

Singer, Nico

Working Paper

A behavioral portfolio analysis of retirement portfolios

Thünen-Series of Applied Economic Theory - Working Paper, No. 104

Provided in Cooperation with:

University of Rostock, Institute of Economics

Suggested Citation: Singer, Nico (2011) : A behavioral portfolio analysis of retirement portfolios, Thünen-Series of Applied Economic Theory - Working Paper, No. 104, Universität Rostock, Institut für Volkswirtschaftslehre, Rostock

This Version is available at:

<http://hdl.handle.net/10419/74660>

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.

Thünen-Series of Applied Economic Theory
Thünen-Reihe Angewandter Volkswirtschaftstheorie

Working Paper No. 104

revised version

**A Behavioral Portfolio Analysis
of Retirement Portfolios in Germany**

by

Nico Singer

Universität Rostock

Wirtschafts- und Sozialwissenschaftliche Fakultät
Institut für Volkswirtschaftslehre

2011

A Behavioral Portfolio Analysis of Retirement Portfolios in Germany*

Nico Singer[†]

First version: February 23, 2009

This version: May 2, 2011

Abstract

To most individuals saving for retirement is the number one financial goal. However, it reveals a complex task and induces serious behavioral problems which cannot be explained by traditional economic theory. This paper investigates the role of behavioral asset selection on retirement portfolios in Germany. Simulated behavioral portfolios show (i) an impact of emotions since pessimism (optimism) induces the most conservative (aggressive) portfolio, (ii) concentrated portfolios with a large position in only one secure asset and a small position in a risky portfolio, and (iii) a large difference to mean-variance portfolios in terms of level of diversification. I conclude that behavioral portfolio theory has remarkably power in understanding, describing and selecting retirement portfolios in Germany. The results have several implication for financial planning, e.g. for an “auto-pilot” solution to encourage people to more retirement saving.

Keywords: behavioral portfolio theory, saving for retirement, rank-dependent utility

JEL Classification: G11

*I would like to thank Alexander Conrad for lively conversations, Thomas Kalinowski for technical support on installing the SCIP-solver, Benjamin Strohner for research assistance and seminar participants at the University of Rostock for constructive comments. An earlier version of the paper circulated under the title “A Behavioral Portfolio Analysis of Retirement Portfolios”.

[†]University of Rostock, Department of Economics, Ulmenstraße 69, 18051 Rostock, Germany. Email: nico.singer@uni-rostock.de. Tel.: ++49 381 498 4390.

1. Introduction

To most individuals saving for retirement is the number one financial goal. Due to demographic changes, tight public budgets, and reduced generosity of occupational pension plans never in the post-World War Two era has been more reason to encourage workers to provide for their own retirement. Many recent studies, among them Oehler and Werner (2008), Bateman et al. (2010), Bridges et al. (2010), Knoll (2010) and Mitchell (2010), point to the shift of responsibility for an adequate old age provision toward individuals' shoulders. In a 2009 OECD study, Antolin and Whitehouse document a pension gap for 11 of 30 OECD countries, among them the United States, the United Kingdom, Germany and France. They define the pension gap as the difference between the replacement rate - the relationship between income in retirement and earnings when working - from the mandatory pension system and the OECD average. For the 11 countries they calculate a pension gap of 18.2% on average. To close this gap, however, reveals a complex task to most people since it requires accurate estimates of uncertain future processes including lifetime earnings, asset returns, tax rates, family and health status, and longevity. As a consequence, people reveal several behavioral problems when confronted with this decision problem, which can not be explained by traditional economic models, such as mean-variance theory (Markowitz, 1952). For an overview of deviations from traditional economic theory in the retirement planning context see for example Mitchell and Utkus (2006), Oehler and Werner (2008) and Knoll (2010).

The aim of this paper is therefore to investigate the role of behavioral portfolio selection on provision for old age. The model used here is a general version of behavioral portfolio theory by Shefrin and Statman (2000) and is employed to return data of retirement investments which are considered as most suitable by German households. To show the difference to traditional portfolio theory I also compute mean-variance portfolios and find that they are not able to describe "real" investors behavior. For behavioral portfolios I constitute a large difference to mean-variance portfolios in terms of level of diversification, an impact of emotions since the security-minded (potential-minded) is the most conservative (aggressive) portfolio, and concentrated portfolios with a large proportion in only one secure asset and a small proportion in risky assets. I conclude that behavioral portfolio theory has remarkably power in understanding, describing and selecting retirement portfolios.

This paper contributes to the existing literature in at least three ways: One, it provides a numerical rather than a probabilistic version of behavioral portfolio theory, that can be applied to a large amount of data and is therefore well-suited for financial planners and financial software. Two, simulated portfolios indicate that behavioral portfolio theory performs better in analyzing retirement investments than traditional theory. Three, related to Hoffmann et al. (2010), this paper successfully demonstrates how behavioral portfolio theory can be applied to real financial problems.

The rest of the paper is organized as follows: Section 2 contains the theoretical background including the underlying decision framework, Shefrin and Statman's (2000) portfolio model, the deterministic equivalent version and a brief description of mean-variance theory. Section 3 descriptively analyzes responses of more than 10,000 German households on the question of suitability and ownership of retirement investments. Based on the results, I collect return data of the seven most suitable retirement investments. Section 4 presents optimal behavioral

and mean-variance portfolios. Section 5 discusses the findings in the light of related studies, provides several conclusions and recommendations for financial planning.

2. Theoretical background

This section is divided into five subsections and starts with a brief introduction to rank-dependent utility theory. Based on this, subsection 2.2 presents SP/A theory, the underlying decision model. Subsection 2.3 shows how Shefrin and Statman (2000) employ SP/A theory to a portfolio selection model. This subsection ends by providing a different formulation of Shefrin and Statman's (2000) single mental accounting behavioral portfolio theory. As this behavioral portfolio model is probabilistic subsection 2.4 provides a deterministic equivalent version that can be treated numerically by mixed-integer programming. Finally, to relate results obtain with behavioral portfolio theory to textbook theory, subsection 2.5 gives a brief introduction to the well-known mean-variance portfolio model.

2.1. Rank-dependent utility

The best way to understand rank-dependent utility, which was independently proposed by Quiggin (1982), Yaari (1987), Allais (1988) and Quiggin (1993), is to start with expected utility

$$EU(R_i) = \sum_{j=1}^m p_j u(R_{ij}), \quad (1)$$

in which $u(R_{ij})$ is the utility of outcome j for lottery R_i and the p_j 's are the outcomes' associated probabilities. As the primary goal of this paper is to study a portfolio selection problem, R_{ij} is interpreted as return of asset i at time j with $i = 1, \dots, n$. Rank-dependent utility assumes that returns are ordered from lowest to highest, i.e. $R_{i,1} \leq \dots \leq R_{i,m}$, and substitutes decision weights, $w(p)$, for probabilities,

$$RDU(R_i) = \sum_{j=1}^m w(p_j) u(R_{ij}), \quad (2)$$

where

$$w(p_j) = h\left(\sum_{k=1}^j p_k\right) - h\left(\sum_{k=1}^{j-1} p_k\right) \quad (3)$$

and the function h transforms decumulative probabilities into the range $[0, 1]$. Let $D_{ij} = \text{prob}(R_i \geq R_{ij})$ denote the decumulative probability distribution function and assuming utility is linear,¹ equation (2) can be equivalently written as

$$RDU(R_i) = \sum_{j=1}^m h(D_{ij})(R_{ij} - R_{i,j-1}) \quad (4)$$

¹The special case of linear utility is what Yaari (1987) assumes in his "dual theory of choice under risk".

with $R_{i,0} = 0$ for all i . In the separate appendix I show that equation (2) and (4) are formally the same. Note, in the special case where h is the identity function, rank-dependent utility collapses to expected utility. In expected utility, risk aversion is equivalent to concave utility, whereas in rank-dependent utility with linear utility function, however, risk aversion is equivalent to a convex probability transformation function, h (see Quiggin, 1993).

2.2. SP/A Theory

SP/A theory developed by Lopes (1987) and Lopes and Oden (1999) is a dual criterion theory of choice under uncertainty in which the process of choosing between alternatives entails integrating two logically and psychologically separate criteria: The SP-criterion in which S stands for security and P stands for potential captures individuals' desire for both risk aversion (security) and risk loving (potential) and is closely related to Friedman and Savage's (1948) observation that individuals who buy insurance policies often buy lottery tickets at the same time. The A-criterion stands for aspiration and operates on a principle of stochastic control (Lopes and Oden, 1999, p. 291), that is individuals are assumed to assess the attractiveness of a lottery by the probability that the lottery fails to achieve an aspiration level.

In the SP/A framework, two emotions operate on the willingness to take risk: fear and hope². Lopes and Oden (1999) model these emotions using rank-dependent utility. Specifically, the SP-criterion is modeled by equation (4) in which the probability transformation function has the following shape: Fear is what Lopes (1987) and Lopes and Oden (1999) call security-mindedness and leads individuals to overweight probabilities attached to unfavorable outcomes, that is modeled by³

$$h_S(D) = D^{1+q_S}, \quad q_S > 0. \quad (5)$$

Hope is what Lopes (1987) and Lopes and Oden (1999) call potential-mindedness and leads individuals to overweight probabilities attached to favorable outcomes, that is modeled by

$$h_P(D) = 1 - (1 - D)^{1+q_P}, \quad q_P > 0. \quad (6)$$

As we have learned from Friedman and Savage (1948) that fear and hope reside within individuals simultaneously, the final shape of the probability transformation function is a convex combination of h_S and h_P :

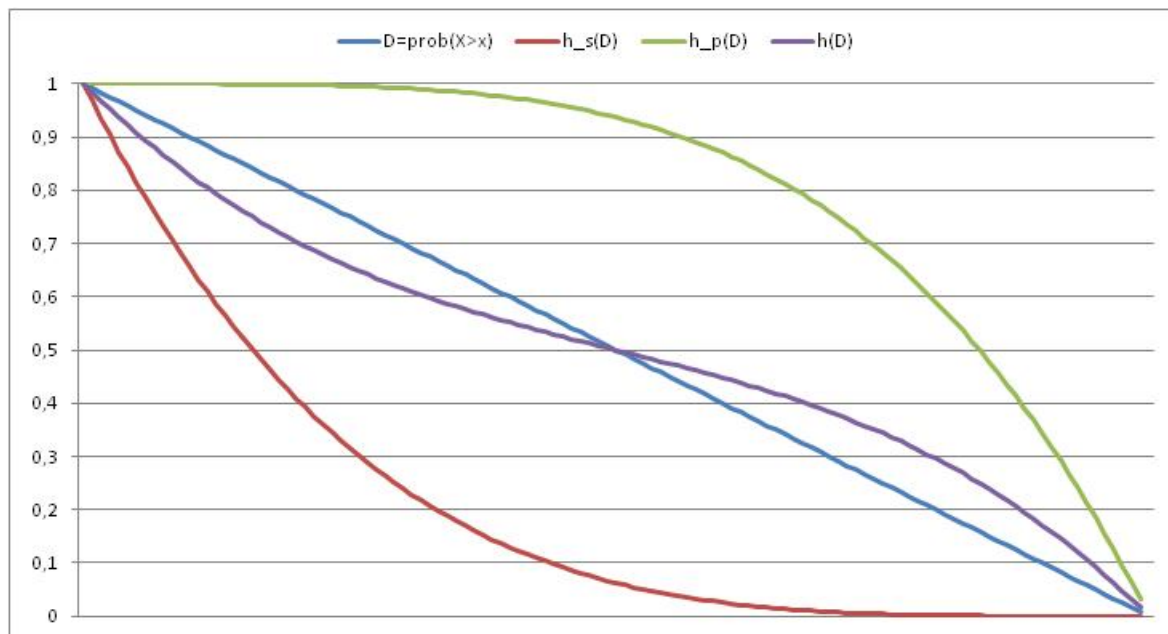
$$h(D) = \lambda h_S(D) + (1 - \lambda) h_P(D), \quad \lambda \in [0, 1]. \quad (7)$$

If $\lambda = 1$, the decision maker is strictly security-minded. If $\lambda = 0$ the investor is strictly potential-minded. If $0 < \lambda < 1$, the decision maker is both with the magnitude of fear and hope depending on λ . Lopes and Oden (1999) call this behavior cautiously hopeful and also distinguish between λ for gains and for losses. The probability transformation function plays a key role in descriptive decision models such as SP/A theory and Cumulative Prospect

²In related literature the terms pessimism and optimism are sometimes used instead.

³For sake of simplicity subscripts are omitted at this point.

Theory (Tversky and Kahneman, 1992). For a detailed discussion of the probability weighting function see for example Prelec (1998). Figure 1 shows decumulative probability distribution functions for security-minded, potential-minded and cautiously hopeful behavior.



Note: The straight line represents the decumulative probability distribution function. $h_S(D)$ shows strictly security-minded behavior with parameters $q_S = 3$ and $\lambda = 1$, while $h_P(D)$ shows strictly potential-mindedness with parameters $q_P = 3$ and $\lambda = 0$. $h(D)$ illustrates cautiously hopeful behavior with parameters $q_P = q_S = 3$ and $\lambda = 0.5$. Outcomes are not specified, but rank from lowest to highest.

Figure 1: Transformed decumulative probability distribution functions

2.3. A stochastic behavioral portfolio model

In their behavioral portfolio theory Shefrin and Statman (2000) develop a single and a multiple mental account version. As this paper focuses on the specific goal of a secure retirement, that is organized in one mental account (see for example Das et al., 2010), the single mental account behavioral portfolio model is of particular interest. In the model, Shefrin and Statman (2000) combine SP/A theory and Telser's (1955) safety-first rule, that is investors select portfolios which maximize expected return subject to the constraint that the probability of not achieving a threshold return is bounded. In this context the threshold return is interpreted as aspiration level. In contrast to Shefrin and Statman (2000) I provide a different definition of the single account version:

Problem 1.

$$\max_x x^T RDU(R) \quad s.t. \tag{8}$$

$$\text{prob}(x^T R < A) \leq \alpha, \tag{9}$$

$$\mathbf{1}^T x = 1, \tag{9}$$

$$x \geq 0, \tag{10}$$

where $\mathbf{1} = (1, \dots, 1)^T \in R^n$, $x = (x_1, \dots, x_n)^T$ being the vector of portfolio weights of n assets with $RDU(R) \in R^n$ being the behavioral mean return vector, A is the aspiration level of portfolio return and the maximum probability of the portfolio failing to reach the return $x^T R$ is α . The SP-criterion is captured in the objective of problem 1 by the vector of rank-dependent utilities of all n assets, i.e. equation (4) is applied to each of the n assets. The aspiration-criterion is captured in the safety-first constraint (8) by the parameter pair (A, α) where α denotes the probability of not achieving aspiration level A . Note, the feasible domain of problem 1 is only determined by the aspiration-criterion and not by the SP-criterion. Equation (9) is the fully invested or budget constraint and (10) is the short sale constraint, which was not explicitly imposed by Shefrin and Statman (2000), but, to my belief, appears essential in the context of retirement planning. Shefrin and Statman (2000, pp. 133) provide a solution to a simplified version of problem 1 under empirical distributed returns. Under the same assumption, the subsequent subsection provides a more general deterministic version of problem 1, which can be solved numerically by mixed-integer linear programming.

2.4. An equivalent deterministic behavioral portfolio model

Many papers dealing with safety-first constraints such as (8) assume normal distributed asset returns (Leibowitz and Henriksson, 1989; Leibowitz and Kogelman, 1991; Albrecht, 1993; Shefrin and Statman, 2000; Das et al., 2010; Singer, 2010), although the normal assumption is mainly rejected in empirical finance. Others employ probability inequalities such as the Chebyshev inequality to get an upper bound of the safety-first constraint (Roy, 1952; Telser, 1955; Kall and Mayer, 2005; Singer, 2010), although Chebyshev's inequality is a rather crude approximation. In this paper, however, we assume the general case of empirical distributed returns, which is closely related to Shefrin and Statman (2000, Theorem 1, pp. 133). This assumption is particularly driven by the fact that SP/A theory is a discrete choice model. Using techniques from stochastic linear programming (Prékopa, 1995; Birge and Louveaux, 1997; Uryasev and Pardalos, 2001; Ruszczyński and Shapiro, 2003; Kall and Mayer, 2005), especially the transformation technique by Raike (1970), problem 1 can be restated as

Problem 2.

$$\begin{aligned} \max_x x^T RDU(R) \quad & s.t. \\ x^T R_j + M(1 - d_j) & \geq A, \quad j = 1, \dots, m, \end{aligned} \quad (11)$$

$$\sum_{j=1}^m p_j d_j \geq 1 - \alpha, \quad (12)$$

$$d_j \in \{0, 1\}, \quad j = 1, \dots, m, \quad (13)$$

$$\mathbf{1}^T x = 1, \quad (14)$$

$$x \geq 0, \quad (15)$$

where R_j is the realized return vector of all assets at time j , d_j 's are binary auxiliary variables, p_j 's are the probabilities from equation (1) and M is a large number (The separate appendix provides a precise definition of M). Problem 2 is the deterministic equivalent of problem 1 and can be treated numerically. As problem 2 has n real (x_1, \dots, x_n) and m binary (d_1, \dots, d_m) decision weights it belongs to the class of mixed-integer linear optimization problems. To solve this problem I use SCIP (**S**olving **C**onstraint **I**nteger **P**rograms) developed by Achterberg (2007), currently the fastest non-commercial mixed-integer programming solver.

2.5. Mean-variance portfolio model

In mean-variance portfolio theory (Markowitz, 1952), investors select efficient portfolios on the basis of portfolio variance, σ^2 , and expected return, μ . The model with short sale constraint states as follows

Problem 3.

$$\begin{aligned} \min_x \sigma^2(x^T R) \quad & s.t. \\ x^T \mu & \geq A, \end{aligned} \quad (16)$$

$$\mathbf{1}^T x = 1, \quad (17)$$

$$x \geq 0, \quad (18)$$

in which A is the minimum desired return and has a similar meaning as the aspiration level in behavioral portfolio theory. The optimal solution to problem 3 minimizes the portfolio variance and achieves at least return A . If short sales are allowed, i.e. constraint (18) is omitted, problem 3 can be solved analytically (see for example Huang and Litzenberger, 1993) otherwise by quadratic programming. For a detailed introduction to mean-variance theory see for example Markowitz (1970) or Elton et al. (2007).

3. Data

To get a better understanding of retirement portfolios in Germany we first analyze survey data of the Spiegel-Verlag survey "Soll und Haben" (Debit and Credit) 2004 in which 10,100

individuals in Germany were asked about their attitudes on the subject of investments. Among other things surveyed, subjects were asked about the suitability and their actual ownership of investments as provision for old age. Table 1 shows responses to both questions. The

	Suitability of investments as provisions for old age		Ownership of investments as provisions for old age	
	# persons	%	# persons	%
Endowment insurance	3630	34.2	2291	18.0
Property used by owners	3562	33.0	1873	16.7
Savings account	733	8.7	1391	16.2
Savings contract/ savings plan with regular deposits/ agreed term	1537	15.2	867	7.4
Savings account with special interest rate	971	10.6	664	7.1
Stocks and equity funds	1173	8.3	896	6.0
Building society savings con- tract	986	9.2	532	4.4
Private pension insurance	2172	18.6	618	4.1
Fixed-interests securities	1391	11.7	575	4.0
Occupational pension	1305	11.0	506	3.4
Property let to tenants	1843	16.3	396	2.5
Gold and other luxury goods	360	3.8	98	0.8

Note: Multiple responses were allowed. Investments are sorted in descending order by ownership.

Table 1: Suitability and ownership of investments for old age provision

first two rows of table 1 reveal that the most popular investments as provision for old age are the endowment insurance and property used by owners. Concerning actual ownership, savings such as savings accounts, savings contracts/plans and savings accounts with special interest rates rank third. Hence, retirement portfolios are dominated by relatively safe assets, which has also been documented by Börsch-Supan and Eymann (2000) who analyzed German household portfolios in the 1980s and 1990s. Only 6% and 4% hold equity and bonds, respectively. Notably, only 4.1% own a private pension insurance, although it ranks third in terms of suitability (18.6%). Moreover, table 1 documents the “self-control problem” (Mitchell and Utkus, 2006; Oehler and Werner, 2008), which supports the view of a wide divergence between individuals’ desire and their actual behaviors. Among all assets, the

percentages of suitability are higher than those of ownership. One reason for the divergence is that subjects are attracted by more assets as they actually own, which implies a lack of diversification in realized retirement portfolios. Clearly, I find that only 9.3% hold three or more assets and only 4% hold four or more assets.

Based on the above findings I collect return data of investments suited for old age provision:

1. Endowment insurance: To approximate the returns of the endowment insurance I use net returns of investments published by the German Insurance Association (GDV).

2. Property: Property used by owners and property let to tenants are approximated as one time series by the index for housing calculated by the Bundesbank, based on data provided by BulwienGesa AG.

3. Savings accounts and other investments with banks: Savings accounts, savings contracts/savings plans with regular deposit/agreed terms are approximated as one time series following Westerheide's (2005) methodology. I extract following time series: The interest rates for savings deposits are composed of deposit rates of banks in Germany with minimum rates of return and with agreed notice of three months. The average rates are calculated as unweighted arithmetic means from the interest rates reported to be within the spread. The spread is ascertained by eliminating the reports in the top 5% and the bottom 5% of the interest rate range. Including rates for savings deposits, I consider the difference between the Bundesbank's interest rate statistics and the new European Central Bank's statistics, with the latter started at January 2003. Moreover, I include savings bonds with fixed maturity of four years, overnight money, savings with/without contract period for varying investment volumes, and fixed-term deposits. All time series are extracted from Deutsche Bundesbank. A representative time series is obtained by calculating the arithmetic mean of all time series mentioned above.

4. Stocks: Investments in stocks, equity funds and other investment funds are represented by the returns of the DAX performance index.

5. Building society savings contract: To approximate returns of the building society savings contract I follow Statistisches Bundesamt Deutschland (Federal Office of Statistics) and use yields on debt securities outstanding issued by residents, also published by Deutsche Bundesbank.

6. Bonds: Fixed-interests securities, private and occupational pensions are approximated as one time series by the returns of the REX performance index, which measures the performance of German government bonds.

7. Gold and other luxury goods: Returns of gold and other luxury goods are approximated by the returns of gold traded at Frankfurt stock exchange.

In total, I collect monthly return data for all assets from January, 1998 to December, 2007, which comprises ten years. The returns of the endowment insurance and property, for which only yearly data is available, are divided equally into monthly returns. Table 2 shows summary statistics of the dataset.

	Min	Max	Mean	S.D.	Skewness	Kurtosis
Endowment insurance	0.3875	0.6317	0.4839	0.1011	0.6132	-1.4415
Property	-0.0930	0.1479	0.0330	0.0684	-0.1608	-0.6400
Savings accounts and other investments with banks	0.1557	0.3548	0.2423	0.0550	0.3222	-0.9532
Stocks	-29.3327	19.3738	0.5341	6.8776	-0.9845	3.2687
Building society savings contract	0.2417	0.4667	0.3522	0.0579	0.1425	-0.9108
Bonds	-1.9184	2.4964	0.3889	0.8708	-0.1866	-0.4473
Gold and other luxury goods	-11.8484	14.7314	0.6150	3.4697	0.2290	2.5230

Table 2: Summary statistics of return distributions of retirement assets

4. Results

Using the ten year dataset described in the previous section problem 2 is solved by mixed-integer programming. The SP-criterion is determined by the parameters q_S that measures the strength of security-mindedness, q_P that measures the strength of potential-mindedness, and λ that determines the strength of security relative to potential. Note, if q_S and q_P are set to zero, problem 1 and 2 collapses to Telser's (1955) model which selects portfolios that maximize expected return subject to the safety-first constraint (8). The aspiration-criterion is determined by the parameters A and α where α denotes the probability of not achieving aspiration level (threshold return) A . As the SP-criterion only impacts the objective function the feasible domain of problem 1 and 2, respectively, is solely determined by the aspiration-criterion, i.e. fear and hope have no impact on the feasibility of problem 1 and 2, respectively. Figure 2 shows feasible (A, α) combinations for the ten year dataset in a reasonable range. Figure 2 documents a positive relation between A and α in the sense that when α decreases A must also decrease to preserve feasibility.

The rest of this section documents optimal portfolio weights of four different scenarios:

1. Security-minded behavior: In this scenario the investor is assumed to be strictly security-minded with parameters $q_S = 0.05$ and $\lambda = 1$. To the best of my knowledge it exists no estimate for q_S based on real data for this scenario.

2. Potential-minded behavior: In this scenario the investor is assumed to be strictly potential-minded with parameters $q_P = 0.05$ and $\lambda = 0$. To the best of my knowledge it exists no estimate for q_P based on real data for this scenario.

3. Cautiously-hopeful behavior: In this scenario I use the parameter estimates by Lopes

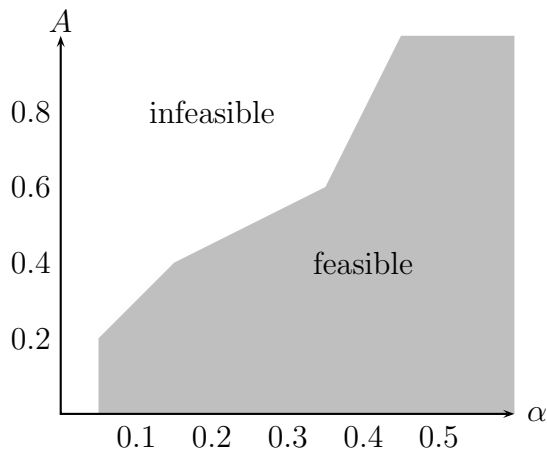


Figure 2: Feasible (A, α) -combinations

and Oden (1999), which are $q_S = q_P = 1.053$ and $\lambda_{gains} = 0.505$ for gains and $\lambda_{losses} = 0.488$ for losses. Undocumented results show that cautiously hopeful behavior modeled by the probability weighting function of Cumulative Prospect Theory (Tversky and Kahneman, 1992) does not significantly differ from that modeled by SP/A theory. As the primary goal of this paper is not the probability weighting function I do not document these results here, but they are available upon request.

4. Mean-variance behavior: In order to compare results from behavioral portfolio theory with those of classic portfolio theory I also calculate optimal portfolios using problem 3. The aspiration level, A , is the same in both problem 2 and 3.

Figure 3 contains optimal portfolios for all scenarios with $A = 0.2$ and various α . Note, as the parameter α is not part of the mean-variance model, all panels of figure 3 show the same mean-variance portfolio. Among all behavioral portfolios, a large concentration of wealth in the endowment insurance can be observed, in the security-minded scenario it reaches even 100%. All other secure investments (property, savings and building society savings contract) are uninvested by all behavioral portfolios. The reason for this result is that the return of the endowment insurance is the highest among all secure investments that meet the aspiration level. This pattern of behavioral portfolios is the same as documented by Shefrin and Statman (2000), that is a combination of risk-free return (endowment insurance) and a lottery ticket (the proportion invested in risky assets). Mean-variance portfolios, in contrast, invest in all secure assets, that is because the key element of mean-variance theory is exploiting correlations between assets. The structural difference between behavioral and mean-variance portfolio theory leads also to the result that mean-variance portfolios are more diversified than behavioral portfolios, a pattern which I observe for all documented and undocumented results.

Among all behavioral portfolios the security-minded is the most conservative one as it invests 100% in the endowment insurance, whereas the cautiously hopeful and potential-

minded portfolios invest a small fraction in risky assets. As can be seen from panel 3(b) and 3(c) the potential-minded portfolio is the most aggressive behavioral portfolio as it invests the largest fraction in risky assets. The reason why the mean-variance portfolio has its largest position in savings is that the mean return of savings, which is 0.2423, meets the aspiration level of $A = 0.2$, and the standard deviation of savings is the lowest among all assets. As the objective in mean-variance problem 3 is to minimize portfolio variance, savings take the largest fraction. Note further, as α increases the proportion invested in safe assets decreases while the proportion in risky assets increases.

Figure 4 contains optimal portfolios for all scenarios for $A = 0.4$ and $\alpha = 0.2$ (panel 4(a)) and $\alpha = 0.3$ (panel 4(b)). Note, it exists no feasible solution for $\alpha = 0.1$, which can also be seen from figure 2. For all behavioral portfolios the same pattern as for $A = 0.2$ can be observed. But, the mean-variance portfolio is not invested in property and savings which is due to the fact that the mean return of property (0.0330) and savings (0.2423) does not achieve the aspiration level (threshold return) of $A = 0.4$, respectively.

5. Discussion

This paper investigates the role of behavioral portfolio theory on provision for old age in Germany. The behavioral portfolio model implemented here is a general version of Shefrin and Statman's (2000) single mental account model which combines SP/A theory and Telser's (1955) safety-first rule. Assuming empirical distributed returns the model is then transformed in its deterministic equivalent, which is solved numerically by mixed-integer linear programming. Using return data of the seven most suitable German retirement investments I simulate portfolios for security-mindedness, potential-mindedness, cautiously hopeful and mean-variance behavior where the latter serves by way of comparison.

The main findings indicate (i) an impact of emotions on behavioral portfolios, since the security-minded (potential-minded) is the most conservative (aggressive) portfolio, (ii) concentrated behavioral portfolios with a large proportion in only one secure asset (endowment insurance) and a small proportion in risky assets and (iii) mean-variance portfolios which are more diversified than behavioral portfolios. To relate the results to others, first, I document the same pattern of behavioral portfolios as documented by Shefrin and Statman (2000), that is a combination of risk-free return (endowment insurance) and a lottery ticket (the proportion invested in risky assets). This pattern is also similar to that in Shiller's (2005) life-cycle portfolios which allocate account balances between stocks and bonds. Second, the large concentration in the endowment insurance is consistent with my own empirical findings in section 3 and those by Börsch-Supan and Eymann (2000) for German household portfolios in the 80s and 90s. Third, the lack of diversification in simulated behavioral portfolios has been widely shown in empirical studies on private portfolio allocation among them Blume and Friend (1975), Kelly (1995), Tyynelä and Perttunen (2002) and Goetzmann and Kumar (2008). Fourth, since mean-variance theory exploits the correlation structure of all assets, it is theoretically not surprising that mean-variance portfolios invest in more than one "riskless" asset. But, allowing for the fact that individual investors ignore correlations (Kroll et al., 1988; Siebenmorgen and Weber, 2003), this result may practically appear puzzling to

most individual investors. Taken as a whole, I conclude that behavioral portfolio theory has remarkably power in understanding, describing and selecting retirement portfolios.

Nevertheless, I am completely aware of at least four shortcomings. One, the fact that I was not able to obtain precise return data for some retirement investments - predominantly secure investments - may induce an approximation bias. Two, as I assume empirical distributed returns the deterministic model is of mixed-integer type for which computing time increases disproportional as input size increases. Three, retirement planning involves long time horizons. Thus, one could argue that dynamic models are better capable in the domain of life cycle saving. Four, according to the SP-criterion of SP/A theory, probability distributions of returns are only transformed in the objective function, whereas they are treated as raw probabilities in the safety-first constraint (8). Consequently, return distributions in the constraints should also be transformed .

These shortcomings leave much room for future research. Another extension involves the aspiration level, A , which is assumed to be fixed. However, in long time decisions investors' aspiration may vary or they wish a benchmark return such as the return of the S&P500 as aspiration level. In such cases the aspiration level can be modeled as a random variable (see for example Singer, 2010). Instead of modeling emotions such as hope and fear, rank-dependent utility can also model the availability heuristic (Tversky and Kahneman, 1973) in which people predict the likelihood of an outcome based on how many outcomes of each type (return increase and decrease) come readily to mind. As recent outcomes are more available than those many years ago, the availability heuristic is modeled by rank-dependent utility with outcomes sorted by date rather than from lowest to highest.

The results in this paper have also implications for professional advisers and for financial planning software. As "[f]inancial planners have a responsibility to guide clients in a manner consistent with how those clients are likely to behave" (Mitchell, 2010, p. 5), this model helps them to understand how their clients behave and guide them on how they should behave, i.e. give a suitable advice. Since this model can simulate several behavioral scenarios it can be used as a powerful tool in institutional and private portfolio management software. It can also be used as an "auto-pilot" that Mitchell and Utkus (2006, p.92) suggest as a default plan to prevent people from inertia and encourages them to more retirement saving.

References

- T. Achterberg. *Constraint Integer Programming*. PhD thesis, Technische Universität Berlin, 2007.
- P. Albrecht. Shortfall returns and shortfall risk. Working Paper 59, University of Mannheim, 1993. Mannheimer Manuskripte zu Risikotheorie, Portfolio Management und Versicherungswirtschaft.
- M. Allais. The general theory of random choices in relation to the invariant cardinal utility function and the specific probability function. The (U, O) model. In *Risk, Decision and Rationality*. Reidel, Dordrecht, 1988.

- P. Antolin and E. Whitehouse. Filling the pension gap: Coverage and value of voluntary retirement savings. Technical report, OECD Social, Employment and Migration Working Papers No. 69, 2009.
- H. Bateman, J. Louviere, S. Thorp, T. Islam, and S. Satchell. Investment decisions for retirement savings. *The Journal of Consumer Affairs*, 44(3):463–482, 2010.
- J. R. Birge and F. Louveaux. *Introduction to stochastic programming*. Springer, New York, NY, 1997.
- M. E. Blume and I. Friend. The asset structure of individual portfolios and some implications for utility functions. *The Journal of Finance*, 30(2):585–603, 1975.
- B. Bridges, R. Gesumaria, and M. V. Leonesio. Assessing the performance of life-cycle portfolio allocation strategies for retirement saving: A simulation study. *Social Security Bulletin*, 70(1), 2010.
- A. Börsch-Supan and A. Eymann. Household portfolios in Germany. discussion paper, Universität Mannheim, 2000.
- S. Das, H. Markowitz, J. Scheid, and M. Statman. Portfolio optimization with mental accounts. *Journal of Financial and Quantitative Analysis*, 45(2):311–334, 2010.
- E. J. Elton, M. J. Gruber, S. J. Brown, and W. N. Goetzmann. *Modern portfolio theory and investment analysis*. Wiley, New York, 2007.
- M. Friedman and L. J. Savage. The utility analysis of choices involving risk. *The Journal of Political Economy*, 56:279–304, 1948.
- W. N. Goetzmann and A. Kumar. Equity portfolio diversification. *Review of Finance*, 12(3):433–463, 2008.
- A. O. I. Hoffmann, H. M. Shefrin, and J. M. Pennings. Behavioral portfolio analysis of individual investors. working paper, Available at SSRN: <http://ssrn.com/abstract=1629786>, 2010.
- C.-f. Huang and R. H. Litzenberger. *Foundations for Financial Economics*. Prentice Hall, Englewood Cliffs, NJ, 1993.
- P. Kall and J. Mayer. *Stochastic linear programming*. Springer, New York, NY, 2005.
- M. Kelly. All their eggs in one basket: Portfolio diversification of US households. *Journal of Economic Behavior and Organization*, 27:87–96, 1995.
- M. A. Z. Knoll. The role of behavioral economics and behavioral decision making in Americans’ retirement savings decisions. *Social Security Bulletin*, 70(4):1–22, 2010.
- Y. Kroll, H. Levy, and A. Rapoport. Experimental tests of the separation theorem and the capital asset pricing model. *American Economic Review*, 78(3):500–519, 1988.

- M. L. Leibowitz and R. D. Henriksson. Portfolio optimization with shortfall constraints: A confidence-limit approach to managing downside risk. *Financial Analysis Journal*, 45(2): 34–41, 1989.
- M. L. Leibowitz and S. Kogelman. Asset allocation under shortfall constraints. *The Journal of Portfolio Management*, Winter:18–23, 1991.
- L. L. Lopes. Between hope and fear: The psychology of risk. *Advances in Experimental Social Psychology*, 20:255–295, 1987.
- L. L. Lopes and G. C. Oden. The role of aspiration level in risky choice. *Journal of Mathematical Psychology*, 43(2):286–313, 1999.
- H. Markowitz. Portfolio selection. *The Journal of Finance*, 7(1):77–91, 1952.
- H. Markowitz. *Portfolio selection*. Yale Univ. Press, New Haven, Conn., 1970.
- J. B. Mitchell. Nudge: An application in retirement withdrawal planning. working paper, Available at SSRN: <http://ssrn.com/abstract=1703945>, 2010.
- O. S. Mitchell and S. P. Utkus. How behavioral finance can inform retirement plan design. *Journal of Applied Corporate Finance*, 18(1):82–94, 2006.
- A. Oehler and C. Werner. Saving for retirement - a case for financial education in Germany and UK? An economic perspective. *Journal of Consumer Policy*, 31:253–283, 2008.
- D. Prelec. The probability weighting function. *Econometrica*, 66(3):497–527, 1998.
- A. Prékopa. *Stochastic Programming*. Kluwer Academic Publishers, 1995.
- J. Quiggin. A theory of anticipated utility. *Journal of Economic Behavior & Organization*, 3(4):323–343, 1982.
- J. Quiggin. *Generalized expected utility theory*. Kluwer Acad. Publ., Boston, 1993.
- W. M. Raike. Dissection methods for solutions in chance constrained programming problems under discrete distributions. *Management Science*, 16(11):708–715, 1970.
- A. D. Roy. Safety first and the holding of assets. *Econometrica*, 20(3):431–449, 1952.
- A. Ruszczyński and A. Shapiro, editors. *Stochastic programming*, volume 10. Elsevier, Amsterdam, 2003.
- H. Shefrin and M. Statman. Behavioral portfolio theory. *Journal of Financial and Quantitative Analysis*, 35(2):127–151, 2000.
- R. J. Shiller. The life-cycle personal accounts proposal for social security: An evaluation. Technical report, Yale ICF Working Paper No. 05-06; Cowles Foundation Discussion Paper No. 1504. Available at SSRN: <http://ssrn.com/abstract=703221>, 2005.

- N. Siebenmorgen and M. Weber. A behavioral model for asset allocation. *Financial Markets and Portfolio Management*, 17(1):15–42, 2003.
- N. Singer. Safety-first portfolio optimization: Fixed versus random target. Thuenen-Series of Applied Economic Theory, working paper 113, University of Rostock, 2010.
- L. G. Telser. Safety first and hedging. *The Review of Economic Studies*, 23(1):1–16, 1955.
- A. Tversky and D. Kahneman. Availability: A heuristic for judging frequency and probability. *Cognitive Psychology*, 5:207–232, 1973.
- A. Tversky and D. Kahneman. Advances in prospect theory. *Journal of Risk and Uncertainty*, 5(4):297–323, 1992.
- M. T. Tyynelä and J. Perttunen. Trading behaviour of Finnish households: Activity, performance and overconfidence. working paper, Available at SSRN: <http://ssrn.com/abstract=302691> or doi:10.2139/ssrn.302691, 2002.
- S. Uryasev and P. M. Pardalos, editors. *Stochastic optimization*, volume 54. Kluwer Academic Publ., Dordrecht, Mass., 2001.
- P. Westerheide. Anlageverhalten privater Haushalte: Auswirkungen von renditeorientierten Portfolioumschichtungen in der Langfristperiode. ZEW working paper, 2005.
- M. E. Yaari. The dual theory of choice under risk. *Econometrica*, 55(1):95–115, 1987.

A. Appendix

A.1. Proof of equation (4)

Proposition. Let $R_1 \leq \dots \leq R_m$ be ordered outcomes of the random variable R , p_j 's are the outcomes' associated probabilities and $D_j = \text{prob}(R \geq R_j)$ denotes the decumulative probability distribution function. Then the expected value, $E(R)$, can be written as

$$E(R) = \sum_{j=1}^m D_j (R_j - R_{j-1}), \quad (19)$$

with $R_0 = 0$.

Proof: As outcomes are sorted from lowest to highest, the decumulative probability distribution function can be restated as $D_j = \sum_{i=j}^m p_i$. Then, equation (19) can be written as

$$\sum_{j=1}^m \sum_{i=j}^m p_i (R_j - R_{j-1}) = \sum_{j=1}^m R_j \sum_{i=j}^m p_i - \sum_{j=1}^m R_{j-1} \sum_{i=j}^m p_i.$$

Extracting the term for $j = m$ from the first addend and rearranging indices in the second addend yields

$$R_m p_m + \sum_{j=1}^{m-1} R_j \sum_{i=j}^m p_i - \sum_{j=1}^{m-1} R_j \sum_{i=j+1}^m p_i,$$

which can be written as

$$R_m p_m + \sum_{j=1}^{m-1} R_j \underbrace{\left(\sum_{i=j}^m p_i - \sum_{i=j+1}^m p_i \right)}_{=p_j} = \sum_{j=1}^m p_j R_j.$$

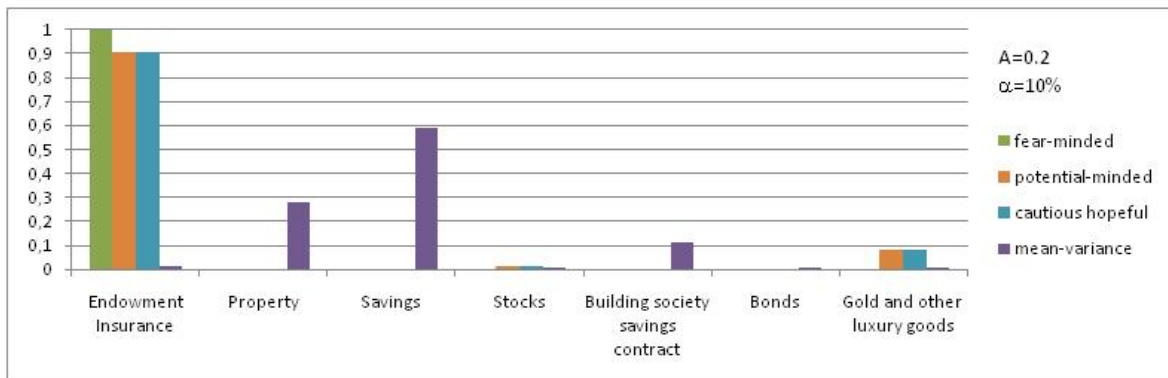
□

A.2. A precise definition for M in problem 2

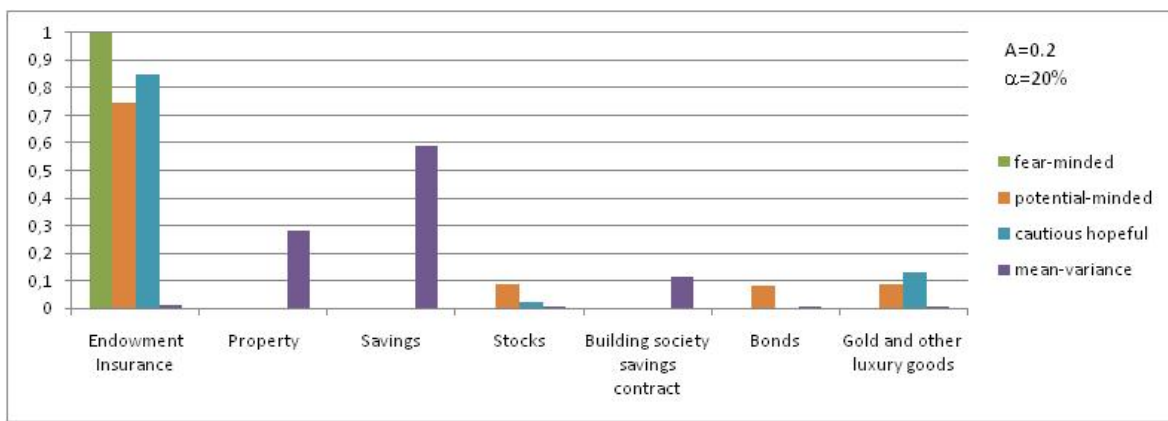
If $d_i = 1$, M drops out. If $d_i = 0$, M has to be chosen such that constraint (11) always holds. The worst can happen is when x takes one for the lowest outcome

$$R_{\min} := \min_{\substack{j=1, \dots, m \\ i=1, \dots, n}} R_{i,j}.$$

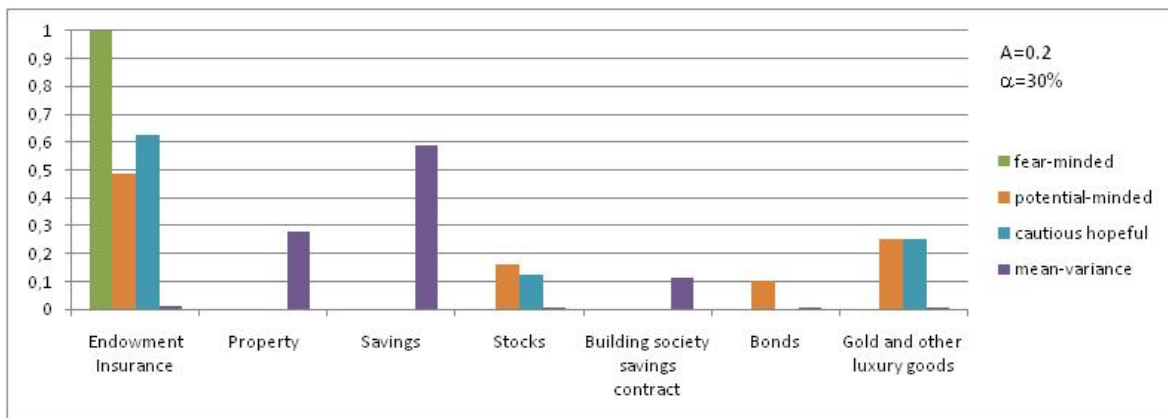
Thus, the left-hand side of (11) takes the value $R_{\min} + M$. To ensure feasibility, M has to be chosen such that $M \geq A - R_{\min}$. In our dataset R_{\min} is -29.33. Hence, M must be at least $A + 29.33$.



(a) $\alpha = 10\%$

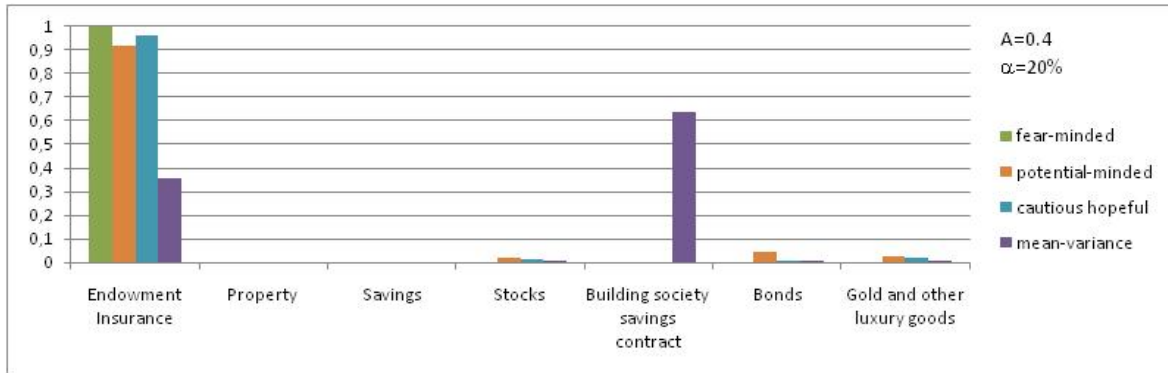


(b) $\alpha = 20\%$

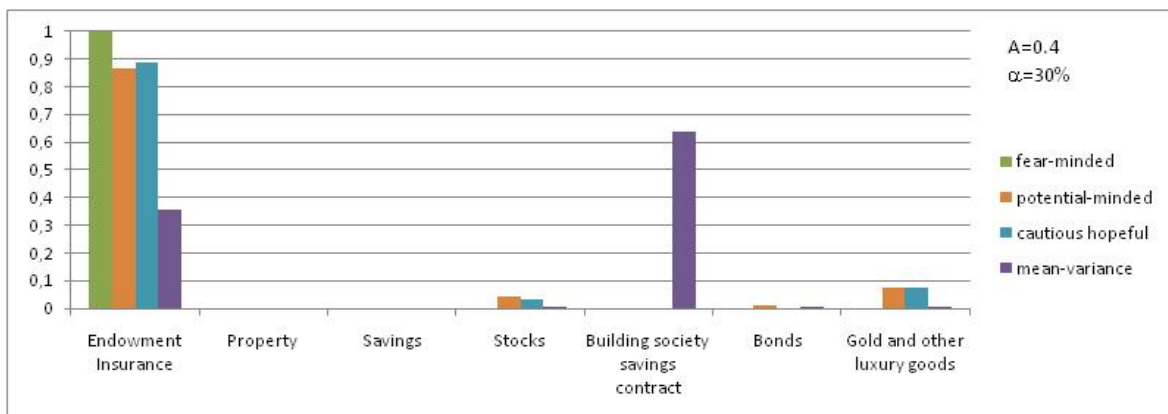


(c) $\alpha = 30\%$

Figure 3: Optimal portfolios for security-minded, potential-minded, cautiously hopeful and mean-variance behavior for $A = 0.2$



(a) $\alpha = 20\%$



(b) $\alpha = 30\%$

Figure 4: Optimal portfolios for security-minded, potential-minded, cautiously hopeful and mean-variance behavior for $A = 0.4$