c and t is denoted by $v(c, t)$, where $v(\cdot)$ is an indirect utility function, increasing and concave. Since c may include the services of professional child minders, concavity implies that bought-in child care is an imperfect substitute for parental attention.

Assuming descending altruism, the utility of partner i may be written as

$$U_i = u(a_{i1}) + u(a_{i2}) + \beta nv(c, t), \quad 0 < \beta < 1, \tag{46}$$

where (a_{i1}, a_{i2}) is i 's consumption stream, and $u(\cdot)$ the instantaneous utility function, also increasing and concave. We will refer to $v(c, t)$ as the quality, and n as the quantity, of children. Since the $nv(c, t)$ term is common to both f 's and m 's utility, children are a local public good. As children are not differentiated by sex, and β is the same for f and m , we are in effect saying that parents love daughters as much as they love sons, and that fathers love their children as much as mothers do. That may or may not be true in practice, but assuming otherwise would give the (author's) game away.

As leisure does not figure in (46), i will throw any time that is left over from child-care activities inelastically on to the labour market. This assumption allows us to focus on the allocation of total work time between market ("labour")

and domestic (identified with child-care) activities. Further assuming that this total is the same for both parties in each period, and normalizing it to unity, f 's and m 's period-1 labour supplies will be, respectively,

$$L_f = 1 - (t_0 + t_f) n \quad (47)$$

and

$$L_m = 1 - n t_m. \quad (48)$$

Therefore,

$$\frac{\partial L_f}{\partial n} = -(t_0 + t_f), \quad \frac{\partial L_f}{\partial t_f} = -n, \quad (49)$$

and

$$\frac{\partial L_m}{\partial n} = -t_m, \quad \frac{\partial L_m}{\partial t_m} = -n. \quad (50)$$

Since L_i cannot be negative, (n, t_f, t_m) must be such that

$$(t_0 + t_f) n \leq 1$$

and

$$n t_m \leq 1.$$

We assume that neither of these restrictions is ever binding (i.e., that the opportunity-cost of looking after children is sufficiently high for neither parent to want to spend more than the whole of his or her total work time in this activity). In period 2, when the children no longer demand attention, the labour supply will be equal to unity for both partners.

When the union is formed, i is endowed with k_i units of a tradeable asset ("money"), and h_i units of human capital. The latter may be partly a reflection of natural talent, and partly the result of previous educational investments or labour experience. From the moment the union is formed, however, human capital will increase only with labour experience (an equivalent assumption would be that human capital depreciates with lack of labour experience). We will assume that i 's human capital accumulates at the positive rate α per unit of labour. Nothing of substance would change if we allowed f and m to differ also in their ability to raise children, or assumed that this ability increases with child care experience.

In the light of these assumptions, i 's wage rate is

$$w_{i1} = (1 + \alpha L_i) h_i$$

in period 1, and

$$w_{i2} = (1 + \alpha) (1 + \alpha L_i) h_i$$

in period 2. The assumption that the wage rate increases with market work implies increasing returns to this activity. The assumptions that a unit of female human capital attracts the same rent (normalized to unity) as a unit of male human capital, and that the wage rates of two equally endowed persons grow

at the same rate per unit of labour irrespective of sex, imply absence of gender discrimination in the labour market. Later in the paper, we will briefly look at what happens if there is.

Partner i then earns

$$y_{i1}(L_i) \equiv w_{i1}L_i \quad (51)$$

in period 1, and

$$y_{i2}(L_i) \equiv w_{i2} \quad (52)$$

in period 2. Notice that not only period-1, but also period-2 earnings are completely determined by the time allocation chosen in period 1, and that $y_{i1}(\cdot)$ and $y_{i2}(\cdot)$ are increasing functions.

5.1 Efficiency

A Pareto-optimal $(a_{f1}, a_{f2}, a_{m1}, a_{m2}, s, t_f, t_m, c, n)$ maximizes

$$\Lambda = \lambda U_f + (1 - \lambda) U_m, \quad 0 \leq \lambda \leq 1, \quad (53)$$

for some λ , subject to (45) – (52), to the resource constraints,

$$\sum_{i=f,m} [a_i^1 - k_i - y_{i1}(L_i)] + nc + s = 0 \quad (54)$$

and

$$\sum_{i=f,m} [a_i^2 - y_{i2}(L_i)] - sr = 0, \quad (55)$$

and to the borrowing constraint

$$-2b \leq s, \quad i = f, m, \quad (56)$$

where r denotes the interest factor, and s the couple's joint savings. The parameter λ may be interpreted as f 's domestic welfare weight.

As U_i depends on t , not on its allocation between t_f and t_m , we can carry out the optimization in two steps. First, we find the (t_f, t_m) which minimizes the opportunity-cost of a child for each possible (n, t) . Second, we look for the $(a_{f1}, a_{f2}, a_{m1}, a_{m2}, s, t, c, n)$ which maximizes Λ . The first step is illustrated in Figure 3. The straight line with absolute slope equal to unity is an isoquant. The convex-to-the-origin curves with absolute slope

$$-\frac{dt_m}{dt_f} = \frac{(1 + 2\alpha [1 - (t_0 + t_f)n])r + \gamma}{[1 + 2\alpha (1 - nt_m)]r + \gamma} \frac{h_f}{h_m}, \quad (57)$$

diminishing as t_m is substituted for t_f , are isocosts. The constant γ is equal to $\alpha(1 + \alpha)$ if (56) is not binding, to 0 if it is binding. Convexity implies that the solution will be at a corner.

Given an (h_f, h_m) pair satisfying

$$\frac{h_f}{h_m} \leq \frac{(1 + 2\alpha)r + \gamma}{(1 + 2\alpha [1 - (t_0 + t)n])r + \gamma}, \quad (58)$$

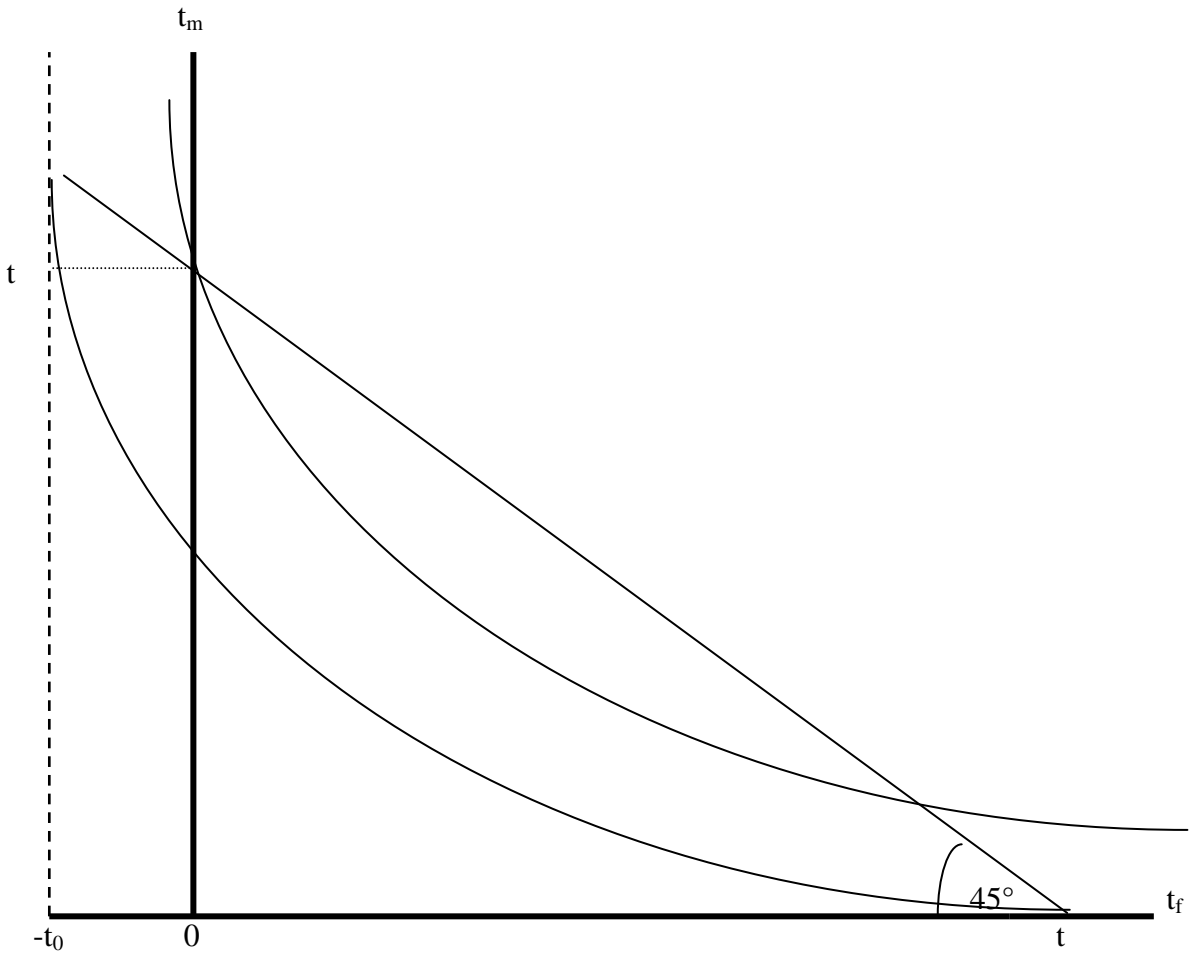


Figure 3. The cost-minimizing division of labour

the opportunity-cost of a child will be minimized by the traditional division of labour,

$$t_f = t, \quad t_m = 0. \quad (59)$$

Given any other (h_f, h_m) , the opportunity-cost will be minimized by the liberated division of labour,

$$t_f = 0, \quad t_m = t. \quad (60)$$

Two implications of this result need commenting. The first is that, in developed countries, where fertility is low, and life expectancy high, division of labour does not entail that the parent who specializes in child-care work will do little else in the course of her or his life. The second is that, for any positive value of t_0 , m may have a comparative advantage in market work even if his human capital endowment is smaller than f 's. If h_f and h_m have the same frequency distribution, the most likely cost-minimizing division of labour is then the traditional one. That is not necessarily true, however, if h_f is less right-skewed than h_m (e.g., because women tend to be more educated than men as appears to be increasingly the case in developed countries).

We now go on to find the (L_f, L_m, s, c, n, t) which maximizes (53), subject to (54) – (56). If the credit constraint is binding, the solution will be only a "local" Pareto optimum, in the sense that the wider economy in which the household is immersed is not at a Pareto optimum.¹³ The first-order conditions yield

$$\frac{u'(a_{i1})}{u'(a_{i2})} = \frac{\mu r + \rho}{\mu}, \quad (61)$$

$$\frac{v_t}{v_c} = - \left(y'_{f1} + \frac{\mu y'_{f2}}{\mu r + \rho} \right) \frac{\partial L_f}{\partial t} - \left(y'_{m1} + \frac{\mu y'_{f2}}{\mu r + \rho} \right) \frac{\partial L_m}{\partial t} \quad (62)$$

and

$$\frac{v}{v_c} = c - \left(y'_{f1} + \frac{\mu y'_{f2}}{\mu r + \rho} \right) \frac{\partial L_f}{\partial n} - \left(y'_{m1} + \frac{\mu y'_{f2}}{\mu r + \rho} \right) \frac{\partial L_m}{\partial n}, \quad (63)$$

where μ is the Lagrange-multiplier of (55), necessarily positive, and ρ that of (56), positive if the couple is credit constrained, zero if it is not. Among other things, these marginal conditions tell us that, at a Pareto optimum, the MRS of present for future consumption, and the MRS of quantity for quality of children, are equalized across the parties. Since λ does not figure in any of them, these conditions also tell us that (U_f, U_m) is separable from (s, n, c, t_f, t_m) at a Pareto optimum.

In view of (49) – (50), and assuming n positive (otherwise, there would be no division of labour),

$$\frac{\partial L_f}{\partial t} < 0, \quad \frac{\partial L_f}{\partial n} < 0 \quad \text{and} \quad \frac{\partial L_m}{\partial t} = \frac{\partial L_m}{\partial n} = 0 \quad (64)$$

¹³That is the sense in which the expression "Pareto efficiency" is generally used in game theory.

if (h_f, h_m) satisfies (58), and the division of labour is consequently (59). Otherwise, the division of labour will be (60), and

$$\frac{\partial L_f}{\partial t} = 0, \quad \frac{\partial L_f}{\partial n} < 0, \quad \frac{\partial L_m}{\partial t} < 0 \quad \text{and} \quad \frac{\partial L_m}{\partial n} < 0. \quad (65)$$

Let us now look at the implications of relaxing some of our assumptions, starting with those about the technology of child care. If we replace the assumption that t_m is a perfect substitute for t_f with the one that the former substitutes for the latter at a diminishing marginal rate, the isoquants become convex to the origin, and the cost-minimizing allocation of the couple's time *may* then be at an interior point.¹⁴ So long as h_f is different from h_m , or t_0 is positive, however, there will be some specialization even if the time allocation is interior, because the isocosts will still be asymmetrical. Allowing for the possibility that not only the ability to raise money, but also the ability to bring up children increases with experience will only make it more likely that the cost-minimizing time allocation is at a corner. The (realistic) assumption that the mother is indispensable at least over the peri-natal period, makes the traditional division of labour more likely than the liberated one. Dropping it (i.e., setting t_0 equal to zero) would remove this asymmetry, but would not affect the efficient solution in any other way.

Allowing for mutual affection between f and m will make no qualitative difference to the results so long as each party cares for its own consumption at least a little more than it cares for the other's. Allowing for the possibility that the labour market discriminates against women, either in the sense that a woman's human capital is remunerated at a lower rate than a man's is (less pay for same work and ability), or in that it accumulates more slowly with labour experience (restricted career opportunities for women), would only make it more likely that the cost-minimizing division of labour is the traditional one.¹⁵

5.2 Equilibrium

As our analysis starts when the matching process ends, we take (k_f, h_f) and (k_m, h_m) as given. Should we impose any restriction on the relative sizes of these endowments? Developing an idea in Becker (1972), Lam (1988) demonstrates the existence and stability of matching equilibria characterized by either positive or negative assortative mating over wage rates and conventional assets. In the more recent literature, the assumption is generally that partners are matched by wealth. In our context, however, wealth depends on the match, and on the ensuing division of labour. Several authors model (k_i, h_i) as the outcome of education and other forms of investment undertaken by either i , or i 's parents, *before* the union was formed. Allowing for some randomness (hormones have

¹⁴But not necessarily. If the elasticity of substitution of t_m for t_f is greater than unity, the isoquants will still cut the axes, and the cost-minimizing division of labour may still be at a corner.

¹⁵In the presence of discrimination, an allocation can be, at best, a local Pareto optimum, even if credit is not rationed.

their part in the mating process), we will take (k_f, h_f) and (k_m, h_m) to be arbitrarily given, subject only to the constraint that f and m would have the same utility in the best alternative to the present match. Assuming, for simplicity, that the best alternative is singlehood, the constraint is then

$$\max_{s_i} \{u(b_i + y_{i1}(1) - s_i) + u(s_i r + y_{i2}(1)) \text{ s.t. } -b \leq s_i\} = U^S, \quad i = f, m. \quad (66)$$

That leaves room for either positive (well-educated boy marries well-educated girl) or negative (rich boy marries well-educated girl) assortment over money and human capital endowments, and only rules out the possibility that a party is superior to the other on all scores. In the course of the exposition, we will examine the implications of different hypotheses about the frequency distributions of h_f and h_m .

In the present subsection, we ask ourselves which game the couple would play if the marriage institution were not available. For simplicity, we will take Cournot-Nash and Nash-bargaining to be the only available alternatives. As both parties have right of veto, the couple will play Nash-bargaining only if (after any appropriate money transfer) neither party would be better-off playing Cournot-Nash instead. If both parties are indifferent between the two games, they will spin a coin. The effect of the marriage institution will be briefly mentioned in the concluding section.

5.2.1 Non-cooperation

Let us start by characterizing the Cournot-Nash equilibrium. Let s_i denote i 's saving. Let c_i be the amount of money that i spends on each child, so that

$$c = c_f + c_m.$$

The woman chooses $(a_{f1}, a_{f2}, s_f, c_f, t_f, n)$ so as to maximize

$$U_f = u(a_{f1}) + u(a_{f2}) + \beta n v(c_f + c_m, t_f + t_m),$$

subject to her own budget constraints,

$$a_{f1} + n c_f + s_f = k_f + y_{f1}(L_f) \quad (67)$$

and

$$a_{f2} = r s_f + y_{f2}(L_f), \quad (68)$$

and to her individual credit constraint,

$$-b \leq s_f, \quad (69)$$

taking (c_m, t_m) as parameters.

The first-order conditions for this optimization yield

$$u'(a_{f1}) = \beta v_c, \quad (70)$$

$$\frac{u'(a_{f1})}{u'(a_{f2})} = \frac{\mu_f r + \rho_f}{\mu_f}, \quad (71)$$

$$\frac{v_t}{v_c} = - \left(y'_{f1} + \frac{\mu_f y'_{f2}}{\mu_f r + \rho_f} \right) \frac{\partial L_f}{\partial t_f} \quad (72)$$

and

$$\frac{v}{v_c} = c_f - \left(y'_{f1} + \frac{\mu_f y'_{f2}}{\mu_f r + \rho_f} \right) \frac{\partial L_f}{\partial n}, \quad (73)$$

where μ_f is the Lagrange-multiplier of (68), and ρ_f that of (69). The derivatives of L_f are given by (49).

The man chooses $(a_{m1}, a_{m2}, s_m, c_m, t_m)$ so as to maximize

$$U_m = u(a_{m1}) + u(a_{m2}) + \beta n v(c_f + c_m, t_f + t_m),$$

subject to

$$a_{m1} + n c_m + s_m = k_m + y_{m1}(L_m), \quad (74)$$

$$a_{m2} = r s_m + y_{m2}(L_m) \quad (75)$$

and

$$-b \leq s_m, \quad (76)$$

taking (c_f, t_f, n) as parameters.

His first-order conditions yield

$$u'(a_{m1}) = \beta v_c, \quad (77)$$

$$\frac{u'(a_{m1})}{u'(a_{m2})} = \frac{r \mu_m + \rho_m}{\mu_m} \quad (78)$$

and

$$\frac{v_t}{v_c} = - \left(y'_{m1} + \frac{\mu_m y'_{m2}}{\mu_m r + \rho_m} \right) \frac{\partial L_m}{\partial t_m}, \quad (79)$$

where μ_m is the Lagrange-multiplier of (75), and ρ_m that of (76). The derivatives of L_m are given by (50).

Using (51) – (52), (70) – (72) and (77) – (79), we find that, at the Cournot-Nash equilibrium, $a_{f1} = a_{m1}$ and $a_{f2} = a_{m2}$. Therefore, $\mu_f = \mu_m$, $\rho_f = \rho_m$ and

$$U_i = U^C, \quad i = f, m,$$

where U^C denotes the Cournot-Nash equilibrium value of utility, the same for both parties. As the woman could always choose $n = 0$ and thus $L_f = 1$, U^C will be at least as large as U^S ,

$$U^S \leq U^C. \quad (80)$$

We can think of this inequality as a participation constraint.

In view of (47) – (48), (51) – (52) and (73), we will also have that

$$\begin{aligned} & \left(1 + 2\alpha [1 - (t_0 + t_f) n] + \frac{\mu\alpha(1 + \alpha)}{r\mu + \rho} \right) h_f \\ = & \left(1 + 2\alpha (1 - nt_m) + \frac{\mu\alpha(1 + \alpha)}{r\mu + \rho} \right) h_m. \end{aligned}$$

where μ is now used to denote the common equilibrium value of μ_f and μ_m , and ρ that of ρ_f and ρ_m . This tells us that, if h_f is equal to h_m , and consequently, in view of (66), k_f equal to k_m , f and m will split everything down the middle. Otherwise, the monetary cost of the children will still be split equally, but the parent with the larger human capital endowment will supply more child care, and less market work, than the one with the larger money endowment. In other words, the parties will specialize *against* their comparative advantages. The opportunity-cost of the children will not be minimized in either case.¹⁶

Notice that the equilibrium will be inefficient even if the parties are not credit constrained ($\rho = 0$), and both face the same the intertemporal trade-off. Comparing (72) and (79) with (62), we do in fact see that the children's MRS of parental attention for money is equated to each parent's, rather than to the couple's, marginal opportunity-cost of providing attention. Comparing (73) with (63), we also see that the MRS of quantity for quality of children is equated to the full cost of an extra child for the mother, rather than for the couple. As the latter is inefficiently large, however, because the parents do not specialize according to their comparative advantages, we cannot be sure that the full cost to the mother in such an inefficient allocation will be smaller than the full cost to the couple in an efficient one. Intuitively, that is because the mother free-rides on the father over the choice of fertility level, but the resulting distortion is traded-off against the one caused by the misallocation of the couple's time. Therefore, we cannot tell whether the couple will have too many or too few children. That would remain true if the man rather than the woman had ultimate control over fertility. If neither party had sole control of fertility, there would be no free-riding with regard to the quantity of children, but the allocation would still be inefficient.

We have already remarked that, given perfect information, it would make no sense for a couple to separate in period 1. If the couple plays Cournot-Nash, the parties will be indifferent between separating or staying together in period 2, because their utility would be the same in either case.

5.2.2 Cooperation

Let us now investigate the circumstances in which f and m would play Nash-bargaining instead of Cournot-Nash. The parties have a common interest in minimizing the opportunity-cost of children, and coordinating their decisions

¹⁶Recall that, so long as t_0 is positive, there will be comparative advantages (in child care for the mother, in market work for the father) even if the parents have the same human capital endowments.

regarding the quality and quantity of the same. Having established that cost minimization requires division of labour, and given that the party who specializes in domestic work (henceforth, the "main childcarer") will earn, in both periods, less than the one who specializes in market work (the "main earner"), because $y_{i1}(\cdot)$ and $y_{i2}(\cdot)$ are increasing functions, neither party will be willing to do the latter unless it receives adequate compensation from the other in period 1, or confidently expects to receive it in period 2. In period 2, however, there are no efficiency gains to be reaped by cooperation, because the children are grown-up, and it will then be in the main earner's interest to renege on any promise it may have made in period 1. In the absence of a contract enforceable through an ordinary court of law, any promise of future payment will then lack credibility, and the compensation will have to be paid in full at front.

Let R_i denote i 's reserve utility. If a Nash-bargaining equilibrium exists, it will maximize

$$N = (U_f - R_f)(U_m - R_m), \quad (81)$$

where R_i denotes i 's reserve utility ($i = f, m$), subject to the Utility-Possibility Frontier (UPF),

$$F(U_f, U_m) \leq 0. \quad (82)$$

In many household economics applications of game theory, R_i reflects only i 's outside options. In Lundberg and Pollak (1996), it is identified with i 's equilibrium utility in the Cournot-Nash game that the couple could have played instead of Nash-bargaining. In our context, however, the couple's money and human capital endowments are irreversibly modified by the choice of (c_f, c_m, t_f, t_m, n) , and there is thus no way to go back to the *status quo ante* once the children are born, and resources expended on them. But it still makes sense to think of either (U^S, U^S) or (U^C, U^C) as the threat-point, because the compensation is delivered at front, and the game is thus played in one shot. In view of (80), we can then write

$$R_i = U^C, \quad i = f, m. \quad (83)$$

Let x_1 be a voluntary transfer (positive, negative or zero) from m to f in period 1. The UPF is traced by choosing $(a_{f1}, a_{f2}, a_{m1}, a_{m2}, s_f, s_m, x_1, c, t, n)$ so as to maximize (53), for each possible value of λ , subject to f 's and m 's budget constraints in the two periods, which now read

$$a_{f1} + cn + s_f = k_f + y_{f1}(L_f) + x_1, \quad (84)$$

$$a_{m1} + x_1 + s_m = k_m + y_{m1}(L_m) \quad (85)$$

$$a_{f2} = s_f r + y_{f2}(L_f), \quad (86)$$

and

$$a_{m2} = s_m r + y_{m2}(L_m), \quad (87)$$

and to their credit constraints, (69) and (76). We have conventionally assigned the monetary cost of the children, cn , entirely to the mother. If x_1 is positive, however, part of this cost will be effectively borne by the father.

At each point of the UPF,

$$\frac{u'(a_{i1})}{u'(a_{i2})} = \frac{\mu_i r + \rho_i}{\mu_i}, \quad i = f, m, \quad (88)$$

$$\frac{v_t}{v_c} = - \left(y'_{f1} + \frac{\mu_f y'_{f2}}{\mu_f r + \rho_f} \right) \frac{\partial L_f}{\partial t} - \left(y'_{m1} + \frac{\mu_m y'_{m2}}{\mu_m r + \rho_m} \right) \frac{\partial L_m}{\partial t} \quad (89)$$

and

$$\frac{v}{v_c} = c - \left(y'_{f1} + \frac{\mu_f y'_{f2}}{\mu_f r + \rho_f} \right) \frac{\partial L_f}{\partial n} - \left(y'_{m1} + \frac{\mu_m y'_{m2}}{\mu_m r + \rho_m} \right) \frac{\partial L_m}{\partial n}, \quad (90)$$

where μ_i is again the Lagrange-multiplier of i 's period-2 budget constraint, and ρ_i that of i 's credit constraint. It can be easily checked that $(\mu_f - \mu_m)$ has the same sign as $(\rho_m - \rho_f)$. As the opportunity-cost of children is minimized, the first derivatives of L_i with respect to t and n are now given by (64) if the initial endowments satisfy (58), by (65) if they do not.

Let j denote the main childcarer, and k the main earner ($j, k = f, m$). At any point of the UPF where

$$\rho_j = \rho_k,$$

(88) – (90) reduces to (61) – (63), and the associated allocation is efficient. At any point where ρ_j is different from ρ_k , by contrast, (61) – (63) will not hold, and the allocation will be inefficient. The economic explanation is that, not only (a_{f1}, a_{f2}) and (a_{m1}, a_{m2}) , but also (c, t, n) will be distorted, because j 's intertemporal trade-off will be different from m 's. If

$$\rho_j < \rho_k,$$

j 's intertemporal trade-off, $\frac{\mu_j r + \rho_j}{\mu_j}$, will be lower than m 's, $\frac{\mu_m r + \rho_m}{\mu_m}$. If we make x_1 larger (more positive for $j = f$, more negative for $j = m$), the difference between the two trade-offs will then increase, because k 's credit ration will get tighter, and j 's slacker. Conversely, if

$$\rho_j > \rho_k,$$

j 's intertemporal trade-off will be higher than k 's. If we make x_1 larger, the difference between the two trade-offs will then decrease (instead of increasing as in the previous case), and eventually become zero.

The case where k is more tightly credit constrained than j everywhere except at $U_j = 0$ is illustrated in Figure 4. The efficiency locus is represented by the continuous concave-to-the-origin curve, and the UPF by the dashed one. The UPF is everywhere steeper than the efficiency locus, and lies below it at all positive values of U_j . The convex-to-the-origin curves are contours of (81). The threat-point of the Nash-bargaining game is \mathbf{C} . As this is the equilibrium of

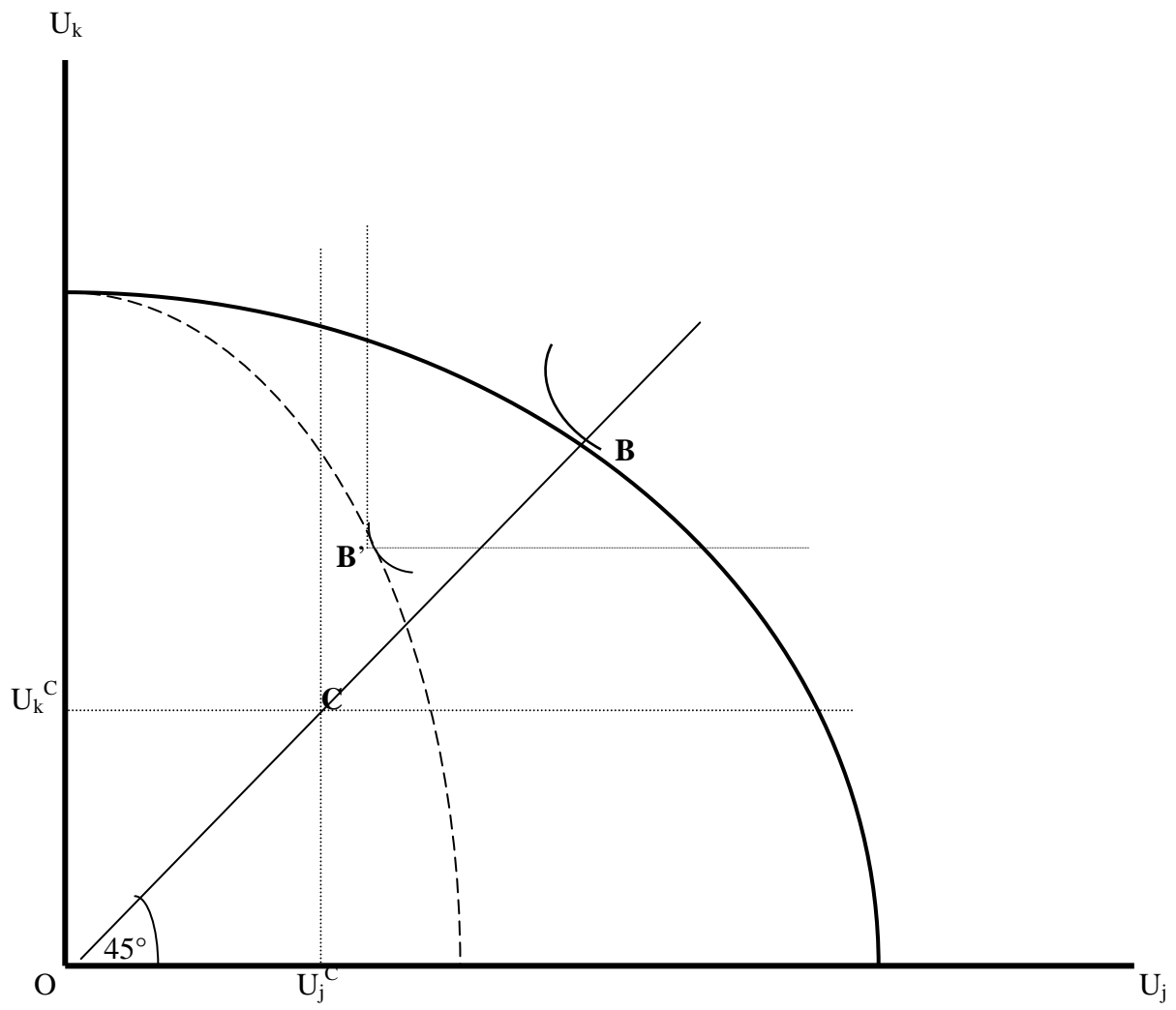


Figure 4. Cournot-Nash and Nash-bargaining equilibria.

the Cournot-Nash game that the couple could play as an alternative to Nash-bargaining, it will lie on the 45° line, and inside the efficiency locus, but not necessarily inside the UPF. If it does, the Nash-bargaining game will have an equilibrium (the couple will play Nash-bargaining). Otherwise, there will be no Nash-bargaining equilibrium (the couple will play Cournot-Nash). In the case illustrated, **C** lies inside the UPF, and the Nash-bargaining game has an equilibrium at point **B'**. At this point,

$$U_j < U_k. \tag{91}$$

In the opposite case (not pictured), where j is more tightly credit constrained than k everywhere except at $U_k = 0$, the UPF is everywhere flatter than the efficiency locus, and

$$U_j > U_k. \tag{92}$$

If neither party were ever credit constrained, the UPF would coincide with the efficiency locus, and the Nash-bargaining game equilibrium would then be at point **B**, where

$$U_j = U_k.$$

Notice that, although it is always the main earner who pays the compensation, and the main childcarer who receives it, the Nash-bargaining equilibrium may favour either or them. In other words, the voluntary redistribution from k to j may stop before (if, at equilibrium, k 's credit ration is tighter than j 's) or after (if j 's credit ration is tighter than k 's) U_j is equated to U_k . Recalling that the man can qualify for the main earner's role even if h_m is smaller than h_f and, consequently, b_m larger than b_f , but the woman can ($k = f$) even if h_f is larger than h_m and, consequently, b_f smaller than b_m , we can then be sure that U_k will be larger than U_j only if the division of labour is the liberated one ($j = m, k = f$).

This result prompts two considerations. One is that, if h_f and h_m have the same frequency distribution, the division of labour is likely to be the traditional one, and we cannot then say whether the Nash-bargaining equilibrium is likely to favour the man or the woman. If the distribution of h_f is sufficiently less right-skewed than the distribution of h_m to make it likely that the division of labour will be the liberated one, however, the Nash-bargaining equilibrium is likely to favour the woman. The other consideration is that, even if utilities are not equalized, and irrespective of who has the higher utility, the parties will still be indifferent between separating or staying together in period 2, because their period-2 incomes would not be affected by the decision.

6 Conclusion

Descending altruism and perfect markets are not sufficient to ensure a Pareto-optimal allocation of resources between different generations of the same family. With or without parental altruism, the existence of self-enforcing and renegotiation-proof family constitutions alleviates the efficiency problem, but

may not eliminate it altogether. Given such constitutions, an exogenous change in the economic environment affects individual behaviour in two ways, (a) by changing the choice of strategy, and (b) by changing the behaviour associated with each strategy. The former applies only to agents with current income close to the threshold below which, in the absence of policy, it is optimal to go-it-alone. The latter applies to all agents. This may make the predictions of the model quite different from those of conventional economic models. We looked at the effects of two environmental changes, the introduction or expansion of an actuarially fair pension scheme, and a tightening of credit conditions.

In the absence of parental altruism, the policy in question has non-positive effects on the fertility and private transfers of inframarginal compliers, those with income sufficiently higher than the threshold, negative effects on those of marginal compliers. The effect on individual saving is negative where inframarginal go-it-aloners are concerned, positive where marginal compliers are concerned, zero where inframarginal compliers are. The effect on aggregate fertility is thus definitely negative, but the effect on aggregate saving may be positive or negative. This is in sharp contrast with the prediction of life-cycle theory, that the pension contribution would displace saving on a one-for-one basis. Parental altruism makes some difference in other respects, but does not change the proposition that the policy may raise aggregate saving. This is in sharp contrast with the prediction of Becker-Barro, that an actuarially fair pension system will cause a reduction in life-cycle saving, but will have no effect on fertility and transfer behaviour because it does not imply a forced transfer to or from the next generation.

Whether the agents are altruistic or not, a reduction in the amount they are allowed to borrow from the credit market may induce some of them to have more children, and to make larger transfers. This prediction contrasts with those of conventional models of either the altruistic or exchange variety. Such results may come as a surprise to anyone who is used to thinking in terms of infinitesimal adjustments of the decision variables to infinitesimal changes in the exogenous ones. In a model where the first and most important decision is about which strategy to follow, however, it is possible that the discreet changes associated with a strategy switch will dominate the infinitesimal ones associated with optimization within a given strategy.

Differentiating parents according to sex, and recognizing that a child requires a certain amount of specifically maternal time (at least over the peri-natal period) allows us to show that an efficient allocation of domestic resources requires division of labour. Depending on relative initial endowments of human capital and conventional assets, the efficient division of labour will be either the traditional (father is the main earner, mother the main childcarer), or the liberated one (mother is the main earner, father the main childcarer). If a person's human capital endowment were purely a reflection of natural talent, the frequency distribution of this endowment would be the same for men and women, and the traditional division of labour would be the most likely pattern of efficient time use. To the extent that human capital endowments are the result of educational investment, however, the frequency distribution may be different for men and

women. If the women's is sufficiently less right-skewed than the men's, and the labour market does not discriminate against women, the liberated division of labour will be as likely as the traditional one.

Division of labour raises the question of compensation. If the couple cannot agree on the level of compensation, there will be no division of labour. A couple will then play Nash-bargaining if this kind of game has an equilibrium. Otherwise, it will play Cournot-Nash. The latter equalizes utilities across the parties, but is inefficient. In the present context, the inefficiency arises from the fact that the parties do not specialize according to their personal comparative advantages, and that the mother free-rides on the father over the choice of number of children. As a consequence, the quantity of children may be either too large or too small, but the quality will be too low in any case, because each child will be raised with too little parental attention, and relatively too much money. In a Nash-bargaining equilibrium, the parties specialize according to their personal comparative advantages, and cooperate over the choice of quantity and quality of children. If the Nash-bargaining equilibrium is efficient, it will equalize utilities. In contrast with what one usually finds (or, rather, with the way Nash-bargaining games are usually constructed), however, the equilibrium may be inefficient, and one party have higher utility than the other.

The reason why a Nash-bargaining equilibrium may not exist, and may be inefficient if it does, is that there may be an insufficient commitment problem. As the main childcarer would earn less (not only while the children are young, but also when they are independent adults) than the main earner, neither party will in fact accept to be the former unless it receives adequate compensation from the latter. In principle, this compensation could be paid either at front, when the children are born, and resources expended on them, or at a later stage, when the children are independent adults. At that later stage, however, it may not be in the main earner's interest to honour any promise it may have made to compensate the main childcarer, because there will be no more efficiency gains to be reaped by division of labour. Any such promise would then lack credibility, and the payment would have to be made at front in full. If the main earner has sufficient assets, or can borrow enough, that will not be a problem. It will be, however, if the main earner is credit constrained, and cannot thus make the required advance payment. Interestingly, although it is always the main earner who pays the compensation, and the main childcarer who receives it, we cannot always say whether this voluntary redistribution will stop before or after utilities are equalized. It will definitely stop before, and the main childcarer will then have lower utility than the main earner, if the latter is the woman (liberated division of labour). If the main earner is the man (traditional division of labour), the redistribution could stop after utilities are equalized, and the equilibrium could thus favour the main childcarer. That is a gender result of sorts. Cigno (2009) shows that marriage will eliminate the insufficient-commitment problem in a community-property jurisdiction, and may alleviate it in a separate-property one.

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