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When do we have borrower or credit volume rationing in competitive credit market with imperfect information?

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Thünen-Series of Applied Economic Theory
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Working Paper No. 117

**When do we have borrower or credit volume rationing in
competitive credit markets with imperfect information?**

von

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2010

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22. Oktober 2010

Abstract	1
1. Introduction	2
2. Basic Model.....	4
2.1 Assumptions	4
2.2 Credit volume	5
2.2.1 Expected profit of an entrepreneur	5
2.2.2 Iso-profit curve	5
2.2.3 Demand curve	6
2.3 Credit rate	7
3. Cournot-Nash equilibrium.....	8
3.1 Social efficient equilibrium	8
3.2 Conditions for a separating contract.....	8
3.3 Alternative assumption sets.....	14
3.3.1 Same return and different success probability	14
3.3.2 Higher return and no lower success probability	16
3.3.3 Lower return and higher success probability	18
4. Wilson's reactive or Grossman's dissembling pooling equilibrium	29
5. Conclusions	30
References	32

Abstract

This paper examines the conditions for credit volume or borrower rationing in a competitive credit market in which the project characteristics are private information of the borrowers. There can only be credit volume rationing if the higher-risk credit applicants have a higher return in the event of a project success than the lower-risk credit applicants. Then the higher-risk borrowers are not rationed and obtain the social efficient credit volume. If the incentive compatibility constraint of the higher risk borrowers is binding, the lower-risk borrowers are credit volume rationed such that the constraint holds as an equation. If credit volume rationing is not sufficient to separate the borrower types, there is additionally a rationing of the low-risk borrowers. If the low-risk borrowers prefer a pooling to a separating contract, then there will not be a Cournot-Nash separating equilibrium, but a Wilson and a Grossmann pooling equilibrium.

JEL-Klassifikation: D82, G21

Keywords: Credit rationing, Credit Size, Loan, Asymmetric Information, Adverse Selection, Non-linear optimization

1. Introduction

Credit rationing means that, in the market equilibrium, demand for credit exceeds supply. In the following this concept is divided into credit volume and borrower rationing. Credit volume rationing takes place if the lenders supply a smaller credit volume than some borrowers demand, although they would be willing to pay a higher credit rate for the higher credit volume. Borrower rationing occurs if, in the equilibrium, some credit applicants are rejected although other credit applicants with the identical risk-return project characteristics are accepted. According to Stiglitz and Weiss (1987) an important reason for credit rationing is asymmetric information, resulting in moral hazard or adverse selection.

Especially in the beginning of a credit relationship, the lenders and the credit applicants don't know each other very well. Then some of the credit applicants' project characteristics are hidden to the lender, whereby adverse selection seems to be the more severe problem than moral hazard. Regarding the empirical studies of, for example Petersen and Rajan (1994) and Berger and Udell (1995) about credit relationships, the adverse selection problem seems to be present in the beginning of a relationship and decreases time by time. As I want to concentrate on this first phase of the credit relationship, I focus on the effects of adverse selection on credit rationing in the following.

Lenders are usually experienced in the field in which they grant credits. By screening, they can collect some information about the risk-return project characteristics of their applicants. However SMEs (small and medium enterprises) are generally young or small and have no or limited reporting obligations. Therefore the lenders are expected to be not able to screen all their applicants' private information. So the credit applicants are assumed to have some private information about the project which they want to have financed with the credit. In fact, according to the empirical findings of Petersen and Rajan (1994), Cole (1998), Blackwell and Winters (1997) credit rationing is a widespread phenomenon for SMEs.

Based on the information about their applicants, the lenders are sorting them into risk classes. By disclosing the risk class in which the credit applicants are rated, the lenders publish their information. The credit applicants, however, are not able to credibly publish their information. The reason is that some borrowers can get better if they pretend to have other project characteristics than they actually have. Thereby the lenders are opposed to borrowers with different project characteristics, although all of them claim to have the same ones. So the borrowers' private information cannot be credibly communicated, although some of them would like to.

As a result, within one risk class, the borrowers have some private information about the risk-return characteristics of their project. The projects differ in their risk-return characteristics, but the project characteristics of an individual credit applicant are not known to the lenders. Because of this asymmetric information the lenders only have the knowledge about the distribution of the project characteristics of their credit applicants.

Reviewing the theoretical literature it is obvious that the assumptions about the distribution of the project characteristics play an important role for the occurrence of credit rationing. They reflect the type of information asymmetry and critically affect the selection effect¹ by

¹ The selection effect is not only affected by the distribution of the project characteristics, but also by the available collateral. Under some assumptions about the project characteristics there can only be credit rationing if the credit applicant cannot supply enough collateral (Besanko/Thakor 1987, Chan/Thakor 1987, Igawa/Kanatas 1990). But collateral does not only serve as a security for the event of a project default. It is also

increasing the credit rate. Depending on the assumptions about the distribution an increase of the credit rate provokes an adverse or a positive selection effect. If an increase of the credit rate provokes the more desired borrower not to apply for a credit, there is an adverse selection effect. If it induces the less desired borrowers not to apply, there is a positive selection effect.

Under asymmetric information two different rationing types can occur, a rationing of the credit volume and a rationing of the borrowers. The first rationing type has first been derived by Jaffee and Russell (1976). They assume different residual values of the enterprise in the event of a project default which are unknown to the lender and argue that the lenders therefore have an incentive to ration the credit volume.

Both Stiglitz and Weiss (1981) and Besanko and Thakor (1987a) regard the credit volume as given. Thereby they only analyze the effect of asymmetric information on the rationing of the borrowers. Their analysis mainly differ by their assumptions about the risk-return characteristics of the credit applicants. Stiglitz and Weiss (1981) assume that the lenders only know the expected profit of their credit applicants, but not their risk respectively their return in the event of a project success. They derive that some borrowers are rationed out. Besanko and Thakor (1987a) assume that good borrowers have a higher success probability than the bad borrowers, but the same return in the event of success. They allow for collateral and find borrower rationing if there is not enough collateral available.

Milde and Riley (1988), Besanko and Thakor (1987b) and De Meza and Webb (1987) treat both, the credit rate and the credit volume, as an action variable. Besanko and Thakor (1987b) analyze a positive selection effect by assuming that good borrowers have a higher success probability and a higher return in the event of a success than bad borrowers. Then the better borrowers receive a higher credit volume than under symmetric information and are not credit volume rationed. De Meza and Webb (1987) assume that the good borrowers have a higher success possibility than the bad borrowers, but the same return in the event of success. Additionally they neglect the possibility of collateral and therefore find a pooling equilibrium. As a consequence an increase of the credit rates leaves the good borrowers with a profit at least as high as the bad borrowers' profit. Due to Milde and Riley's (1988) varying assumptions about the production functions, the credit applicants with either the lower or the higher quality projects are willing to pay a larger credit rate for a larger credit. Hence, they can establish a separating, but not a rationing equilibrium.

Subsumed, there is sure strong empirical evidence for credit volume and borrower rationing. But to my best knowledge it has not been analyzed yet under which conditions there is credit volume rationing or borrower rationing or both. This is the goal of the paper.

In section two I introduce the basic model and derive the iso-profit curve of the credit applicants and the lenders. In section three I analyze the social efficient equilibrium and derive the conditions for a separating contract and a Cournot-Nash equilibrium. At the same time I differentiate the possible assumptions about the risk-return characteristics of the projects of the credit applicants. In section four I discuss Wilson's reactive and Grossman's disassembling pooling equilibrium and in the last section I conclude.

eligible to set incentives. A higher probability of a project default namely increases the probability to pay the collateral, wherefore it is more expensive for the riskier borrowers. Therefore they have a stronger incentive to avoid collateral. Given only the expected return of the credit applicants is known, Bester (1985) clearly illustrates this relationship. See Coco (2000) for more detailed overview.

2. Basic Model

2.1 Assumptions

The projects of the entrepreneurs are completely financed by a credit which they obtain from the lenders. There are two types of entrepreneurs, i and j , differing in the risk or the return of their project. Because of asymmetric information, the lenders cannot distinguish the two entrepreneur types. The project risk is determined by the success probability of the project, $p^k \in [0,1]$ with $k = i, j$. In the unsuccessful state which occurs with a probability $1 - p^k$, the return is zero. In the event of a project success, the entrepreneur k has a project return, $R^k \in R^+$, which is a function of the invested credit volume $v \in R^+$. For each entrepreneur i , the marginal project return is positive, $R'_v > 0$, but decreasing with the credit volume, $R''_{vv} < 0$. If $R^i(v) < R^j(v), \forall v$, the marginal return of the low-risk project is continuously smaller than the marginal return of the high-risk project, $R'_v(v) < R'_v(v)$ and $R''_{vv} \leq R''_{vv}$ for all v . The probability that a credit application is granted may vary with the borrower type $\tau^k \in [0,1]$, $k = i, j$.

The assumptions about the project characteristics seem to be critical for the equilibrium result. Contrary to the literature I do not state to one set of project characteristics. Rather I analyze the effect of the variation of the possible assumption alternatives. The good borrowers' project is defined to yield at least the same expected profit for the lenders than the bad borrowers' project. As the profit of the lenders does only depend on the probability p^k that the credit is repaid, the good borrowers have a higher success probability than the bad borrowers, $p^G > p^B$. If the lenders' expected profit of the projects of both borrower types is on the same level, $p^G = p^B$, the project of the good borrowers is defined to have a higher return in the event of a project success for each credit volume, $R^G(v) > R^B(v), \forall v$. Therewith result the following three assumption sets:

$$(I) \quad p^G > p^B \wedge R^G(v) = R^B(v), \forall v$$

The first case has already been analyzed by Webb and De Meza (1987) and Besanko and Thakor (1987a) in a similar assumption environment.

$$(II) \quad p^G \geq p^B \wedge R^G(v) > R^B(v), \forall v$$

The second case is only known as a special case from Besanko and Thakor 1987b in which the good borrowers have a higher success probability than the bad borrowers, and a higher return in the event of a project success: $p^G > p^B \wedge R^G(v) > R^B(v), \forall v$.

$$(III) \quad p^G > p^B \wedge R^G(v) < R^B(v), \forall v$$

The third case is only known as a special case from Stiglitz and Weiss (1981) in which both project types yield the same expected return:

$$p^G > p^B \wedge p^G R^G(v) = p^B R^B(v), \forall v \Rightarrow R^G(v) < R^B(v), \forall v.$$

Based on their available information, the lenders assign the entrepreneurs' projects to risk classes. However within a risk class, the success probability of the entrepreneurs' project is

not observable to the lenders. The lenders cannot detect who is a good and who is a bad borrower. He only knows the good resp. bad borrowers' proportion, λ resp. $(1 - \lambda)$ and the good resp. bad borrowers' success probability, p^G resp. p^B . Based on this information the credit rate is determined under a competitive credit market in which the lenders earn zero expected profits. The lenders finance their credit at a fixed deposit rate ρ . The entrepreneurs are assumed to know the success probability of their project. All entrepreneurs and lenders are risk neutral.

The game has two stages. In the first stage, the lenders simultaneously offer contracts. In the second stage, each entrepreneur either selects one of the contracts offered, or chooses not to apply for a credit.

2.2 Credit volume

The borrowers know the credit rate which they have to pay choosing any credit volume. Therefore, I first analyze the optimal credit volume $v^*(r)$ for the borrowers, dependent on the credit rate r under asymmetric information. Then, based on the determined optimal credit volume function, I detect the cost covering credit rate $r(v^*)$ for the lenders. Starting from the expected profit of an entrepreneur, I formulate the properties of an entrepreneur's iso-profit curve. By means of them I derive the characteristics of the entrepreneurs' credit demand curves.

2.2.1 Expected profit of an entrepreneur

The credit is granted with probability τ , the project is successful with probability p^k . If the project is successful, the entrepreneur earns a return $R^k(v)$ minus the credit costs. So, the expected profit of an entrepreneur k , given r , is

$$\Pi_e^k(\tau, r, v) = \tau p^k [R^k(v) - (1 + r)v].$$

As there are no opportunity costs, this result does not depend on whether the entrepreneur has been accepted as a borrower or not. Consequently, with respect to the credit volume, the first order condition is $R_v^k(v^*) = (1 + r)$. So the entrepreneur's expected profit maximizing with respect to the credit volume v^* only depends on the credit rate and the borrower's marginal return. The latter is positive, $R_v^k > 0$, and decreasing, $R_{vv}^k < 0$, in the credit volume, wherefore there exists an interior solution³. Obviously the entrepreneurs' expected profit decreases with the credit rate, $\Pi_r^k = -\tau v p < 0$.

2.2.2 Iso-profit curve

As a consequence, the entrepreneurs' iso-profit curve⁴ is

$$^2 \frac{\partial \Pi_e(\tau, r, v; p)}{\partial v} = \tau [R_v(v; p) - (1 + r)] p \begin{cases} > 0; v < v^* \\ < 0; v > v^* \end{cases} \text{ with } R_v(v^*; p) - (1 + r) = 0$$

$$^3 \frac{\partial^2 \Pi_e(\tau, r, v; p)}{\partial v^2} = \tau [R_{vv}(v; p)] p < 0$$

$$^4 \left. \frac{dr}{dv} \right|_{\Pi_e} = - \frac{\partial \Pi_e / \partial v}{\partial \Pi_e / \partial r} = - \frac{\tau [R_v(v; p) - (1 + r)] p}{-\tau v p} = \frac{R_v(v; p) - (1 + r)}{v}$$

$$\left. \frac{dr}{dv} \right|_{\Pi_e} = \frac{R_v^k(v) - (1+r)}{v}.$$

An increase of the credit rate decreases the profit of the borrowers. This loss can be compensated by a higher credit volume as long as it has not reached the optimal credit volume v^* . As the marginal return decreases with the credit volume, the compensation increases with the level of the credit rate. In other words, the borrower's marginal willingness to pay for a higher credit volume is positive and decreasing with his credit volume as long as $v < v^*$. It has its maximum at $v = v^*$ and then becomes negative for $v > v^*$.

2.2.3 Demand curve

For each credit rate, the borrower wants to get on the lowest-possible iso-profit curve to earn the maximum profit. The peak of each iso-profit curve describes the profit maximizing credit volume for a given credit rate. So the credit demand curve is just the geometric place of all iso-profit curve maxima. From these iso-profit curve maxima, a total differentiation with respect to the optimal credit volume v^* and the credit rate r results in⁵

$$\frac{dv^*}{dr} = \frac{1}{R_{vv}^k(v^*)} < 0.$$

As the marginal return decreases with the credit volume, $R_{vv}^k < 0$, the borrower's optimal credit volume v^* decreases with the credit rate r .

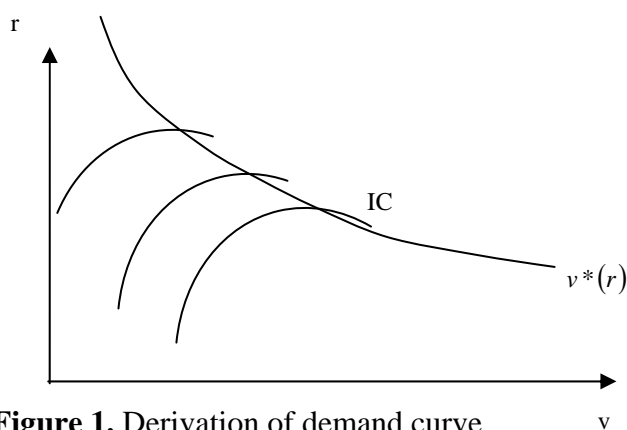


Figure 1. Derivation of demand curve

To analyze the difference between the demand curves of the good and the bad borrowers, their willingness to pay for a marginal higher credit volume is compared:

$$\left. \frac{dr}{dv} \right|_{\Pi_e^G} = \frac{R_v^G(v) - (1+r)}{v} \text{ vs. } \frac{R_v^B(v) - (1+r)}{v} = \left. \frac{dr}{dv} \right|_{\Pi_e^B}.$$

⁵ The total differential of the profit maximization condition $\partial \Pi_e(\tau, r, v; p) / \partial v = \tau [R_v(v^*; p) - (1+r)] p = 0$ is

$$R_{vv}(v^*; p) dv^* - dr = 0 \Leftrightarrow \frac{dv^*}{dr} = \frac{1}{R_{vv}} < 0.$$

If the good and the bad borrowers' marginal returns are identical, i.e. $R_v^G(v) = R_v^B(v)$, they have identical iso-profit curves and therefore identical demand curves. Then they demand the same credit volume for each credit rate and there is no separating contract. This is due to the fact that identical returns in the event of a project success do neither have an adverse nor a positive effect.

If the marginal return of the good borrowers is lower (higher) than the marginal return of the bad borrowers, i.e. $R_v^G(v) < R_v^B(v)$ ($R_v^G(v) > R_v^B(v)$), the iso-profit curve of the bad (good) borrowers is steeper than the iso-profit curve of the good (bad) borrowers. Then it crosses the iso-profit curve of the good (bad) borrowers from below. For each credit rate r , the bad (good) borrowers therefore demand a higher credit volume than the good (bad) borrowers. So the credit demand curve of the bad (good) borrowers is to the right of the credit demand curve of the good (bad) borrowers and does not cross it.

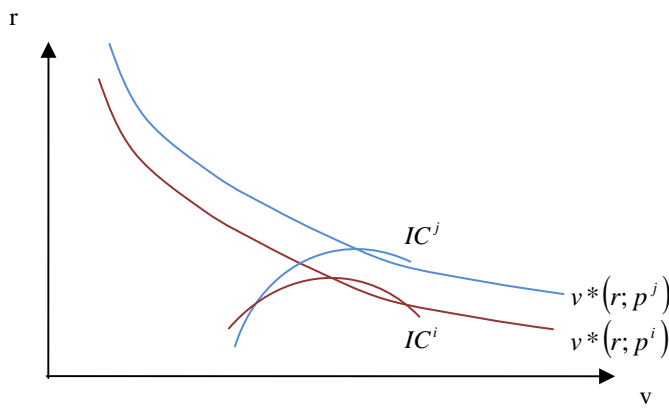


Figure 2. Iso-profit and demand curves of different borrower types i and j

2.3 Credit rate

Because the credit market is competitive, in a Cournot-Nash equilibrium the lenders make zero profits. However, because of asymmetric information, the lenders cannot perceive distinct risky projects within one risk class. They can only separate the borrower types by self selection contracts or pool them.

In a separating equilibrium the lenders get a repayment $(1 + r^k)v^k$ with probability p^k , but always have to pay deposit costs of $(1 + \rho)v^k$. They can vary the credit rate, the credit volume and ration the borrowers such that the borrowers self select their contract corresponding to their type.

$$\Pi_l(\tau^k, r^k, v^k) = \tau^k [p^k(1 + r^k) - (1 + \rho)]v^k = 0, \quad k = i, j$$

However under specific circumstances it might not be possible to establish a separating equilibrium. In a pooling equilibrium, if there exists one, the lenders offer the same contract to all borrowers:

$$\Pi_l(r, v) = \lambda [p^G(1 + r) - (1 + \rho)]v + (1 - \lambda) [p^B(1 + r) - (1 + \rho)]v = [\bar{p}(1 + r) - (1 + \rho)]v$$

with the average success probability $\bar{p} = \lambda p^G + (1 + \lambda)p^B$. Based on the zero expected profit condition the pooling credit rate is

$$r = \frac{1 + \rho}{\bar{p}} - 1.$$

The credit rate is determined by the deposit rate and the expected average success probability. As it is assumed that the credit volume does not affect the success probability, the credit rate does not depend on the credit volume.

3. Cournot-Nash equilibrium

Based on the distribution of the project characteristics of the credit applicants, the lenders offer a menu of contracts. From this menu the credit applicants choose their optimal contract. However a determination of an equilibrium requires not only more specific assumptions about the strategic behavior of the lenders and the credit applicants, but also a specified equilibrium concept. Most of the conclusions of the theoretical credit rationing literature are built on the Cournot-Nash equilibrium which is described and used in the following.

Definition (Cournot-Nash equilibrium). The contract offers of the competing lenders are assumed to be independent of the lenders' strategies. Credit applicants choose contracts to maximize expected utility. The credit market is competitive. Then the Cournot-Nash equilibrium is a set of contracts S^* such that,

- (1) no contract in the equilibrium set makes negative expected profits; and
- (2) there is no contract outside the equilibrium set that, if offered, will make a positive profit.

3.1 Social efficient equilibrium

A Cournot-Nash equilibrium is not necessarily social efficient. For social efficiency of the investment, the expected social marginal return of the invested capital must equal the social marginal cost. So the contract is optimal for the borrowers when the slope of their iso-profit curve equals the slope of the lender's offer curve:

$$\left. \frac{dr^k}{dv^k} \right|_{\Pi_i^k} = \frac{R_v^k(v^k) - (1 + r^k)}{v^k} = - \frac{p^k(1 + r^k) - (1 + \rho)}{p^k v^k} = \left. \frac{dr^k}{dv^k} \right|_{\Pi_i^k} \Leftrightarrow R_v^k(v^k) = \frac{1 + \rho}{p^k}.$$

Then the borrowers' expected marginal return just covers the lenders' marginal deposit costs. Though, this result is based on lenders who know the borrowers' type or can reveal the type without any informational costs. But because of asymmetric information this supposition is not given wherefore the social efficient solution might not be achieved.

3.2 Conditions for a separating contract

Under asymmetric information the success probability and the return in the event of a project success is private information of the borrowers. An equilibrium under which the borrowers completely reveal their private information to the lenders is called a separating equilibrium. For the existence of a separating equilibrium, a separating contract is necessary. For a separating contract it must hold:

$$\max_{\substack{\tau^i, v^i, r^i \\ \tau^j, v^j, r^j}} \lambda \left(\tau^i p^i \left[R^i(v^i) - (1+r^i)v^i \right] \right) + (1-\lambda) \left(\tau^j p^j \left[R^j(v^j) - (1+r^j)v^j \right] \right)$$

$$(NPC-k) \quad \tau^k \left\{ p^k (1+r^k) - (1+\rho) \right\} v^k = 0, \quad k = i, j$$

$$(IC-i) \quad \tau^i p^i \left[R^i(v^i) - (1+r^i)v^i \right] \geq \tau^j p^j \left[R^i(v^j) - (1+r^j)v^j \right]$$

$$(IC-j) \quad \tau^j p^j \left[R^j(v^j) - (1+r^j)v^j \right] \geq \tau^i p^i \left[R^j(v^i) - (1+r^i)v^i \right]$$

$$(PC-k) \quad \tau^k p^k \left[R^k(v^k) - (1+r^k)v^k \right] \geq 0, \quad k = i, j$$

$$(BRC-k) \quad 0 \leq \tau^k \leq 1, \quad k = i, j$$

$$(NNC-k) \quad v^k \geq 0, \quad k = i, j$$

The degree of borrower rationing, the credit volume and the credit rate are chosen such that the total expected profit of the entrepreneurs is maximized. In a separating equilibrium the type i and j borrowers are revealed and might pay a different credit rate. Because there is a perfect competition among the lenders, they make zero profits (NPC-k) with each borrower type k. This condition is uniform with the requirements of a Cournot-Nash equilibrium.

If there is a separating equilibrium, at least one of the incentive compatible constraints of the two different borrower types i and j, (IC-i) and (IC-j), is assumed to be not binding. This will apply if the profit of the type i borrowers revealing their type is at least as high as the profit pretending to be of type j:

$$\tau^{i^*} \left\{ p^i \left[R^i(v^{i^*}) - (1+r^{i^*})v^{i^*} \right] \right\} \geq p^i \left[R^i(v^{j^*}) - (1+r^{j^*})v^{j^*} \right] \Leftrightarrow \tau^{i^*} \geq \frac{R^i(v^{j^*}) - (1+r^{j^*})v^{j^*}}{R^i(v^{i^*}) - (1+r^{i^*})v^{i^*}}.$$

As the borrower rationing is constrained to $0 \leq \tau^i \leq 1$, the transformation of the equality above results in:

$$1 \geq \tau^{i^*} \geq \min \left\{ 1; \frac{R^i(v^{j^*}) - (1+r^{j^*})v^{j^*}}{R^i(v^{i^*}) - (1+r^{i^*})v^{i^*}} \right\}.$$

Later it is shown that (IC-i) is an inequality for a very broad range of assumptions sets. In the following the incentive compatible constraint of the type i borrowers is assumed to be not binding.

The participation condition (PC-k) of each borrower of type k is assumed to hold. Of course, there are possible assumptions sets under which an application for a credit might not yield a positive profit for the borrowers. But it is relatively simple to solve this case and it does not help to gain new insights about the credit rationing phenomenon. Therefore the participation conditions are assumed to hold and suppressed in the following.

It is possible to ration no, some or all type i or j borrowers (BRC-k), i.e. the rationed borrowers of the type are not granted a credit. The credit volume can never be negative, the credit rate is determined by (NPC-k).

$$\tau^k \left[p^k (1+r^k) - (1+\rho) \right] v^k = 0 \Leftrightarrow (1+r^k)v^k = \frac{(1+\rho)v^k}{p^k}, \quad k = i, j.$$

Setting this into the maximization condition and suppressing (IC-i) and (PC-k) results in the Lagrangian:

$$L(\tau^i, \tau^j, v^i, v^j, \mu) = \lambda \tau^i \left[p^i R^i(v^i) - (1 + \rho)v^i \right] + (1 - \lambda) \tau^j \left[p^j R^j(v^j) - (1 + \rho)v^j \right] - \mu \left\{ \tau^i p^j \left[R^j(v^i) - \frac{1 + \rho}{p^i} v^i \right] - \tau^j p^j \left[R^j(v^j) - \frac{1 + \rho}{p^j} v^j \right] \right\}.$$

Depending on the assumptions about the project characteristics it must be checked if any of the incentive compatible conditions of the type i or j borrowers is binding or not. In the following I differentiate between the cases in which it is possible to separate the two borrower types and in which it is not.

In the first case both borrower types always demand the same contracts. Then the incentive compatible conditions of both borrower types are binding at the same time and a separating equilibrium cannot exist. As a consequence the inequalities disappear and the problem reformulates to a simple linear programming. Then there is the same credit rate and the same credit volume for both borrower types.

In the second case it is possible to separate the borrower types by a corresponding contract design. Then at most one of the incentive compatible conditions of the borrower types is binding. However it is not clear if any of the incentive compatible condition is binding in the optimum at all. As a consequence it is an optimization with inequalities as constraints. For the possibility of the so-called non-linear programming the following conditions for the Lagrangian are necessary:

(i) First order conditions: $\frac{\partial L}{\partial v^k} \stackrel{!}{\leq} 0$ and $v^k \geq 0$, $k = i, j$ ($= 0$ for $v^k > 0$)

(ii) First order conditions: $\frac{\partial L}{\partial \tau^k} \stackrel{!}{=} 0$ if $0 < \tau^{k*} < 1$, $k = i, j$

$$\frac{\partial L}{\partial \tau^k} \stackrel{!}{\leq} 0 \text{ if } \tau^{k*} = 0, k = i, j$$

$$\frac{\partial L}{\partial \tau^k} \stackrel{!}{\geq} 0 \text{ if } \tau^{k*} = 1, k = i, j$$

(iii) Complementary slackness conditions (C-S):

$$\mu^* \left\{ \tau^{i*} p^j \left[R^j(v^{i*}) - \frac{1 + \rho}{p^i} v^{i*} \right] - \tau^{j*} p^j \left[R^j(v^{j*}) - \frac{1 + \rho}{p^j} v^{j*} \right] \right\} \stackrel{!}{=} 0$$

(iv) Non-negativity of the Lagrange multiplier: $\mu^* \stackrel{!}{\geq} 0$ ($= 0$, if

$$\tau^{i*} p^j \left[R^j(v^{i*}) - \frac{1 + \rho}{p^i} v^{i*} \right] < \tau^{j*} p^j \left[R^j(v^{j*}) - \frac{1 + \rho}{p^j} v^{j*} \right]).$$

(v) Incentive compatible conditions (IC-j)

If none of the incentive compatible conditions is binding, i.e.

$\tau^j p^j [R^j(v^j) - (1+r^j)v^j] > \tau^i p^j [R^j(v^i) - (1+r^i)v^i]$, the complementary slackness multiplier is zero $\mu^* = 0$. If the incentive compatible condition of borrower type j is binding, i.e.

$\tau^j p^j [R^j(v^j) - (1+r^j)v^j] = \tau^i p^j [R^j(v^i) - (1+r^i)v^i]$, the complementary slackness multiplier is larger than zero, $\mu^* > 0$.

In the following, the incentive compatible condition of the type i borrowers is assumed to be not binding. Because of its arithmetical complexity, the non-linear program requires the derivation of the Lagrangian, which is done below.

a) Optimal credit volume of the type j borrowers

$$\frac{\partial L}{\partial v^j} = [(1-\lambda) + \mu] \tau^j [p^j R_v^j(v^j) - (1+\rho)] \stackrel{!}{=} 0 \Leftrightarrow R_v^j \stackrel{!}{=} \frac{1+\rho}{p^j}.$$

Borrower j is willing to increase his credit volume until his expected marginal profit is zero. Then he can take out the social efficient credit volume and is not credit volume rationed.

b) Optimal credit volume of the type i borrowers

$$\frac{\partial L}{\partial v^i} = \tau^{i*} \left\{ \lambda [p^i R_v^i(v^i) - (1+\rho)] - \mu \left[p^j R_v^j(v^i) - p^j \frac{(1+\rho)}{p^i} \right] \right\} \stackrel{!}{=} 0$$

$$p^i R_v^i(v^{i*}) - (1+\rho) \stackrel{!}{=} \frac{\mu}{\lambda} \left[p^j R_v^j(v^{i*}) - p^j \frac{(1+\rho)}{p^i} \right]$$

The optimal credit volume of the type i borrowers is only social efficient if the incentive compatibility constraint of type j is not binding, $\mu = 0$. If it is binding, $\mu > 0$, the asymmetric information causes a lower credit volume than the type i borrowers would prefer and thereby informational costs accrue.

c) Rationing of type j borrowers

The derivation with respect to borrower j rationing results in

$$\frac{\partial L}{\partial \tau^j} = [(1-\lambda) + \mu] [p^j R^j(v^{j*}) - (1+\rho)v^{j*}] > 0.$$

A rationing of the type j borrowers causes costs for borrower type j. As the incentive constraint of the type i borrowers is not binding, they are not interested in pretending to be a type j borrower. This is why a rationing of the type j borrower neither benefits the lenders nor the borrowers. Later it will be shown that under the assumption sets one and two the incentive compatible constraint is not binding and under assumption set three only for a small range, wherefore it is suppressed in the future.

d) Lagrange multiplier

$$\frac{\partial L}{\partial \tau^i} = \lambda p^i \left\{ R^i(v^{i*}) - \frac{1+\rho}{p^i} v^{i*} \right\} - \mu^* p^j \left\{ R^j(v^{i*}) - \frac{1+\rho}{p^i} v^{i*} \right\}.$$

Whether the type i borrowers are rationed or not depends on the Lagrange multiplier, which reflects the binding of the incentive compatible constraint of the type j borrowers. If this constraint is not binding, this multiplier will be zero. Then the derivation will be positive and there won't be any rationing of the type i borrowers. If, however, in the optimum the constraint is binding, then the Lagrange multiplier will become positive and is

$$\mu^* = \lambda \frac{p^i \left[R^i(v^{i*}) - \frac{(1+\rho)}{p^i} v^{i*} \right]}{p^j \left[R^j(v^{i*}) - \frac{1+\rho}{p^i} v^{i*} \right]} = \lambda \frac{p^i \left[R^i(v^{i*}) - (1+r^{i*}) v^{i*} \right]}{p^j \left[R^j(v^{i*}) - (1+r^{i*}) v^{i*} \right]}.$$

The Lagrange multiplier μ^* is equal to the relation of the expected profit of the type i borrowers revealing their true type and the expected profit of the type j borrowers pretending to be type i borrowers, weighted with the share of the type i borrowers.

e) Slackness condition and incentive compatible constraint of type j borrowers

As the incentive compatible constraint of the type j borrowers is not necessarily binding a simple linear program is not possible. Therefore the problem is non-linear and the introduction of a slackness condition is required:

$$\mu^* \left\{ \tau^{i*} p^j \left[R^j(v^{i*}) - \frac{1+\rho}{p^i} v^{i*} \right] - p^j \left[R^j(v^{j*}) - \frac{1+\rho}{p^j} v^{j*} \right] \right\} = 0.$$

This condition implies that the Lagrange multiplier μ^* depends on the incentive compatible constraint of the type j borrowers. If the type j borrowers make a lower profit by pretending to be a type i borrower, it is an inequality and not binding. So the term within the bracket is non-zero, wherefore the Lagrange multiplier must be zero to fulfill the slackness condition. This entails that both borrower types obtain their demanded credit volume and that no borrowers are rationed.

If the incentive compatible condition of the type j borrowers is binding, it holds as an equation. The term within the brackets will be zero and the slackness condition is fulfilled. Then the multiplier can be bigger than zero, which must be factored in by the type i borrowers before they choose their optimal credit volume. Given that borrower rationing is not optimal, then the type i borrowers will choose a credit volume which equates the type j borrowers' incentive compatible constraint. If borrower rationing is optimal it is required to set the resulting Lagrange multiplier from the calculation of the first order conditions,

$$\mu^* = \lambda \frac{p^i \left[R^i(v^{i*}) - \frac{1+\rho}{p^i} v^{i*} \right]}{p^j \left[R^j(v^{i*}) - \frac{1+\rho}{p^i} v^{i*} \right]},$$

into the first order condition with respect to the credit volume of borrower type i. Then there results:

$$\frac{p^i \left[R_v^i(v^{i*}) - \frac{1+\rho}{p^i} \right]}{p^i \left[R^i(v^{i*}) - \frac{1+\rho}{p^i} v^{i*} \right]} = \frac{p^j \left[R_v^j(v^{i*}) - \frac{(1+\rho)}{p^i} \right]}{p^j \left[R^j(v^{i*}) - \frac{1+\rho}{p^i} v^{i*} \right]} \Leftrightarrow \frac{\partial \Pi_i^i / \Pi_i^i}{\partial v^i / v^i} = \frac{\partial \Pi_i^j / \Pi_i^j}{\partial v^i / v^i}.$$

Lemma 1 (Optimal credit volume under asymmetric information). Regard the contract designed to reveal the type i and j borrowers in the Cournot-Nash equilibrium. Then by a one percent change in the credit volume for the type i borrowers, the percentage change in the expected profit chosen by type i borrowers is equal to the percentage change in the expected profit chosen by type j borrowers.

f) Rationing of type i borrowers

Borrower rationing can be an eligible instrument to induce the type j borrowers to reveal their type. However this exclusion of type i borrowers causes opportunity costs for the rejected type i borrowers. As a consequence this instrument will only be used if the incentive compatible constraint of the type j borrowers is binding and holds as an equation.

Transforming (IC-j) it results in the rationing of the type i borrowers:

$$\tau^{i*} = \frac{R^j(v^{j*}) - (1+r^{j*})v^{j*}}{R^j(v^{i*}) - (1+r^{i*})v^{i*}}.$$

As the borrower rationing is constrained to $0 \leq \tau^i \leq 1$ and because of the assumption that the incentive compatible constraint of the good borrowers is not binding:

$$\min \left\{ 1; \frac{R^i(v^{j*}) - (1+r^{j*})v^{j*}}{R^i(v^{i*}) - (1+r^{i*})v^{i*}} \right\} \leq \tau^{G*} = \min \left\{ 1; \frac{R^j(v^{j*}) - (1+r^{j*})v^{j*}}{R^j(v^{i*}) - (1+r^{i*})v^{i*}} \right\}.$$

Given the optimal credit volumes and credit rates are chosen, if the profit of the type j borrowers pretending to be a type i borrower is higher than revealing their true type, then type i borrowers are rationed. The higher this profit difference is, the more the type i borrowers must be rationed.

Both, credit volume and borrower rationing are feasible instruments to separate the borrower types. With it, they interact. On the one hand the credit volume rationing of the type i borrowers decreases the profit of the type j borrowers pretending to be a type i borrower. Thus, less type i borrowers have to be rationed. On the other hand, the rationing of the type i borrowers helps to equate the incentive compatible constraint of the type j borrowers. Thereby the type i borrowers have to be less credit volume rationed.

Proposition 1.

- (1) If neither the incentive compatible constraint of the type i nor the type j borrowers is binding, the Cournot-Nash equilibrium is social efficient and there cannot be borrower or credit volume rationing.
- (2) If the incentive compatible constraint of the type j borrowers is binding, but of the type i borrowers is not, then the Cournot-Nash equilibrium will not be social efficient and there may be borrower or credit volume rationing for type i borrowers.
 - i. The type j borrowers obtain the social efficient credit volume and are not borrower or credit volume rationed.

- ii. The type i borrowers obtain a smaller than the social efficient credit volume. Without borrower rationing, for the type i's optimal credit volume it holds that the profit of the type j borrowers pretending to be a type i borrower is as high as revealing their true type. With borrower rationing, for the credit volume of the type i borrowers it holds $\frac{\partial \Pi_i^i / \Pi_i^i}{\partial v^i / v^i} = \frac{\partial \Pi_i^j / \Pi_i^j}{\partial v^i / v^i}$.

3.3 Alternative assumption sets

The results of the last section are very general and do only explain that there can be credit rationing. However it does not become clear under which conditions there is credit rationing. So in this section I will answer under which conditions there is a separating, a pooling or no equilibrium, an under- or overinvestment and credit volume or borrower rationing.

In the literature it is also broadly discussed when there is credit rationing. However different authors conclude different propositions about the credit rationing. Summarizing the results of their models, it is obvious that the assumptions about the risk and return properties of the projects play an important role in their deliberations. Depending on these assumptions it is determined which credit applicant types have an incentive to select a contract and if, which one.

Therefore I will differentiate between three different assumption sets and follow the definition of a good and a bad borrower. In the first assumption set, both borrower types have the same return, but the good borrowers have a higher success probability than the bad borrowers. In the second assumption set, the good borrowers have a higher return than the bad borrowers, but not a lower success probability. In the third assumption set, the good borrowers have lower return and a higher success probability than the bad borrowers.

3.3.1 Same return and different success probability

Assumption set one. In the first assumption set, the project of the good borrowers has a higher success probability than the project of the bad borrowers. The return in the event of a project success is a function of the invested credit volume. Both borrower types have the same return function:

- (1) $p^G > p^B$
- (2) $R^G(v) = R^B(v) = R(v), \forall v$.

This implies that the project of the good borrowers stochastically dominates the projects of the bad borrowers in the first-order sense. A similar assumption set is used in Webb and De Meza (1987) and Besanko and Thakor (1987a). Both borrower types get the same return in the event of a project success. However, in their model, the return is not a function of the credit volume. They assume that the credit volume is fixed. So they cannot analyze if there is any credit volume rationing.

At first I check if a separating equilibrium can exist. This can only apply if

$$\begin{aligned} \text{(IC-G)} \quad & p^G [R(v^G) - (1 + r^G)v^G] \geq p^G [R(v^B) - (1 + r^B)v^B] \\ \text{(IC-B)} \quad & p^B [R(v^B) - (1 + r^B)v^B] \geq p^B [R(v^G) - (1 + r^G)v^G], \end{aligned}$$

and if at least one of the two incentive compatible constraints holds with a strict inequality. Shortening the incentive compatibility constraints by the success probability results in

$$\begin{aligned} \text{(IC-G)} \quad & R(v^G) - (1 + r^G)v^G \geq R(v^B) - (1 + r^B)v^B \\ \text{(IC-B)} \quad & R(v^B) - (1 + r^B)v^B \geq R(v^G) - (1 + r^G)v^G. \end{aligned}$$

Summarized you can write

$$R(v^G) - (1 + r^G)v^G \geq R(v^B) - (1 + r^B)v^B \geq R(v^G) - (1 + r^G)v^G,$$

which means that the inequalities hold as equations:

$$R(v^G) - (1 + r^G)v^G = R(v^B) - (1 + r^B)v^B.$$

So there cannot be a separating, but a pooling equilibrium. In a pooling equilibrium there is only one contract with a universal credit rate \bar{r} which reflects the borrowers' average success probability $\bar{p} = \lambda p^G + (1 - \lambda)p^B$. As the good and the bad borrowers have the same return function in the event of a project success, they have the same iso-profit curve. Given, the universal credit rate \bar{r} , then the borrowers' credit volume will be optimal if thereby the slope of the borrowers' iso-profit curve equals the slope of the lenders' cost curve:

$$\left. \frac{dr}{dv} \right|_{\Pi_b} = \frac{R_v(v^*) - (1 + r)^!}{v^*} = \frac{\bar{p}(1 + r) - (1 + \rho)}{\bar{p}v^*} = \left. \frac{dr}{dv} \right|_{\Pi_l} \Leftrightarrow R_v(v) - \frac{(1 + \rho)^!}{\bar{p}} = 0 \Leftrightarrow R_v(v) - (1 + \bar{r})^! = 0.$$

As both borrower types have the same iso-profit curve, they are not willing to pay a higher credit rate to obtain a higher credit volume. So no borrowers are credit volume rationed or rationed out.

The bad borrowers pay a lower credit rate than it would be justified by the success probability of their project. Therefore they demand a higher credit volume than is social efficient. The good borrowers pay a higher credit rate than it would be justified by the success probability of their project. Therefore they demand a lower credit volume than is social efficient.

As there is a pooling contract, a selection effect could be present if a raise of the credit rate induces one of the two borrower types to refrain from a credit application, given that the other borrower type applies for a credit. As both, the good and the bad borrowers, have the same return in the event of success, the bad borrower only makes zero profits i.f.f. the good borrower makes zero profits. As a consequence there is neither positive nor adverse selection. The following proposition summarizes these insights.

Proposition 2 (Assumption set one). Assume that the project characteristics correspond to the assumption set 1. Then

- (1) there is a pooling Nash equilibrium
- (2) no borrower type is credit volume rationed
- (3) no borrower type is borrower rationed
- (4) the bad borrowers are overfinanced and the good borrowers are underfinanced.

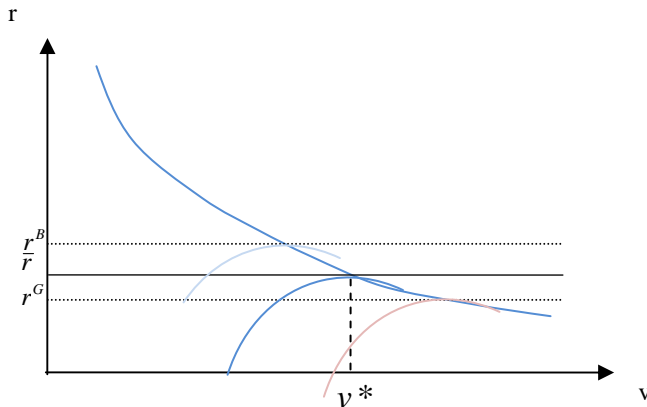


Figure 3. Pooling equilibrium under assumption set one

3.3.2 Higher return and no lower success probability

Assumption set two. In the second assumption set, the project of the good borrowers has the same or a higher success probability than the project of the bad borrowers. The return of the good borrowers in the event of a project success is higher than the return of the bad borrowers for each credit volume:

- (1) $p^G \geq p^B$
- (2) $R^G(v) > R^B(v), \forall v.$

Besanko and Thakor (1987b) have a similar assumption set. Their borrower type with the higher success probability has a higher return in the event of a project success. Furthermore Besanko and Thakor assume continuous borrower types with an identical non-random end-of period wealth which they can use as collateral. However this wealth is not sufficient to cover the project risk of all borrower types. So in their model, the borrower types with low success probabilities demand a lower credit volume than the borrower types with high success probabilities. As they however have the same wealth, their collateral relative to the credit volume is higher. So the higher-risk borrower types are able to eliminate their lower success probability by their collateral and therefore pay a lower credit rate than the lower-risk borrowers.

The higher the success probability of a borrower type, the higher is his willingness to pay for a higher credit volume. Under asymmetric information the lenders use this relationship by increasing the credit volume above the social efficient level and demand a higher credit rate, adequate to the consequential higher risk. However the conclusions of Besanko and Thakor (1987b) are limited to the presence of wealth which is eligible to serve as collateral costlessly.

If there is no wealth, the borrower types with a risky project do not have the possibility to eliminate their risk by collateral. Then they are riskier for the lender and no longer have to pay a lower credit rate than the less risky borrowers. So the results of Besanko and Thakor (1987b) are not suitable for the event in which the borrowers do not have any wealth. To guarantee an adequate comparison to the other assumption sets an analysis of the assumption set 3 without wealth is required.

The analysis follows two steps. In the first step I assume that none of the incentive compatible constraints are binding. Based on this assumption I derive the optimal credit volume in the

equilibrium. In the next step I analyze the incentive compatible constraints and show that really none of the incentive compatible constraints is binding.

As the good borrowers have a higher return in the event of a project success than the bad borrowers, for each credit volume, $R^G(v) > R^B(v), \forall v$, the good borrowers have a steeper iso-profit curve than the bad borrowers:

$$\frac{R_v^G(v) - (1+r)}{v} > \frac{R_v^B(v) - (1+r)}{v}.$$

So, for each credit rate, the good borrowers demand a higher credit volume than the bad borrowers, $v^{G*} > v^{B*}$.

If none of the incentive compatible constraints is binding, then according to the slackness condition $\mu^* = 0$. So there are no information asymmetry costs and it is sufficient to equate the slope of the iso-profit curves and the cost curves for the good and the bad borrowers

$$\frac{R_v^G(v^{G*}) - (1+r^G)}{v^{G*}} = \frac{p^G(1+r^G) - (1+\rho)}{p^G v^{G*}} \Leftrightarrow R_v^G(v^{G*}) = \frac{1+\rho}{p^G},$$

$$\frac{R_v^B(v^{B*}) - (1+r^B)}{v^{B*}} = \frac{p^B(1+r^B) - (1+\rho)}{p^B v^{B*}} \Leftrightarrow R_v^B(v^{B*}) = \frac{1+\rho}{p^B}.$$

If $p^G > p^B$, it is obvious that in a separating equilibrium the good borrowers will pay a lower competitive credit rate than the bad borrowers. Because of the lower credit rate and the higher marginal return in the event of a project success, the good borrowers demand a higher credit volume than the bad borrowers. If both borrower types have the same project success probability, they will pay the same credit rate $r = [(1+\rho)/p] - 1$. Then the good borrowers again demand a higher credit volume than the bad borrowers, because of their higher marginal return in the event of a project success.

Now I have derived the separating equilibrium if none of the incentive compatible constraints is binding. That in fact none of the incentive compatible conditions is binding, can be shown by simplifying the incentive compatible conditions of the borrower types to⁶:

$$(IC-G) \quad p^G [R^G(v^{G*}) - (1+r^G)v^{G*}] \geq p^G [R^G(v^{B*}) - (1+r^B)v^{B*}]$$

$$(IC-B) \quad p^B [R^B(v^{B*}) - (1+r^B)v^{B*}] \geq p^B [R^B(v^{G*}) - (1+r^G)v^{G*}].$$

After some transformations and setting in the non-profit condition, it results:

$$R^G(v^{G*}) - R^G(v^{B*}) \geq \frac{1+\rho}{p^G} v^{G*} - \frac{1+\rho}{p^B} v^{B*} \geq R^B(v^{G*}) - R^B(v^{B*}).$$

The good borrowers have a higher marginal return in the event of a project success, $R_v^G(v) > R_v^B(v)$. This is why they demand a higher credit volume than the bad borrowers, $v^{G*} > v^{B*}$. As $v^{G*} > v^{B*}$, it is $\lim_{v^B \rightarrow v^G} R^i(v^G) - R^i(v^B) = R_v^i$. Because of assumption

⁶ If none of the incentive compatible constraints is binding, then no borrowers are rationed, $\tau^{G*} = \tau^{B*} = 1$.

$R_v^G < R_v^B, \forall v$, it strictly holds $R^G(v^{G*}) - R^G(v^{B*}) > R^B(v^{G*}) - R^B(v^{B*})$. That means, that none of the incentive compatible conditions is binding, $\mu^* = 0$. Thereby, the good borrowers are not rationed, $\tau^{G*} = 1$, and both borrower types can demand their optimal credit volumes. Thereby it is easy to conclude the following proposition.

Proposition 3 (Assumption set two). Assume that the project characteristics correspond to the assumption set two. Then

- (1) there is a separating Nash equilibrium
- (2) no borrower type is credit volume rationed
- (3) no borrower type is borrower rationed
- (4) no borrower type is under- or overfinanced

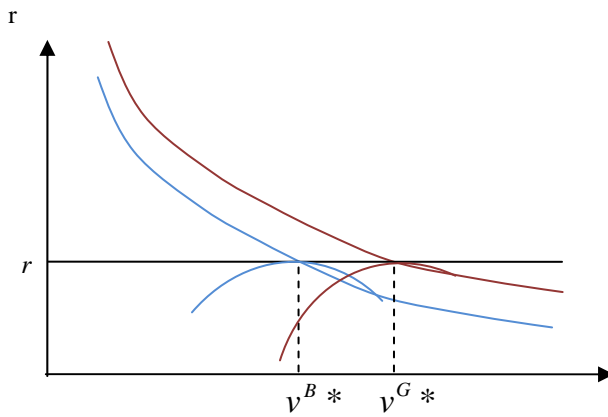


Figure 4a. Separating equilibrium under assumption set two and same success probability

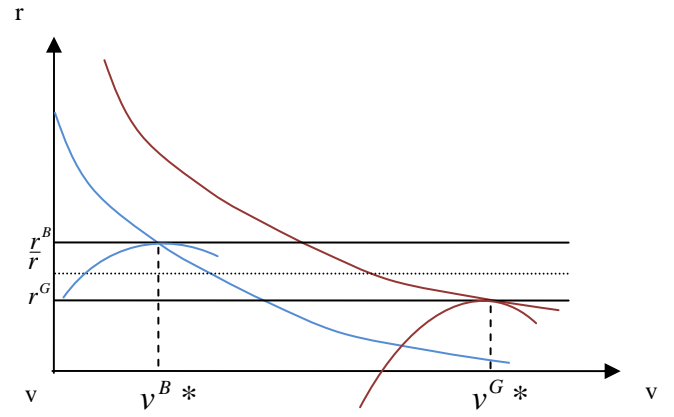


Figure 4b. Separating equilibrium under assumption set two and different success probabilities

3.3.3 Lower return and higher success probability

The last assumption set alternative is the most interesting one because, in my opinion, it is the most realistic one. It reflects the idea that entrepreneurs will only put through a riskier project if they expect it to yield a higher return in the event of a project success. This implies that for a critical credit rate \hat{r} which yields into zero profits for the good borrowers,

$$\tau^G p^G [R^G(v) - (1 + \hat{r})v] = 0 \Leftrightarrow R^G(v) = (1 + \hat{r})v,$$

the bad borrowers still have positive profits:

$$\tau^B p^B [R^B(v) - (1 + \hat{r})v] > 0 \Rightarrow R^B(v) > R^G(v).$$

From this idea the assumption set three necessarily follows.

Assumption set three. In the third assumption set, the project of the good borrowers has a higher success probability than the project of the bad borrowers. For each credit volume, the return of the good borrowers in the event of a project success is lower than the return of the bad borrowers:

- (1) $p^G > p^B$
(2) $R^G(v) < R^B(v), \forall v.$

Surprisingly, in the theoretical literature this assumption alternative has not been analyzed yet. To my best knowledge, only Stiglitz and Weiss (1981) used an assumption set in which the borrowers with a lower success probability have a higher return in the event of a project success. However their model is a special case. In contrary to my model they assume a fixed credit volume. Moreover in their model the expected profit of the good borrowers equals the expected profit of the bad borrowers:

$$p^G > p^B \wedge p^G R^G = p^B R^B \Rightarrow R^G < R^B.$$

In my model the assumption set is more generalized than in the Stiglitz and Weiss model. This helps to gain some new interesting insights about credit rationing and even about the existence of a Cournot-Nash equilibrium.

i) Conditions for a separating contract

A separating contract in which the good and the bad borrowers apply for a credit can only exist if the participation and the incentive compatible constraints hold⁷. However in contrary to the assumption sets one and two this is not self-evident under assumption set three. As a consequence, the optimal credit volume of the borrowers or the degree of borrower rationing must factor in these constraints. This means that the social efficient solution is not necessarily optimal. To derive the optimal design of the separating contract, the first order conditions of the maximization problem are regarded. But before it must be determined under which conditions a separating contract exists.

Lemma 2. If the participation conditions hold and the incentive compatible constraint of the good borrowers is not binding, then a separating contract will exist.

Proof. From the first order conditions above it can be seen that if the incentive compatible constraint of the good borrowers is not binding, then the bad borrowers will never be rationed, i.e. $\tau^{B*} = 1$. As the good borrowers are assumed to participate, it won't matter if the incentive constraint of the bad borrowers will be binding or not. A sufficient rationing of the good borrowers, i.e. a sufficient small τ^{G*} ,

$$0 < \tau^{G*} \leq \frac{R^B(v^{B*}) - (1 + r^{B*})v^{B*}}{R^B(v^{G*}) - (1 + r^{G*})v^{G*}},$$

will always prevent that the bad borrowers choose the contract for the good borrowers. Q.e.d.

To know if a separating contract does exist it must be analyzed under which conditions the incentive compatible condition of the good borrowers will not be binding. Rationing is a way to overcome the binding of any incentive compatible constraint. From this insight the following lemma can be derived.

⁷ The participation conditions hold by assumption.

Lemma 3. Under assumption set three the incentive compatible constraint of the good borrowers can only be binding if the good borrowers have to be rationed to guarantee a separating contract. The incentive compatible constraint of the good borrowers will not be binding if for the rationing of the good borrowers τ^{G^*} holds:

$$\tau^{G^*} \geq \frac{R^G(v^{B^*}) - (1 + r^{B^*})v^{B^*}}{R^G(v^{G^*}) - (1 + r^{G^*})v^{G^*}}.$$

Proof. For any credit rate, the good borrowers maximize their expected profit by getting on the lowest possible iso-profit curve. From assumption $R_v^G(v) < R_v^B(v)$, the iso-profit curve of the bad borrowers is steeper than the iso-profit curve of the good borrowers. From the analysis of the properties of the demand curve it is clear that the optimal credit volume, v^{G^*} , is at the peak of the most convenient iso-profit curve. Now regard the iso-profit curve of the bad borrowers which goes through this point. It always cuts the iso-profit curve of the good borrowers from below. As a consequence, for the regarded credit rate, the bad borrowers can get on a lower iso-profit curve by choosing a higher credit volume v^{B^*} . So, given a credit rate, the good borrowers cannot get better by demanding the contract designed for the bad borrowers. As $p^G > p^B$, in a separating equilibrium the bad borrowers have to pay a higher credit rate than the good borrowers. So the good borrowers never have an incentive to choose the contract for the bad borrowers, wherefore their incentive compatible constraint is not binding. As a consequence $R^G(v^{G^*}) - (1 + r^{G^*})v^{G^*} > R^G(v^{B^*}) - (1 + r^{B^*})v^{B^*}$.

However if the demanded credit volume of the good borrowers is not low enough relative to the demanded credit volume of the bad borrowers, it will be optimal to ration the good borrowers. To understand why there might not be a separating contract if there is rationing of the good borrowers, remember the incentive compatibility constraint of the good borrowers. From the analysis in “3.2 Conditions for a separating equilibrium” and because of

$$\frac{R^G(v^{B^*}) - (1 + r^{B^*})v^{B^*}}{R^G(v^{G^*}) - (1 + r^{G^*})v^{G^*}} < 1,$$

it is clear that for the rationing of the good borrowers τ^{G^*} it must hold

$$\tau^{G^*} \geq \frac{R^G(v^{B^*}) - (1 + r^{B^*})v^{B^*}}{R^G(v^{G^*}) - (1 + r^{G^*})v^{G^*}}.$$

If too many good borrowers must be rationed to guarantee the incentive compatible constraint of the bad borrowers, i.e. τ^{G^*} is too small, then the good borrowers will gain more by pretending to be bad borrowers. As the rationing of the good borrowers is a necessary condition to guarantee the incentive compatibility constraint of the bad borrowers, there cannot be a separating equilibrium. Q.e.d.

In the following the assumption set is analyzed for which the incentive compatible constraint of the good borrowers is not binding.

a) Optimal credit volume of the bad borrowers

$$\frac{\partial L}{\partial v^B} = p^B \left[R_v^B(v^{B*}) - \frac{1+\rho}{p^B} \right] \stackrel{!}{=} 0$$

The bad borrowers do not have to take into account the incentive compatible constraint of the good borrowers as it is not binding. So their optimal credit volume is the same as under symmetric information which is social efficient at the same time.

b) Slackness condition and incentive compatible constraint of the bad borrowers

The slackness condition is

$$\mu^* \left\{ \tau^{G*} p^B \left[R^B(v^{G*}) - \frac{1+\rho}{p^G} v^{G*} \right] - p^B \left[R^B(v^{B*}) - \frac{1+\rho}{p^B} v^{B*} \right] \right\} \stackrel{!}{=} 0.$$

If the incentive compatible constraint of the bad borrowers is not binding, then the term in the bracket will be an inequality and the Lagrange multiplier must be zero, $\mu^* = 0$. If the incentive compatible condition of the bad borrowers is binding, then it is an equation and can be bigger than zero, $\mu^* > 0$.

Complicating, τ^G is bounded to the interval $[0,1]$. If there is borrower rationing, you can set the first derivation of the Lagrange approach with respect to τ^G to zero. That is because the choice of τ^G helps to manage it. As by assumption, there are no negative profits, the lower boundary is not relevant. However, if the optimal choice of borrower rationing were larger than one, then it wouldn't have been possible because of the upper boundary. This has implications for the Lagrange multiplier and the optimal credit volume of the good borrowers which will be discussed later.

First assume that there is borrower rationing. Then the first order condition with respect to the rationing of the good borrowers τ^G set to zero results in the Lagrange multiplier μ^* :

$$\frac{\partial L}{\partial \tau^G} = \lambda p^G \left[R^G(v^{G*}) - \frac{1+\rho}{p^G} v^{G*} \right] - \mu^* p^B \left[R^B(v^{G*}) - \frac{1+\rho}{p^G} v^{G*} \right] \stackrel{!}{=} 0 \Leftrightarrow \mu^* = \lambda \frac{p^G \left[R^G(v^{G*}) - \frac{1+\rho}{p^G} v^{G*} \right]}{p^B \left[R^B(v^{G*}) - \frac{1+\rho}{p^G} v^{G*} \right]}.$$

It is notable that the Lagrange multiplier depends on the optimal credit volume of the good borrowers, but not on the rationing of the good borrowers or the credit volume of the bad borrowers. If the derivation cannot be set to zero as there is no borrower rationing, it can only be said that it will be non-negative. Then the Lagrange multiplier must be derived by another way which is done later. In the next step the optimal credit volume of the good borrowers is derived.

c) Optimal credit volume of the good borrowers

$$\frac{\partial L}{\partial v^G} = p^G \left[R_v^G(v^{G*}) - \frac{1+\rho}{p^G} \right] - \frac{\mu^*}{\lambda} p^B \left[R_v^B(v^{G*}) - \frac{(1+\rho)}{p^G} \right] \stackrel{!}{=} 0$$

If $\mu^* = 0$, the second term of the first order condition will be zero. Then the good borrowers will have the same optimal credit volume as under symmetric information which is social

efficient at the same time. If $\mu^* > 0$, the second term will always be positive. Then the good borrowers are willing to pay a higher credit rate to get a higher credit volume than they are actually granted. As it is not possible they are credit volume rationed. So under asymmetric information, the optimal credit volume of the good borrowers can be smaller than under symmetric information depending on the binding of the incentive compatible condition of the bad borrowers.

To understand the necessity of the credit volume rationing of the good borrowers, suppose that they obtain the same credit volume as under symmetric information. Then the bad borrowers would have an incentive to pretend to be good borrowers, which would result in losses for the lenders. The stronger the restrictiveness of the bad borrowers' incentive compatible constraint, the higher is the credit volume rationing. The degree of the binding is determined by the Lagrange multiplier.

The Lagrange multiplier will not only depend on if the incentive compatible constraint of the bad borrowers is binding or not. Given that it is binding, it will furthermore depend on if there is borrower rationing or not. If there is no borrower rationing and $\mu^* > 0$, then the Lagrange multiplier will be determined by the first order condition with respect to the good borrowers' credit volume:

$$\mu^* = \lambda \frac{p^G \left[R_v^G(v^{G*}) - \frac{1+\rho}{p^G} \right]}{p^B \left[R_v^B(v^{G*}) - \frac{(1+\rho)}{p^G} \right]}.$$

However the bad borrowers' choice of their optimal credit volume does not depend on the Lagrange multiplier. Whereas the good borrowers have to factor in the incentive compatible constraint of the bad borrowers, whereby they choose a credit volume which fulfills:

$$R^B(v^{G*}) - \frac{1+\rho}{p^G} v^{G*} = R^B(v^{B*}) - \frac{1+\rho}{p^B} v^{B*}.$$

If there is borrower rationing, then the Lagrange multiplier results from the first order condition with respect to the borrower rationing:

$$\mu^* = \lambda \frac{p^G \left[R^G(v^{G*}) - \frac{1+\rho}{p^G} v^{G*} \right]}{p^B \left[R^B(v^{G*}) - \frac{1+\rho}{p^G} v^{G*} \right]}.$$

Set into the first order condition with respect to the credit volume of the good borrowers results in:

$$\frac{\partial \Pi_G^G / \Pi_G^G}{\partial v^G / v^G} = \frac{\partial \Pi_G^B / \Pi_G^B}{\partial v^G / v^G}$$

The good borrowers marginally increase their credit volume as long as they thereby have a higher profit in percent than the bad borrowers. So, from **Lemma 1**, by a one percent change in the credit volume for the good borrowers, the percentage change in the expected profit chosen by the good borrowers is equal to the percentage change in the expected profit chosen by type bad borrowers.

d) Rationing of good borrowers

On the other side it is interesting how the borrower types' choice of the credit volume affects the rationing of the good borrowers. In the event of a project success, because of the assumption $R_v^G(v) < R_v^B(v), \forall v$, the marginal profit of the bad borrowers who pretend to be good borrowers is always higher than the marginal profit of the good borrowers who reveal their type. Whether the good borrowers are rationed or not, will depend on the difference between the profits of the good and the bad borrowers, both borrower types revealing their type. If the profits of the bad borrowers are too low compared to the profits of the good borrowers, then it will be necessary to ration the good borrowers. As the programming is non-linear, the incentive compatible constraint of the bad borrowers is directly analyzed and can be transformed to

$$\tau^{G^*} \leq \frac{R^B(v^{B^*}) - (1 + r^{B^*})v^{B^*}}{R^B(v^{G^*}) - (1 + r^{G^*})v^{G^*}}.$$

The rationing of the good borrowers is defined to be $\tau^{G^*} \in [0,1]$. If the incentive compatible constraint of the bad borrowers is not binding, the constraint can be neglected and no good borrowers are rationed, $\tau^{G^*} = 1$. If it is binding, then it holds as an equality.

So the inequality can be rewritten by $\tau^{G^*} = \min \left\{ 1; \frac{R^B(v^{B^*}) - (1 + r^{B^*})v^{B^*}}{R^B(v^{G^*}) - (1 + r^{G^*})v^{G^*}} \right\}$.

The denominator of the second term within the brackets can be replaced by the first order condition with respect to the credit volume of the good borrowers:

$$\tau^{G^*} = \min \left\{ 1; \frac{R^B(v^{B^*}) - (1 + r^{B^*})v^{B^*}}{R^G(v^{G^*}) - (1 + r^{G^*})v^{G^*}} \frac{R_v^G(v^{G^*}) - (1 + r^{G^*})}{R_v^B(v^{G^*}) - (1 + r^{G^*})} \right\}$$

The good borrowers are rationed if the right term within the brackets is smaller than the left term.

Proposition 4 (Conditions for the rationing of the good borrowers). If the optimal credit volume of the good borrowers is smaller and small enough relative to the optimal credit volume of the bad borrowers, then the bad borrowers won't have an incentive to pretend to be good borrowers and there won't be rationing of the good borrowers. If not, there is a rationing of the good borrowers:

$$\tau^{G^*} = \frac{R^B(v^{B^*}) - (1 + r^{B^*})v^{B^*}}{R^G(v^{G^*}) - (1 + r^{G^*})v^{G^*}} \frac{R_v^G(v^{G^*}) - (1 + r^{G^*})}{R_v^B(v^{G^*}) - (1 + r^{G^*})} \text{ as long as}$$

$$\frac{R^G(v^{B^*}) - (1+r^{B^*})v^{B^*}}{R^G(v^{G^*}) - (1+r^{G^*})v^{G^*}} \leq \tau^{G^*} \leq \min \left\{ 1, \frac{R^B(v^{B^*}) - (1+r^{B^*})v^{B^*}}{R^B(v^{G^*}) - (1+r^{G^*})v^{G^*}} \right\}.$$

Proof. Again I temporarily assume that information asymmetry is absent. Regard the first order conditions with respect to the optimal credit volumes of the good and the bad borrowers:

$$\frac{\partial L}{\partial v^B} = p^B \left[R_v^B(v^{B^*}) - \frac{1+\rho}{p^B} \right] \stackrel{!}{=} 0$$

$$\frac{\partial L}{\partial v^G} = p^G \left[R_v^G(v^{G^*}) - \frac{1+\rho}{p^G} \right] \stackrel{!}{=} 0$$

Under these assumptions there are two effects with respect to the demand of the borrower types for the credit volume. First, for each credit rate, the bad borrowers demand a higher credit volume than the good borrowers. Second, in a separating equilibrium the bad borrowers have to pay a higher credit rate than the good borrowers, wherefore they would demand a lower credit volume. So, under assumption set three it remains unclear if the good or the bad borrowers demand a higher credit volume. This requires a distinction of cases.

It has been shown that it is never optimal to ration bad borrowers. Moreover in some cases the credit volume of the good borrowers is rationed. Determining the possible rationing of the good borrowers requires formal analysis, regarding the incentive compatible constraints and setting $\tau^{B^*} = 1$:

$$(IC-G) \quad \tau^G \left[R^G(v^G) - (1+r^G)v^G \right] \geq R^G(v^B) - (1+r^B)v^B$$

$$(IC-B) \quad R^B(v^B) - (1+r^B)v^B \geq \tau^G \left[R^B(v^G) - (1+r^G)v^G \right].$$

(IC-G) and (IC-B) can be transformed to

$$(IC-G) \quad \tau^G R^G(v^G) - R^G(v^B) \geq \tau^G (1+r^G)v^G - (1+r^B)v^B$$

$$(IC-B) \quad \tau^G (1+r^G)v^G - (1+r^B)v^B \geq \tau^G R^B(v^G) - R^B(v^B).$$

which results in

$$\tau^G R^G(v^G) - R^G(v^B) \geq \tau^G (1+r^G)v^G - (1+r^B)v^B \geq \tau^G R^B(v^G) - R^B(v^B).$$

Neglecting the middle term,

$$\tau^G R^G(v^G) - R^G(v^B) \geq \tau^G R^B(v^G) - R^B(v^B)$$

$$\Leftrightarrow \tau^G \left[R^B(v^G) - R^G(v^G) \right] \leq R^B(v^B) - R^G(v^B)$$

$$\Leftrightarrow \tau^G \leq \frac{R^B(v^B) - R^G(v^B)}{R^B(v^G) - R^G(v^G)}$$

As it is not clear which borrower type demands a higher credit volume, a case differentiation is needed. In the first case, if $v^{G^*} > v^{B^*}$, then the right side of the inequality is always smaller than one, as

$$\frac{R^B(v^B) - R^G(v^B)}{R^B(v^G) - R^G(v^G)} < 1 \Leftrightarrow R^B(v^G) - R^B(v^B) > R^G(v^G) - R^G(v^B).$$

By $\lim_{v^G \rightarrow v^B} R^i(v^G) - R^i(v^B) = R_v^i$ and the assumption $R_v^B > R_v^G$, the good borrowers always have to be rationed:

$$\tau^G \leq \frac{R^B(v^B) - R^G(v^B)}{R^B(v^G) - R^G(v^G)} < 1.$$

In the second case, if $v^{B^*} > v^{G^*}$, then the right side of the inequality is always bigger than one as

$$\frac{R^B(v^B) - R^G(v^B)}{R^B(v^G) - R^G(v^G)} > 1 \Leftrightarrow R^B(v^B) - R^B(v^G) > R^G(v^B) - R^G(v^G).$$

By $\lim_{v^B \rightarrow v^G} R^i(v^B) - R^i(v^G) = R_v^i$ and the assumption $R_v^B > R_v^G$, the inequality always holds.

So, if there is rationing, it's because of the second condition, implied by the inequality for a separating contract. Thereby it is checked up to which degree of good borrower rationing, the bad borrowers have an incentive to reveal their type:

$$(1 + r^B)v^B - \tau^G(1 + r^G)v^G \leq R^B(v^B) - \tau^G R^B(v^G)$$

$$\tau^G \leq \frac{R^B(v^B) - (1 + r^B)v^B}{R^B(v^G) - (1 + r^G)v^G}$$

If the success probability of the good borrowers is high enough, then the credit rate for them will be small enough and therefore the bad borrowers will earn more by choosing the contract for the good borrowers. This necessitates a rationing of the good borrowers.

Summarized and including the borrower rationing condition, $0 \leq \tau^G \leq 1$ and the condition which guarantees that the good borrowers have an incentive to pretend to be good borrowers, for a separating equilibrium, the degree of the rationing of the good borrowers, τ^G , must hold

$$\frac{R^G(v^{B^*}) - (1 + r^{B^*})v^{B^*}}{R^G(v^{G^*}) - (1 + r^{G^*})v^{G^*}} \leq \tau^G \leq \min \left\{ 1, \frac{R^B(v^B) - (1 + r^B)v^B}{R^B(v^G) - (1 + r^G)v^G} \right\}.$$

The left side of the inequalities is always smaller than the right side of the inequalities as it is never optimal for the good borrowers to pretend to be bad borrowers and

$$\frac{R^G(v^B) - (1+r^B)v^B}{R^G(v^G) - (1+r^G)v^G} < \frac{R^B(v^B) - (1+r^B)v^B}{R^B(v^G) - (1+r^G)v^G}$$

$$\frac{R^B(v^G) - (1+r^G)v^G}{R^G(v^G) - (1+r^G)v^G} < \frac{R^B(v^B) - (1+r^B)v^B}{R^G(v^B) - (1+r^B)v^B}$$

$$\frac{R^G(v^G) - R^G(v^G) + R^B(v^G) - (1+r^G)v^G}{R^G(v^G) - (1+r^G)v^G} < \frac{R^G(v^B) - R^G(v^B) + R^B(v^B) - (1+r^B)v^B}{R^G(v^B) - (1+r^B)v^B}$$

$$\frac{R^B(v^G) - R^G(v^G)}{R^G(v^G) - (1+r^G)v^G} + 1 < \frac{R^B(v^B) - R^G(v^B)}{R^G(v^B) - (1+r^B)v^B} + 1$$

$$\frac{R^B(v^G) - R^G(v^G)}{R^G(v^G) - (1+r^G)v^G} < \frac{R^B(v^B) - R^G(v^B)}{R^G(v^B) - (1+r^B)v^B} \text{ as}$$

in the numerator $R^B(v^G) - R^G(v^G) < R^B(v^B) - R^G(v^B) \Leftrightarrow R^G(v^B) - R^G(v^G) < R^B(v^B) - R^B(v^G)$
and in the denominator $R^G(v^G) - (1+r^G)v^G > R^G(v^B) - (1+r^B)v^B$ because of
 $R^G(v^G) - (1+r^G)v^G > R^G(v^G) - (1+r^B)v^G > R^G(v^B) - (1+r^B)v^B$.

The right side of the inequality above is bigger than one if in the credit volume optimum of the good and the bad borrowers, the bad borrowers have an incentive to reveal their type. Otherwise it is smaller than one and the good borrowers have to be rationed to prevent that the bad borrowers pretend to be good borrowers. Q.e.d.

Setting in the non-profit condition of the bad borrowers and transforming the inequality, that means that there exists a critical success probability of the bad borrowers:

$$p^B < \hat{p}^B \equiv \frac{(1+\rho)v^{B*}}{R^B(v^{B*}) - [R^G(v^{G*}) - (1+r^{G*})v^{G*}] \frac{R^B(v^{G*}) - (1+r^{G*})v^{G*}}{R^G(v^{G*}) - (1+r^{G*})v^{G*}}}$$

If the success probability of the bad borrowers is smaller than this critical success probability, that is if $p^B < \hat{p}^B$, then it is optimal to ration good borrowers. As the share of the good borrowers μ is not contained in this inequality, it has no impact on the rationing of the good borrowers.

ii) Conditions for a separating Cournot-Nash equilibrium

A separating contract will only lead to a separating equilibrium if it is chosen by the borrowers. That this is not self-evident is shown in the following. Despite of the existence of a separating contract it may be possible that the borrowers prefer another contract. In the following lemma it becomes clear that there will only be a separating contract if the good borrowers prefer it to a pooling contract.

Lemma 4 (Good borrowers responsible for separating equilibrium). There can only exist a separating equilibrium if the good borrowers prefer a separating to a pooling contract.

Proof. From assumption $R_v^G(v) < R_v^B(v)$, the bad borrower's iso-profit curve is steeper than the good borrower's iso-profit curve. Focus on a situation in which the good borrowers are

indifferent between a separating and a pooling contract. In both times, the iso-profit curve of the good borrowers cuts iso-profit curve of the bad borrowers from above.

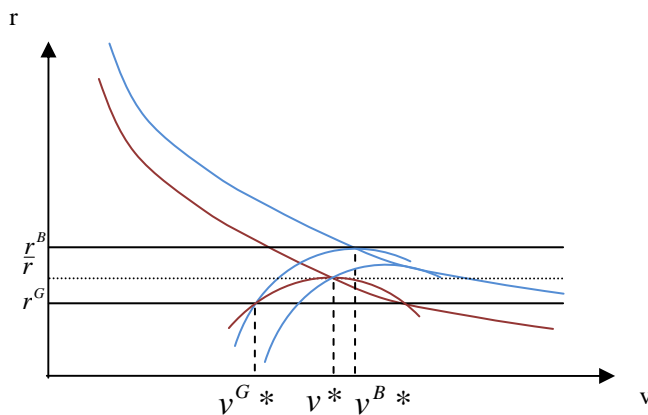


Figure 5. Good borrowers who prefer a pooling to a separating contract

Start from a separating contract in which the bad borrowers are indifferent between the contract for the bad and the contract for the good borrowers. Both, the good and the bad borrowers are interested in a lower iso-profit curve. Then for the lenders the bad borrowers would cost more than they pay by their credit rate. So it's not possible to move to a lower iso-profit curve without the help of the good borrowers.

The good borrowers might get better by paying a higher credit rate for a higher credit volume. Only then the bad borrowers get on a lower iso-profit curve and there is a possibility of a non-separating equilibrium. Q.e.d.

For a pooling equilibrium it is not only necessary that the good borrowers prefer a pooling to a separating contract. The bad borrowers must also prefer a pooling to a separating contract. This is guaranteed by the following lemma.

Lemma 5 (Bad borrowers' choice if good borrowers prefer a pooling contract). If the good borrowers prefer a pooling to a separating contract, the bad borrowers also prefer a pooling contract to revealing their type.

Proof. In a separating equilibrium, the iso-profit curve of the bad borrowers is steeper and always cuts the iso-profit curve of the good borrowers from below. If the good borrowers prefer a pooling to a separating contract, then this pooling contract will always be below the iso-profit curve of the bad borrowers choosing the separating contract. So the bad borrowers always get better by choosing the pooling contract. Q.e.d.

From the two lemmas above the separating equilibrium condition follows. A separating contract can only be offered if both borrower types demand it. This will only sufficiently apply if the good borrowers prefer the separating to the pooling contract. That the bad borrowers prefer the separating to the pooling contract is neither a sufficient nor a necessary condition.

Proposition 5 (Separating equilibrium condition). A separating equilibrium can only exist if the good borrowers prefer the separating to the pooling contract, i.e.

$$\Pi_e^G(\tau^{G^*}, r^{G^*}, v^{G^*}) \geq \Pi_e^G(r^*, v^*).$$

Up to now it is clear under which conditions there is a separating equilibrium. However it has not been discussed yet what happens if the conditions for a separating equilibrium do not hold.

Given Nash-type behavior in the insurance market, Rothschild and Stiglitz (1976) conclude that a Cournot-Nash equilibrium cannot exist. Following their arguments, I can derive the next proposition if the borrowers prefer a pooling to a separating contract.

Proposition 6. If the borrowers prefer a pooling to a separating contract, there is no Cournot-Nash equilibrium.

Proof. Regard **Figure 6**. Starting from the pooling contract, (r^*, v^*) , the lenders have an incentive to offer a defective contract by which they can make positive profits. This contract lies below the good borrowers' iso-profit curve through the point (v^{G^*}, r^{G^*}) , above the good borrowers' cost curve r^{G^*} and above the bad borrowers' iso-profit curve through the pooling contract point (r^*, v^*) . This contract would only be preferred by the good borrowers. Perfect competition under the lenders forces down the profits for this defective contract to a credit rate which yields in zero profits for the lenders, (v^{G^*}, r^{G^*}) .

As this defective contract only attracts the good borrowers, the bad borrowers continue to demand the old pooling contract (r^*, v^*) . This causes the pooling contract to imply negative expected profits. Consequently, the lenders stop to offer a pooling contract. Then the bad borrowers also choose the defective contract and pretend to be a good borrower. Because of asymmetric information the lenders cannot distinguish the good and the bad borrowers. The bad borrowers cause higher costs, whereby the defective contract makes losses and also disappears. As a result no contract is offered and there is no equilibrium. Q.e.d.

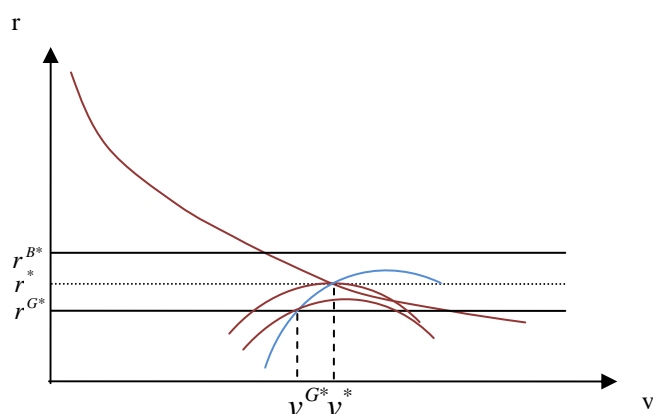


Figure 6. Good borrowers who prefer a defective to a pooling contract which leads to the non-existence of a Cournot-Nash equilibrium

The results of this section can be summarized to **Proposition 7**.

Proposition 7 (Assumption Set 4). Under assumption set three and if a separating contract exists, then there will

- (1) not be a Cournot-Nash equilibrium if $\Pi_e^G(\tau^{G^*}, r^{G^*}, v^{G^*}) < \Pi_e^G(r^*, v^*)$.
- (2) be a Cournot-Nash separating equilibrium if $\Pi_e^G(\tau^{G^*}, r^{G^*}, v^{G^*}) \geq \Pi_e^G(r^*, v^*)$.

If there is a Cournot-Nash separating equilibrium, then

- (1) the bad borrowers will never be credit volume rationed; the good borrowers will be credit volume rationed if under symmetric information

$$R^B(v^{G^*}) - \frac{1+\rho}{p^G} v^{G^*} > R^B(v^{B^*}) - \frac{1+\rho}{p^B} v^{B^*}.$$

Then they will get a credit volume which equates the bad borrowers' incentive compatible constraint, if no good borrowers are rationed and a credit volume which will equate the non-zero $\frac{\partial \Pi_G^G / \Pi_G^G}{\partial v^G / v^G} \neq \frac{\partial \Pi_G^B / \Pi_G^B}{\partial v^G / v^G}$, if

good borrowers are rationed;

- (2) the bad borrowers will never be rationed; the good borrowers will be rationed if under symmetric information $R^B(v^{G^*}) - \frac{1+\rho}{p^G} v^{G^*} > R^B(v^{B^*}) - \frac{1+\rho}{p^B} v^{B^*}$ and if the good

borrowers prefer it to a merely rationing of the credit volume;

- (3) the bad borrowers will be socially efficient financed; the good borrowers will be underfinanced if under symmetric information $R^B(v^{G^*}) - \frac{1+\rho}{p^G} v^{G^*} > R^B(v^{B^*}) - \frac{1+\rho}{p^B} v^{B^*}$.

4. Wilson's reactive or Grossman's dissembling pooling equilibrium

If the separating equilibrium condition does not hold, a Cournot-Nash equilibrium cannot exist. However the non-existence is very sensitive to the behavior assumed for the lenders and borrowers. Wilson (1977) and Grossman (1979) modify the behavior assumptions and thereby derive another equilibrium type. I will discuss under which behavior assumptions an equilibrium exists and which contract or contracts are chosen.

Wilson (1977) makes the behavior assumption that the lenders anticipate possible losses by a defective contract and show that then there is no non-existence of an equilibrium. To describe this behavior, Wilson added a third stage to the game. After having observed the contracts offered by the other lenders, the lenders can choose not to offer some or all of the contracts.

Transferred to my model this means, if some lenders choose to offer a defecting contract, the other lenders won't offer a pooling contract. Consequently, the bad borrowers then also demand the defective contract, it induces losses. As all lenders know this, no lender offers a defective contract and all lenders offer the pooling contract.

Grossman's (1979) model is also based on Rothschild and Stiglitz's non-existence of a Nash equilibrium. He changes the behavior assumptions about the high-risk borrowers and assumes that they always dissemble their true type and pretend to be low-risk borrowers. Transferred to my model this means that the bad borrowers demand the defective contract, intended to attract the good borrowers, although they rather profit by a pooling contract. As a

consequence, both borrower types demand the defective contract which thereby suffers losses. So, in the equilibrium the pooling contract would be offered.

Proposition 8 (Pooling equilibrium). Under assumption three and if the good borrowers prefer a pooling to a separating contract, then there will be a Wilson or a Grossmann pooling equilibrium.

Proposition 9 (Assumption set three and Wilson or Grossmann pooling equilibrium). Under assumption three and if there is a Wilson or a Grossmann pooling equilibrium, then

- (1) the bad borrowers will be credit volume rationed, the good borrowers not
- (2) no borrower type will be borrower rationed
- (3) the good borrowers will be underfinanced, the bad borrowers either under- or overfinanced

Proof. First, given the credit rate of the pooling contract, the bad borrowers would prefer to pay a higher credit rate to obtain a higher credit volume, as can easily be seen on **Figure 6**, whereas the good borrowers obtain their optimal credit volume. Second, in a pooling equilibrium the borrower types are not separated by a corresponding contract design, wherefore there is no borrower rationing. Third, under symmetric information, the good borrowers prefer a contract on a lower iso-profit curve, pay a lower credit rate and demand a higher credit volume. The bad borrowers however would have to pay a higher credit rate. This higher credit rate can more than offset the higher demand for credit volume, given a credit rate, compared to the pooling contract. Therefore the effect is unclear.

5. Conclusions

Credit rationing means that, in the market equilibrium, demand for credit exceeds supply. In the following this concept is divided into credit volume and borrower rationing. Credit volume rationing takes place if the lenders supply a smaller credit volume than some borrowers demand, although they would be willing to pay a higher credit rate for the higher credit volume. Borrower rationing occurs if, in the equilibrium, some credit applicants are rejected although other credit applicants with the identical risk-return project characteristics are accepted. The assumptions about the project characteristics are critical for the equilibrium result. They determine if there is a separating, a pooling or even no Cournot-Nash equilibrium, which borrower types are rationed, and if their credit volumes are rationed or socially efficient. The good borrowers are defined to yield at least the same expected profit for the lenders than the bad borrowers' project. If because of $p^G = p^B$ the lenders' expected profit of the projects of both borrower types is on the same level, the project of the good borrowers is defined to have a higher return in the event of a project success for each credit volume, $R^G(v) > R^B(v), \forall v$.

In the first assumption set, the good borrowers have a higher success probability than the bad borrowers, but the same return in the event of a project success. There is a pooling equilibrium in which the good and the bad borrowers pay the same credit rate and demand the same credit volume. Although there is no credit rationing, the bad borrowers are overfinanced and the good borrowers underfinanced. The good borrowers are underfinanced, but not credit volume rationed as they do not want to pay a higher credit rate for a higher credit volume.

In the second assumption set, the good borrowers have a higher return in the event of a project success than the bad borrowers, but no lower success probability. The equilibrium is social

efficient as the borrower types can be revealed without any informational costs. So, there is a separating equilibrium, no credit rationing and all borrowers obtain the social efficient credit volume. As $p^G \geq p^B$ and $R^G(v) > R^B(v), \forall v$, the good borrowers demand a higher credit volume than the bad borrowers.

In the third assumption set, the good borrowers have a higher success probability than the bad borrowers, but a lower return in the event of a project success. There will be a separating equilibrium if the good borrowers prefer the separating to the pooling contract and their incentive compatible constraint is not binding. If there is a separating equilibrium, the bad borrowers will not be rationed and obtain the social efficient credit volume. If the incentive compatibility constraint of the higher risk borrowers is not binding, the good borrowers will not be credit volume rationed. If it is binding, the good borrowers will be credit volume rationed. If there is no borrower rationing, the good borrowers will choose such a credit volume that the bad borrowers' incentive compatible constraint holds as an equation. If, based on this condition, the rationing of the credit volume of the good borrowers is not sufficient to separate the borrower types, there is additionally a rationing of the good borrowers. Then in a separating equilibrium by a one percent change in the credit volume for the low-risk borrowers, the percentage change in the expected profit chosen by good borrowers is equal to the percentage change in the expected profit chosen by bad borrowers. If the good borrowers prefer a pooling to a separating contract or the required rationing of the good borrowers is too high, then there will not be a Cournot-Nash equilibrium, but a Wilson and a Grossmann pooling equilibrium.

Credit volume and borrower rationing is a widespread phenomenon. Reviewing the results of this paper, credit rationing can arise if the risk-return characteristics of the project are private information of the credit applicants and the credit applicants with a higher success probability have a lower return in the event of a project success than the credit applicants with a lower success probability. Moreover there can only be a rationing of the good borrowers if they are credit volume rationed, but not vice versa. An empirical confirmation of these propositions might be interesting for future research.

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