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## Article

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# The BIP Trilogy (Bipolarization, Inequality and Polarization): One Saga but Three Different Stories

*Joseph Deutsch, Alessio Fusco, and Jacques Silber*

## Abstract

Inequality, bi-polarization and polarization are related but distinct concepts aiming at analysing the income distribution. This paper first recalls the main differences between these three notions of inequality, bipolarization and polarization and then suggests using the so-called Shapley decomposition to show that the various income sources have a different impact on these three types of indicators. Our empirical investigation, based on 2008 data for Luxembourg, shows in fact that, in the case of the so-called zero income Shapley decomposition, inequality in Luxembourg is strongly related to income from capital while bipolarization is mainly due to the distribution of income from work and polarization to that of the income from work and transfers.

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**Keywords** Bi-polarization; income sources; inequality; Luxembourg; polarization; Shapley decomposition procedure

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## 1 Introduction

A growing literature attempted during the past twenty years to propose measures of polarization and to apply them to the income data of various countries. It is however very important to make a distinction not only between the study of inequality and that of polarization, but also between the concept of income *bi-polarization* and that of income *polarization*. As stressed by the title of this paper, the ‘saga’ (‘a long detailed account’<sup>1</sup>) is the same because the three notions refer to a specific aspect of the same income distribution but the “story” (‘narrative designed to instruct the reader’<sup>2</sup>) is not necessarily identical.

A similar distinction seems to have been made in the field of international relations. Mansfield (1993, pages 109–110) thus wrote that ‘Hegemony (or unipolarity) is characterized by a ‘wide’ power disparity between the largest state in the system and all other states; bipolarity is characterized by the ‘approximate’ equality of the two largest states and a ‘wide’ power disparity between the smallest pole and any remaining state; and multi-polarity is characterized by the ‘approximate’ equality of more than two particularly powerful states and a ‘wide’ power disparity between the smallest pole and any other state in the system.’ Mansfield (1993) stressed however that an analysis of the distribution of power in international relations should not be limited to counting the number of major powers because ‘for the purposes of explaining patterns of balancing behavior, the onset of war, and many aspects of the international political economy, scholars are likely to find it useful to analyze both features of the distribution of power that have been used repeatedly in studies of international relations: (1) polarity; and (2) concentration.’

As far as income distributions are concerned, the focus of inequality, as is well known, is on disparities in the distribution of income. However since inequality is maximal when one individual ‘eats the whole pie’, the relative frequency of the observations at income zero will be close to one so that there should be quite a strong link between the idea that a distribution is very unequal and that of viewing a distribution as being uni-polar.

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<sup>1</sup> See [www.merriam-webster.com/dictionary/saga](http://www.merriam-webster.com/dictionary/saga).

<sup>2</sup> See <http://dictionary.reference.com/browse/story>.

In contrast bi-polarization refers to the case where there is a significant number of individuals who are very poor but there exists also a non negligible share of the population that is quite rich. Such a gap between the “poor” and the “rich” implies evidently that there is no sizeable middle class. The analysis of bi-polarization is hence linked to that of the importance of the middle class.

One may however wonder, when looking at the distribution of incomes in a given country, whether the question of bi-polarization is not an odd issue, given that income distributions often seem to follow a log-normal distribution with a single peak.<sup>3</sup> The lognormal functional form, originally proposed by Gibrat (1931) and thoroughly examined by Aitchinson and Brown (1969), has indeed attractive analytical properties and has been very popular in the literature on income distributions (see, for example, Lambert, 2009, for its recent application to the issue of pro-poor growth). But it has also been shown not to be the best choice to fit well observed data. Salem and Mount (1974) thus preferred using the gamma distribution, originally introduced by Ammon (1895), while Bartels and van Metelen (1975) preferred using the two-parameter Weibull distribution. Better fits were in fact obtained with three-parameter distributions, such as those proposed by Singh and Maddala (1976) and Dagum (1977). McDonald (1984) even proposed a four-parameter distribution. Bandourian et al. (2003) offer a nice comparison of these different models. Whatever model one uses, it should however be clear that all these distribution have a single peak, so that the question previously raised would seem to be a serious one. But one should first remember that the index of bipolarization we suggest to use (see Section 2.1. below) compares in fact the dispersion of the incomes above the median income with that of those below it, and then normalizes this difference with respect to the overall dispersion of incomes. Computing an index of bipolarization is hence relevant even with a single-peak distribution.

The issue of bipolarization becomes even more relevant once one accepts the idea that the overall income distribution is in fact a mixture of several distributions. For example, if one finds out that it is composed of three (or four) distributions, bipolarization and its link with the concept of middle class (or of an upper and a lower middle class) becomes a relevant question, even when the overall distribution has a single peak. Modeling an income distribution as a

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<sup>3</sup> We wish to thank an editor for raising this issue.

mixture distribution has in fact become quite popular, especially in studies of the world income distribution. One of the first works of this type was that of Paap and van Dijk (1998) who looked at the 1960–1989 period. More recently, several authors criticized the so-called twin-peaks hypothesis, labeled sometimes the two components hypothesis, which stressed (see, Quah, 1996) that the world distribution of income was polarizing into twin peaks of rich and poor countries.<sup>4</sup> Thus Holzmann et al. (2007), in an analysis of the world’s cross-national distribution of income and of its evolution between 1970 and 2003, found that this distribution which seemed to have only two components in 1970–1975 consisted “of three components from 1976 onwards, a low, average and high mean-income group, with group means diverging over time”. Pittau et al. (2009) who modeled the cross-country distribution of per capita income as a mixture distribution, using the Penn World Data for the period 1960–2000, found also evidence of three components densities, with little cross-component mobility, so that they interpreted the multiple mixture components as representing convergence clubs. Finally Grier and Maynard (2010), in a similar study, concluded that, “as early as the 1950s and 1960s, the per capita income distribution is most likely composed of two components and there is a significant amount of inter-group mobility. By 1980, the distribution changes into one that is most likely composed of three components, with a large gap opening between the means of the poorest and richest components and a much lower rate of inter-group mobility.”

The concept of mixture of distributions allows one in fact to make the link between the notions of bi-polarization and polarization, even though originally, in the economic literature, polarization referred to quite a different issue, that of the potential for social conflict. Esteban et al. (1994), who introduced the topic, and later Duclos et al. (2004), argued that the concept of polarization is derived from the combination of the two notions of *identification* and *alienation*. In the case of income polarization, identification is related to the idea that individuals identify themselves with those with similar income levels. Alienation on the contrary assumes that an individual will feel more alienated with respect to another individual, the greater the distance between their incomes. There is thus a clear connection between the notions of polarization and multipolarity.

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<sup>4</sup> Here again we are grateful to an editor who reminded us to link the issue of bipolarization to that of twin peaks.

As mentioned previously, bipolarization, inequality and polarization (BIP) refer to the same ‘saga’ but are theoretically supposed to highlight different aspects of the income distribution. However, are such theoretical differences observed empirically? Answering this question is in fact the main aim of this paper where decomposition by income component will be the empirical tool used to determine whether these three concepts tell similar or different stories. The focus of this paper is thus on the differential impact of various income sources on inequality, bi-polarization and polarization.

Numerous studies have been devoted during the past thirty years to the study of the ways various income sources affect income inequality (see, Lerman, 1999, for a survey of the literature on this topic) and this is why we will not cover income inequality decomposition by factor components in our methodological section but simply report the results. There have on the contrary been very few attempts to estimate the contribution of various income sources to the bi-polarization or polarization of income. Deutsch and Silber (2010) analyzed the impact of various income sources on bi-polarization in Israel on the basis of the bi-polarization index  $P_G$  that had been introduced by Deutsch et al. (2007). Araar (2008) proposed some analytical methods to decompose the Duclos, Esteban, and Ray (2004) polarization index  $DER$  by income sources and population subgroups and applied his approach to data from China and Nigeria. Finally Apouey (2010) quantified the contribution of various determinants to social polarization in health.

The approach taken in the present paper, in analyzing the impact of different income sources on inequality, bi-polarization and polarization, is based on the systematic use of the concept of Shapley decomposition that has been proposed by Chantreuil and Trannoy (1999, 2013), Shorrocks (1999; 2013) and Sastre and Trannoy (2001, 2002). More precisely we apply the Shapley decomposition procedure to analyze the marginal impact of each income source on inequality, bi-polarization and polarization. Our empirical investigation, based on 2008 data for Luxembourg, shows in fact, when using the so-called zero income Shapley decomposition (see, Section 3 below for more details), that inequality in Luxembourg is strongly related to income from capital while bi-polarization is mainly due to the distribution of income from work and polarization to that of the income from work and from transfers. Similar conclusions are drawn when including seven types of income sources. The paper is organized as follows. Section 2 reviews the literature on the measurement of bi-polarization and

polarization and Section 3 shows how to apply the Shapley decomposition procedure to analyze the impact of income sources on inequality, bi-polarization and polarization. Section 4 applies then this technique to a detailed analysis of the differential impact of various income sources on inequality, bi-polarization and polarization in Luxembourg in 2008. Concluding comments are finally given in Section 5.

## **2 On the Measurement of Bi-polarization and Polarization**

### **2.1 Measures of Bi-polarization**

The concept of bi-polarization is clearly related to that of the middle class (see, Wolfson, 1994) and there have been quite a few attempts at measuring the importance of the middle class, since a sizable middle class is supposed to be an important factor in economic development (see, Thurow, 1984, Foster and Wolfson, 1992 and 2010, Landes, 1998, Easterly, 2001, Birdsall, 2007a and 2007b, and Pressman, 2007). There is however no agreement on how the relative importance of the middle class should be measured. Thurow (1984), for example, defined the middle class as including those households whose income ranges from 75% to 125% of the median household income. Blackburn and Bloom (1985) used a wider interval (60% to 225% of the median). Other ranges were also proposed (e.g., Davis and Huston, 1992, Lawrence, 1984). Birdsall et al. (2000) adopted Thurow's approach but for Birdsall (2007a) the middle class should include people at or above the equivalent of \$10 day in 2005, and at or below the 90<sup>th</sup> percentile of the income distribution in their own country.

The concept of bi-polarization stresses in fact two notions. The first one, “increasing spread”, implies that moving from the middle position (the median) to the tails of the income distribution makes an income distribution more polarized. More precisely a rank preserving increment in incomes above the median or a rank preserving reduction in income below the median will widen the distribution, that is, extend the distance between the two groups (those above and below the median) and hence increase the degree of bi-polarization (the rich become richer and the poor poorer). The second concept, “increased bipolarity”, refers to the case where the incomes below the median or those above the median get closer to each

other. What is happening here is a “bunching” of the two groups in the sense that the gaps between the incomes below the median (or those above the median) have been reduced and such a “bunching” is assumed to increase bi-polarization.

To measure bi-polarization Foster and Wolfson (1992; 2010) recommended using an index  $P_{FW}$  which is defined as

$$P_{FW} = (G^B - G^W)(\mu / m) \quad (1)$$

where  $\mu, m, G^B$  and  $G^W$  are respectively the mean, the median, the between and within groups Gini indices of the distribution and where it is assumed that there are only two income groups, those with an income below or above the median income.

While  $P_{FW}$  is a relative bi-polarization index, there have also been propositions to define absolute (see, Chakravarty et al., 2007) or intermediate bi-polarization indices (see, Chakravarty and D'Ambrosio, 2010, and Lasso de la Vega et al., 2010). Some suggestions have also been made to extend Foster and Wolfson's Bi-polarization Index  $P_{FW}$  (see, Wang and Tsui, 2000, and Rodriguez and Salas, 2003).

More recently, Deutsch et al. (2007) proposed an alternative approach to the measurement of bi-polarization, related to previous work by Berrebi and Silber (1989) on the measurement of the flatness of an income distribution. They then derived a bi-polarization index  $P_G$  defined as

$$P_G = (G_B - G_W) / G = (\Delta_B - \Delta_W) / \Delta \quad (2)$$

where  $\Delta, \Delta_B$  and  $\Delta_W$  are the mean differences for the whole distribution and the between and within groups mean differences. Deutsch et al. (2007) assumed that there were two groups, those with an income below or above the median income, so that these two groups do not overlap and as a consequence the overall Gini index  $G$  is equal to the sum of the indices  $G_B$  and  $G_W$ .

They also proved that  $P_G$  had the two most desirable properties of a polarization index: it obeys the axioms of *Non-Decreasing Spread* and *Non-Decreasing Bipolarity* (see, Appendix A, for a detailed proof). Moreover it is easy to show that the bi-polarization index  $P_G$  is invariant not only to a multiplication of all incomes by a constant but also to equal additions to all incomes since the mean



differences  $\Delta$ ,  $\Delta_B$  and  $\Delta_W$  that appear in (2) are invariant to equal additions to all incomes.

Finally one may note that there is quite an important similarity between the definition of the bi-polarization index  $P_G$  given in (2) and the Foster and Wolfson (1992; 2010) bi-polarization index  $P_W$  defined in (1).

## 2.2 On the Measurement of Polarization

Here we are interested in studying the extent to which a population is clustered around a small number of distant poles so that we do not limit ourselves any more to two groups. Various measures of polarization have appeared in the literature, that correspond to this case (see, for example, Esteban and Ray, 1994, Zhang and Kanbur, 2001, Reynal-Querol, 2002, Duclos, Esteban and Ray, 2004, Lasso de la Vega and Urrutia, 2006, Esteban, Gradin and Ray, 2007, Gagliarano and Mosler, 2009 and Poggi and Silber, 2010).

The notion of polarization in a multi-group context, which was originally proposed by Esteban and Ray (1994), was in fact an attempt at capturing the degree of potential conflict inherent in a given distribution. The idea is that political or social conflict is more likely, the more homogenous, separate and of a similar size the groups are, so that it is polarization, and not inequality, that matters for conflict. For Esteban and Ray (1994) society may be considered as an amalgamation of groups, where two individuals drawn from the same group are assumed to be “similar” whereas two persons belonging to different groups are supposed to be “different” with respect to a given set of attributes.

To describe the potential for social conflict Esteban and Ray (1994) have introduced the notions of “identification” and “alienation”. The first notion refers to the fact that if each group consists of very similar individuals, it is then likely that their objectives will also be very similar, and, as a consequence, they will form a stronger unit because of their mutual sense of identification. In addition, if there is a clear difference between groups, then this heterogeneity across groups will in a certain way contribute to tensions by increasing the probability that the objectives of two or more groups will be conflicting. This is therefore the identification-alienation framework introduced by these authors.

A distinction can be made between the case where the groups are defined a priori, whether on the basis of their income or of some other criterion (e.g.

ethnicity) or that where we let the data define the groups. Defining the groups from the onset was the approach taken by Esteban and Ray (1994), Zhang and Kanbur (2001) or Poggi and Silber (2010). Such an approach raises however several questions: how many groups should be defined? How big should a group be to be considered as relevant? To overcome these difficulties Duclos et al. (2004) suggested computing the degree of polarization on the basis of the density function so that the number of groups considered would be endogenous. Their approach is based on the use of continuous distributions and they approximate the strength of group identification of a person via the value of the density function evaluated at the person's income. Let  $f$  be such a density. The effective antagonism of an individual with income  $x$  towards an individual with income  $y$  is a nonnegative function of the identification  $I = f(y)$  and of the alienation  $a = |x - y|$ . Polarization is, then, assumed to be proportional to the sum of all effective antagonisms and the authors derive axiomatically a new measure of polarization  $DER$  defined as

$$DER = P_{\alpha}(f) = \iint f(x)^{1+\alpha} f(y) |x - y| dx dy \quad (3)$$

with  $0.25 \leq \alpha \leq 1$ .

To compute  $DER$  in applied work Duclos et al. (2004) first note that expression (3) may also be written as

$$DER = P_{\alpha}(F) = \int_y f(y)^{\alpha} a(y) dF(y) \quad (4)$$

with

$$a(y) = \mu + y(2F(y) - 1) - 2 \int_{-\infty}^y x dF(x) \quad (5)$$

Assume now a random sample of  $n$  i.i.d. observations on the income  $y_i$  ( $i = 1, \dots, n$ ) which are drawn from the distribution  $F(y)$  and which are classified in increasing order ( $y_1 \leq \dots \leq y_i \leq \dots \leq y_n$ ). Duclos et al. (2004) then state that a natural estimator of  $P_{\alpha}(F)$  is

$$P_{\alpha}(\hat{F}) = (1/n) \sum_{i=1}^n \hat{f}(y_i)^{\alpha} \hat{a}(y_i) \quad (6)$$

with

$$\hat{\alpha}(y_i) = \hat{\mu} + y_i[(1/n)(2i-1) - 1] - (1/n)[2\sum_{j=1}^{i-1} y_j + y_i] \quad (7)$$

In (7)  $\hat{\mu}$  is the sample meanwhile in (6)  $\hat{f}(y_i)^\alpha$  is estimated non-parametrically using kernel estimation procedures.

### 3 Using the Shapley Decomposition Procedure to Measure the Marginal Contribution of Various Income Sources to Inequality, Bi-polarization and Polarization

Although when introduced in the literature (see, Chantreuil and Trannoy, 1999, 2013, and Sastre and Trannoy, 2001, 2002) the so-called Shapley decomposition procedure was applied to the breakdown of income inequality, Shorrocks (1999; 2013) has shown that this decomposition could be applied to any function.<sup>5</sup> For example, this technique was subsequently applied to the study of variations in poverty across Russian regions (Kolenikov and Shorrocks, 2005), the decomposition of the sources of disparities in the relative wealth position of Mexican Americans (Cobb-Clark and Hildebrand, 2006), the breakdown of the R-square of a regression (Israeli, 2007) or of the Likelihood Ratio Index of a logit regression (D'Ambrosio et al., 2011), the decomposition of changes in the wage distribution (Devicienti, 2010) or in poverty/inequality indices due to changes in tax-benefit policy (Bargain and Callan, 2010). Another application, directly relevant for this paper, is the one by Deutsch and Silber (2010) who used the Shapley decomposition procedure to measure the exact impact of an income source on the bi-polarization index  $P_G$ .

Before explaining briefly the mechanism underlying the Shapley decomposition rule, let us recall that, in the case where the total income variable is the sum of a set of income components, a straightforward and intuitive way to assess the contribution of each component taken separately to, say, inequality is to apply a before/after calculation. Each source contribution is then equal to its first round marginal impact corresponding to the difference between overall inequality

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<sup>5</sup> See Auvray and Trannoy (1992) and Rongve (1994) for the first applications.

and the inequality obtained when the income component (or when the inequality from that income component) is eliminated.

For Sastre and Trannoy (2002), this approach, to which they refer as a ‘local method’<sup>6</sup> suffers from two drawbacks. First, the sum of the first round marginal impacts of the various factors does not result in an exact decomposition of the overall index (see also Shorrocks, 1999). A solution to this problem would be to eliminate each factor in turn. In this case, the decomposition is exact; however, and this is the second drawback of this method, a path dependency problem arises: the contribution of each factor (except when there are only two income sources) clearly depends on their order in the elimination process.

The Shapley decomposition allows overcoming this path dependency issue. Indeed, the idea of the Shapley decomposition procedure is precisely to average the contribution of each income component over all the possible sequences allowing us to eliminate the different income sources. Before giving a concrete example, it is important to note that Sastre and Trannoy (2002) mention two possibilities to ‘eliminate a variable’. The first one simply consists in removing a variable from the computation, assuming that the income from that source is equal to zero for all observations. We then compare a situation where a given income source does not exist and that where it is present. They refer to this option as the *zero income decomposition*. The second one amounts to supposing that a given income source is equally distributed, implying that each individual (household) is attributed the mean value of this income source. Here the two situations compared are that where all the individuals receive the same amount of income from a given source and that where the distribution of this income source is assumed to be unequally distributed. This method is named the *equalized income decomposition*. When decomposing inequality, Sastre and Trannoy (2002) tend to recommend not using the zero income decomposition as it yields highly volatile results, depending on the level of aggregation of the income components.

Let us now explain the algorithm in more details. We take as illustration the case of the equalized income assumption and suppose, to simplify, that there are

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<sup>6</sup> Sastre and Trannoy (2002) use this term by contrast to “global methods” where “contributions must be computed for all income components defined at the outset”. The sum of the components should also add up to the inequality to be “explained”. Shorrocks’s (1982) decomposition rule or Lerman and Yitzhaki’s (1985) decomposition method belong to this global method.

only two income sources,  $x$  and  $y$  with means  $\bar{x}$  and  $\bar{y}$ . The inequality index  $I_G$  may therefore be written as  $I_G(x, y)$ ,  $x$  and  $y$  being the vectors of the two incomes sources received by the various individuals. We first want to compute the contribution of income source  $x$ . The inequality linked to this component may be that existing when this component is the first or the second to be eliminated. If it is eliminated first, the function  $I_G(x, y)$  will become equal to  $I_G(x = \bar{x}, y)$  since the inequality of variable  $x$  has been eliminated, so that in this case the contribution  $C(x)$  of  $x$  to the function  $I_G(x, y)$  may be expressed as  $C(x) = [I_G(x, y) - I_G(x = \bar{x}, y)]$ . Note that in the case where  $I_G(x = \bar{x}, y)$  is higher than  $I_G(x, y)$  the first round contribution  $C(x)$  can be negative. If the variable  $x$  is the second one to be eliminated, the function  $I_G(x, y)$  will be written as  $I_G(x, y = \bar{y})$ . Since both elimination sequences are possible and assuming the probability of these two sequences is the same, we may conclude that the contribution  $C(x)$  of income source  $x$  to the function  $I_G(x, y)$  is equal to

$$C(x) = (1/2)\{I_G(x; y) - I_G(x = \bar{x}; y)\} \\ + (1/2)\{I_G(x; y = \bar{y}) - I_G(x = \bar{x}; y = \bar{y})\} \quad (8)$$

Similarly one can prove that the contribution  $C(y)$  of the variable  $y$  to the function  $I_G(x, y)$  may be expressed as

$$C(y) = (1/2)\{I_G(x; y) - I_G(x; y = \bar{y})\} \\ + (1/2)\{I_G(x = \bar{x}; y) - I_G(x = \bar{x}; y = \bar{y})\} \quad (9)$$

Combining (10) and (11) we observe that

$$I_G(x, y) = C(x) + C(y) \quad (10)$$

since  $I_G(x = \bar{x}; y = \bar{y})$  is assumed to be equal to 0.

The same type of decomposition may be extended to the case of more than two determinants and it is then possible to determine the exact marginal impact of each

of the different variables (income sources) on the inequality index  $I_G$ . The zero income Shapley decomposition follows the same algorithm but instead of replacing the value of an income source by its mean, we simply give it a value of zero.<sup>7</sup>

As already mentioned, Shorrocks (1999) has shown that the Shapley decomposition can be applied to any function. In this paper we apply it to the analysis of bi-polarization and polarization.

As far as the bi-polarization index  $P_G$  is concerned, one should note that the equalized income Shapley decomposition procedure would give the same result as the zero income Shapley decomposition procedure. The reason is that the bi-polarization index  $P_G$ , as indicated previously, is invariant not only to a multiplication of all incomes by a constant but also to equal additions to all incomes.

#### **4 On the Differential Impact of Various Income Sources on Inequality, Bi-polarization and Polarization: the Case of Luxembourg**

The empirical analysis uses data from the Socio-Economic Panel “Liewen zu Lëtzebuerg” (PSELL-3) which is the component for Luxembourg of the EU *Statistics on Income and Living Conditions* (EU-SILC). PSELL-3 was launched in 2003, with an initial sample of 3500 households that were representative of the population living in private households in Luxembourg. In this paper, we analyse the data for the year 2008 composed of 3779 households.

The concept of income used in the PSELL dataset is quite broad as it comprises earnings from work including company cars, all social benefits received in cash, income from investment and property and inter-households payments. It is however not comprehensive as it excludes non-monetary income components such as imputed rents, the value of goods produced for own consumption and non-cash employee income (with the exception of company car). In the first stage of our

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<sup>7</sup> It should be clear that the interpretation of the contribution of a given component (determinant) to the value of a given index (Gini index, bi-polarization or polarization index) does not depend on the range/bound of this index. We thank an anonymous reader for asking us to clarify this point.

analysis we make a distinction between three broad income sources which together constitute total unadjusted gross income: income from work, income from capital and social transfers.<sup>8</sup> In a second stage we further refine our analysis and use seven income sources (see Table B1 in Appendix B).

Table 1 displays the relative contribution of each of the three broad income sources to the values of the Gini index  $I_G$ , the bi-polarization index  $P_G$  and the polarization index  $DER$ , following the methodology presented in the previous sections. The Shapley contributions of the different income sources (in percentage terms) to these three indices are quite different.

Regarding the decomposition of the Gini inequality index, we examine the two types of Shapley decomposition, the zero income and the equalized income decompositions. The two cases give strikingly different results. In the first case it turns out that the marginal contribution of income from capital was 80.7% in 2008. In other words, had there not been any income from capital, inequality would have been much smaller (around 20% of the actual value of the Gini index). In the second case, what contributes most to inequality is income from work whose relative contribution represents almost 80% of the value of the Gini index. In other words, had income from work been equally distributed, inequality would have been much smaller (around 20% of the actual value of the Gini index). The relative contribution of each source can be easily compared to its share in the income distribution when computing the ratio of the former to the latter. In the case of the zero decomposition, the relative contribution of income from capital is substantially higher than its share in total income (25 times!), whereas the contribution of income from work is extremely small compared to its income share. The equalized income decomposition gives results that are much more in line with the shares of the various income sources in total income. In 2008, the results show that income from work contributes to overall inequality slightly more

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<sup>8</sup> The definition of each component can be found in the EU-SILC 2008 guidelines. See [http://circa.europa.eu/Public/irc/dsis/eusilc/library?l=/guidelines\\_questionnaire/operation\\_guidelines/silc065\\_version/\\_EN\\_1.0\\_&a=d](http://circa.europa.eu/Public/irc/dsis/eusilc/library?l=/guidelines_questionnaire/operation_guidelines/silc065_version/_EN_1.0_&a=d)

Table 1: Shapley Decomposition Procedure Giving the Contribution of Three Aggregated Income Sources to the Gini Inequality Index, the Bi-polarization Index  $P_G$  and the DER Polarization Index, 2008

	Absolute Contribution	Relative Contribution (RC)	Income Share (IS)	RC/IS
<b>Gini inequality index, zero income decomposition</b>				
Work	-0.011	-3.0	73.2	-0.04
Capital	0.297	80.7	3.3	24.5
Transfers	0.082	22.4	23.5	0.95
Total	0.368	100	100	
<b>Gini inequality index, equalized income decomposition</b>				
Work	0.292	79.4	73.2	1.08
Capital	0.02	5.3	3.3	1.61
Transfers	0.056	15.3	23.5	0.65
Total	0.368	100	100	
<b>Bi-polarization PG index</b>				
Work	0.467	136	73.2	1.86
Capital	-0.008	-2.2	3.3	-0.66
Transfers	-0.116	-33.8	23.5	-1.44
Total	0.343	100	100	
<b>Polarization DER (<math>\alpha=0.5</math>) index</b>				
Work	0.04	25.2	73.2	0.34
Capital	0.06	37.3	3.3	11.31
Transfers	0.06	37.5	23.5	1.6
Total	0.159	100	100	
<b>Polarization DER (<math>\alpha=1</math>) index</b>				
Work	0.029	39	73.2	0.53
Capital	0.016	21.1	3.3	6.39
Transfers	0.029	39.9	23.5	1.7
Total	0.074	100	100	

Source: PSELL-3, CEPS/INSTEAD, authors' calculation.



than its share in the income distribution whereas income from capital contributes 61% more than its weight in total income and transfers contributes only 65% of their share in total gross income.

One may also note that in the case of an equalized income decomposition, income from work has a positive contribution while in that of a zero income decomposition its contribution is negative (though small). This implies that when total income includes only income from capital and transfers the Gini index of such a total income will be greater than when income from work is also included in total income. This indicates that income of work is more equally distributed than the sum of the two other income sources. In Appendix C we show in more details where such a negative contribution comes from by “unpacking” the overall contribution of income from work.

One should not be surprised to observe a dependence of the contributions of the income sources on the type of Shapley decomposition that is implemented. In the first case we are wondering what would have happened to inequality if a given income source did not exist. In the second case we do not simulate the disappearance of an income source. We simply ask what inequality would have been, had a given income source been equally distributed between the households. These two questions are totally different and hence one should not be amazed by the fact that the answers in the two cases are different.

We now examine the results of the Shapley decomposition of the bi-polarization index  $P_G$ . As mentioned previously the results of such a breakdown will be identical, whatever the type of Shapley decomposition that is implemented. It appears that here income from capital does not play an important role since its contribution is negative and around 2.2% of the value of the index  $P_G$ . This implies that if there had been no income from capital the index  $P_G$  would have been 2.2% higher. The main impact on bi-polarization turns out to be that of income from work. The Shapley contribution of this income source is greater than 100% (around 136%). As a consequence we can conclude that if this income source did not exist, the index  $P_G$  would have been negative and its magnitude would have been equal to 36% of the actual magnitude of the bi-polarization index. Finally, results show that income from work contribute to overall bi-polarization close to twice its share in the income distribution.

A negative value of the index  $P_G$  implies that the between groups Gini index is smaller than the within groups Gini index. Income from work has therefore a great impact on between groups inequality (the two groups being those with an income higher and lower than the median income) and this is how it affects bi-polarization. Finally we also observe that income from transfers has a negative impact on bi-polarization (the absolute value of its Shapley contribution is equal more or less to 40%). This means that if there had been no transfers, bi-polarization would have been quite higher (around 135% of its actual value). Since such a higher bi-polarization means, as was mentioned previously, that there would have been either a greater positive difference between the “between” and the “within” groups Gini index (the numerator of the  $P_G$  index) or/and a smaller overall Gini index (the denominator of the  $P_G$  index), we may conclude that when transfers are included in total income, either the Gini index of total income is higher, or, what is more likely, the between groups Gini index is smaller (there are less differences between the rich and the poor) and the within groups index is higher (there is more dispersion in the Gini index of the poor and of the rich). In any case it should be clear that *bi-polarization is mainly due to income from work*.

As far as polarization is concerned, we have made a distinction between the case where the polarization sensitivity parameter  $\alpha$  is equal to 0.5 and that where it is equal to 1. When  $\alpha = 0.5$ , Table 1 shows that income from work contributes less than income from capital and transfers. However when  $\alpha = 1$ , it appears that the main Shapley contributions are those of income from work and income from transfers (these two contributions are of somehow similar magnitude and represent together around 80% of the value of the index  $DER$ ), income from capital having a marginal contribution of around 20%. Since, when examining the case of zero income decomposition, we had found that when  $\alpha = 0$  (the case of the Gini index), the main Shapley contribution was that of income from capital, we can state that as the polarization sensitivity parameter  $\alpha$  increases, the contribution of income from capital decreases.

We may therefore conclude that, although the three income sources contribute to polarization, as we increase the sensitivity polarization parameter the two main influences on polarization become those of income from work and income from transfers. To understand these results we have to remember that the polarization index  $DER$  measures really the extent to which there are local poles in the income

distribution. These local poles are hence mainly due to income from work and income from transfers, income from capital playing a smaller role.

These findings should help us understanding the differences that exist between the concepts of inequality, bi-polarization and polarization. We base our conclusions on the case where the alternative in the Shapley decomposition is the absence of a given income source. In such a case the main contribution to inequality in Luxembourg is income from capital. In the case of bi-polarization the main role is played by income from work whereas polarization, whose degree is assumed to measure the extent to which there are local poles in the distribution of income, tends to be mainly related to income from work and income from transfers.

These conclusions should be confirmed by a more detailed analysis based on seven income sources. Table 2 gives the Shapley contributions of these seven sources to the Gini index. When using the zero income decomposition the source which contributes most to inequality is income from self-employment. It is followed by unemployment benefits and income from capital. When using the equalized income decomposition, income from employment contributes the most to inequality.

Table 3 gives the Shapley contributions of the seven income sources to bi-polarization. The Shapley contribution of income from employment is around 140% which means that if there had been no income from employment, the index  $P_G$  would have been negative and its magnitude in absolute value would have been around 40% of the value of  $P_G$ . Since, as mentioned previously, a negative value of the index  $P_G$  implies that the between groups Gini index is smaller than the within groups Gini index, we can conclude that income from employment has a great impact on between groups inequality and this is how it affects bi-polarization. If we now look at the marginal impact of income from self-employment we observe that its Shapley value is 16.6% which means that if there had been no income from self-employment bi-polarization would have been smaller and equal to more or less 85% of its actual value. The Shapley contribution of income from rent or capital is around 8% which means that if this source did not exist, bi-polarization would have been a bit smaller (92% of its actual value).

*Table 2: Shapley Decomposition Procedure Giving the Contribution of Seven Income Sources to the Gini Inequality Index, 2008*

	Absolute Contribution	Relative Contribution (RC)	Income Share (IS)	RC/IS
<b>Gini inequality index, zero income decomposition</b>				
income from employment	-0.194	-52.6	67.1	-0.8
income from self-employment	0.155	42.1	6.09	6.9
income from unemployment benefits	0.126	34.2	1.67	20.5
income from old age and survivors allowances	0.028	7.7	13.69	0.6
income from rent and capital	0.112	30.5	3.33	9.2
income from family allowances	0.046	12.5	4.67	2.7
income from other transfers	0.094	25.6	3.45	7.4
<b>Total</b>	<b>0.368</b>	<b>100.0</b>	<b>100</b>	
<b>Gini inequality index, equalized income decomposition</b>				
income from employment	0.248	67.5	67.1	1.0
income from self-employment	0.042	11.3	6.09	1.9
income from unemployment benefits	0.006	1.5	1.67	0.9
income from old age and survivors allowances	0.033	9.0	13.69	0.7
income from rent and capital	0.018	5.0	3.33	1.5
income from family allowances	0.014	3.8	4.67	0.8
income from other transfers	0.007	1.8	3.45	0.5
<b>Total</b>	<b>0.368</b>	<b>100.0</b>	<b>100</b>	

*Source:* PSELL-3, CEPS/INSTEAD, authors' calculation.

Table 3: Shapley Decomposition Procedure Giving the Contribution of Seven Income Sources to the Bipolarization  $P_G$  index, 2008

	Absolute Contribution	Relative Contribution (RC)	Income Share (IS)	RC/IS
income from employment	0.483	140.8	67.1	2.1
income from self-employment	0.057	16.6	6.09	2.7
income from unemployment benefits	-0.053	-15.5	1.67	-9.3
income from old age and survivors allowances	-0.036	-10.4	13.69	-0.8
income from rent and capital	0.027	8.0	3.33	2.4
income from family allowances	-0.054	-15.9	4.67	-3.4
income from other transfers	-0.081	-23.6	3.45	-6.8
Total	0.343	100.0	100	

Source: PSELL-3, CEPS/INSTEAD, authors' calculation.

If we now look at the Shapley contributions of the seven income sources to the value of the polarization index  $DER$ , we make, here also, a distinction between the case where the polarization sensitivity parameter  $\alpha$  is equal to 0.5 and that where it is equal to 1. In the former case (see, Table 4) we observe that all the income sources have generally a positive Shapley contribution. Moreover all these contributions are smaller than 100% which means that, whatever the source, we can say that, had the source under study not existed, polarization would have been smaller. Table 4 shows also that when the parameter  $\alpha$  is equal to 0.5, income from family allowances contributes the most to polarization.

If we now take the case where the polarization sensitivity parameter  $\alpha$  is equal to 1 (see Table 4), we first observe that only one source has a negative contribution, income from self-employment, but the magnitude of its contribution is small. We can nevertheless say that when the Shapley contribution is negative, this implies that had there been no income from self-employment polarization would have been higher. Income from self-employment seems therefore to smooth

*Table 4: Shapley Decomposition Procedure Giving the Contribution of Seven Income Sources to the Polarization DER Index (with  $\alpha=0.5$  and  $\alpha=1$ ), 2008*

	Absolute Contribution	Relative Contribution (RC)	Income Share (IS)	RC/IS
<b>Polarization DER (<math>\alpha=0.5</math>) index</b>				
income from employment	0.021	13.2	67.1	0.2
income from self-employment	0.001	0.3	6.09	0.1
income from unemployment benefits	0.022	13.5	1.67	8.1
income from old age and survivors allowances	0.025	15.6	13.69	1.1
income from rent and capital	0.029	18.0	3.33	5.4
income from family allowances	0.044	27.4	4.67	5.9
income from other transfers	0.019	11.9	3.45	3.5
<b>Total</b>	<b>0.159</b>	<b>100.0</b>	<b>100</b>	
<b>Polarization DER (<math>\alpha=1</math>) index</b>				
income from employment	0.033	45.4	67.1	0.7
income from self-employment	-0.008	-11.2	6.09	-1.8
income from unemployment benefits	0.005	6.8	1.67	4.1
income from old age and survivors allowances	0.007	9.8	13.69	0.7
income from rent and capital	0.009	11.8	3.33	3.6
income from family allowances	0.024	32.9	4.67	7.1
income from other transfers	0.003	4.4	3.45	1.3
<b>Total</b>	<b>0.074</b>	<b>100.0</b>	<b>100</b>	

*Source: PSELL-3, CEPS/INSTEAD, authors' calculation.*

the income distribution in the sense that it decreases the extent to which there are local poles. This might indicate that self-employed individuals may come from all strata of the population.

All the other income sources have a positive Shapley contribution, but smaller than 100%. This implies that, had one of these sources not existed, polarization

would have been smaller. The most important contributions are those of income from employment and from family allowances which are therefore those that have an impact on the existence of local poles in the distribution of income.

To summarize, when using the zero income decomposition of the Gini inequality index, the main contribution to inequality in Luxembourg is income from self-employment and capital. In the case of bi-polarization the main role is played by income from work whereas polarization whose degree is assumed to measure the extent to which there are local poles in the distribution of income tends to be mainly related to income from work and income from family allowances. As a whole these conclusions confirm those drawn on the basis of only three broad income sources.

## **5 Concluding Comments**

This paper confirms that inequality, bi-polarization and polarization are indeed three different concepts. Such a conclusion is based on an empirical analysis that attempted to look at the impact of different income sources on inequality, bi-polarization and polarization on the basis of the concept of Shapley decomposition, using the “Liewen zu Lëtzebuerg” socio-economic panel for the year 2008. When making a distinction between only three broad income sources, income from work, from capital and from transfers, and implementing a Shapley breakdown, we found, when using the zero income decomposition, that inequality in Luxembourg is strongly related to income from capital. On the other hand when using the equalized income decomposition we concluded that income from work plays the main role. We also observed that the extent of bi-polarization is mainly a consequence of the distribution of income from work. Finally polarization whose degree is assumed to measure the extent to which there are local poles in the distribution of income tends to be mainly related to income from work and income from transfers. Similar conclusions were derived on the basis of seven types of income sources. It thus appears that a given income source has generally a different impact on inequality, bi-polarization and polarization, so that it seems to be of utmost importance to make a clear distinction between these three concepts.

One may however wonder, since Luxembourg is a small country and probably an atypical one, and given that many individuals who work in Luxembourg live in

a neighbouring country, whether the results presented in this paper can be generalized. We believe they are, and for several reasons. First the shares in total income of the three main sources we distinguished (income from work and capital and transfers) are not exceptional. This is even true when making a distinction between seven income sources. Second the overall Gini index does not have an extremely low or high value. On the contrary it is well within the usual range of Gini indices for countries with a similar level of development. Third our goal was not only to present empirical results but also to draw the attention of researchers on the need to make a clear distinction between the concepts of inequality, bipolarization and polarization. Our argument will however become certainly stronger if additional empirical studies, with a focus on different countries, confirm our findings.

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## References

- Aitchinson, J. and J. A. C. Brown (1969). *The Lognormal Distribution with Special Reference to Its Uses in Economics*, Cambridge University Press.
- Ammon, O. (1995). *Die Gesellschaftsordnung und ihre Natürlichen Grundlagen*, Iena, Germany.
- Apouey, B. H. (2010). On Measuring and Explaining Socioeconomic Polarization in health with an Application to French Data, *Review of Income and Wealth* 56(1): 141-170. <http://ideas.repec.org/a/bla/revinw/v56y2010i1p141-170.html>
- Araar, A. (2008). On the Decomposition of Polarization Indices: Illustrations with Chinese and Nigerian Household Surveys, CIRPÉE Working Paper 08-06, Université Laval, Québec, Canada. <http://ideas.repec.org/p/lvl/lacict/0806.html>
- Auvray, C., and Trannoy, A. (1992). Décomposition par source de l'inégalité des revenus à l'aide de la valeur de Shapley. *Journées de Microéconomie Appliquées*. Sfax.



- Bandourian, R., J. B. McDonald and R. S. Turley (2003). A Comparison of Parametric Models of Income Distribution Across Countries and Over Time, *Estadistica* 55 : 135-152. [http://papers.ssrn.com/sol3/papers.cfm?abstract\\_id=324900](http://papers.ssrn.com/sol3/papers.cfm?abstract_id=324900)
- Bargain O. and T. Callan, (2010). Analysing the Effects of Tax-benefit Reforms on Income Distribution: A Decomposition Approach, *Journal of Economic Inequality*, Springer, vol. 8(1), pages 1-21, March. <http://ideas.repec.org/a/kap/jecin/v8y2010i1p1-21.html>
- Bartels, C. P. A. and H. Van Metelen (1975). *Alternative Probability Density Functions of Income*, Vrije Universiteit Amsterdam, Research Memorandum 29.
- Berrebi, Z. M. and J. Silber (1989). Deprivation, the Gini Index of Inequality and the Flatness of an Income Distribution, *Mathematical Social Sciences* 18: 229-237. [http://dx.doi.org/10.1016/0165-4896\(89\)90032-2](http://dx.doi.org/10.1016/0165-4896(89)90032-2)
- Birdsall, N. (2007a). Do No Harm: Aid, Weak Institutions, and the Missing Middle in Africa, Working Paper No 113, Center for Global Development. <http://ideas.repec.org/a/bla/devpol/v25y2007i5p575-598.html>
- Birdsall, N. (2007b). Reflections on the Macro Foundations of the Middle Class in the Developing World, Working Paper 130, Center for Global Development. <http://ideas.repec.org/p/cdl/glinre/qt4nt1n232.html>
- Birdsall, N., C. Graham, and S. Pettinato (2000). Stuck in the Tunnel: Is Globalization Muddling the Middle Class? Working Paper No. 14, Center on Social and Economic Dynamics. <http://ideas.repec.org/p/fth/brooki/14.html>
- Blackburn, M. and D. Bloom (1985). What is Happening to the Middle Class? *American Demographics* 7(1): 18-25.
- Chakravarty, S. R. and C. D'Ambrosio (2010). Polarization Orderings of Income Distributions, *Review of Income and Wealth* 56(1): 47-64. <http://ideas.repec.org/a/bla/revinw/v56y2010i1p47-64.html>
- Chakravarty, S., A. Majumder and S. Roy (2007). A Treatment of Absolute Indices of Polarization, *The Japanese Economic Review* 58(2): 273-293. <http://ideas.repec.org/a/bla/jecrev/v58y2007i2p273-293.html>
- Chantreuil, F. and A. Trannoy (1999). Inequality Decomposition Values: The Trade-Off Between Marginality and Consistency. THEMA Discussion Paper, Université de Cergy-Pontoise. <http://ideas.repec.org/p/ema/worpaper/99-24.html>
- Chantreuil, F. and A. Trannoy (2013). Inequality Decomposition Values: The Trade-off between Marginality and Efficiency, *Journal of Economic Inequality* 11(1): 83-98. <http://ideas.repec.org/a/kap/jecin/v11y2013i1p83-98.html>

- Cobb-Clark, D. and V. Hildebrand, (2006). The Wealth of Mexican Americans, *Journal of Human Resources*, 41(4), 841-873.  
<http://ideas.repec.org/a/uwp/jhriss/v41y2006i4p841-868.html>
- Dagum, C. (1977). A New Model for Personal Income Distribution: Specification and Estimation, *Economie Appliquée* 30: 413-437.
- D'Ambrosio, C., J. Deutsch and J. Silber (2011). Multidimensional Approaches to Poverty Measurement: An Empirical Analysis of Poverty in Belgium, France, Germany, Italy and Spain, based on the European Panel, *Applied Economics* 43(8): 951-962.  
<http://ideas.repec.org/a/taf/applec/v43y2009i8p951-961.html>
- Davis, J.C. and J.H. Huston (1992). The Shrinking Middle-income Class: A Multivariate Analysis, *Eastern Economic Journal* 18(3): 277-285.  
<http://ideas.repec.org/a/eej/eeconj/v18y1992i3p277-285.html>
- Deutsch, J., M. Hanoka and J. Silber (2007). On the Link between the Concepts of Kurtosis and Bipolarization, *Economics Bulletin* 4(36): 1-6.  
<http://ideas.repec.org/a/ebl/ecbull/eb-07d30002.html>
- Deutsch and Silber, J. (2010). Analyzing the Impact of Income Sources on Changes in Bi-Polarization, in *The Measurement of Individual Well-Being and Group Inequalities: Essays in Memory of Z. M. Berrebi*, edited by J. Deutsch and J. Silber, Routledge Economics, Taylor and Francis Group, pp. 127-152.
- Devicienti F. (2010). Shapley-Value Decompositions of Changes in Wage Distributions: A Note, *Journal of Economic Inequality*, n°8: 35-45  
<http://ideas.repec.org/a/kap/jecinq/v8y2010i1p35-45.html>
- Duclos, J., Esteban, J.M. and Ray, D. (2004). Polarization: Concepts, Measurement, Estimation, *Econometrica* 72(6): 1737-1772.  
<http://ideas.repec.org/a/ecm/emetrp/v72y2004i6p1737-1772.html>
- Easterly, W. (2001). The Middle Class Consensus and Economic Development, *Journal of Economic Growth* 6(4): 317-335.  
<http://ideas.repec.org/a/kap/jecgro/v6y2001i4p317-35.html>
- Esteban, J. and D. Ray (1994). On the Measurement of Polarization, *Econometrica* 62(4): 819-851. <http://ideas.repec.org/a/ecm/emetrp/v62y1994i4p819-51.html>
- Esteban, J.M., Gradin, C., and Ray, D. (2007). An Extension of a Measure of Polarization, with an application to the income distribution of five OECD countries, *Journal of Economic Inequality* 5(1): 1-19. <http://ideas.repec.org/a/kap/jecinq/v5y2007i1p1-19.html>
- Foster, J. E. and M. C. Wolfson (1992). Polarization and the Decline of the Middle Class: Canada and the U.S., mimeo.

- Foster, J. E. and M. C. Wolfson (2010). Polarization and the Decline of the Middle Class: Canada and the U.S., *Journal of Economic Inequality* 8(2): 247-273.  
<http://ideas.repec.org/a/kap/jecinq/v8y2010i2p247-273.html>
- Gibrat, R. (1931). *Les Inégalités Economiques*, Sirey, Paris.
- Gigliarano, C., and K. Mosler (2009). Constructing indices of multivariate polarization, *Journal of Economic Inequality* 7: 435-460.  
<http://ideas.repec.org/a/kap/jecinq/v7y2009i4p435-460.html>
- Grier, K. and N. Maynard (2010). Beyond Twin Peaks: Development and Polarization in the World Income Distribution, mimeo, University of Oklahoma.
- Holzmann, H. S. Vollmer and J. Weisbrod (2007). Twin Peaks or Three Components? – Analyzing the World’s Cross-Country Distribution of Income, Ibero America Institute for Economic Research (IAI), Discussion Paper 158, Faculty of Economics, University of Göttingen. <http://ideas.repec.org/p/got/iaidps/162.html>
- Israeli, O. (2007). A Shapley-based Decomposition of the R-Square of a Linear Regression, *Journal of Economic Inequality*, 5(2): 199-212.  
<http://ideas.repec.org/a/kap/jecinq/v5y2007i2p199-212.html>
- Kolenikov, S. and A. Shorrocks (2005). A Decomposition Analysis of Regional Poverty in Russia, *Review of Development Economics*, Wiley Blackwell, vol. 9(1): 25-46, 02.  
<http://ideas.repec.org/a/bla/rdevec/v9y2005i1p25-46.html>
- Lambert, P. J. (2009). Pro-poor Growth and the Lognormal Income Distribution, ECINEQ Working Paper Series No. 130.  
<http://ideas.repec.org/p/inq/inqwps/ecineq2009-130.html>
- Landes, D. (1998). *The Wealth and Poverty of Nations: Why Some Are So Rich and Some So Poor*. Norton (New York NY).
- Lasso de la Vega, M.C. and A. M. Urrutia (2006). An Alternative Formulation of the Esteban-Gradín-Ray Extended Measure of Polarization, *Journal of Income Distribution* 15(Index Issue): 42-54.
- Lasso de la Vega, C., A. Urrutia and H. Diez (2010). Unit Consistency and Bipolarization of Income Distributions, *Review of Income and Wealth* 56(1): 65-83.  
<http://ideas.repec.org/a/bla/revinw/v56y2010i1p65-83.html>
- Lawrence, R.Z. (1984). Sectoral Shifts and the Size of the Middle Class, *The Brookings Review* Fall: 3-11. <http://www.jstor.org/stable/20079849>
- Lerman, R. (1999). *How Income Sources Affect Income Inequality? In Handbook on Income Inequality Measurement*, edited by J. Silber. Boston, Dordrecht, London: Kluwer Academic Publishers.

- McDonald, J. B. (1984). Some Generalized Functions for the Size Distribution of Income, *Econometrica* 52: 647-663.  
<http://ideas.repec.org/a/ecm/emetrp/v52y1984i3p647-63.html>
- Mansfield, E. D. (1993). Concentration, Polarity, and the Distribution of Power, *International Studies Quarterly* 37(1): 105-128.  
<http://www.jstor.org/stable/2600833>
- Paap, R. and H. K. van Dijk (1998). Distribution and Mobility of Wealth of Nations, *European Economic Review* 42: 1269-1293.  
<http://ideas.repec.org/a/eee/eecrev/v42y1998i7p1269-1293.html>
- Pittau, M. G., R. Zelli and P. A. Johnson (2009). Mixture Models, Convergence Clubs, and Polarization, *Review of Income and Wealth* 56(1): 102-122.  
<http://ideas.repec.org/a/bla/revinw/v56y2010i1p102-122.html>
- Poggi, A., and Silber, J. (2010). On Polarization and Mobility: A Look at Polarization in the Wage-career Profile in Italy, *Review of Income and Wealth* 56(1): 123-140.  
<http://ideas.repec.org/a/bla/revinw/v56y2010i1p123-140.html>
- Pressman, S. (2007). The Decline of the Middle Class: An International Perspective, *Journal of Economic Issues* 41: 181-201. <http://www.jstor.org/stable/25511161>
- Quah, D. T. (1996). Twin Peaks: Growth and Convergence in Models of Distribution Dynamics, Center for Economic Performance, Discussion paper No. 280. London School of Economics and Political Science. <http://www.jstor.org/stable/2235377>
- Reynal-Querol, M. (2002). Ethnicity, Political Systems and Civil War, *Journal of Conflict Resolution* 46(1): 29-54. <http://www.jstor.org/stable/3176238>
- Rodriguez, J.G. and R. Salas (2003). Extended Bi-polarization and Inequality Measures, *Research on Economic Inequality* 9: 69-83.
- Rongve I. (1994). Sources of Inequality in Canada, PhD thesis. The University of British Columbia. [https://circle.ubc.ca/bitstream/id/17502/ubc\\_1994-95390x.pdf](https://circle.ubc.ca/bitstream/id/17502/ubc_1994-95390x.pdf)
- Salem, A. B. and T. D. Mount (1974). A Convenient Descriptive Model of Income Distribution: The Gamma Density, *Econometrica* 42: 1115-1127.  
<http://ideas.repec.org/a/ecm/emetrp/v42y1974i6p1115-27.html>
- Sastre, M. and A. Trannoy (2001). Une décomposition de l'évolution de l'inégalité en France avec une perspective internationale, 1985-1995, in *Conseil d'Analyse Economique, Les Inégalités économiques*, pp.315-332, Paris.
- Sastre, M. and A. Trannoy (2002). Shapley Inequality Decomposition by Factor Components: Some Methodological Issues, *Journal of Economics*, Supplement 9: 51-89. <http://ideas.repec.org/a/kap/jeczfn/v9y2002i1p51-89.html>

- Shorrocks, A. F. (1999). Decomposition Procedures for Distributional Analysis: A Unified Framework Based on the Shapley Value, mimeo, University of Essex.
- Shorrocks, A. F. (2013). Decomposition Procedures for Distributional Analysis: A Unified Framework Based on the Shapley Value, *Journal of Economic Inequality* 11(1): 99-126. <http://link.springer.com/article/10.1007%2Fs10888-011-9214-z?LI=true>
- Silber, J. (1989). Factors Components, Population Subgroups and the Computation of the Gini Index of Inequality, *The Review of Economics and Statistics*, LXXI:107-115. <http://ideas.repec.org/a/tpr/restat/v71y1989i1p107-15.html>
- Singh, S. K. and G. S. Maddala (1976). A function for the Size Distribution of Incomes, *Econometrica* 44: 963-970. <http://ideas.repec.org/a/ecm/emetrp/v44y1976i5p963-70.html>
- Thurow, L.C. (1984, February 05). The Disappearance of the Middle Class, The New-York Times, E2.
- Wang, Y. Q. and K.Y. Tsui (2000). Polarization Orderings and New Classes of Polarization Indices, *Journal of Public Economic Theory* 2(3): 349-363. <http://ideas.repec.org/a/bla/jpbect/v2y2000i3p349-63.html>
- Wolfson, M. C. (1994). When Inequalities Diverge, *American Economic Review* 84(2): 353-358. <http://ideas.repec.org/a/aea/aecrev/v84y1994i2p353-58.html>
- Zhang, X. and R. Kanbur (2001). What Difference Do Polarization Measures Make? An Application to China, *Journal of Development Studies* 37(3): 85-98. <http://ideas.repec.org/a/taf/jdevst/v37y2001i3p85-98.html>

## Appendix A: The Index $P_G$ and the Two Properties of “Increasing Spread” and “Increasing Bipolarity”

For non-overlapping groups the  $P_G$  index may be written as

$$P_G = (G_B - G_W)(G_B + G_W) \quad (\text{A-1})$$

where  $G_B$  and  $G_W$  refer respectively to the between groups (of equal size) and within groups Gini indices.

Since a Gini index  $G$  may be written as

$$G = (1/2)(\Delta / \bar{y}) \quad (\text{A-2})$$

where  $\Delta$  is the mean difference and  $\bar{y}$  the mean income, we can also express (A-1) as

$$P_G = \{[(1/2)(\Delta_B / \bar{y})] - [(1/2)(\Delta_W / \bar{y})]\} / \left\{ \left[ (1/2) \left( \frac{\Delta_B}{\bar{y}} \right) \right] + \left[ (1/2) \left( \frac{\Delta_W}{\bar{y}} \right) \right] \right\} \quad (\text{A-3})$$

$$\leftrightarrow P_G = (\Delta_B - \Delta_W) / (\Delta_B + \Delta_W) \quad (\text{A-4})$$

Note that this result shows that  $P_G$  has an interesting property: it is invariant to both a multiplication of all incomes by a constant but also to equal additions to all incomes, since a mean difference is invariant to equal additions to all incomes. This result was not stressed in the original paper of Deutsch et al. (2007).

Since the groups are of equal size, the between groups Gini index  $G_B$  may be written as

$$G_B = (1/2)((\bar{y}_R - \bar{y}_P) / (\bar{y}_R + \bar{y}_P)) \quad (\text{A-5})$$

where  $\bar{y}_R$  and  $\bar{y}_P$  are the mean incomes of the “rich” (those whose income is higher than the median) and of the “poor” (those whose income is lower than the median).

Since

$$G_B = (1/2)(\Delta_B / \bar{y}) \quad (\text{A-6})$$

we end up, combining (A-5) and (A-6) with

$$\Delta_B = 2G_B \bar{y} = (1/2)(\bar{y}_R - \bar{y}_P) \quad (\text{A-7})$$

since  $\bar{y} = (1/2)(\bar{y}_R + \bar{y}_P)$ .

In addition, we know (see, for example, Silber, 1989) that the within groups Gini index may be expressed as

$$G_W = (1/2)(\Delta_W/\bar{y}) = (f_P s_P G_P) + (f_R s_R G_R) \quad (\text{A-8})$$

where  $f_P, f_R, s_P, s_R, G_P, G_R$  are respectively the shares in the total population of the poor and rich, the shares in total income of the poor and rich and the Gini indices among the poor and rich.

But

$$G_P = (1/2)(\Delta_P/\bar{y}_P) \quad (\text{A-9})$$

and

$$G_R = (1/2)(\Delta_R/\bar{y}_R) \quad (\text{A-10})$$

where  $\Delta_R$  and  $\Delta_P$  are respectively the mean income differences among the “poor” and the “rich”.

Combining (A-8), (A-9) and (A-10) we get

$$\begin{aligned} \Delta_W = 2\bar{y}G_W = (2\bar{y})\{ & (f_P) \left( f_P \left( \frac{\bar{y}_P}{\bar{y}} \right) \right) ((1/2)(\Delta_P/\bar{y}_P)) \\ & + (f_R) \left( f_R \left( \frac{\bar{y}_R}{\bar{y}} \right) \right) ((1/2)(\Delta_R/\bar{y}_R)) \} \end{aligned} \quad (\text{A-11})$$

$$\text{since } s_P = \left( f_P \left( \frac{\bar{y}_P}{\bar{y}} \right) \right) \text{ and } s_R = \left( f_R \left( \frac{\bar{y}_R}{\bar{y}} \right) \right)$$

so that we end up with

$$\Delta_W = f_P^2 \Delta_P + f_R^2 \Delta_R = \left(\frac{1}{4}\right)\Delta_P + \left(\frac{1}{4}\right)\Delta_R \quad (\text{A-12})$$

since  $f_P = f_R = (1/2)$  (the two groups of “poor” and “rich” are of equal size).

Combining (A-4), (A-7) and (A-12) we get

$$P_G = \{[(1/2)(\bar{y}_R - \bar{y}_P)] - [(\frac{1}{4})\Delta_P + (\frac{1}{4})\Delta_R]\} / \{[(1/2)(\bar{y}_R - \bar{y}_P)] + [(\frac{1}{4})\Delta_P + (\frac{1}{4})\Delta_R]\} \quad (\text{A-13})$$

$$\leftrightarrow P_G = [2(\bar{y}_R - \bar{y}_P) - (\Delta_P + \Delta_R)] / [2(\bar{y}_R - \bar{y}_P) + (\Delta_P + \Delta_R)] \quad (\text{A-14})$$

Expression (A-14) shows clearly that  $P_G$  is a function of the mean incomes of the “rich” and of the “poor” and of the mean differences of the incomes of the “rich” and of the “poor”.

The axiom of increasing spread (IS) states that bi-polarization will increase if either  $\bar{y}_R$  increases or  $\bar{y}_P$  decreases. It is best to assume that this increase in the mean income of the “rich” or this decrease in the mean income the poor occurs, for a given value of the mean differences of the income of the “rich” and of the “poor”, so that IS does not take place at the same time as there is a change in bipolarity.

Similarly there will be increased bi-polarity (IB) if  $\Delta_R$  or/and  $\Delta_P$  decreases. It is here again best to assume that such a change occurs without any change in the mean income of the “rich” and the “poor” so that increased bipolarity (IB) will occur without affecting the spread (IB).

We can now easily compute the derivative of the bi-polarization index  $P_G$  with respect to the mean income of the “rich” and of the “poor” as well as with respect to the mean difference of the incomes of the “rich” and of the “poor” and, as expected, we can easily prove that

$$\frac{\partial P_G}{\partial \bar{y}_R} > 0; \frac{\partial P_G}{\partial \bar{y}_P} < 0; \frac{\partial P_G}{\partial \Delta_R} < 0; \frac{\partial P_G}{\partial \Delta_P} < 0.$$

The bi-polarization index  $P_G$  therefore obeys the principles of “increasing spread” and “increasing bipolarity”.



## Appendix B

*Table B1: Income Components in the PSELL dataset.*

3 sources	7 components	single component
income from work	Employment	gross employee cash or near cash income
		gross non cash employee income
	self-employment	gross cash benefits or losses from self-employment (including royalties)
Capital	Capital	income from rental of a property or land
		interests/dividends from capital investments
Transfers	unemployment benefits	unemployment benefits
	old age and survivors benefits	old-age benefits
		survivors benefits
	family allowances	family/children related allowances
	other transfers	sickness benefits
		disability benefits
		education related allowances
social exclusion not elsewhere classified		
	housing allowances	
	regular inter-household cash transfers received	
	income received by under 16	

*Reading note:* ‘income from work’ is the sum of ‘income from employment’ and ‘income from self-employment’. ‘Income from employment’ is the sum of two income components, namely ‘gross employee cash or near cash income’ and ‘gross non cash employee income’; ‘Income from self-employment’ includes only one component, ‘gross cash benefits or losses from self-employment’. The sum of each column is equal to the ‘total unadjusted gross income’.

## Appendix C: The Shapely Decomposition Procedure and Negative Contributions. An Illustration

We give here a simple illustration of the case where negative contributions may be observed when “unpacking” the overall contribution of a factor. This illustration uses the data from which Table 1 was derived. More specifically we “unpack” the negative contribution of income from work (-0.011) in the zero income decomposition of the Gini index in Table 1.

Let  $a$  represent income from work,  $b$  income from transfers and  $c$  income from capital.

Therefore  $I(a \neq 0, b \neq 0, c \neq 0)$  will be the Gini index of total income, assuming each of the three income sources (income from work, transfers and capital) are different from zero while  $I(a=0, b \neq 0, c=0)$ , for example, refers to the Gini index of total income, assuming income from work and capital are equal to zero.

Working with the original database the following results were obtained:

$$I(a \neq 0, b \neq 0, c \neq 0) = 0.3676$$

$$I(a=0, b \neq 0, c \neq 0) = 0.6088$$

$$I(a \neq 0, b=0, c \neq 0) = 0.5155$$

$$I(a=0, b=0, c \neq 0) = 0.8983$$

$$I(a \neq 0, b \neq 0, c=0) = 0.3639$$

$$I(a=0, b \neq 0, c=0) = 0.6212$$

$$I(a \neq 0, b=0, c=0) = 0.5275$$

$$I(a=0, b=0, c=0) = 0 \text{ (as expected).}$$

Since the contribution  $C(a)$  may be written as

$$\begin{aligned} C(a) = & \{ (2/6) [I(a \neq 0, b \neq 0, c \neq 0) - I(a=0, b \neq 0, c \neq 0)] \\ & + (1/6) [I(a \neq 0, b=0, c \neq 0) - I(a=0, b=0, c \neq 0)] \\ & + (1/6) [I(a \neq 0, b \neq 0, c=0) - I(a=0, b \neq 0, c=0)] \\ & + (2/6) [I(a \neq 0, b=0, c=0) - I(a=0, b=0, c=0)] \} \end{aligned}$$

we derive that

$$C(a) = \{ (2/6) [0.3676 - 0.6088] + (1/6) [0.5155 - 0.8983] + (1/6) [0.3639 - 0.6212] + (2/6)[0.5275 - 0] \}$$

$$C(a) = \{ (2/6) * -0.2412 + (1/6) * -0.3828 + (1/6) * -0.2573 + (2/6) * 0.5275 \}$$

$$C(a) = -0.01125$$

which is exactly the value of the (marginal) contribution of income from work that appears in Table 1 in the case of a zero income decomposition.

In this example, drawn from our data, income from work is more equally distributed (Gini = 0.5275) than income from transfers (0.6212) and income from capital (0.8983). It appears then clearly that the contribution of income from work is negative in all cases except when income from work is eliminated last.

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